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Signed Column Reactions for Combined Footing Design in Response Spectrum Analysis

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Abstract:- The loss of sign in the output results of Response Spectrum Method has been a subject of discussion in literature, highlighting the issues caused, manifesting primarily in: (i) the loss of relative signs of forces acting on the structural members and (ii) the loss of relative signs of the support reactions. The method of Dominant Mode Signage (DMS) had been presented as a remedy in some of the structural analysis packages to tackle the issue. The intention of this paper is to elaborate an alternate manual method of applying signs to the support reactions for those using structural analysis packages not yet equipped with the DMS facility, presenting an easy step-by-step process fit for utilising on a day-to-day basis.

It also demonstrates why a plot of the envelope of maximum footing pressures on soil on the footing is not symmetric in case of the Response Spectrum Method approach, even for a symmetric building with symmetric loading and supported on a symmetric combined footing, while it is rendered symmetric for the Seismic Coefficient Method approach (ie., the Static method), and as demonstrated, for the proposed method too.

Keywords:- *Response Spectrum Method, support reactions, loss of sign.*

I. INTRODUCTION

One of the issues that many have come across ever since the method of seismic analysis has been switched from Static Analysis (also known as Seismic Coefficient method or SCM for short) to Response Spectrum method (RSM) is the loss of sign of seismic forces. The reason is to be squarely placed on the SRSS and CQC methods (stipulated in IS:1893-2016 [1], as with most of the seismic codes around the world) that are used for combining the structural member forces (viz., bending moments BM, shear forces SF, axial forces AF, as well as the support reactions SR, and even deflections) obtained separately for each modal load (ie., lateral load corresponding to each mode). This can be briefly explained as follows: if R1 and R2 are two resulting modal forces (be it BM, SF, AF or SR) corresponding to modes 1 and 2, then the SRSS combination of the two modal forces will be

$$R_{SRSS} = \sqrt{R_1^{\,2} + R_2^{\,2}} \qquad (1)$$

and thus, whatever the signs of R1 and R2 may be, the resulting RSRSS is positive. This is the same case with the CQC method of combination also. The resulting RSRSS values enter the load combinations with a '±' sign as:

$$\begin{aligned} &1.0{*}R_{DL} + 1.0{*}R_{LL} \\ &1.0{*}R_{DL} + 0.8{*}R_{LL} \pm\ 0.8{*}RS_{RSS} \\ &1.0{*}R_{DL} \pm 1.0{*}R_{SRSS} \end{aligned} \qquad (2)$$

(where '*' represents multiplication), which are the service combinations for foundation design [1]. RDL and RLL represents the forces pertaining to the Dead loads and Live loads respectively. The last two combinations in (2) are duplicated in application: one with a '+' sign and another with a '–' in place of the '±'. This 'classical' approach, based on (1), shall be referred to as the Classical RSM.

The issue of the loss of sign during SRSS or CQC combination manifests primarily in two different aspects of design: (i) the loss of relative signs of forces acting on the structural members (Fig. 2b compared to Fig. 2d, for example) and (ii) the loss of relative signs of the support reactions.

The method of Dominant Mode Signage (DMS) was one of the remedies introduced as a new feature in some of the structural analysis packages (eg., STAAD.Pro [2]). Since this paper focuses only on the impact of the loss of signs of support reactions, discussion of its impact on the design of structural members is not covered. An introduction to the latter and the DMS method as a remedy to it can be found in [3], while an elaborate explanation on DMS with a real building example is given in [4]. Nevertheless, the final design load combinations based on the Classical RSM approach is somehow doing well enough to keep the buildings safe, it seems. Any further inference on this aspect is beyond the scope of this paper.

