Computing Incremental Risk Charge

Tim Xiao

ABSTRACT

In this paper, we present a methodology for calculating IRC. First, a Merton-type model is introduced for simulating default and migration. The model is modified to incorporate concentration. The calibration is also elaborated. Second, a simple approach to determine market data, including equity, in response to default and credit migration is presented. Next, a methodology toward constant level of risk is described. The details of applying the constant level of risk assumption and aggregating different subportfolios are addressed. Finally, the empirical and numerical results are presented.

Keywords: Incremental risk charge (IRC), constant level of risk, liquidity horizon, constant loss distribution, Merton-type model, concentration.

1 Introduction

The Basel Committee on Banking Supervision requires banks to compute Incremental Risk Charge (IRC) as a supplemental to value as risk. IRC is part of the new rules developed in response to the financial crisis.

IRC captures the loss due to default and migration events at a 99.9% confidence level over a one-year capital horizon. The liquidity of position is explicitly modeled in IRC through liquidity horizon and constant level of risk.

The constant level of risk assumption in IRC reflects the view that securities and derivatives held in the trading book are generally more liquid than those in the banking book and may be rebalanced more frequently than once a year (see Aimone [2018]).

IRC assumes a constant level of risk over a one-year capital horizon which may contain shorter liquidity horizons. This constant level of risk assumption implies that a bank would rebalance, or rollover, its positions over the one-year capital horizon in a manner that maintains the initial risk level, as indicated by the profile of exposure by credit rating and concentration.

This paper proposes a methodology consisting of two Monte Carlo simulations. The first Monte Carlo simulation simulates default, migration, and concentration in an integrated way. Combining with full re-valuation, the loss distribution at the first liquidity horizon for a subportfolio can be generated. The second Monte Carlo simulation is the random draws based on the constant level of risk assumption. It convolutes the copies of the single loss distribution to produce one year loss distribution. The aggregation of different subportfolios with different liquidity horizons is addressed. Moreover, the methodology for equity is also included, even though it is optional in IRC.

2 Simulation of Default and Credit Migration

2.1 Simulation Model

Most of the portfolio models of credit risk used in the banking industry is based on the conditional independence framework. In these models, defaults and credit migration of individual borrowers depend on a set of common systematic risk factors describing the state of the economy. Merton-type models have become very popular. The Merton-type model (or standardized Merton model) is

$$z_i = \beta_i \phi + \sqrt{1-\beta_i^{\,2}}\,\varepsilon_i \tag{3}$$

where

ϕ, ε_i	The independent standard normally random variables
ϕ	The systematic risk
ε_i	The idiosyncratic risk for issuer/obligor i
β_i	The weighted correlation reflecting the impact of systematic risk factor on issuer/obligor i.
z_i	The normalized asset return or creditworthiness indicator for issuer/obligor i

This model becomes the most popular one in default and migration risk modeling and yields the core of the Basel II capital rule (see Heitfield [2003]).

The IRC encompasses all positions subject to a capital charge for specific interest rate risk according to the internal models with exception of securitization and nth-to-default credit derivatives. Equity is optional. For IRC-covered positions, the IRC captures default risk and credit migration risk only.

2.2 Simulation model for multiple-liquidity-horizon subportfolios

Liquidity horizons are determined for each position to reflect actual practice and experience during periods of both systematic and idiosyncratic stresses. The total portfolio shall be divided into the subportfolios based on different liquidity horizons. Let’s assume that there are two subportfolios with different liquidity horizons: 3 month and 6 month. To model different liquidity periods, one can use the above model (3) but calibrate different β_i's, such as, β_{3m_i} and β_{6m_i}, for different periods.

Alternatively, one can also use a multiple-period model as:

$$z_{3m} = \beta_i \phi_{3m} + \sqrt{1-\beta_i^2}\,\varepsilon_{3m\_i} \qquad \text{For 3 month} \qquad (4)$$

$$z_{6m} = \beta_i \frac{\phi_{6m} + \gamma \cdot \phi_{3m}}{\sqrt{1+\gamma^2}} + \sqrt{1-\beta_i^2}\,\varepsilon_{6m\_i} \qquad \text{For 6 month} \qquad (5)$$

where β_i is unique for different periods under issuer i and γ is an exponentially declining weight (see Dunn [2008]).

