

An optimized algorithm for peak detection in noisy periodic and quasi-periodic signals

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ABSTRACT

This paper represents a modified version (called T-AMPD) of automatic multiscale-based peak detection (AMPD) algorithm, which was previously proposed by Scholkmann et al for peak detection of noisy periodic and quasi-periodic signals, in order to optimize the algorithm and reduce the computational cost while still maintaining the accuracy. By introducing a new parameter n_0 , the number of expected peaks, to the algorithm and some other adjustments, the T-AMPD outperforms the original algorithm in processing time. Experiments in some real-world signals show that the modified algorithm is greatly better than the original one in runtime performance. Specifically, our T-AMPD was compared to the newest version of AMPD which is available in an R programming package. As a result, our T-AMPD is 2 to 25 times faster than the original AMPD in computational time, while the accuracy remains unchanged.

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1. INTRODUCTION

The peaks and troughs detection is a very important task in many real-world applications in different fields such as bioinformatics [1], mass spectrometry [2], image processing [3] and signal processing [4], [5]. Because trough detection is equivalent to peak detection when we implement the peak detection for the additive inverse of the data, the algorithms are usually designed for peak detection. In signal processing, peak detection is the key step. However, the challenge of the peak detection is usually caused by noise where the light peaks are detected and the strong peaks are ignored.

Nowadays, with the explosion of smart devices and automatic intelligent control systems along with the ever-increasing volume of data, signal analysis is more difficult because of the increase in noise as well as computational requirements [6], [7]. Many solutions have been proposed for peak detection in signal processing in the literature ranging from the adaptive threshold method (ATM) [8], wavelet transform techniques [9], [10], hidden Markov models [11], k-means clustering [12], and entropy-based techniques [13] to the automatic chromatographic peak detection (ACPD) [14], the peak of Shannon energy envelope (PSEE) [15], mountaineer's method [16], automatic multi-scale peak detection (AMPD) [17], [18] and ATPD [19]. But most of them are designed for a specific type of signals. Only a few algorithms have been developed for general signals.

Some algorithms are designed only for specific signals. For instance, ACPD is designed for chromatographic signals [20]–[22], and the PSEE [23]–[25] are designed for electrocardiogram (ECG) signals, while the ATM and others [16], [26]–[28] are designed for photoplethysmography (PPG) signals. Some other algorithms (but fewer) are designed for peak detection in general such as AMPD and ATPD [13],

[29], [30]. In which, the AMPD has more advantages because it operates without any parameters and quite robust regardless high and low frequency noise and detects peaks accurately in quasi-periodic signals.

Besides the accuracy, the computational complexity of peak detection algorithms is also main concern for researchers, especially in big data analysis for real-world problems today. Therefore, our aim is to modify the original AMPD, an algorithm that is suitable for many type of signals without any parameters, to improve its runtime performance and the memory storage requirements. The modified version is then compared to the original AMPD in both peak detection accuracy and runtime performance. Finally, some suggestions for improving peak detection algorithms are recommended.

2. METHOD

In this section, we take an overview of the original AMPD and point out some necessary optimizations for it. Finally, some improvements are proposed.

2.1. Overview of the original AMPD

Given $X = (x_1, x_2, \dots, x_N)$ is a sequence of observations from a univariate signal which the periodic or quasi-periodic peaks are involved. At the first stage, the AMPD calculates the local maxima scalogram (LMS) $L \times N$ matrix where $L = \lfloor N/2 \rfloor - 1$ (notice that $\lfloor z \rfloor$ is the smallest integer that is not less than z) and N is the length of X . For each k in $\{1, 2, \dots, L\}$, the k -th rows of LMS allocates all the local maxima positions of X at scale k (the level of zoom for peak finding) at time t by assigning its element, $m[k, t]$, equal to zero if $x_t > x_{t-k}$ and $x_t > x_{t+k}$ for each $t = k + 1$ to $N - k$. Otherwise, $m[k, t] = \alpha + r$ (where r is a random number from $[0, 1]$ -uniformly distribution and $\alpha \in \mathbb{R}$ is a constant). In [17], α is fixed by 1. The k -th row of the MLS is calculated as in Figure 1 for $t = k + 1$ to $N - k$.