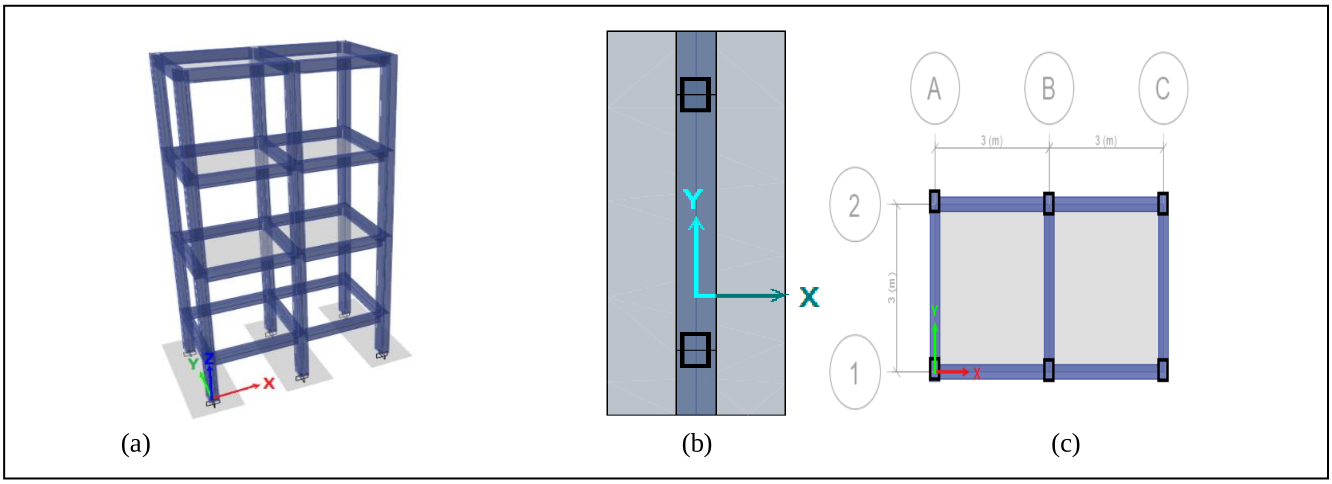

Fig. 1: (a) The ETABS model, (b) plan view with dimensions and grid designations, and (c) the plan view of footing provided for the B frame

II. THE SAMPLE BUILDING FOR INVESTIGATION

In order to understand how the loss of signs of support reactions affects foundation design, a simple sample structure modelled in the analysis package of ETABS [5] (Fig. 1a, shown along with its combined footings – the footings are not part of the ETABS model, but only shown for illustration) is selected for investigation. Considering the middle frame (Fig. 2a, designated grid B in Fig.1b), with its foundation being a beam-slab combined footing (Fig. 1c) of dimensions 1.8x4.5 m, placed symmetric to the columns, and modelled with a soil sub-grade modulus corresponding to a Safe Bearing Capacity (SBC) of 140 kN/m^2. The footing is analysed using the software package of SAFE [5]. The building is a three storied one with 2 by 1 number of bays in plan (Fig. 1b). It has a storey height of 3.6 m, and 2 m from plinth beam to support point. The plan dimensions are as in Fig. 1b. It has 300x400 mm columns, 250x400 mm beams and 110 mm thick slabs, all of M25 concrete (ie, of grade 25 MPa). The beams support masonry wall loads of 14.72 kN/m (while at the roof, only the periphery beams support parapet loads of 2.88 kN/m). The floor slabs support a finishing load of 1 kN/m^2 (and 2 kN/m^2 instead on the roof); and a Live Load of 3 kN/m^2 on the floors (and 1.5 kN/m^2 on the roof). The structure is assumed to be located in seismic zone IV on Type I soil [1].