2.3 Calibration of β_i

The most popular approaches to calibrate the asset correlation are Maximum Likelihood Estimation or regression based on time series default data. Alternatively, in the new Basel Capital Accord, a formula for derivation of risk weighted asset correlation for corporate, sovereign, and bank exposures is given as (see Tasche [2004] and Basel [2003]):

$$\beta_i = 0.12 \times \lambda_i + 0.24 \times (1-\lambda_i) \qquad (6)$$

Where $\lambda_i = \dfrac{1-e^{-50\times PD_i}}{1-e^{-50}}$

2.4 Concentration

The phenomenon we need to model is that concentration will result a higher IRC number, comparing to non-concentration case. Furthermore, the more concentration a portfolio has, the higher IRC result it generates. To achieve this, we model the effect of issuer and market concentration as well as clustering of default and migration by introducing another parameter, the ***concentration parameter***.

Our methodology is based on a simple mechanism for coupling issuer/market concentrations to migrations and defaults. In the simulation framework (3) or (4) and (5), the probability of a migration or default increases with the asset volatility. Since the effect of increasing concentration within a sector is to increase the probability of migration/default events within that sector, we model increased concentration as an increase in the volatility of the systematic risk driver. All positions sensitive to that risk driver will have an increased probability of migration/default events occurring. The modified simulation model is

$$z_i = \beta_i (1 + \sqrt{|\rho_i|})\phi_t + \sqrt{1 - \beta_i^{\,2}}\,\varepsilon_i \tag{7a}$$

Where ρ_i is the weighted concentration factor depending on correlation between issuer default and migration events and

$$\phi_t = \frac{x_t + \gamma \cdot x_{t-1} + \cdots + \gamma^k \cdot x_{t-k}}{\sqrt{1 + \gamma^2 + \cdots + \gamma^{2k}}} \tag{7b}$$

where if one uses (3), $\gamma = 0$ and $\phi_t = x_t = \phi$. Otherwise, γ is time declining weight and $x_t, \cdots, x_{t-k}$ are independent standard normally random variables representing systematic risks in different time periods.

2.5 Calibration of ρ_i

The calibration is based on credit migration matrix. It can be derived using either analytic closed-form or Monte-Carlo simulation. In theory, one can use Pearson's product moment or Kendall's τ .

2.6 Determination of default and credit migration

The simulated asset return z_i, combined with migration/default thresholds, is used to ascertain when default or migration is deemed to occur. The calculation of the thresholds of credit migration and default is based on credit migration probability (see JP Morgan [1997]). Using a BBB issuer as an example and given migration matrix, we can calculate the thresholds as: z_D^{BBB}, z_{CCC}^{BBB}, z_B^{BBB}, z_{BB}^{BBB}, z_{BBB}^{BBB}, z_A^{BBB}, z_{AA}^{BBB}. The rating bands and thresholds are shown in Figure 1

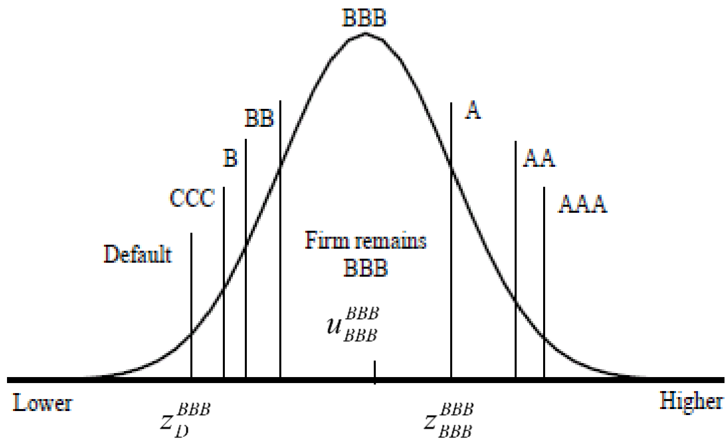

Figure 1 Credit migration rating thresholds (for BBB)