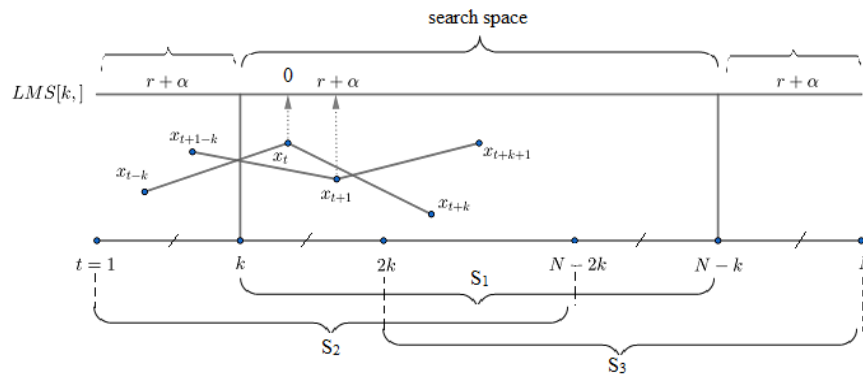


Figure 1. LMS calculation of the AMPD

The first row of LMS has the highest resolution of the potential peak points and it reduces to the “low resolution” for each a half of the length of the signal. The second step is cutting the LMS by the r -th row to get LMS_r matrix containing the r first rows of LMS, where r is the scale with greatest number of potential peaks (i.e. the greatest number of zeros). At the final stage of AMPD, the column-wise standard deviation of each column of LMS_r is calculated. The position of the column where the standard deviation is equal to zero identify the maxima locations of X . The Figure 2 (Figure 1 in [17]) shows the steps of the AMPD algorithm and Figure 3 (Figure 2 in [17]) visualizes the AMPD process by an example.

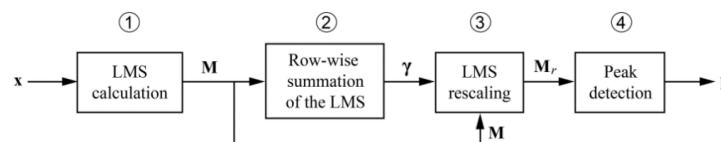


Figure 2. Diagram of AMPD algorithm (Figure 1. in [17])

2.2. Some necessary optimizations for AMPD

There are two main observations are necessary optimized for the AMPD. Firstly, the most computational expensive in AMPD comes from calculating the LMS matrix. It has an $\mathcal{O}(N^2)$ upper complexity

bound of memory requirements. Moreover, the LMS using the scales up to $L = \lfloor N/2 \rfloor - 1$ is unnecessary. In the example given in Figures 3(a) to (f) (see [17]) for more detail), we can see that the number of necessary scales in the LMS is at the dramatically low rate (i.e. only first 55 significant scales against 1500 in total). Secondly, the calculation of column-wise standard deviations to find the columns with zero value in the final stage is equivalent to finding the intersection of r first rows of the LMS matrix. Therefore, the calculation of uniform random number for filling the void of the LMS is very computationally expensive.