III. THE STEPS FOR THE PROPOSED APPROACH

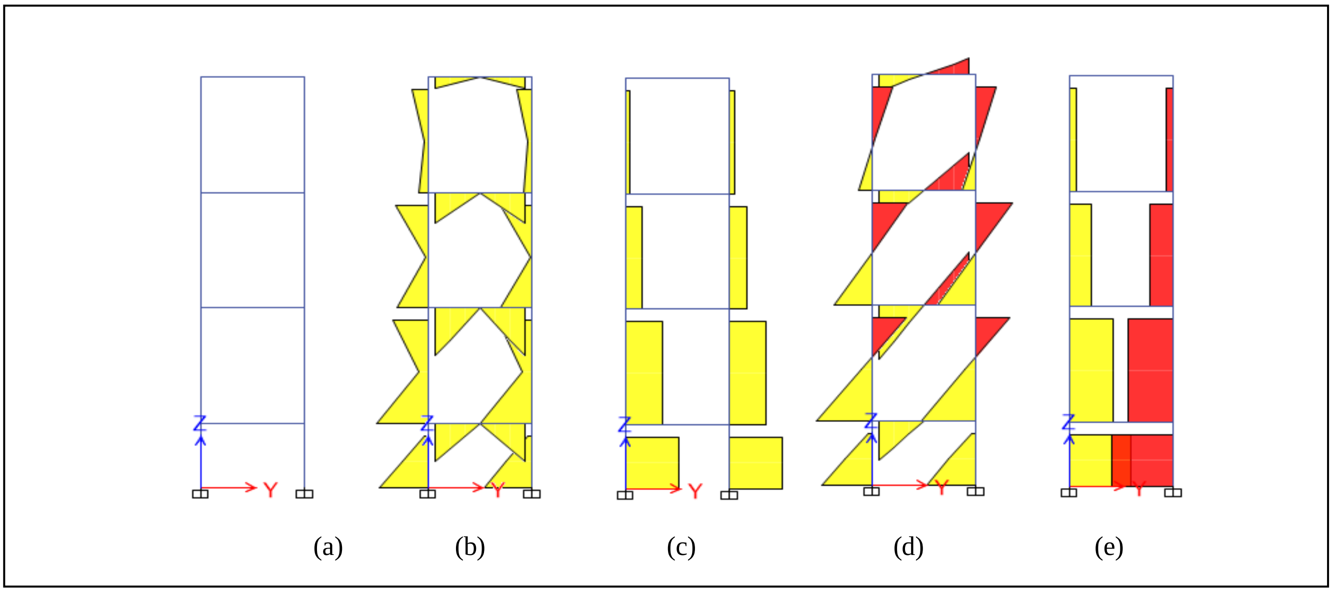

Fig. 2: (a) The grid B frame, (b) the BM’s and (c) the axial forces from the RSM approach, and (d) the BM’s and (e) the axial force from the SCM approach.

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Fig. 3 illustrates how the support reactions add up to the finial resulting support reactions in both the Classical RSM as well as in the proposed method which shall be referred to as 'Signed RSM'. Shown with bold arrows (Fig. 3a) are the support reactions SR_1 and SR_2 from the modal loads corresponding the 1st and 2nd modes, which are then combined by SRSS method to get the resultants, being positive for all components of the reaction, for all supports (denoted SR_{SRSS} in Fig.3b). In the Classical RSM approach, this, as it is, is combined with the gravity load reactions SR_{DL+LL} as the two cases of $R_{DL+LL} \pm R_{SRSS}$: one with $+SR_{SRSS}$, and the other with $-SR_{SRSS}$ (Fig.3c): note that there are combination cases with both Dead Load (DL) and Live Load (LL), as well as with DL alone (2), but here the label SR_{DL+LL} represents both.

What was meant to be highlighted (Fig. 3) is that, the resulting reactions of both the columns when combined by SRSS (or CQC, as the case might be) are always positive (SR_{SRSS} in Fig. 3b) and as these support reactions enter a load combination representing the $\pm SR_{SRSS}$ case, all the reactions are together positive, and then together negative in the corresponding '+' and '–' cases respectively. What is being lost is not just the sign, but also the relative sign of the individual support reaction resultants of one from that of another. What is desired is to assign signs to each support reaction in the SR_{SRSS} on some basis as to what these signs are expected be. One of the approaches, along the lines of [6] is to take the signs of each resultant force from the analysis for the lateral load pertaining to the dominant mode (in that direction), SR_1 in this case, and apply it to the resultant as

$$\sim SR_{SRSS} = sign(SR_1) * \sqrt{SR_1^{\ 2} + SR_2^{\ 2}}$$

where $\sim SR_{SRSS}$ represents the signed SR_{SRSS}. The function 'sign(SR_1)' gives the sign of SR_1 as values of +1, 0 or –1 only, depending on whether SR_1 is positive, zero or negative respectively (represented in the Fig.3b as "sgn-SR_1") – thus rendering (3) to be nothing more than changing the sign of SR_{SRSS}, to obtain $\sim SR_{SRSS}$. This is done separately for each component of the support reactions (viz., moments about the x and y axes: Mx and My; the vertical force: Fz; etc. – see Fig. 1a and 1b for Cartesian directions). This is what is being done by the DMS facility available in some of the analysis packages. Note that the term Dominant Mode refers to that mode (in that Cartesian direction) with the highest modal mass participation.

