

# PARTICLE ACCELERATION IN PULSARS AND PULSAR WIND NEBULAE

ELENA AMATO  
INAF-OSSERVATORIO ASTROFISICO DI ARCETRI



# PLAN OF THE LECTURES

- LECTURE 1: PULSARS AND THEIR MAGNETOSPHERES
  - BRIEF HISTORICAL NOTES
  - THE GOLDREICH AND JULIAN MAGNETOSPHERE
  - GAPS, PLASMA SUPPLY, MULTIPLICITY
  - FAST PULSARS AND UHECRS
  - THE PULSAR WIND
- LECTURE 2: PULSAR WIND NEBULAE
  - DYNAMICS
  - PARTICLE ACCELERATION
  - RECENTS FROM GAMMA-RAYS
  - PARTICLE ESCAPE



# LECTURE 1

- LECTURE 1: PULSARS AND THEIR MAGNETOSPHERES
  - BRIEF HISTORICAL NOTES
  - THE GOLDREICH AND JULIAN MAGNETOSPHERE
  - GAPS, PLASMA SUPPLY, MULTIPLICITY
  - FAST PULSARS AND UHECRS
  - THE PULSAR WIND



# NEUTRON STARS

- 1933: DISCOVERY OF THE NEUTRON (Chadwick 1933)
- 1934: NEUTRON STARS PROPOSED (Baade & Zwicky 1934) AS EXPLANATION FOR SUPERNOVA ENERGY RELEASE
- 1939: FIRST MEANINGFUL NEUTRON STAR EQUATION OF STATE (Tolman, Oppenheimer & Volkoff 1939): MAXIMUM MASS SMALLER THAN FOR WHITE DWARFS
- 1939–1959: NO HOPE OF OBSERVATION →FORGOTTEN PROBLEM
- 1959: REVISED MASS ESTIMATE (Cameron 1959)



# IN GRAVITATIONAL COLLAPSE

CONSERVATION OF ANGULAR MOMENTUM

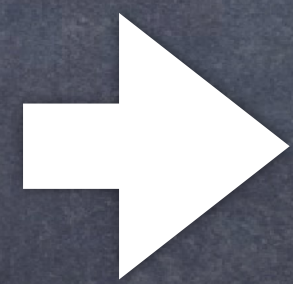
$$M_f R_f^2 \Omega_f = M_i R_i^2 \Omega_i$$

CONSERVATION OF MAGNETIC FLUX

$$B_i R_i^2 = B_f R_f^2$$

$$R_i \approx 10^{11} \text{cm}$$

$$R_f \approx 10^6 \text{cm}$$

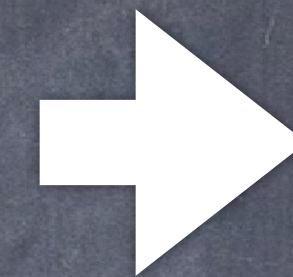


$$\left( \frac{R_i}{R_f} \right)^2 = 10^{10}$$



$$P_i \sim 10^5 \text{s}$$

$$B_i \sim 100 \text{G}$$



$$P_f < 0.1 \text{ms}$$

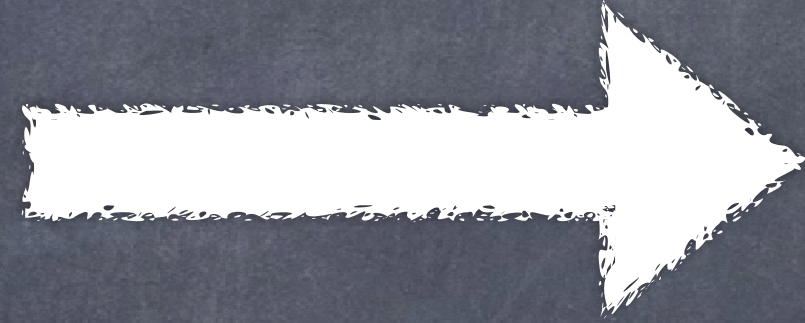
$$B_f \sim 10^{12} \text{G}$$



# EXTREME ENVIRONMENT

## QED EFFECTS

$$B_{\text{QED}} = \frac{m_e^2 c^3}{\hbar e} \approx 5 \times 10^{13} \text{ G}$$



$$\frac{B_{\text{NS}}}{B_{\text{QED}}} = 0.01 - 1$$

MAGNETARS....

## GR EFFECTS

$$R_{\text{GR}} = \frac{2GM}{c^2}$$



$$\xi_{\text{NS}} = \frac{R_{\text{GR}}}{R_{\text{NS}}} = 0.34 \left( \frac{M_{\text{NS}}}{1.4 M_{\odot}} \right) \left( \frac{R_{\text{NS}}}{10 \text{ km}} \right)^{-1}$$

FOR COMPARISON:  $\xi_{\text{BH}} = 1$



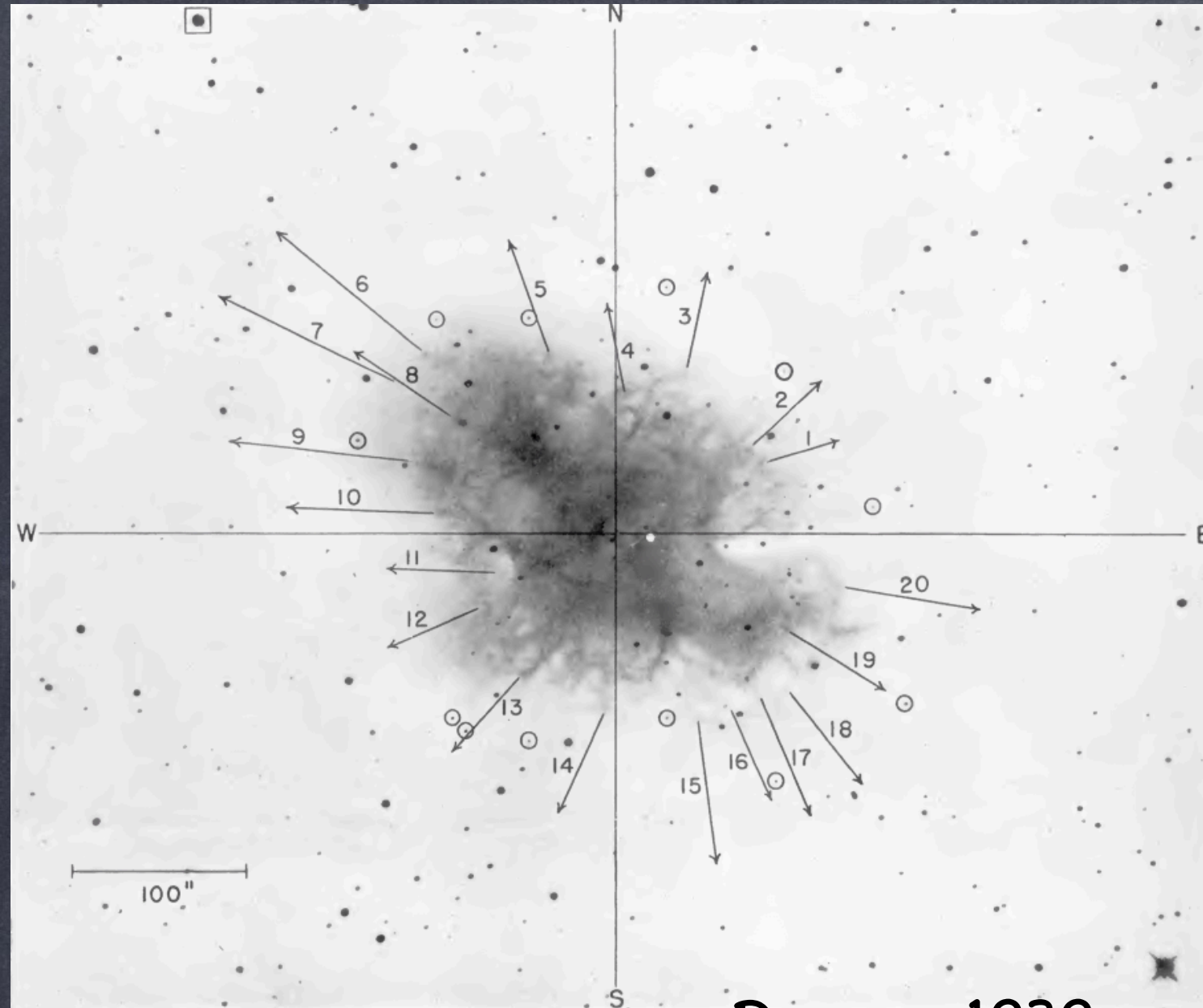
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- 1959: REVISED MASS ESTIMATE (Cameron 1959)
- 1969: A BRILLIANT IDEA



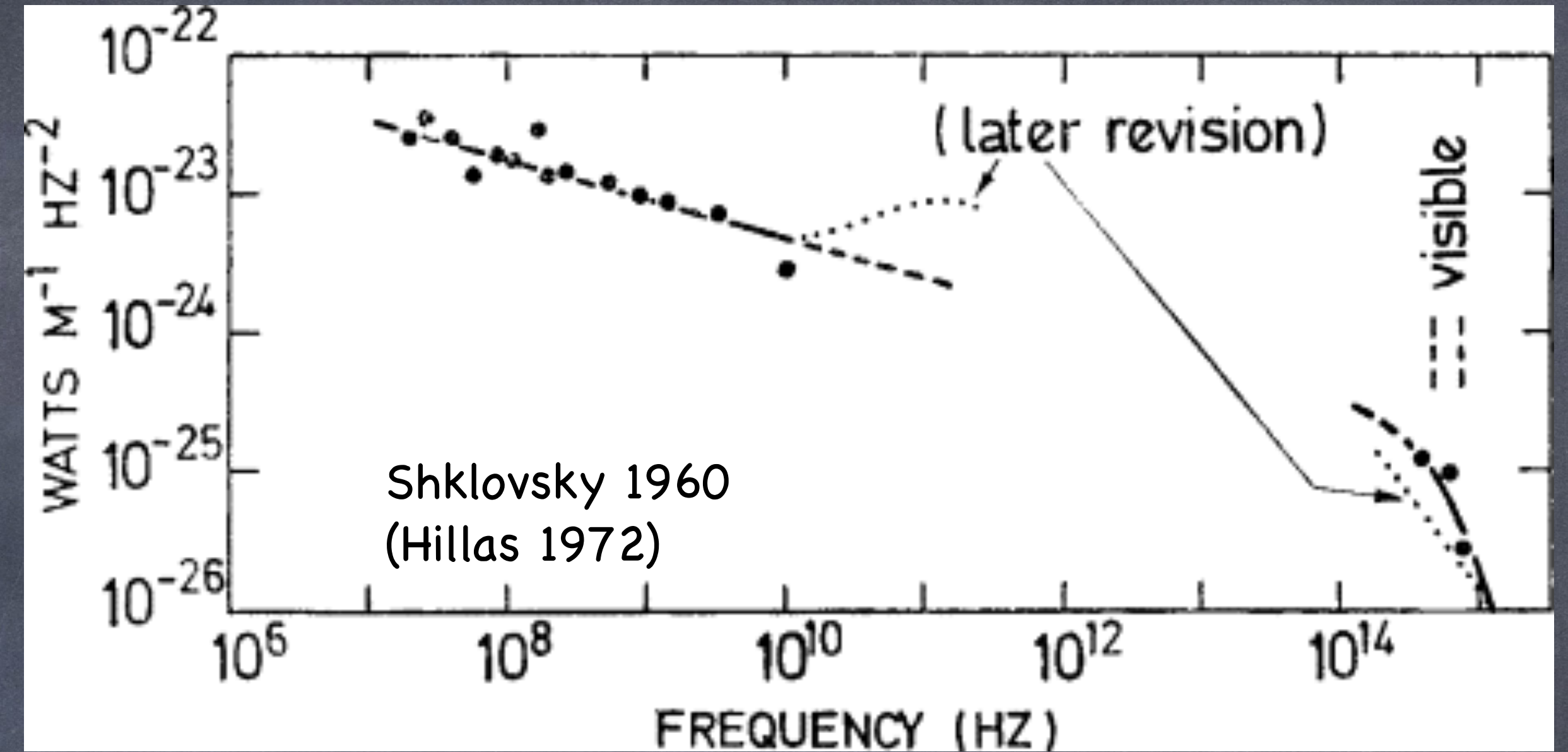
# ACTIVITY IN THE CRAB NEBULA

## ACCELERATED EXPANSION

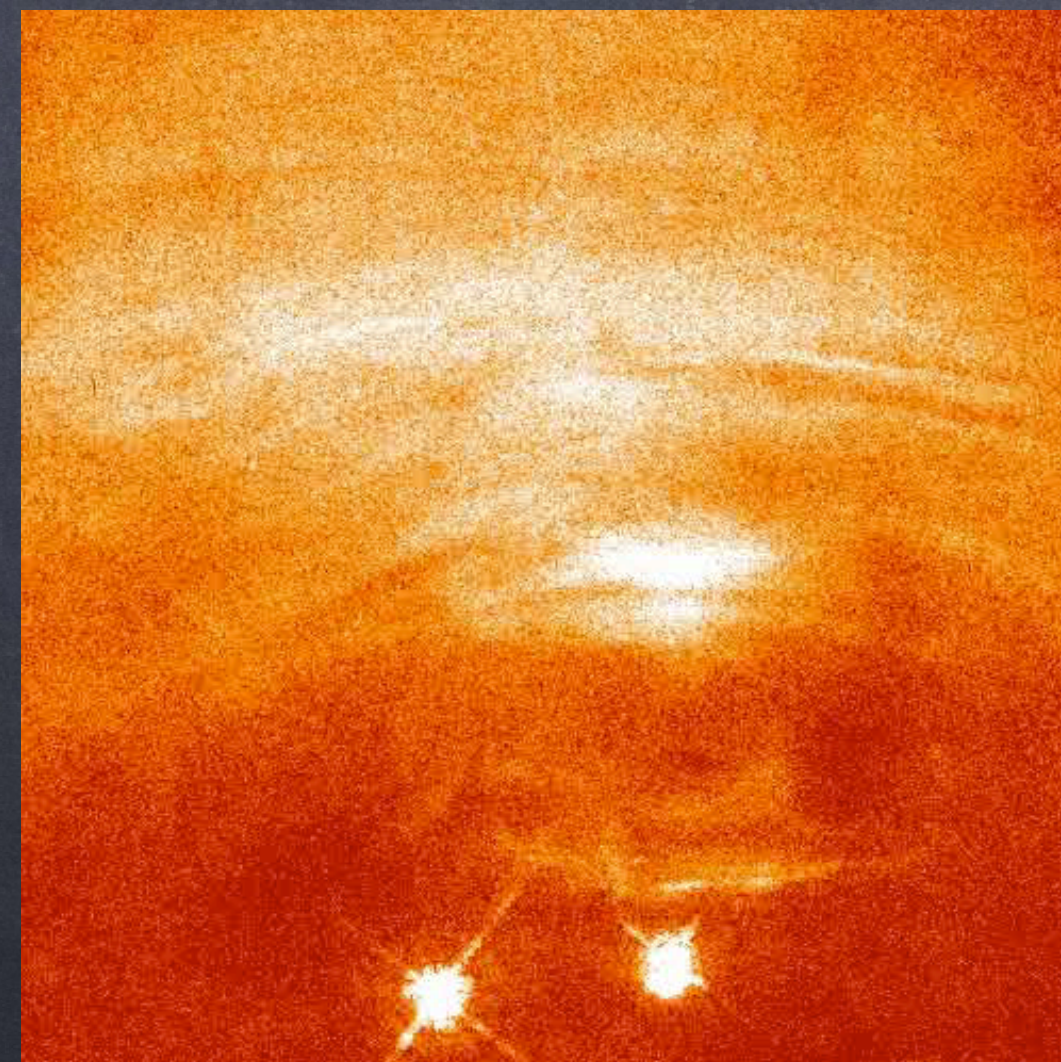


Duncan 1939

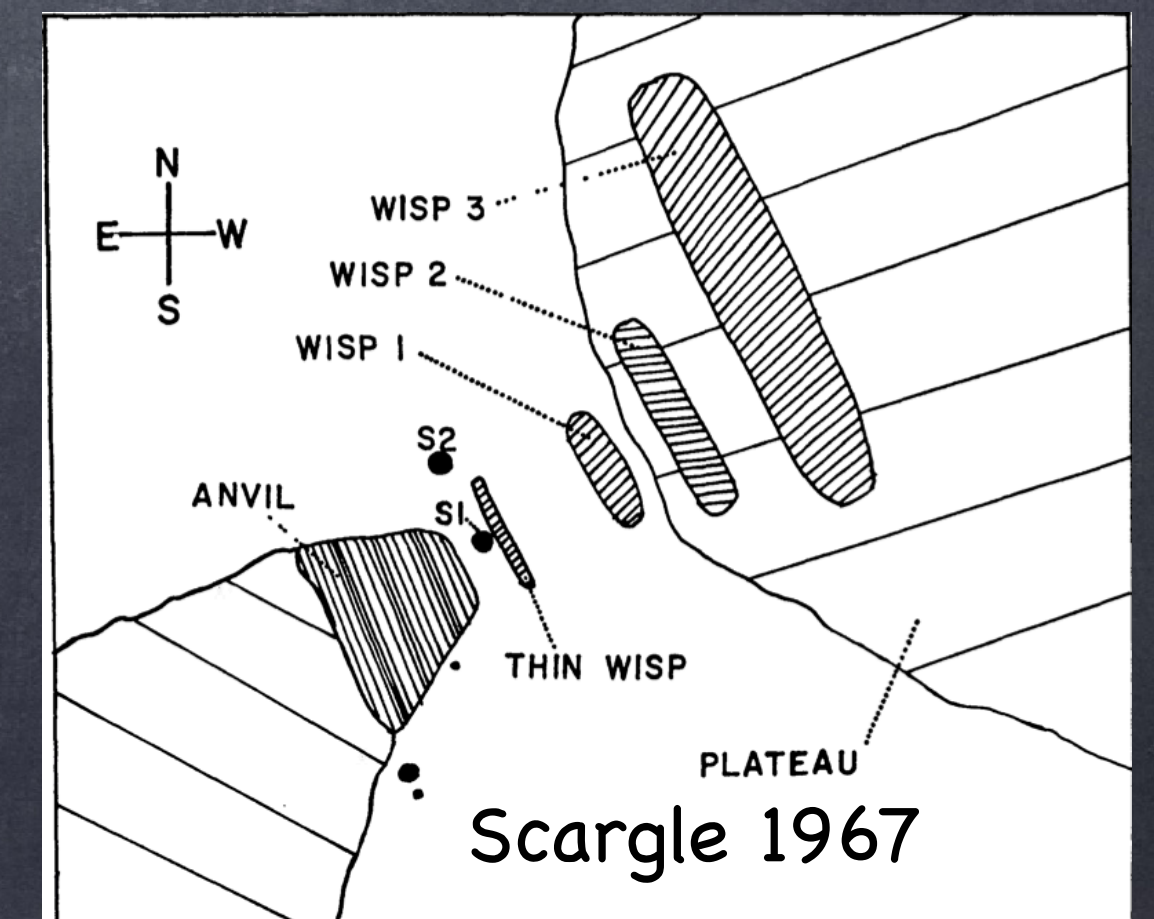
## NON-THERMAL EMISSION



Shklovsky 1960  
(Hillas 1972)



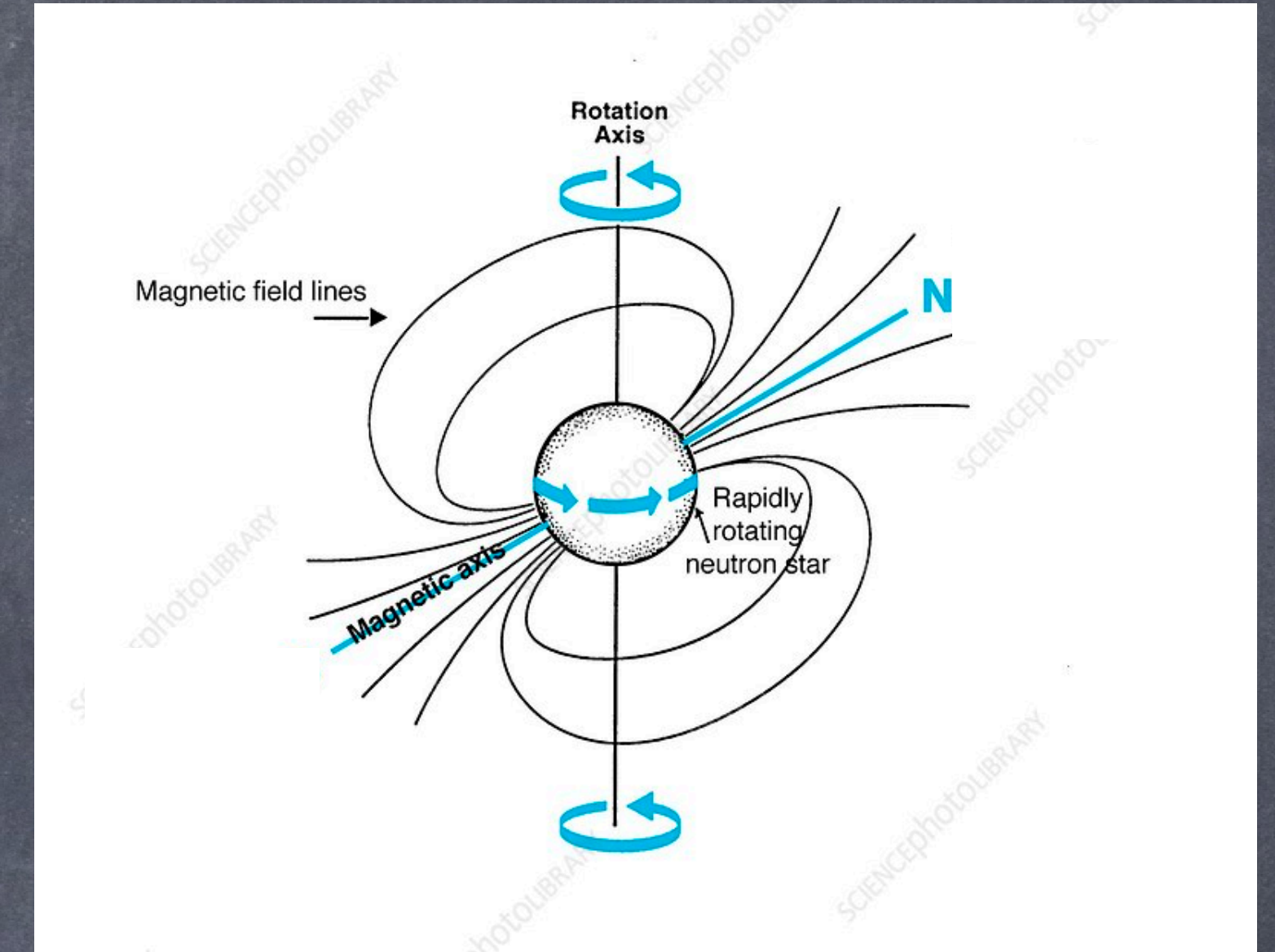
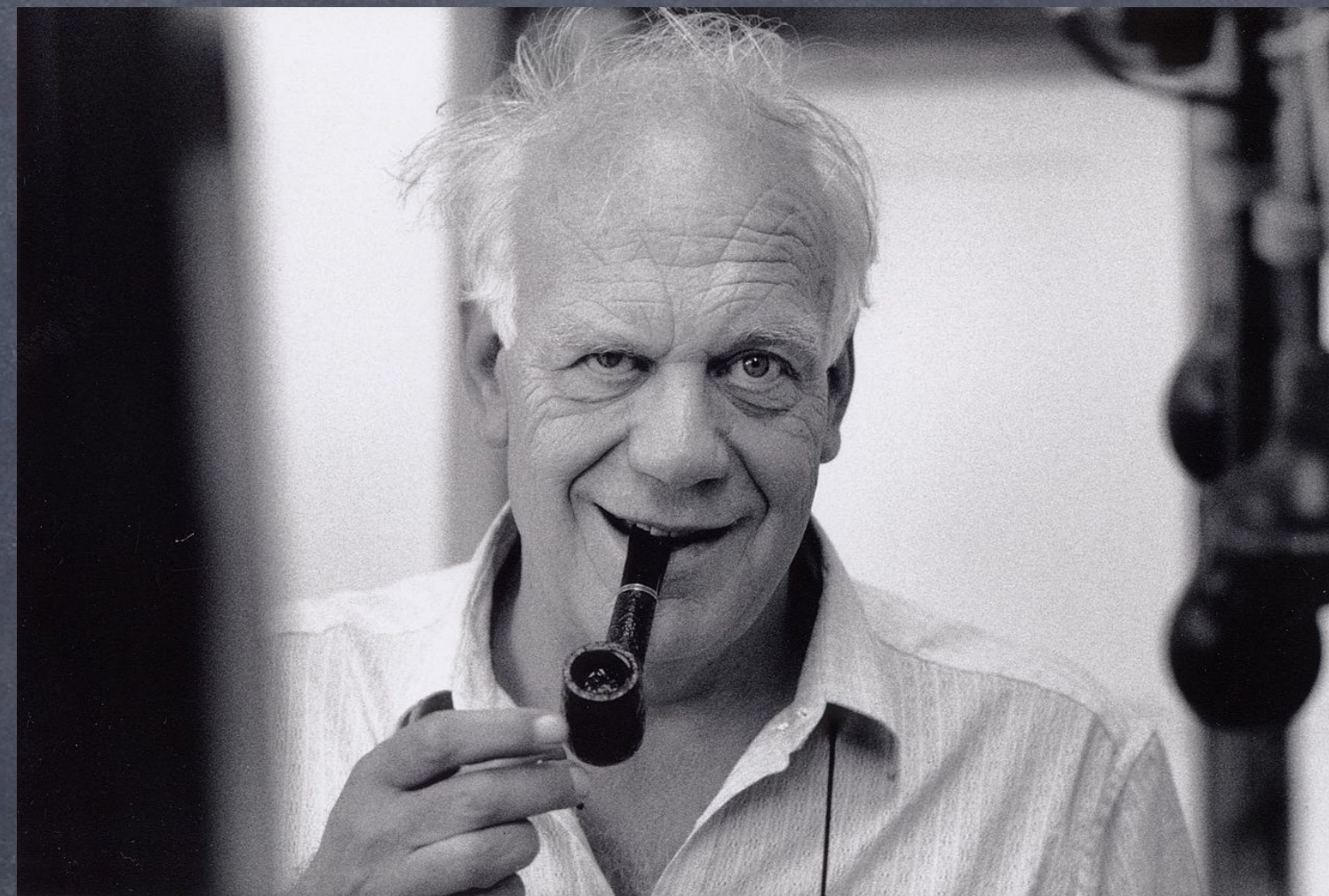
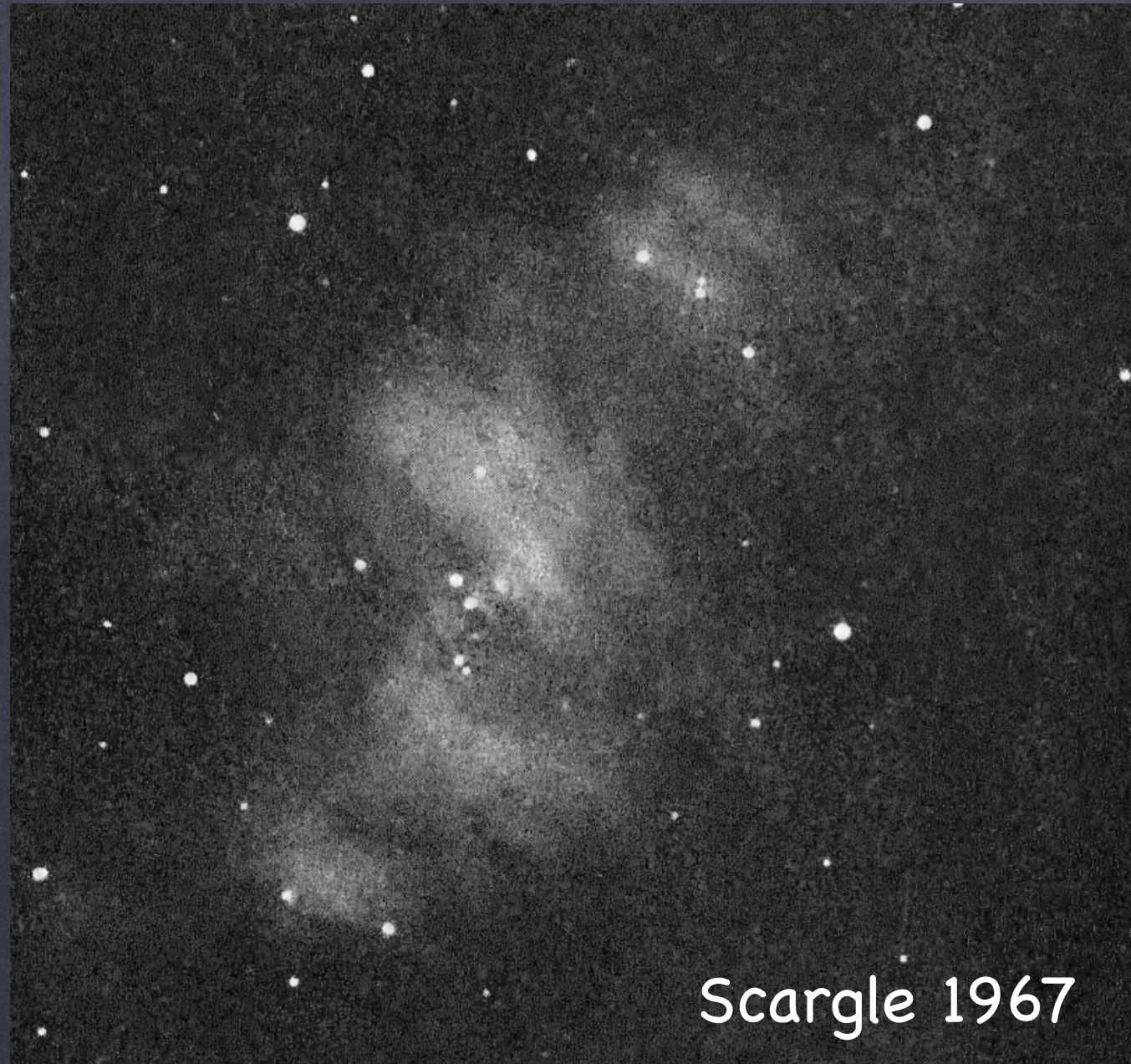
## VARIABILITY





# "ENERGY EMISSION FROM A NEUTRON STAR"

Pacini 1967

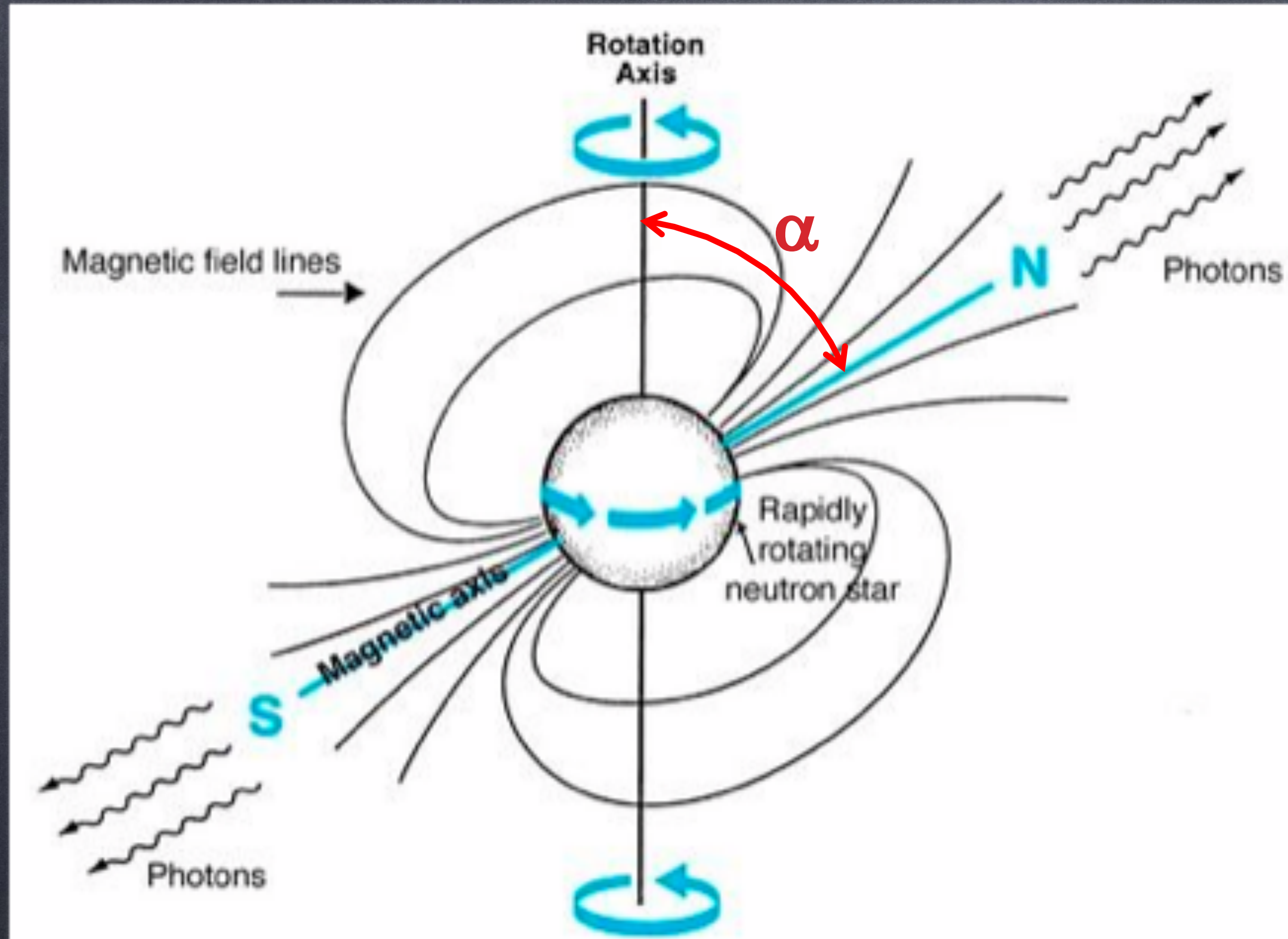


$$\dot{E} = \frac{B_{\star}^2 R_{\star}^6 \Omega^4 \sin^2 \chi}{6c^3}$$





# OBLIQUE ROTATING DIPOLE



## MAGNETIC MOMENT

$$\vec{\mu} = \mu_0 \left[ \sin \alpha \left( \cos \Omega t \underline{e}_1 + \sin \Omega t \underline{e}_2 \right) + \cos \alpha \underline{e}_3 \right]$$

$$\vec{B} = \frac{3\underline{e}_R (\vec{\mu} \cdot \underline{e}_R) - \vec{\mu}}{R^3} \quad \mu_0 = \frac{B_\star R_\star^3}{2}$$

## LARMOR FORMULA

$$\frac{dE}{dt} = -\frac{2}{3c^3} \ddot{j}^2$$

$$\dot{E} = \frac{2}{3c^3} \frac{B_\star^2 R_\star^6}{4} \Omega^4 \sin^2 \alpha$$

$$\dot{E} = 10^{40} \frac{B_{12}^2}{P_{-3}^4} \text{ erg/s}$$



# IN GRAVITATIONAL COLLAPSE

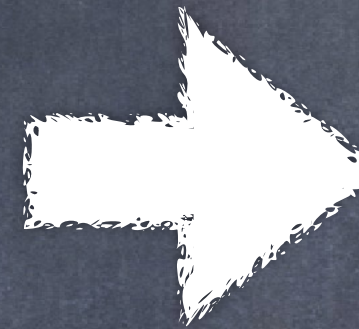
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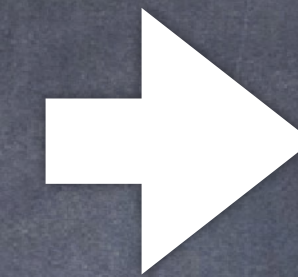
CONSERVATION OF MAGNETIC FLUX

$$B_i R_i^2 = B_f R_f^2$$

$$\begin{array}{l} R_i \approx 10^{11} \text{cm} \\ R_f \approx 10^6 \text{cm} \end{array} \quad \Rightarrow \quad \left( \frac{R_i}{R_f} \right)^2 = 10^{10}$$



$$\begin{array}{l} P_i \sim 10^5 \text{s} \\ B_i \sim 100 \text{G} \end{array}$$



$$\begin{array}{l} P_f < 0.1 \text{ms} \\ B_f \sim 10^{12} \text{G} \end{array}$$

$$\begin{aligned} \frac{\dot{E}_{\text{NS}}}{\dot{E}_{\star}} &= \left( \frac{\Omega_{\text{NS}}}{\Omega_{\star}} \right)^4 \left( \frac{B_{\text{NS}}}{B_{\star}} \right)^2 \left( \frac{R_{\text{NS}}}{R_{\star}} \right)^6 \\ &= \left( \frac{R_{\star}}{R_{\text{NS}}} \right)^8 \left( \frac{R_{\star}}{R_{\text{NS}}} \right)^4 \left( \frac{R_{\text{NS}}}{R_{\star}} \right)^6 \sim 10^{30} \end{aligned}$$

$$\dot{E}_{\text{NS}} \approx 10^{40} \text{ erg/s}$$



# THE FIRST PULSAR

Hewish & Bell 1968

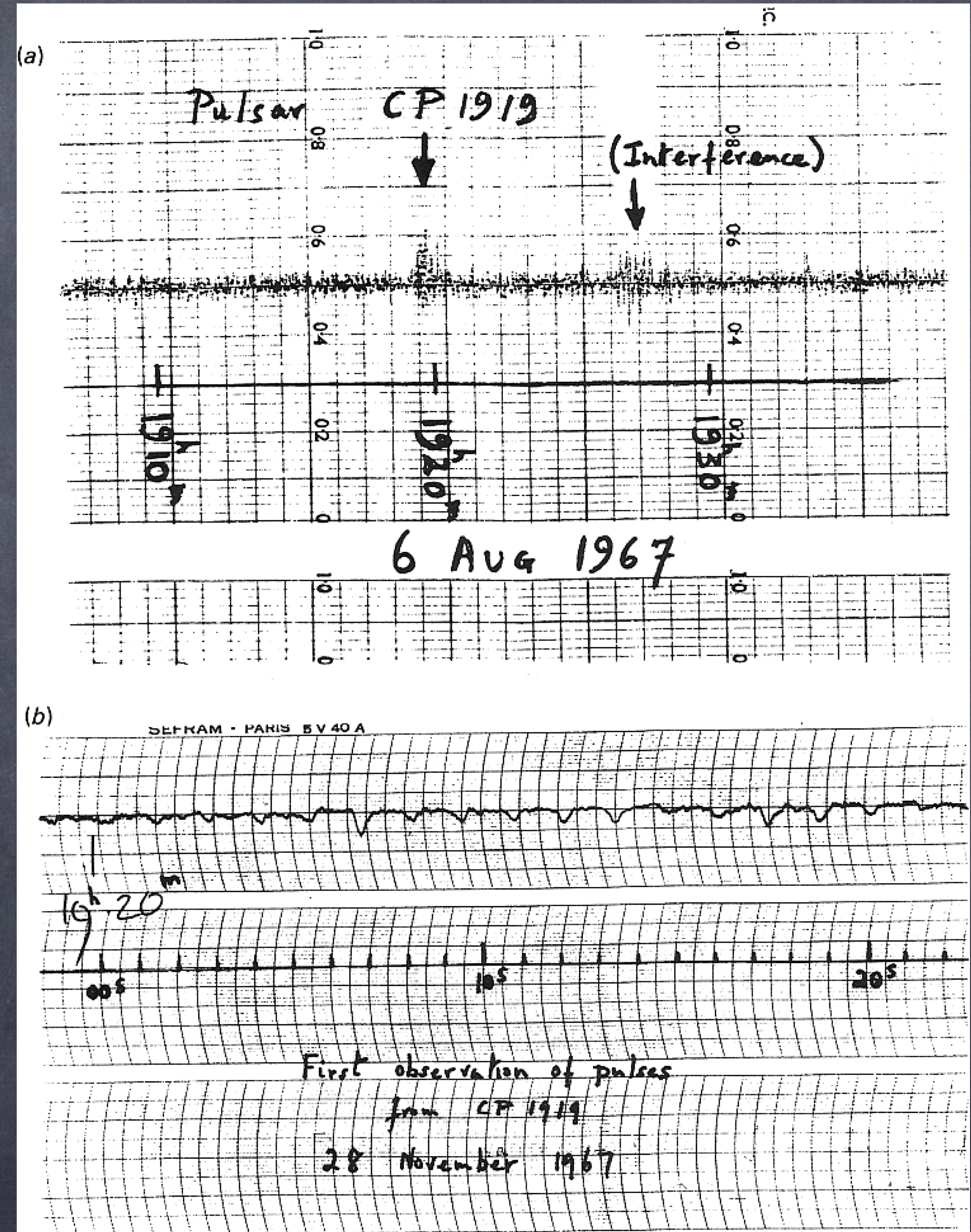


$P=1.33s$  AND KEEPING SIDEREAL TIME

IF A ROTATING OBJECT:

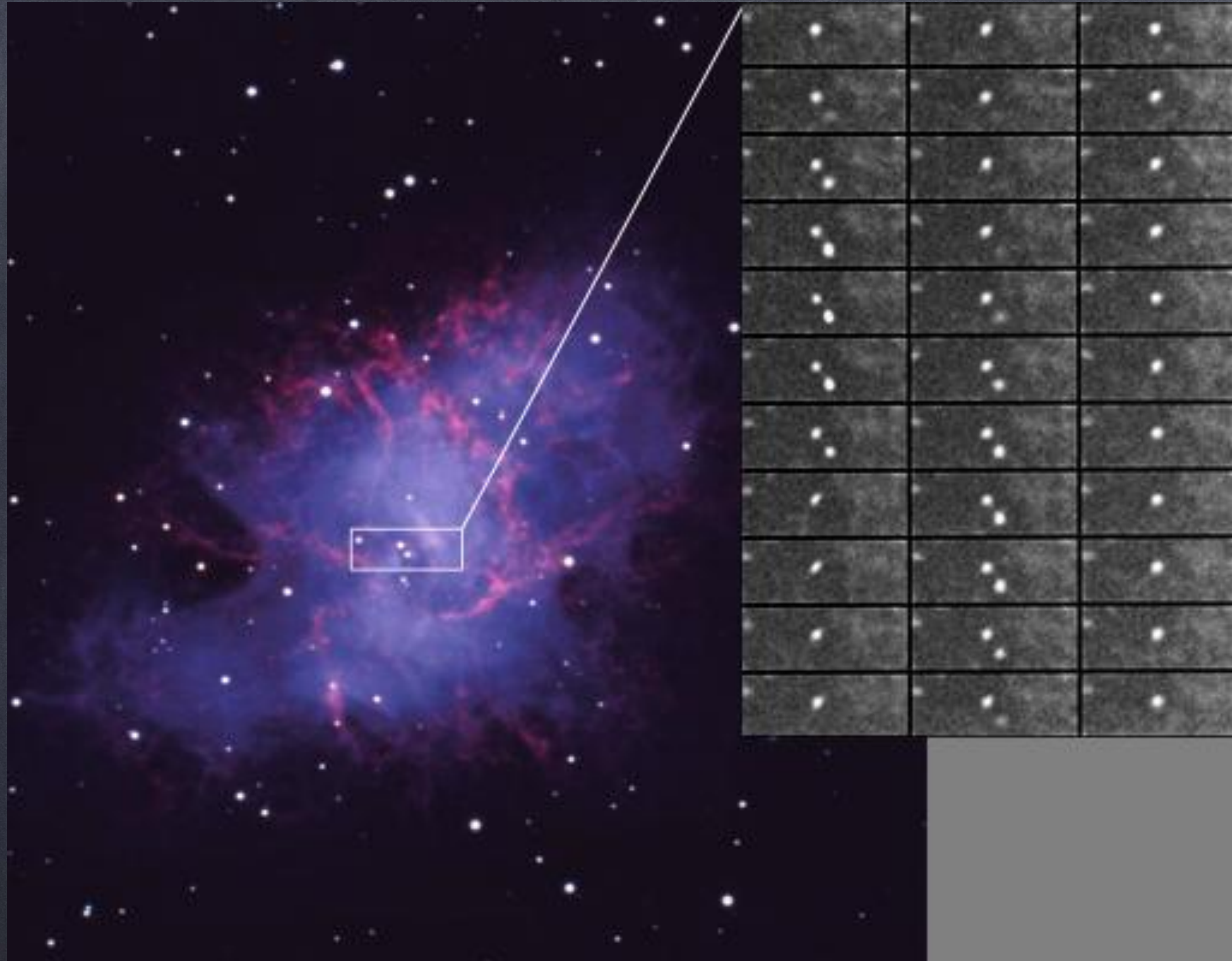
$$\frac{\Omega^2 R^2}{R} < \frac{G M}{R^2} \quad \Rightarrow \quad P > 1/\sqrt{R^3/GM}$$

FOR A WHITE DWARF:  $P > 10$  S



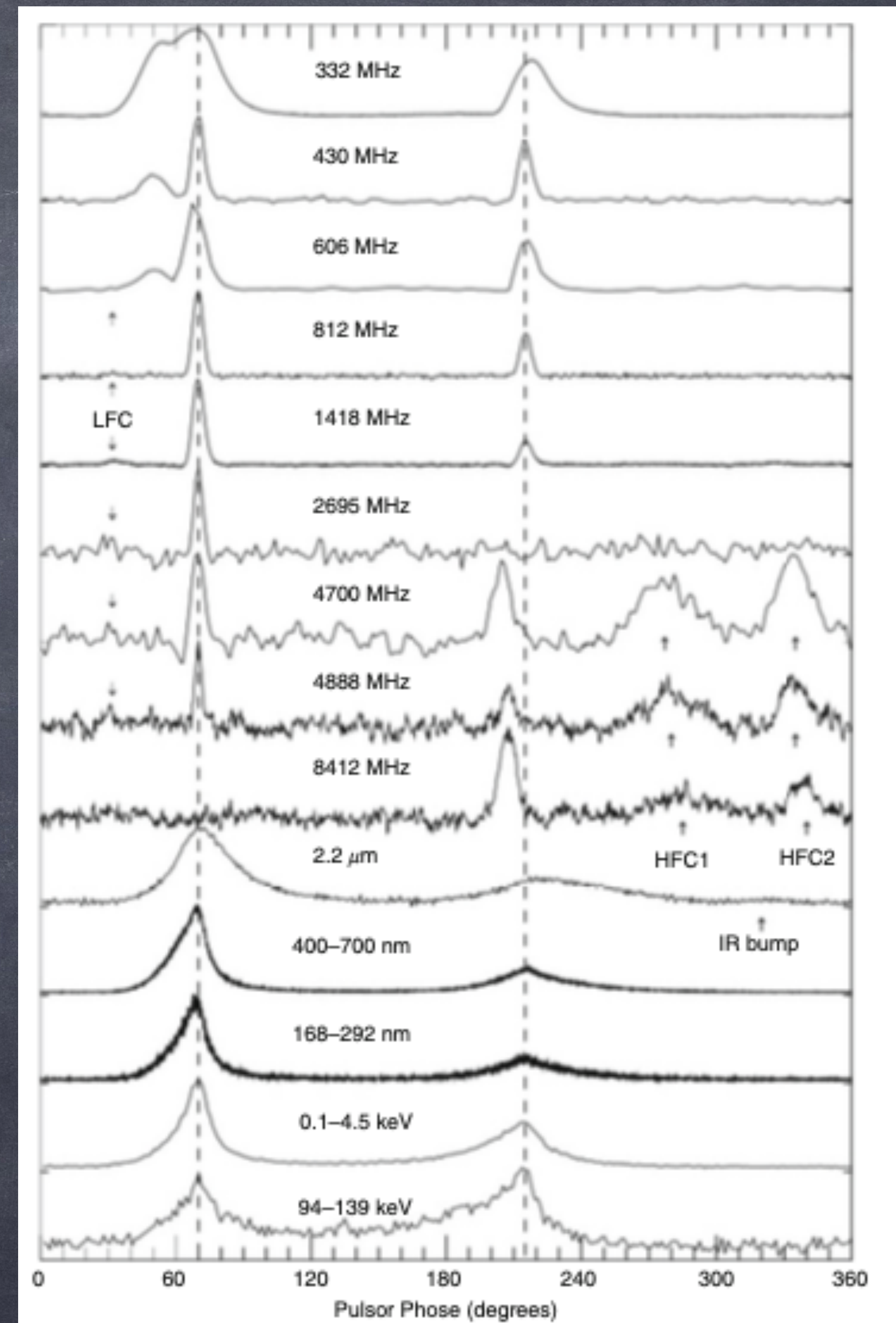


# THE CRAB PULSAR



$P=33\text{ms}$

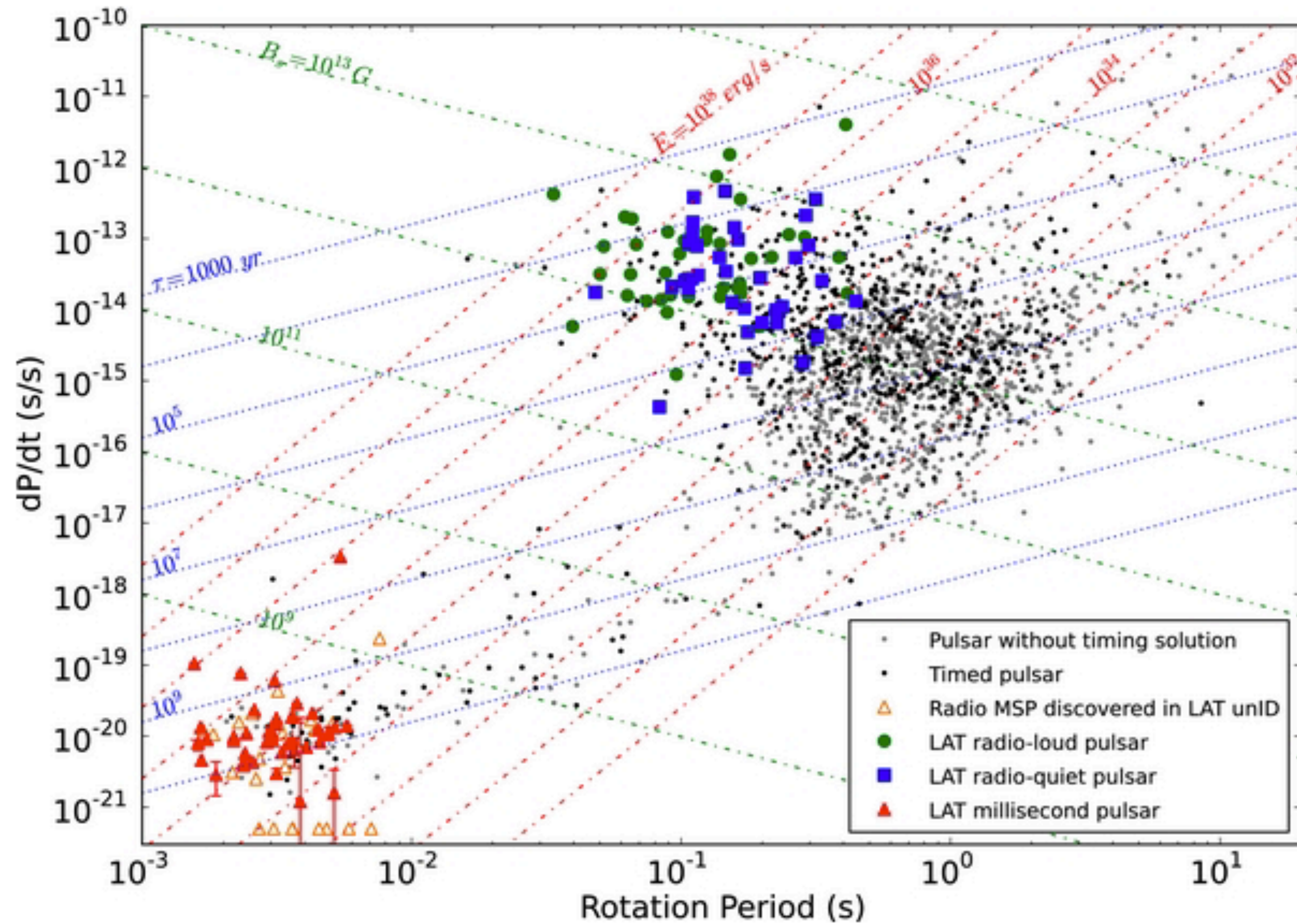
(Staelin & Reifenstein 1968)



Moffett and Hankins 1996



# P-PDOT DIAGRAM



Abdo+ 2013



# PULSAR SPIN DOWN

$$\dot{E} = \frac{B_{\star}^2 R_{\star}^6 \Omega^4 \sin^2 \chi}{6c^3}$$



ROTATIONAL ENERGY LOSS



$$\dot{E} = I_{\star} \Omega \dot{\Omega}$$



$$\frac{B_{\star}^2 R_{\star}^6 \Omega^4 \sin^2 \chi}{6c^3} = I_{\star} \Omega \dot{\Omega}$$

$$\dot{\Omega} = -K_{\text{sd}} \Omega^n$$



$$\Omega(t) = \frac{\Omega_0}{(1 + t/\tau)^{1/(n-1)}}$$

$$\tau = \frac{P_0}{(n-1)\dot{P}_0}$$

$$\dot{E} = \frac{\dot{E}_0}{(1 + t/\tau)^{\frac{n+1}{n-1}}}$$

$$n = \frac{\ddot{\Omega} \Omega}{\dot{\Omega}^2}$$

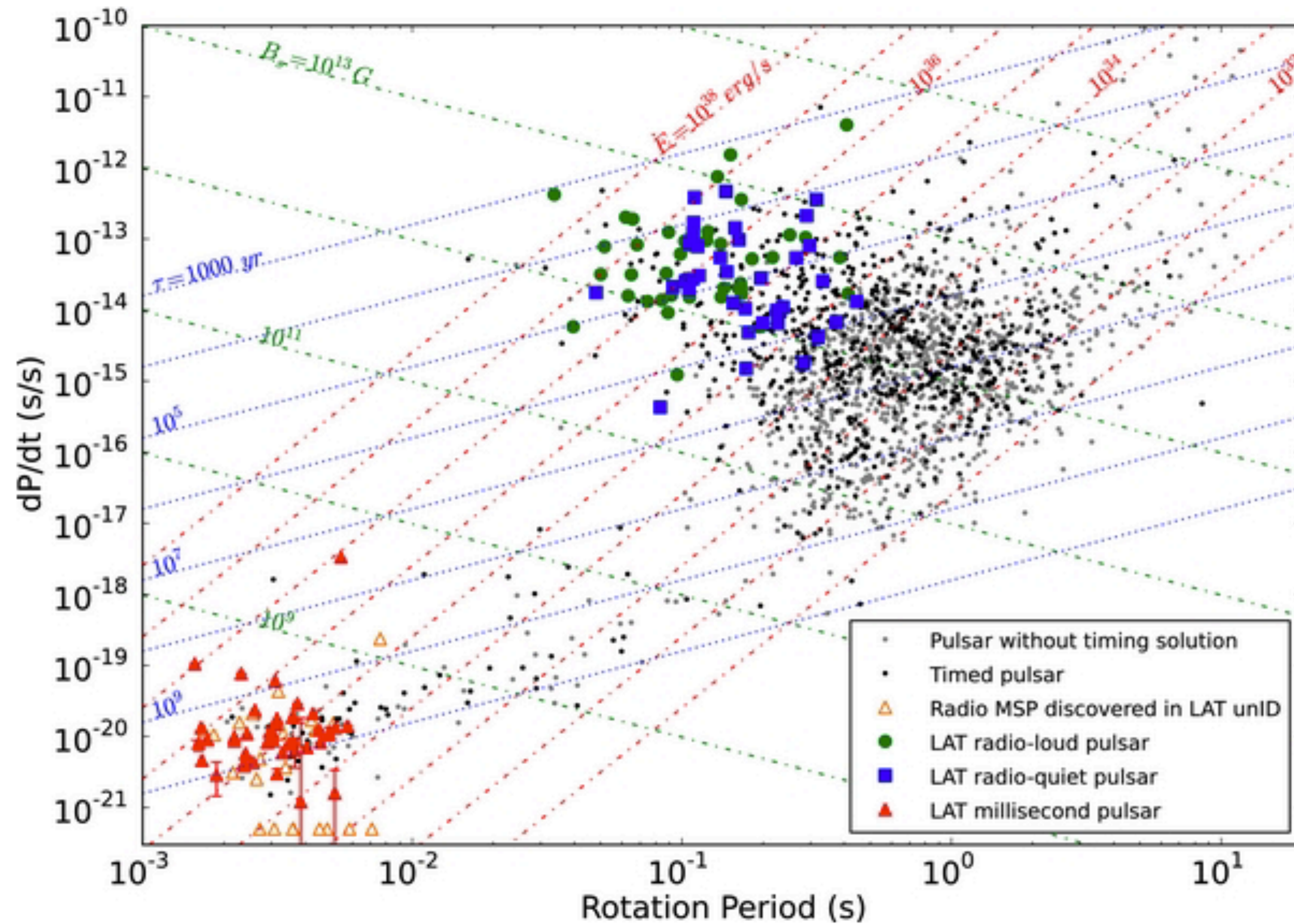
**NOTE**

$$K_{\text{sd}} = \frac{6 I_{\star} c^3}{B_{\star}^2 R_{\star}^6 \sin^2 \chi}$$

$$n = 3 + \frac{\Omega}{\dot{\Omega}} \left( 2 \frac{\dot{B}_{\star}}{B_{\star}} + 2 \dot{\chi} \cot \chi + 6 \frac{\dot{R}_{el}}{R_{el}} - \frac{\dot{I}}{I} \right)$$