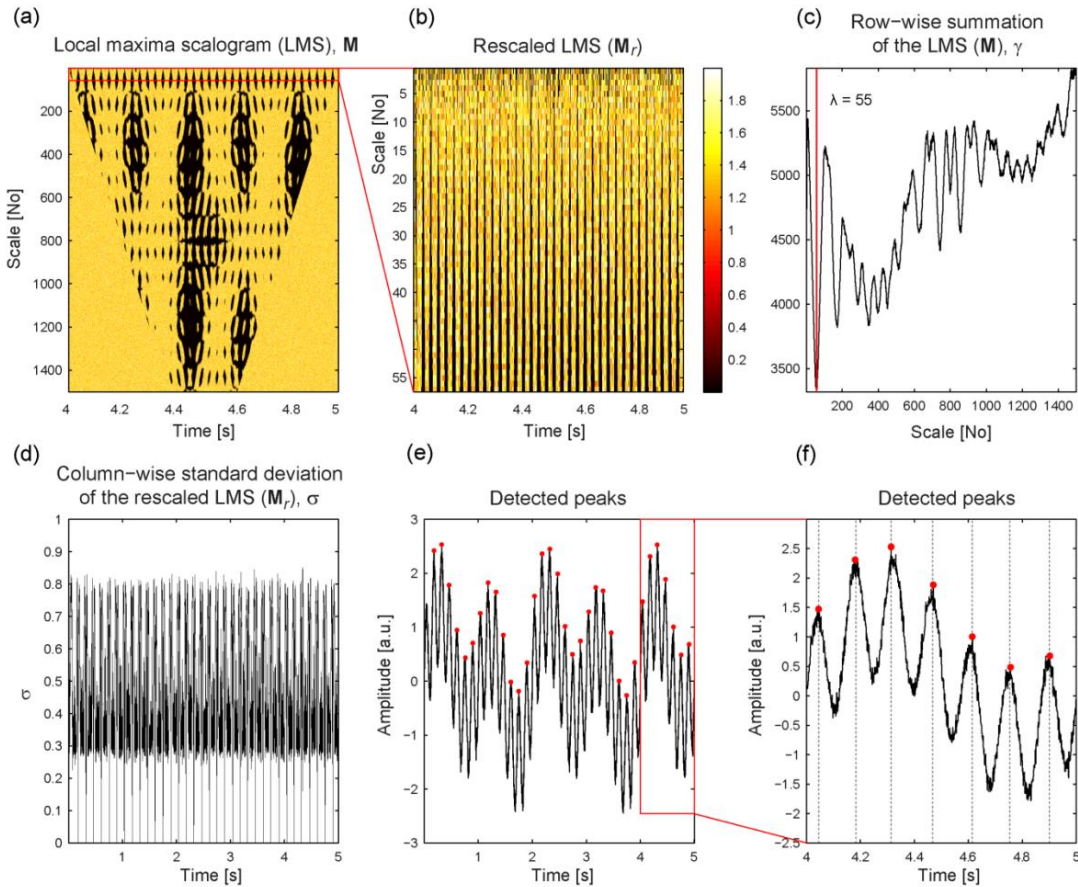


Figure 3. An example of applying the AMPD to a simulated signal, (a) LMS, (b) rescaled LMS, (c) number of maxima for each scale, (d) standard deviation for each column of rescaled LMS, (e) true peaks, and (f) zoomed true peaks

2.3. Some improvements of the T-AMPD

In this section, we propose some solutions to overcome the obstacles of the original AMPD mentioned in section 2.2, the modified algorithm called T-AMPD. For L valued reducing, we can consider setting it with a fixed number. For example, if the length of X is greater than 5000 then set $L = \lfloor 5000/2 \rfloor - 1$, otherwise, $L = \lfloor N/2 \rfloor - 1$ as used in the **R** package *ampd* in [31]. In other respects, if we call S_1 the search space at the scale k then $S_1 = \{x_k, \dots, x_{N-k}\}$ (see Figure 1). Suppose that n_0 ($n_0 \geq 1$) is the number of desirable peaks need to be detected in the signal and k is the last scale that $MLS[k,]$ containing all of these peaks then they are all of S_1 . Therefore, the highest frequency of quasi-periodic signal is equal to:

$$f = \frac{N-2k}{n_0-1} \tag{1}$$

for $n_0 \geq 2$ in general.

