MATHEMATICAL SCIENCES

TOPOLOGICAL SPACES GENERATED BY DISCRETE SUBSPACES

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Abstract

In [3] the authors initiated a systematic study of the property of a space to be generated by its discrete subsets. Discretely generated properties seems to be interesting in themselves due to their good categorical behaviour: discrete generability is hereditary; each compact space of countable tightness is discretely generated; Frechet-Urysohn is discretely generated. Such a property turns out to be not only interesting, but also the base for many nice questions.

Discrete generability has surprising relationships with classical properties. According to the article [1] sequential spaces is generated by discrete sets, but in this work we will show an example which sequential space is not discrete generated space. A lot of properties are preserved by making product. The product of Hausdorff spaces is a Hausdorff space, the product of compacts is a compact, the countable product of metrizable spaces is metrizable. However it is not case of the discrete generability. We will show a simple counter-example. Additionally, the product of Frechet-Urysohn spaces are not discretely generated space was shown by example.

Keywords: Discretely generated space; Frechet-Urysohn space; sequential space.

Definition 1. A topological space X is called discretely generated if for every subset $A \subset X$ we have $\overline{A} = \bigcup \left\{ \overline{D} : D \subset A \text{ is discrete subspace of } X \right\}$. It is natural to say that the topology of a spaces X is determined by discrete subspaces if for every $A \subset X$ the closure of A is the union of the closures of discrete subspaces of A. We will also call such spaces discretely generated [3].

Definition 2. Let X be a topological space. A space X is generated by discrete subspaces if for every $A \subset X$ subset and any $x \in \overline{A}$ point there exists a discrete $D \subset A$ such that $x \in \overline{D}$. We will also call such spaces discretely generated.

Definition 3. A Frechet-Urysohn topological space (also Frechet) is one with the following property: For every subset X of domain of the space and for any point x in the closure of X there is a sequence of points from X converging to x.

Definition 4. A topological space X is called a sequential space if a set $A \subset X$ is closed if and only if together with any sequence it contains all its limits.

Remark. There is a space which is sequential but not generated by discrete subspaces.

Let

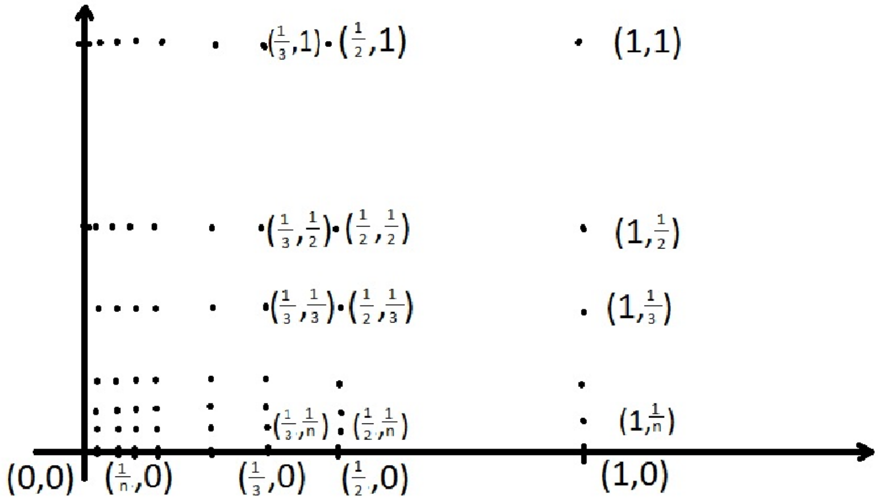
$$X = \{\langle 0,0 \rangle\} \bigcup \{\left\langle \frac{1}{i}, 0 \right\rangle : \mathrm{i} \geq 1\} \bigcup \{\left\langle \frac{1}{i}, \frac{1}{j} \right\rangle : j \geq k\}$$

and topologise X by the decaring the following families of basic open neighbourhoods:

1. each $\left\langle \frac{1}{i}, \frac{1}{j} \right\rangle$ is isolated;

2. basic open neighbourhoods of $\left\langle \frac{1}{i}, 0 \right\rangle$ are of the form $\{\left\langle \frac{1}{i}, 0 \right\rangle\} \bigcup \{\left\langle \frac{1}{i}, \frac{1}{j} \right\rangle : \mathrm{j} \geq \mathrm{k}\}$ some $k \geq 1$;

3. basic open neighbourhoods of $\langle 0,0 \rangle$ are of the form $\left\{\langle 0,0 \rangle\right\} \bigcup \left\{ \cup_{i \geq l} U_i \right\}$ where $l \geq 1$ and U_i is a basic open neighbourhood of $\left\langle \frac{1}{i}, 0 \right\rangle$ for each $i \geq j$.