# P-PDOT DIAGRAM



FOR  $n = 3$

$$\dot{E} = 4\pi^2 I_{\star} \dot{P} P^{-3}$$

$$\dot{E} \approx 5 \times 10^{31} \text{ erg/s } P^{-3} \dot{P}_{-15}$$

$$B_{\star} = \frac{(6I_{\star}c^3)^{1/2}}{2\pi R_{\star}^3} \sqrt{P \dot{P}}$$

$$B_{\star} \approx 10^{12} G \sqrt{P \dot{P}_{-15}}$$

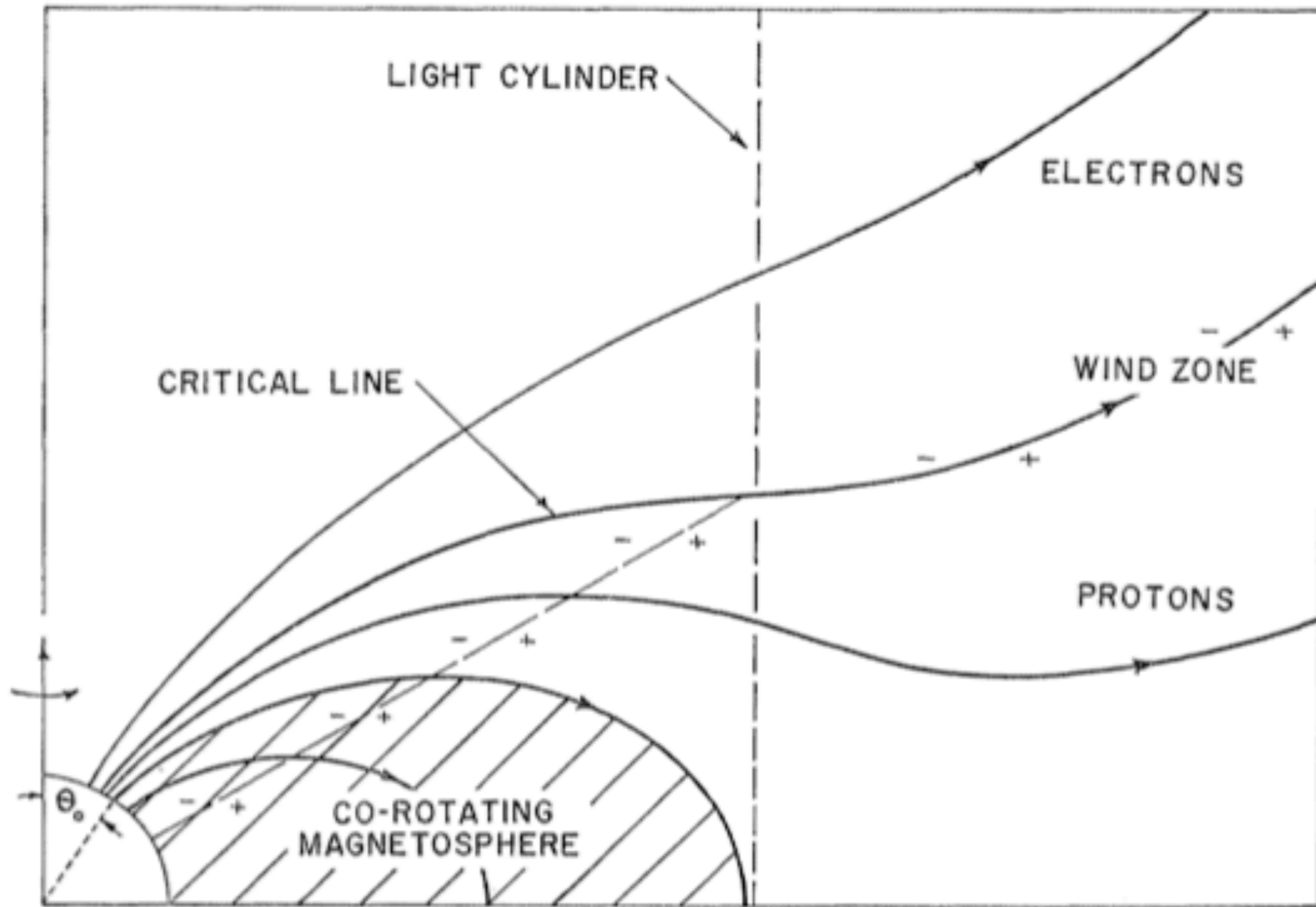
$$\tau = \frac{P}{2\dot{P}} \approx 3 \times 10^7 \text{ yr } P \dot{P}_{-15}^{-1}$$



PULSARS AS UNIPOLAR INDUCTORS



# NO VACUUM PHYSICS



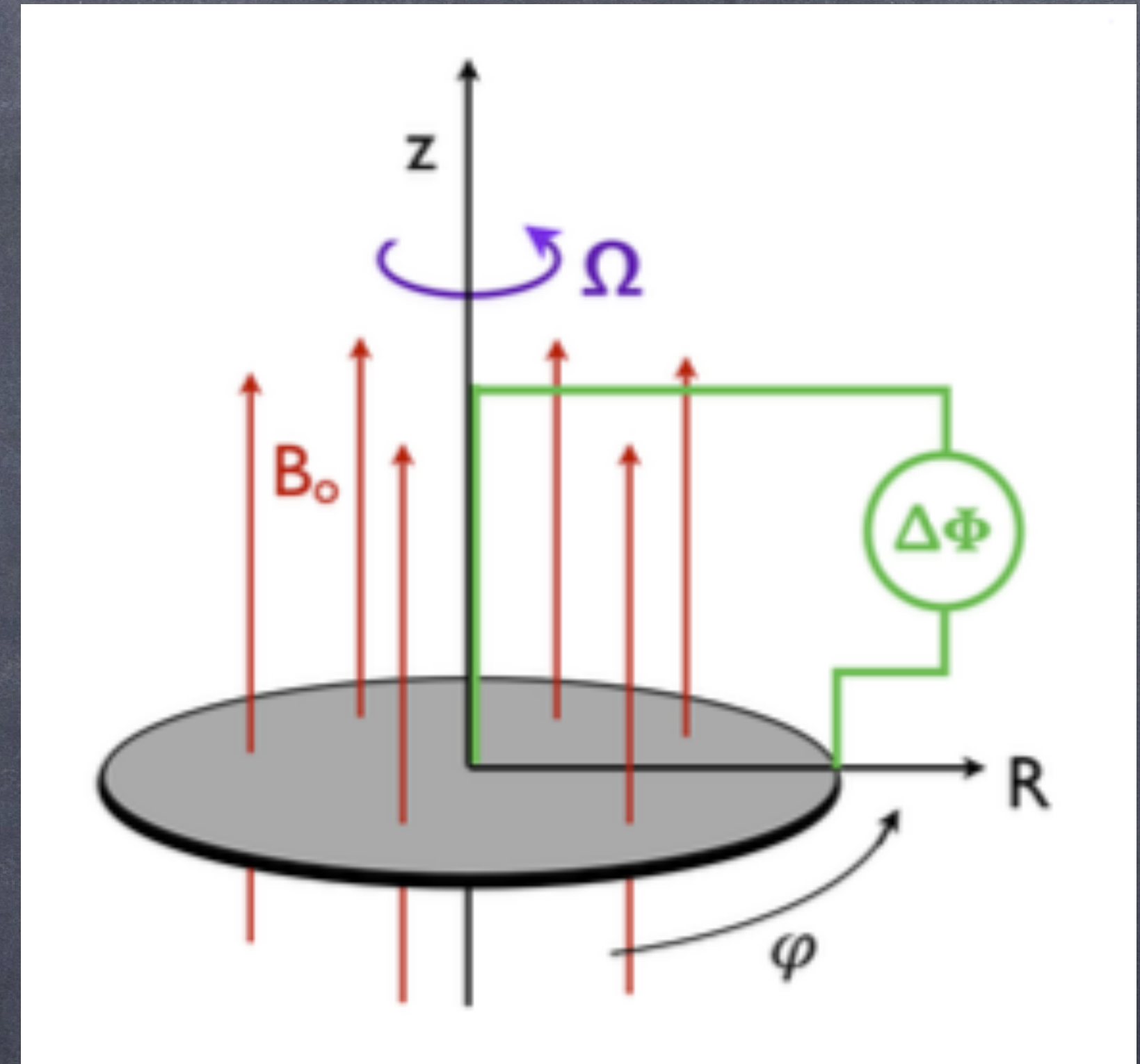
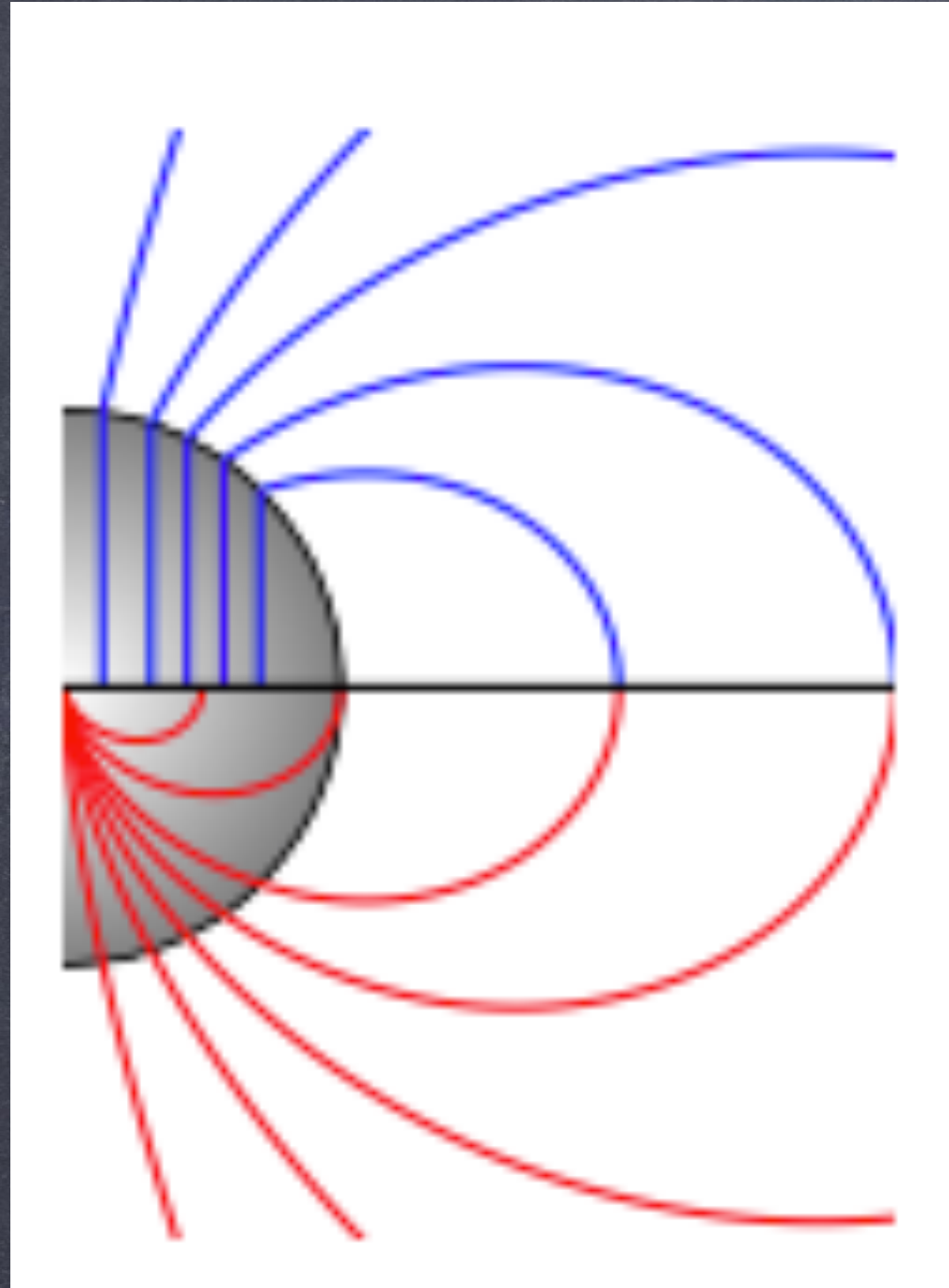
EVEN AN ALIGNED  
DIPOLE SPINS DOWN

$$\dot{E} = \frac{\mu^2 \Omega^4}{c^3} (1 + \sin^2 \chi)$$

$$\left( \dot{E}_{\text{vac}} = \frac{2}{3} \frac{\mu^2 \Omega^4}{c^3} \sin^2 \chi \right)$$

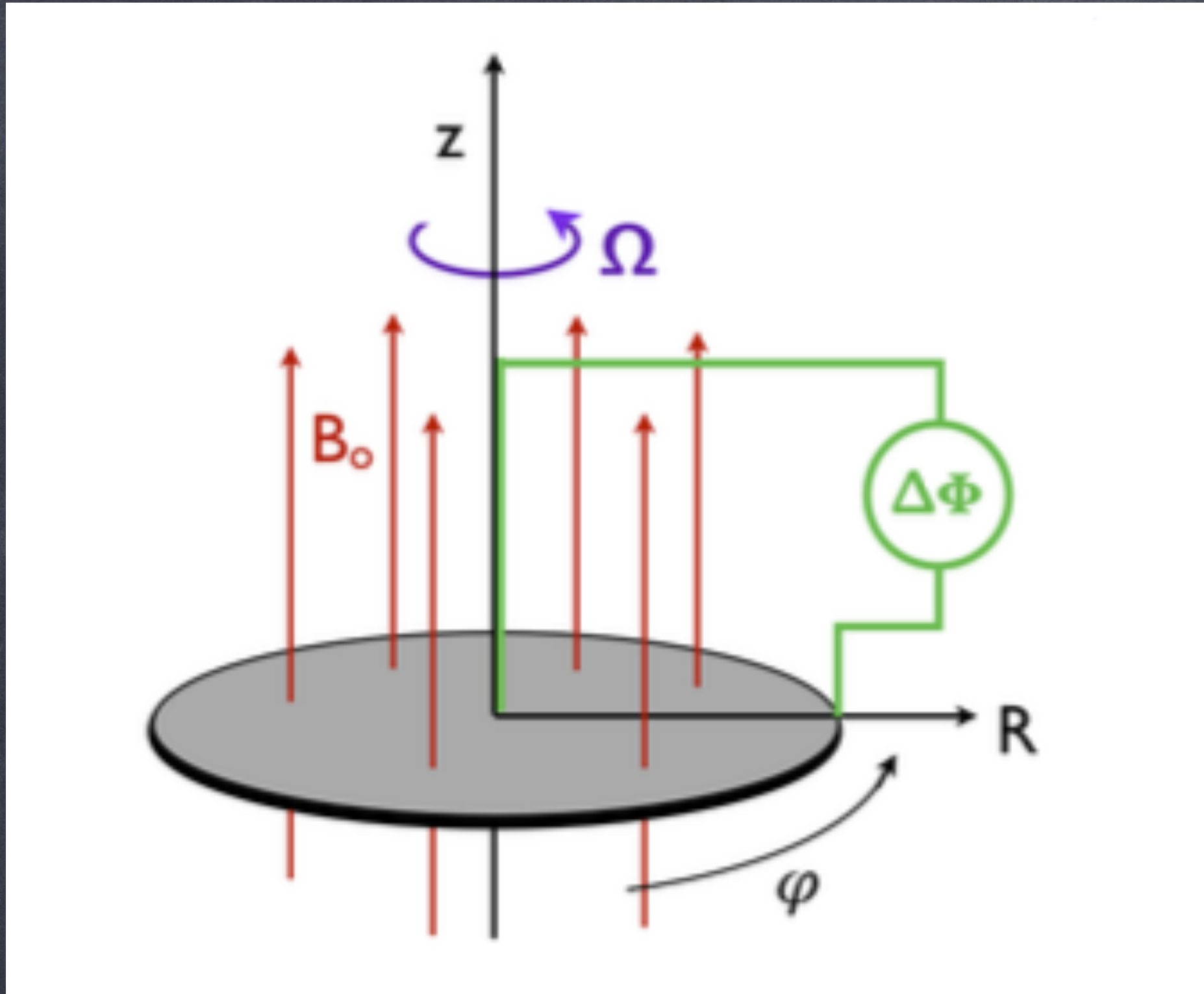


# A SIMPLE PROBLEM....





# FARADAY DISK

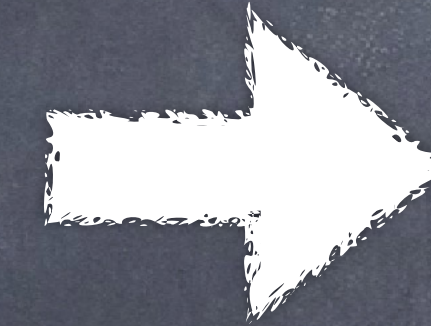


FREE CHARGES IN THE DISK  
ROTATE WITH  $\vec{v} = \Omega R \underline{e}_\phi$

$$\vec{B} = B_0 \underline{e}_z$$

$$\vec{\Omega} = \Omega \underline{e}_z$$

$$\vec{E} + \frac{\vec{v}}{c} \wedge \vec{B} = 0$$



$$\vec{E} = -\frac{\Omega R}{c} B_0 \underline{e}_r$$

POTENTIAL DIFFERENCE  
BETWEEN CENTER AND  
PERIPHERY

$$\Delta\Phi = \int_0^{R_d} \vec{E} \cdot d\vec{r} = \frac{\Omega R_d^2}{2c} B_0$$

CHARGE DENSITY IN THE DISK

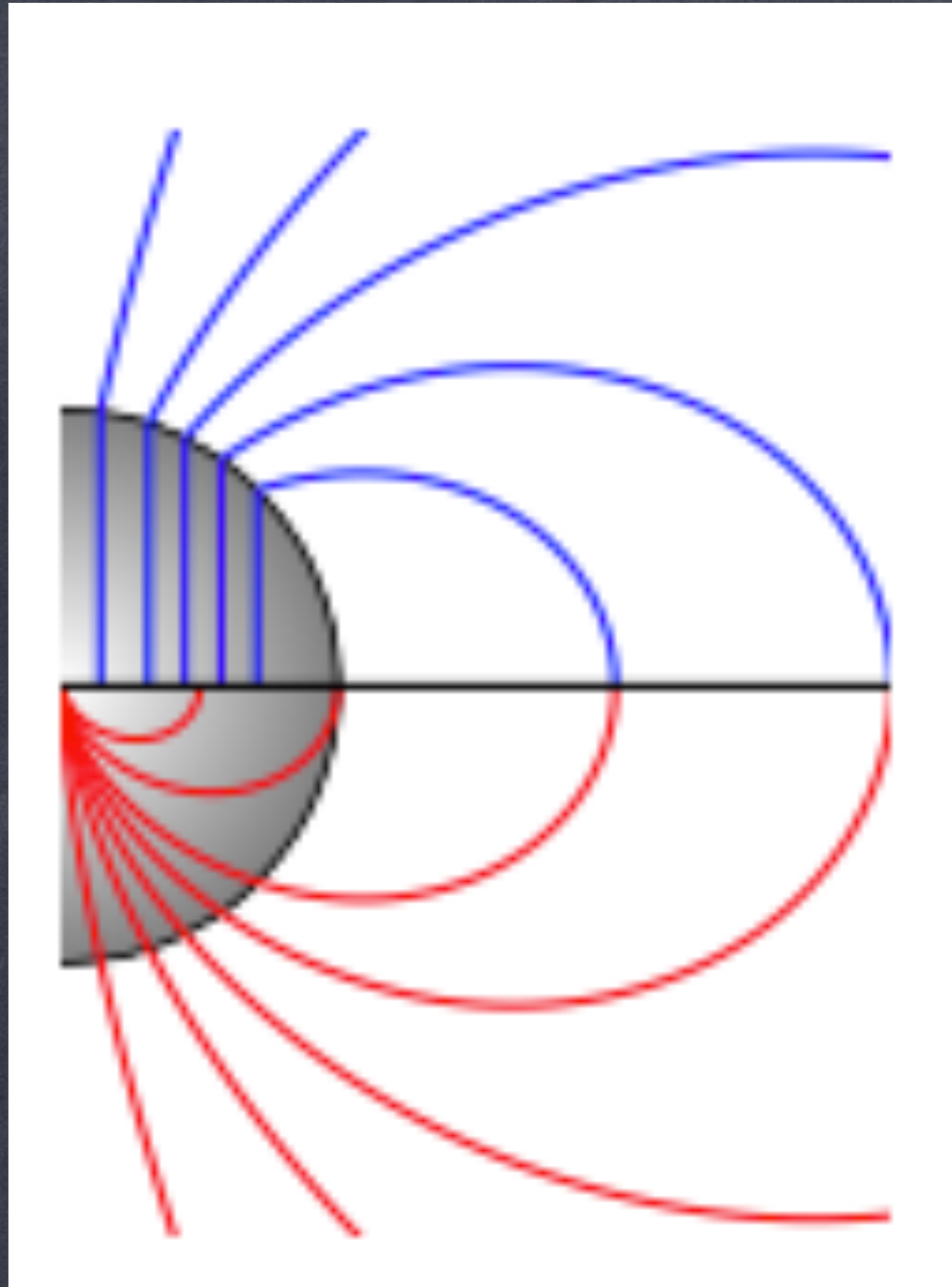
$$4\pi\rho_e = \vec{\nabla} \cdot \vec{E} = \frac{1}{r} \frac{\nabla}{\nabla_r} (rE_r) = -\frac{2\Omega B_0}{c}$$

$$\Delta\Phi^{FD} = \frac{B_0 \Omega R_d^2}{2c}$$

$$\rho_e^{FD} = -\frac{\Omega B_0}{2\pi c}$$



# PULSAR



$$\vec{\Omega} = \Omega \underline{e}_z \quad \vec{\mu} = \mu \underline{e}_z \quad \mu = \frac{B_\star R_\star^3}{2}$$

FREE CHARGES IN THE STAR  
ROTATE WITH  $\vec{v} = \Omega R \underline{e}_\phi$

$$\vec{E} + \frac{\vec{v}}{c} \wedge \vec{B} = 0 \quad \rightarrow \quad \vec{E}^{in} = - \left( \frac{\vec{\Omega} \wedge \vec{R}}{c} \right) \wedge \vec{B}$$

IF VACUUM OUTSIDE

BOUNDARY CONDITION AT STAR SURGACE

$$\nabla^2 \Phi = 0 \quad \vec{E}^{out} = - \vec{\nabla} \Phi$$

$$E_\theta^{in}(R_\star) = E_\theta^{out}(R_\star)$$

$$\Phi \rightarrow \vec{E}^{out}$$

$$(E_r^{out} - E_r^{in})_{R_\star} = 4\pi\sigma_e \neq 0$$

FINITE CHARGE SURFACE DENSITY  
AT STAR SURGACE

$$(\vec{E} \cdot \vec{B})_{R_\star} \neq 0$$

$$eE_\parallel = e \frac{B_\star \Omega R_\star}{c} \cos^2 \theta \approx \frac{0.2}{P} \text{erg/cm}$$

$$\frac{eE_\parallel}{GM_\star m_e / R_\star^2} \approx 8 \times 10^{12} \frac{B_{12}}{P_{100}}$$

AND A LARGE ENOUGH  
FIELD TO EXTRACT IT!



# PULSAR ELECTRODYNAMICS

$$B_R = B_\star \left( \frac{R_\star}{R} \right)^3 \cos \theta$$

$$B_\theta = \frac{B_\star}{2} \left( \frac{R_\star}{R} \right)^3 \sin \theta$$

$$B_\phi = 0$$

$$E_R^{in} = B_\star \left( \frac{R_\star}{R} \right)^3 \frac{\Omega R}{c} \sin^2 \theta$$

$$E_\theta^{in} = -\frac{B_\star}{2} \left( \frac{R_\star}{R} \right)^3 \frac{\Omega R}{c} \sin \theta \cos \theta$$

$$\Phi(R, \theta) = \Phi_0 \left( \frac{R_\star}{R} \right)^{l+1} P_l(\cos \theta)$$

$$E_\theta^{out}(R_\star) = \frac{1}{R_\star} \frac{\partial \Phi}{\partial \theta}(R_\star) = E_\theta^{in}(R_\star)$$

$$\Phi(R, \theta) = -\frac{B_\star \Omega R_\star^2}{3c} \left( \frac{R_\star}{R} \right)^3 (3 \cos^2 \theta - 1)$$



$$E_R^{out} = -\frac{B_\star \Omega R_\star^2}{2cR} \left( \frac{R_\star}{R} \right)^3 (3 \cos^2 \theta - 1)$$

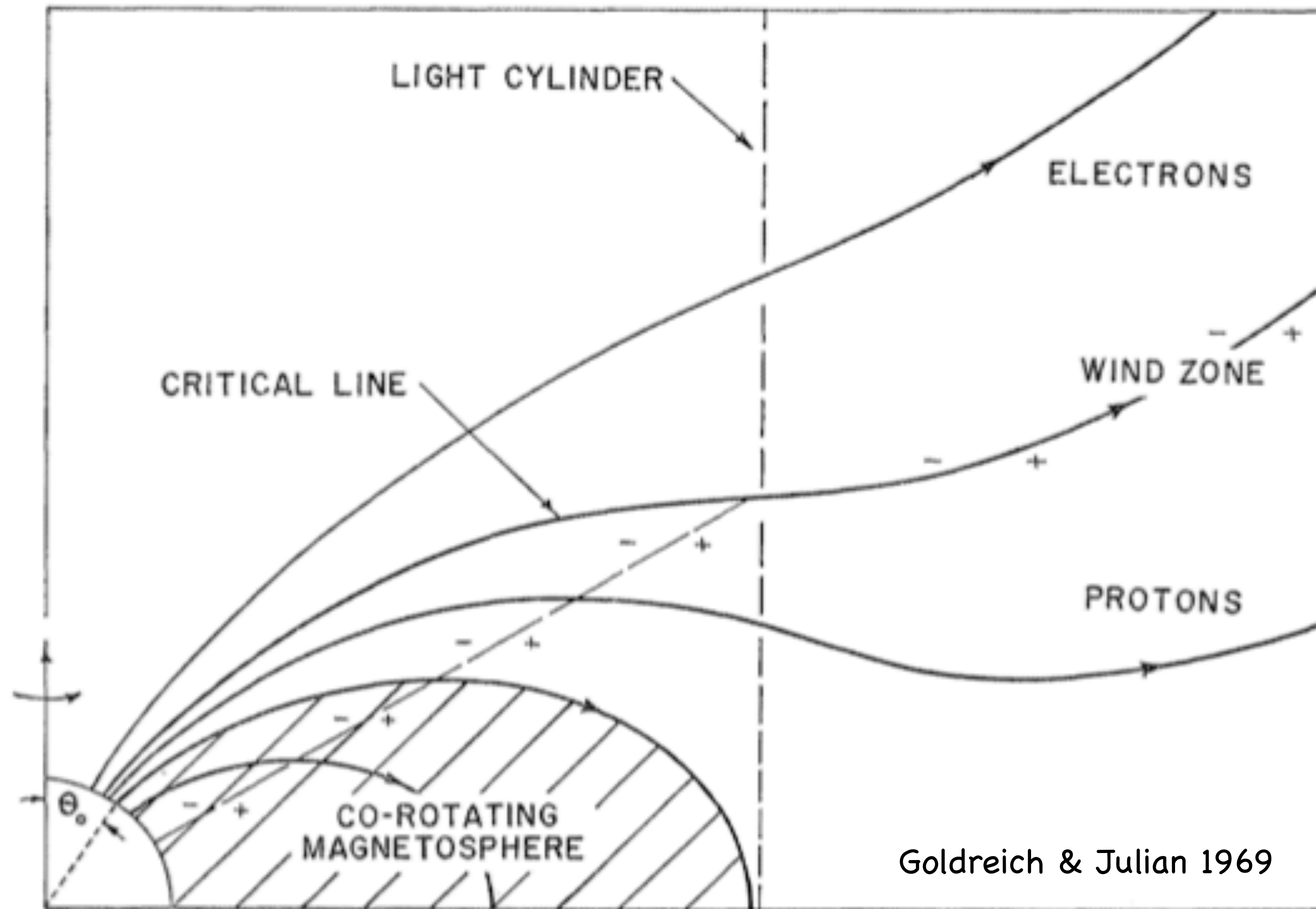
$$E_\theta^{out} = -\frac{B_\star \Omega R_\star^2}{cR} \left( \frac{R_\star}{R} \right)^3 \sin \theta \cos \theta$$

$$4\pi\sigma_e = E_R^{out}(R_\star) - E_R^{in}(R_\star) = -\frac{B_\star \Omega R_\star}{c} \cos^2 \theta$$





# THE GOLDREICH AND JULIAN MAGNETOSPHERE



STAR DEVELOPS  
COROTATING  
MAGNETOSPHERE

$$\vec{E} = -\frac{\Omega R \sin \theta}{c} \mathbf{e}_\phi \wedge \vec{B}$$

THROUGHOUT COROTATION  
REGION

COROTATION ONLY POSSIBLE  
UNTIL  $v < c$

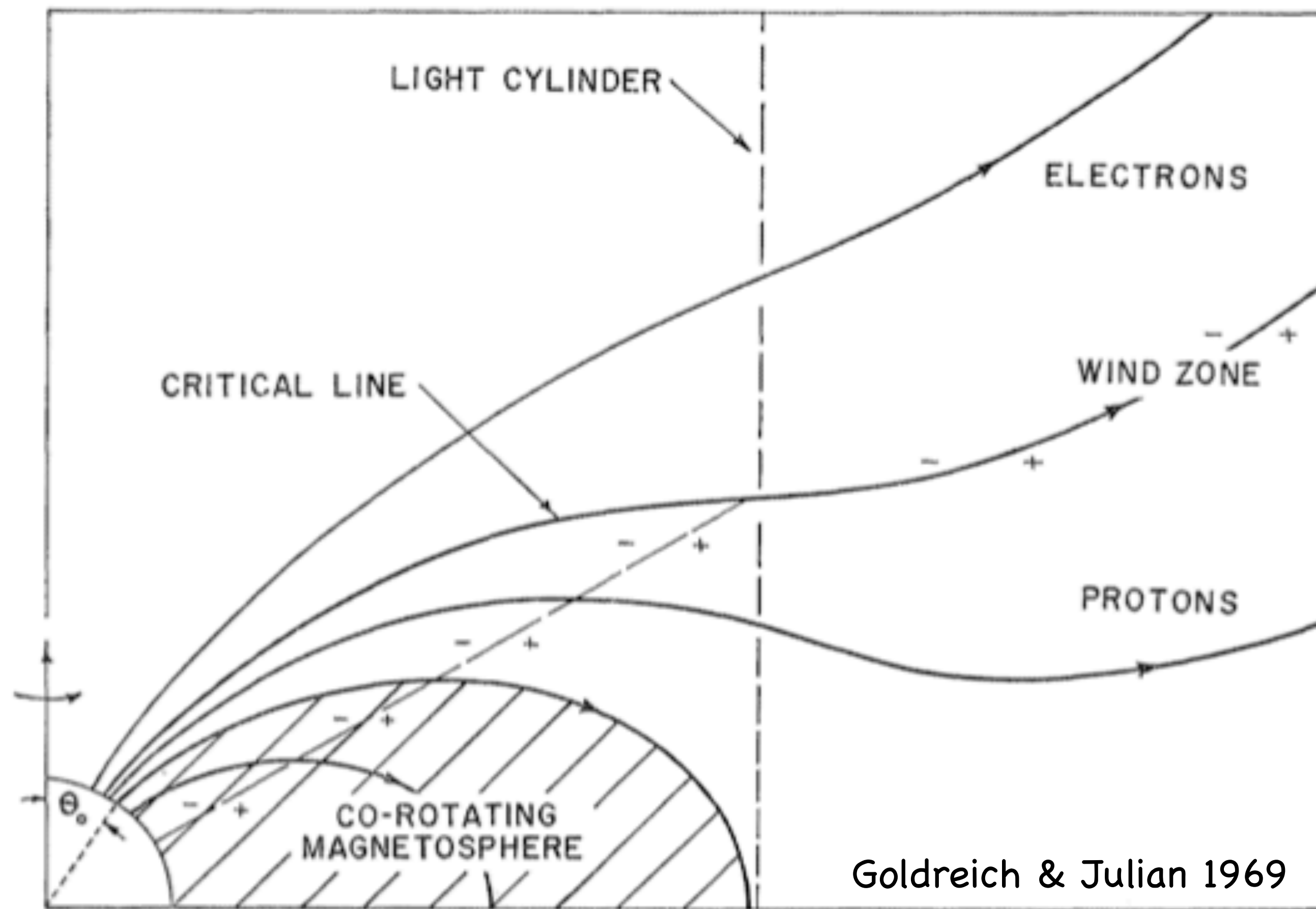
**LIGHT CYLINDER**

$$R_L = \frac{c}{\Omega} = 5 \times 10^8 P_{100} \text{ cm}$$

PARTICLES FLOWING ALONG OPEN FIELD  
LINES MAY REACH INFINITY



# THE STRUCTURE OF THE MAGNETOSPHERE



$$B_{\star} = \frac{2\mu_0}{R_{\star}^3} \quad \vec{B} = B_{\star} \left( \frac{R_{\star}}{R} \right)^3 \left[ \cos \theta \underline{e}_R - \frac{\sin \theta}{2} \underline{e}_{-\theta} \right]$$

LAST CLOSED FIELD LINE  
DEFINES **POLAR CAP**

$$\theta_{pc} = \sqrt{\frac{R_{\star}}{R_L}}$$

$$\vec{E} = -\frac{\Omega R \sin \theta}{c} \underline{e}_{-\phi} \wedge \vec{B}$$

CHARGE  
DENSITY

$$\rho_{GJ} = -\frac{\vec{\Omega} \cdot \vec{B}}{2\pi c} \frac{1}{\left[ 1 - \left( \frac{R}{R_L} \right)^2 \sin^2 \theta \right]}$$

NOTE:  
COMPARE WITH  
FARADAY DISK

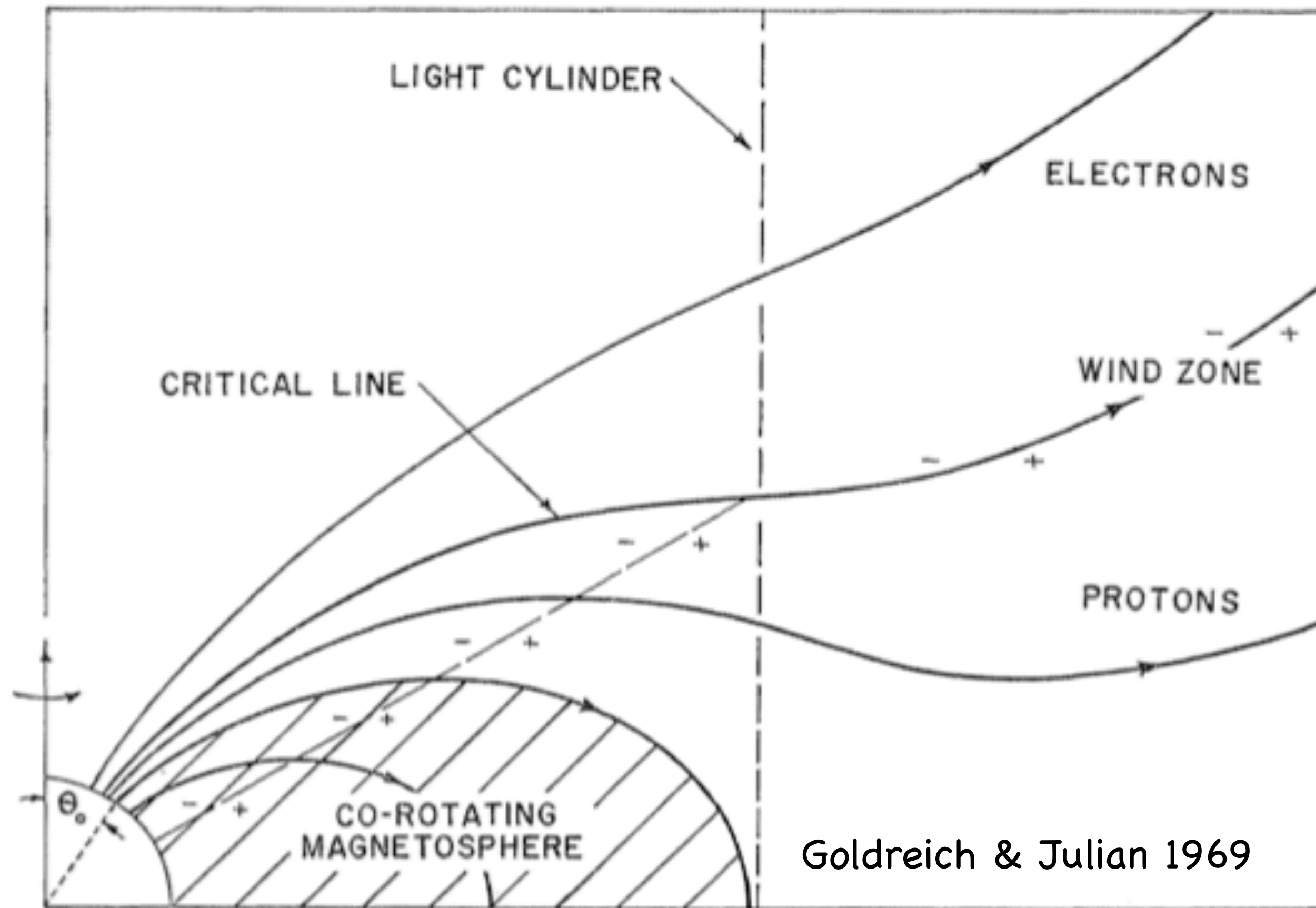
$$\rho_e^{FD} = -\frac{\Omega B_0}{2\pi c}$$

**CRITICAL LINE:** WHERE  
 $\vec{\Omega} \cdot \vec{B}$  CHANGES SIGN

$$\theta_c = \arccos \sqrt{\frac{1}{3}}$$



# THE PULSAR POTENTIAL DROP



**NOTE:**

COMPARE WITH FARADAY DISK

$$\Delta\Phi^{FD} = \frac{B_0 \Omega R_d^2}{2c}$$

## POTENTIAL DROP ACROSS PC

$$\Delta\Phi_{pc} = -\frac{B_\star \Omega R_\star^2}{2c} \sin^2 \theta_{pc}$$

NOTICE:

$$\Delta\Phi_{pc} = -\frac{B_\star \Omega R_\star^2}{2c} \frac{R_\star}{R_L} \approx -\sqrt{\dot{E}/c}$$

$$E_{max} = e\Phi = 6.5 \times 10^{14} \frac{B_{12}}{P_{100}^2} \text{ eV}$$

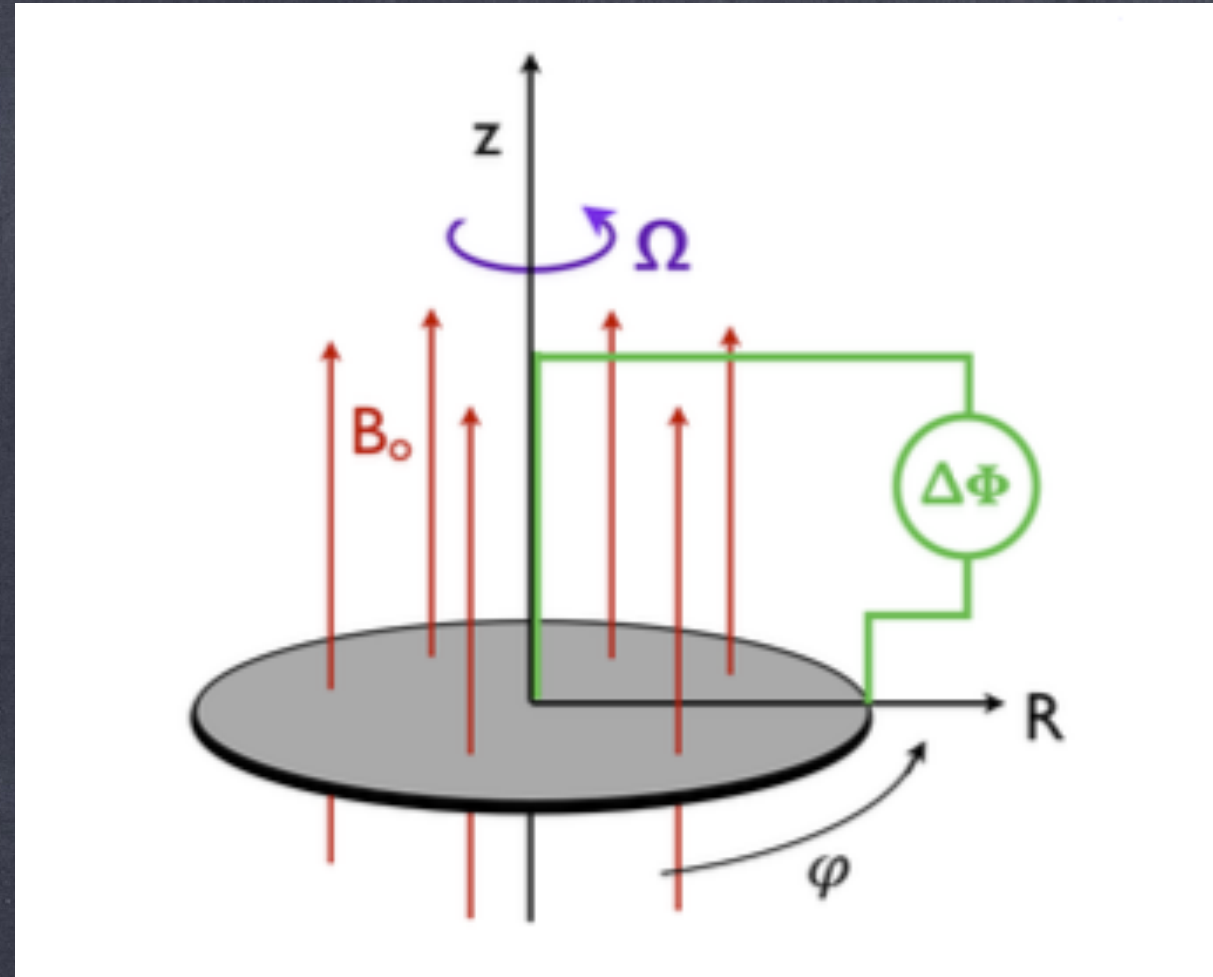
## POTENTIAL DROP BETWEEN POLE AND INFINITY

$$\Delta\Phi_c = -\frac{B_\star \Omega R_\star^2}{3c} = \frac{R_L}{R_\star} \Delta\Phi_{pc}$$

$$E_{max} = e\Phi = 2 \times 10^{17} \frac{B_{12}}{P_{100}^2} \text{ eV}$$

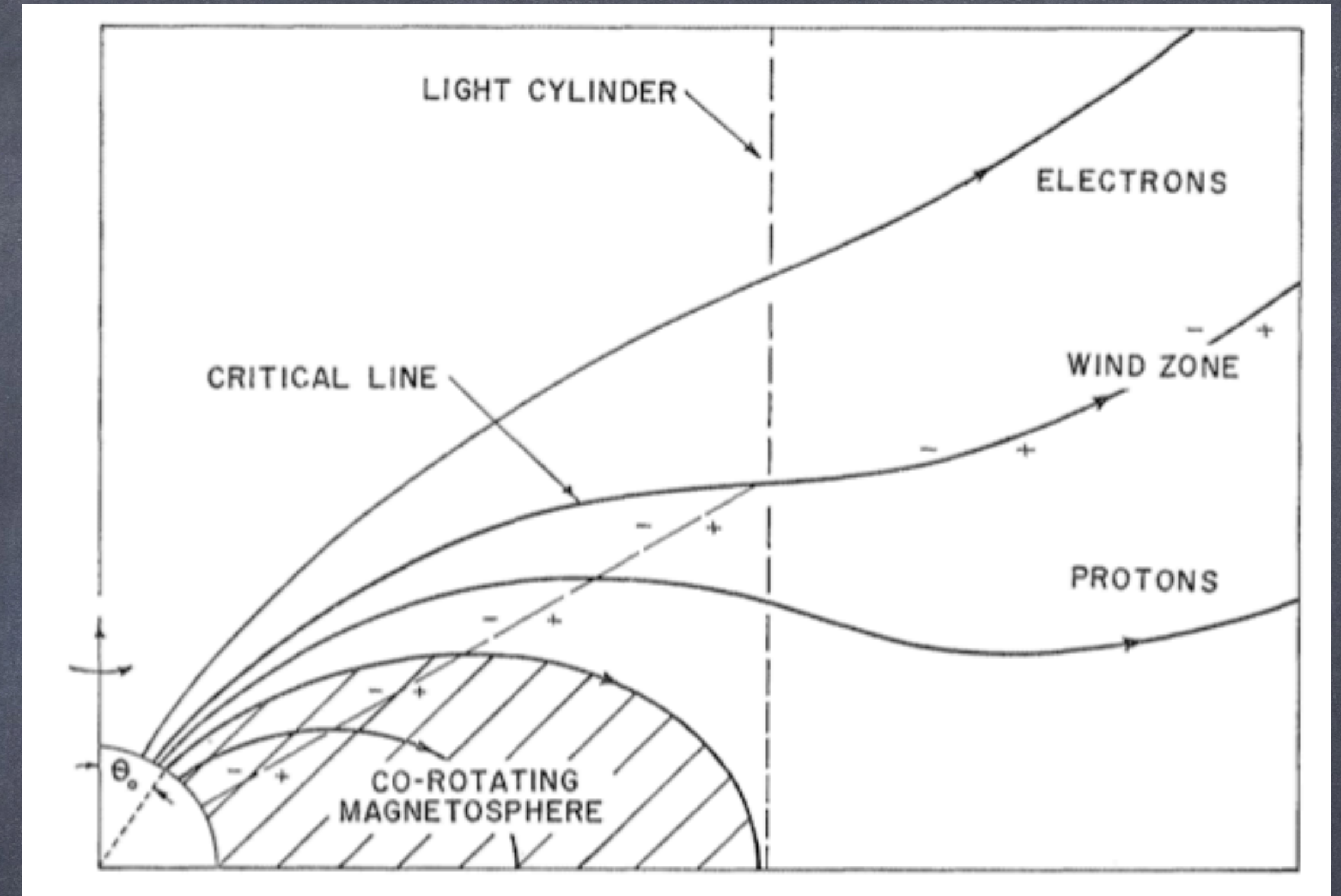


# PULSAR BASIC ELECTRODYNAMICS



$$\Delta\Phi = \frac{B_0 \Omega R_d^2}{2c}$$

$$\rho_e = -\frac{\Omega B_0}{2\pi c}$$



$$\Delta\Phi_{\text{Tot}} = \frac{B_{\star} \Omega R_{\star}^2}{3c}$$

$$\Delta\Phi_{\text{pc}} = \frac{B_{\star} \Omega^2 R_{\star}^3}{2c^2}$$

$$\rho_e = -\frac{\vec{\Omega} \cdot \vec{B}}{2\pi c} \left[ 1 - \left( \frac{R}{R_L} \right)^2 \sin^2 \theta \right]^{-1}$$



# THOUGHTS ON PULSAR POTENTIAL DROP

- $\Phi_{pc}$  IS THE ACTUAL POTENTIAL AVAILABLE
- $\Phi_{pc}$  IS A "MEASURED" QUANTITY, ONCE YOU MEASURE  $\dot{E}$
- PULSARS CAN EASILY REACH THE KNEE:  
-FOR CRAB PERIOD OF 33ms  $E_{max} \approx 60 \text{ PeV}$
- NEW BORN (FAST SPINNING) MAGNETARS CAN BE ZEVATRONS IN PRINCIPLE
- POTENTIAL DROPS LARGER THAN  $\Phi_{pc}$  AND UP TO SOME FRACTION OF  $\Phi_c$  CAN BE ACHIEVED IN THE MAGNETOSPHERE IF FOR SOME REASONS MORE FIELD LINES ARE OPEN

## POTENTIAL DROP ACROSS PC

$$\Delta\Phi_{pc} = -\frac{B_{\star}\Omega R_{\star}^2}{2c} \sin^2 \theta_{pc}$$

NOTICE:

$$\Delta\Phi_{pc} = -\frac{B_{\star}\Omega R_{\star}^2}{2c} \frac{R_{\star}}{R_L} \approx -\sqrt{\dot{E}/c}$$

$$E_{max} = e\Phi = 6.5 \times 10^{14} \frac{B_{12}}{P_{100}^2} \text{ eV}$$

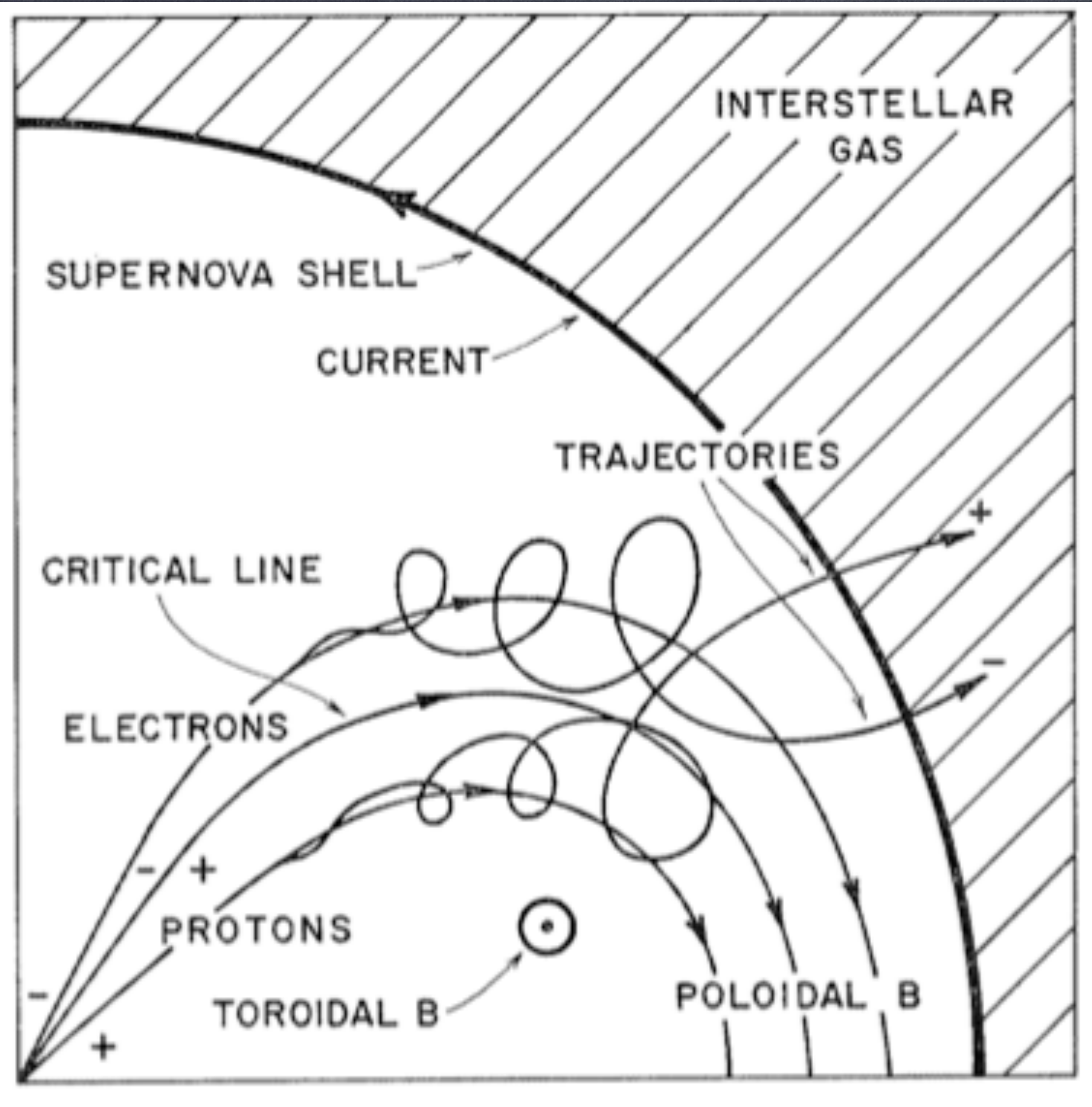
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$$E_{max} = e\Phi = 2 \times 10^{17} \frac{B_{12}}{P_{100}} \text{ eV}$$



# FLAWS OF THE GJ MODEL



- VACUUM FIELD NOT SELF-CONSISTENT:
  - NEGLECT OF DISPLACEMENT CURRENT (Deutsch 1955)
  - NEGLECT OF MONOPOLE TERM (Michel 1969)
- CHARGE SUPPLY UNCLEAR
- CHARGE SEPARATED FLOW OR QUASI NEUTRAL PLASMA?

$$\left( \frac{G M_{\star} m_e}{R_{\star} k_B} \approx 10^9 \text{ K} \right)$$

Goldreich & Julian 1969



# MORE THOUGHTS

$$\dot{E} = \frac{\mu^2 \Omega^4}{c^3} (1 + \sin^2 \chi)$$

- WHY DOES AN ALIGNED ROTATOR SPIN DOWN?

$$\dot{E}_{\text{vac}} = \frac{2}{3} \frac{\mu^2 \Omega^4}{c^3} \sin^2 \chi$$

- ENERGY LOSS BY A GJ FLUX OF PARTICLES LEAVING THE STAR POLAR CAP AT PULSAR DROP

$$\dot{E}_{\text{part}} = \dot{N}_{GJ} E_{\text{drop}}$$

$$\dot{N}_{GJ} = \frac{\Omega B}{2\pi c e} \left( \pi R_{\star}^2 \frac{R_{\star}}{R_L} \right) c \approx \frac{\sqrt{c} \dot{E}}{e}$$

$$E_{\text{drop}} = e \sqrt{\dot{E}/c}$$

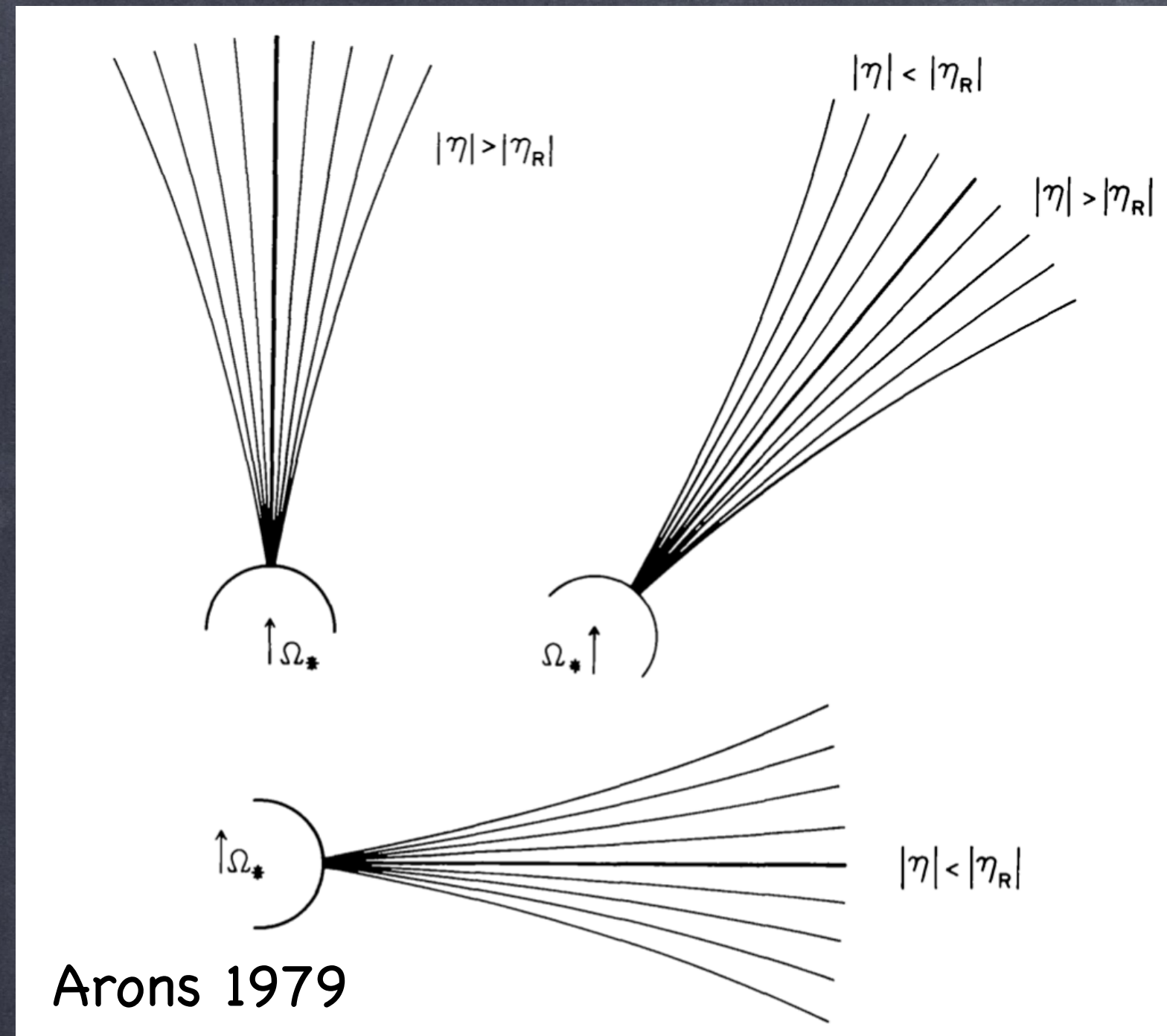


$$\dot{E}_{\text{part}} = \dot{E}$$



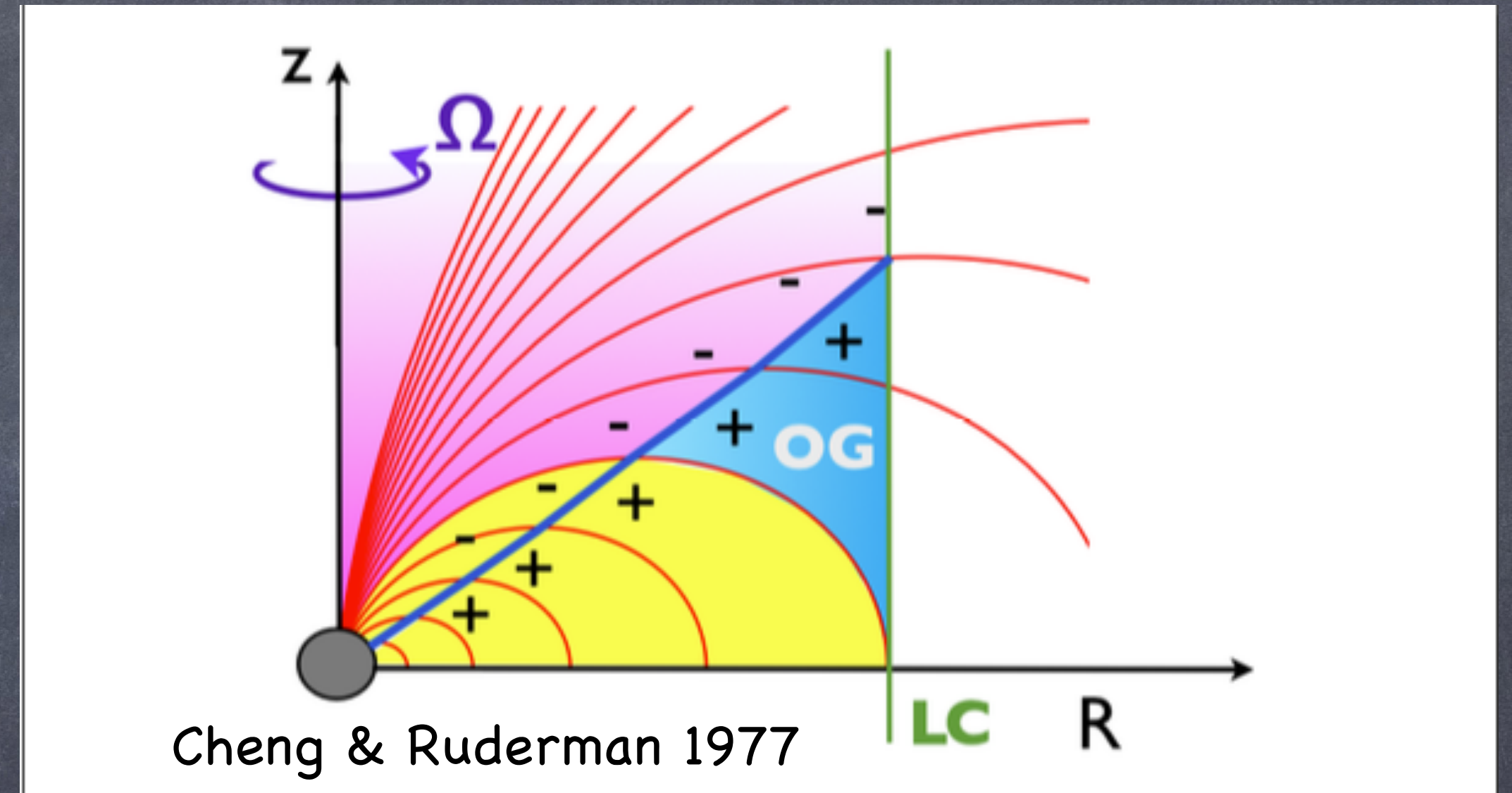
# SPACE CHARGE LIMITED FLOW

## POLAR CAPS



$$\rho_{GJ}^{\star} = J_{\star}/c \quad J = J_{\star} \frac{B}{B_{\star}} \quad J_{GJ} = c\rho_{GJ} \propto B_z$$