Another approach [7], found especially useful in handling the issue of support reactions, is to take the signs of the resultant forces of the SCM analysis (SR_{SCM}) and apply them to those of the Response Spectrum results, as

$$\sim SR_{SRSS} = sign(SR_{SCM}) * \sqrt{SR_1^{\ 2} + SR_2^{\ 2}}$$

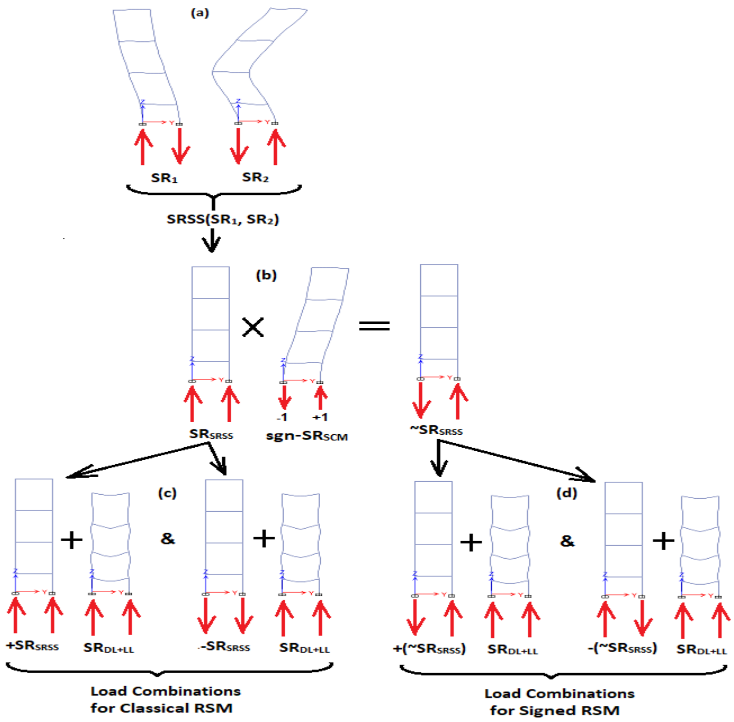

Fig. 3: (a) Support reactions of the two modal loads, (b) SRSS of the modal support reactions, SRSRSS, multiplied with signs of support reactions of SCM, sgs-SRSCM, and resulting signed SRSS support reaction, ~SRSRSS; (c) combining SRSS resultant support reactions, SRSRSS, with those of gravity loads, SRDL+LL, and (d) combining signed SRSS resultant support reactions, ~SRSRSS, with those of gravity loads, SRDL+LL.

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Nevertheless, both the above approaches give you mostly the same signs to be applied for nearly all the building cases. Since the intention of this paper is to elaborate a manual method of applying signs to support reactions to be used in those analysis packages not yet equipped with the DMS facility, only the second approach [7] shall be elaborated, referred to earlier in this paper as Signed RSM. The method derived along the lines of [6] is not elaborated for manual implementation, primarily because it is difficult (or rather cumbersome) to extract the SR_1 component alone (for determining the sign) in some of the analysis packages. The basic approach of Signed RSM is illustrated in Fig. 3b and 3d – the Classical RSM approach (Fig. 3c) had been already explained earlier in the paper. In the Signed RSM approach, the resulting SR_{SRSS} is multiplied by the signs from the results of the SCM analysis, sgn-SR_{SCM}, to obtain ~SR_{SRSS} (Fig. 3b). The resulting signed ~SR_{SRSS} is then combined with the gravity load reactions SR_{DL+LL} in two separate cases (Fig. 3d): one with +(~SR_{SRSS}), and the other with –(~SR_{SRSS}). The combining of SR_1 and SR_2 is done automatically by the software; the step of multiplying them with the appropriate sign and combing them with reactions of gravity loads, as per (2) is done manually, and then is input into a foundation design spreadsheet or a specialised foundation design package like SAFE, as the case may be.