If the normalized asset of the issuer is smaller than z_D^{BBB}, it defaults. If the normalized asset is between z_D^{BBB} and z_{CCC}^{BBB}, it migrates to CCC, and so on. We use an effective middle value to represent each band:

$$
\begin{aligned}
u_D^{BBB} &= \Phi^{-1}\left[\tfrac{1}{2}((\Phi(z_D^{BBB})+0)\right] \\
u_{CCC}^{BBB} &= \Phi^{-1}\left[\tfrac{1}{2}((\Phi(z_{CCC}^{BBB})+\Phi(z_D^{BBB}))\right] \\
u_B^{BBB} &= \Phi^{-1}\left[\tfrac{1}{2}((\Phi(z_{CCC}^{BBB})+\Phi(z_B^{BBB}))\right] \\
u_{BB}^{BBB} &= \Phi^{-1}\left[\tfrac{1}{2}((\Phi(z_B^{BBB})+\Phi(z_{BB}^{BBB}))\right] \\
u_{BBB}^{BBB} &= \Phi^{-1}\left[\tfrac{1}{2}((\Phi(z_{BB}^{BBB})+\Phi(z_{BBB}^{BBB}))\right] \\
u_A^{BBB} &= \Phi^{-1}\left[\tfrac{1}{2}((\Phi(z_{BBB}^{BBB})+\Phi(z_A^{BBB}))\right] \\
u_{AA}^{BBB} &= \Phi^{-1}\left[\tfrac{1}{2}((\Phi(z_A^{BBB})+\Phi(z_{AA}^{BBB}))\right] \\
u_{AAA}^{BBB} &= \Phi^{-1}\left[\tfrac{1}{2}((\Phi(z_{AA}^{BBB})+1)\right]
\end{aligned}
\tag{8}
$$

2.7 Calibration of transition matrix, default probability (PD), and loss given default (LGD)

The straight forward cohort approach is used to estimate transition matrices based on obligors' rating history, which has become the industry standard. The PD estimate is based on EDF data that is used for calculation of PD benchmarked against internal default history. Internal data is used for LGD parameter benchmarked against relevant external proxy data.

3 Constant Level of Risk

The constant level of risk reflects recognition by regulators that securities/derivatives held in the trading book are generally much more liquid than those in the banking book, where a buy-and-hold assumption over one year may be reasonable. It implies that IRC should be modeled under the assumption that banks rebalance their portfolio several times over the capital horizon in order to maintain a constant risk profile as market conditions evolve.

There are several ways to interpret constant level of risk: constant loss distribution or constant risk metrics (e.g. VaR). We believe the constant loss distribution assumption is the most rigorous. Under this assumption, the same metrics (e.g. VaR, moments, etc.) can be achieved for each liquidity horizon.

The liquidity horizon for a position or set of positions has a floor of **three months**. Let us use three months as an example. We interpret constant level of risk to mean that the bank holds its portfolio constant for the liquidity horizon, then rebalances by selling any default, downgraded, or upgraded positions and replaces them so that the portfolio is returned to the level of risk it had at the beginning. The process is repeated 4 times over the capital horizon resulting 4 independent and identical loss distributions.

An intuitive explanation is shown in Figure 2. A generic path with appears in red; P&L contributions from each liquidity horizon appear in blue. In this schematic, the position experiences downgrade, upgrade or default, resulting in a loss or profit. This position is then removed and replaced at the end of each liquidity horizon by rebalancing. The final P&L for the path will be the summary of all losses and profits.

In addition, one needs to consider the reinvestment of all cash flows realized during the liquidity horizon and rollover of expired deals.

4 Aggregation and Time Horizon Correlation

First we need to divide the portfolio into the subportfolios based on liquidity horizons. If there is only one single-liquidity-horizon subportfolio, the rebalance at the end of each liquidity horizon washes out the time horizon correlation. However, if there are multiple subportfolios, the time horizon correlations need to be addressed.

To elaborate the details, we assume there are two subportfolios with liquidity horizons: 3 months and 6 months. Based on the default and migration simulation and full re-valuation, we can generate loss distributions at first liquidity horizons for 3-month and 6-month subportfolios as PL_{3m}, and PL_{6m}.

There are two approaches to achieve the correlative aggregation: copula approach or correlation matrix approach.

4.1 Copula approach

We conduct the second Monte Carlo simulation by generate 4 standard normal random draws for scenario j: $x_1^j, x_2^j, .x_3^j, x_4^j$. These random draws represent a Monte-Carlo path.