## OUTER GAPS



- CHARGE DENSITY LOCALLY NEEDED CAN BE SUPPLIED FROM THE STAR ONLY FOR FIELD LINES WITH DECREASING  $B_{\parallel}/B$  A FRACTION OF THE TOTAL FOR OBLIQUE ROTATORS
- UNSCREENED ELECTRIC FIELD IS LEFT IN ALL CASES IF PARTICLES FROM STAR SURFACE ONLY

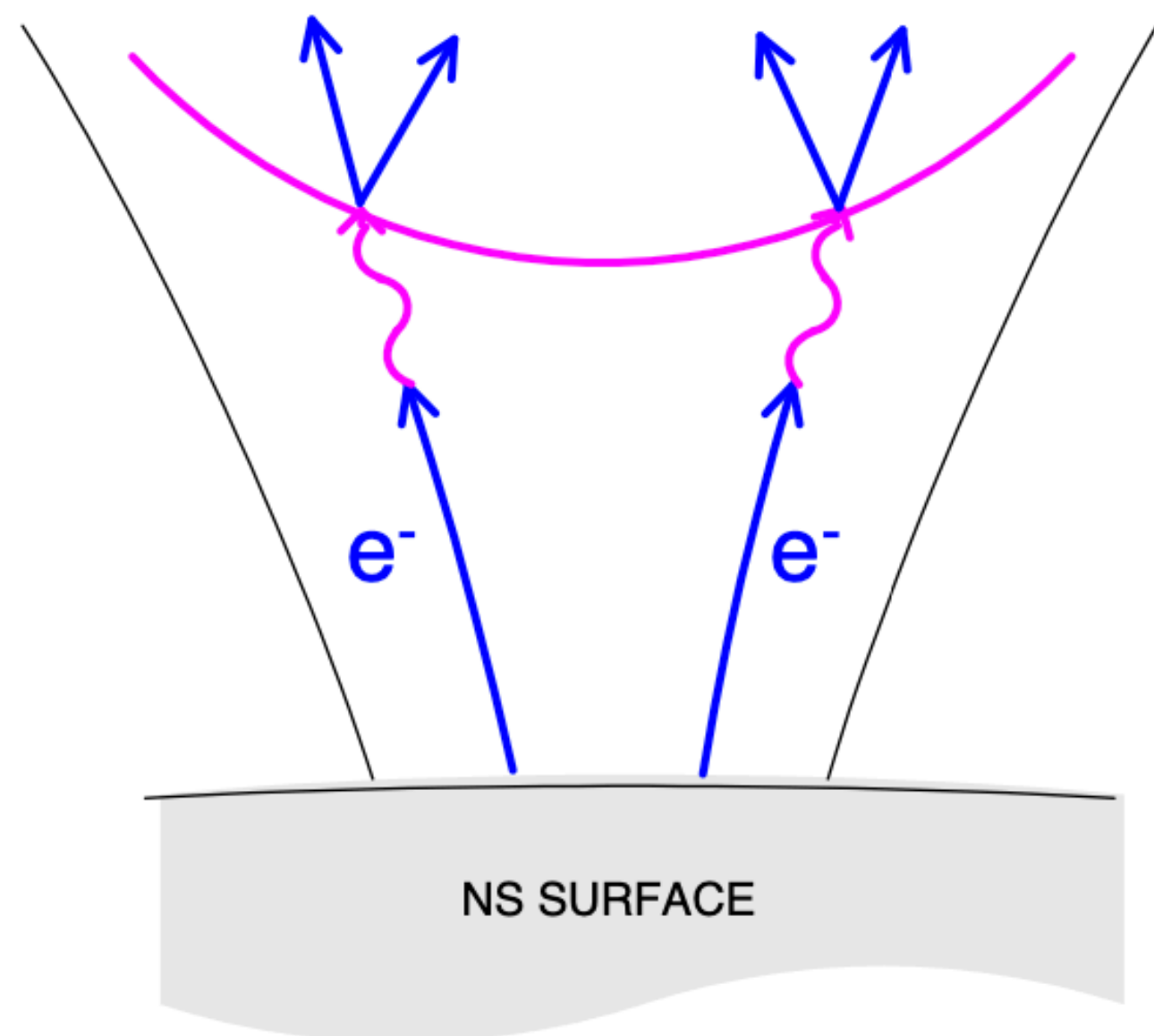


# SPACE CHARGE GAPS AND VACUUM GAPS

## SPACE CHARGE "GAP"

$$T_s > T_{e,i}$$

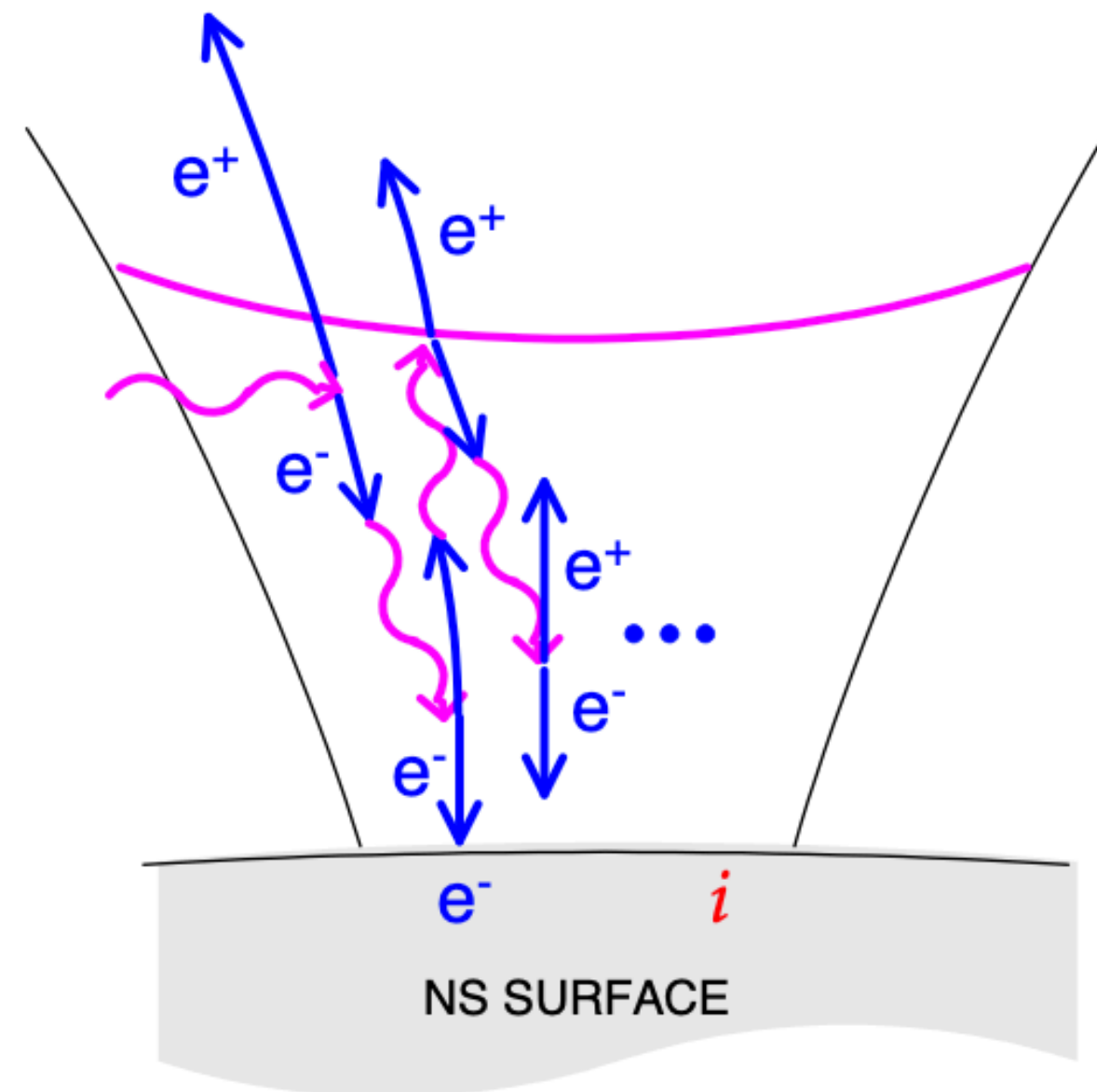
$$\Omega \cdot B > 0$$



## VACUUM GAP

$$T_s < T_{e,i}$$

$$\Omega \cdot B < 0$$

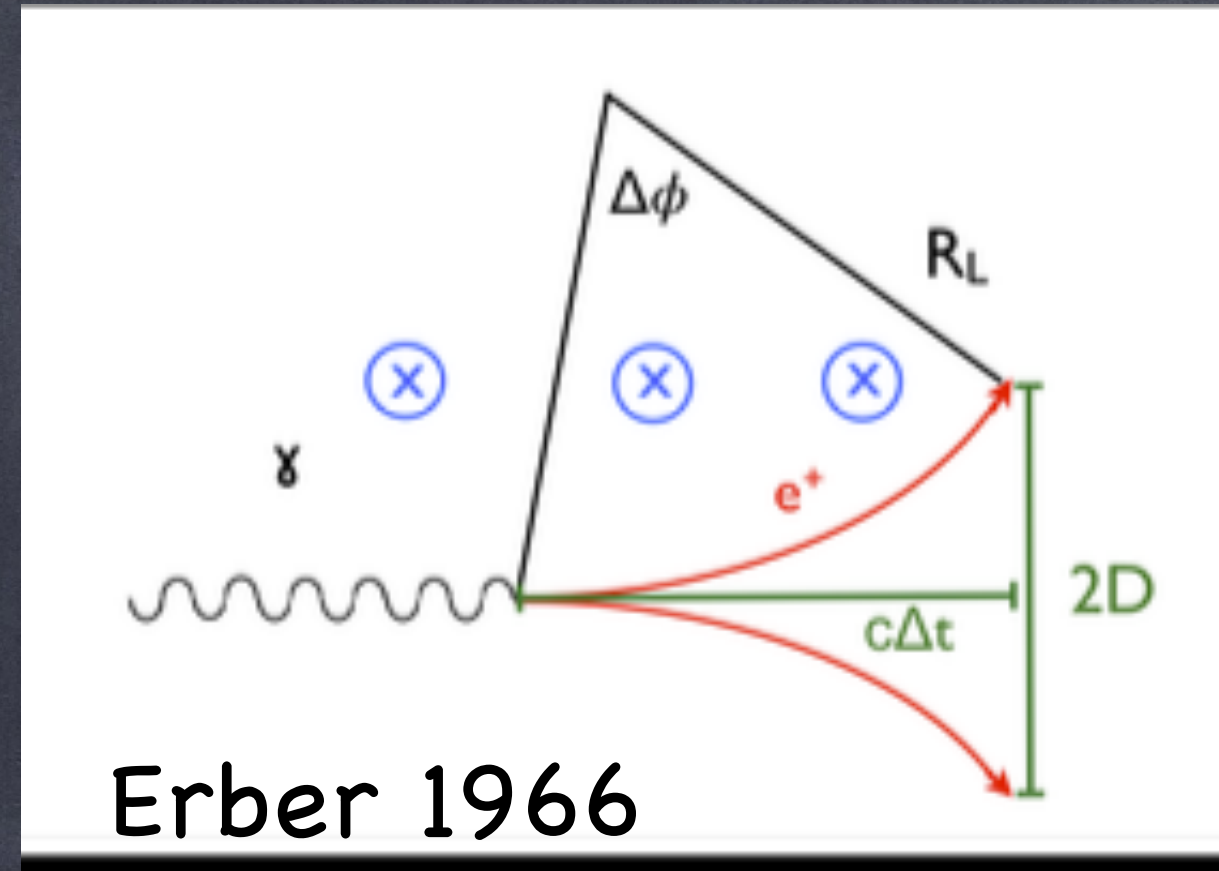


$$T_e \approx 3.6 \times 10^5 \text{ K} \left( \frac{Z}{26} \right)^{0.8} \left( \frac{B_\star}{10^{12} \text{ G}} \right)^{0.4}$$

$$T_i \approx 3.5 \times 10^5 \text{ K} \left( \frac{B_\star}{10^{12} \text{ G}} \right)^{0.73}$$



# PAIR PRODUCTION



$$h\nu > 2m_e c^2 \Rightarrow \text{PHOTON CAN CREATE PAIR} \quad \begin{cases} h\nu \rightarrow 2m_e c^2 \gamma \\ h\nu/c \rightarrow 2m_e c^2 \gamma \beta \end{cases} \Rightarrow \Delta E \sim \frac{2 (m_e c^2)^2}{h\nu}$$

$$\text{IN VACUUM PAIR LIVES ONLY} \quad \Delta t \sim \hbar/\Delta E$$

$$\text{IF B-FIELD} \quad \Delta\phi = \frac{c\Delta t}{r_L} \Rightarrow D \sim c \Delta t \Delta\Phi$$

$$\text{PAIR BECOMES REAL IF } D > \lambda_C = \frac{h}{m_e c} \quad D \approx \frac{\lambda_c}{4\pi} \left( \frac{h\nu}{m_e c^2} \right) \left( \frac{B}{B_{QED}} \right) = \chi \frac{\lambda}{4\pi}$$

$$\chi = \frac{\epsilon_\gamma}{m_e c^2} \frac{B}{B_{QED}} = 0.4 \left( \frac{\epsilon_\gamma}{10 \text{ MeV}} \right) \left( \frac{B}{10^{12} \text{ G}} \right) \quad \chi \gg 1 \Rightarrow \text{EXTREMELY EFFECTIVE PAIR CREATION}$$

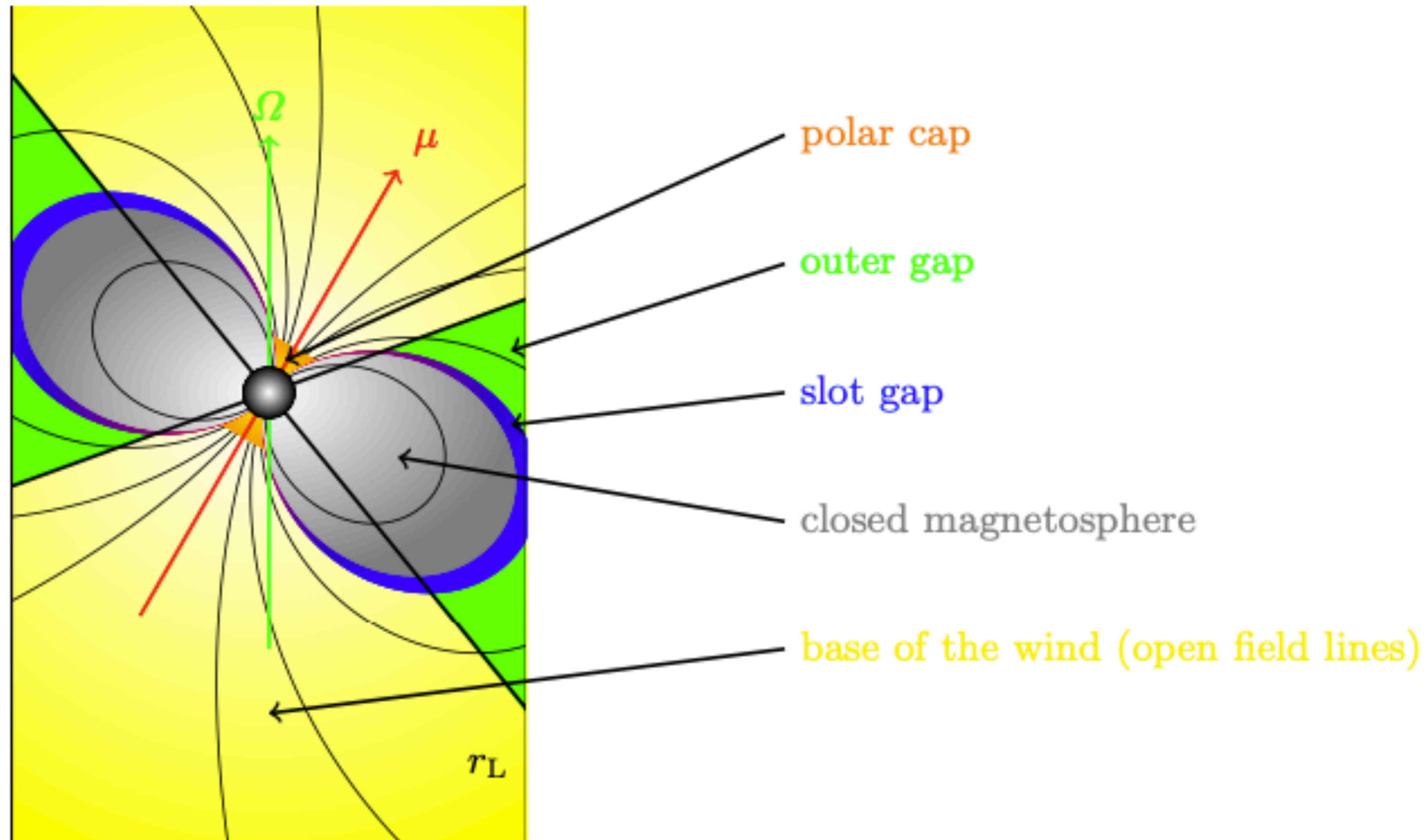
$$\chi < 1 \Rightarrow \text{PAIR CREATION SUPPRESSED AS } \exp(-4/3\chi)$$

$$\text{PAIR CREATION LENGTH:} \quad l_p = \frac{4.4}{\alpha_{fs}} \lambda_c \frac{B_{QED}}{B} \exp \left( \frac{4}{3\chi} \right)$$

RAPIDLY CHANGING QUANTITY IN THE MAGNETOSPHERE



# ALL GAPS



Petri 2017

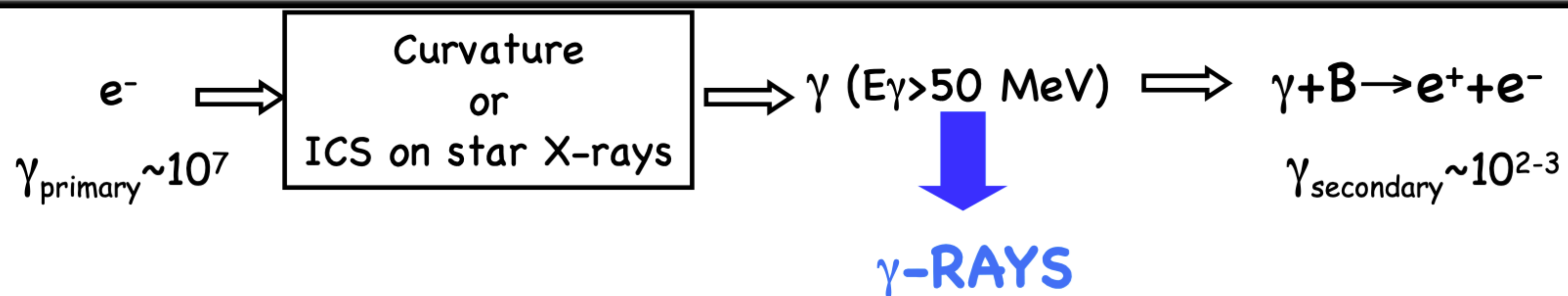
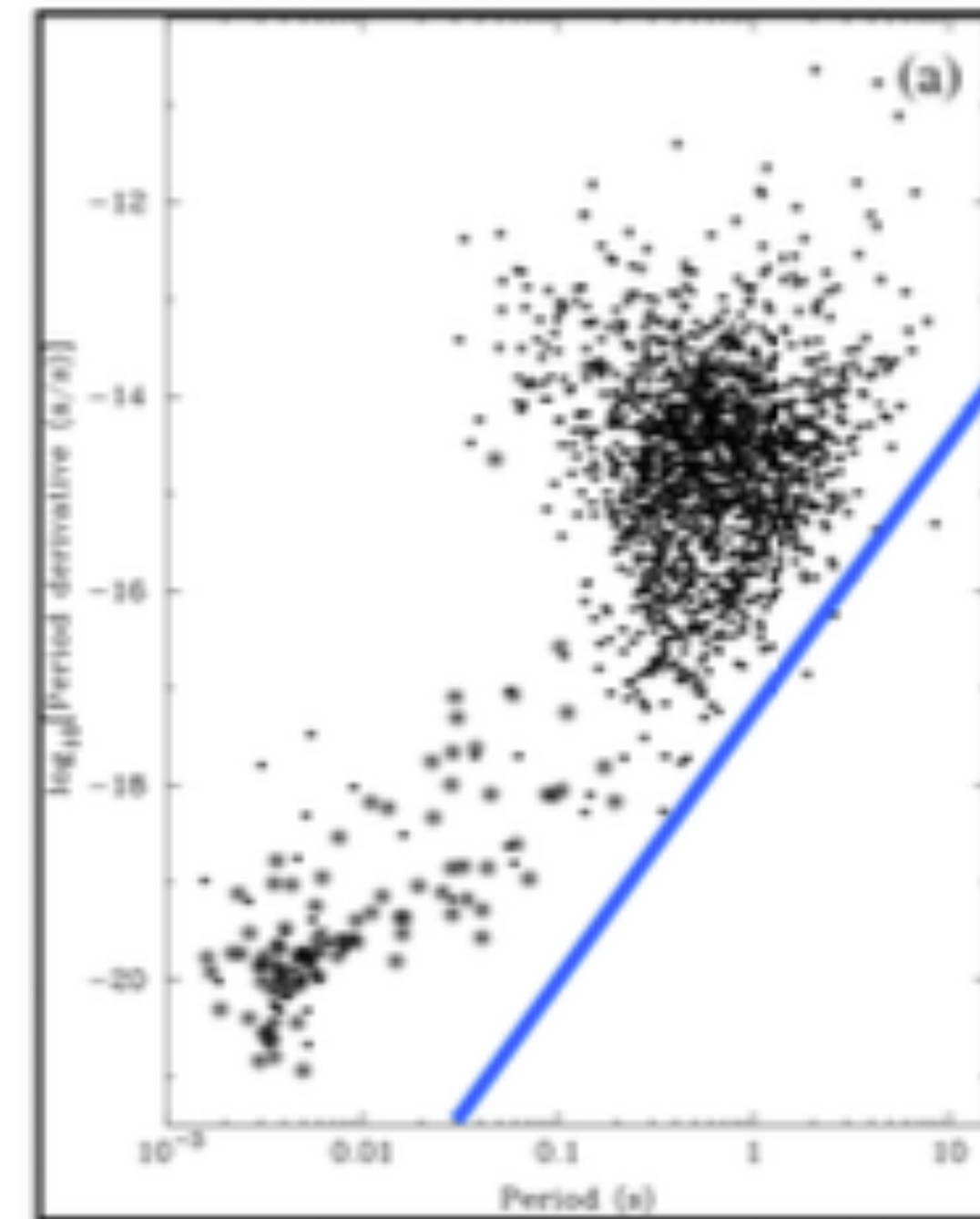
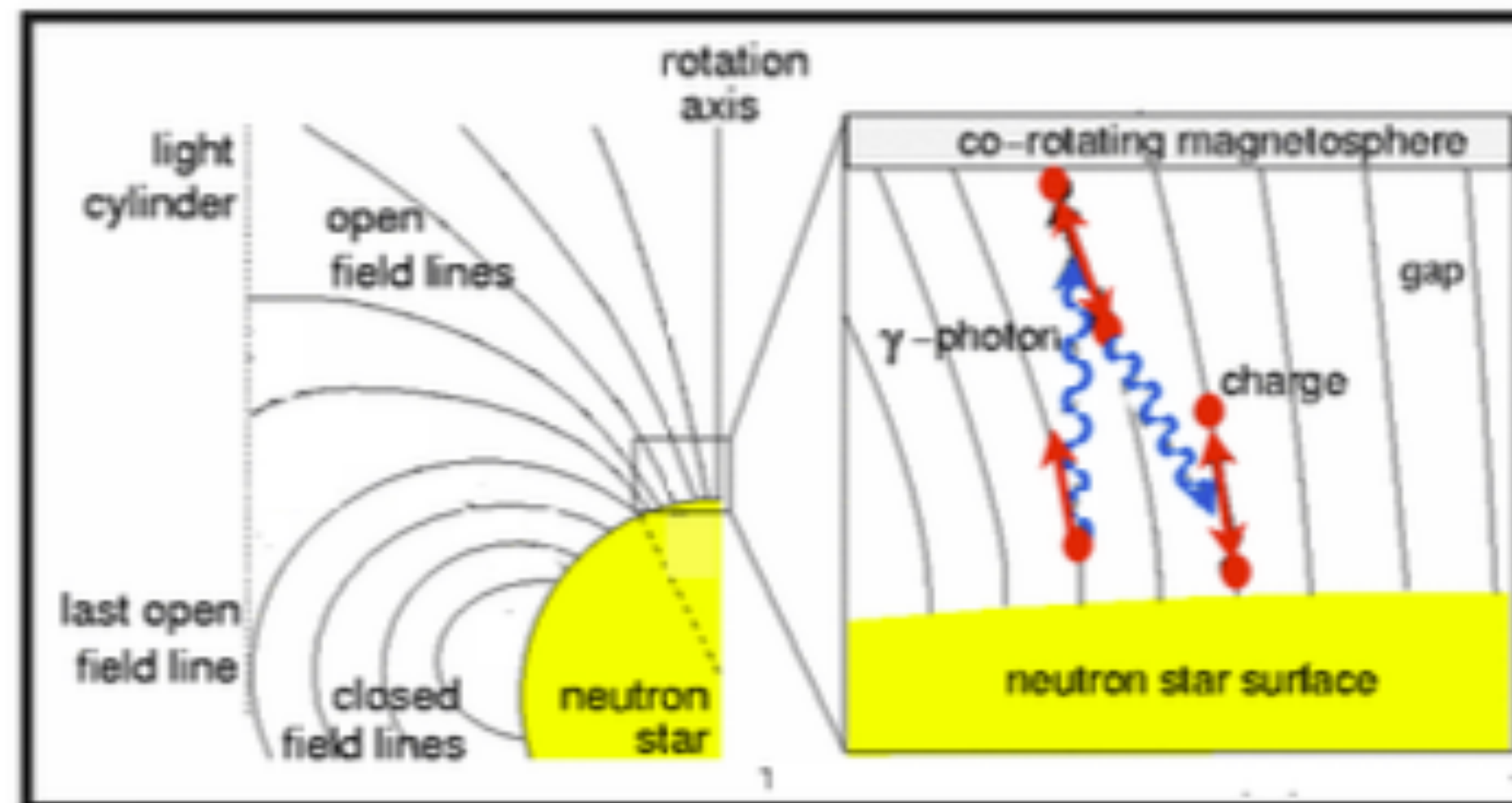


# DIAGNOSTICS OF THE CASCADE

DEATHLINE

GAMMA-RAYS

MULTIPLICITY

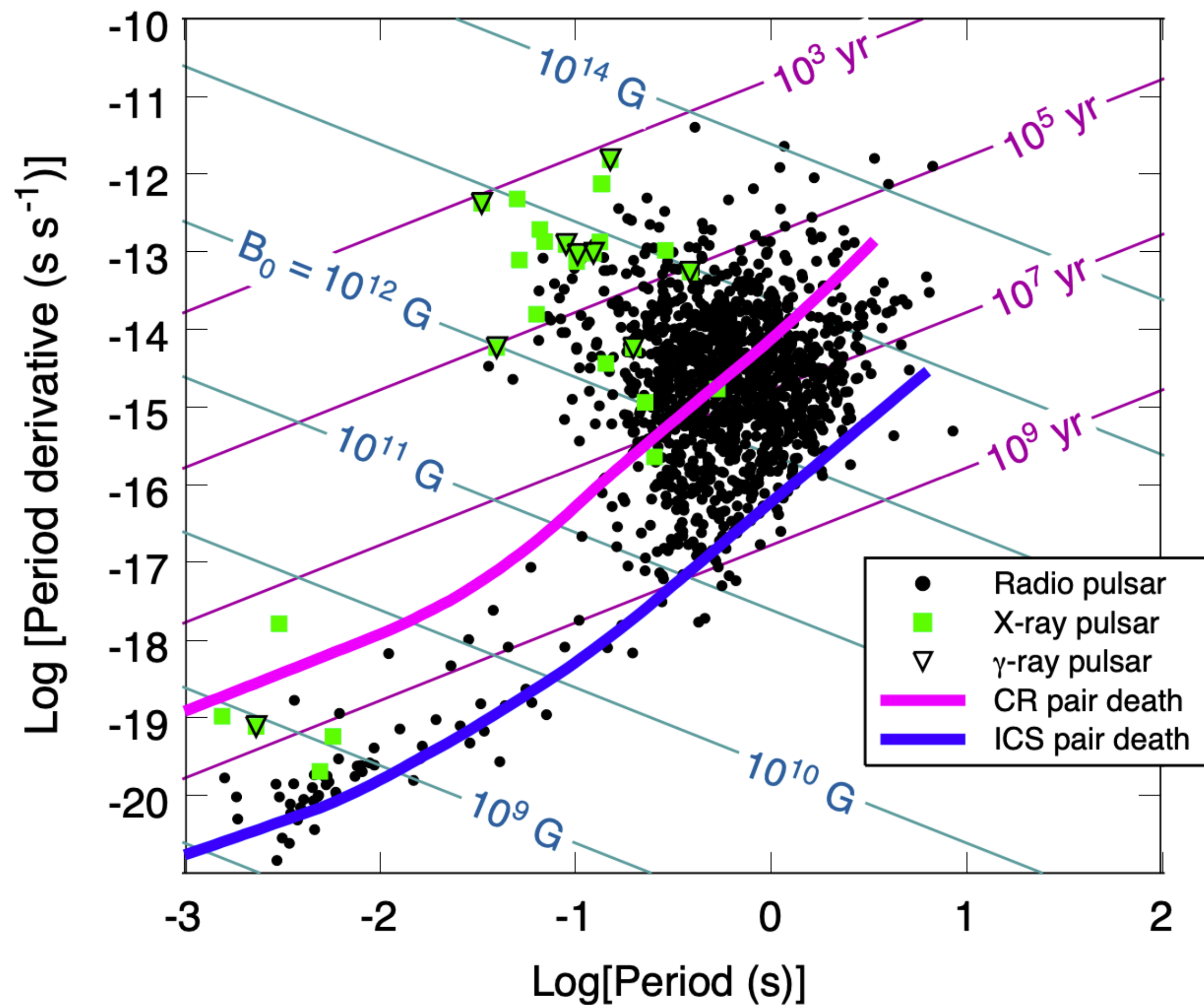


$$L_{\text{radio}} \leq 10^{-10} \dot{E}$$

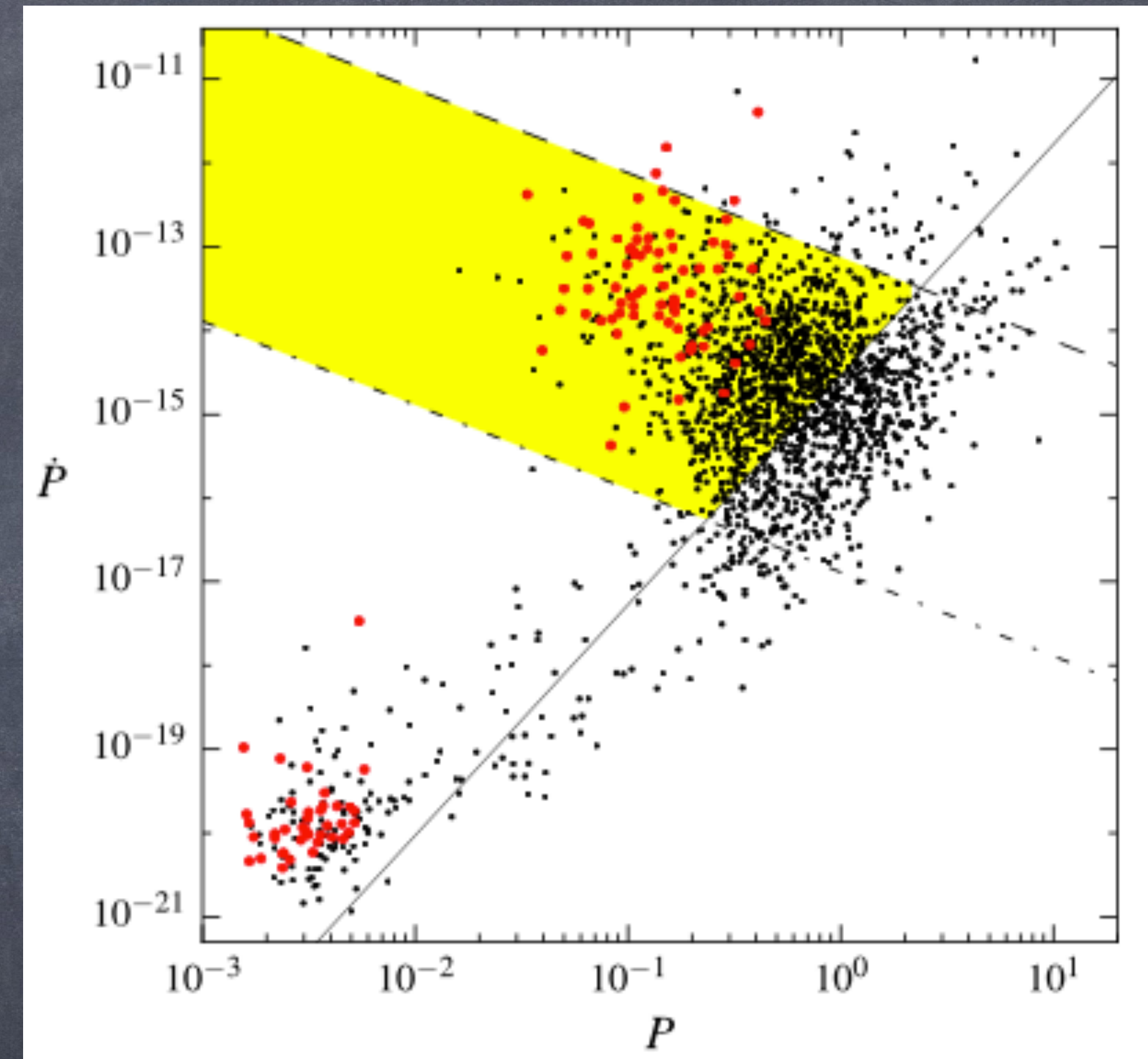
$$L_\gamma \sim 10^{-2} \dot{E}$$



# DEATHLINE AND MULTIPLICITY



Harding 2007

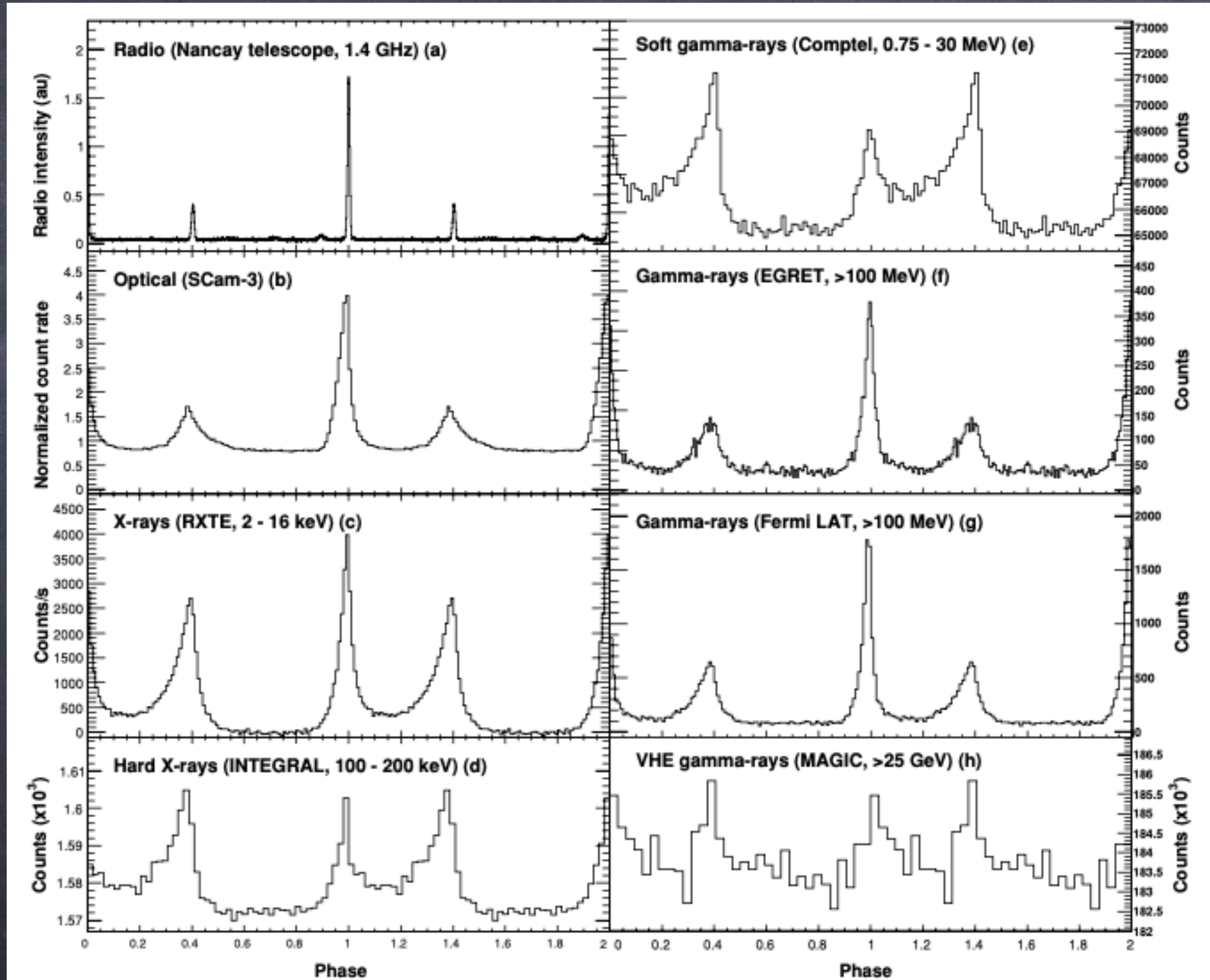


Timokhin & Harding 2015

HIGHEST MULTIPLICITY FOR SYNCHROTRON/CURVATURE POWERED GAPS:  $K < \text{few} \times 10^5$   
BUT.... PWNe



# GAMMA-RAYS

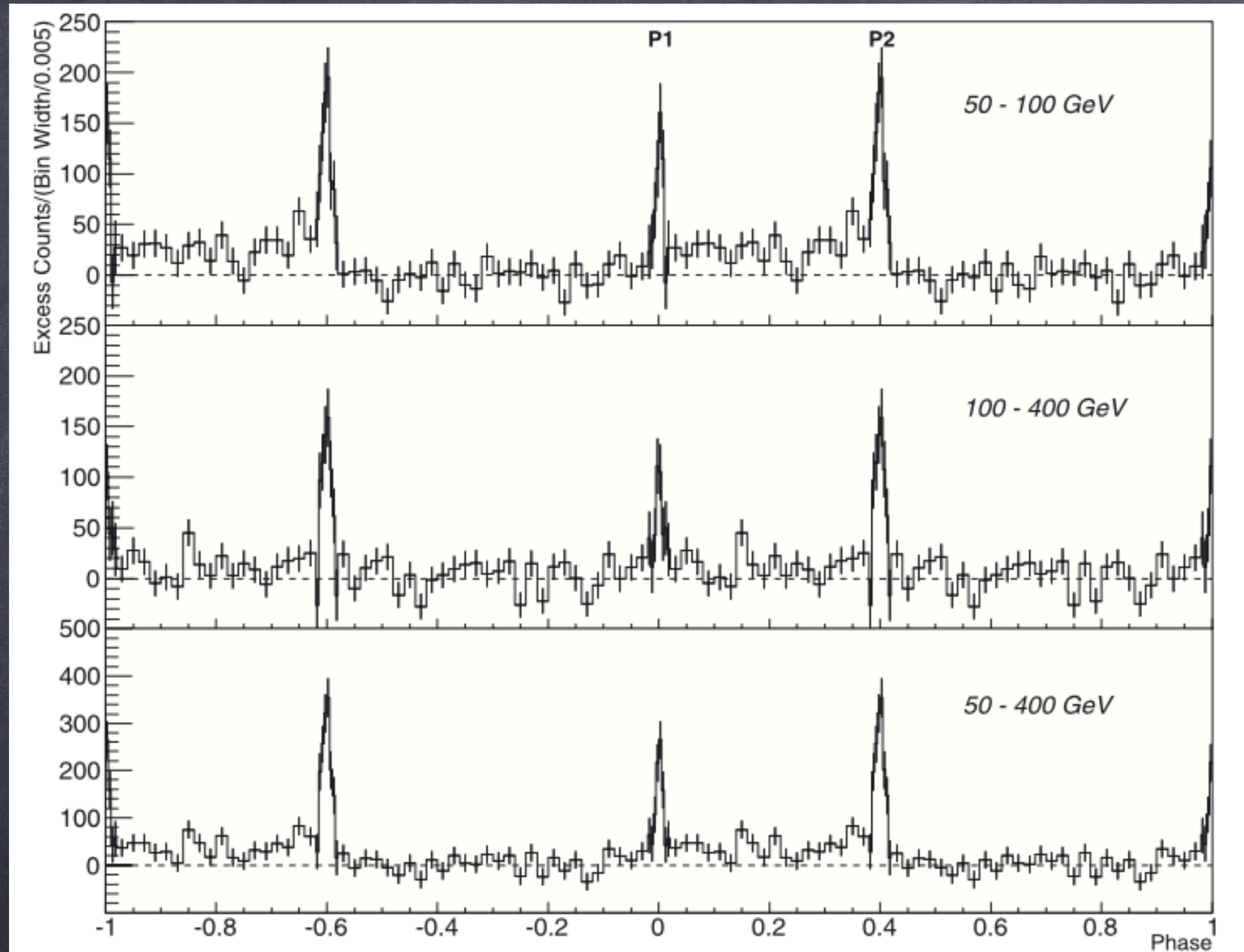


CRAB

Abdo et al 2010



# GAMMA-RAYS



DETECTION  
UP TO 1.5 TeV

GAMMA-RAYS FROM  
OUTER  
MAGNETOSPHERE  
OR EVEN WIND....

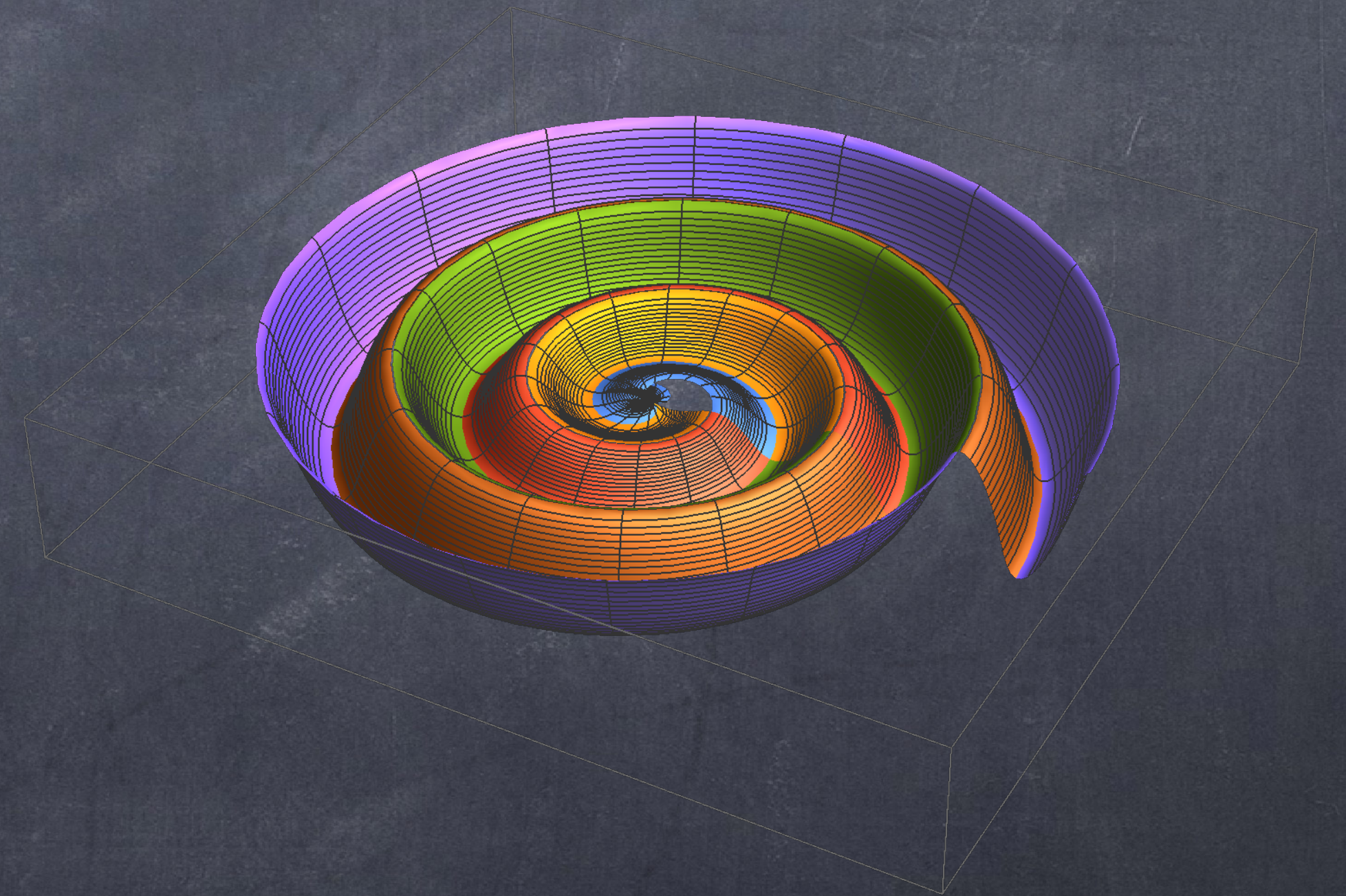
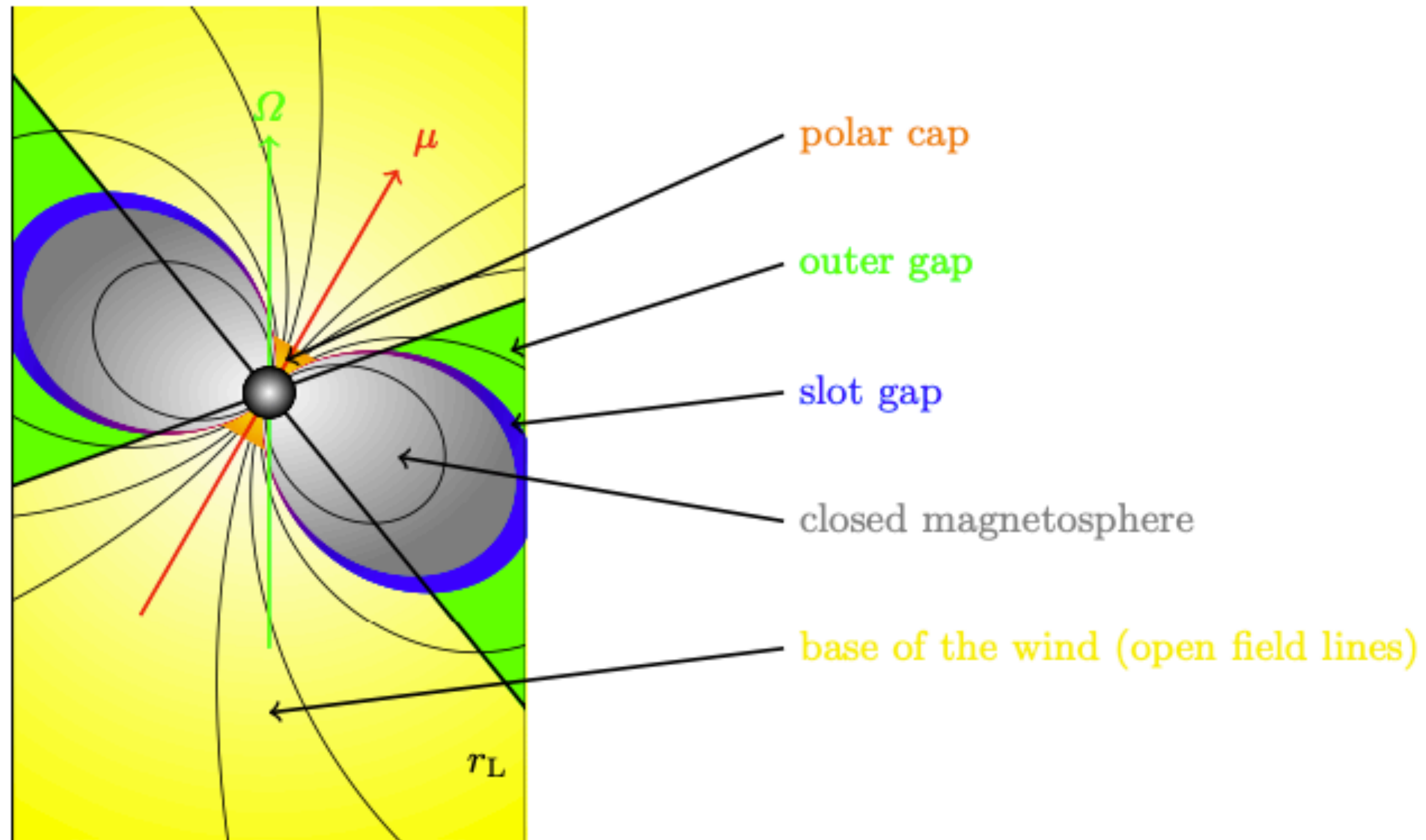
Aleksic et al 2014, 2016