IV. THE MANUAL PROCEDURE FOR IMPLEMENTING THE PROCEDURE

Sl. No	Load Case	*FX (kN)*	*FY (kN)*	*FZ (kN)*	*MX (kNm)*	*MY (kNm)*	*MZ (kNm)*
1	DL	0	3.936	346	–2.469	0	0
2	LL	0	–0.246	33.33	0.146	0	0
3	RSx	30.407	0	0	0	35.9	0
4	EQx	–30.67	0	0	0	–36.5	0
5	sgn-EQx	–1	0	0	0	–1	0
6	~RSx	–30.407	0	0	0	–35.9	0
7	RSy	0	16.84	76.55	28.28	0	0
8	EQy	0	–16.4	–88.49	28.51	0	0
9	sgn-EQy	0	–1	–1	1	0	0
10	~RSy	0	–16.84	–76.55	28.28	0	0

Table 1a. Support reactions from Load Cases for support B1

Sl. No	Load Case	*FX (kN)*	*FY (kN)*	*FZ (kN)*	*MX (kNm)*	*MY (kNm)*	*MZ (kNm)*
1	DL	0	–3.936	346	2.469	0	0
2	LL	0	0.246	33.33	–0.146	0	0
3	RSx	30.4074	0	0	0	35.9	0
4	EQx	–30.678	0	0	0	–36.5	0
5	sgn-EQx	–1	0	0	0	–1	0
6	~RSx	–30.407	0	0	0	–35.9	0
7	RSy	0	16.84	76.55	28.28	0	0
8	EQy	0	–16.4	88.49	28.51	0	0
9	sgn-EQy	0	–1	1	1	0	0
10	~RSy	0	–16.84	76.55	28.28	0	0

Table 1b. Support reactions from Load Cases for support B2

In the building model, the results both the methods, viz., (a) the Classical RSM, and (b) the Signed RSM, are being discussed and shown how the footing pressures on soil of the combined footing vary for each. For comparison, these shall also then be compared with the pressures for (c) the SCM approach. The RSM (analysed using the first twelve modes obtained by Eigen extraction) has been scaled for its base shear V_b to match that of the SCM analysis, so as to be able to fairly compare the pressures on soil from both.

Table 1(a-b) lists the support reactions for the different basic Load Cases, which are then added together as per (2) to form the support reactions for Load Combinations in Table 2(a-c). In the tables, FX, FY, FZ stands for support reaction forces along the global X, Y and Z directions respectively, and MX, MY, MZ stands for support reaction moments, about the global X, Y and Z directions respectively. RSx and RSy represent reactions for the RSM cases applied in the global X and Y directions respectively, while EQx and EQy represents reactions for the SCM case. The rest of the naming conventions (viz., DL, LL, sgn-EQx, ~RSx, etc.) have already been discussed. The naming of the Load Combinations in Table 2 is a representation of (2), which are easily decipherable on closer look, in light of the notations defined for Table 1. The support points of designation B1 and B2 are located as in Fig. 1b. For the sake of limiting space, only FZ, MX and MY are being tabulated in Table 2. Note that in Table 1, the 5^{th} and 9^{th} row have entries that are signs of the 4^{th} and 8^{th} rows respectively, while rows 6^{th} and 10^{th} are values of rows 3^{rd} and 7^{th} signed by corresponding 5^{th} and 9^{th} rows respectively.

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V. DISCUSSION OF FOOTING ANALYSIS RESULTS

Load Combination		Support B1				Support B2		
		FZ (kN)	*MX (kNm)*	*MY (kNm)*		*FZ (kN)*	*MX (kNm)*	*MY (kNm)*
DL+LL		379.3	–2.323	0		379.3	2.323	0
DL+0.8LL+0.8RSx		372.7	–2.352	28.71		372.7	2.352	28.717
DL+0.8LL–0.8RSx		372.7	–2.352	–28.72		372.7	2.352	–28.717
DL+0.8LL+0.8RSy		433.9	20.272	0		433.9	24.976	0
DL+0.8LL–0.8RSy		311.4	–24.98	0		311.4	–20.272	0
DL+RSx		346	–2.469	35.89		346	2.468	35.897
DL–RSx		346	–2.469	–35.9		346	2.468	–35.897
DL+Rsy		422.5	25.812	0		422.5	30.749	0
DL–Rsy		269.4	–30.75	0		269.4	–25.812	0