Claim. X is not discretely generated.

Let $A = \{\left\langle \frac{1}{i}, \frac{1}{j} \right\rangle : i, j \geq 1\}$. Clearly $\langle 0,0 \rangle \in \overline{A} \setminus A$. And we have to find discrete subset of A where its elements converge to point $\langle 0,0 \rangle$. Suppose that $\left\langle x_n = \left\langle i_n^{-1}, j_n^{-1} \right\rangle \right\rangle_{n-1}^{\infty}$ is any sequence in $D \subset A$. Sequence makes up isolated points and it is obvious that D is discrete subset. If $\{i \in \mathbb{N} : (\exists n)(i_n = i)\}$ is finite, then it is easy to construct an open neighbourhood of $\langle 0,0 \rangle$ not containing any element of the sequence. Otherwise by passing to a subsequence we can assume that $i_n < i_{n+1}$ for all n, and from here construct a basic open neighbourhood of $\langle 0,0 \rangle$ disjoint from this subsequence. As a result there is not any discrete subset which $\langle 0,0 \rangle \in \overline{D}$. So X is not generated by discrete subsets.

Claim. X is sequential.

Let $A \subseteq X$ contain all the limits of convergent sequences in A. We wish to show that A is closed; i.e., $\overline{A} \subseteq A$. Suppose that $x \in \overline{A}$. Note that if $x \neq \langle 0,0 \rangle$, then as there is a countable base at x it is srtaightforward to show that there is a sequence in A converging to x, and so $x \in A$. So let's assume that $\langle 0,0 \rangle \in \overline{A} \setminus A$. We can show that the set

$$\{i \geq 1 : \text{every open neighbourhood of } \left\langle \frac{1}{i}, 0 \right\rangle \text{ intersects A}\}$$

is infinite (since otherwise we could construct an open neighbourhood of $\langle 0,0 \rangle$ disjoint from A), and so we may enumerate it increasingly as $\langle i_n \rangle_{n=1}^{\infty}$. But then for each $n \geq 1$ we have that $\langle i_n^{-1}, 0 \rangle \in \overline{A}$, and therefore by the above observation $\langle i_n^{-1}, 0 \rangle \in A$. It is then easy to see that $\langle \langle i_n^{-1}, 0 \rangle \rangle_{n=1}^{\infty}$ is a sequence in A converging to $\langle 0,0 \rangle$, and so $\langle 0,0 \rangle \in A$, contradicting our assumption that it is not. As a result A is closed. So X is sequential space.

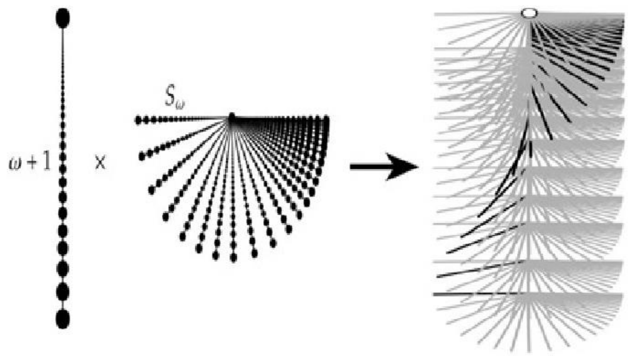
Theorem 1. The product of discretely generated spaces are not discretely generated.

Proof.

To show this we will construct X and Y topological spaces generated by discrete subsets which their product is not discretely generated.

Consider that X is the space $\omega + 1$, i.e. an infinite sequence of isolated points which converges to a point ∞. Further take the space S_ω as topological space Y, i.e. a space with domain $(\omega + \omega) \bigcup \{\infty\}$ and with the weakest possible topology such that for each n the sequence (n, i) converges to ∞ when i goes to ∞. Both these spaces has discretely generated property. Yet their product $X \times Y$ does not.