UHECRS FROM PULSARS



# LOCATIONS OF PARTICLE ACCELERATION





# PARTICLE ACCELERATION IN THE MAGNETOSPHERE

DIRECT E-FIELD ACCELERATION IN GAP OF SIZE  $\xi R_L$  WITH POTENTIAL DIFFERENCE  $\Phi$  VS CURVATURE

$$\frac{d\gamma}{dt} = \frac{Ze\Phi}{Am_p c^2} \frac{2\pi}{\xi P} - \frac{8\pi^2}{3cP^2} \frac{Z^2 e^2}{Am_p c^2} \gamma^4$$

$$\Phi = \frac{B_\star \Omega R_\star^2}{2c} \frac{R_\star}{R_L} \approx \sqrt{\dot{E}/c}$$

$$\gamma_{\text{curv}} = \left( \frac{3\pi B R_\star^3}{2ZecP\xi} \right)^{1/4} \sim 1.1 \times 10^8 Z_{26}^{-1/4} \xi^{-1/4} B_{13}^{1/4} P_{-3}^{-1/4} R_{\star,6}^{3/4}$$

$$\dot{E} = \kappa \dot{N}_{GJ} m_e \Gamma c^2 \left( 1 + \frac{m_i}{\kappa m_e} \right) (1 + \sigma)$$

$$\sigma = \frac{B^2}{4\pi n_\pm m_e c^2 \Gamma^2}$$

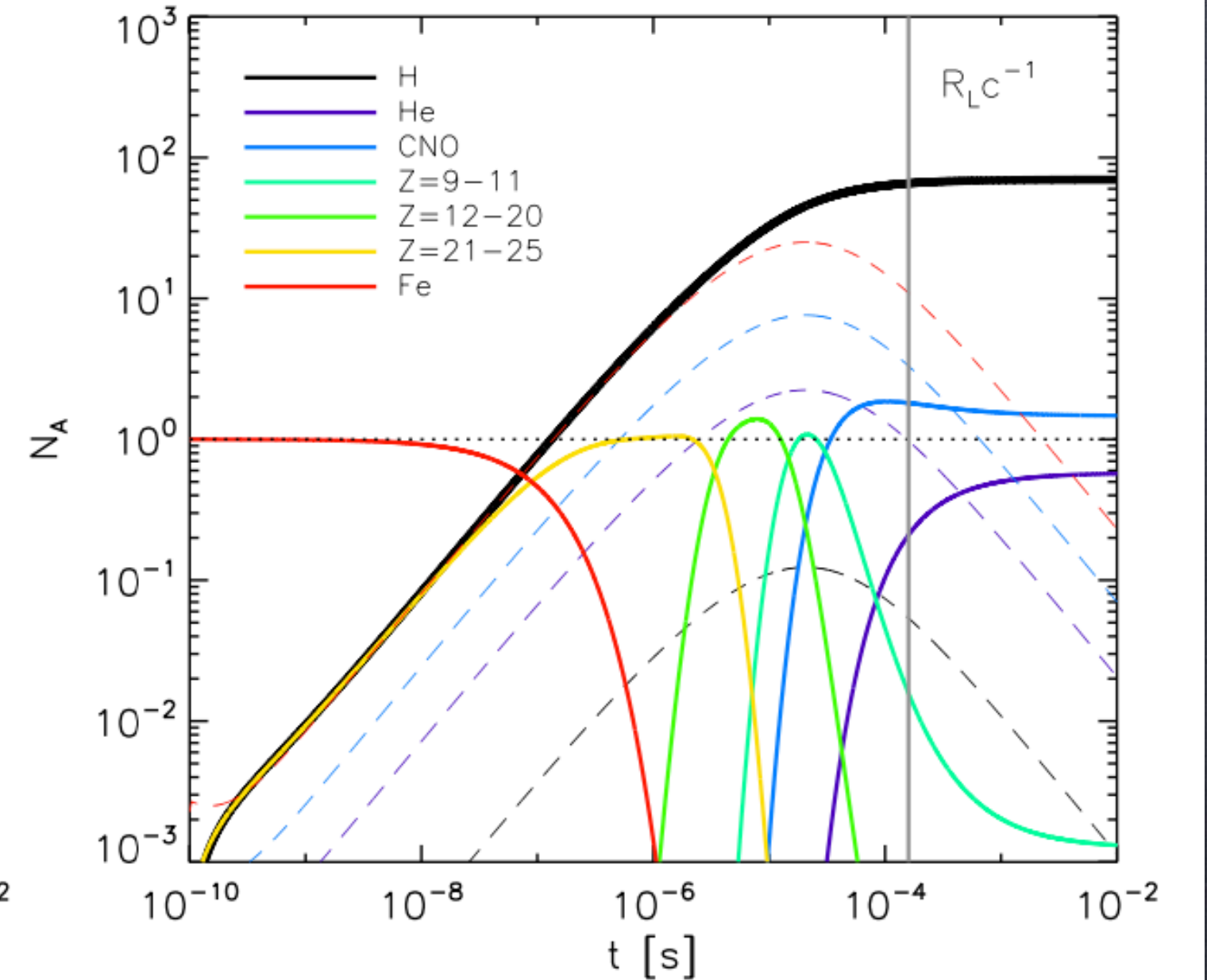
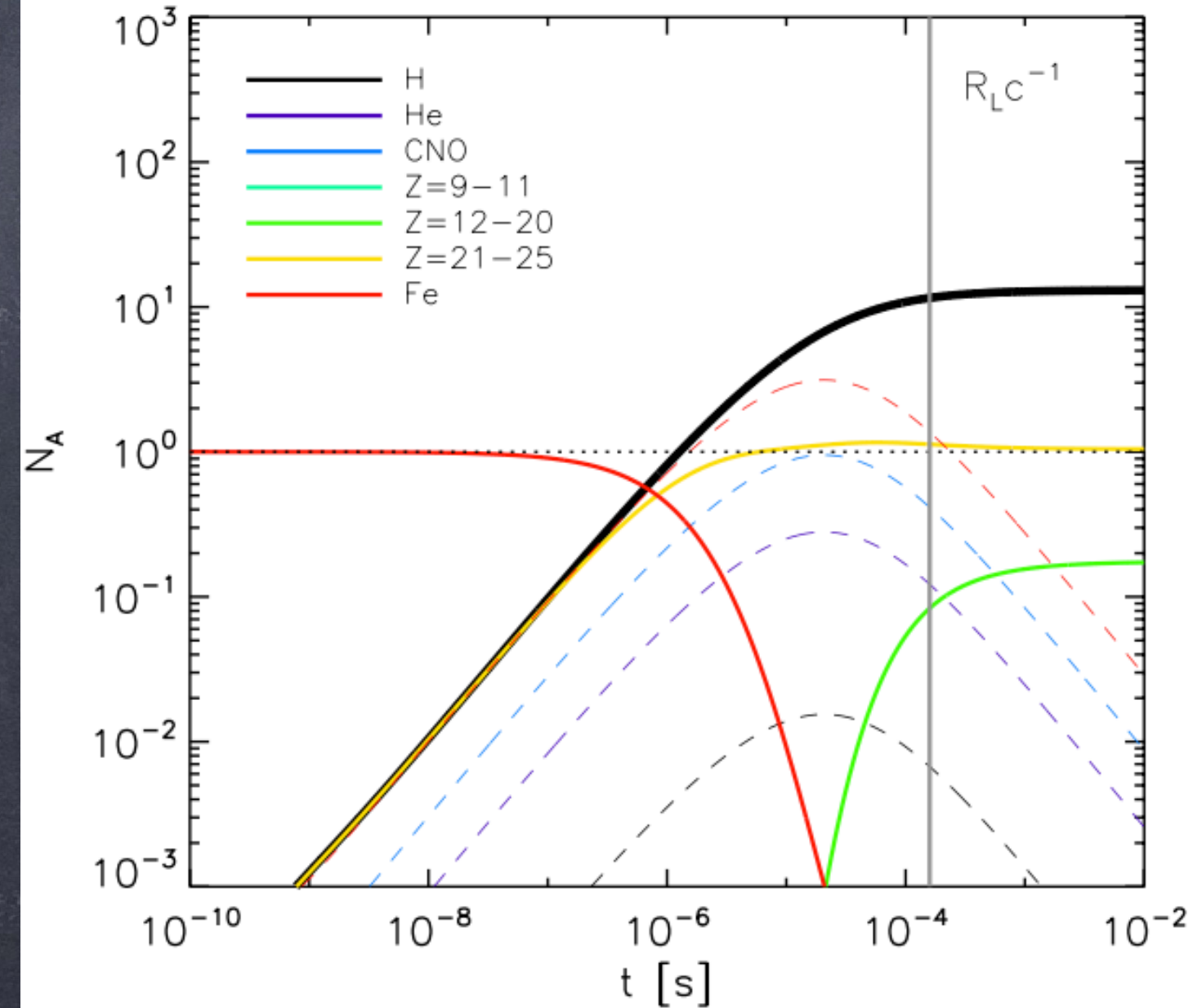
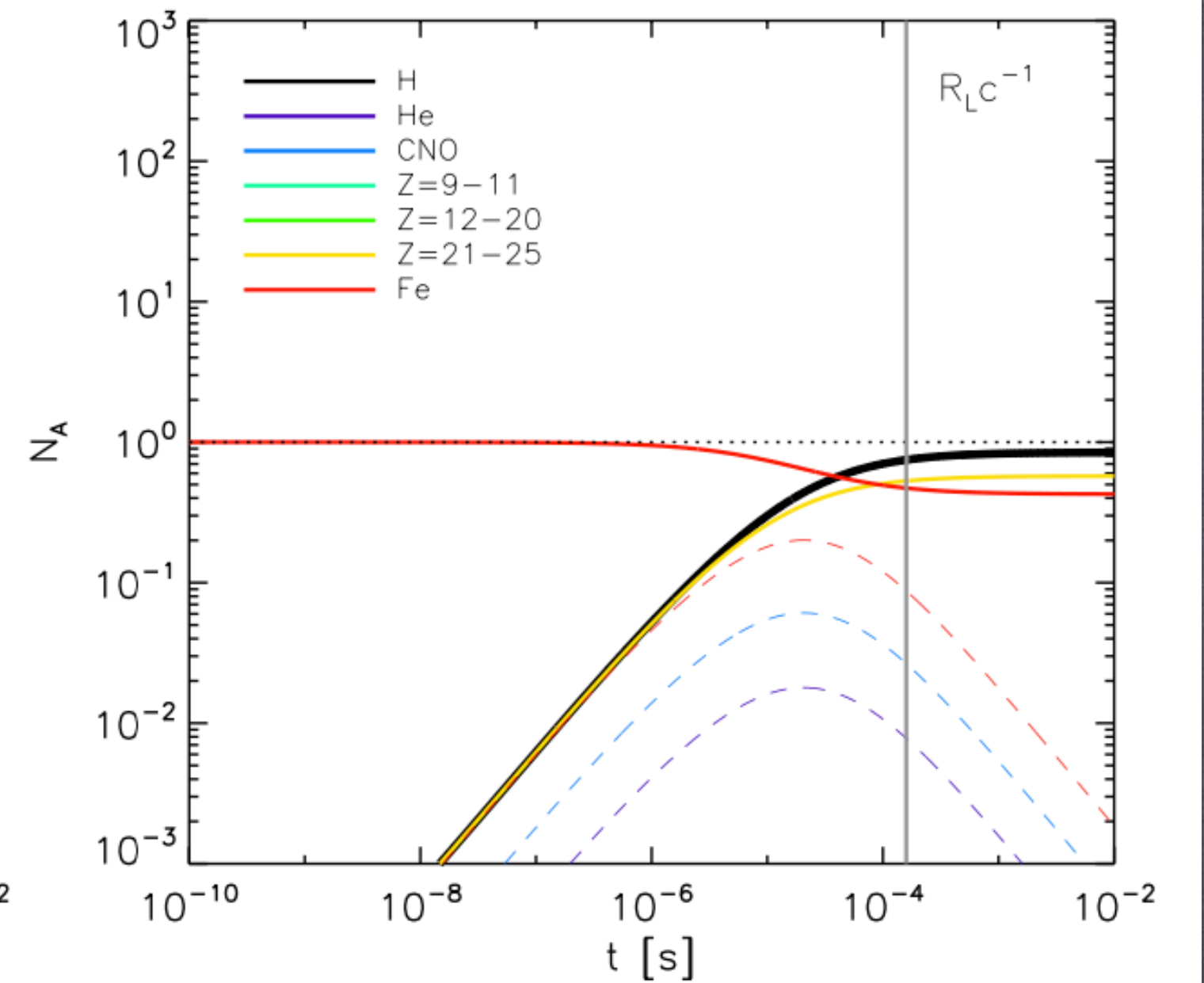
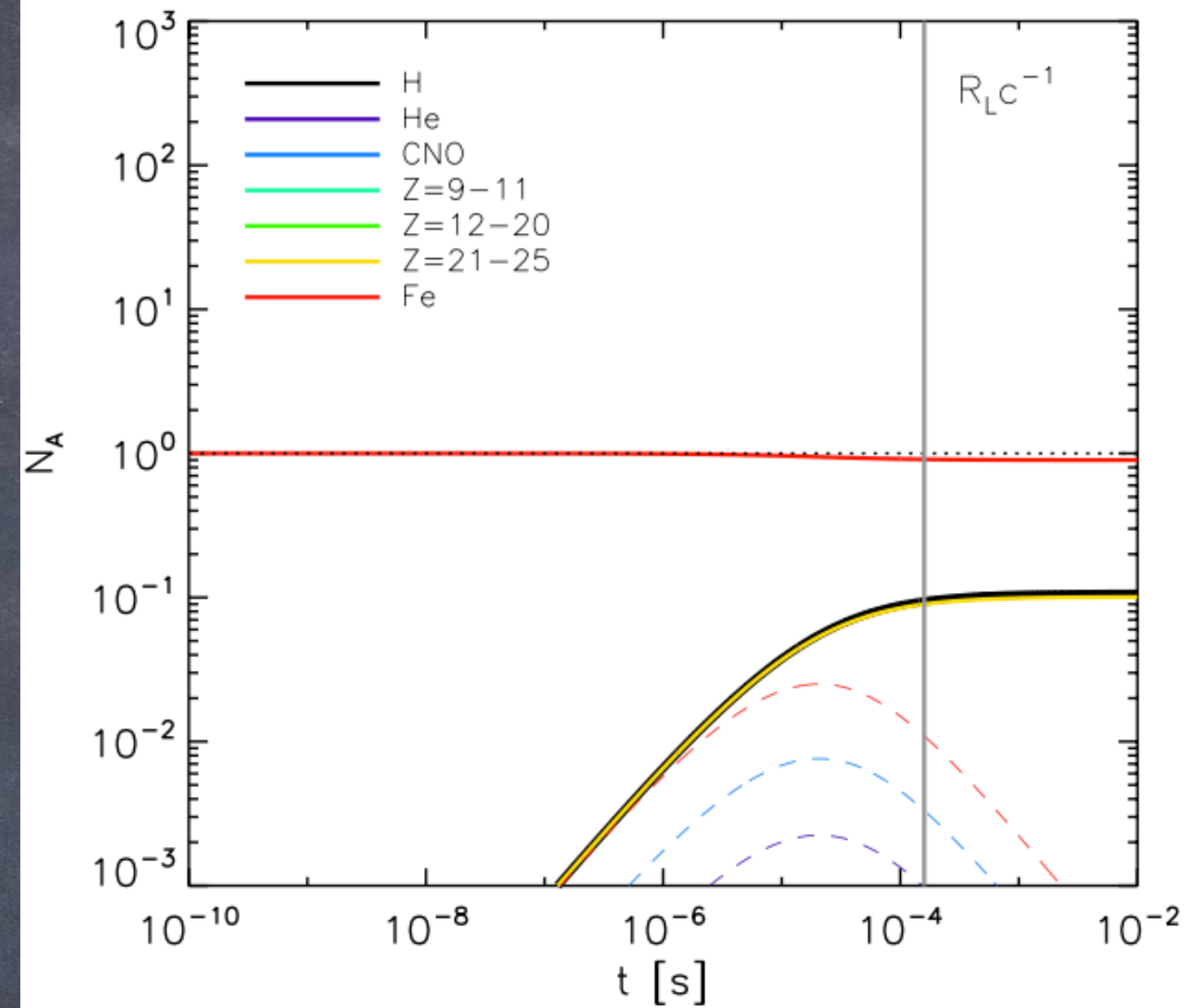
$$\gamma_{\text{max}} = \min(\gamma_w, \gamma_\Phi, \gamma_{\text{curv}})$$



# UHECR COMPOSITION

## PURE IRON EXTRACTION AND PHOTODISINTEGRATION

$$\frac{dN_A}{dt} + \frac{N_A}{t_A} = \frac{N_{A+1}}{t_{A+1}}$$





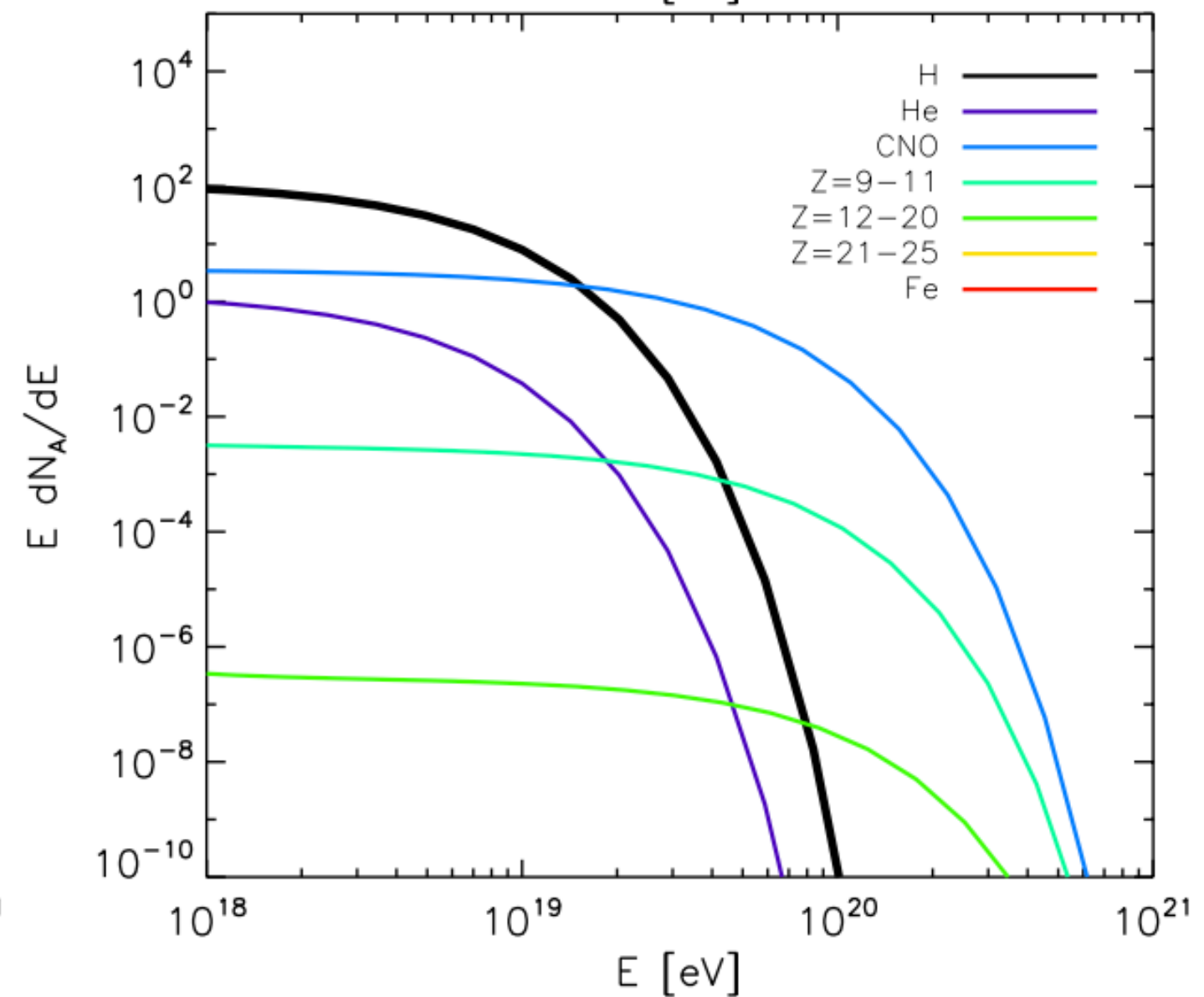
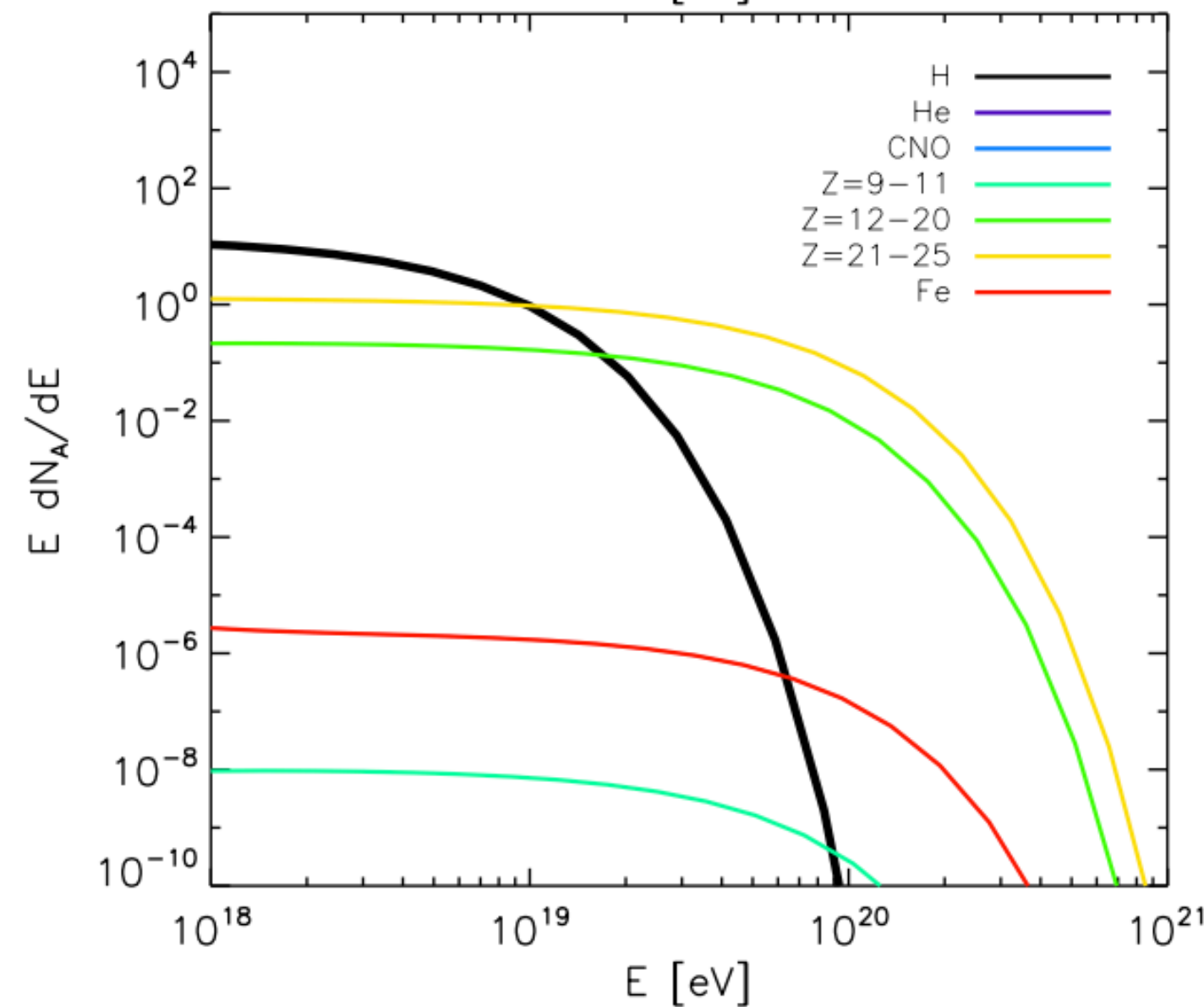
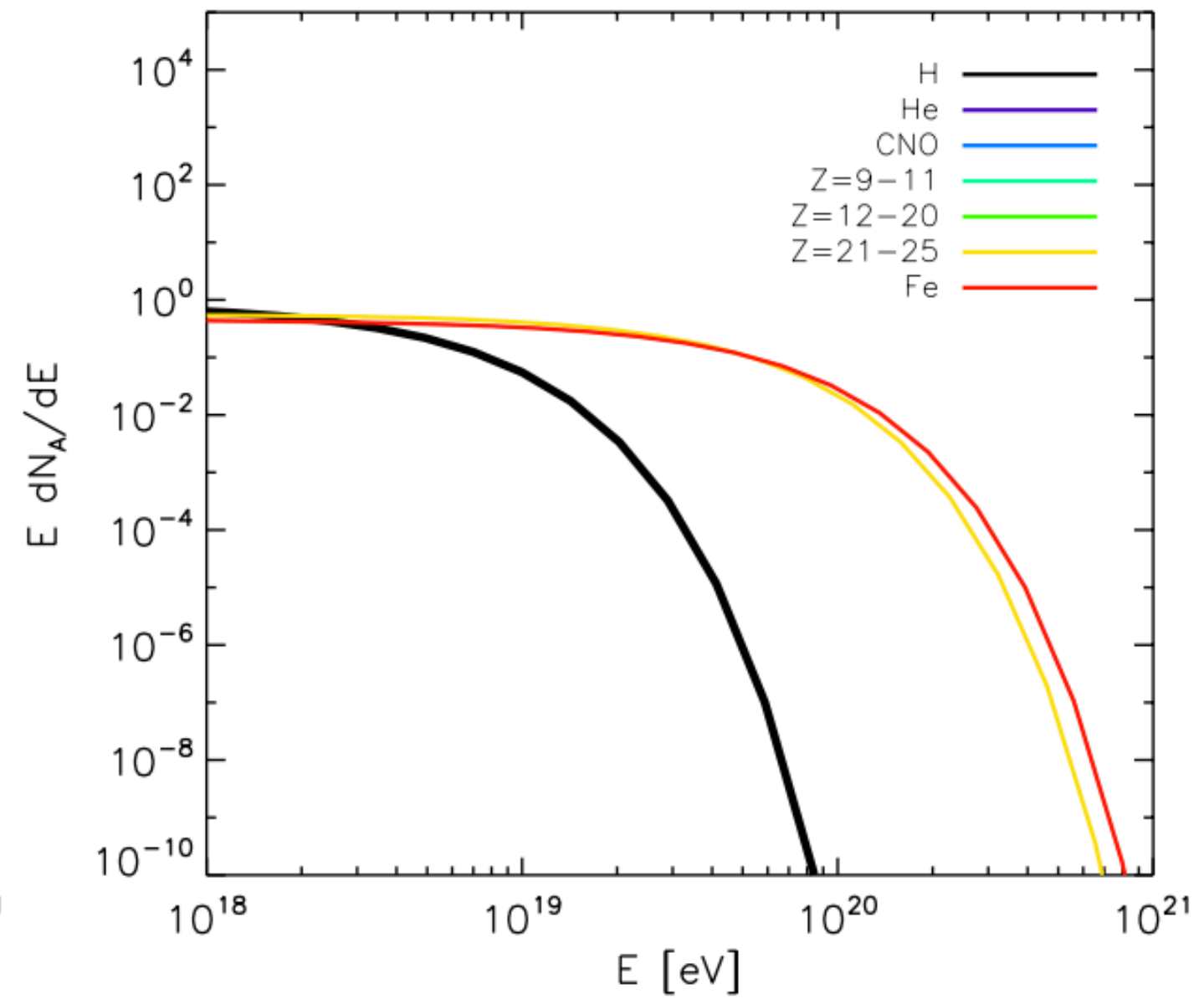
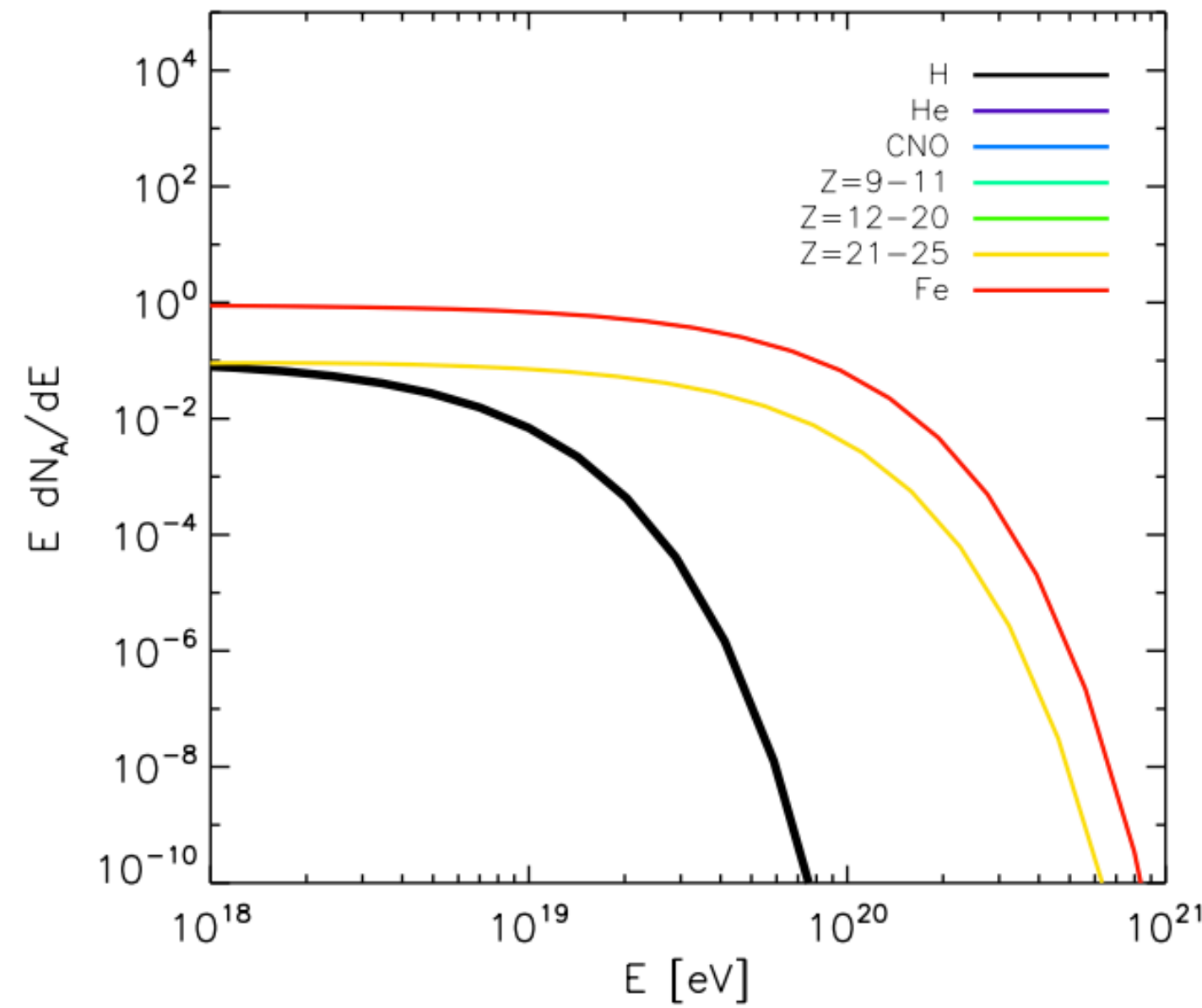
# PARTICLE SPECTRUM

$$E_{\text{CR}}(t) = E_0 (1 + t/t_{\text{sd}})^{-1}$$

$$\sim 1.2 \times 10^{20} \text{ eV } \eta A_{56} \kappa_4 I_{45} B_{13}^{-1} R_{\star,6}^{-3} t_{7.5}^{-1}$$

$$\frac{dN_{\text{CR}}}{dE} = \int_0^\infty dt \dot{N}_{\text{GJ}}(t) \delta(E - E_{\text{CR}}(t)) = \frac{\dot{N}_{\text{GJ}}(0) t_{\text{sd}}}{E}$$

$$t_{\text{sd}} = \frac{9 I c^3 P_i^2}{8 \pi^2 B^2 R^6} \sim 3.1 \times 10^7 \text{ s } I_{45} B_{13}^{-2} R_{\star,6}^{-6} P_{i,-3}^2$$





# THE PULSAR EQUATION

PERFECT CONDUCTOR

FORCE-FREE

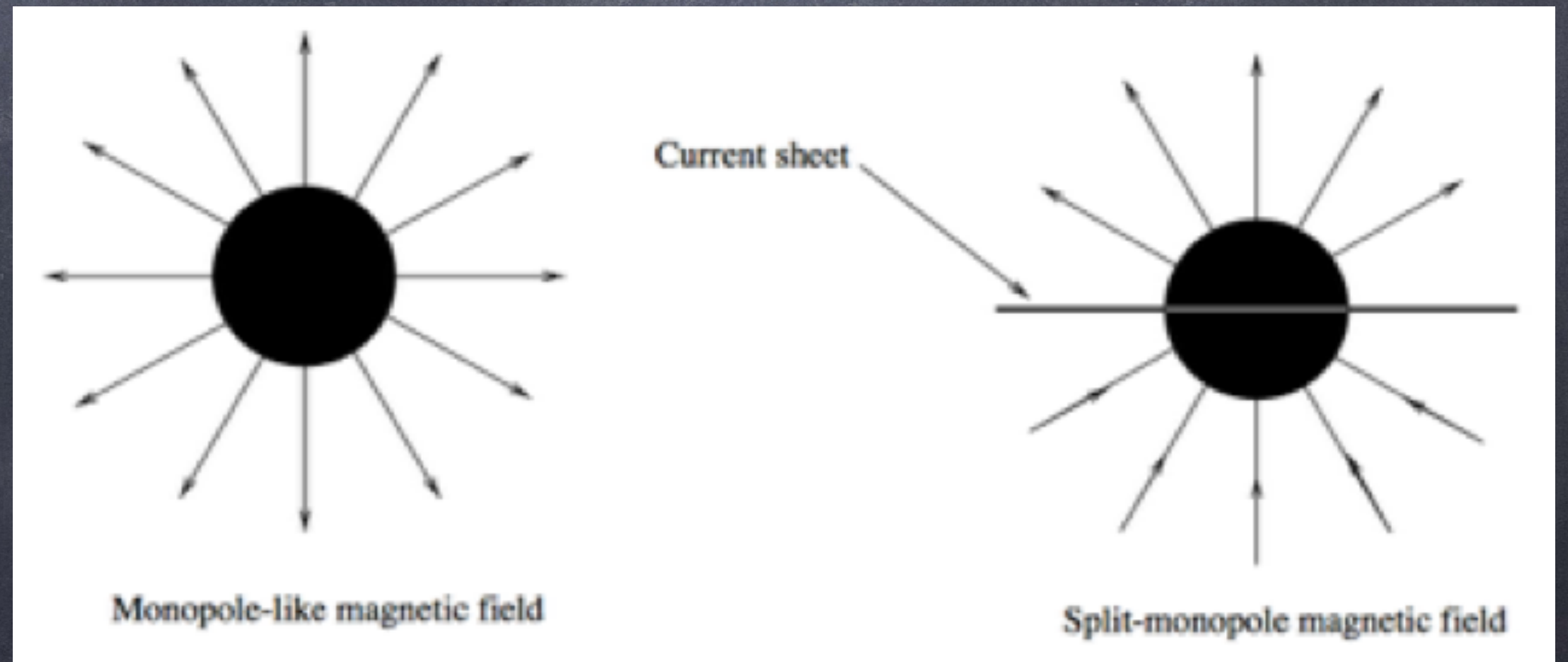
$$\begin{cases} \vec{E} = -\frac{\vec{v}}{c} \wedge \vec{B} \\ \rho_e \vec{E} + \frac{\vec{J}}{c} \wedge \vec{B} = 0 \end{cases}$$

$$\begin{cases} \vec{E} = -\vec{\nabla} \Phi \\ \vec{B} = \frac{c}{\Omega} \frac{\vec{\nabla} \Phi}{r} \wedge \vec{e}_\Phi \\ \rho_e = -\frac{\nabla^2 \Phi}{4\pi} \end{cases}$$

$$\Phi_{zz} + \Phi_{rr} - \frac{R_L^2 + r^2}{R_L^2 - r^2} \frac{\Phi_r}{r} = -\frac{A(\Phi) A'(\Phi)}{R_L^2 - r^2}$$

$$A(\Phi) = rB_\phi \quad \text{UNKNOWN}$$

EXACT SOLUTION EXISTS  
FOR MONOPOLE





# LECTURE 2



# PLAN OF THE LECTURES

## • LECTURE 1: PULSARS AND THEIR MAGNETOSPHERES

- BRIEF HISTORICAL NOTES
- THE GOLDREICH AND JULIAN MAGNETOSPHERE
- GAPS, PLASMA SUPPLY, MULTIPLICITY
- FAST PULSARS AND UHECRS
- THE PULSAR WIND

## • LECTURE 2: PULSAR WIND NEBULAE

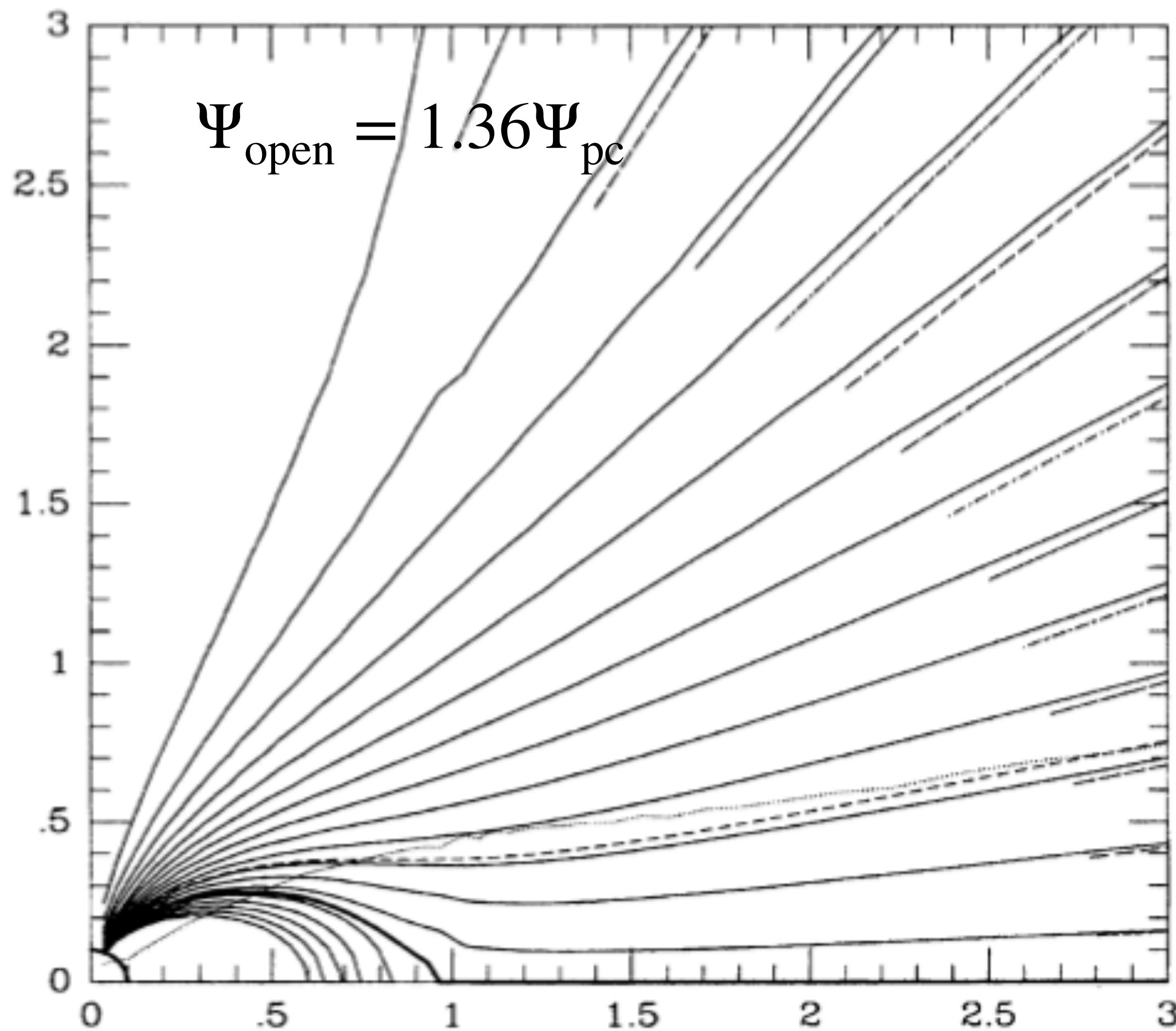
- DYNAMICS
- PARTICLE ACCELERATION
- RECENT NEWS FROM GAMMA-RAYS
- PARTICLE ESCAPE



# THE PULSAR WIND



# THE ALIGNED ROTATOR



Contopoulos, Kazanas, Fendt 2001

$$F(R, \theta) \propto \frac{\sin^2 \theta}{R^2}$$

ALIGNED ROTATING DIPOLE  
VERY SIMILAR TO SPLIT MONOPOLE

$$\vec{B} = B_L \left[ \left( \frac{R_L}{R} \right)^2 \underline{e}_R - \left( \frac{R_L}{R} \right) \sin \theta \underline{e}_\phi \right]$$

$$\vec{E} = -B_L \left( \frac{R_L}{R} \right) \sin \theta \underline{e}_\theta$$

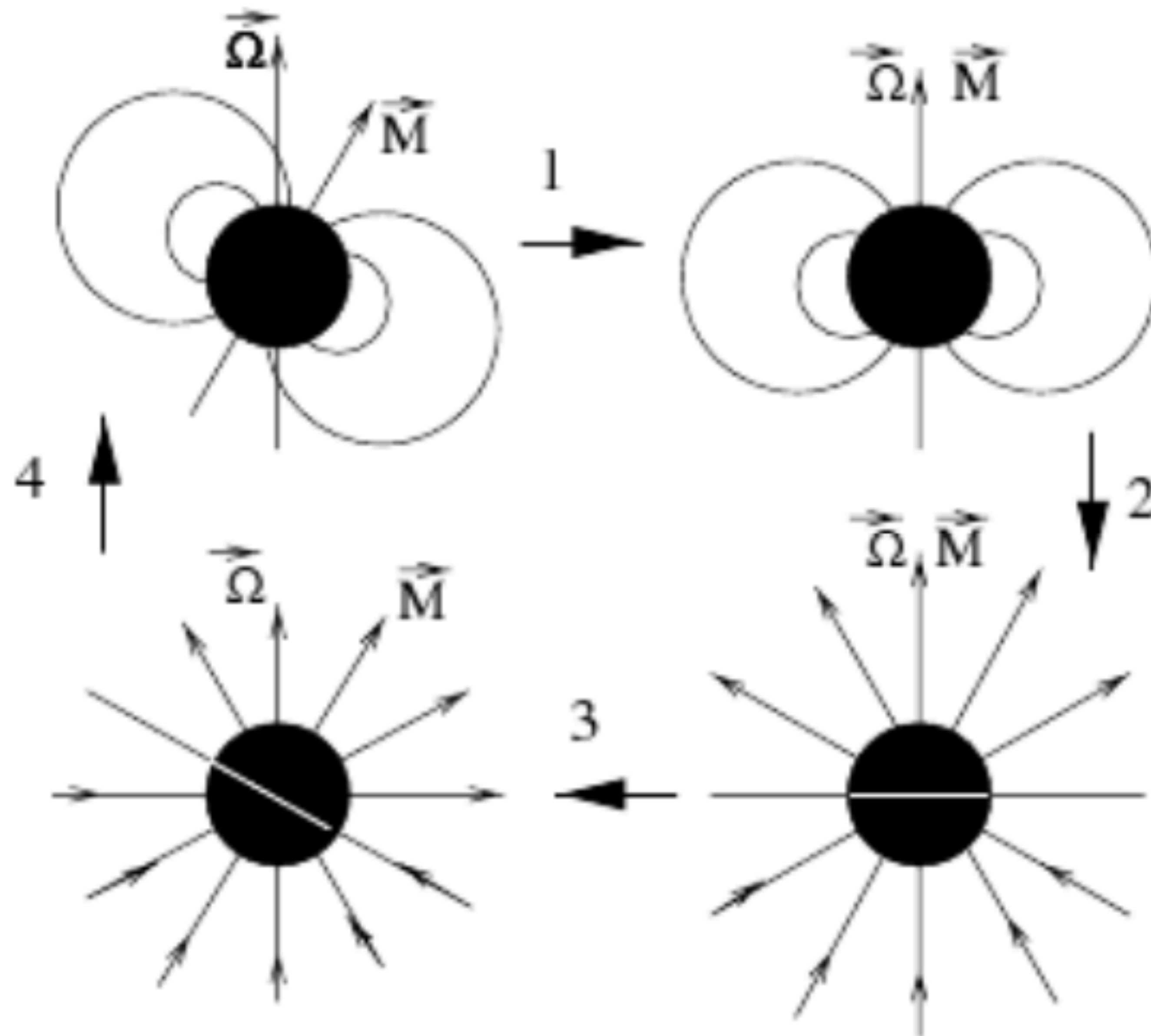
YET ANOTHER WAY TO LOOK AT THE STAR  
SPIN-DOWN: POYNTING FLUX THROUGH THE  
LIGHT CYLINDER SURFACE:

$$\dot{E} = \pi S_L R_L^2 \quad S_L = \frac{B_L^2}{4\pi} c \pi R_L^2 \quad B_L = B_\star \left( \frac{R_\star}{R_L} \right)^3$$

$$\dot{E} \approx \frac{B_\star^2 \Omega^4 R_\star^6}{c^3}$$



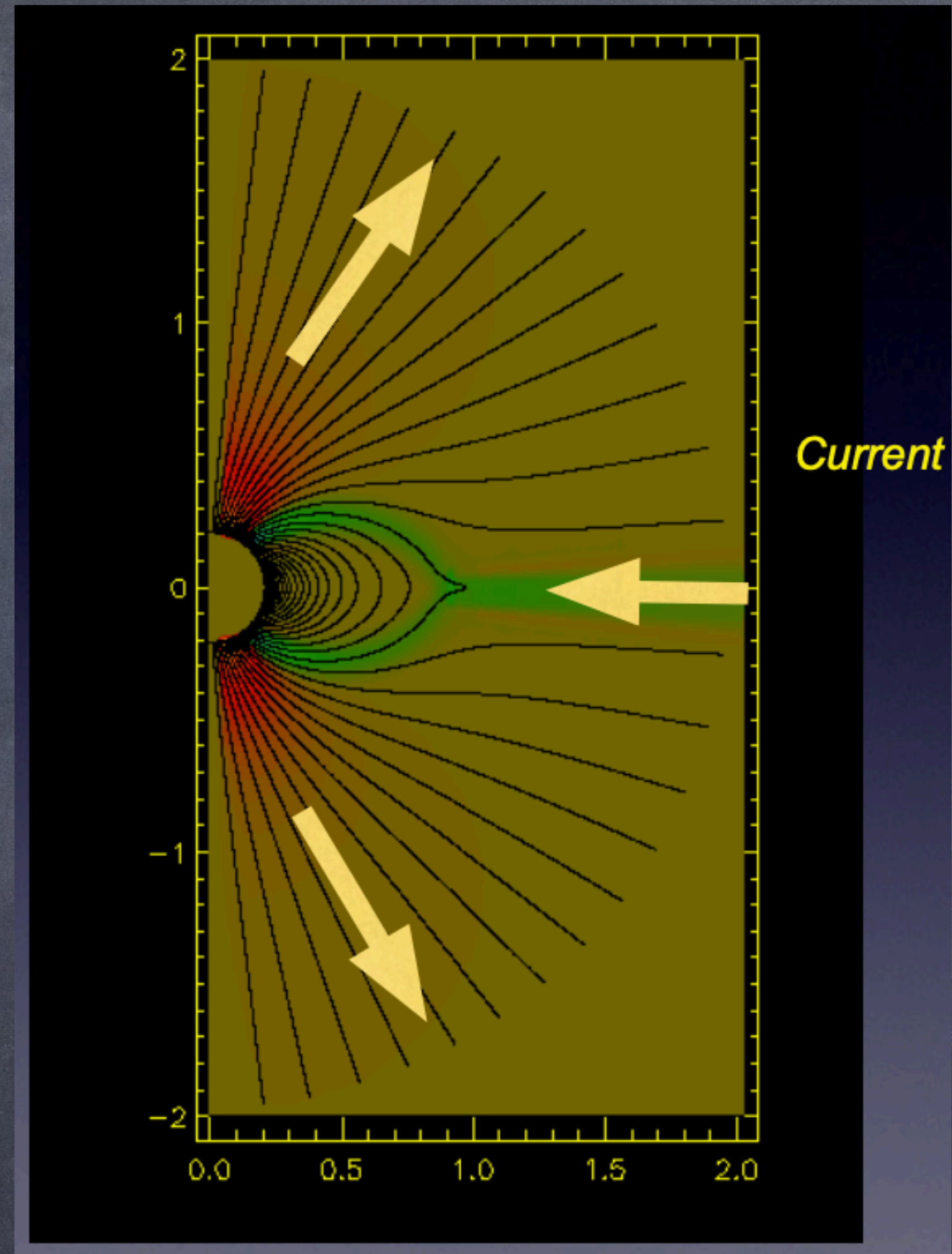
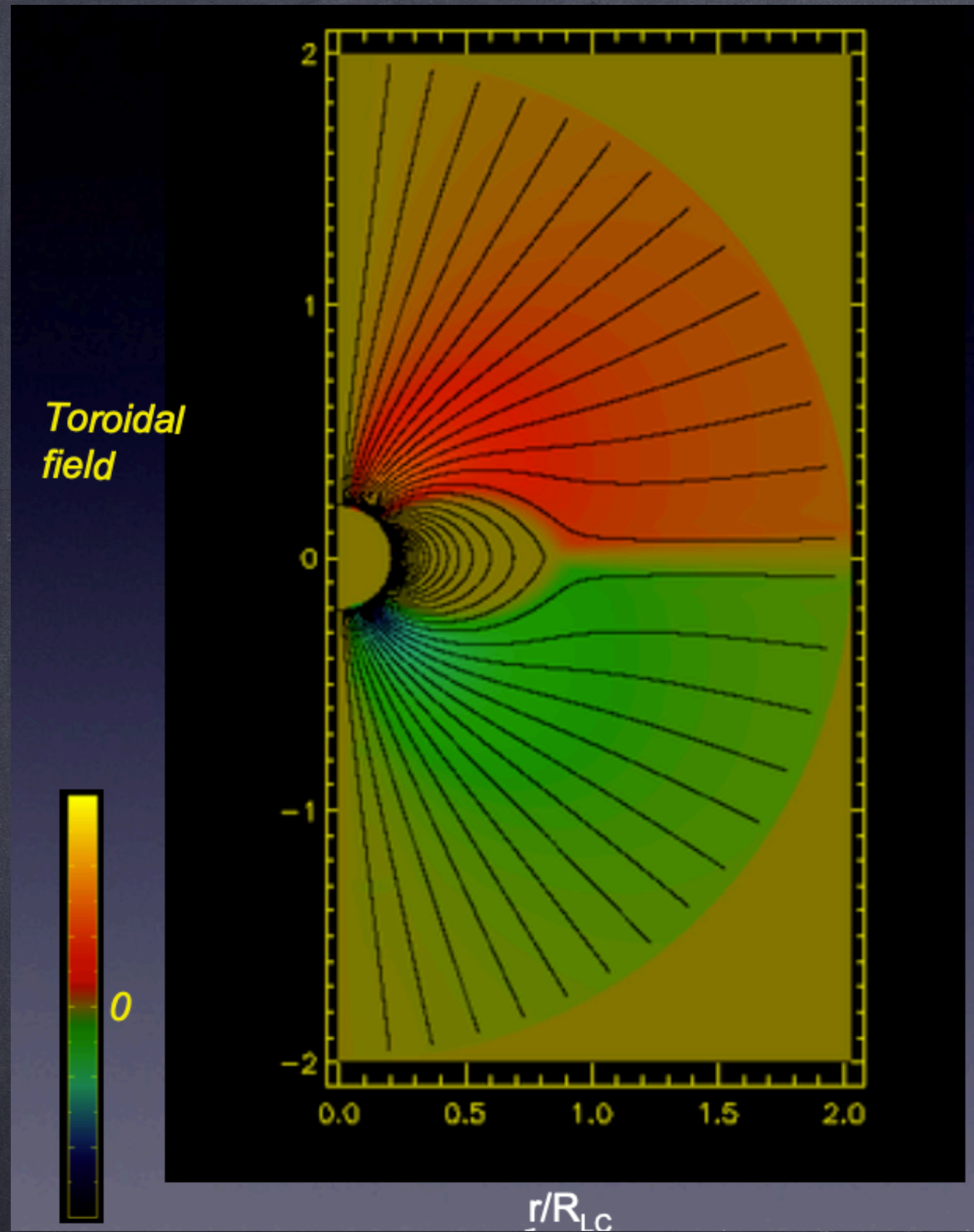
# DIPOLES AND MONOPOLES



Bogovalov, 1999; 2000



# TIME-DEPENDENT FORCE-FREE SOLUTION

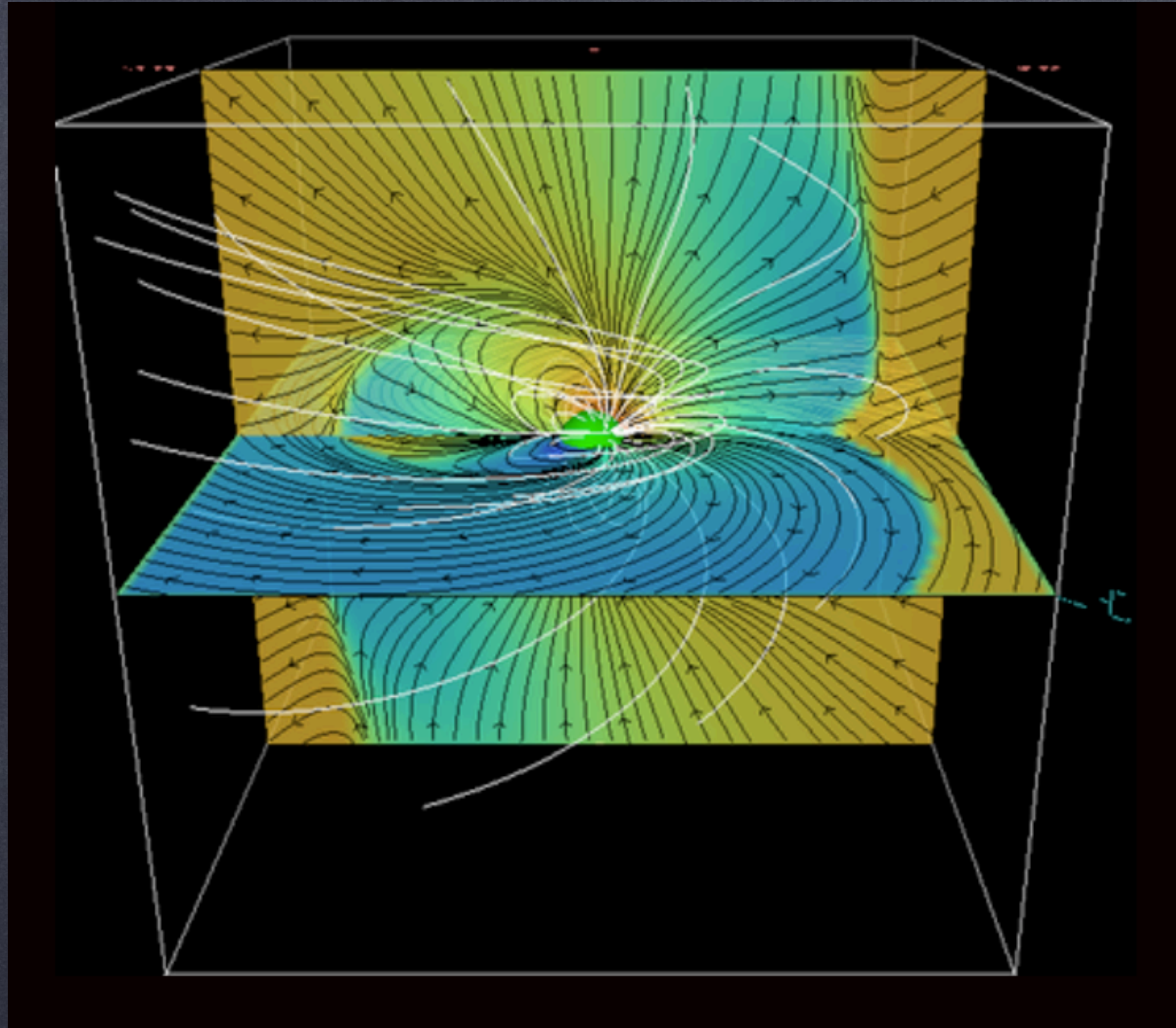


Spitkovsky 2006

$$F(R, \theta) \propto \frac{\sin^2 \theta}{R^2}$$

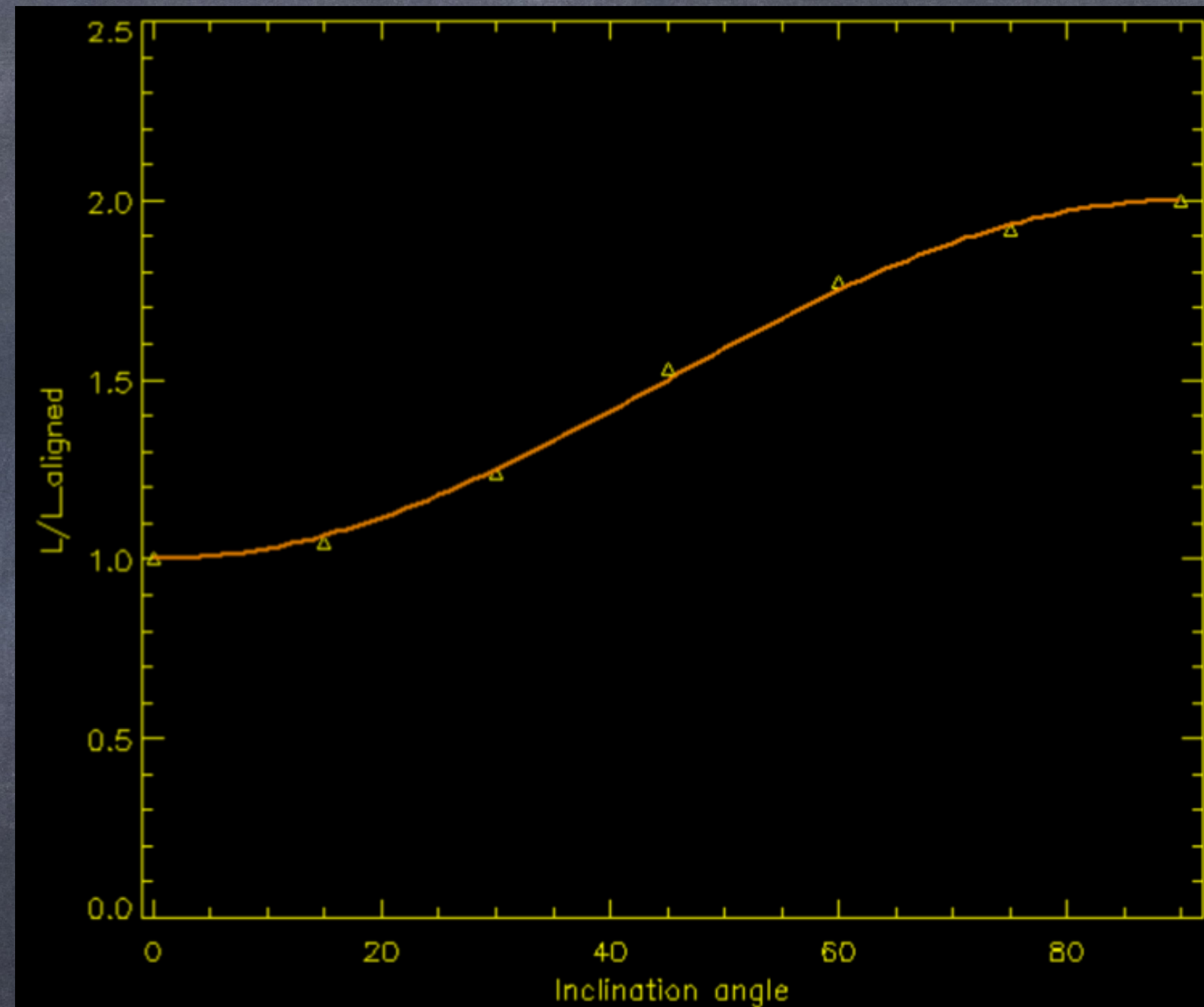


# TIME-DEPENDENT FORCE-FREE OBLIQUE ROTATOR



Spitkovsky 2006

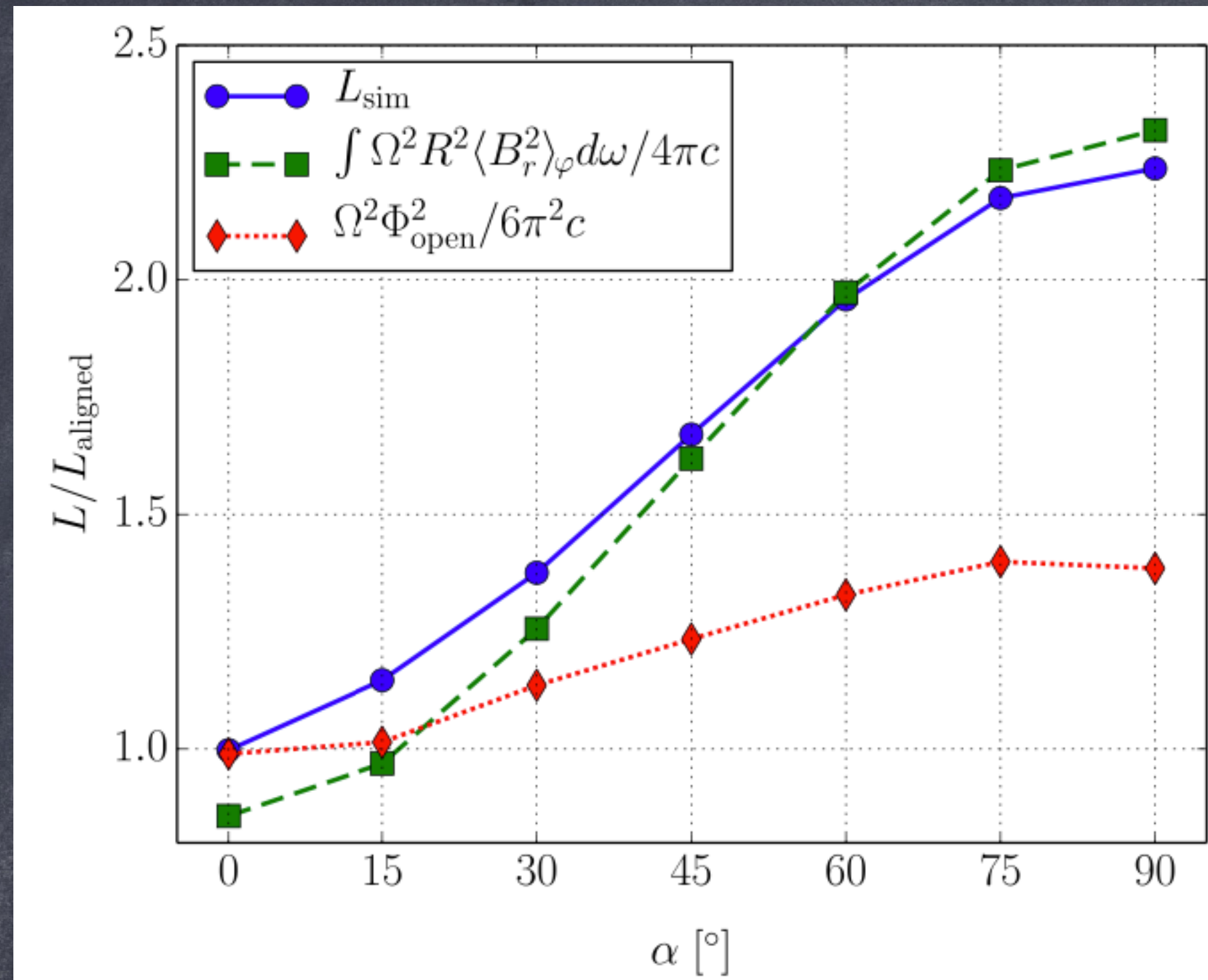
$$\dot{E} = \frac{\mu_0^2 \Omega^4}{c^3} (1 + \sin^2 \theta)$$



$$\left( \dot{E}_{\text{vac}} = \frac{2}{3} \frac{\mu_0^2 \Omega^4}{c^3} \sin^2 \theta \right)$$