4.1.1 Three-month Subportfolio

The P&L distribution of three-month subportfolio is PL_{3m}. The four draws of loss distribution are $PL_{3m}\left(\Phi(x_1^j)\right), PL_{3m}\left(\Phi(x_2^j)\right), PL_{3m}\left(\Phi(x_3^j)\right), PL_{3m}\left(\Phi(x_4^j)\right)$, where Φ is the accumulative normal. The total P&L of the three-month subportfolio for scenario j is

$$PL^j_{total\_3m} = \sum_{i=1}^{4} PL_{3m}\left(\Phi(x_i^j)\right) \tag{18}$$

4.1.2 Six-month Subportfolio

The P&L distribution of the six-month subportfolio is PL_{6m}. We can calculate correlation $\rho(PL_{3m}, PL_{6m})$ between PL_{3m} and PL_{6m} using Pearson product-moment. The two correlated random draws are $x^j_{6m\_1} = \rho(PL_{3m}, PL_{6m})x_1^j + \sqrt{1-\rho(PL_{3m}, PL_{6m})^2}\,x_2^j$ and $x^j_{6m\_2} = \rho(PL_{3m}, PL_{6m})x_3^j + \sqrt{1-\rho(PL_{3m}, PL_{6m})^2}\,x_4^j$. The two draws of loss distribution are $PL_{6m}\left(\Phi(x^j_{6m\_1})\right)$, $PL_{6m}\left(\Phi(x^j_{6m\_2})\right)$. The total P&L of the six-month subportfolio for scenario j is

$$PL^j_{total\_6m} = \sum_{i=1}^{2} PL_{6m}\left(\Phi(x^j_{6m\_i})\right) \tag{19}$$

Summing up (18) and (19), we can get the total P&L for scenario j as

$$PL^j_{total} = PL^j_{total\_6m} + PL^j_{total\_3m} \tag{20}$$

4.2 Correlation matrix approach

Based on the four 3-month independent identical loss distributions: PL_{3m}, $PL_{3m}, PL_{3m}, PL_{3m}$, and two 6-month independent identical loss distributions: PL_{6m}, PL_{6m}, we can construct a 6×6 pair-wise sample correlation matrix Σ. Applying the Cholesky decomposition to the correlation matrix Σ, we have $\Sigma = LL^T$, where L is a lower triangular matrix.

We conduct the second Monte Carlo simulation by generating 4 independent standard normal random draws: $x_1^j, x_2^j,.x_3^j, x_4^j$ for the four 3-month periods in a year and 2 independent standard normal random draws x_5^j, x_6^j for the two 6-month periods to construct a path/scenario j.

The random draw vector is $X = \begin{bmatrix} x_1^j & x_2^j & x_3^j & x_4^j & x_5^j & x_6^j \end{bmatrix}$. We can obtain correlative random draw vector

$$\tilde{X} = \begin{bmatrix} \tilde{x}_1^j & \tilde{x}_2^j & \tilde{x}_3^j & \tilde{x}_4^j & \tilde{x}_5^j & \tilde{x}_6^j \end{bmatrix} \text{ by } \tilde{X}^T = L \times X^T \tag{21}$$

The total P&L for scenario j is

$$PL_{total}^j = PL_{total\_3m}^j + PL_{total\_6m}^j = \sum_{i=1}^{4} PL_{3m}\left(\Phi(\tilde{x}_i^j)\right) + \sum_{i=5}^{6} PL_{6m}\left(\Phi(\tilde{x}_i^j)\right) \tag{22}$$

The final IRC will be 99.9% VaR based on distribution PL_{total}^j. In general, the correlation matrix approach is more generic and can be easily extended to any number of subportfolios.

References

Alessandro Aimone, 2018, "ING's market risk charge edges higher," Risk Quantum, 2018

Basel Committee on Banking Supervision, July 2009 (a), "Guidelines for Computing Capital for Incremental Risk in the Trading Book."

Gary Dunn, April 2008, "A multiple period Gaussian Jump to Default Risk Model."

FinPricing, Market Data Solution, https://finpricing.com/lib/FxAccumulator.html

Erik Heitfield, 2003, "Dealing with double default under Basel II," Board of Governors of the Federal Reserve System.

Jongwoo Kim, Feb 2009, "Hypothesis Test of Default Correlation and Application to Specific Risk," RiskMetrics Group.

Dirk Tasche, Feb 17, 2004, “The single risk factor approach to capital charges in case of correlated loss given default rates.”