Consider a subset $A = \{(n, (n,i)) : n, i \in \omega\}$ and $A \subset X \times Y$ thus one converging sequence in S_ω is taken in each floor. The closure of A contains all points $(n, \infty) \in \overline{A} \setminus A$, so also the point $(\infty, \infty) \in \overline{A} \setminus A$ is there. To show $X \times Y$ is discretely generated we have to find discrete subset of A which $(\infty, \infty) \in \overline{D}$. However there is no sequence of points from A converging to (∞, ∞). It means that there is no a discrete subset which satisfying the condition. If a sequence converges to ∞ within the projection to S_ω it intersects one sequence from S_ω at infinitely many points. Therefore the sequence can not converge to ∞ within the projection to $\omega+1$. As a result the topological space $X \times Y$ consisted of discretely generated spaces is not generated by discrete subspaces.



The product of Frechet-Urysohn spaces are not discretely generated.

We let X denote the set of positive integers. We will now construct a collection of subsets of X. The collection $\Im$. By use of the continuum hypothesis and transfinite induction we will construct an infinite collection, $\Im$, of infinite subsets of X such that:

1. $S_1 \in \Im, S_2 \in \Im \Rightarrow S_1 \bigcap S_2$ is finite;

2. $\Im$ is a maximal collection of infinite subsets of X with respect to property 1;

3. given any subset $A \subset X$, either A is covered by a finite number of members of $\Im$ or else there exists an $S \in \Im$ such that $S \subset A$.

We well-order the collection $A = \{A_\alpha : \alpha < \Omega\}$ of all subsets of X. For each α we will construct a countable collection $\Im_\alpha$ of infinite subsets of X having property 3_α:

(3_α) Given any $A_\beta, \beta \le \alpha$, either A_β is covered by a finite number of members of $\Im_\alpha$ or there is an $S \in \Im_\alpha$ such that $S \subset A_\beta$.

We begin with a collection $\Im_1^{'}$ which is a partition of X into an infinite number of infinite sets. If A_1 is covered by a finite subcollection of $\Im_1^{'}$, let $\Im_1 = \Im_1^{'}$. If A_1 is covered by no finite subcollection of $\Im_1^{'}$, put the set of $\Im_1^{'}$ into one-to-one correspondence with the positive integers, $\Im_1^{'} = \{T_n : n = 1, 2, ...\}$; then for each N, $A_1 \setminus \bigcup_1^N T_n$ is infinite. Pick $a_1 \in A_1 \setminus T_1$ and for each $N > 1$ pick

$$a_N \in A_1 \setminus (\bigcup_{n=1}^{N} T_n \cup \{a_1, a_2, ..., a_{N-1}\})$$

Let $S_1 = \{a_1, a_2, ..., a_n, ...\}$ and let $\Im_1 = \Im_1^{'} \cup \{S_1\}$.

Let $S_\alpha = \{a_1^\alpha, a_2^\alpha, ..., a_n^\alpha, ...\}$ and let $\Im_\alpha = \Im_\alpha^{'} \cup \{S_\alpha\}$.

The sets $\Im_\alpha$ each have property 1 and property 3_α and, for $\beta < \alpha$, $\Im_\beta < \Im_\alpha$.

Let $\Im^{'} = \bigcup_{\alpha < \Omega} \Im_\alpha$. Then $\Im^{'}$ has property 1 and property 3. Let $\Im$ be any collection of infinite subsets of X which contains $\Im^{'}$ and is maximal with respect to property 1. $\Im$ has properties 1, 2, and 3.

For each $S \in \Im$ let S_0 and S_e be any two disjoint infinite subsets of S whose union is S. For example if

$$S = \{a_1, a_2, a_3, a_4, ..., a_n, ...\}$$

Let

$$S_0 = \{a_1, a_3, a_5, ..., a_{2n+1}, ...\}$$

$$S_e = \{a_2, a_4, a_6, ..., a_{2n}, ...\}$$

Let $\Im_e = \{S_e : S \in \Im\}$ and $\Im_0 = \{S_0 : S \in \Im\}$. Then $\Im_e \cup \Im_0$ is a maximal, almost disjoint (i.e., satisfying

property 1) collection of infinite subsets of X, and if $A \subset X$ is not covered by a finite collection of elements $\Im_e \cup \Im_0$ there exist $S_1 \in \Im_e$ and $S_2 \in \Im_0$ such that $A \supset S_1 \cup S_2$.