Because k is the last scale that $MLS[k,]$ containing all of the true peaks, there is not any peak in $S_2 \setminus S_1$ and $S_3 \setminus S_1$. It means that $f \geq k$. Now from (1) we have:

$$k \leq \frac{N}{n_0+1} \tag{2}$$

In (2) implies that L should be equal to $\lfloor N/(n_0 + 1) \rfloor - 1$.

In general cases, the computational resource in this proposal with $n_0 = 2$ is dramatically cut down in comparison to the original algorithm. The suitable L scales with different n_0 are illustrated as in Figure 4 where an example of MLS calculation was given by [17]. When $n_0 = 1$ then $L = \lfloor N/2 \rfloor - 1$ as in the original AMPD. But this is unusual case in most of real-world applications. Notice that if n_0 is greater than the number of true peaks in the signal, high rate of false positive errors can happen. From the number of peaks is unknown and depend on the highest frequency of the signal, we can choose n_0 by small number (i.e. $n_0 = 2, 3, \dots$) or $n_0 = \lfloor N/f_{max} \rfloor$ if the information about the highest frequency of the signal, f_{max} , is known. This case is available in many facts (i.e., ECG, PPG signals).

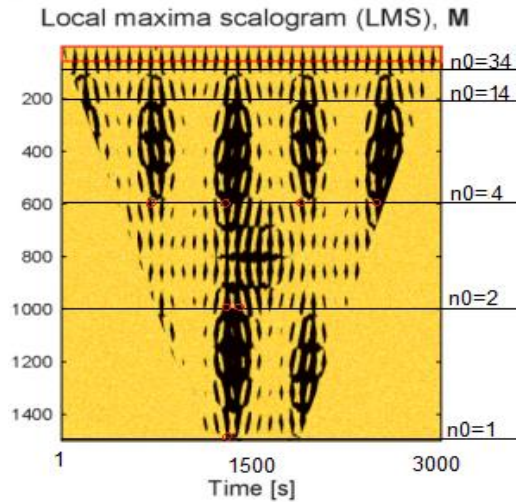


Figure 4. The suitable L with different n_0 (an example of [17])

For optimization of the LMS calculation, for each k in 1 to L , the original AMPD (i.e. in [17] and another modified AMPD in [18]) finds the local maxima in the search space S_1 by taking each t in $k + 1$ to $N - k$ and checking whether $x_t > x_{t-k}$ and $x_t > x_{t-k+1}$. If it is true, the value $LMS[k, t] = 0$ and $LMS[k, t] = r + \alpha$ otherwise. This way causes much waste of computer resources while it is equivalent to comparing the elements of S_1 with S_2 and S_3 correspondingly (see the Figure 1 for more details). From S_1, S_2 and S_3 have the same length, instead of using two for-loops for finding the local maxima, we can compare the elements of S_1 with S_2 and S_1 with S_3 simultaneously using the built-in operator for comparing two vectors which is available in many programming language such as **R** or **MATLAB** (i.e. and $S_1 > S_3$ directly). This reduces the runtime complexity from $\mathcal{O}(N^2)$ of the original AMPD to $\mathcal{O}(N)$ of the T-AMPD. The positions of X which matched the condition are then stored in a vector, called $p.list[k]$, with auto length (i.e. number of these positions). This length is surely less than N . Hence, we have a list of k vectors containing the potential peaks for each scale instead of a $L \times N$ matrix.

Finally, let r be the first index of the vector among the vectors in the list where $p.list[r]$ has the greatest length in the vector list, then r is the scale with the most potential peaks. The intersection of all these first r vectors give the vector, called \mathbf{p} , including the indices of the true detected peaks.