Table 2a. Support reactions from Load Combinations for Classical RSM approach

Load Combination		**Support B1**				**Support B2**		
		FZ (kN)	***MX (kNm)***	***MY (kNm)***		***FZ (kN)***	***MX (kNm)***	***MY (kNm)***
DL+LL		379.3	–2.323	0		379.3	2.323	0
DL+0.8LL+0.8(~RSx)		372.7	–2.352	–28.72		372.7	2.352	–28.7
DL+0.8LL–0.8(~RSx)		372.7	–2.352	28.72		372.7	2.352	28.72
DL+0.8LL+0.8(~RSy)		311.4	20.27	0		433.9	24.98	0
DL+0.8LL–0.8(~RSy)		433.9	–24.98	0		311.4	–20.3	0
DL+(~RSx)		346	–2.469	–35.9		346	2.469	–35.9
DL–(~RSx)		346	–2.469	35.9		346	2.469	35.9
DL+(~Rsy)		269.4	25.81	0		422.5	30.75	0
DL–(~Rsy)		422.5	–30.75	0		269.4	–25.8	0

Table 2b. Support reactions from Load Combinations for Signed RSM approach

Load Combination		**Support B1**				**Support B2**		
		FZ (kN)	***MX (kNm)***	***MY (kNm)***		***FZ (kN)***	***MX (kNm)***	***MY (kNm)***
DL+LL		379.3	–2.32	0		379.3	2.323	0
DL+0.8LL+0.8EQx		372.7	–2.35	–29.2		372.7	2.352	–29.2
DL+0.8LL–0.8EQx		372.7	–2.35	29.2		372.7	2.352	29.2
DL+0.8LL+0.8EQy		301.9	20.46	0		443.5	25.16	0
DL+0.8LL–0.8EQy		443.5	–25.2	0		301.9	–20.5	0
DL+EQx		346	–2.47	–36.5		346	2.469	–36.5
DL–EQx		346	–2.47	36.5		346	2.469	36.5
DL+EQy		257.5	26.04	0		434.5	30.98	0
DL–EQy		434.5	–31	0		257.5	–26	0

