New Idea to compute the Geometric Series and its Derivative

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Abstract: This paper presents the summations of single terms and successive terms of geometric series and computation of the first derivative of geometric series in a different way. This idea will be useful for researchers who are involving in finding the scientific solutions.

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Introduction
In the earlier days, geometric series served as a vital role in the development of differential and integral calculus and as an introduction to Taylor series and Fourier series. This article presents the summations of single terms and successive terms of geometric series [1-4] and computation of the first derivative of geometric series in a different way.

Summation of one term of geometric series:

$$1 = \frac{x-1}{x-1}, \quad x = \frac{x^2-x}{x-1}, \quad x^2 = \frac{x^3-x^2}{x-1}, \quad x^3 = \frac{x^4-x^3}{x-1}, \cdots, \quad x^n = \frac{x^{n+1}-x^n}{x-1}.$$

Summation of two successive terms of geometric series:

$$1+x = \frac{x^2-1}{x-1}, x+x^2 = \frac{x^3-x}{x-1}, \; x^2+x^3 = \frac{x^4-x^2}{x-1}, \cdots, \; x^{n-1}+x^n = \frac{x^{n+1}-x^{n-1}}{x-1}.$$

Summation of three successive terms of geometric series:

$$1+x+x^2 = \frac{x^3-1}{x-1}, \quad x+x^2+x^3 = \frac{x^4-x}{x-1}, \cdots, \quad x^{n-2}+x^{x-1}+x^n = \frac{x^{n+1}-x^{n-2}}{x-1}.$$

Similarly, we can continue these expressions up to multiple successive terms of geometric series.

Summation of various successive terms of geometric series:

$$\sum_{i=k}^{n} x^i = x^k + x^{k+1} + x^{k+2} + \cdots + x^{n-1} + x^n = \frac{x^{n+1}-x^k}{x-1}.$$

$$\sum_{i=-k}^{n} x^i = x^{-k} + x^{-k+1} + x^{-k+2} + \cdots + x^{n-1} + x^n = \frac{x^{n+1}-x^{-k}}{x-1}.$$

$$\sum_{i=1}^{n} x^i = 1 + x + x^2 + x^3 + \cdots + x^{n-1} + x^n = \frac{x^{n+1}-1}{x-1}.$$

First Derivative of Geometric Series

The first derivative of geometric series [5] is found here without using the differential calculus:

$$\sum_{i=0}^{n-1} x^i + \sum_{i=1}^{n-1} x^i + \sum_{i=2}^{n-1} x^i + \cdots + \sum_{i=n-2}^{n-1} x^i + \sum_{i=n-1}^{n-1} x^i$$
$$= \frac{x^n - 1}{x-1} + \frac{x^n - x}{x-1} + \frac{x^n - x^2}{x-1} + \cdots + \frac{x^n - x^{n-2}}{x-1} + \frac{x^n - x^{n-1}}{x-1}.$$

Here, $\sum_{i=0}^{n-1} x^i + \sum_{i=1}^{n-1} x^i + \sum_{i=2}^{n-1} x^i + \cdots + \sum_{i=n-2}^{n-1} x^i + \sum_{i=n-1}^{n-1} x^i = \sum_{i=0}^{n-1} (i+1)x^i$ and

$$\frac{x^n - 1}{x-1} + \frac{x^n - x}{x-1} + \frac{x^n - x^2}{x-1} + \cdots + \frac{x^n - x^{n-2}}{x-1} + \frac{x^n - x^{n-1}}{x-1} = \frac{nx^n - \sum_{i=0}^{n-1} x^i}{x-1}$$
$$= \frac{nx^n - \left(\frac{x^n - 1}{x-1}\right)}{x-1} = \frac{(nx - n - 1)x^n + 1}{(x-1)^2}.$$

Thus, $$\sum_{i=0}^{n-1} (i+1)x^i = \frac{(nx - n - 1)x^n + 1}{(x-1)^2}, \qquad (x \neq 1).$$

This result denotes the first derivative [5] of geometric series.

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