3. RESULTS AND DISCUSSION

To demonstrate the efficiency of T-AMPD in comparison to the original AMPD, we run both algorithms on several signals and compare them in accuracy (i.e. false positive detection errors (FPDEs) and false negative detection errors (FNDEs)) and runtime performance. The algorithms are analysed in **R** on a 2.60GHz Intel Core i7-4510U CPU with 8GB RAM running on Window 10 v.20H2. For the up-to-date of the algorithms, the original AMPD is implemented by the `ampd` function in the **R** package called `ampd` [31] which has been optimized by its authors. Furthermore, the splitting size for peak detection of AMPD in `ampd` package is restricted by 5000 for signals with the length greater than 5000 and adapted for at least 10 local maxima are included (see the reference manual of the package in [31] for more details). Hence, T-AMPD will be implemented with $n_0 = 1$ which is equivalent to the original AMPD, $n_0 = 10$ for the common

requirement of the AMPD and to the first one which meets at least one false positive detection error (the T-AMPD with different n_0 is denoted by T-AMPD(n)). The runtime for each case is 95% confident interval with 100 times of run-simulation. The first time series for comparison is the total monthly number of sunspots [17]. The time series includes 3177 values (< 5000) ranging from 1/1947 to 9/2013 is enclosed in the **R** package called *datasets*. The results of comparison are given in Table 1.

Table 1. Comparison of T-AMPD and AMPD for sunspot data

Algorithm	Accuracy		Runtime (secs) (95% Conf.)
	FPDEs	FNDEs	
AMPD	0	0	0.6286 ± 0.018
T-AMPD(1)	0	0	0.4757 ± 0.013
T-AMPD(10)	0	0	0.1366 ± 0.0027
T-AMPD(65)	0	0	0.0251 ± 0.0008
T-AMPD(66)	1	0	0.0242 ± 0.0007

We can see that with only $n_0 = 1$, the runtime of T-AMPD is much reduced as about 75% the runtime of AMPD and decreases as n_0 increases. With $n_0 = 10$, the runtime ratio of T-AMPD and AMPD is at a very impressive rate, The T-AMPD is 4 times faster than the AMPD. The first false positive detection error is caught when n_0 is up to 66 where the runtime is very small with 25 times faster. Figure 5 shows all 23 true peaks of the signal and the value of n_0 when the first FPDE is found.

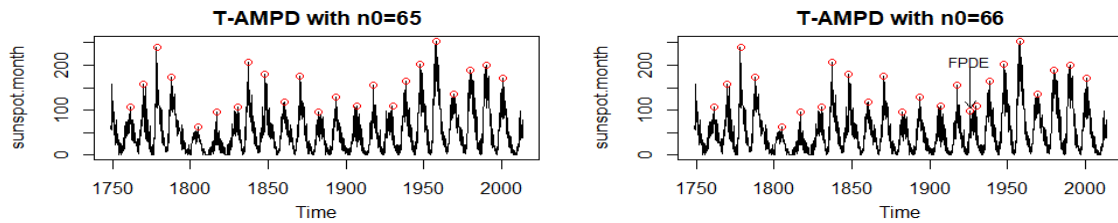


Figure 5. T-AMPD for monthly sunspot data

The second time series for comparison is the length of day (LOD) data. In this data, the duration of a day according to the SI units definition (1 day=86,400s) is compared with the astronomically determined duration of a day on earth. We can obtain the data from the International Earth Rotation and Reference Service (IERS) at <https://hpiers.obspm.fr/iers/eop/eopc04/> with 1461 days (< 5000) from 2008 to 2011. The analysis shows that the results of T-AMPD with n_0 from 1 to 207 have identical detection rates to AMPD where all of them have one false negative detection error. All the T-AMPDs with n_0 from 208 to 729 have no false detection occurred (see Figure 6). The false positive detection error only occurs when n_0 from 730 above.