We will now define two topological spaces X_e and X_0. Let

$$X_e = X \cup \Im_e$$

and

$$X_0 = X \cup \Im_0$$

Let $X_e^*(X_0^*)$ be the one point compactification of $X_e(X_0)$.

Lemma. X_e^* and X_0^* are Frechet spaces.

The proof of this lemma was given in the article [2].

Theorem 2. $X_e^* \times X_0^*$ is not generated by discrete subspaces.

Proof.

We will show that the diagonal $B = \{(x, x) : x \in X\}$ has $(\infty_e, \infty_0) \in \overline{A} \setminus A$ as a limit point. It is obvious that we have to find D discrete subset of B which its closure containing this limit point, but no discrete subset from B whose elements converges to (∞_e, ∞_0).

(∞_e, ∞_0) is a limit point of B. Let $S \in \Im_0$, $S = \{x_1, x_2, ...\}$. Then $x_n \to S$ in X_0^*, but x_n is eventually outside each basic compact set in X_e^*. Thus $x_n \to \infty_e$ in X_e^*. Thus $(\infty_e, S) \in \overline{B}$ for each $S \in \Im_0$. Let S_n be a sequence of distinct elements in $\Im_0$. Then for each n, $(\infty_e, S_n) \in \overline{B}$ and $(\infty_e, S_n) \to (\infty_e, \infty_0)$ in $X_e^* \times X_0^*$ as $n \to \infty$

No sequence in B coverges to (∞_e, ∞_0). Let (x_n, x_n) be a sequence of distinct points in B. Since $\Im_e \cup \Im_0$ is a maximal almost disjoint collection of infinite sets in X the sequence $(x_n : n = 1, 2, ...)$ is either frequently in some $S \in \Im_e$ (and thus has a subsequence convergent to $S \in X_e^*$) or it is frequently in some $S \in \Im_0$ (and has a subsequence convergent to $S \in X_0^*$). Thus $(x_n : n = 1, 2, ...)$ cannot tend both to ∞_e in X_e and ∞_0 in X_0.

Theorem is proved.

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MATHEMATICAL MODELING OF THE CURVILINEAR MOTION OF THE CAR, TAKING INTO ACCOUNT THE ELASTICITY AND DEFORMABILITY OF TIRES

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Abstract

The paper proposes a mathematical model of a curvilinear motion of the car, taking into account an elastic characteristic of tires and the characteristics of their interaction with the road, as well as the design features of the front suspension and steering system based on the dynamics of rolling systems. The obtained mathematical model makes it possible to fully study the dynamics and stability of systems, taking into account design parameters.

Keywords: mathematical model, motion, rolling, deformation, elasticity, tire, potential forces, kinetic energy.

Introduction

An increase in the number of vehicles and an increase in traffic intensity lead to the need to improve their reliability and safety.

The modernization of vehicles, the active introduction of automatic control elements in various components of vehicles requires an assessment of the impact of all introduced improvements on the behavior of the car.

On the one hand, the development of computer technologies associated with analytical transformations makes it possible to consider vehicle models with a large number of degrees of freedom.

One of the most important of these problems is to determine the dependence of certain dynamic properties of systems, for example, stability or the nature of the loss of stability, on certain parameters of the problem.

Recent study [1] has shown that a reasonable reduction in the number of degrees of freedom and parameters taken into account does not significantly affect a number of practically important motion parameters. This indicates the need for the most complete study of the properties of simple car models and an increase in the number of degrees of freedom and the number of parameters taken into account only if necessary.

Of particular importance among the elements of the car, which largely determine its dynamics, is the pneumatic tire due to its role as a link between the crew and the road. All relevant forces for acceleration, braking, curvilinear motion are realized through the force interaction between the tire and the road. Despite the boundless number of works, the subject of which was the pneumatic tire, due to the complexity of its properties, the study of the influence of tire properties on the dynamics of the car continues to be one of the most urgent problems of mechanics.