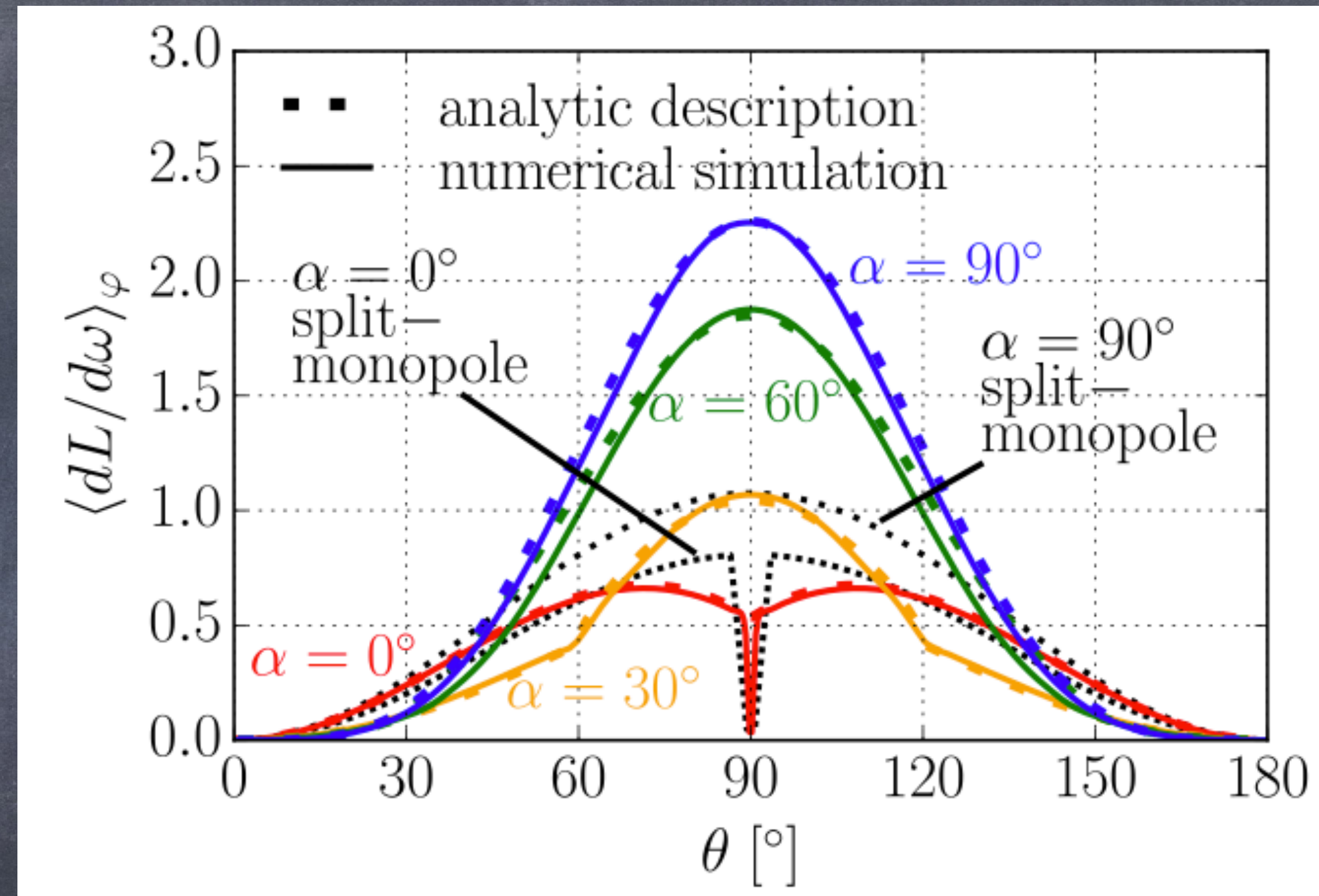
# TIME-DEPENDENT MHD OBLIQUE ROTATOR



Tchekovskoy, Philippov, Spitkovsky 2016

OUTFLOW IS EVEN MORE  
CONCENTRATED IN EQUATORIAL  
PLANE

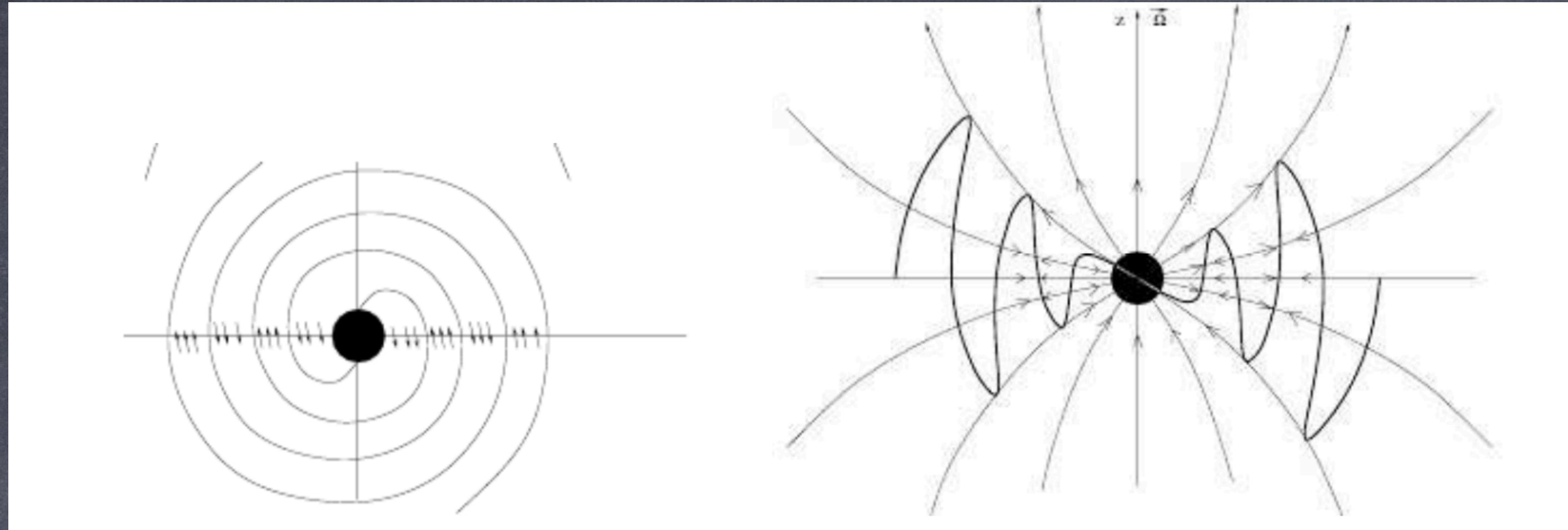
$$S = \frac{\Omega^2}{4\pi c} \langle B_r^2 \rangle_\phi r^2 \sin^2 \theta$$



$$\langle B_r^2 \rangle_\phi = \frac{1}{2} B_0^2 \left( \frac{r}{r_0} \right)^{-4} \sin^2 \theta$$



# THE CURRENT SHEET



$$\underline{e}_R = \sin \theta \cos \phi \underline{e}_1 + \sin \theta \sin \phi \underline{e}_2 + \cos \theta \underline{e}_3$$

$$\vec{\mu} = \mu \left[ \sin \chi \cos \Omega t \underline{e}_1 + \sin \chi \sin \Omega t \underline{e}_2 + \cos \chi \underline{e}_3 \right]$$

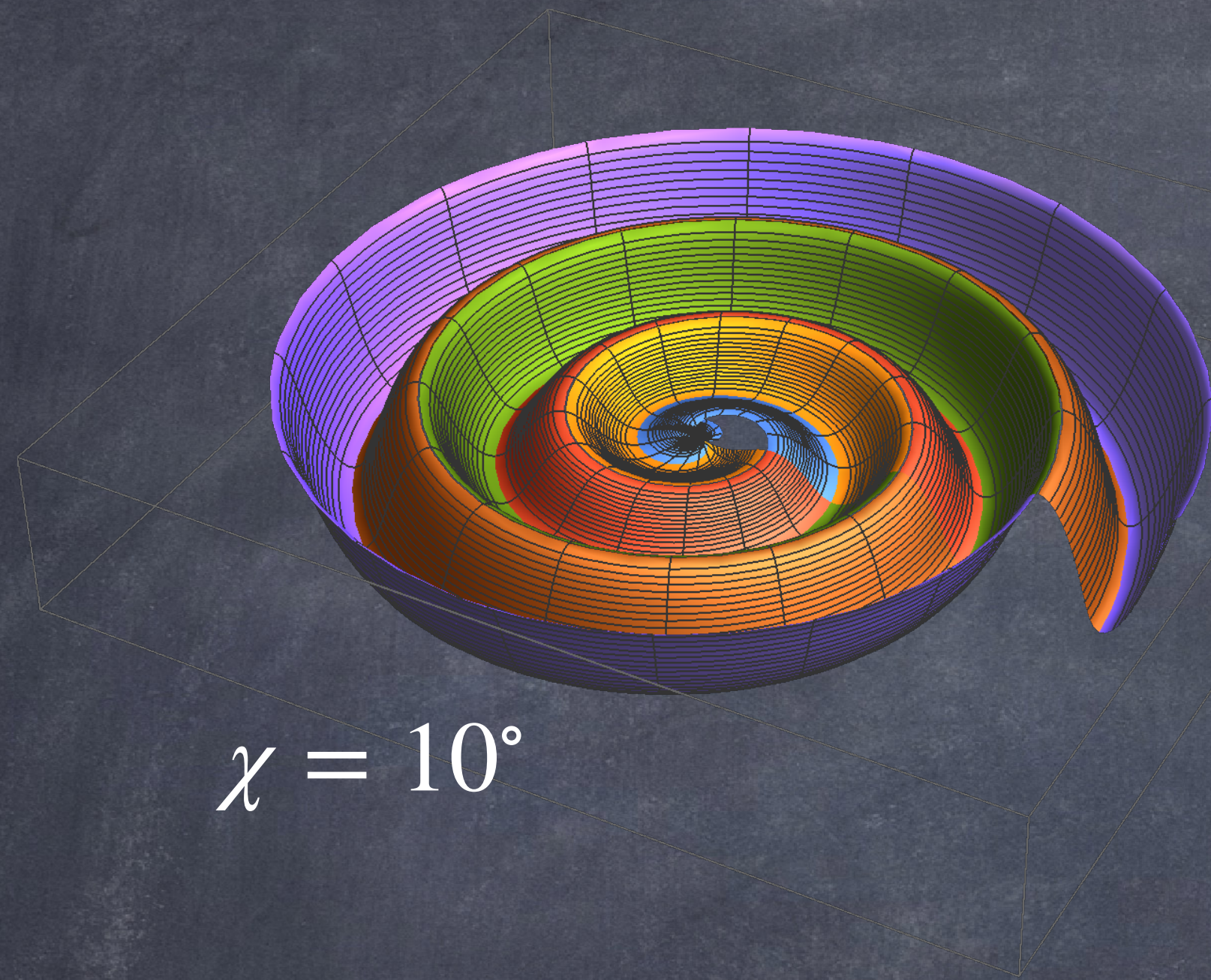
$$\vec{\mu} \cdot \underline{e}_R = \mu \left\{ \sin \theta \sin \chi \cos \phi \cos \Omega t + \sin \theta \sin \chi \sin \phi \sin \Omega t + \cos \theta \cos \chi \right\}$$

$$\vec{\mu} \cdot \underline{e}_R = 0 \quad \longleftrightarrow \quad \Psi(t, \theta, \phi) = \sin \theta \sin \chi \cos(\phi - \Omega t) + \cos \theta \cos \chi = 0$$

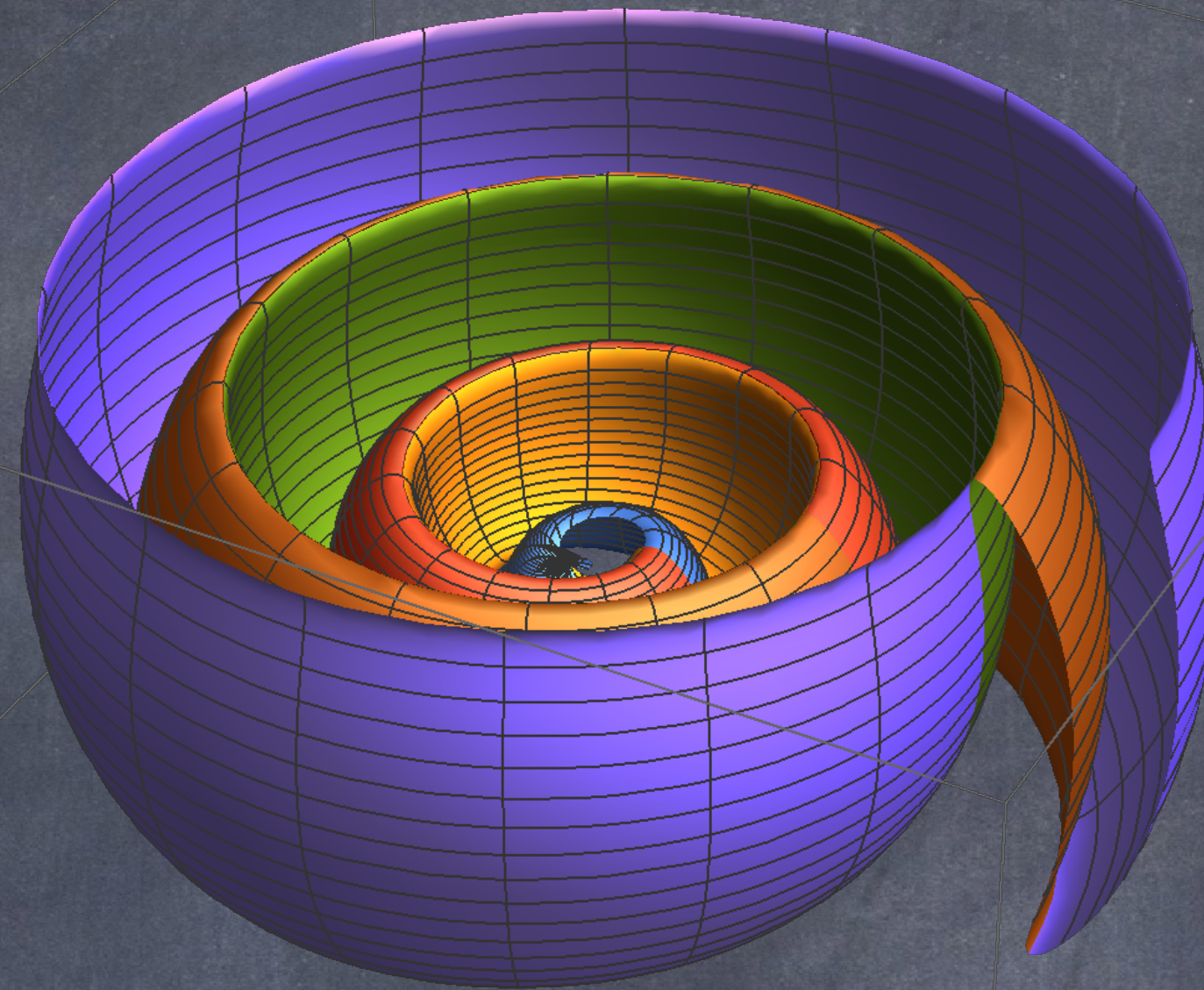
PATTERN PROPAGATES AT SPEED V



# THE CURRENT SHEET



$\chi = 10^\circ$



$\chi = 30^\circ$

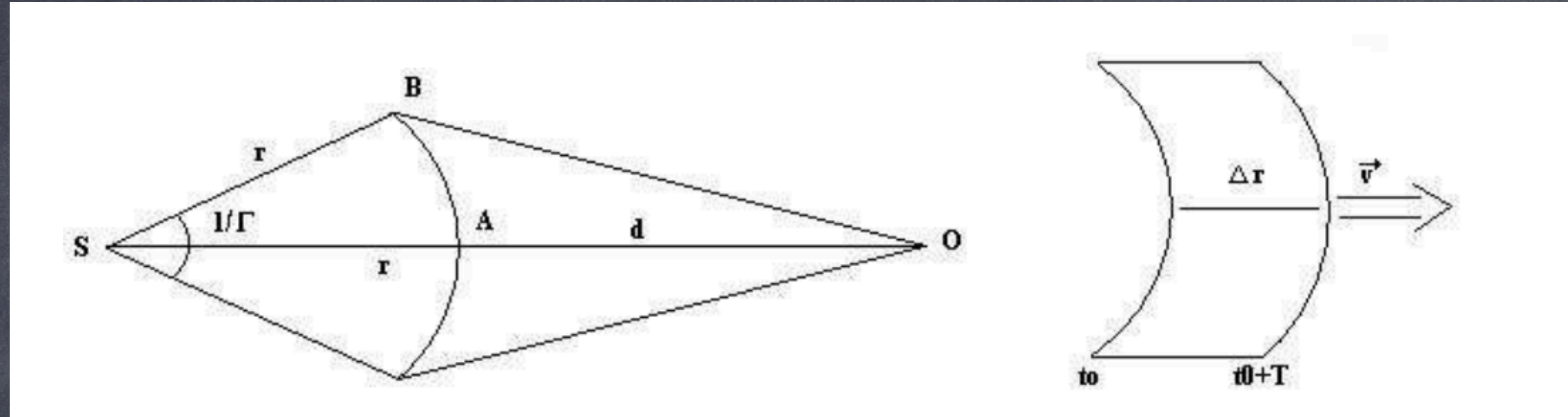
$$\Psi(t, R, \theta, \phi) = \sin \theta \sin \chi \cos \left[ \phi - \Omega \left( t - \frac{R}{V} \right) \right] + \cos \theta \cos \chi = 0$$



$$r_s(\theta, \phi, t) = \frac{v}{\Omega} \left\{ \Omega t - \phi + 2\pi l \pm \arccos \left[ -\frac{\cos \chi \cos \theta}{\sin \chi \sin \theta} \right] \right\}$$



# EMISSION FROM THE SHEET



$$\Delta t_1 \approx \frac{r}{2\Gamma_w^2 c}$$

$$\Delta t_2 = \frac{\Delta R}{2\Gamma_w^2 c}$$

$$\Delta t_1 = t_{BO} - t_{AO} = \frac{\overline{BO} - \overline{AO}}{c} = \frac{1}{c} \left[ \sqrt{(r+d)^2 + r^2 - 2r(r+d) \cos(1/\Gamma_w)} - d \right]$$

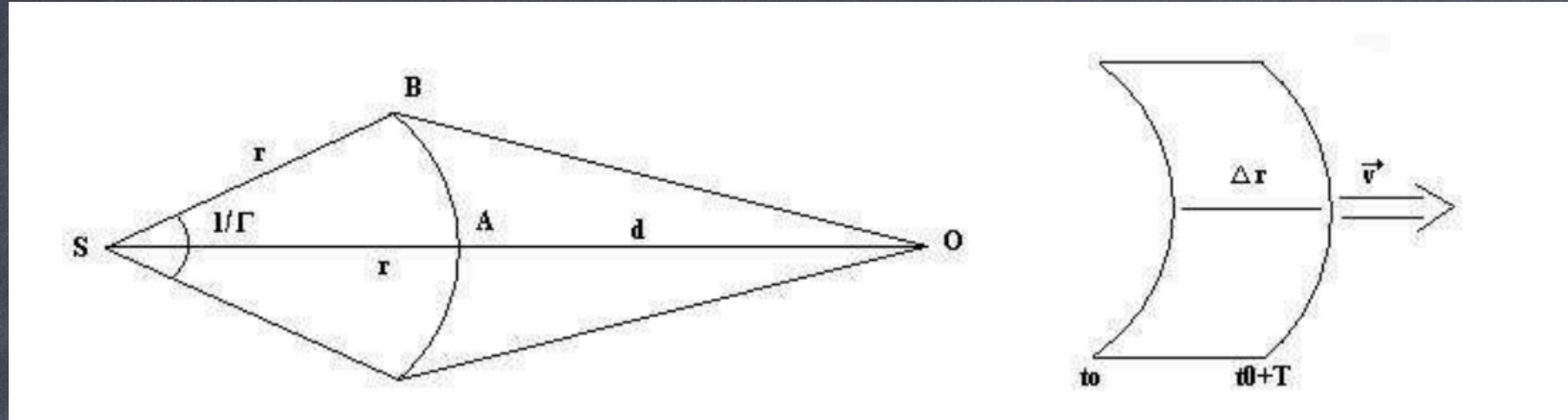
$$d \gg r \quad \Delta t_1 = \frac{d}{c} \left[ \sqrt{\left(1 + \frac{r}{d}\right)^2 + \left(\frac{r}{d}\right)^2 - 2\frac{r}{d} \left(1 + \frac{r}{d}\right) \left(1 - \frac{1}{2\Gamma_w^2}\right)} - 1 \right] = \frac{d}{c} \left[ \sqrt{1 + \frac{r}{d\Gamma_w^2}} - 1 \right] \approx \frac{r}{2\Gamma_w^2 c}$$

$$t_0 \rightarrow t_A = \frac{d}{c} \qquad t_0 + T \rightarrow t_B = \frac{d - vT}{c} + T$$

$$\Delta t_2 = t_B - t_A = \left[ d - \Delta R + \frac{c}{v} \Delta R - d \right] \frac{1}{c} = \Delta R \left( 1 - \frac{v}{c} \right) \approx \frac{\Delta R}{2\Gamma_w^2 c^2}$$



# EMISSION FROM THE SHEET

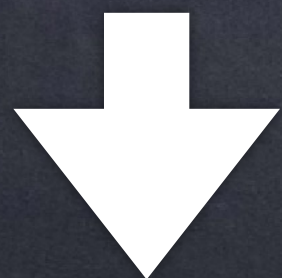


$$\Delta t_1 \approx \frac{r}{2\Gamma_w^2 c}$$

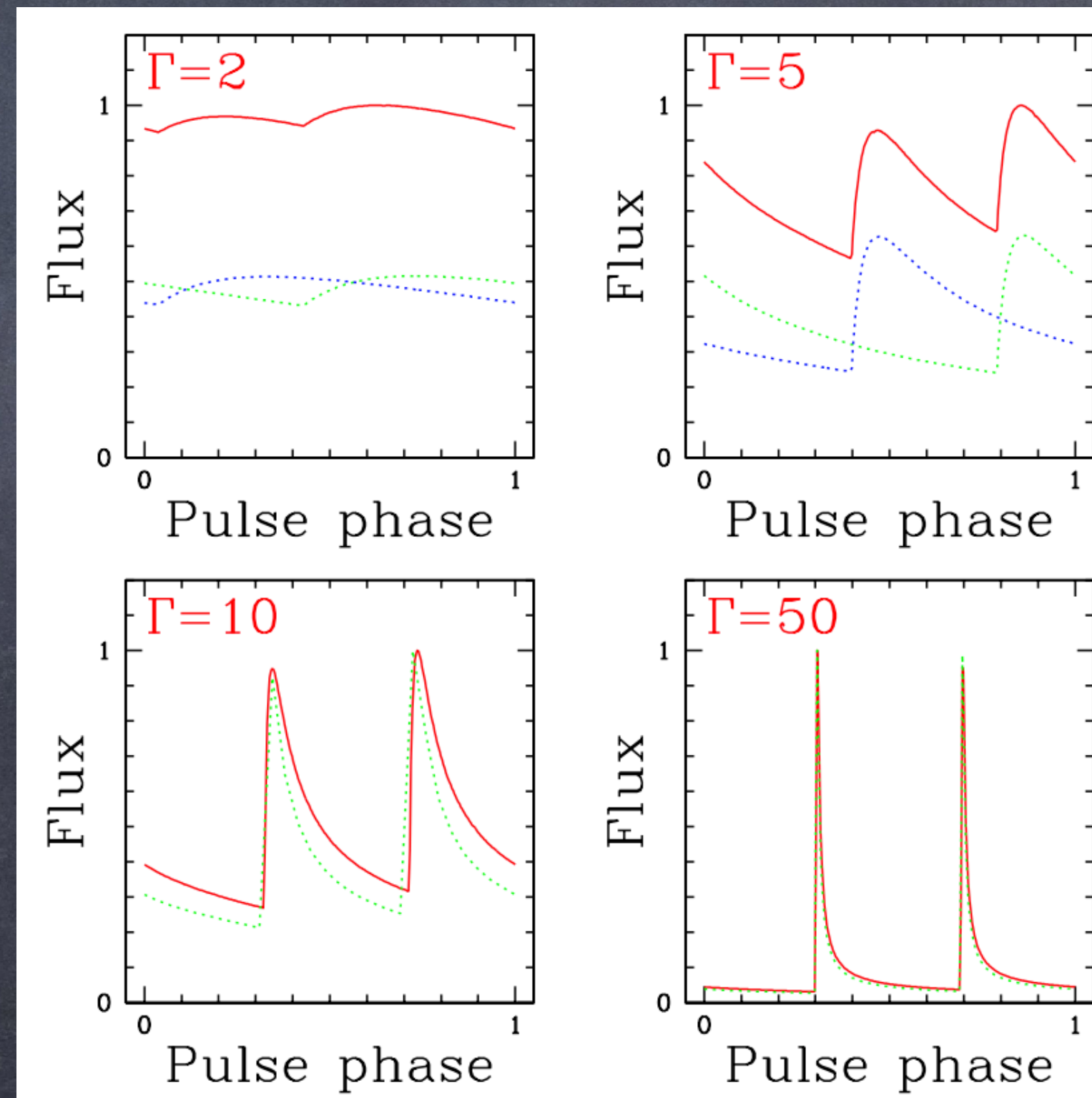
$$\Delta t_2 = \frac{\Delta R}{2\Gamma_w^2 c}$$

PULSED EMISSION IF

$$\Delta t_1, \Delta t_2 < P$$



$$r, \Delta R < \Gamma_w^2 R_L$$

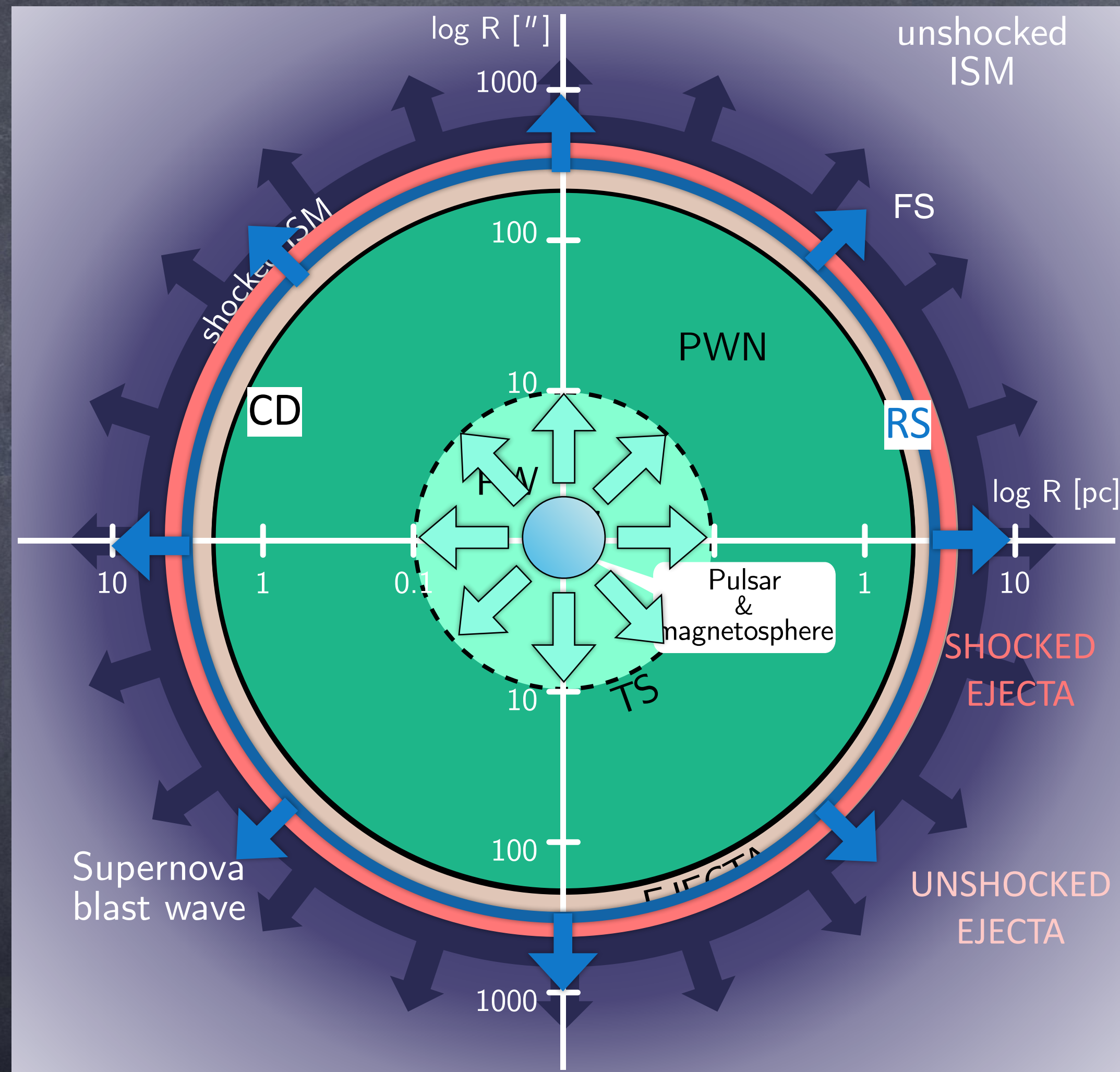




PULSAR WIND NEBULAE



# BASIC PICTURE FOR YOUNG SYSTEMS



Adapted from Kennel & Coroniti 1984



# RELATIVISTIC MHD EQUATIONS

$$\frac{\partial}{\partial x^\mu} (T_m^{\mu\nu} + \sigma^{\mu\nu}) = F^\nu$$

$$\frac{\partial}{\partial x^\mu} (n u^\mu) = 0$$

$$\vec{E} = -\frac{\vec{v} \wedge \vec{B}}{c}$$

$$T_m^{\mu\nu} = w u^\mu u^\nu + P^{\mu\nu}$$

$$P^{\mu\nu} = -p g^{\mu\nu}$$

$$w = e + p$$

$$\sigma^{\mu\nu} = -\frac{1}{4\pi} \left[ F^\mu{}_\lambda F^{\nu\lambda} - \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} \right]$$

## SPHERICAL SYMMETRY

$$\vec{u} = u \vec{e}_R$$

$$\vec{B} = B \vec{e}_\phi$$

$$\vec{E} = -\frac{\vec{u} \wedge \vec{B}}{\gamma} = B \frac{u}{\gamma} \vec{e}_\theta$$



$$\frac{\partial}{\partial x^\mu} (T_m^{\mu\nu} + \sigma^{\mu\nu}) = F^\nu$$

$$\frac{1}{c} \frac{\partial}{\partial t} (T_{00}) + (\vec{\nabla} \cdot \vec{T}_0) = 0$$

$$\frac{1}{c} \frac{\partial}{\partial t} (T_{0r}) + (\vec{\nabla} \cdot \vec{T})_r = 0$$

$$\frac{1}{c} \frac{\partial}{\partial t} (T_{0\vartheta}) + (\vec{\nabla} \cdot \vec{T})_{\vartheta} = 0$$

$$\frac{1}{c} \frac{\partial}{\partial t} (T_{0\varphi}) + (\vec{\nabla} \cdot \vec{T})_{\varphi} = 0$$



$$(\vec{\nabla} \cdot \vec{T}_0) = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 T_{0r})$$

$$(\vec{\nabla} \cdot \vec{T})_r = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 T_{rr}) - \frac{T_{\vartheta\vartheta} + T_{\varphi\varphi}}{r}$$

$$(\vec{\nabla} \cdot \vec{T})_{\vartheta} = 0$$

$$(\vec{\nabla} \cdot \vec{T})_{\varphi} = 0$$



# MATTER ENERGY MOMENTUM TENSOR

$$T_m^{\mu\nu} = w u^\mu u^\nu + P^{\mu\nu}$$

$$P^{\mu\nu} = -p g^{\mu\nu}$$

$$w = e + p$$

$$u^\mu = (\gamma, u \vec{e}_r)$$

## SPHERICAL SYMMETRY

$$T_m^{00} = w \gamma^2 - p$$

$$T_m^{0r} = w \gamma u$$

$$T_m^{rr} = w u^2 + p$$

$$T_m^{\vartheta\vartheta} = T_m^{\varphi\varphi} = p$$

$$T_m^{0\vartheta} = T_m^{0\varphi} = 0$$

$$T_m^{r\vartheta} = T_m^{r\varphi} = T_m^{\varphi\vartheta} = 0$$

$$w = nmc^2 + \epsilon + p = nmc^2 + \frac{p}{\Gamma - 1} + p = nmc^2 + \frac{\Gamma}{\Gamma - 1} p$$



# E.M. FIELDS ENERGY MOMENTUM TENSOR

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{bmatrix} \quad F_{\mu\nu} = g_{\mu\sigma} g_{\nu\rho} F^{\sigma\rho} = \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{bmatrix}$$

$$\sigma^{\mu\nu} = -\frac{1}{4\pi} \left[ F^{\mu}{}_{\lambda} F^{\nu\lambda} - \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} \right]$$

$$\sigma^{00} = \frac{\vec{E}^2 + \vec{B}^2}{8\pi} \quad \text{ENERGY DENSITY}$$

$$\sigma^{0i} = \frac{1}{4\pi} (\vec{E} \wedge \vec{B})_i \quad \text{POYNTING VECTOR} \rightarrow \text{ENERGY FLUX}$$

$$\sigma^{ij} = -\frac{1}{4\pi} \left( E_i E_j + B_i B_j - \frac{1}{2} \delta_{ij} (\vec{E}^2 + \vec{B}^2) \right) \quad \text{STRESS TENSOR} \rightarrow \text{TORQUE}$$



# E.M. FIELDS ENERGY MOMENTUM TENSOR

$$\vec{u} = u\vec{e}_r \quad \vec{B} = B\vec{e}_\varphi \quad \vec{E} = -\frac{\vec{u} \wedge \vec{B}}{\gamma} = B\frac{u}{\gamma}\vec{e}_\vartheta$$

## SPHERICAL SYMMETRY

$$\sigma^{00} = \frac{\vec{E}^2 + \vec{B}^2}{8\pi}$$

$$\sigma^{0i} = \frac{1}{4\pi}(\vec{E} \wedge \vec{B})_i$$

$$\sigma^{ij} = -\frac{1}{4\pi}\left(E_i E_j + B_i B_j - \frac{1}{2}\delta_{ij}(\vec{E}^2 + \vec{B}^2)\right)$$

$$\sigma^{00} = \frac{E_\vartheta^2 + B_\varphi^2}{8\pi}$$

$$\sigma^{0r} = \frac{E_\vartheta B_\varphi}{4\pi}$$

$$\sigma^{rr} = \frac{E_\vartheta^2 + B_\varphi^2}{8\pi}$$

$$\sigma^{\vartheta\vartheta} = -\sigma^{\varphi\varphi} = -\frac{E_\vartheta^2 - B_\varphi^2}{8\pi}$$

$$\sigma^{0\vartheta} = \sigma^{0\varphi} = \sigma^{r\vartheta} = \sigma^{r\varphi} = \sigma^{\vartheta\varphi} = 0$$



# TOTAL ENERGY MOMENTUM TENSOR

$$\vec{u} = u \vec{e}_R$$

$$\vec{B} = B \vec{e}_\phi$$

$$\vec{E} = -\frac{\vec{u} \wedge \vec{B}}{\gamma} = B \frac{u}{\gamma} \vec{e}_\theta$$

$$\sigma^{00} = \frac{B^2}{8\pi} \left( 1 + \frac{u^2}{\gamma^2} \right) \quad \sigma^{0r} = \frac{E_\theta B_\phi}{4\pi} = \frac{B^2}{4\pi} \frac{u}{\gamma}$$

$$\sigma^{rr} = \frac{[E_\theta^2 + B_\phi^2]}{8\pi} = \frac{B^2}{8\pi} \left( 1 + \frac{u^2}{\gamma^2} \right)$$

$$\sigma^{\theta\theta} = -\sigma^{\varphi\varphi} = -\frac{[E_\theta^2 - B_\phi^2]}{8\pi} = \frac{B^2}{8\pi} \left( 1 - \frac{u^2}{\gamma^2} \right)$$

$$T_m^{00} = w \gamma^2 - p$$

$$T_m^{0r} = w \gamma u^r$$

$$T_m^{rr} = w u_r^2 + p$$

$$T_m^{\vartheta\vartheta} = T^{\varphi\varphi} = p$$

$$T^{00} = w \gamma^2 - p + \frac{B^2}{8\pi} \left( 1 + \frac{u^2}{\gamma^2} \right)$$

$$T^{0r} = w \gamma u + \frac{B^2}{4\pi} \frac{u}{\gamma}$$

$$T^{rr} = w u^2 + p + \frac{B^2}{8\pi} \left( 1 + \frac{u^2}{\gamma^2} \right)$$

$$T^{\theta\theta} = p + \frac{B^2}{8\pi} \left( 1 - \frac{u^2}{\gamma^2} \right)$$

$$T^{\varphi\varphi} = p - \frac{B^2}{8\pi} \left( 1 - \frac{u^2}{\gamma^2} \right)$$



# RELATIVISTIC MHD EQUATIONS IN SPHERICAL SYMMETRY

$$\begin{aligned}\frac{1}{c} \frac{\partial}{\partial t} (T^{00}) + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 T_{0r}) &= 0 \\ \frac{1}{c} \frac{\partial}{\partial t} (T^{0r}) + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 T_{rr}) - \frac{T_{\vartheta\vartheta} + T_{\varphi\varphi}}{r} &= 0\end{aligned}$$

## ENERGY CONSERVATION

$$\frac{1}{c} \frac{\partial}{\partial t} \left( w \gamma^2 - p + \frac{B^2}{8\pi} \left( 1 + \frac{u^2}{\gamma^2} \right) \right) + \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( w \gamma u + \frac{B^2}{4\pi} \frac{u}{\gamma} \right) \right] = 0$$

## MOMENTUM CONSERVATION

$$\frac{1}{c} \frac{\partial}{\partial t} \left[ w \gamma u + \frac{B^2}{4\pi} \frac{u}{\gamma} \right] + \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( w u^2 + p + \frac{B^2}{8\pi} \left( 1 + \frac{u^2}{\gamma^2} \right) \right) \right] - \frac{2p}{r} = 0$$

$$\begin{aligned}T^{00} &= w \gamma^2 - p + \frac{B^2}{8\pi} \left( 1 + \frac{u^2}{\gamma^2} \right) \\ T^{0r} &= w \gamma u + \frac{B^2}{4\pi} \frac{u}{\gamma}\end{aligned}$$

$$\begin{aligned}T^{rr} &= w u^2 + p + \frac{B^2}{8\pi} \left( 1 + \frac{u^2}{\gamma^2} \right) \\ T^{\vartheta\vartheta} &= p + \frac{B^2}{8\pi} \left( 1 - \frac{u^2}{\gamma^2} \right) \\ T^{\varphi\varphi} &= p - \frac{B^2}{8\pi} \left( 1 - \frac{u^2}{\gamma^2} \right)\end{aligned}$$

## INDUCTION

$$\frac{1}{c} \frac{\partial B}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{r u B}{\gamma} \right) = 0$$

## CONTINUITY

$$\frac{1}{c} \frac{\partial n}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 n u) = 0$$



# RELATIVISTIC MHD EQUATIONS IN SPHERICAL SYMMETRY

$$\frac{1}{c} \frac{\partial n}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 n u) = 0$$

CONTINUITY

$$\frac{1}{c} \frac{\partial B}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{r u B}{\gamma} \right) = 0$$

INDUCTION

$$\frac{1}{c} \frac{\partial}{\partial t} \left( w \gamma^2 - p + \frac{B^2}{8\pi} \left( 1 + \frac{u^2}{\gamma^2} \right) \right) + \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( w \gamma u + \frac{B^2}{4\pi} \frac{u}{\gamma} \right) \right] = 0$$

ENERGY CONSERVATION

$$\frac{1}{c} \frac{\partial}{\partial t} \left[ w \gamma u + \frac{B^2}{4\pi} \frac{u}{\gamma} \right] + \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( w u^2 + p + \frac{B^2}{8\pi} \left( 1 + \frac{u^2}{\gamma^2} \right) \right) \right] - \frac{2p}{r} = 0$$

MOMENTUM CONSERVATION



# CLASSICAL LIMITS



# CLASSICAL LIMIT

$$\gamma \rightarrow 1$$

$$\frac{1}{c} \frac{\partial}{\partial t} (n\gamma) + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 n u) = 0 \rightarrow \frac{\partial n}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 n v) = 0$$

$$\frac{\partial n}{\partial t} + v \frac{\partial n}{\partial r} + \frac{n}{r^2} \frac{\partial}{\partial r} (r^2 v) = 0 \Leftrightarrow \frac{dn}{dt} + n \vec{\nabla} \cdot \vec{v} = 0$$

$$\frac{1}{c} \frac{\partial}{\partial t} (n\gamma) + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 n u) = 0 \Leftrightarrow \frac{dn}{dt} + n \vec{\nabla} \cdot \vec{v} = 0$$

**CONTINUITY**

$$\frac{1}{c} \frac{\partial B}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{ruB}{\gamma} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{rvB}{c} \right) \Leftrightarrow \frac{1}{c} \frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \wedge \vec{E} = 0$$

$$\frac{1}{c} \frac{\partial B}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{ruB}{\gamma} \right) \Leftrightarrow \frac{1}{c} \frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \wedge \vec{E} = 0$$

**INDUCTION**



$$\gamma \rightarrow 1$$

$$\gamma \approx 1 + \frac{1}{2} \frac{v^2}{c^2}$$

$$\gamma^2 \approx 1 + \frac{v^2}{c^2}$$

# CLASSICAL LIMIT: MOMENTUM

$$\frac{1}{c} \frac{\partial}{\partial t} \left[ w \gamma u + \frac{B^2}{4\pi} \frac{u}{\gamma} \right] + \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( w u^2 + p + \frac{B^2}{8\pi} \left( 1 + \frac{u^2}{\gamma^2} \right) \right) \right] - \frac{2p}{r} = 0$$

$$w \gamma u = \left[ \rho c^2 \gamma + \rho c^2 (\gamma - 1) + (\varepsilon + p) \gamma \right] u \rightarrow (\rho c^2 + \varepsilon + p) \frac{v}{c}$$

$$\frac{1}{c} \frac{\partial}{\partial t} \left[ \rho c v + \frac{B^2}{4\pi} \frac{v}{c} \right] + \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( \rho v^2 + p + \frac{B^2}{8\pi} \right) \right] - \frac{2p}{r} = 0$$

$$\frac{\partial(\rho v)}{\partial t} + \frac{1}{r^2} \frac{\partial(r^2 \rho v^2)}{\partial r} + \frac{\partial p}{\partial r} + \frac{2p}{r} + \frac{2B}{8\pi r} \frac{\partial(Br)}{\partial r} - \frac{2p}{r} = 0$$

$$v \frac{\partial \rho}{\partial t} + \rho \frac{\partial v}{\partial t} + \frac{v}{r^2} \frac{\partial(r^2 \rho v)}{\partial r} + \rho v \frac{\partial v}{\partial r} + \frac{\partial p}{\partial r} - B J_{\vartheta} = 0$$

$$-\frac{1}{r} \frac{\partial}{\partial r} (r B_{\varphi}) \vec{e}_{\vartheta} = 4\pi \vec{J}$$

$$v \frac{\partial \rho}{\partial t} + \frac{v}{r^2} \frac{\partial(r^2 \rho v)}{\partial r} = 0$$

$$\rho \frac{\partial v}{\partial t} + \rho v \frac{\partial v}{\partial r} + \frac{\partial p}{\partial r} - B J_{\vartheta} = 0$$

$$\rho \frac{d\vec{v}}{dt} = -\vec{\nabla} p + \vec{J} \wedge \vec{B}$$

**MOMENTUM  
CONSERVATION**



# CLASSICAL LIMIT: ENERGY

$$\gamma \rightarrow 1$$

$$\gamma \approx 1 + \frac{1}{2} \frac{v^2}{c^2}$$

$$\gamma^2 \approx 1 + \frac{v^2}{c^2}$$

$$\frac{1}{c} \frac{\partial}{\partial t} \left( w \gamma^2 - p + \frac{B^2}{8\pi} \left( 1 + \frac{u^2}{\gamma^2} \right) \right) + \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( w \gamma u + \frac{B^2}{4\pi} \frac{u}{\gamma} \right) \right] = 0$$

$$w \gamma^2 = [\rho c^2 \gamma + \rho c^2 (\gamma - 1) + (\varepsilon + p) \gamma] \gamma \rightarrow \rho c^2 \left( 1 + \frac{v^2}{2c^2} \right) + (\varepsilon + p)$$

$$w \gamma u = [\rho c^2 \gamma + \rho c^2 (\gamma - 1) + (\varepsilon + p) \gamma] u \rightarrow \left[ \rho c^2 \left( 1 + \frac{v^2}{2c^2} \right) + \varepsilon + p \right] \frac{v}{c}$$

$$\frac{1}{c} \frac{\partial}{\partial t} \left\{ \left[ \rho c^2 + \rho c^2 \frac{v^2}{2c^2} \right] + (\varepsilon + p) - p + \frac{B^2}{8\pi} \right\} + \frac{1}{r^2} \frac{\partial}{\partial r} \left\{ r^2 \left[ \left( \rho c^2 + \rho c^2 \frac{v^2}{2c^2} + (\varepsilon + p) \right) \frac{v}{c} \right] + \frac{B^2}{4\pi} \right\} = 0$$

$$\begin{cases} \frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho v) = 0 \\ \left\{ \frac{1}{c} \frac{\partial}{\partial t} \left[ \frac{1}{2} \rho v^2 + \varepsilon + \frac{B^2}{8\pi} \right] + \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left[ \frac{1}{2} \rho v^2 + \varepsilon + p \right] \frac{v}{c} + \frac{B^2}{4\pi} \frac{v}{c} \right] \right\} = 0 \end{cases}$$



# $\gamma \rightarrow 1$ CLASSICAL LIMIT: ENERGY

$$\gamma \approx 1 + \frac{1}{2} \frac{v^2}{c^2}$$

$$\gamma^2 \approx 1 + \frac{v^2}{c^2}$$

$$\frac{1}{c} \frac{\partial}{\partial t} \left( w \gamma^2 - p + \frac{B^2}{8\pi} \left( 1 + \frac{u^2}{\gamma^2} \right) \right) + \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( w \gamma u + \frac{B^2}{4\pi} \frac{u}{\gamma} \right) \right] = 0$$

$$\begin{cases} \frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho v) = 0 \\ \frac{1}{c} \frac{\partial}{\partial t} \left[ \frac{1}{2} \rho v^2 + \varepsilon + \frac{B^2}{8\pi} \right] + \frac{1}{r^2} \frac{\partial}{\partial r} \left\{ r^2 \left[ \frac{1}{2} \rho v^2 + \varepsilon + p \right] \frac{v}{c} + \frac{B^2}{4\pi} \frac{v}{c} \right\} = 0 \end{cases}$$

$$\begin{aligned} \frac{\partial}{\partial t} \left( \frac{B^2}{8\pi} \right) + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 v \frac{B^2}{4\pi} \right) &= \frac{1}{4\pi} \vec{B} \cdot \frac{\partial \vec{B}}{\partial t} + \frac{c}{4\pi} \vec{\nabla} \cdot (\vec{B} \wedge \vec{E}) = \\ &= -\frac{c}{4\pi} \vec{B} \cdot (\vec{\nabla} \wedge \vec{E}) + \frac{c}{4\pi} \vec{\nabla} \cdot (\vec{B} \wedge \vec{E}) = -\frac{c}{4\pi} \vec{E} \cdot (\vec{\nabla} \wedge \vec{B}) = -c \vec{E} \cdot \vec{J} \end{aligned}$$

$$\frac{\partial}{\partial t} \left[ \frac{1}{2} \rho v^2 + \varepsilon \right] + \vec{\nabla} \cdot \left[ \left( \frac{1}{2} \rho v^2 + \varepsilon + p \right) \vec{v} \right] = -c \vec{E} \cdot \vec{J}$$

**ENERGY  
CONSERVATION**



# RELATIVISTIC MHD EQUATIONS IN SPHERICAL SYMMETRY: CLASSICAL LIMIT

$$\frac{1}{c} \frac{\partial n}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 n u) = 0 \quad \text{CONTINUITY}$$

$$\gamma \rightarrow 1$$

$$\gamma \rightarrow 1 + \frac{1}{2} \frac{v^2}{c^2}$$

$$\frac{\partial n}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 n v) = 0$$

$$\frac{1}{c} \frac{\partial B}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{r u B}{\gamma} \right) = 0 \quad \text{INDUCTION}$$

$$\gamma^2 \rightarrow 1 + \frac{v^2}{c^2}$$

$$\frac{1}{c} \frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \wedge \vec{E} = 0$$

## ENERGY CONSERVATION

$$\frac{1}{c} \frac{\partial}{\partial t} \left( w \gamma^2 - p + \frac{B^2}{8\pi} \left( 1 + \frac{u^2}{\gamma^2} \right) \right) + \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( w \gamma u + \frac{B^2}{4\pi} \frac{u}{\gamma} \right) \right] = 0$$

$$\frac{\partial}{\partial t} \left[ \frac{\rho v^2}{2} + \epsilon \right] + \vec{\nabla} \cdot \left[ \left( \frac{\rho v^2}{2} + \epsilon + p \right) \vec{v} \right] = -c \vec{E} \cdot \vec{J}$$

## MOMENTUM CONSERVATION

$$\frac{1}{c} \frac{\partial}{\partial t} \left[ w \gamma u + \frac{B^2}{4\pi} \frac{u}{\gamma} \right] + \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( w u^2 + p + \frac{B^2}{8\pi} \left( 1 + \frac{u^2}{\gamma^2} \right) \right) \right] - \frac{2p}{r} = 0$$

$$\rho \left( \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v} \right) = -\vec{\nabla} p + \vec{J} \wedge \vec{B}$$



# STEADY STATE RELATIVISTIC MHD EQUATIONS IN SPHERICAL SYMMETRY

$$\frac{1}{c} \frac{\partial n}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 n u) = 0 \quad \text{CONTINUITY}$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 n u) = 0$$

$$\frac{1}{c} \frac{\partial B}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{r u B}{\gamma} \right) = 0 \quad \text{INDUCTION}$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left( \frac{r u B}{\gamma} \right) = 0$$

## ENERGY CONSERVATION

$$\frac{1}{c} \frac{\partial}{\partial t} \left( w \gamma^2 - p + \frac{B^2}{8\pi} \left( 1 + \frac{u^2}{\gamma^2} \right) \right) + \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( w \gamma u + \frac{B^2}{4\pi} \frac{u}{\gamma} \right) \right] = 0$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( w \gamma u + \frac{B^2}{4\pi} \frac{u}{\gamma} \right) \right] = 0$$

## MOMENTUM CONSERVATION

$$\frac{1}{c} \frac{\partial}{\partial t} \left[ w \gamma u + \frac{B^2}{4\pi} \frac{u}{\gamma} \right] + \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( w u^2 + p + \frac{B^2}{8\pi} \left( 1 + \frac{u^2}{\gamma^2} \right) \right) \right] - \frac{2p}{r} = 0$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( w u^2 + p + \frac{B^2}{8\pi} \left( 1 + \frac{u^2}{\gamma^2} \right) \right) \right] - \frac{2p}{r} = 0$$



# STEADY STATE RELATIVISTIC MHD EQUATIONS IN SPHERICAL SYMMETRY

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 n u) = 0 \quad \text{CONTINUITY}$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left( \frac{r u B}{\gamma} \right) = 0 \quad \text{INDUCTION}$$

$$\gamma \rightarrow 1$$

$$\gamma \rightarrow 1 + \frac{1}{2} \frac{v^2}{c^2}$$

$$\gamma^2 \rightarrow 1 + \frac{v^2}{c^2}$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 n v) = 0$$

$$\vec{\nabla} \wedge \vec{E} = 0$$

## MOMENTUM CONSERVATION

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( w \gamma u + \frac{B^2}{4\pi} \frac{u}{\gamma} \right) \right] = 0$$

$$\rho (\vec{v} \cdot \vec{\nabla}) \vec{v} + \vec{\nabla} p - \vec{J} \wedge \vec{B} = 0$$

## ENERGY CONSERVATION

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( w u^2 + p + \frac{B^2}{8\pi} \left( 1 + \frac{u^2}{\gamma^2} \right) \right) \right] - \frac{2p}{r} = 0$$

$$\vec{\nabla} \cdot \left[ \left( \frac{\rho v^2}{2} + \epsilon + p \right) \vec{v} \right] + c \vec{E} \cdot \vec{J} = 0$$



# NEBULAR DYNAMICS 1

$$(1) \quad \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 n u) = 0 \quad (3) \quad \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( w u^2 + p + \frac{B^2}{8\pi} \left( 1 + \frac{u^2}{\gamma^2} \right) \right) \right] - \frac{2p}{r} = 0$$

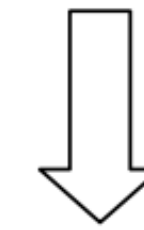
$$(2) \quad \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{r u B}{\gamma} \right) = 0 \quad (4) \quad \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( w \gamma u + \frac{B^2}{4\pi} \frac{u}{\gamma} \right) \right] = 0$$

$$(3) + (2) \rightarrow \frac{\partial}{\partial r} (\mu n u^2 r^2) + \frac{\partial}{\partial r} \left( \frac{B^2 r^2}{8\pi} \right) + r^2 \frac{\partial p}{\partial r} = 0$$



$$(3b) \quad \frac{\partial}{\partial r} (\mu n u^2 r^2) - \frac{\gamma}{u} \frac{\partial}{\partial r} (\mu n \gamma u r^2) + r^2 \frac{\partial p}{\partial r} = 0$$

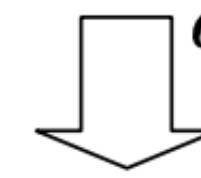
$$(4) + (2) \rightarrow \frac{\partial}{\partial r} (\mu n \gamma u r^2) + \frac{B u r}{4\pi \gamma} \frac{\partial}{\partial r} (B r) = 0$$



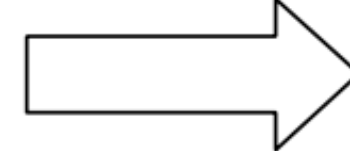
$$\frac{B r}{4\pi} \frac{\partial}{\partial r} (B r) = -\frac{\gamma}{u} \frac{\partial}{\partial r} (\mu n \gamma u r^2)$$



$$(3c) \quad u \frac{\partial}{\partial r} (\mu n u r^2) + \mu n u r^2 \frac{\partial u}{\partial r} - \frac{\gamma}{u} \left[ \gamma \frac{\partial}{\partial r} (\mu n u r^2) + \mu n u r^2 \frac{\partial \gamma}{\partial r} \right] + r^2 \frac{\partial p}{\partial r} = 0$$



$$\gamma^2 = 1 + u^2 \Rightarrow \gamma \frac{\partial \gamma}{\partial r} = u \frac{\partial u}{\partial r}$$



$$(3d) \quad \frac{u^2 - \gamma^2}{u} \frac{\partial}{\partial r} (\mu n u r^2) + r^2 \frac{\partial p}{\partial r} = 0$$



# NEBULAR DYNAMICS 2

$$(3d) \quad \frac{u^2 - \gamma^2}{u} \frac{\partial}{\partial r} (\mu n u r^2) + r^2 \frac{\partial p}{\partial r} = 0 \quad \Rightarrow \quad (3e) \quad \frac{\partial}{\partial r} (\mu n u r^2) - r^2 u \frac{\partial p}{\partial r} = 0$$

$$(3e) + (1) \rightarrow (3f) \quad \frac{\partial \mu}{\partial r} = \frac{1}{n} \frac{\partial p}{\partial r}$$

RECALLING  $\mu = mc^2 + \frac{\Gamma}{\Gamma - 1} \frac{p}{n}$   $(3e) \rightarrow (3g) \quad \frac{\Gamma}{\Gamma - 1} \left( \frac{1}{n} \frac{\partial p}{\partial r} - \frac{p}{n^2} \frac{\partial n}{\partial r} \right) = \frac{1}{n} \frac{\partial p}{\partial r}$

$$\frac{\partial}{\partial r} \left( \frac{p}{n^\Gamma} \right) = 0$$

$$(4) \quad \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( \mu n \gamma u_r + \frac{B^2}{4\pi} \frac{u_r}{\gamma} \right) \right] = 0 \quad (4a) \quad \frac{\partial}{\partial r} \left[ n u r^2 \left( \mu \gamma + \frac{B^2}{4\pi n \gamma} \right) \right] = 0$$

$$\left( \mu \gamma + \frac{B^2}{4\pi n \gamma} \right) = \cos t$$



# SUMMARIZING....

$$\frac{\partial}{\partial r} (r^2 n u) = 0 \quad \frac{\partial}{\partial r} \left( \mu \gamma + \frac{B^2}{4\pi n \gamma} \right) = 0 \quad \frac{\partial}{\partial r} \left( \frac{p}{n^\Gamma} \right) = 0 \quad \frac{\partial}{\partial r} \left( \frac{r u B}{\gamma} \right) = 0$$

$$\mu = mc^2 + \frac{\Gamma}{\Gamma - 1} \frac{p}{n mc^2} \Rightarrow \mu \rightarrow mc^2 \quad \text{FOR A COLD WIND}$$

$$\Rightarrow \gamma + \frac{B^2}{4\pi n mc^2 \gamma} = \text{const} \quad \text{ENERGY PER PARTICLE ALONG A STREAMLINE}$$

$$F(r, \theta) = n mc^2 \gamma^2 (1 + \sigma) = n mc^2 \gamma \left[ \gamma_0 (1 + \sigma_0) \right] \approx n mc^2 \gamma \gamma_0 \sigma_0 \propto \sin^2 \theta$$

**ANISOTROPIC PULSAR WIND!!!!!!**



# NEBULAR DYNAMICS 3

$$n u r^2 = n_2 u_2 r_s^2$$

$$\frac{u B r}{\gamma} = \frac{u_2 B_2 r_s}{\gamma_2}$$

$$\frac{p}{n^\Gamma} = \frac{p_2}{n_2^\Gamma}$$

$$\gamma \left( m c^2 + \frac{\Gamma}{\Gamma - 1} \frac{p}{n} + \frac{B^2}{4 \pi n \gamma^2} \right) = \cos t$$

$$\begin{cases} z = \frac{r}{r_s} \\ v = \frac{u}{u_2} \end{cases}$$

$$n = \frac{n_2}{v z^2}$$

$$p = \frac{p_2}{(v z^2)^\Gamma}$$

$$B = B_2 \frac{\gamma}{\gamma_2} \frac{1}{v z}$$

$$\gamma \left( m c^2 + \frac{\Gamma}{\Gamma - 1} \frac{p_2}{n_2} \frac{1}{(v z^2)^{\Gamma-1}} + \frac{B_2^2}{4 \pi n_2 \gamma_2^2} \frac{v z^2}{v^2 z^2} \right) = \cos t$$

$$\gamma \frac{B_2^2}{4 \pi n_2 \gamma_2^2} \left( \frac{4 \pi n_2 \gamma_2^2 m c^2}{B_2^2} + \frac{\Gamma}{\Gamma - 1} \frac{4 \pi p_2 \gamma_2^2}{B_2^2} \frac{1}{(v z^2)^{\Gamma-1}} + \frac{1}{v} \right) = \cos t$$

$$\begin{cases} \delta = \frac{4 \pi n_2 m c^2 \gamma_2^2}{B_2^2} \\ \Delta = \frac{\Gamma}{\Gamma - 1} \frac{4 \pi p_2 \gamma_2^2}{B_2^2} \end{cases}$$

$$\gamma \left( \delta + \frac{\Delta}{(v z^2)^{\Gamma-1}} + \frac{1}{v} \right) = \gamma_2 (\delta + \Delta + 1)$$



# TERMINAL VELOCITY 1

$$\lim_{z \rightarrow \infty} v(z) = v_{\infty}$$

$$\gamma \left( \delta + \frac{\Delta}{(vz^2)^{\Gamma-1}} + \frac{1}{v} \right) = \gamma_2 (\delta + \Delta + 1)$$

$$\begin{cases} \delta = \frac{4\pi n_2 mc^2 \gamma_2^2}{B_2^2} \\ \Delta = \frac{\Gamma}{\Gamma-1} \frac{4\pi p_2 \gamma_2^2}{B_2^2} \end{cases} \quad \Gamma = \frac{4}{3}$$

