

# Differential Calculus for the Summation of Geometric Series with Binomial Expansions

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**Abstract:** This paper presents a computational technique for finding the  $n$ th derivative of a geometric series using binomial coefficients, and some useful results. These results refer to the methodological advances which are useful for researchers who are working in computational science. Computational science is a rapidly growing multi-and inter-disciplinary area where science, engineering, computation, mathematics, and collaboration use advance computing capabilities to understand and solve the most complex real life problems.

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## 1. Introduction

In the earlier days, geometric series [1-5] served as a vital role in the development of differential and integral calculus and as an introduction to Taylor series and Fourier series. In calculus, the derivative of a function is the instantaneous rate of change of the function with respect to one of its independent variables. The geometric series and combinatorics [6-10] have significant applications in computational science. Computational science is a rapidly growing inter-disciplinary area where science, engineering, computation, mathematics, and collaboration use advance computing capabilities to understand and solve the most complex real life problems.

## 2. The First Derivative of Geometric Series

Differentiation is the derivative [7] of a function with respect to an independent variable. In this section, a geometric series is considered as the function of independent variable  $x$  as follows:

The function of geometric series is  $f(x) = \sum_{i=0}^r x^i = 1 + x + x^2 + x^3 + \dots + x^r = \frac{x^{r+1} - 1}{x - 1}$ .

Let  $N = \{0, 1, 2, 3, 4, 5, \dots\}$  be the set of natural numbers including zero element.

The first derivative of geometric series is give below:

$$f^1(x) = 1 + 2x + 3x^2 + 4x^3 \dots + rx^{r-1} = f^1\left(\frac{x^{r+1} - 1}{x - 1}\right) = \frac{(rx - r - 1)x^r + 1}{(x - 1)^2}$$
$$\Rightarrow V_0^1 + V_1^1x + V_2^1x^2 + V_3^1x^3 \dots + V_{r-1}^1x^{r-1} = \frac{(rx - r - 1)x^r + 1}{(x - 1)^2}, (x \neq 1).$$

By substituting  $x = 2$  in  $f^1(x)$ , we get the mathematical equation as follows:

$$1 + 2(2) + 3(2)^2 + 4(2)^3 + \dots + r2^{r-1} = \frac{(r - 1)2^r + 1}{(2 - 1)^2} = (r - 1)2^r + 1.$$

$$\text{For } x = 3, \quad 1 + 2(3) + 3(3)^2 + 4(3)^3 + \dots + r3^{r-1} = \frac{(2r-1)3^r + 1}{(3-1)^2} = \frac{(2r-1)3^r + 1}{2^2}.$$

$$\text{For } x = 4, \quad 1 + 2(4) + 3(4)^2 + 4(4)^3 \dots + r4^{r-1} = \frac{(3r-1)4^r + 1}{(4-1)^2} = \frac{(3r-1)4^r + 1}{3^2}.$$

$$\text{Similarly, for any number } k \text{ that is equal to } x, \quad \sum_{i=0}^{r-1} V_i^1 = \frac{(kr-r-1)k^r + 1}{(k-1)^2}.$$

### 3. The $n^{\text{th}}$ Derivative of Geometric Series

$y = f(x) = \sum_{i=0}^r x^i = \frac{x^{r+1} - 1}{x - 1}$ . The derivatives of  $y$  are given below.

$$\frac{1}{1!} \frac{dy}{dx} = \sum_{i=0}^{r-1} V_i^1 x^i \Rightarrow \frac{1}{2!} \frac{d^2 y}{dx^2} = \sum_{i=0}^{r-2} V_i^2 x^i \Rightarrow \frac{1}{3!} \frac{d^3 y}{dx^3} = \sum_{i=0}^{r-3} V_i^3 x^i \Rightarrow \dots \frac{1}{n!} \frac{d^n y}{dx^n} = \sum_{i=0}^{r-n} V_i^n x^i.$$

The  $n^{\text{th}}$  derivative [7] of geometric series is

$$\frac{1}{n!} \frac{d^n y}{dx^n} = \sum_{i=0}^{r-n} V_i^n x^i = \frac{1}{n!} f^n(x) = \frac{1}{n!} f^n \left( \frac{x^{r+1} - 1}{x - 1} \right).$$

$$\sum_{i=0}^{r-1} V_i^1 x^i = \frac{1}{1!} f^1 \left( \frac{x^{r+1} - 1}{x - 1} \right); \sum_{i=0}^{r-2} V_i^2 x^i = \frac{1}{2!} f^2 \left( \frac{x^{r+1} - 1}{x - 1} \right); \& \sum_{i=0}^{r-3} V_i^3 x^i = \frac{1}{3!} f^3 \left( \frac{x^{r+1} - 1}{x - 1} \right)$$

are first, second, and third derivatives respectively.

### 4. Conclusion

In this article, the  $n^{\text{th}}$  derivative ( $n = 1, 2, 3, \dots$ ) of a geometric series using binomial coefficients is introduced in an innovative way. These results refer to the methodological advances which are useful for researchers who are working in science, economics, engineering, management, computation, and medicine [11].

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