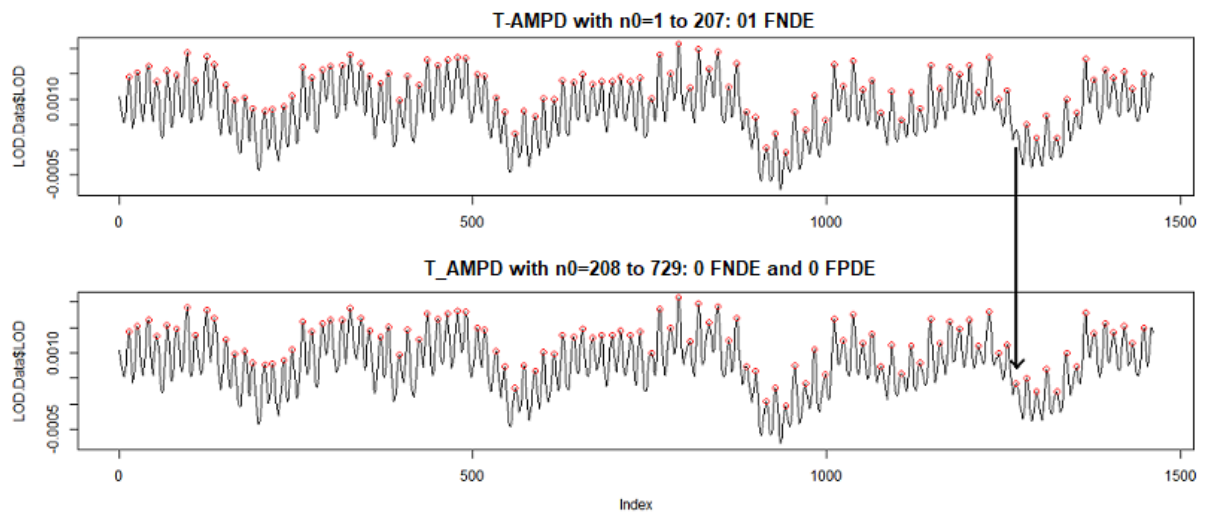


Figure 6. T-AMPD for LOD data

The runtime performance of T-AMPD against AMPD is given in Table 2. The dramatical improvement in runtime of T-AMPD is the same to the sunspot analysis case. We can see that with $n_0 = 10$ (for general cases), the T-AMPD is more than 5 times faster than the original AMPD. From these peaks we are easily to estimate the average period of the longest days is 13.6 days which is adapted with the value reported in previous literature [32].

Table 2. Comparison of T-AMPD and AMPD for LOD data

Algorithm	Accuracy		Runtime (secs) (95% Conf.)
	FPDEs	FNDEs	
AMPD	0	1	0.1471 ± 0.0057
T-AMPD(1)	0	1	0.1011 ± 0.004
T-AMPD(10)	0	1	0.0273 ± 0.001
T-AMPD(207)	0	1	0.0028 ± 0.00026
T-AMPD(208)	0	0	0.0025 ± 0.00035
T-AMPD(729)	0	0	0.00153 ± 0.00015
T-AMPD(730)	7	0	0.00138 ± 0.00014

Finally, we analysis with a low-noise electrocardiography (ECG) time series with 200000 values (> 5000) downloaded from <https://plux.info/sensors/277-electrocardiogram-ecg.html>. Unfortunately, the original AMPD cannot run on our device in default setting because it cannot allocate the vector of too large size needed for the matrix LMS. Our T-AMPD still run well regardless the size of the scanning window though the runtime is a bit long. For fairly comparison, we set the *splitting=TRUE* for AMPD in order to limit the scanning window by 5000 which ensures the algorithm run normally, and our T-AMPD is also limited the scanning window in the same size. The results are given in Table 3.