LET'S USE THE JUMP CONDITIONS

$$n_2 = \frac{n_1 u_1}{u_2} \quad B_2 = B_1 \frac{\gamma_2}{u_2} \quad \frac{p_2}{n_1 mc^2 \gamma_1^2} = \frac{1}{4u_2 \gamma_2} \left[ 1 + \sigma \left( 1 - \frac{\gamma_2}{u_2} \right) - \frac{\gamma_2}{\gamma_1} \right]$$

$$\delta = \frac{4\pi n_1 mc^2 \gamma_1^2}{B_1^2} \frac{B_1^2}{B_2^2} \frac{n_2}{n_1} \frac{\gamma_2^2}{\gamma_1^2} = \frac{1}{\sigma} \frac{u_2^2}{\gamma_2^2} \frac{n_1 u_1}{n_1 u_2} \frac{\gamma_2^2}{\gamma_1^2} = \frac{1}{\sigma} \frac{u_2}{\gamma_1}$$

ALWAYS NEGLIGIBLE IF  $\gamma_1 > 1/\sigma$

$$\begin{cases} \sigma \rightarrow \infty \Rightarrow \delta \rightarrow \frac{1}{\gamma_1 \sqrt{\sigma}} \\ \sigma \rightarrow 0 \Rightarrow \delta \rightarrow \frac{1}{2\sqrt{2}\gamma_1 \sigma} \end{cases}$$

$$\Delta = \frac{4\pi n_1 mc^2 \gamma_1^2}{B_1^2} \frac{4p_2}{n_1 mc^2} \frac{B_1^2}{B_2^2} \frac{\gamma_2^2}{\gamma_1^2} = \frac{1}{\sigma} \frac{u_2^2}{\gamma_2^2} \frac{\gamma_2^2}{\gamma_1^2} \frac{4p_2}{n_1 mc^2} = \frac{u_2}{\sigma \gamma_2} \left[ 1 + \sigma \left( 1 - \frac{\gamma_2}{u_2} \right) - \frac{\gamma_2}{\gamma_1} \right]$$



# TERMINAL VELOCITY 2

$$\gamma \left( \delta + \frac{\Delta}{(vz^2)^{\Gamma-1}} + \frac{1}{v} \right) = \gamma_2 (\delta + \Delta + 1) \quad \begin{matrix} z \rightarrow \infty \\ \Rightarrow \\ \delta \approx 0 \end{matrix} \quad \frac{\gamma_\infty}{v_\infty} = \gamma_2 (\Delta + 1)$$

$$\gamma^2 = 1 + u^2 \quad \Rightarrow \quad (1 + u_2^2 v_\infty^2) = \gamma_2^2 v_\infty^2 (\Delta + 1)^2 \quad \Rightarrow \quad v_\infty = \frac{1}{\sqrt{\gamma_2^2 (\Delta + 1)^2 - u_2^2}}$$

$$\Delta = \frac{u_2}{\sigma \gamma_2} (1 + \sigma) - 1 - \frac{u_2}{\sigma \gamma_1}$$

$\gamma_1 > 1/\sigma$

$$v_\infty = \frac{\sigma}{u_2 \sqrt{(1 + \sigma)^2 - \sigma^2}}$$

$$u_\infty = \frac{\sigma}{\sqrt{(1 + 2\sigma)}}$$

$$\beta_\infty = \frac{u_\infty}{\gamma_\infty} = \frac{\sigma}{1 + \sigma}$$

$$\sigma \rightarrow \infty \quad \Rightarrow \quad \beta_\infty \rightarrow 1$$

$$\sigma \rightarrow 0 \quad \Rightarrow \quad \beta_\infty \rightarrow \sigma$$



# RELATIVISTIC SHOCK JUMP CONDITIONS

$$\begin{aligned}
 (1) \quad \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 n u) &= 0 & (3) \quad \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( w u^2 + p + \frac{B^2}{8\pi} \left( 1 + \frac{u^2}{\gamma^2} \right) \right) \right] - \frac{2p}{r} &= 0 \\
 (2) \quad \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{ruB}{\gamma} \right) &= 0 & (4) \quad \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( w \gamma u + \frac{B^2}{4\pi} \frac{u}{\gamma} \right) \right] &= 0
 \end{aligned}$$

EXTRACT  $r$  AND INTEGRATE ACROSS INFINITELY  
THIN DISCONTINUITY

$$n_1 u_1 = n_2 u_2 \qquad \frac{u_1 B_1}{\gamma_1} = \frac{u_2 B_2}{\gamma_2}$$

$$\mu = \frac{w}{n}$$

$$\mu_1 n_1 u_1^2 + p_1 + \frac{B_1^2}{8\pi} \left( 1 + \frac{u_1^2}{\gamma_1^2} \right) = \mu_2 n_2 u_2^2 + p_2 + \frac{B_2^2}{8\pi} \left( 1 + \frac{u_2^2}{\gamma_2^2} \right)$$

$$\mu_1 n_1 u_1 \gamma_1 + \frac{B_1^2}{4\pi} \frac{u_1}{\gamma_1} = \mu_2 n_2 u_2 \gamma_2 + \frac{B_2^2}{4\pi} \frac{u_2}{\gamma_2}$$



# RELATIVISTIC SHOCK 1

$$\begin{aligned}
 (1) \quad n_1 u_1 &= n_2 u_2 & (2) \quad \frac{u_1 B_1}{\gamma_1} &= \frac{u_2 B_2}{\gamma_2} \\
 (3) \quad \mu_1 n_1 u_1^2 + p_1 + \frac{B_1^2}{8\pi} \left( 1 + \frac{u_1^2}{\gamma_1^2} \right) &= \mu_2 n_2 u_2^2 + p_2 + \frac{B_2^2}{8\pi} \left( 1 + \frac{u_2^2}{\gamma_2^2} \right) \\
 (4) \quad \mu_1 n_1 u_1 \gamma_1 + \frac{B_1^2}{4\pi} \frac{u_1}{\gamma_1} &= \mu_2 n_2 u_2 \gamma_2 + \frac{B_2^2}{4\pi} \frac{u_2}{\gamma_2}
 \end{aligned}$$

$$\begin{aligned}
 (1) \Rightarrow (3) \rightarrow (3a) \quad \mu_1 u_1 + \frac{p_1}{n_1 u_1} + \frac{B_1^2}{8\pi n_1 u_1} &= \mu_2 u_2 + \frac{p_2}{n_1 u_1} + \frac{B_2^2}{8\pi n_1 u_1} \\
 (1) \Rightarrow (4) \rightarrow (4a) \quad \mu_1 \gamma_1 + \frac{B_1^2}{4\pi n_1 u_1} \frac{u_1}{\gamma_1} &= \mu_2 \gamma_2 + \frac{B_2^2}{4\pi n_1 u_1} \frac{u_2}{\gamma_2}
 \end{aligned}$$

$$(5) \quad Y = \frac{B_2}{B_1} = \frac{u_1 \gamma_2}{u_2 \gamma_1}$$

$$(5) + (2) \Rightarrow \begin{cases} (3a) \rightarrow (3b) \quad \mu_1 u_1 + \frac{p_1}{n_1 u_1} + \frac{B_1^2}{8\pi n_1 u_1} = \mu_2 u_2 + \frac{p_2}{n_2 u_2} + \frac{Y^2 B_1^2}{8\pi n_1 u_1} \\ (4a) \rightarrow (4b) \quad \mu_2 = \frac{1}{\gamma_2} \left[ \mu_1 \gamma_1 + \frac{B_1^2}{4\pi n_1 \gamma_1} - Y \frac{B_1^2}{4\pi n_1 \gamma_1} \right] \end{cases}$$



# RELATIVISTIC SHOCK 2

$$(3b) \mu_1 u_1 + \frac{p_1}{n_1 u_1} + \frac{B_1^2}{8\pi n_1 u_1} = \mu_2 u_2 + \frac{p_2}{n_1 u_1} + \frac{Y^2 B_1^2}{8\pi n_1 u_1}$$

$$(4b) \mu_2 = \frac{1}{\gamma_2} \left[ \mu_1 \gamma_1 + \frac{B_1^2}{4\pi n_1 \gamma_1} - Y \frac{B_1^2}{4\pi n_1 \gamma_1} \right]$$

$\Gamma =$  ADIABATIC INDEX

$$\mu_2 = \frac{w_2}{n_2} = mc^2 + \varepsilon_2 + p_2 \quad \varepsilon_2 = \frac{p_2}{\Gamma_2 - 1} \Rightarrow \mu_2 = mc^2 + \frac{p_2}{n_2} \frac{\Gamma_2}{\Gamma_2 - 1}$$

$$\frac{p_2}{n_2} = \frac{\Gamma_2 - 1}{\Gamma_2} (\mu_2 - mc^2) \Rightarrow (6) \frac{p_2}{n_1 u_1} = \frac{1}{u_2} \frac{\Gamma_2 - 1}{\Gamma_2} (\mu_2 - mc^2)$$

$$(3b) \rightarrow (3c) \mu_1 u_1 + \frac{p_1}{n_1 u_1} + \frac{B_1^2}{8\pi n_1 u_1} = \frac{\mu_2}{u_2} \left( u_2^2 + \frac{\Gamma_2 - 1}{\Gamma_2} \right) - \frac{\Gamma_2 - 1}{\Gamma_2} \frac{mc^2}{u_2} + \frac{Y^2 B_1^2}{8\pi n_1 u_1}$$



# RELATIVISTIC SHOCK 3

$$(4b) \mu_2 = \frac{1}{\gamma_2} \left[ \mu_1 \gamma_1 + \frac{B_1^2}{4\pi n_1 \gamma_1} - Y \frac{B_1^2}{4\pi n_1 \gamma_1} \right]$$

$$(3c) \mu_1 u_1 + \frac{p_1}{n_1 u_1} + \frac{B_1^2}{8\pi n_1 u_1} = \frac{\mu_2}{u_2} \left( u_2^2 + \frac{\Gamma_2 - 1}{\Gamma_2} \right) - \frac{\Gamma_2 - 1}{\Gamma_2} \frac{mc^2}{u_2} + \frac{Y^2 B_1^2}{8\pi n_1 u_1}$$

use (4b) and divide by  $mc^2 \gamma_1$

$$(3d) \frac{\mu_1}{mc^2} \frac{u_1}{\gamma_1} + \frac{p_1}{n_1 mc^2 u_1 \gamma_1} + \frac{B_1^2}{8\pi n_1 mc^2 u_1 \gamma_1} =$$

$$\frac{1}{u_2 \gamma_2} \left( u_2^2 + \frac{\Gamma_2 - 1}{\Gamma_2} \right) \left( \frac{\mu_1}{mc^2} + \frac{B_1^2}{4\pi n_1 u_1 mc^2 \gamma_1} (1 - Y) \right) - \frac{\Gamma_2 - 1}{\Gamma_2} \frac{1}{u_2 \gamma_1} + \frac{Y^2 B_1^2}{8\pi n_1 u_1 mc^2 \gamma_1}$$

$$\sigma = \frac{B_1^2}{4\pi n_1 u_1 \gamma_1 mc^2}$$

$$\gamma_1 \gg 1 \Rightarrow u_1 \approx \gamma_1$$

$$(3e) \frac{\mu_1}{mc^2} + \frac{p_1}{n_1 mc^2 \gamma_1^2} + \frac{\sigma}{2} =$$

$$\frac{1}{u_2 \gamma_2} \left( u_2^2 + \frac{\Gamma_2 - 1}{\Gamma_2} \right) \left( \frac{\mu_1}{mc^2} + \sigma (1 - Y) \right) - \frac{\Gamma_2 - 1}{\Gamma_2} \frac{1}{u_2 \gamma_1} + \frac{\sigma}{2} Y^2$$



# RELATIVISTIC SHOCK 4

$$(3e) \frac{\mu_1}{mc^2} + \frac{p_1}{n_1 mc^2 \gamma_1^2} + \frac{\sigma}{2} =$$

$$\frac{1}{u_2 \gamma_2} \left( u_2^2 + \frac{\Gamma_2 - 1}{\Gamma_2} \right) \left( \frac{\mu_1}{mc^2} + \sigma(1 - Y) \right) - \frac{\Gamma_2 - 1}{\Gamma_2} \frac{1}{u_2 \gamma_1} + \frac{\sigma}{2} Y^2$$

**STRONG  
SHOCK**

$$\frac{p_1}{n_1 mc^2} \rightarrow 0, \quad \mu_1 \rightarrow mc^2, \quad Y = \frac{B_2}{B_1} = \frac{u_1 \gamma_2}{u_2 \gamma_1} \rightarrow \frac{\gamma_2}{u_2} = \frac{\sqrt{1 + u_2^2}}{u_2}, \quad \Gamma_2 = \frac{4}{3}$$

$$(3e) \rightarrow (3f) \quad 1 + \frac{\sigma}{2} = \frac{1}{u_2 \sqrt{1 + u_2^2}} \left( u_2^2 + \frac{1}{4} \right) \left( 1 + \sigma - \sigma \frac{\sqrt{1 + u_2^2}}{u_2} \right) + \frac{\sigma}{2} \frac{1 + u_2^2}{u_2^2}$$

$$1 - \frac{\sigma}{2u_2^2} = \left( u_2^2 + \frac{1}{4} \right) \left( \frac{1 + \sigma}{u_2 \sqrt{1 + u_2^2}} - \frac{\sigma}{u_2^2} \right) \rightarrow 1 - \frac{\sigma}{u_2^2} \left( \frac{1}{2} - \frac{1}{4} \right) + \sigma = \left( u_2^2 + \frac{1}{4} \right) \left( \frac{1 + \sigma}{u_2 \sqrt{1 + u_2^2}} \right)$$

$$u_2^2 - \frac{\sigma}{4(1 + \sigma)} = \left( u_2^2 + \frac{1}{4} \right) \left( \frac{u_2}{\sqrt{1 + u_2^2}} \right) \Rightarrow \left( u_2^2 - \frac{\sigma}{4(1 + \sigma)} \right)^2 (1 + u_2^2) = u_2^2 \left( u_2^2 + \frac{1}{4} \right)^2$$



# RELATIVISTIC SHOCK 5

$$\left(u_2^2 - \frac{\sigma}{4(1+\sigma)}\right)^2 (1 + u_2^2) = u_2^2 \left(u_2^2 + \frac{1}{4}\right)^2 \rightarrow u_2^2 = \frac{1}{2} \left\{ \frac{8\sigma^2 + 10\sigma + 1}{8(1+\sigma)} \pm \sqrt{\left(\frac{8\sigma^2 + 10\sigma + 1}{8(1+\sigma)}\right)^2 - \frac{\sigma^2}{2(1+\sigma)}} \right\}$$

compute p

$$(6) \frac{p_2}{n_1 u_1} = \frac{1}{4u_2} (\mu_2 - mc^2)$$

$$(4b) \mu_2 = \frac{1}{\gamma_2} \left[ \mu_1 \gamma_1 + \frac{B_1^2}{4\pi n_1 \gamma_1} - \frac{\gamma_2}{u_2} \frac{B_1^2}{4\pi n_1 \gamma_1} \right] = \frac{mc^2 \gamma_1}{\sqrt{1+u_2^2}} \left[ 1 + \sigma - \frac{\sqrt{1+u_2^2}}{u_2} \sigma \right]$$

$$\frac{p_2}{n_1 u_1} = \frac{1}{4u_2} \left\{ \frac{mc^2 \gamma_1}{\sqrt{1+u_2^2}} \left[ 1 + \sigma - \frac{\sqrt{1+u_2^2}}{u_2} \sigma \right] - mc^2 \right\}$$

$$\frac{p_2}{n_1 mc^2 u_1 \gamma_1} = \frac{1}{4u_2 \gamma_2} \left\{ 1 + \sigma \left[ 1 - \frac{\gamma_2}{u_2} \right] - \frac{\gamma_2}{\gamma_1} \right\}$$



# LARGE AND SMALL $\sigma$

$$u_2^2 = \frac{1}{2} \left\{ \frac{8\sigma^2 + 10\sigma + 1}{8(1 + \sigma)} \pm \sqrt{\left( \frac{8\sigma^2 + 10\sigma + 1}{8(1 + \sigma)} \right)^2 - \frac{\sigma^2}{2(1 + \sigma)}} \right\} \quad \gamma_2 = \sqrt{1 + u_2^2}$$

$$\frac{p_2}{n_1 m c^2 \gamma_1^2} = \frac{1}{4u_2 \gamma_2} \left[ 1 + \sigma \left( 1 - \frac{\gamma_2}{u_2} \right) - \frac{\gamma_2}{\gamma_1} \right] \quad \frac{B_2}{B_1} = \frac{\gamma_2}{u_2}$$

$$\sigma \rightarrow \infty$$

$$u_2^2 \rightarrow \sigma + \frac{1}{8} + \frac{1}{64\sigma}$$

$$\gamma_2^2 \rightarrow \sigma + \frac{9}{8} + \frac{1}{64\sigma}$$

$$\frac{p_2}{n_1 m c^2 \gamma_1^2} \rightarrow \frac{1}{8\sigma} - \frac{1}{32\sigma^2}$$

$$\frac{B_2}{B_1} = \frac{N_2}{N_1} \rightarrow 1 + \frac{1}{2\sigma}$$

IF MAGNETIC FIELD  
DOMINATES:

• PLASMA DOES NOT  
SLOW DOWN

• P STAYS LOW

• B INCREASES SLIGHTLY

$$\sigma \rightarrow 0$$

$$u_2^2 \rightarrow \frac{1 + 9\sigma}{8}$$

$$\gamma_2^2 \rightarrow \frac{9}{8}(1 + \sigma)$$

$$\frac{p_2}{n_1 m c^2 \gamma_1^2} \rightarrow \frac{2}{3}(1 - 7\sigma)$$

$$\frac{B_2}{B_1} = \frac{N_2}{N_1} \rightarrow 3 - 12\sigma$$



# NEBULAR DYNAMICS 3

$$n u r^2 = n_2 u_2 r_s^2$$

$$\frac{u B r}{\gamma} = \frac{u_2 B_2 r_s}{\gamma_2}$$

$$\frac{p}{n^\Gamma} = \frac{p_2}{n_2^\Gamma}$$

$$\gamma \left( m c^2 + \frac{\Gamma}{\Gamma - 1} \frac{p}{n} + \frac{B^2}{4 \pi n \gamma^2} \right) = \cos t$$

$$\begin{cases} z = \frac{r}{r_s} \\ v = \frac{u}{u_2} \end{cases}$$

$$n = \frac{n_2}{v z^2}$$

$$p = \frac{p_2}{(v z^2)^\Gamma}$$

$$B = B_2 \frac{\gamma}{\gamma_2} \frac{1}{v z}$$

$$\gamma \left( m c^2 + \frac{\Gamma}{\Gamma - 1} \frac{p_2}{n_2} \frac{1}{(v z^2)^{\Gamma-1}} + \frac{B_2^2}{4 \pi n_2 \gamma_2^2} \frac{v z^2}{v^2 z^2} \right) = \cos t$$

$$\gamma \frac{B_2^2}{4 \pi n_2 \gamma_2^2} \left( \frac{4 \pi n_2 \gamma_2^2 m c^2}{B_2^2} + \frac{\Gamma}{\Gamma - 1} \frac{4 \pi p_2 \gamma_2^2}{B_2^2} \frac{1}{(v z^2)^{\Gamma-1}} + \frac{1}{v} \right) = \cos t$$

$$\begin{cases} \delta = \frac{4 \pi n_2 m c^2 \gamma_2^2}{B_2^2} \\ \Delta = \frac{\Gamma}{\Gamma - 1} \frac{4 \pi p_2 \gamma_2^2}{B_2^2} \end{cases}$$

$$\gamma \left( \delta + \frac{\Delta}{(v z^2)^{\Gamma-1}} + \frac{1}{v} \right) = \gamma_2 (\delta + \Delta + 1)$$



# TERMINAL VELOCITY

$$\gamma \left( \delta + \frac{\Delta}{(vz^2)^{\Gamma-1}} + \frac{1}{v} \right) = \gamma_2 (\delta + \Delta + 1) \quad \begin{matrix} z \rightarrow \infty \\ \Rightarrow \\ \delta \approx 0 \end{matrix} \quad \frac{\gamma_\infty}{v_\infty} = \gamma_2 (\Delta + 1)$$

$$\gamma^2 = 1 + u^2 \quad \Rightarrow \quad (1 + u_2^2 v_\infty^2) = \gamma_2^2 v_\infty^2 (\Delta + 1)^2 \quad \Rightarrow \quad v_\infty = \frac{1}{\sqrt{\gamma_2^2 (\Delta + 1)^2 - u_2^2}}$$

$$\Delta = \frac{u_2}{\sigma \gamma_2} (1 + \sigma) - 1 - \frac{u_2}{\sigma \gamma_1}$$

$\gamma_1 > 1/\sigma$

$$v_\infty = \frac{\sigma}{u_2 \sqrt{(1 + \sigma)^2 - \sigma^2}}$$

$$u_\infty = \frac{\sigma}{\sqrt{(1 + 2\sigma)}}$$

$$\beta_\infty = \frac{u_\infty}{\gamma_\infty} = \frac{\sigma}{1 + \sigma}$$

$$\begin{array}{ll} \sigma \rightarrow \infty & \Rightarrow \beta_\infty \rightarrow 1 \\ \sigma \rightarrow 0 & \Rightarrow \beta_\infty \rightarrow \sigma \end{array}$$



IN COLD WIND:

$$u = \Gamma = \text{const}$$

$$p \ll nmc^2\Gamma^2$$

$$\frac{\partial}{\partial r}(nur^2) = 0$$



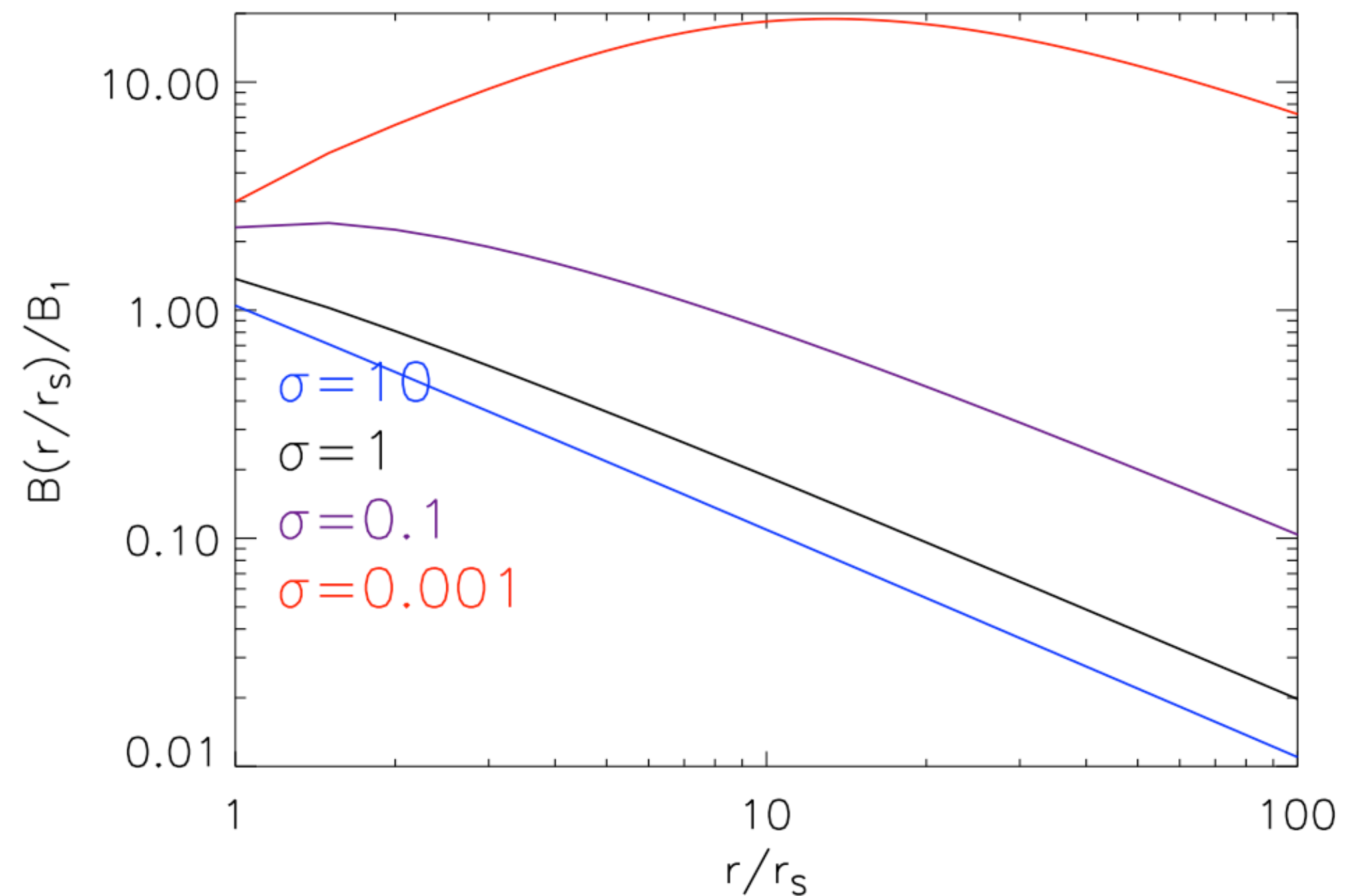
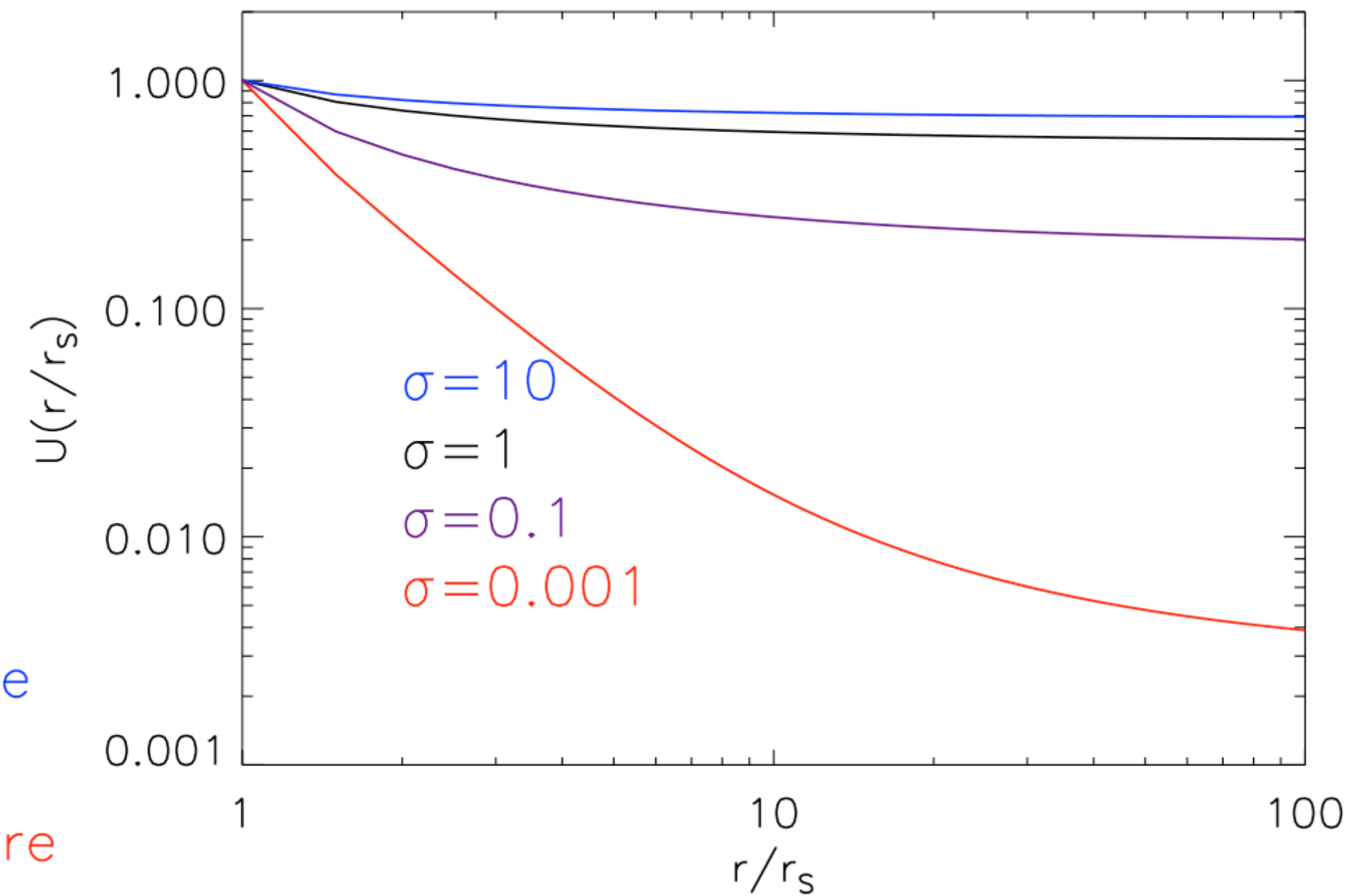
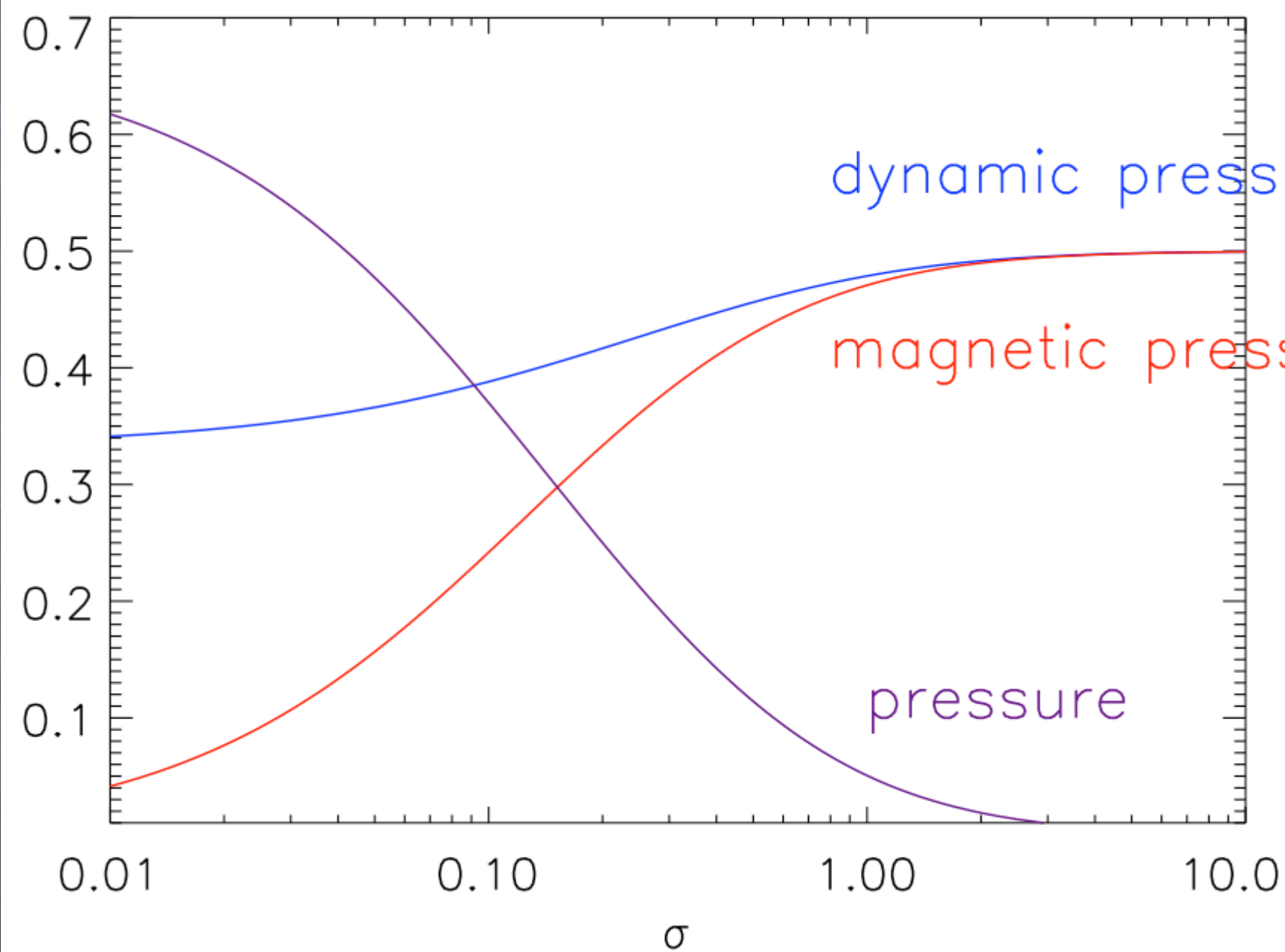
$$n \propto r^{-2}$$

$$\frac{\partial}{\partial r}\left(\frac{ruB}{\Gamma}\right) = 0$$



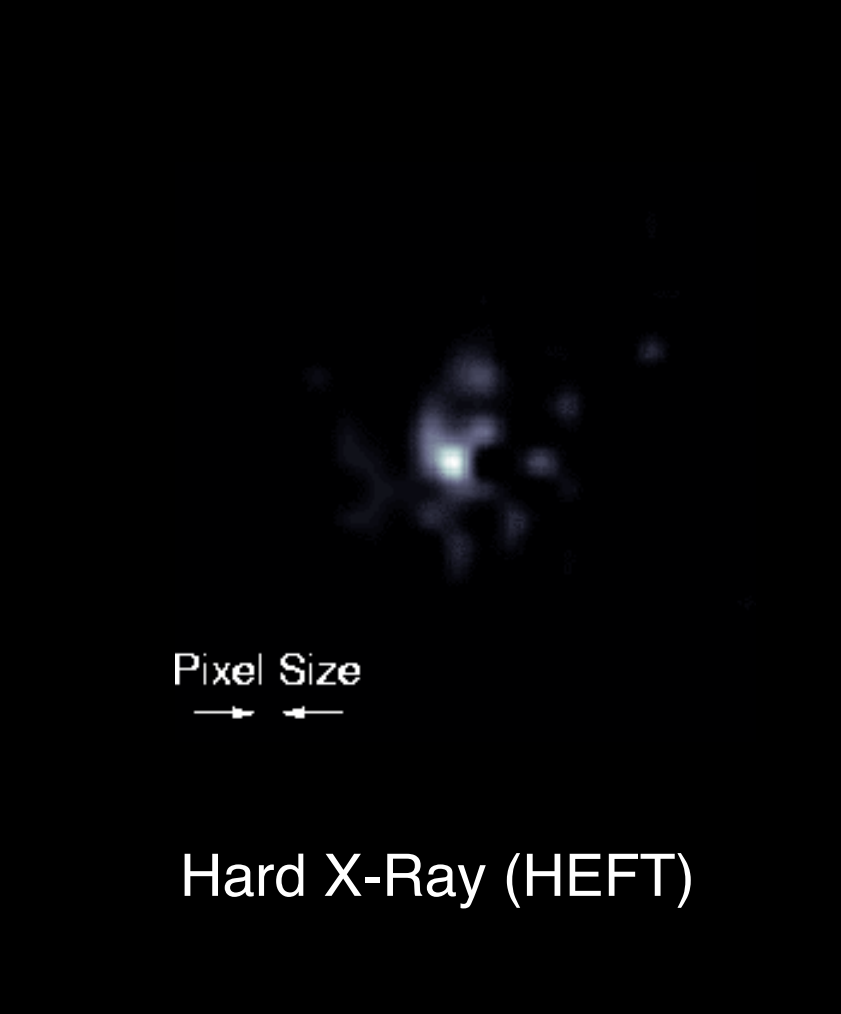
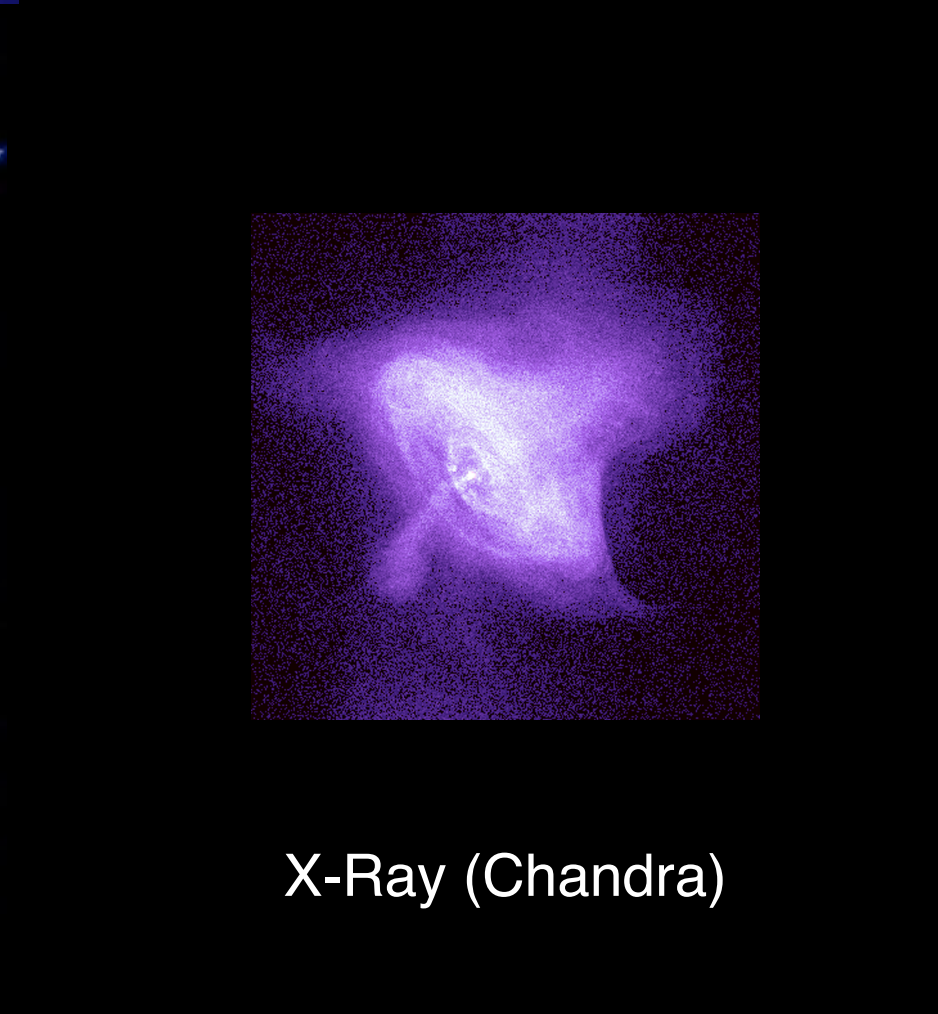
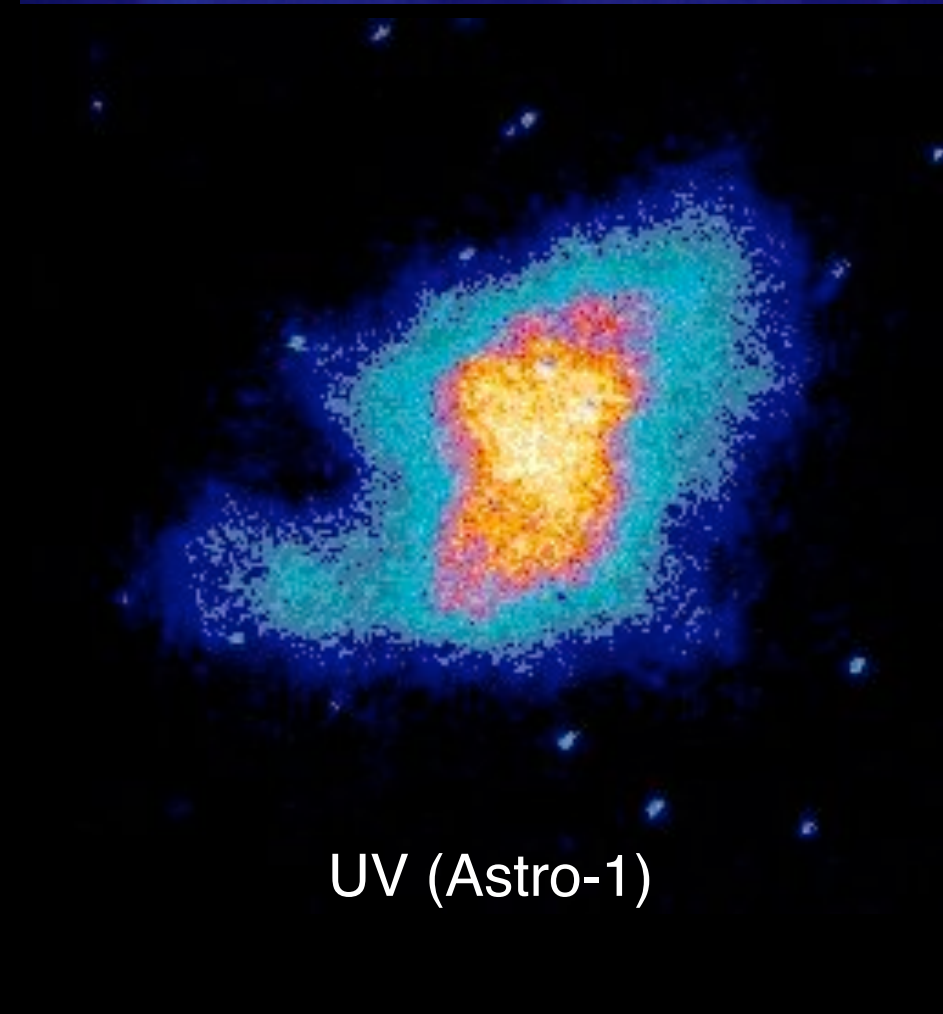
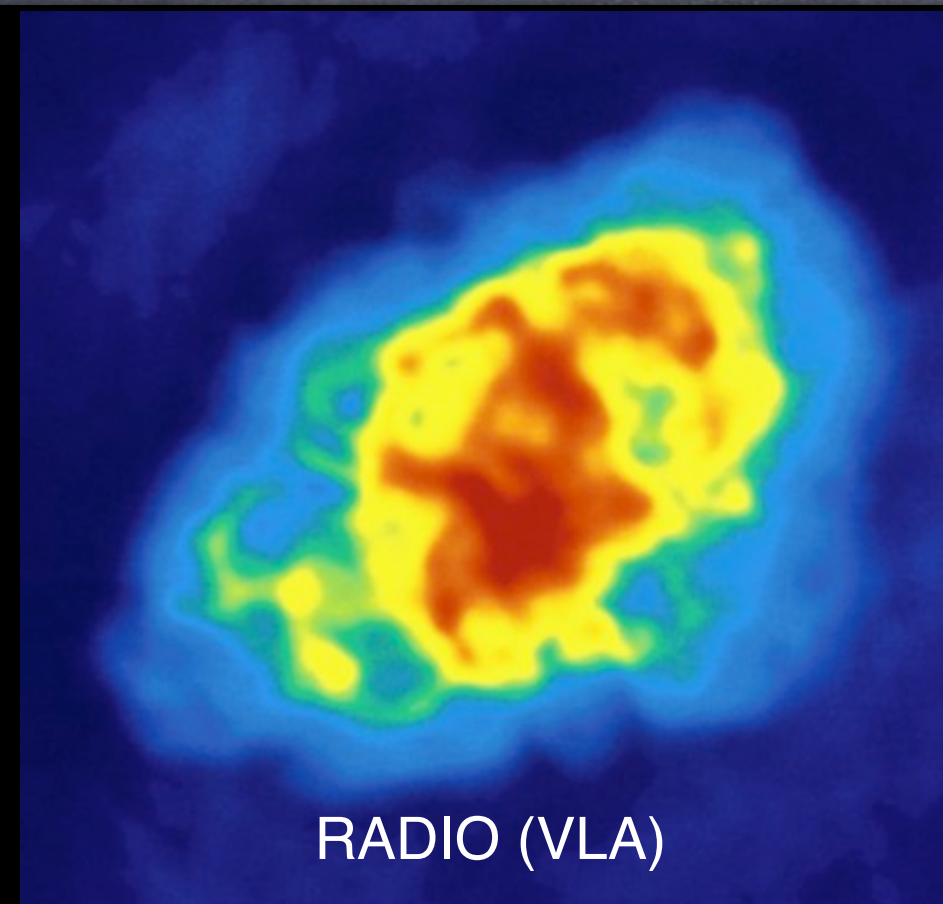
$$B \propto r^{-1}$$

# NEBULAR DYNAMICS



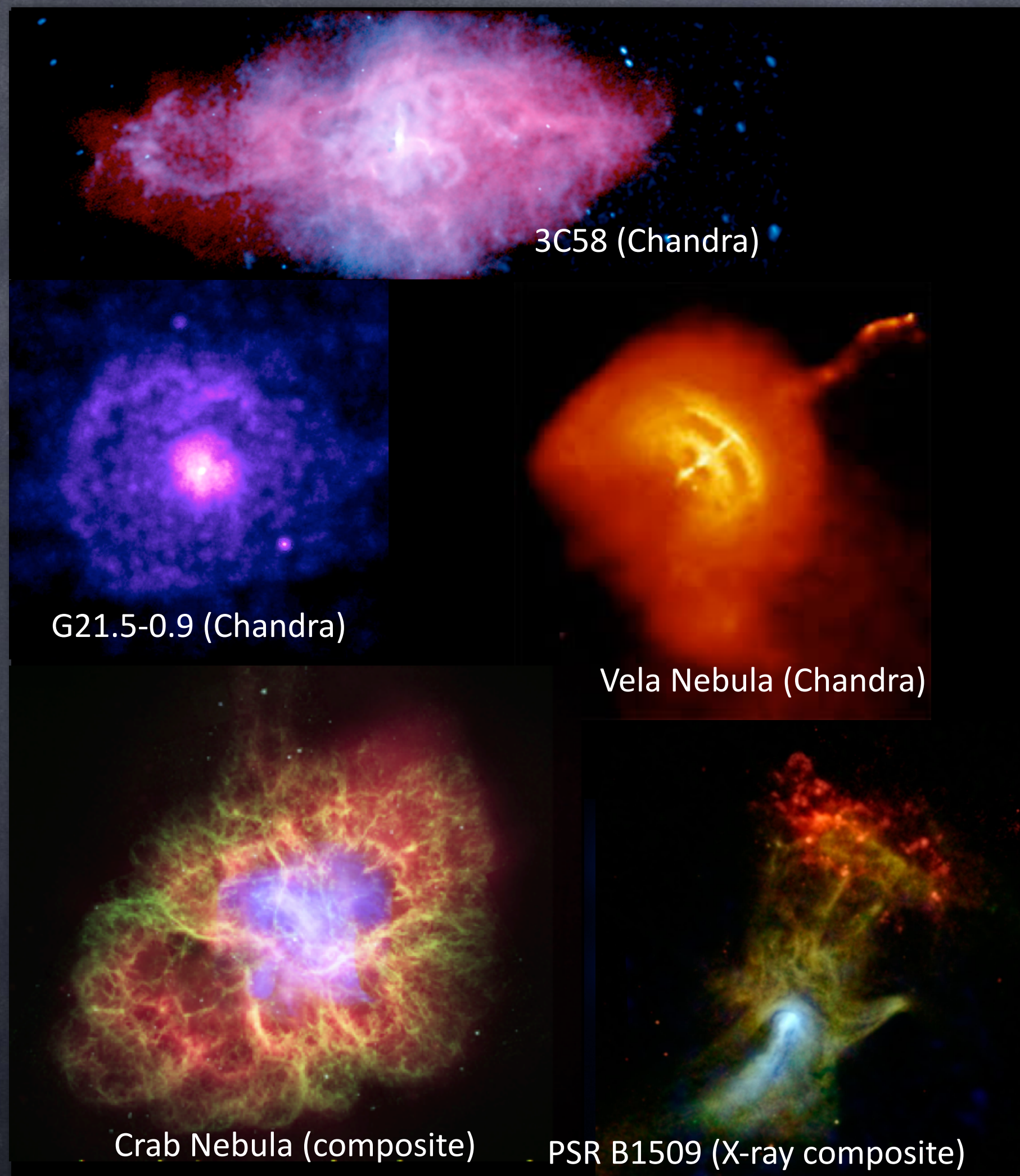


# "THE CRAB NEBULA AT DIFFERENT FREQUENCIES





# THE CRAB NEBULA AS A PROTOTYPE



## PULSAR WIND NEBULAE OR PLERIONS

### SNRs WITH

- CENTER FILLED MORPHOLOGY
- BROAD NON THERMAL SPECTRUM
- FLAT RADIO SPECTRUM

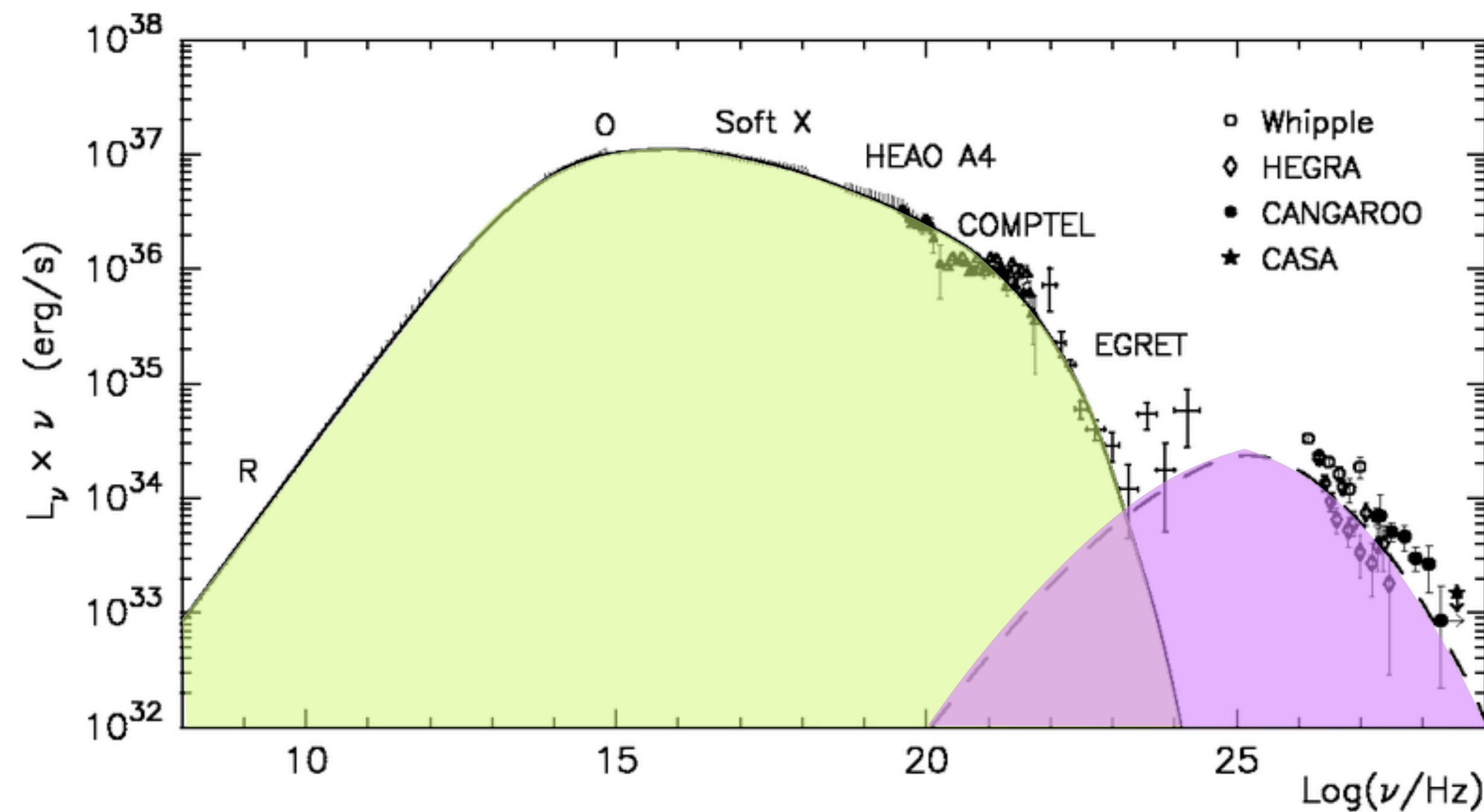
$$F_{\nu} \propto \nu^{-\alpha}, \quad \alpha < 0.5$$



# THE CRAB NEBULA SPECTRUM

## BROAD BAND NON-THERMAL SPECTRUM

CRAB NEBULA spectrum [adapted from Atoyan & Aharonian 1996]

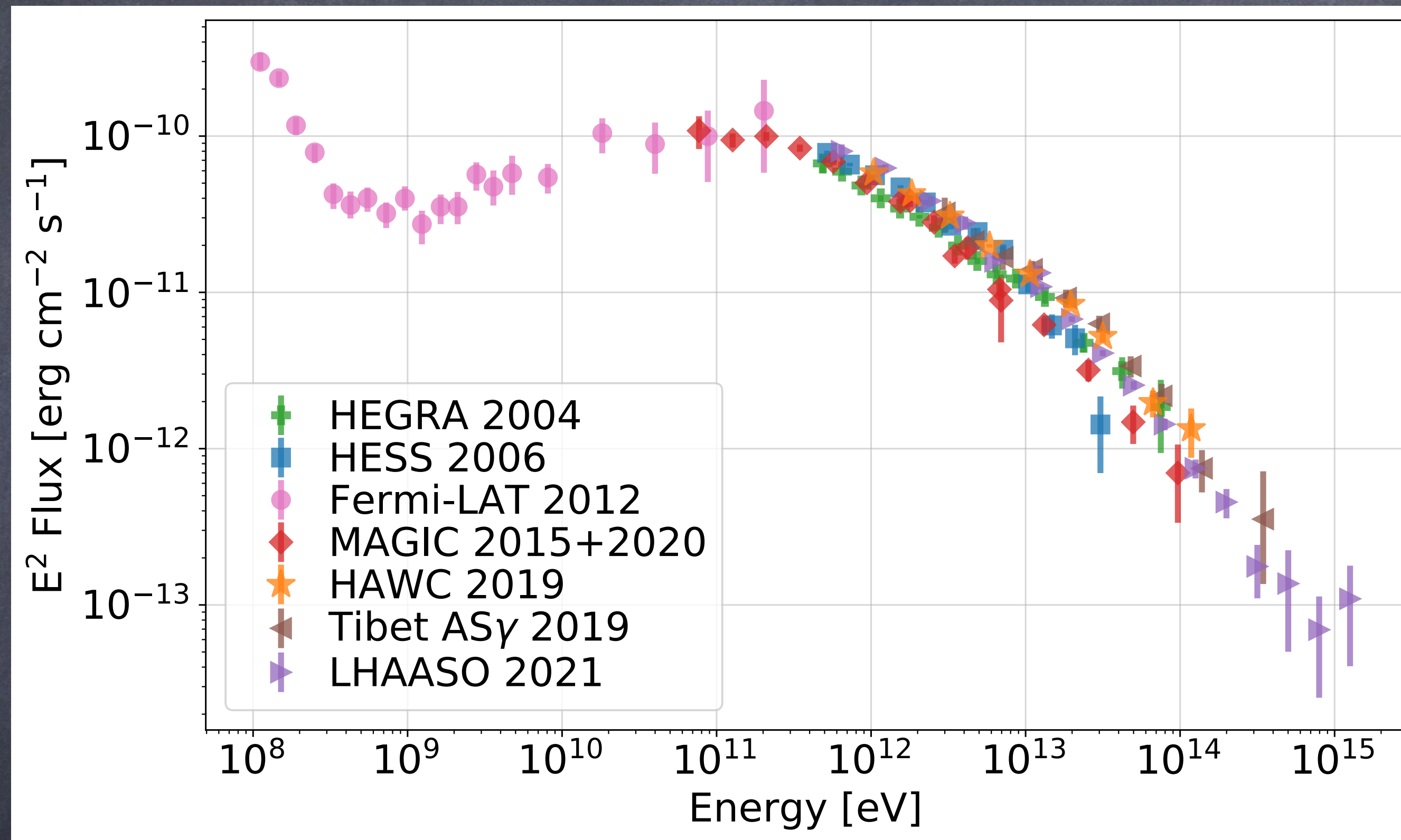


synchrotron radiation by relativistic particles in the nebular B field

Inverse Compton scattering with local photon field



# THE CRAB NEBULA IN GAMMA-RAYS



THE ONLY  
ESTABLISHED  
GALACTIC  
PEVATRON!!!