TABLE 2C. SUPPORT REACTIONS FROM LOAD COMBINATIONS FOR SCM APPROACH

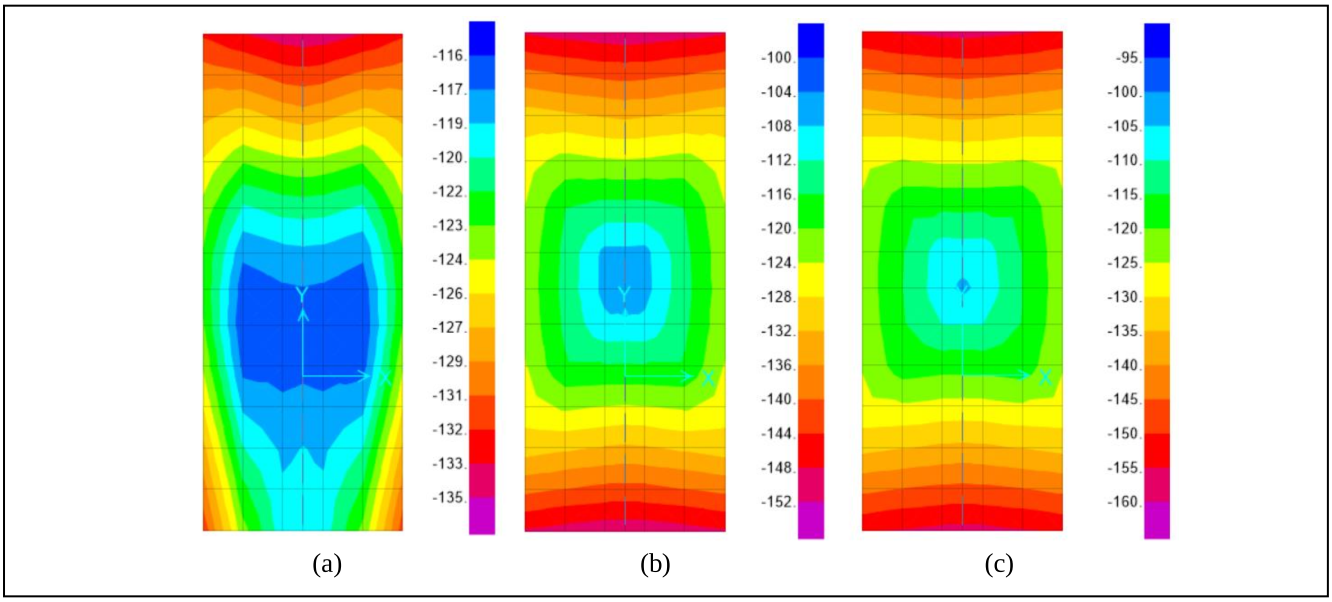

Fig. 4. Envelope of footing pressure on soil on the footing of central frame (see Fig. 1b) from (a) Classical RSM approach, (b) Signed RSM approach, and (c) SCM approach

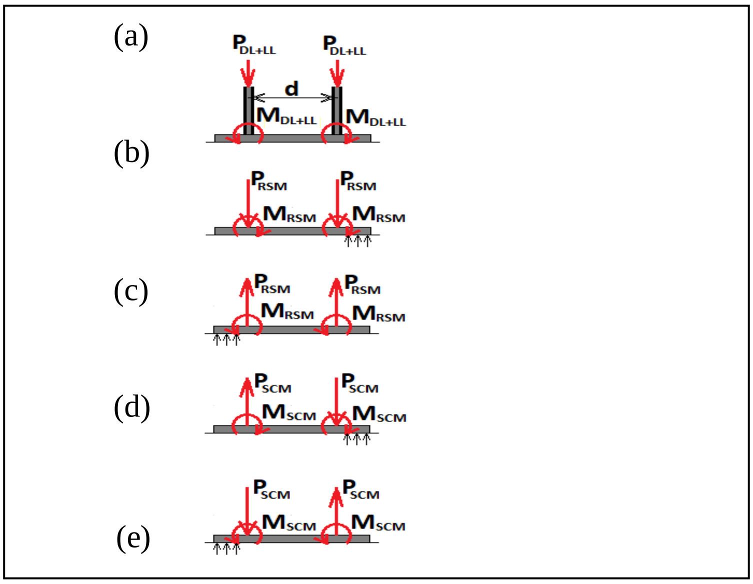

Fig. 5: The schematic representation of the components of footing pressures on soil for: (a) gravity load case (and also shown are the location of the columns), (b) +RSM case, (c) –RSM case, (d) +SCM case, and (e) –SCM case. Shown in bold arrows at column locations are the forces and moments. The three small arrows below the footing show the location of the maximum pressure for the seismic cases (b-e).

On application of the final support reactions from Table 2(a-c) on the footing modelled in SAFE (Fig.1c), and analysing, the envelope of maximum footing pressures on soil is plotted for comparison (Fig. 4). The maximum pressure is represented by purple (seen at the edges), which gradually changes shades as it decreases to the minimum represented by blue. The values are shown against the corresponding shade of colour in the palette on the right side of the pressure contour plot for each. The maximum pressures on soil are 134.54, 150.83 and 156.73 kN/m^2 for the Classical RSM, the Signed RSM, and the SCM approaches respectively (Fig. 4a-c). It may be noted that the maximum pressure on soil value of 134.54 kN/m^2 for Classical RSM has increased to 150.83 kN/m^2 for the Signed RSM. Thus, based on the SBC of the soil, the size of the footing, finalised based on Classical RSM, has to be increased, in case the Signed RSM method is adopted. It may also be noted that the value of pressure on soil for the Signed RSM is closer to that for the SCM than to that for the Classical RSM. This is due to the similarity in signs of forces between those of the Signed RSM and of the SCM approaches. The higher value of pressure on soil for the SCM compared to the Signed RSM is probably because of the difference in the pattern of distributing the base shear along the height of the structure: while RSM distributes the base shear based on the combination of forces corresponding to the actual mode shapes (Fig. 3a), the SCM distributes the same based on an assumed parabolic mode shape [1], which in turn, lumps more lateral load towards the top of the structure than that done by the RSM, thus slightly increasing the base moment compared to the RSM.