Table 3. Comparison of T-AMPD and AMPD for ECG data

Algorithm	Accuracy		Runtime (secs) (95% Conf.)
	FPDEs	FNDEs	
AMPD	0	0	238.2067 ± 0.9802
T-AMPD(1)	0	0	144.036 ± 0.7275
T-AMPD(10)	0	0	27.1262 ± 0.4094

For the ECG signal with a large number of values as in this case, we can see in Table 3 that T-AMPD has approximately a duration of 144.036s (~2.4 mins) which is approximately a half of AMPD runtime with $n_0 = 1$, and it is only approximately 27.12s with $n_0 = 10$ which is more than 8 times faster than AMPD. Figure 7 shows the detected true peaks of the ECG signal run by T-AMPD at $n_0 = 10$. We can see all 229 true peaks are detected from 200000 observations without any errors, exactly the same as the original AMPD.

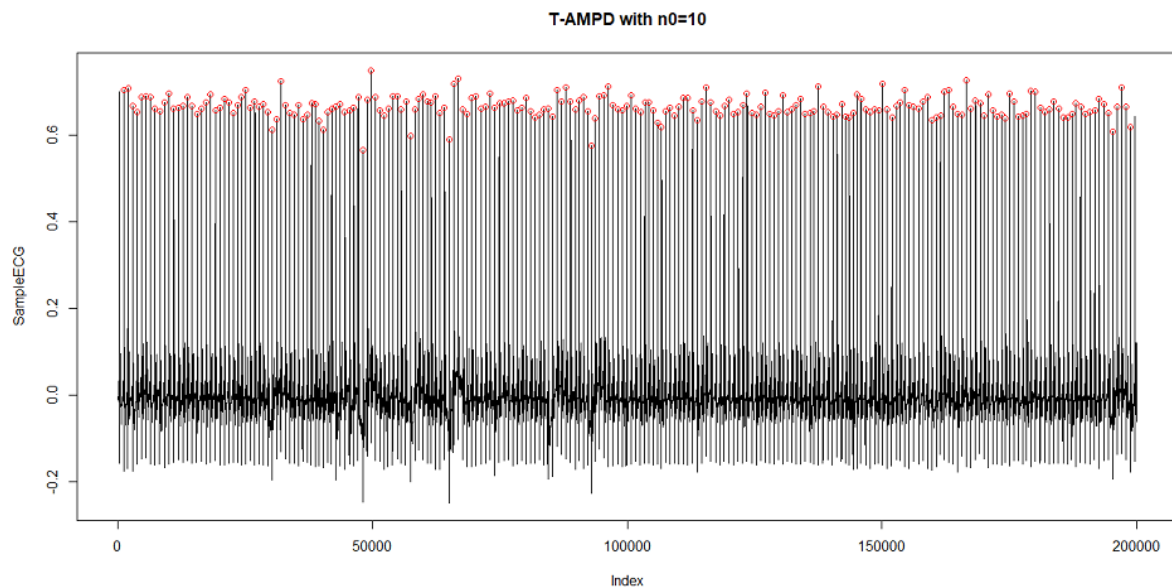


Figure 7. T-AMPD for ECG data with all 229 true peaks are detected at $n_0 = 10$

4. CONCLUSION

In this paper, we have proposed some important adjustments for the AMPD algorithm in improving its runtime performance while maintaining the accuracy by introducing the desirable number of detected peaks parameter, called n_0 . The experimental results show that even with $n_0 = 1$, our algorithm (T-AMPD) greatly improves the computation time with at least 2 times faster in comparison to the AMPD's runtime, and it gets even much better with larger but reasonable values of n_0 . There is a way that the T-AMPD can be improved, that is we can fix "the best" n_0 for the algorithm. This "the best" n_0 as commented in section 2.3 that it depends on the highest frequency f_{max} of the signal. In most practical areas, this f_{max} can be approximated by technical devices.

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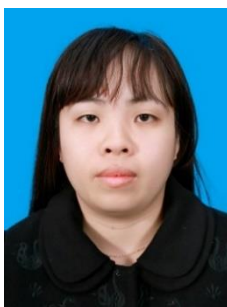
BIOGRAPHIES OF AUTHORS






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