Amato & Olmi 2021

FOR ICS ON CMB

$$\epsilon_\gamma \approx 0.37 (E_e/\text{PeV})^{1.3} \text{ PeV}$$

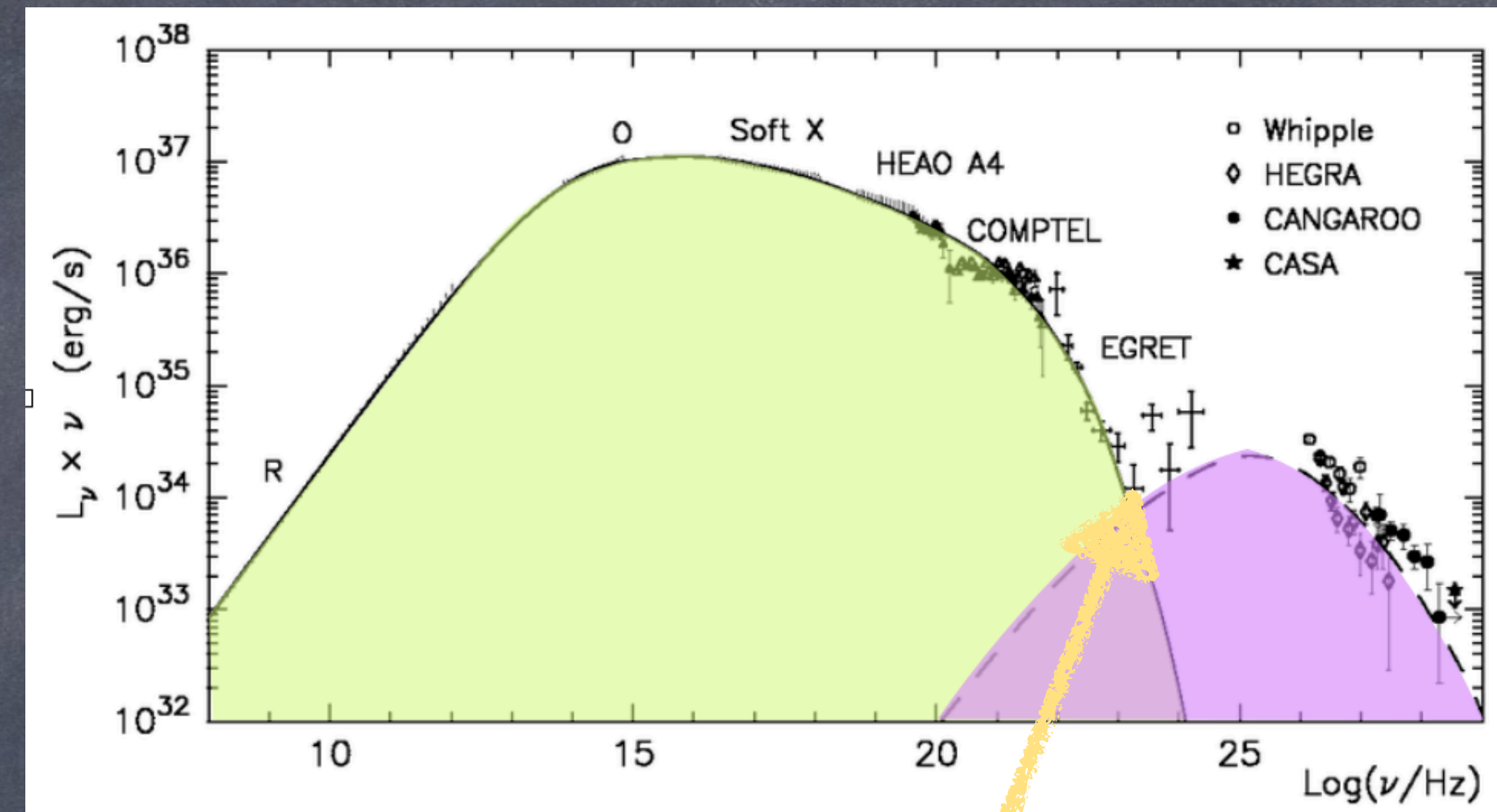
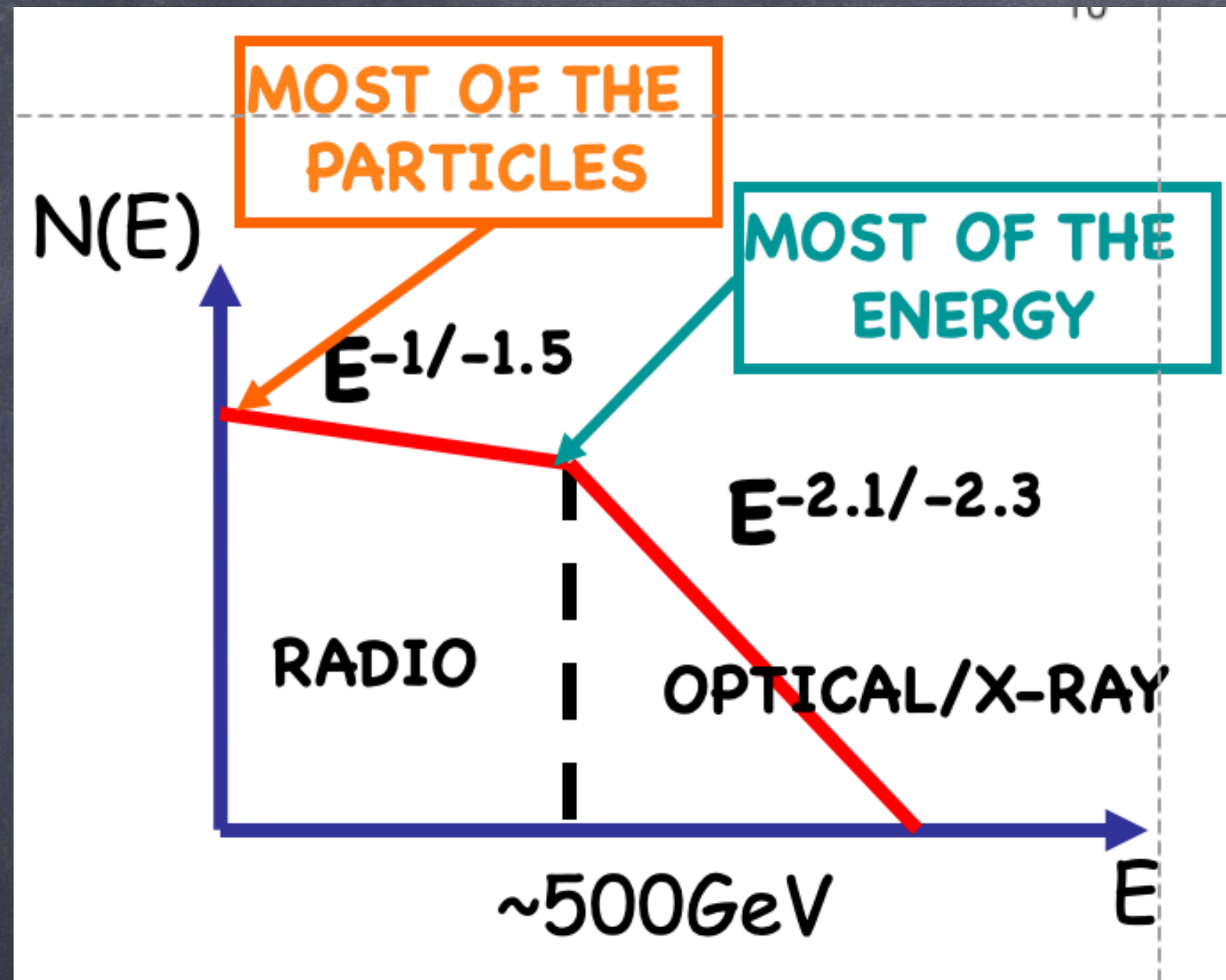
HIGHEST ENERGY  
LHAASO  
DATA POINT



$$E_e \approx 2.4 \text{ PeV}$$



# EMITTING PARTICLES



$$B_{\text{NEB}} \approx 100 \mu\text{G}$$

PeV ELECTRONS

$$L_{\text{NEB}} > 20 \% \dot{E}$$

**EXTRAORDINARY  
ACCELERATOR!**

ONE ZONE MODELS

(Pacini & Salvati 1973, EA+ 2000, Bucciantini+ 2011....)

BUT....



# BIG OPEN QUESTION

## WHAT WE KNOW:

- MOST EFFICIENT ACCELERATORS IN NATURE  $\epsilon_{\text{acc}} \lesssim 30\%$

- ENERGY FLUX THAT LEAVES THE PSR

$$\dot{E} = \kappa \dot{N}_{GJ} m_e \Gamma c^2 \left( 1 + \frac{m_i}{\kappa m_e} + \sigma \right)$$
$$\sigma = \frac{B^2}{4\pi n_{\pm} m_e c^2 \Gamma^2}$$

## WE DO NOT KNOW:

- WHAT THE ACCELERATION MECHANISM(S) IS (ARE)  
POSSIBILITIES DEPEND ON

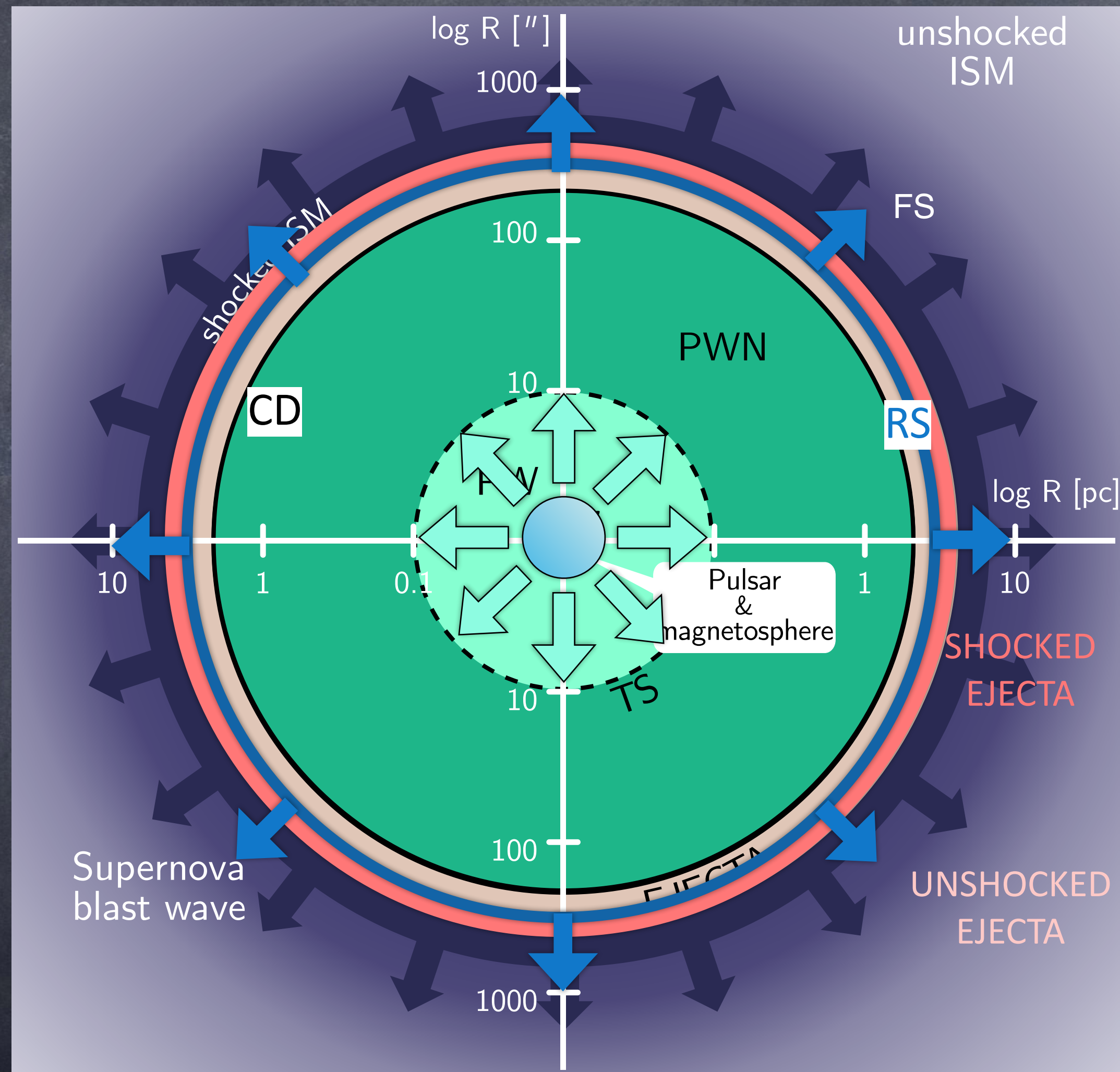
WIND COMPOSITION (IONS?  $\kappa$ ?)

WIND MAGNETIZATION ( $\sigma$ ?)

IN PRINCIPLE BOTH  
DEPEND  
ON LOCATION



# BASIC PICTURE OF A PWN



Adapted from Kennel & Coroniti 1984  
[Del Zanna & Olmi 2017]



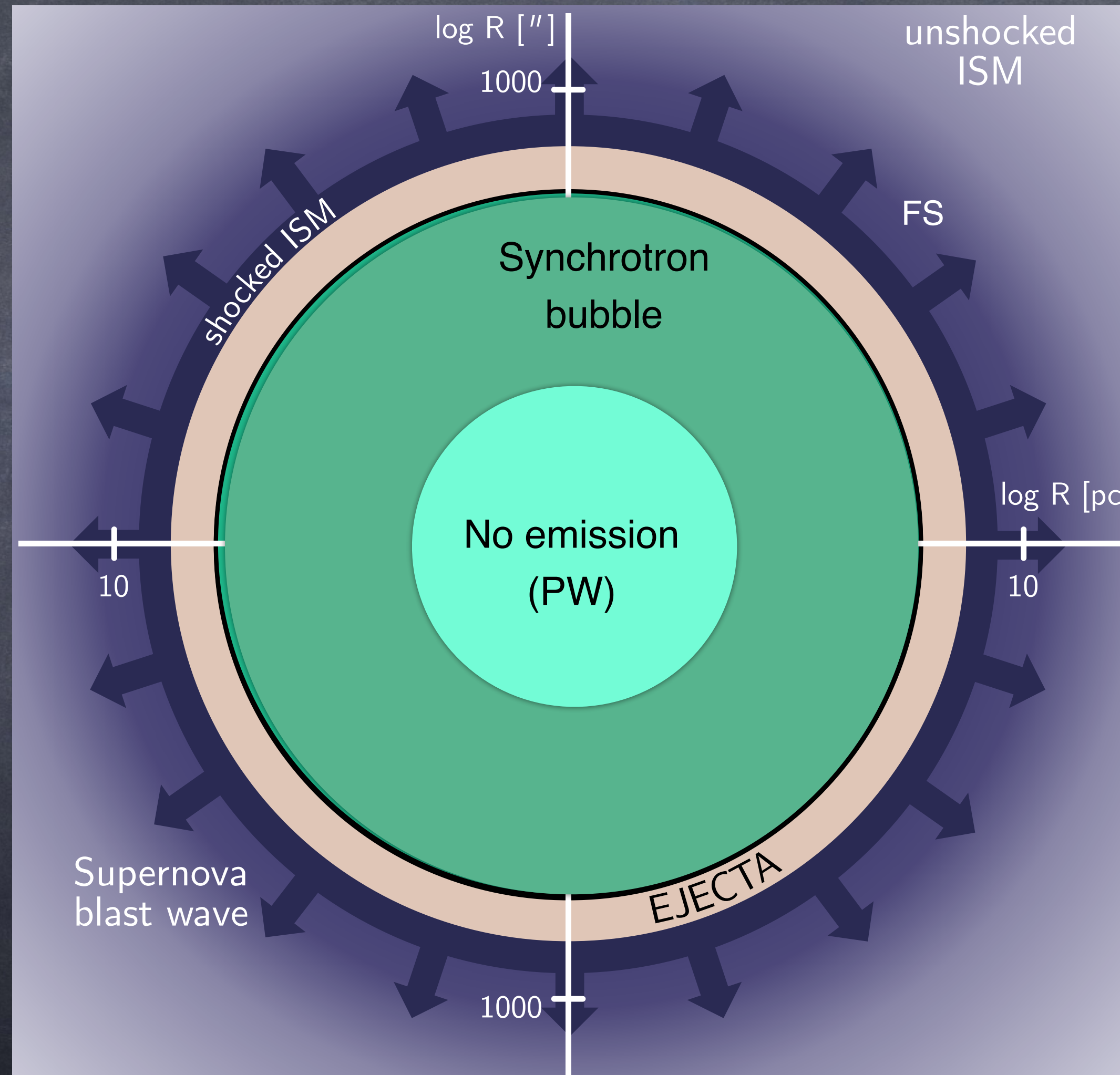
$$\frac{\dot{E}}{4\pi c R_{TS}^2} = P_{PWN} = \frac{\dot{E} t}{4\pi R_N^3}$$



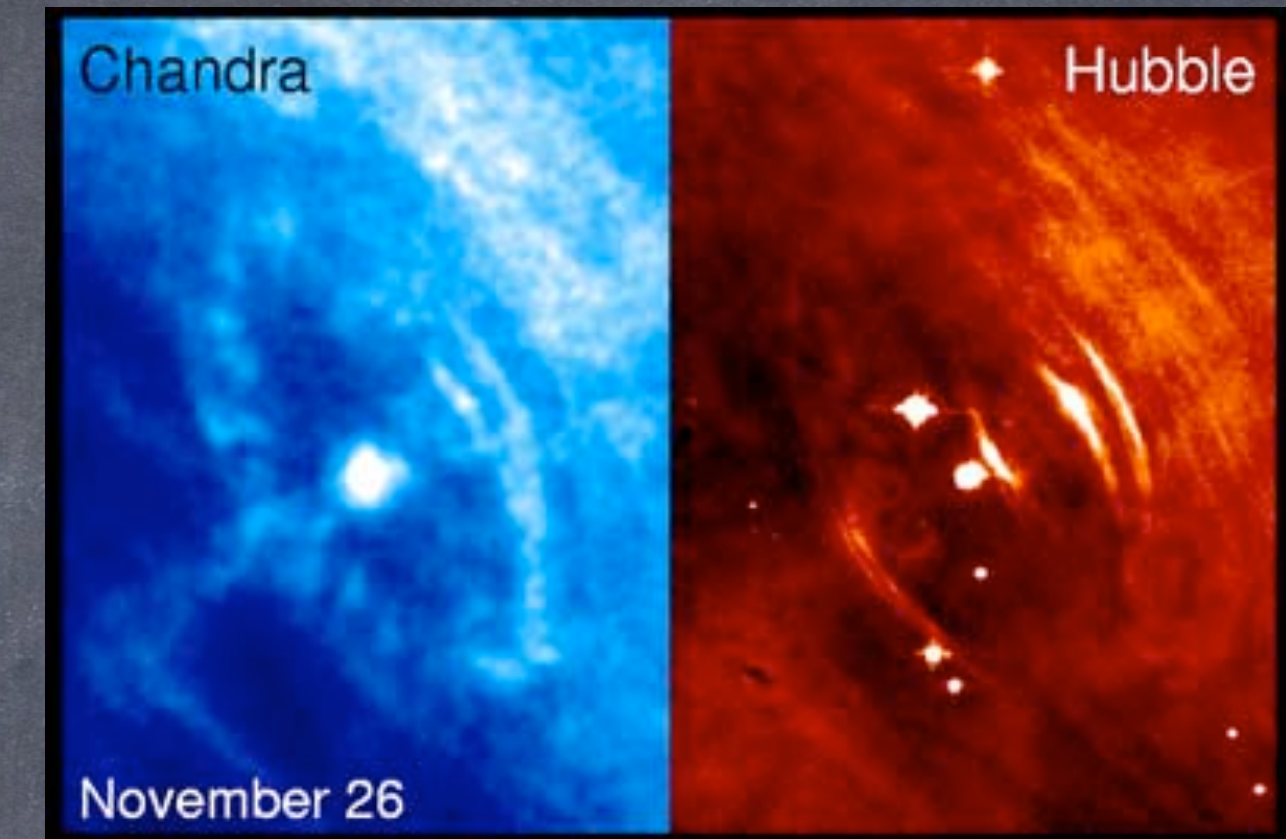
$$R_{TS} = \left( \frac{v_N}{c} \right)^{1/2} R_N$$



# THE TERMINATION SHOCK

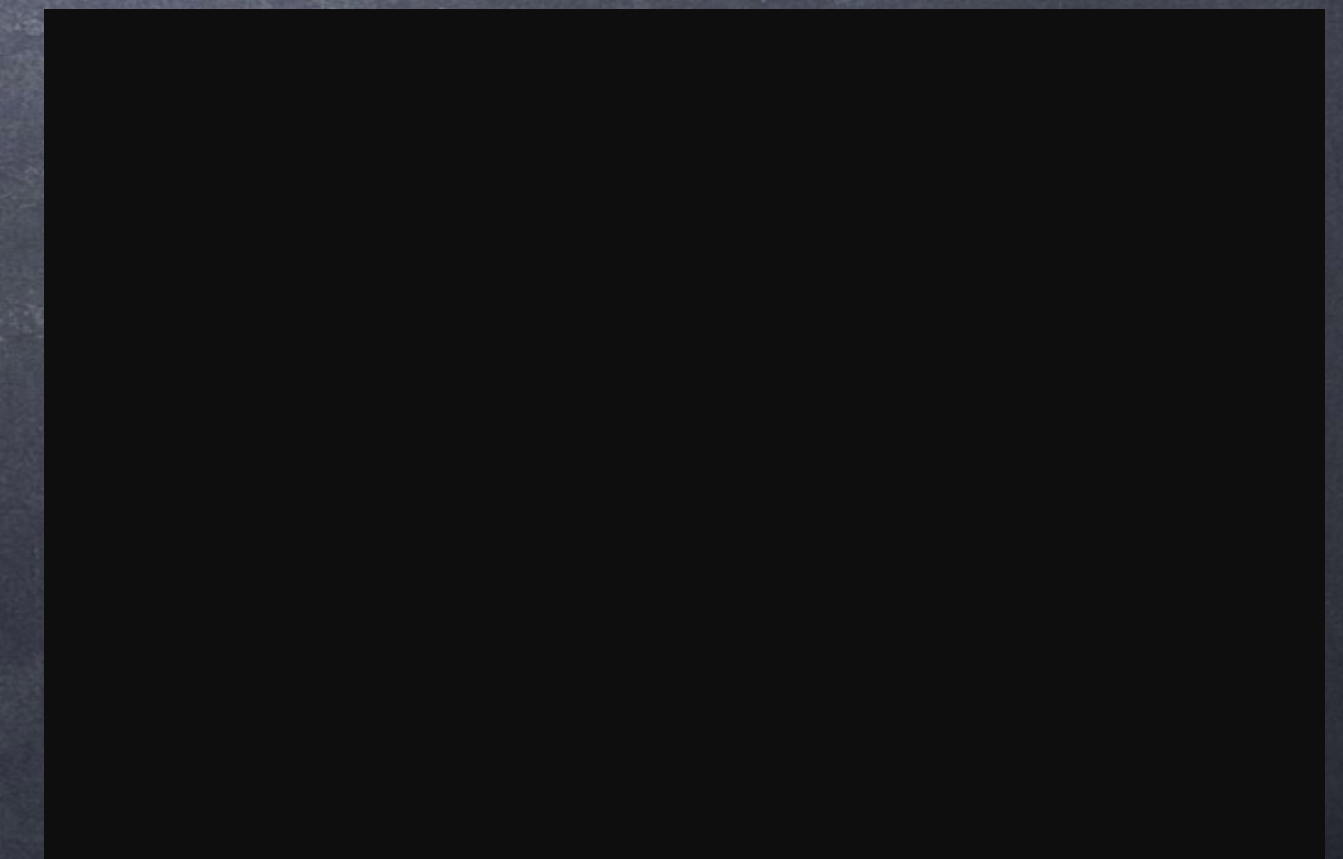


Adapted from Kennel & Coroniti 1984  
[Del Zanna & Olmi 2017]



$$R_{TS} = \left( \frac{v_N}{c} \right)^{1/2} R_N$$

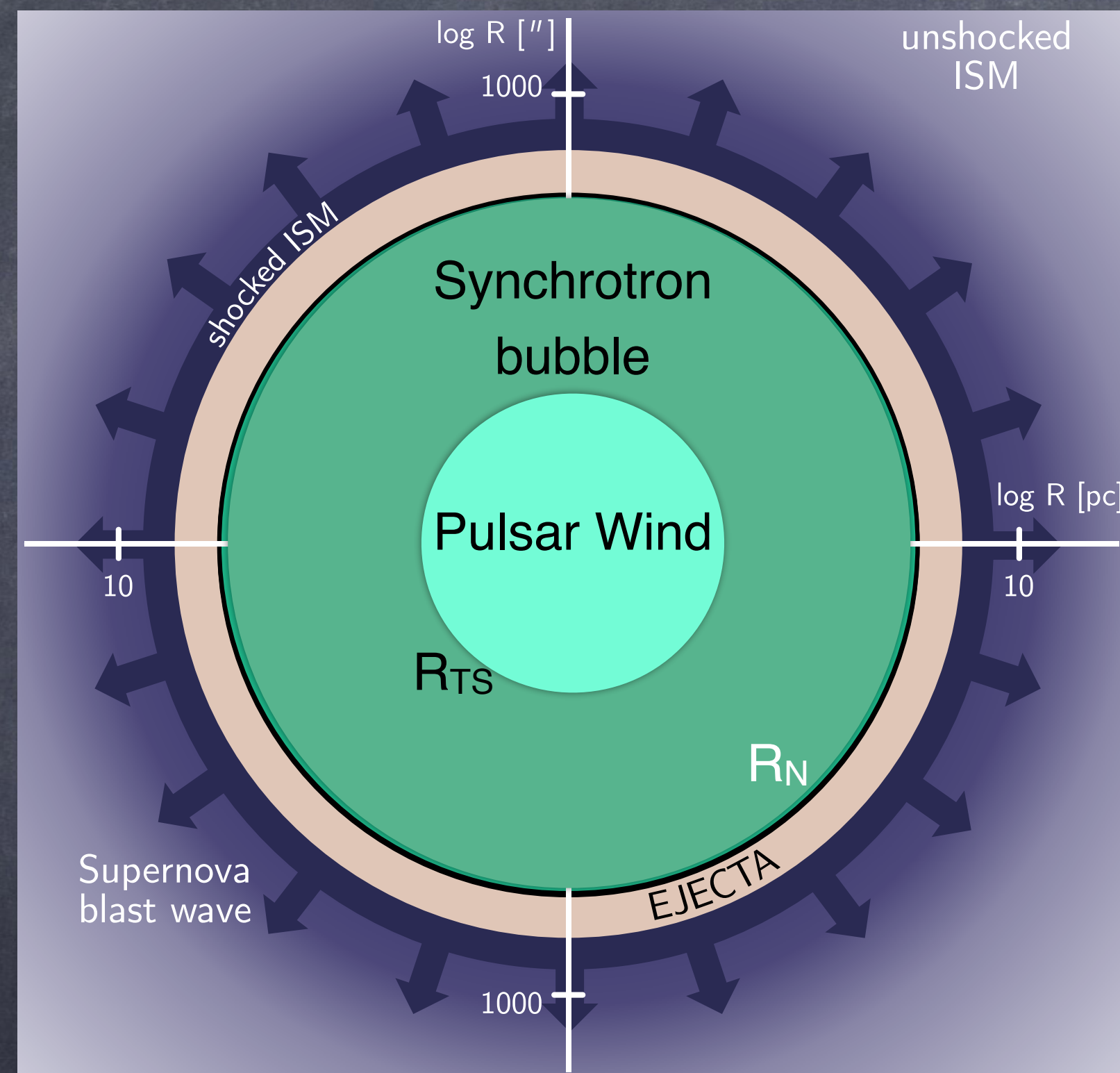
DISSIPATION AND  
PARTICLE  
ACCELERATION AT TS





# 1D/2D STATIC MODELS OF PWNE

[Rees & Gunn 1974, Kennel & Coroniti 1984, Emmering & Chevalier 1987, Begelman & Li 1992]



- particle spectral index(es)  $\rightarrow \gamma = 2.3$

- wind Lorentz factor  $\rightarrow \Gamma = 3 \times 10^6$

- wind magnetization  $\rightarrow \sigma = v_N/c \approx 3 \times 10^{-3}$

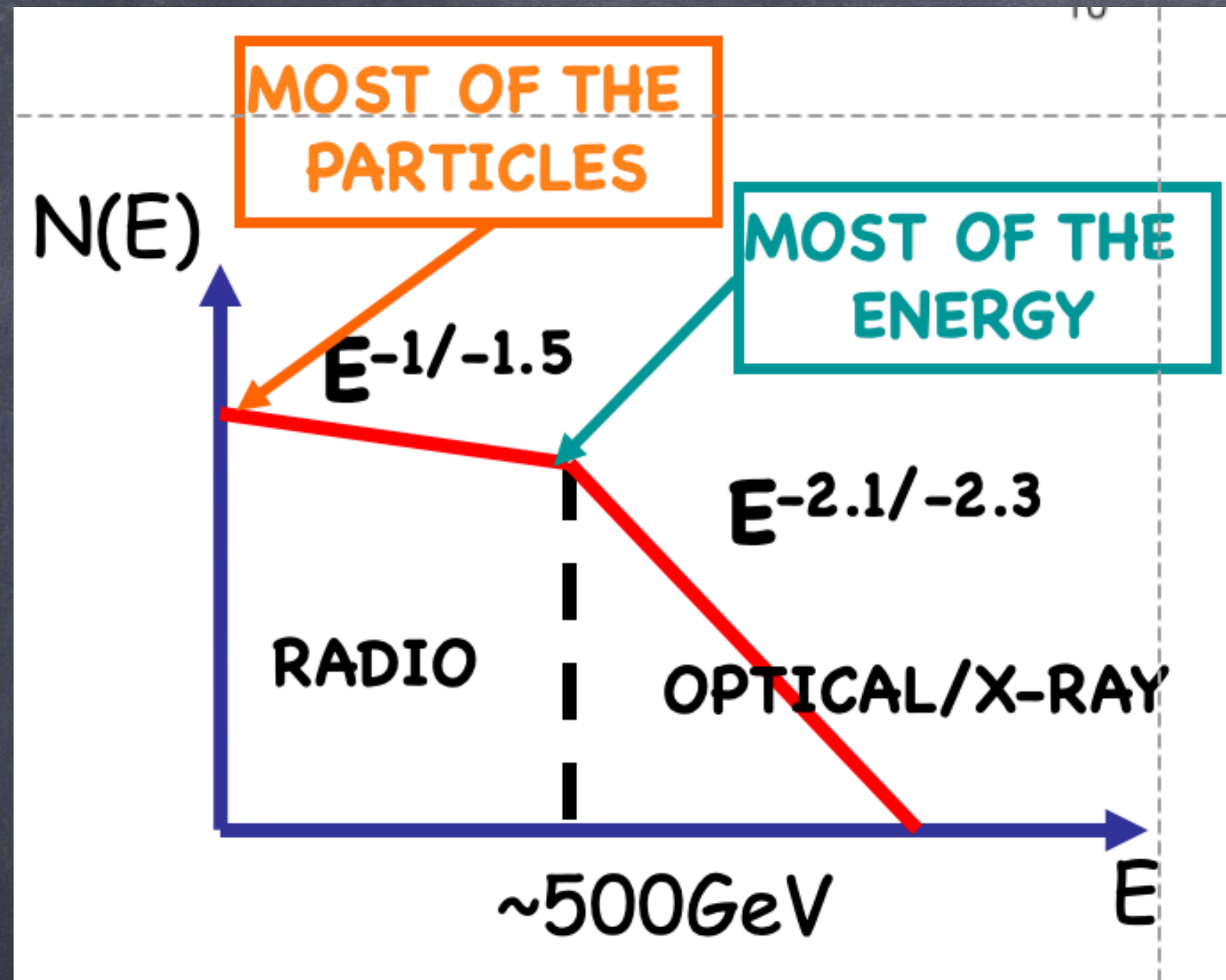
- particle injection rate  $\rightarrow \dot{N} \approx 10^{38} s^{-1}$



FROM DYNAMICS AND RADIATION MODELING OF OPTICAL /X-RAY EMISSION

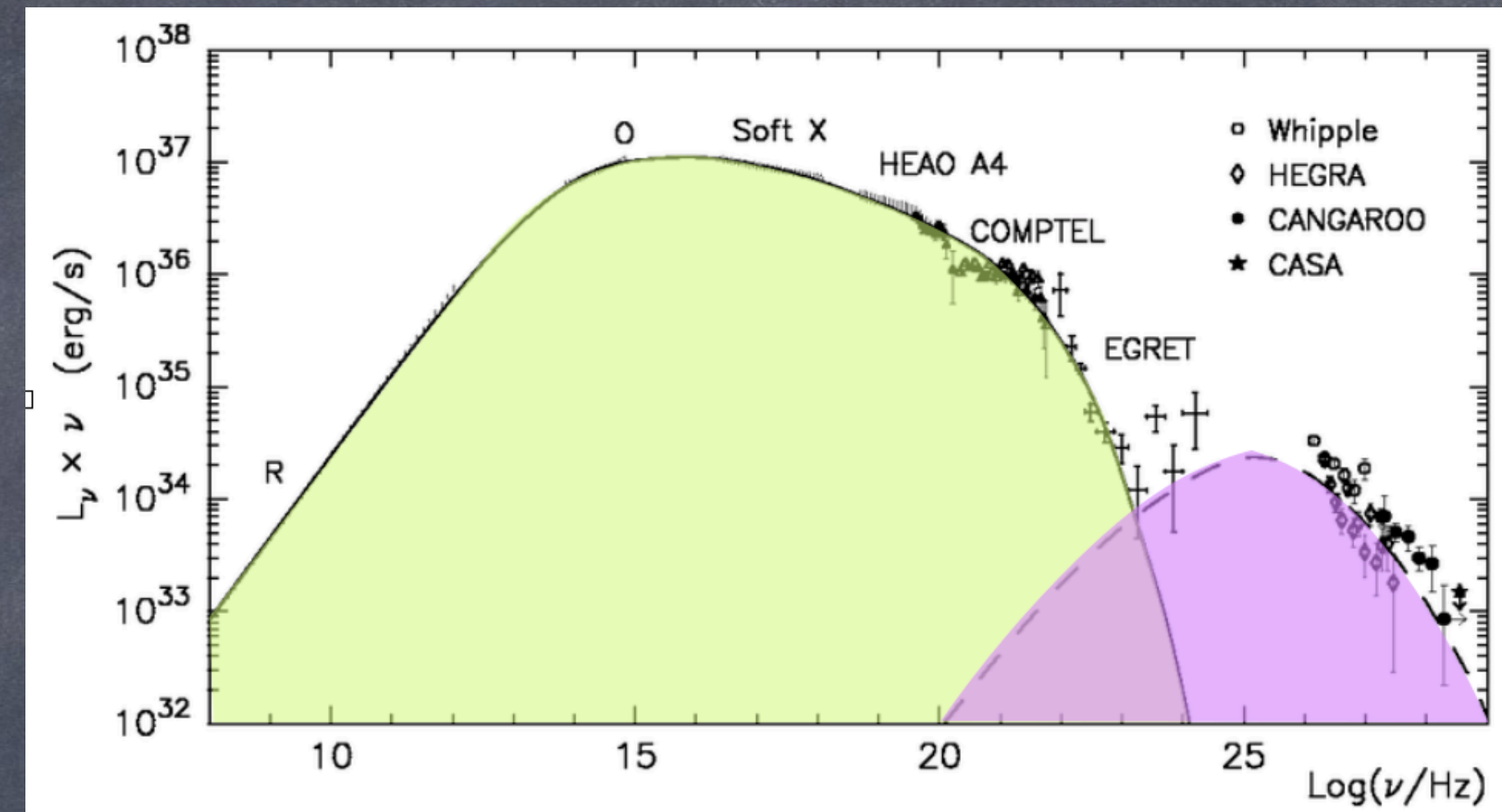


# EMITTING PARTICLES



## ONE ZONE MODELS

(Pacini & Salvati 1973, EA+ 2000, Bucciantini+ 2011....)



RADIO EMITTING PARTICLES  
HAVE LONG LIFETIMES:  
DO NOT NEED TO BE PART OF  
THE FLOW

IF PART OF THE FLOW

$$\kappa \approx 10^6 \quad \Gamma \approx 10^4$$

OTHERWISE

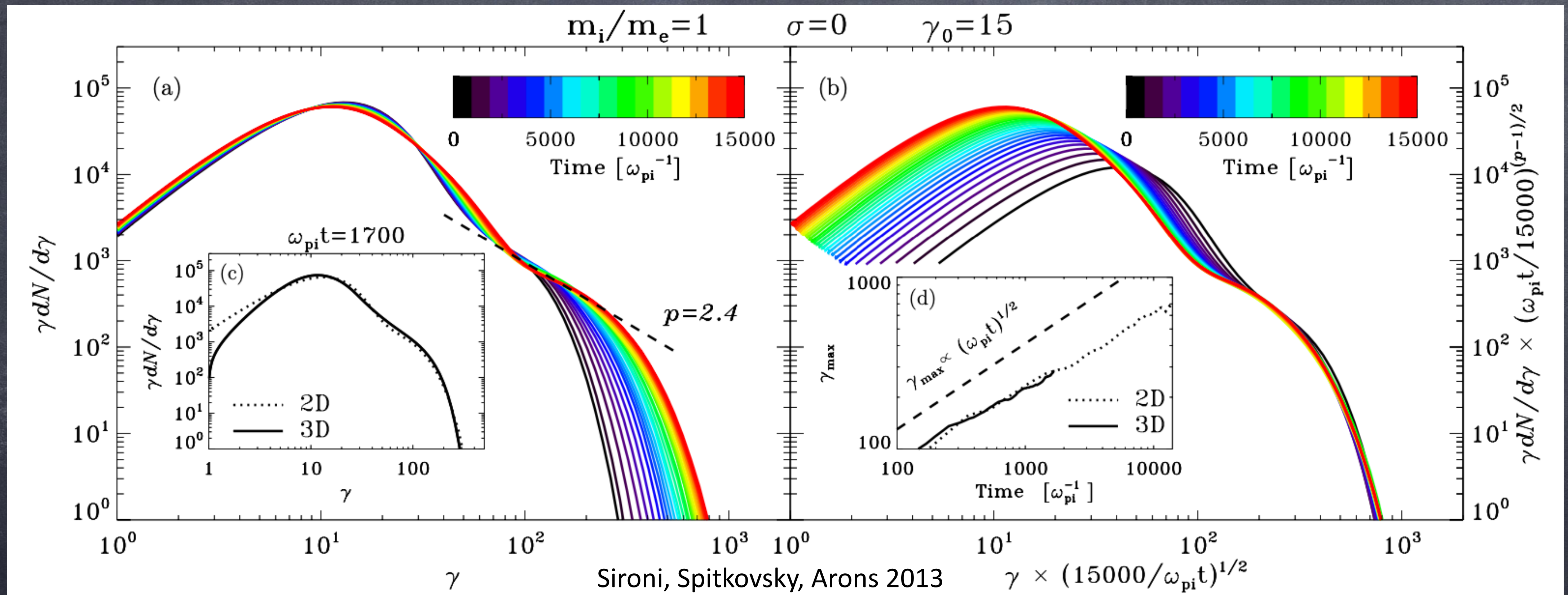
$$\kappa \approx 10^3-10^4 \quad \Gamma \approx 10^6-10^7$$



# PARTICLE ACCELERATION MECHANISMS



# FERMI ACCELERATION (RELATIVISTIC UNMAGNETIZED!)

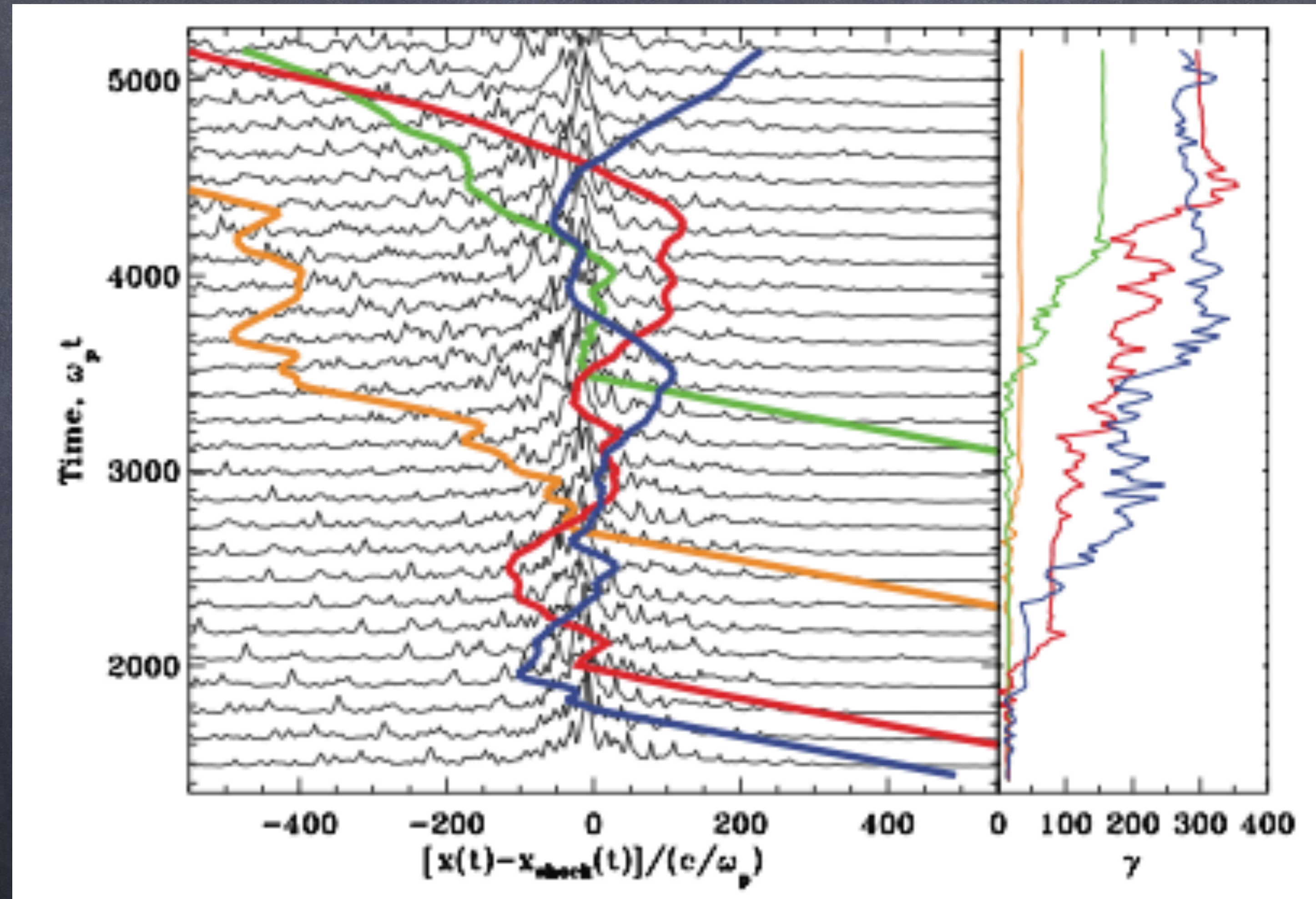


POWER-LAW DEVELOPS BUT SLOW PROCESS!

SCATTERING ON SMALL-SCALE TURBULENCE:  $E_{\text{MAX}} \propto t^{1/2}$

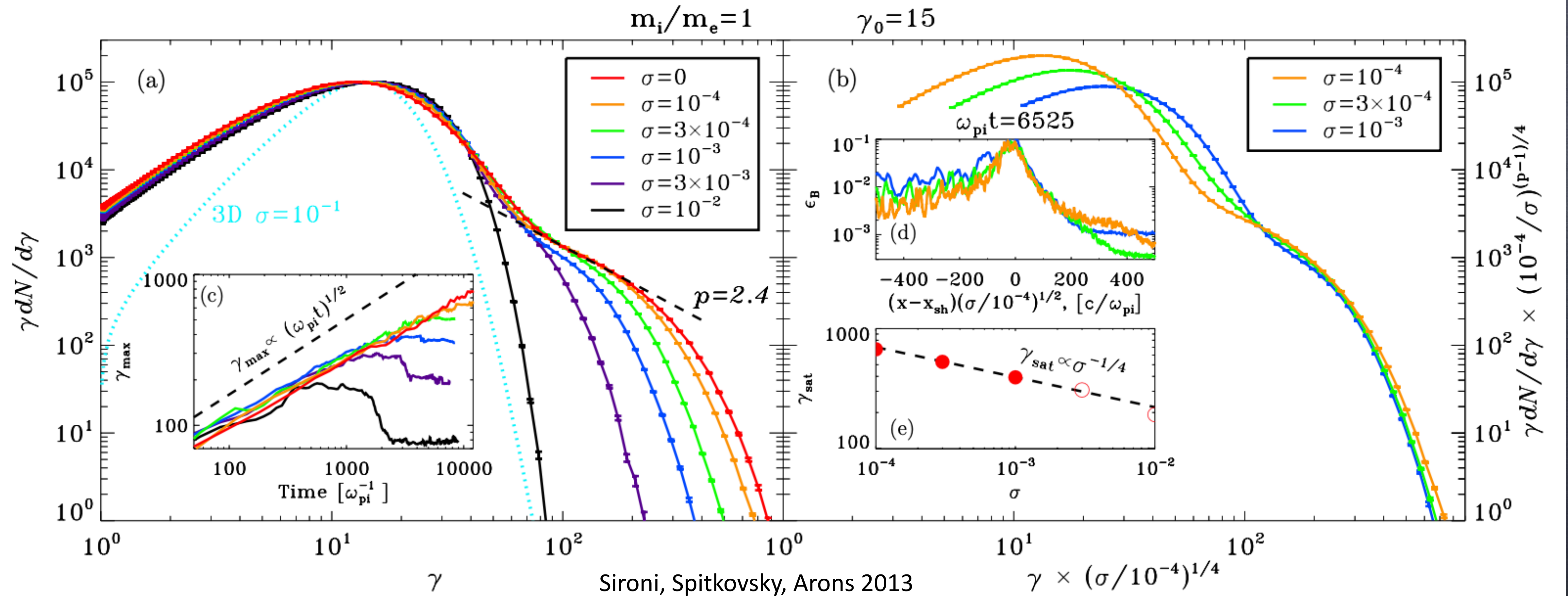


# FERMI ACCELERATION (RELATIVISTIC MAGNETIZED!)





# FERMI ACCELERATION (RELATIVISTIC MAGNETIZED!)

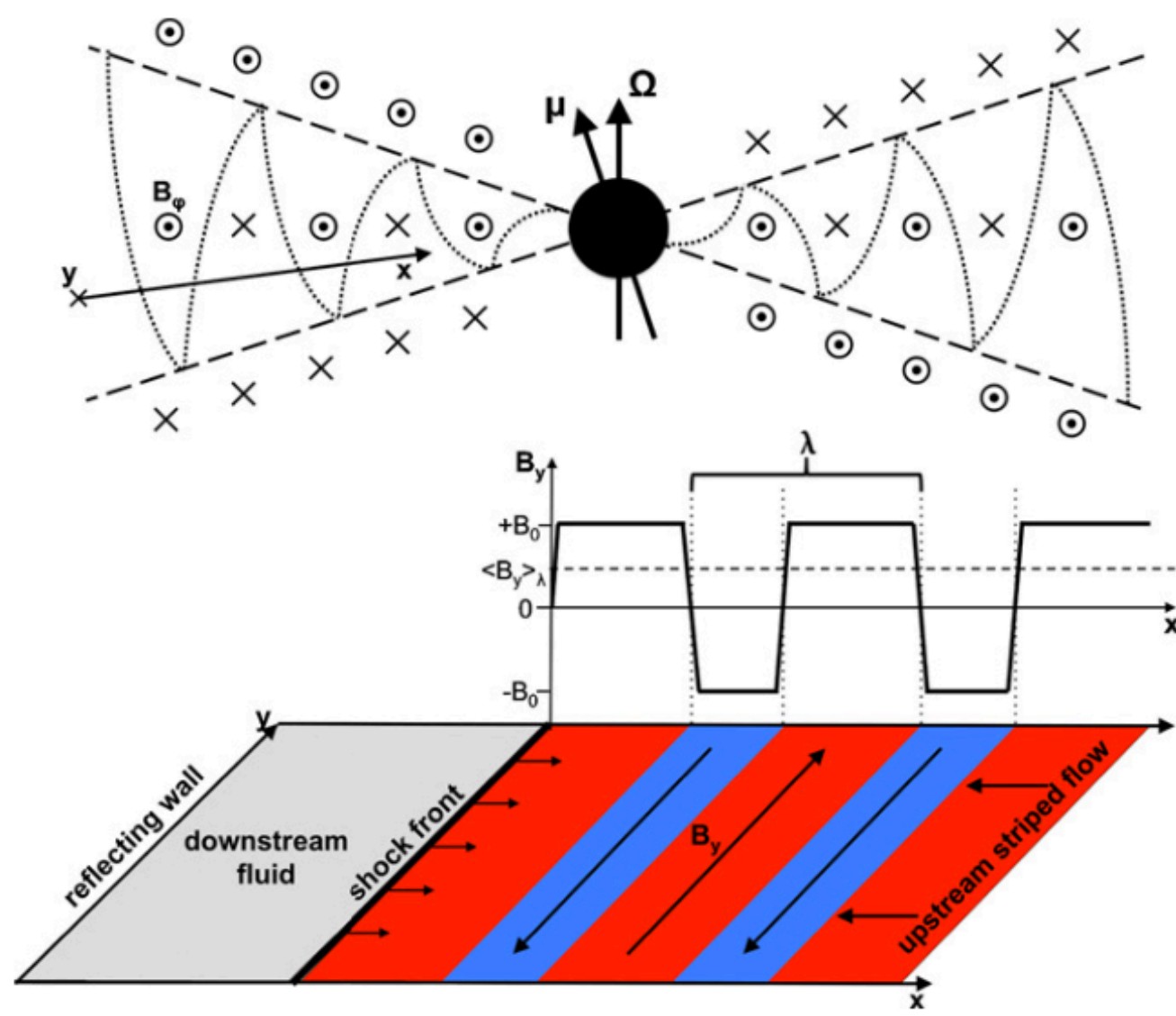


ACCELERATION COMPLETELY SUPPRESSED FOR  $\sigma > 10^{-3}$

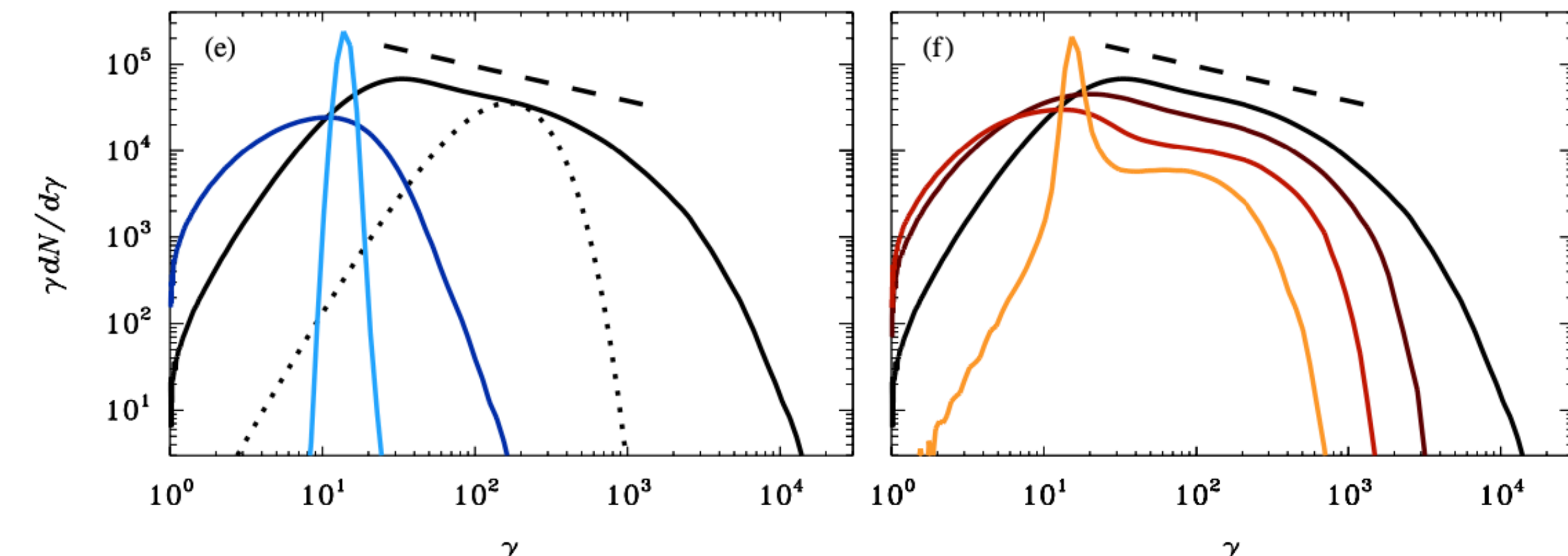
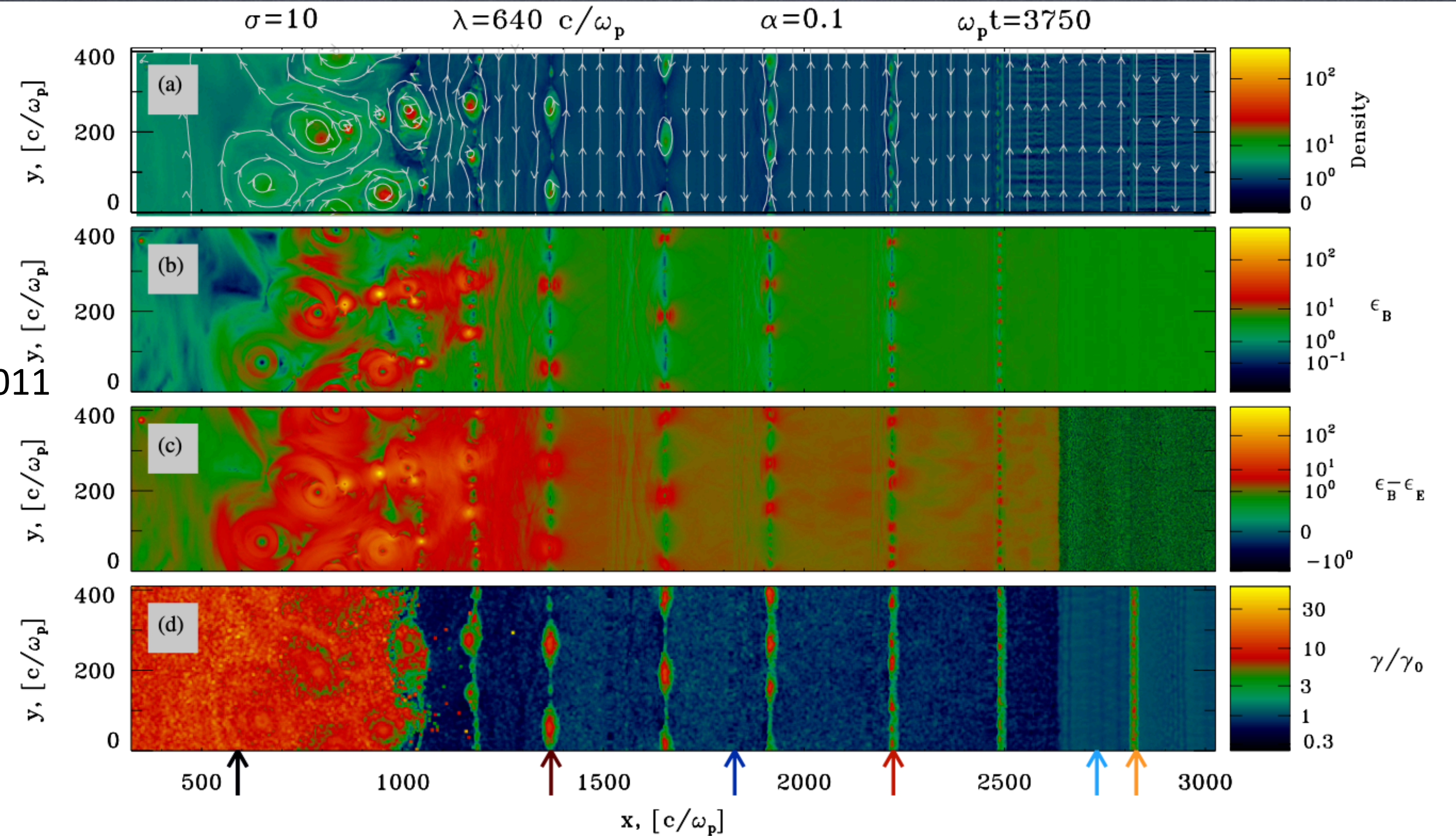
$$E_{\text{MAX}} \approx \sigma^{-1/4}$$