As to how the contribution of the column loads from the various load cases add up to give the resulting footing pressures of the combined footing in case of SR_{SRSS}, it is as follows: for the $+SR_{SRSS}$ case, the vertical seismic forces from both the columns push the footing downwards (Fig. 3c, marked $+SR_{SRSS}$, shown as the reaction of the supports acting upwards) with the moments from both the columns acting clockwise; for the $–SR_{SRSS}$ case, both the columns pull the footing upwards with the moments from both the columns acting anticlockwise (Fig. 3c, marked $–SR_{SRSS)}$. Both these cases of loading are in superposition to the forces and moments due to the gravity loads in the Load Combinations. An illustration of this, as well as that for the SCM (the two already established methods) is given in Fig. 5, separating the contributing components from the column forces for each. Equations (5) to (8) corresponds to Fig. 5b-e respectively, but with the forces shown in Fig. 5a added to each.

$$q_{\max(+RSM)} = \frac{2*P_{DL+LL}}{A} + \frac{2*P_{RSM}}{A} + \frac{2*M_{RSM}}{Z} \tag{5}$$

$$q_{\max(-RSM)} = \frac{2*P_{DL+LL}}{A} + \frac{2*(-P_{RSM})}{A} - \frac{2*(-M_{RSM})}{Z} \tag{6}$$

$$q_{\max(+SCM)} = \frac{2*P_{DL+LL}}{A} + \frac{2*M_{SCM}}{Z} + \frac{P_{SCM}*d}{Z} \tag{7}$$

$$q_{\max(-SCM)} = \frac{2*P_{DL+LL}}{A} - \frac{2*(-M_{SCM})}{Z} - \frac{-(P_{SCM}*d)}{Z} \tag{8}$$

In (5) to (8), P_{DL+LL} corresponds to the vertical forces from gravity loads – in it, M_{DL+LL}, the moments, are not included since they cancel out (Fig 5a). While P_{RSM} and M_{RSM} pertains to RSM and P_{SCM} and M_{SCM} to SCM, the q_{max}'s represent the pressure on soil at the location marked by three small arrows in Fig. 5b-e – viz., $q_{max(+RSM)}$ and $q_{max(–RSM)}$ – representing the maximum pressure on soil for the RSM cases, and $q_{max(+SCM)}$ and $q_{max(–SCM)}$ correspondingly for the SCM cases.

As evident in (5) to (8), $q_{max(+RSM)}$ is not equal to $q_{max(–RSM)}$, but $q_{max(+SCM)}$ turns out to be equal to $q_{max(–SCM)}$, demonstrating that a plot of the envelope of maximum pressures on soil on the footing is not symmetric (Fig.4a) for the Classical RSM, even for a symmetric building with symmetric loading and supported on a symmetric combined footing (such as the one investigated here), while the plot of maximum pressures on soil is rendered symmetric for the SCM approach (Fig 4b), and consequently for the Signed RSM too (Fig. 4c) – this aspect happens to be a convincing point in favour of the Signed RSM approach.

VI. CONCLUSION

Having demonstrated how the application of Classical RSM approach results in an un-conservative estimate of the support reactions (at least in some of the cases) and how the Signed RSM fairs in that matter, the paper urges the structural designers to implement the latter (in case of structural packages not equipped with the DMS facility) and develop spreadsheets specifically for the purpose. Once having done so, the process of converting RSM based support reactions to Signed RSM based reactions is only a matter of copy-paste from the analysis package output to the spreadsheet, followed by another copy-paste of the resulting load combinations with Signed RSM reactions from the spreadsheet to a foundation design spreadsheet or a foundation design package. On the other hand, it may also be preferred to be done using a scripting language facility provided by the analysis package, if available.

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