# FORCED MAGNETIC RECONNECTION



Sironi, Spitkovsky 2011



IN PRINCIPLE VERY FLAT SPECTRA AT LOW ENERGY

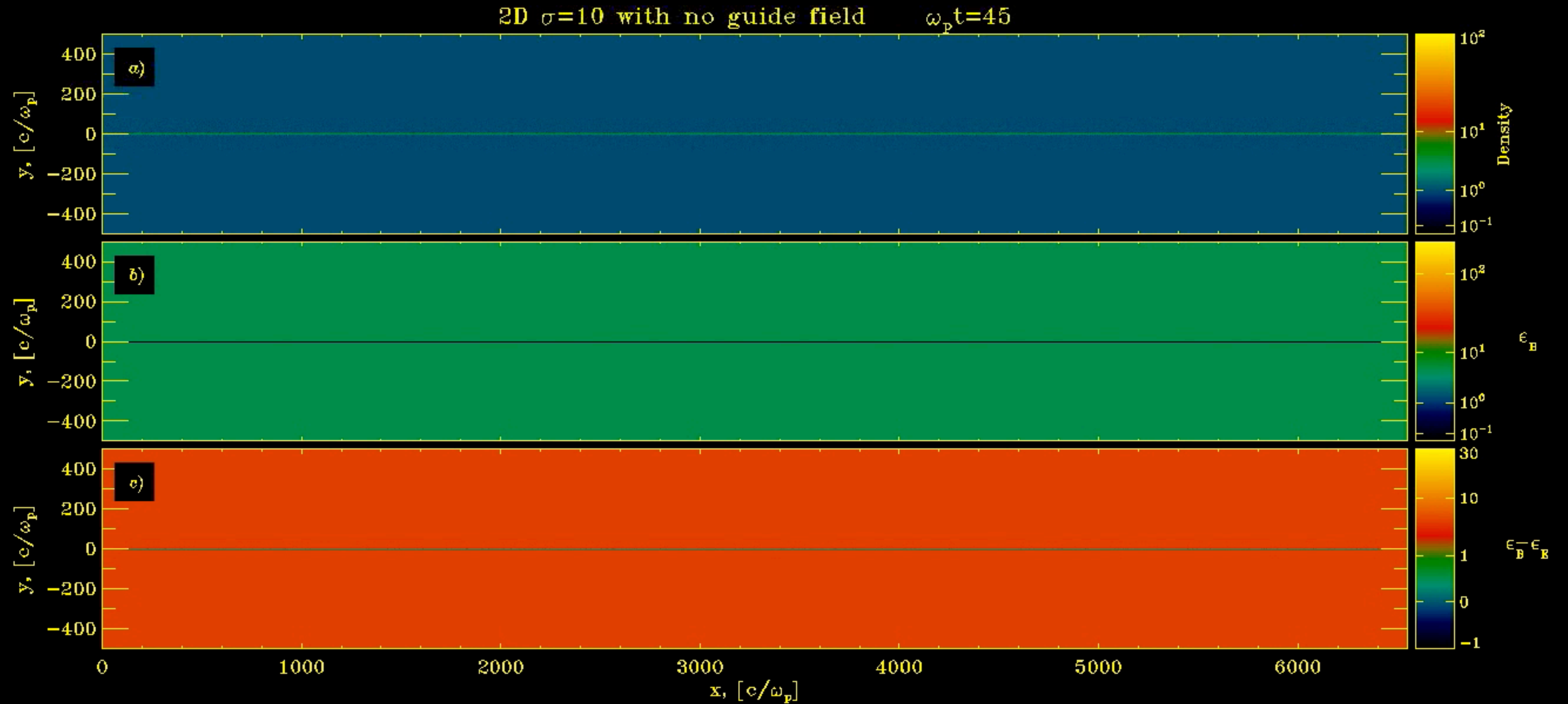
FERMI ACCELERATION IN UNMAGNETIZED PLASMA AFTERWARDS

RESULTS DEPEND ON

$$\sigma \text{ AND } \frac{\lambda}{r_L \sigma}$$

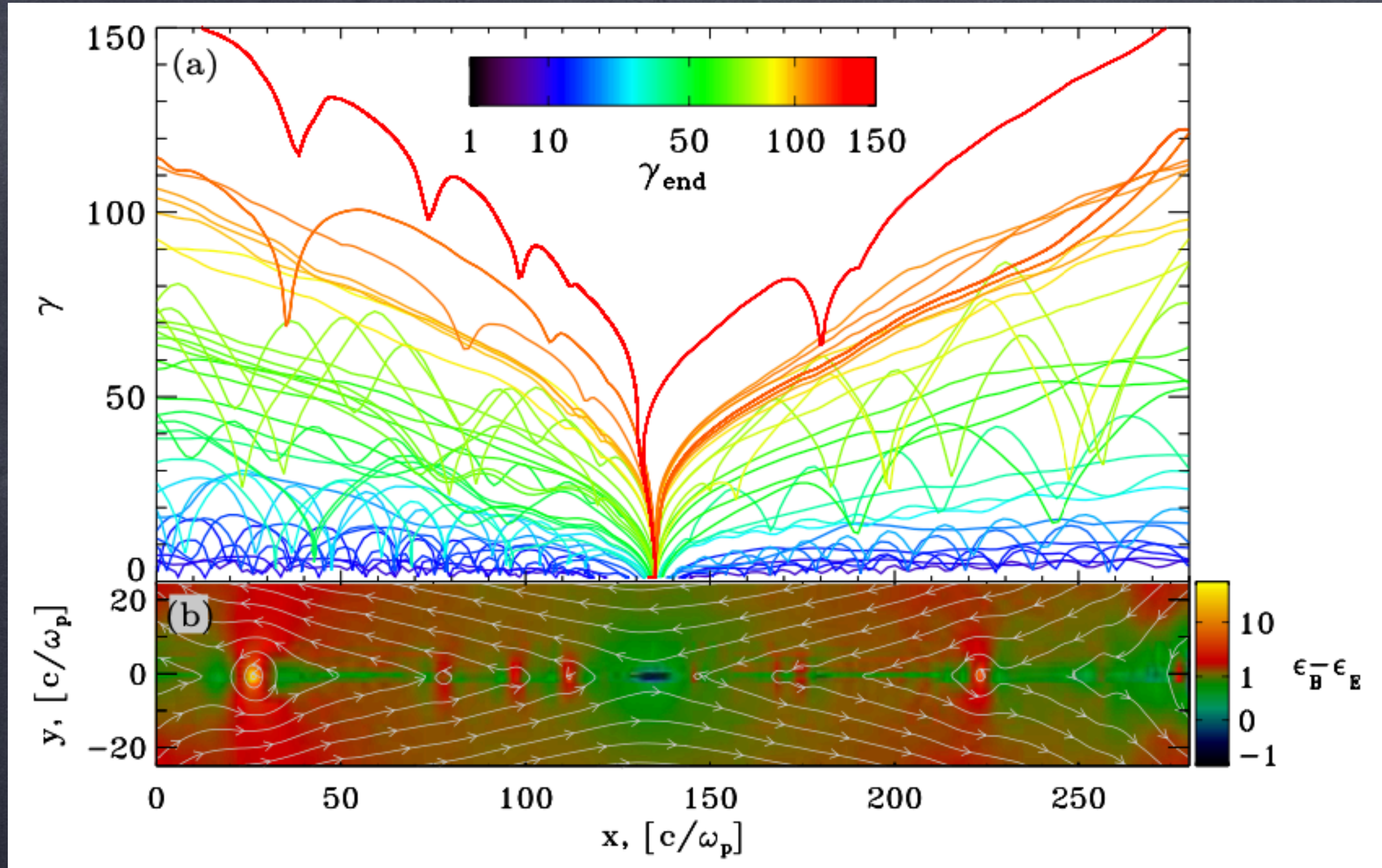


# FORCED MAGNETIC RECONNECTION





# FORCED MAGNETIC RECONNECTION



INTERACTION WITH X-POINT

DC ACCELERATION

THEN ADVECTION INTO MAJOR ISLANDS

SUCH LARGE  $K$  DIFFICULT TO ACCOUNT FOR

IF REALIZED, RECONNECTION BEFORE THE SHOCK

BROAD SPECTRUM



$$\sigma > 30$$

$$\frac{\lambda}{r_L \sigma} > \text{few} \times 10$$



BUT

$$\frac{\lambda}{r_L \sigma} = 4\pi\kappa \frac{R_L}{R_{TS}}$$



$$\kappa > \text{few} \times 10^7$$





# FORCED MAGNETIC RECONNECTION

$$\frac{\lambda}{r_L \sigma} = ?$$

$$\lambda = 2\pi R_L$$

$$r_L = \frac{mc^2 \gamma_{TS}}{e B_{TS}}$$

$$4\pi R_{TS}^2 \gamma_{TS} (1 + \sigma_{TS}) mc^3 n_{TS} = \frac{c}{4\pi} 4\pi B_L^2 R_L^2$$

$$n_{TS} = \kappa n_{GJ} \left( \frac{R_L}{R_{TS}} \right)^2$$

ENERGY CONSERVATION  
ALONG STREAMLINE

$$n_{GJ} = \frac{\Omega B_L}{2\pi e c}$$

$$\gamma_{TS} (1 + \sigma_{TS}) mc^2 n_{GJ} \kappa = \frac{B_L^2}{4\pi}$$

$$\gamma_{TS} = \frac{e B_L}{2 m c \kappa \Omega (1 + \sigma_{TS})}$$

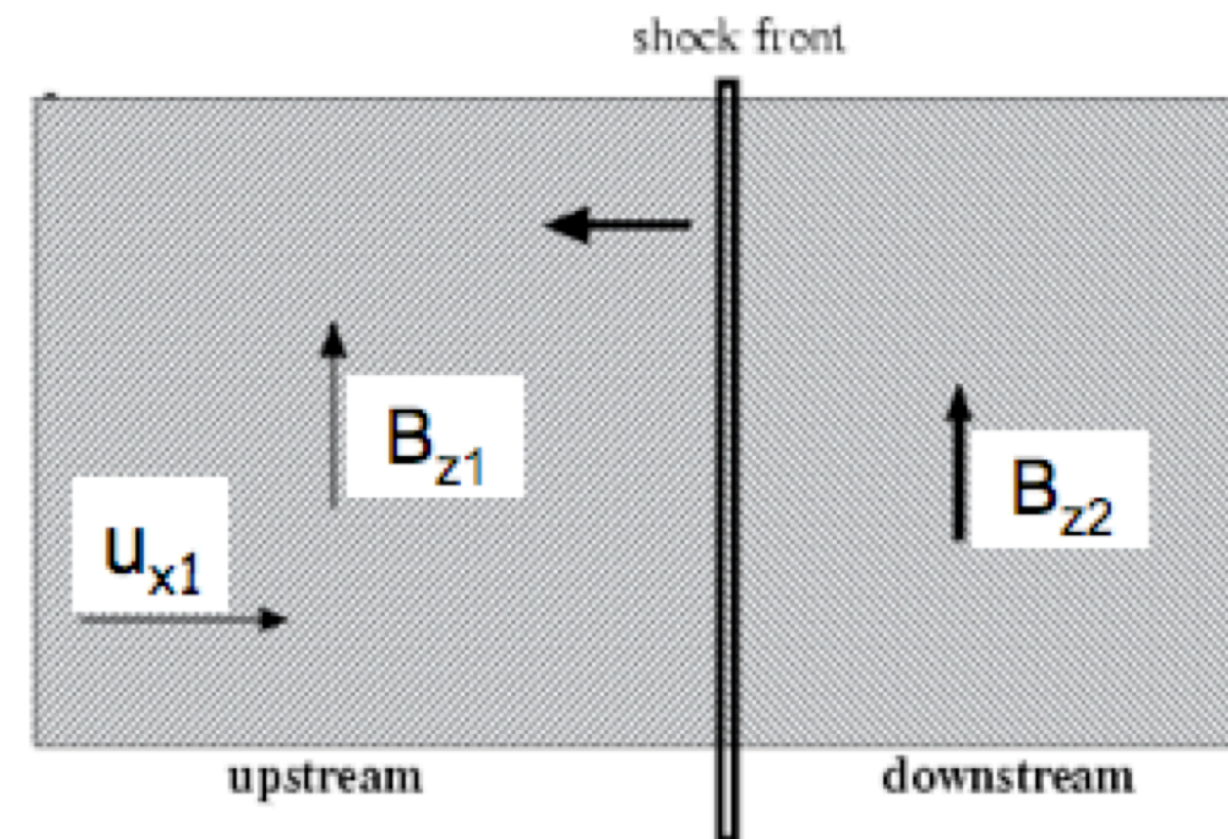
$$r_L = \frac{mc^2}{e} \frac{e B_L}{2 m c \kappa \Omega (1 + \sigma_{TS})} \frac{R_{TS}}{B_L R_L}$$

$$\frac{\lambda}{r_L \sigma} = \frac{2\pi R_L}{\sigma} \frac{2 R_L \kappa \Omega (1 + \sigma_{TS})}{c R_{TS}} = \frac{4\pi \kappa R_L}{R_{TS}}$$

$$r_L = \frac{c R_{TS}}{2(1 + \sigma_{TS}) \kappa \Omega R_L}$$

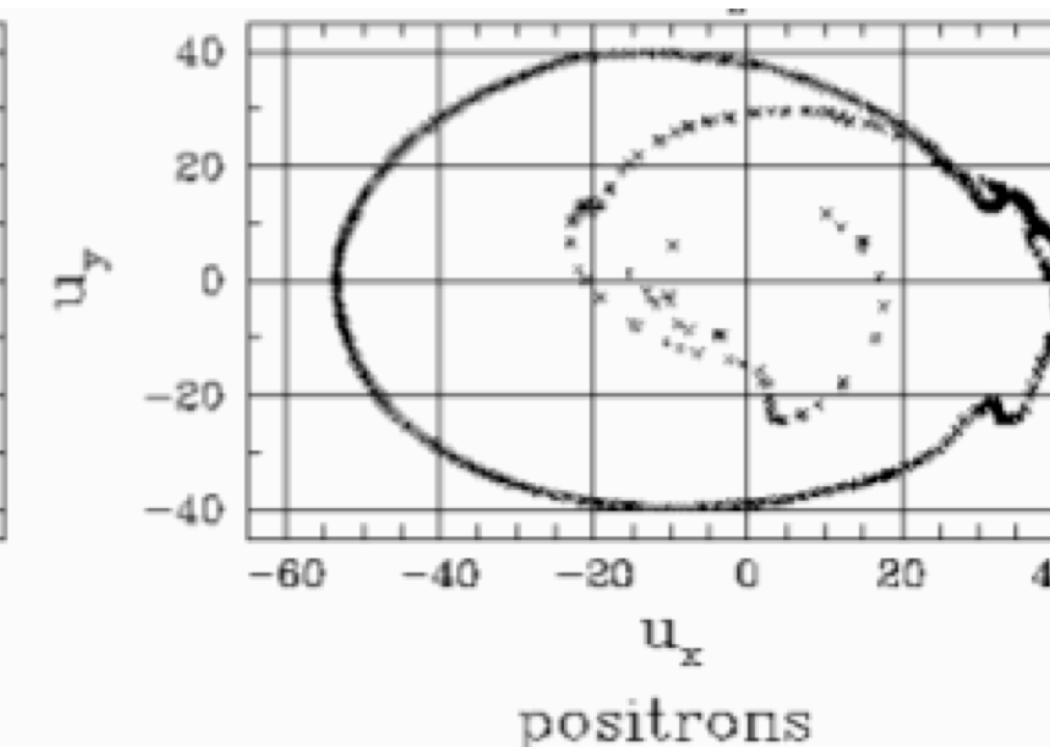
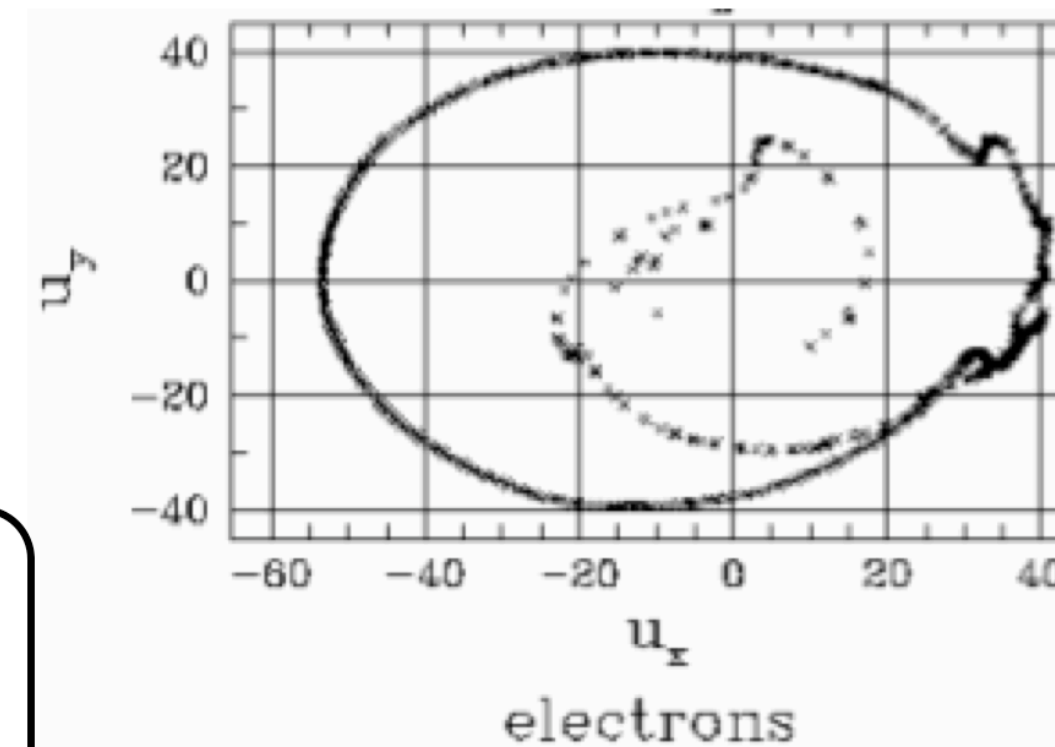


# RESONANT CYCLOTRON ABSORPTION IN ION DOPED PLASMA



Magnetic reflection mediates the transition

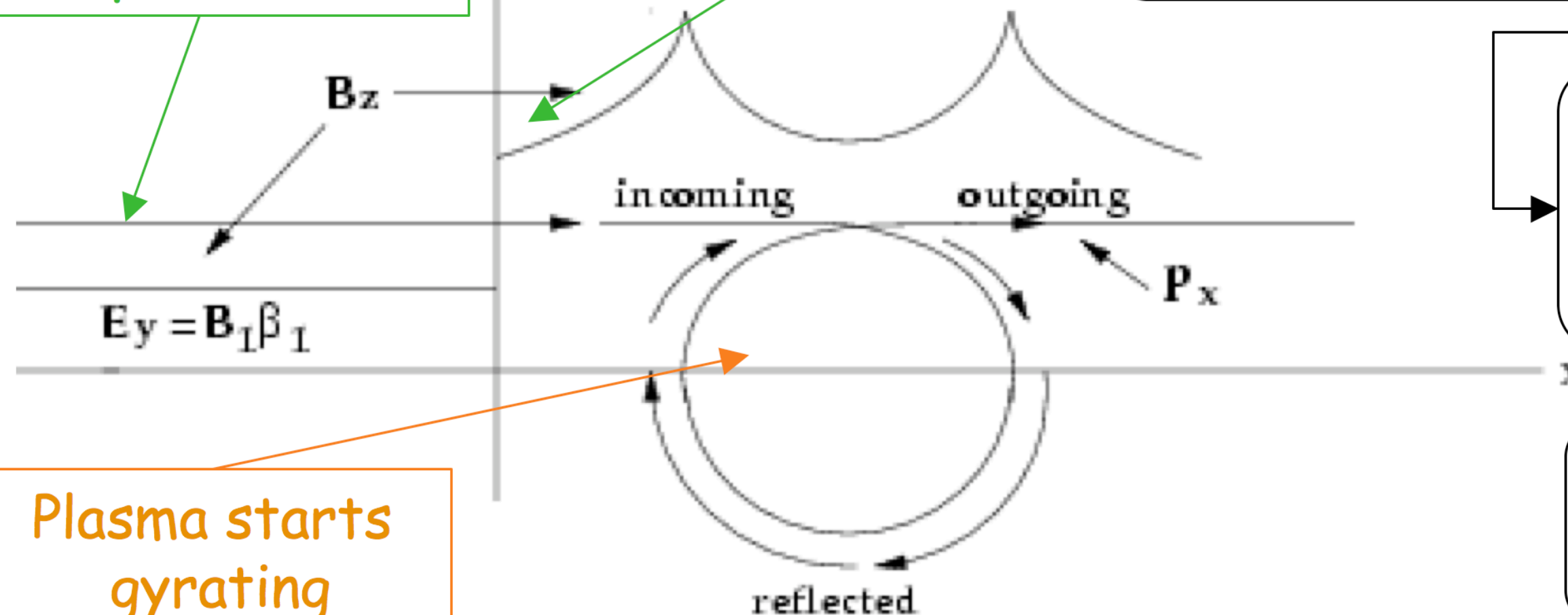
Configuration at the leading edge  
~ cold ring in momentum space



Drifting  $e^+e^-p$  plasma

B increases

Coherent gyration leads to  
collective emission of cyclotron waves



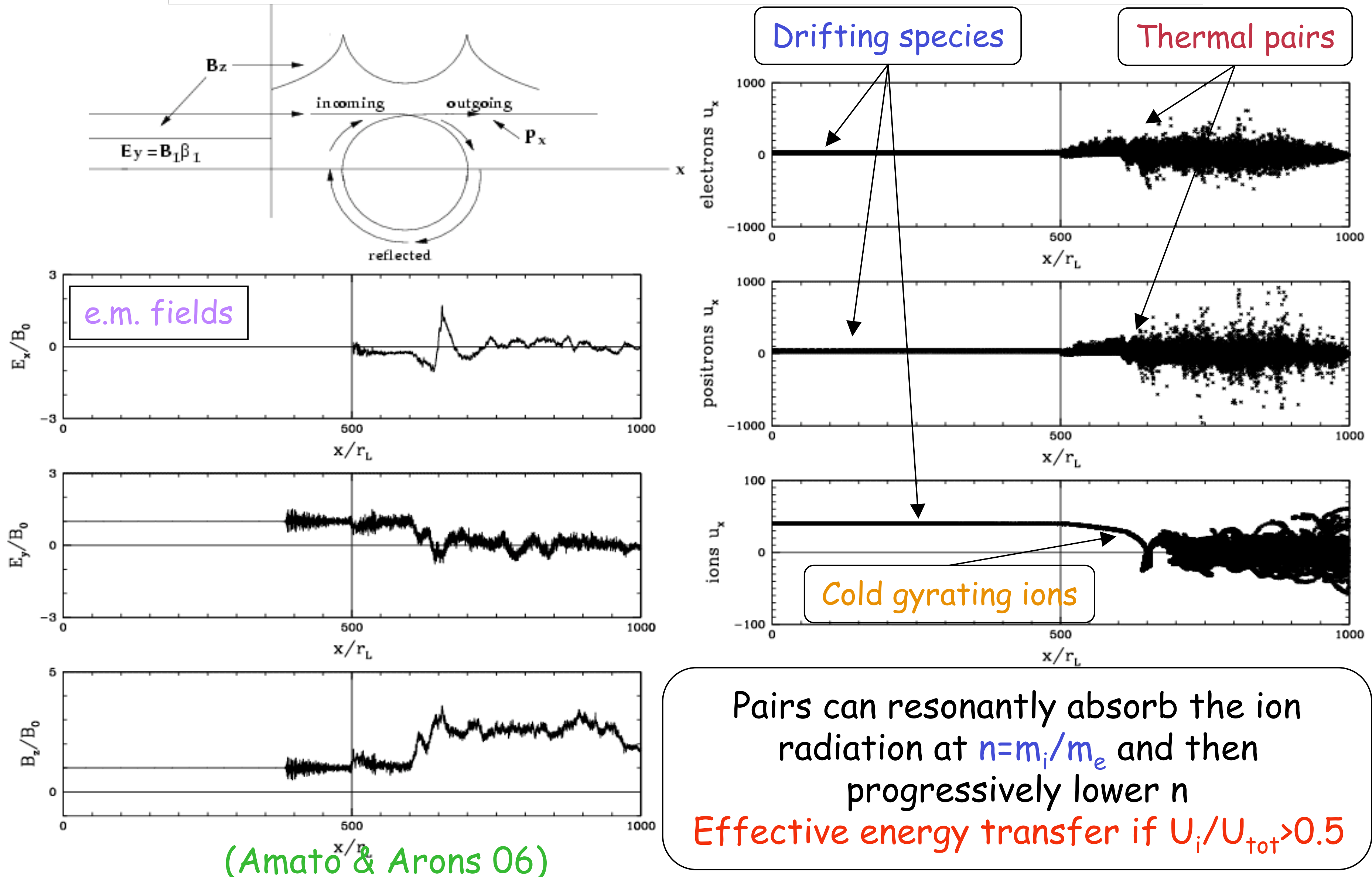
Plasma starts  
gyrating

Pairs thermalize to  
 $kT \sim m_e \Gamma c^2$  over  
 $10-100 \times (1/\Omega_{ce})$

Ions take their time:  
 $m_i/m_e$  times longer<sup>20</sup>

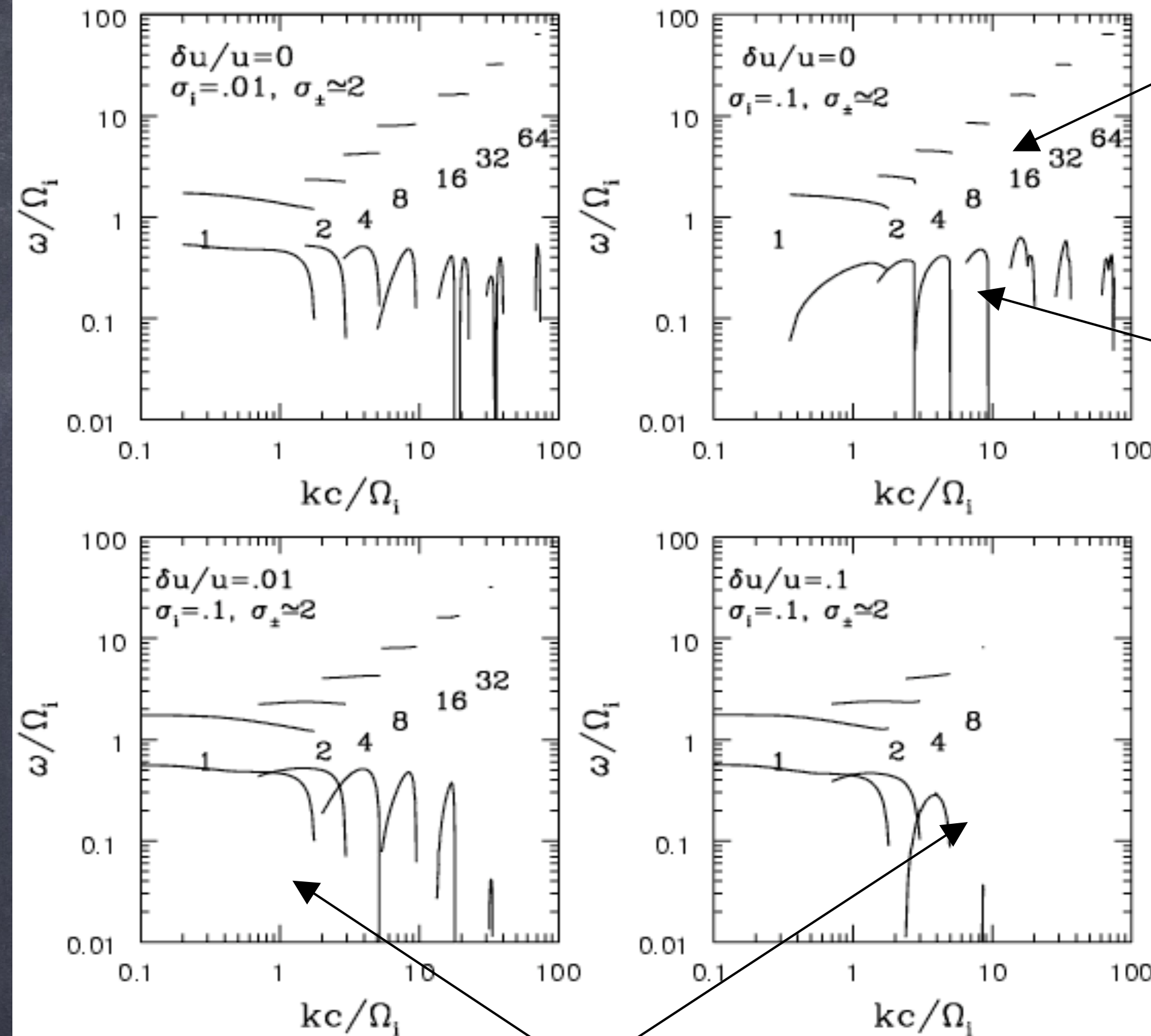


# LEADING EDGE OF THE SHOCK





# Subtleties of the RCA process



frequency

Ion cyclotron frequency  
( $m_e/m_i \Omega_{ce}$ )

Growth-rate

Electrons **initially** need  
 $n \sim m_i/m_e$   
for resonant absorption  
Then lower n

however  
growth-rate  $\sim$  independent of  
harmonic number  
(Hoshino & Arons 92)  
as long as ion plasma cold  
(Amato & Arons 06)

Spectrum is cut off at  $n \sim u/\delta u$

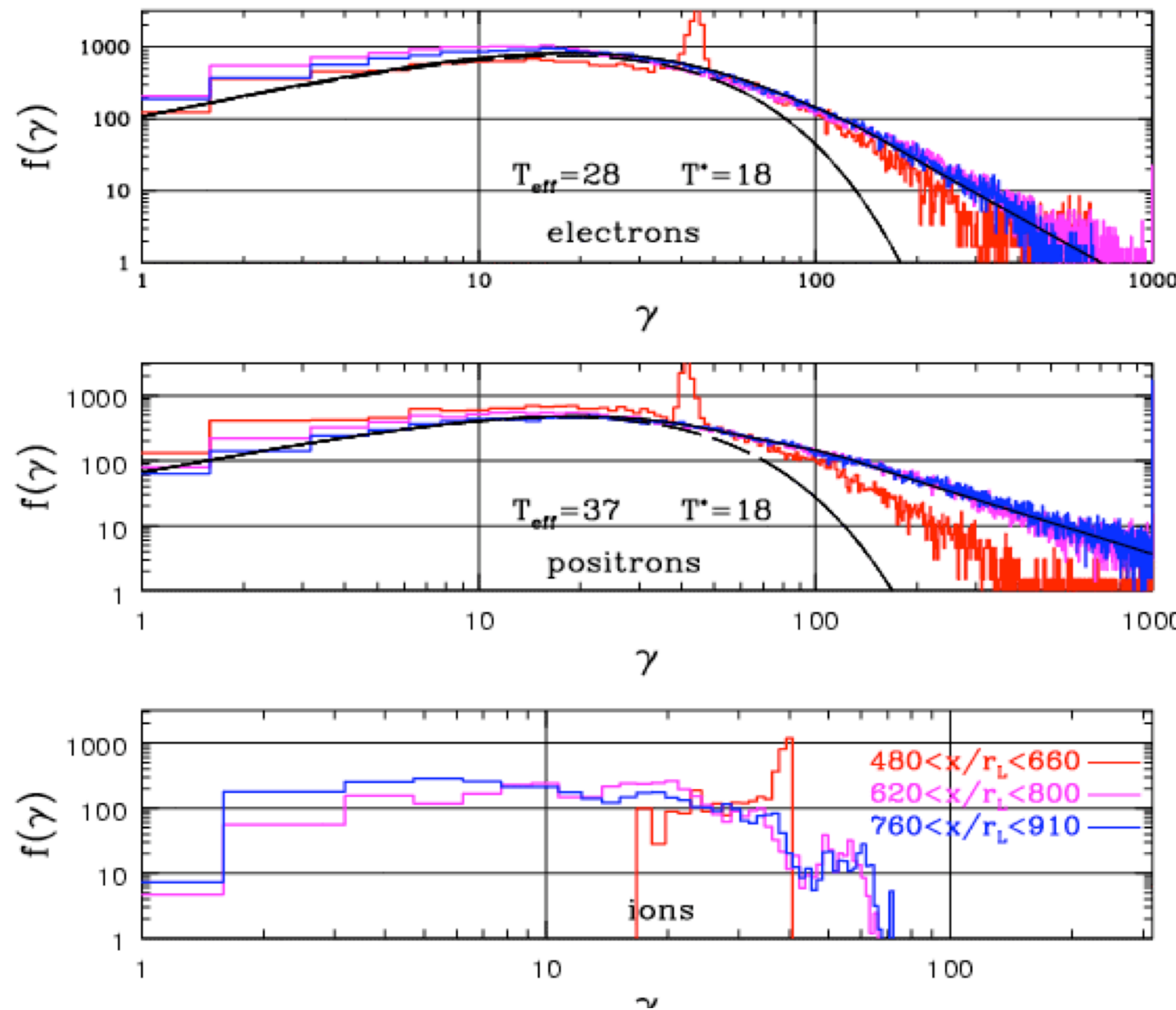
In order for the process to work the **pulsar wind** must be  
really **very cold** ( $\delta u/u < m_e/m_i$ )!!!!



# PARTICLE SPECTRA AND ACCELERATION EFFICIENCY

IF

- IONS CARRY MOST OF THE ENERGY:  $\kappa < m_i/m_e$
- WIND SUFFICIENTLY COLD:  $\delta u/u < m_e/m_i$



## ACCELERATION EFFICIENCY:

- $\sim \text{few\%}$  for  $U_i/U_{\text{tot}} \sim 60\%$
- $\sim 30\%$  for  $U_i/U_{\text{tot}} \sim 80\%$

## SPECTRAL SLOPE:

- $> 3$  for  $U_i/U_{\text{tot}} \sim 60\%$
- $< 2$  for  $U_i/U_{\text{tot}} \sim 80\%$

## MAXIMUM ENERGY:

- $\sim 20\% m_i c^2 \Gamma$  for  $U_i/U_{\text{tot}} \sim 60\%$
- $\sim 80\% m_i c^2 \Gamma$  for  $U_i/U_{\text{tot}} \sim 80\%$



# PARTICLE ACCELERATION MECHANISMS

## SUMMARY

### FERMI MECHANISM

- ✓ EFFICIENT AT UNMAGNETIZED  $e^+e^-$  RELATIVISTIC SHOCKS [Spitkovsky 08]
- NO ACCELERATION AT  $\sigma > 0.001$  SUPERLUMINAL SHOCKS [Sironi & Spitkovsky 09, 11]
- TOO SLOW TO GUARANTEE MAXIMUM ENERGY OBSERVED IN CRAB [Pelletier+ 17]
- ✓ POSSIBLY EFFICIENT AT HIGHLY TURBULENT MODERATELY MAGNETIZED SHOCKS [Lemoine 17, Giacinti & Kirk 18, Cerutti & Giacinti 20]
- ✓ RIGHT SPECTRUM FOR X-RAYS

### DRIVEN MAGNETIC RECONNECTION:

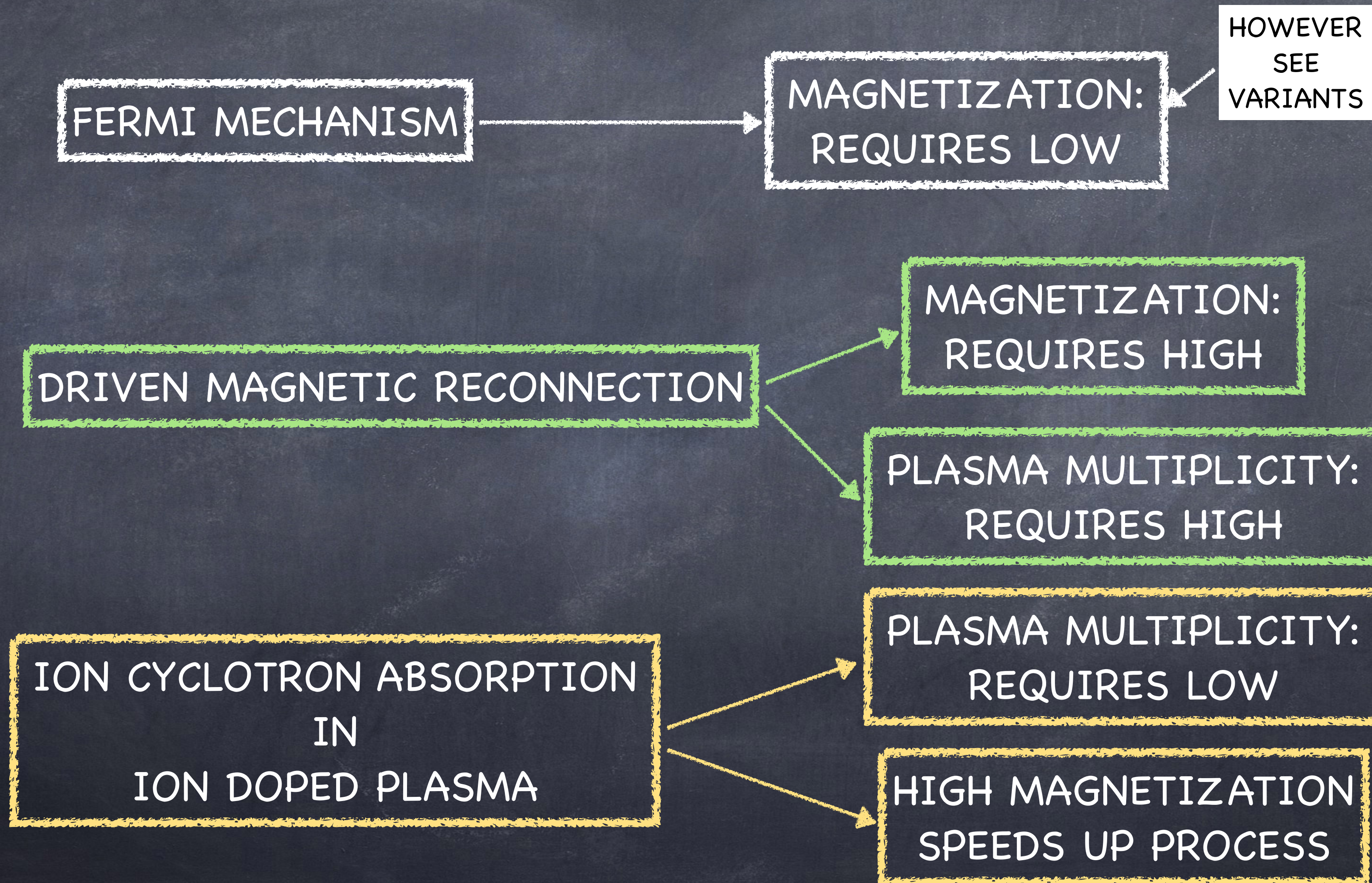
- ✓ BROAD AND HARD PARTICLE SPECTRA IF  $\sigma \geq 30$  AND  $\kappa > 10^8$  [Sironi & Spitkovsky 11b]
- FOR THIS LARGE  $\kappa$  WIND LIKELY TO DISSIPATE BEFORE SHOCK [Kirk & Skjeraasen 03]

### RESONANT CYCLOTRON ABSORPTION:

- ✓ SPECTRA AND ACCELERATION EFFICIENCY DEPEND ON ENERGY FRACTION IN IONS:  
 $U_i/U_{\text{TOT}} = 0.8-0.6$ ,  $\gamma = 1.5-3$ ,  $\epsilon_{\text{ACC}} = 0.3-0.03$  [Hoshino+ 92, EA & Arons 06; Stockem+ 12]
- ✓ HIGHER  $\sigma$  IMPLIES FASTER ACCELERATION
- NO ACCELERATION IF  $\kappa > m_i/m_e$



# PARTICLE ACCELERATION MECHANISMS: SUMMARY OF REQUIREMENTS





# PARTICLE ACCELERATION MECHANISMS (MORE RECENTLY PROPOSED)

## SHOCK CORRUGATION

- FORMULATED TOGETHER WITH  $B$  DISSIPATION [Lemoine 17, Lyutikov+12]
- INTERESTING SCENARIO FOR SPEEDING UP FERMI PROCESS

## TURBULENT ACCELERATION AT THE SHOCK

- ASSUMES DIFFERENT TURBULENCE LEVELS AT DIFFERENT SHOCK LATITUDES [Giacinti & Kirk 18]
- PRODUCES HARD (STEEP) SPECTRA FOR LOW (HIGH) TURBULENCE LEVEL
- INTERESTING LATITUDE DEPENDENCE OF SPECTRAL INDEX
- ACCELERATES ONE SIGN OF CHARGES PREFERENTIALLY
- ANISOTROPIC FIELD HELPS PROVIDING THE TURBULENCE [Cerutti & Giacinti 20]
- SPECTRUM HARDENS WITH INCREASING MAGNETIZATION

## ACCELERATION BY HIGH $\sigma$ TURBULENCE

- ENERGY DEPENDENT ANISOTROPY OF PARTICLE DISTRIBUTION MIMICS FLAT PARTICLE SPECTRA AT LOW ENERGY [Comisso+ 18,19,20, Luo+21]
- WHERE? ON WHAT SCALES? MAXIMUM ENERGY?
- IMPORTANT BROAD IMPLICATIONS!



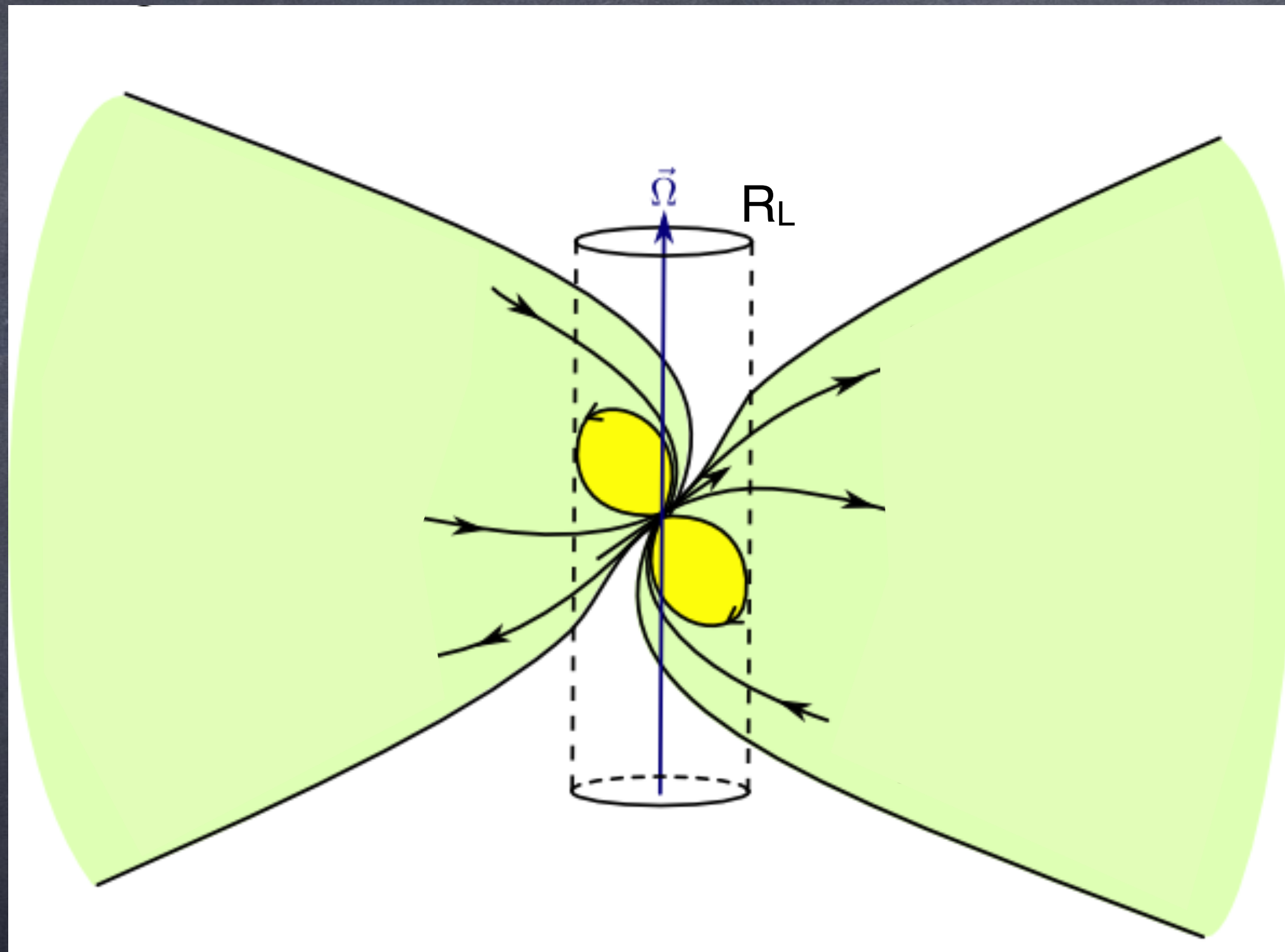
BACK TO PWNe



# THE $\sigma$ PROBLEM

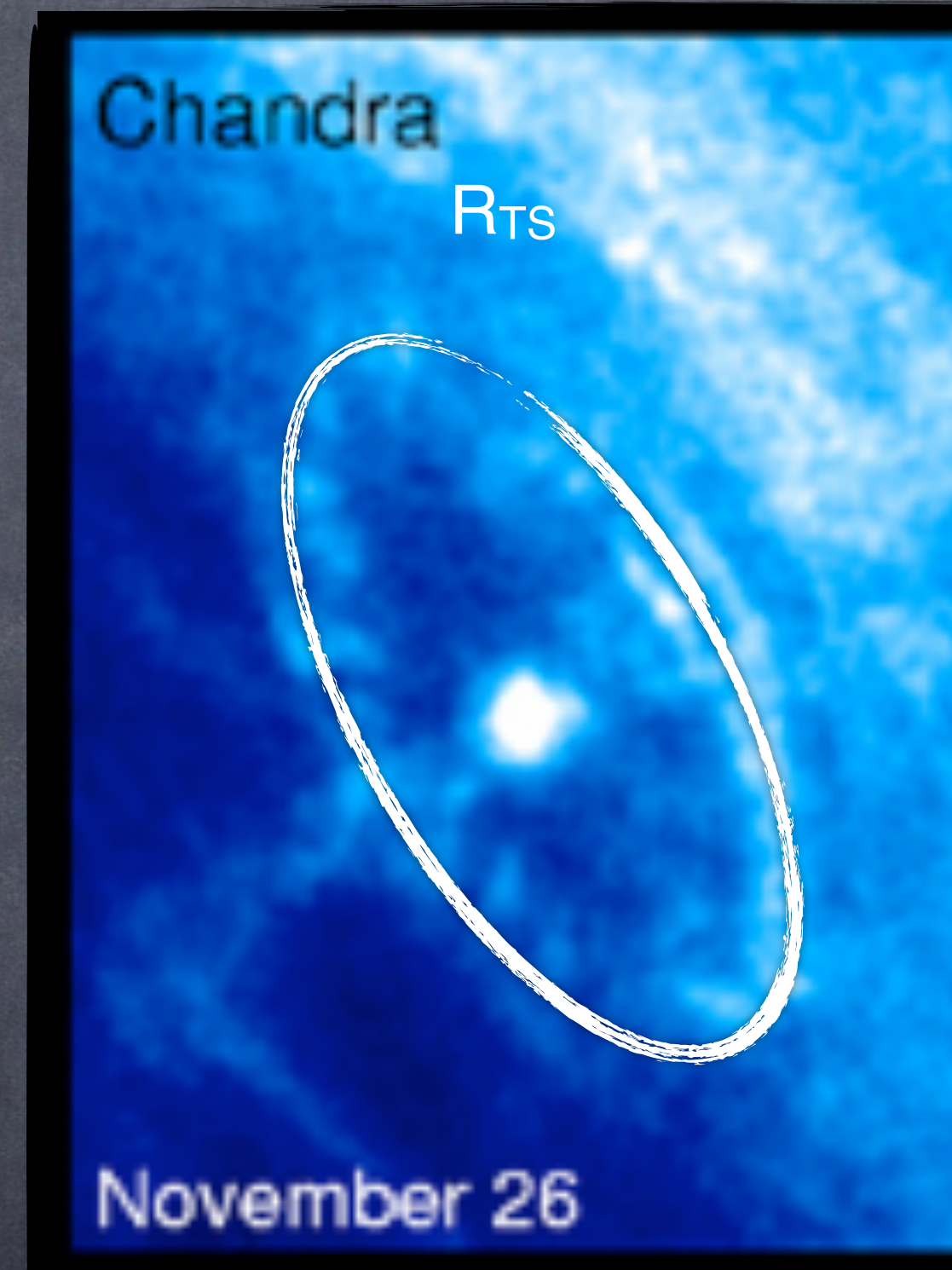
FROM PULSAR THEORIES:

$$\sigma \sim 10^4 @ R_L$$



FROM 1D PWN MODELS:

$$\sigma \sim 10^{-3} @ R_{TS}$$

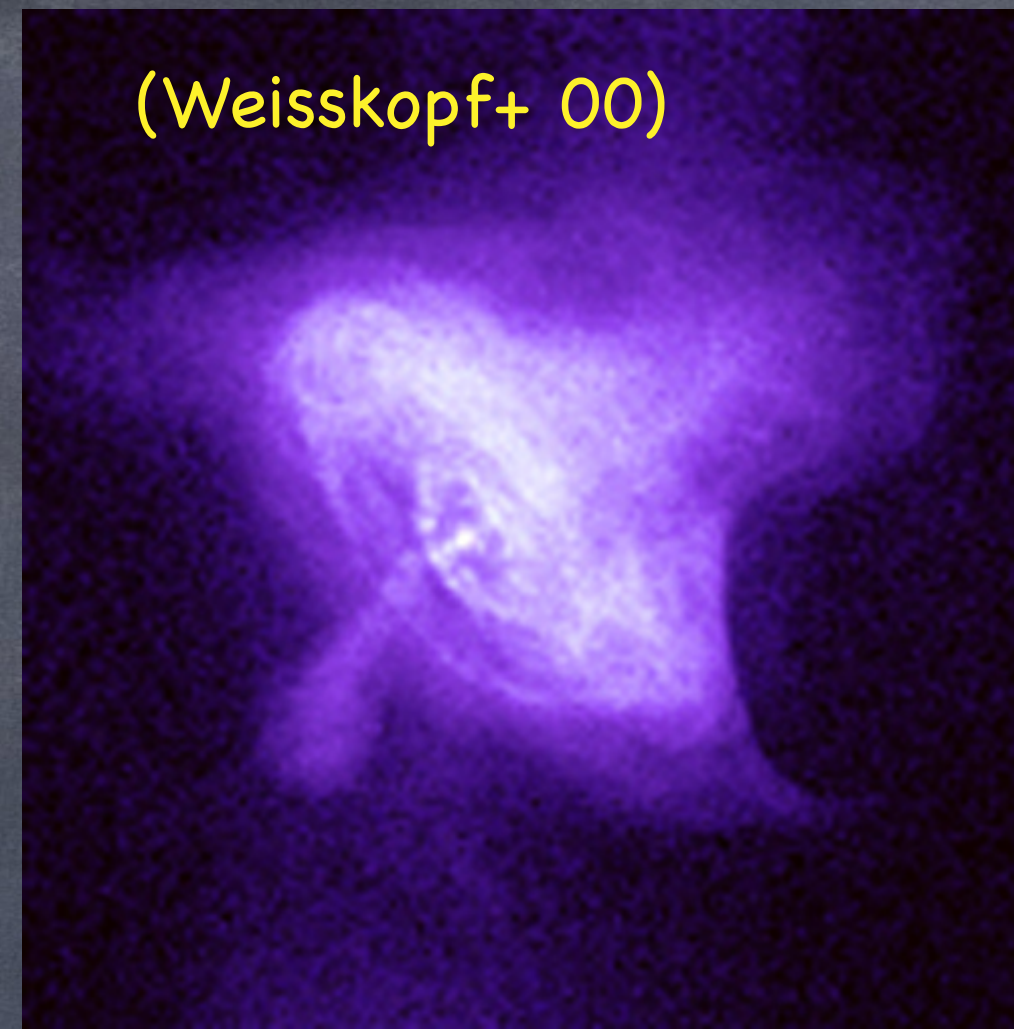
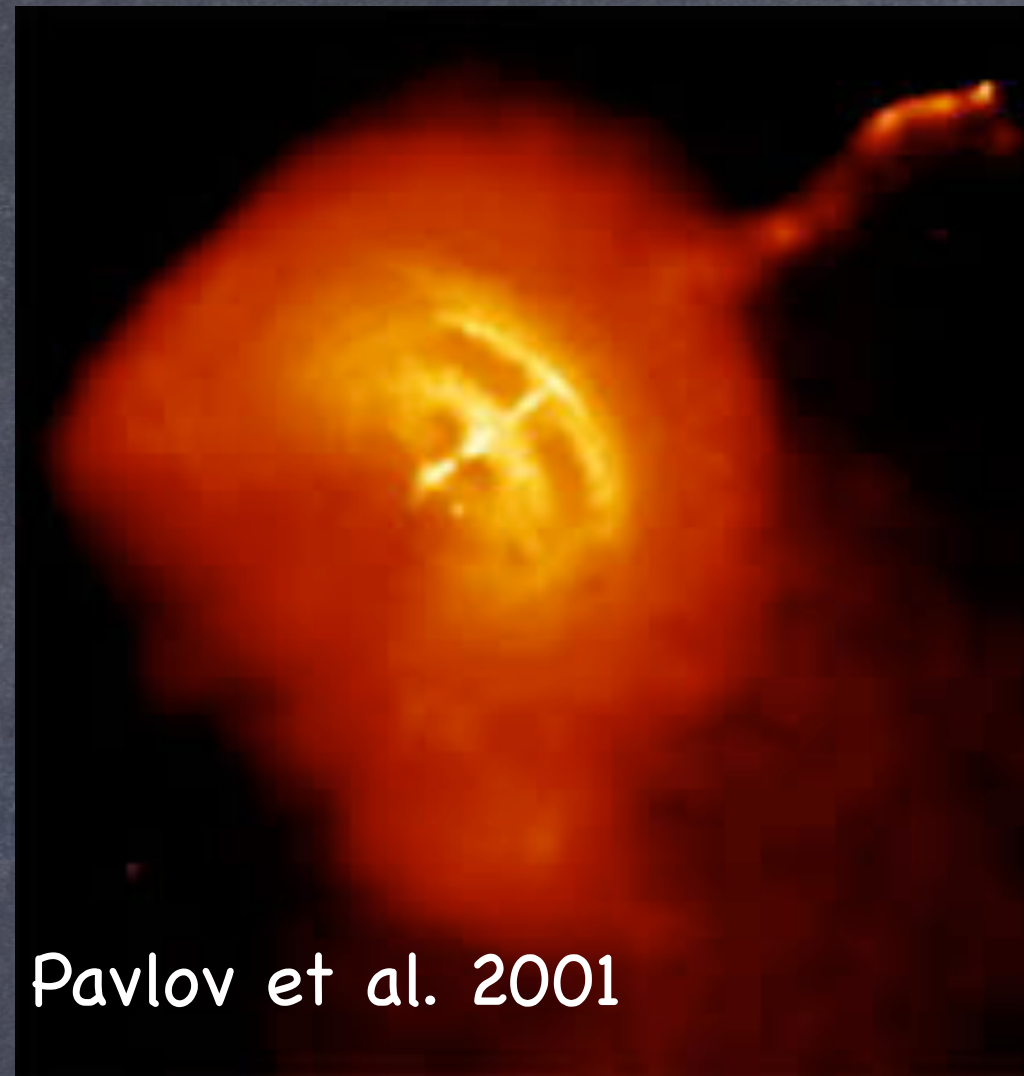


$$R_{TS} \sim 10^9 R_L$$

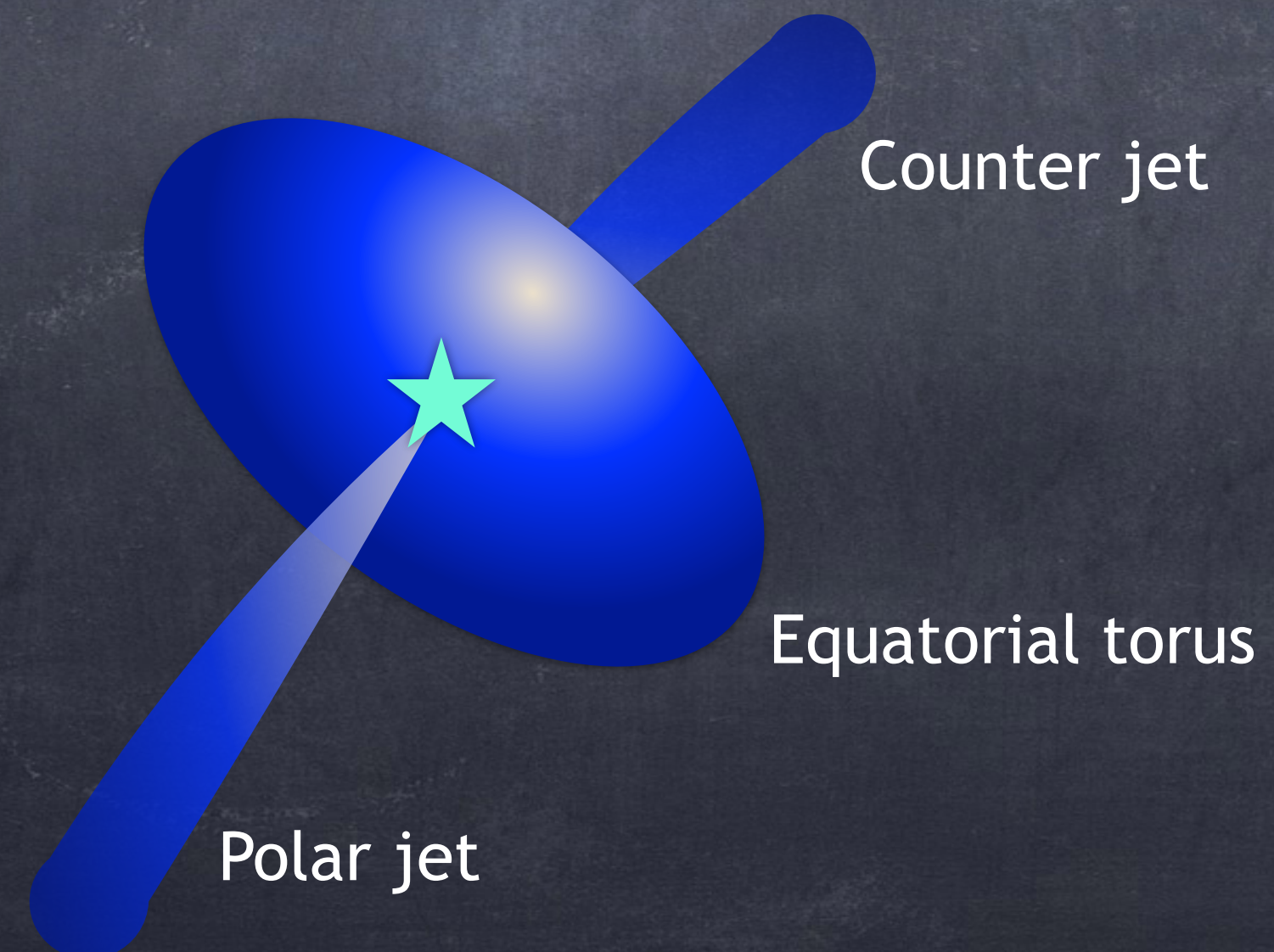
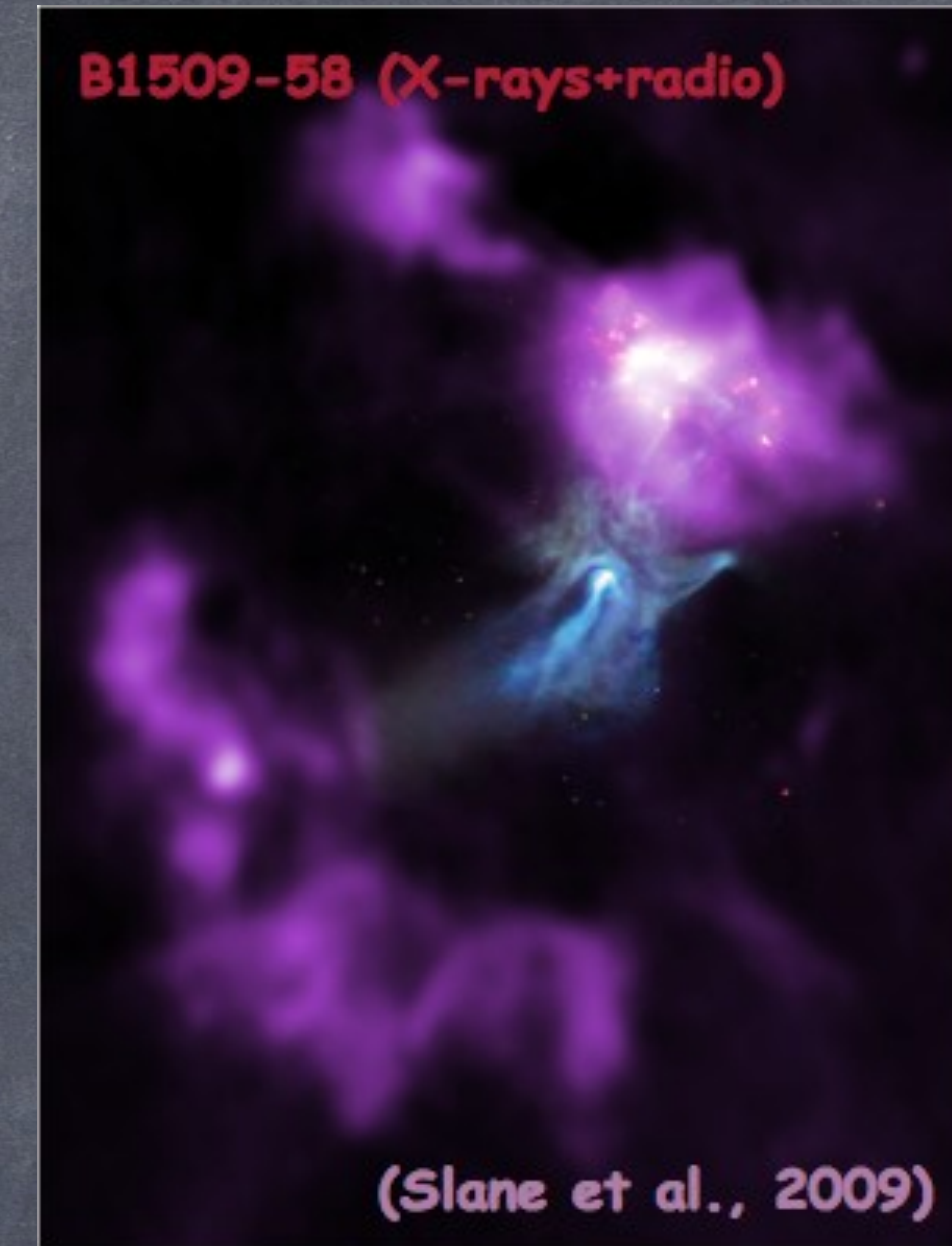
BUT ENERGY CONVERSION  
DIFFICULT TO EXPLAIN



# CHANDRA'S VIEW OF PWNE



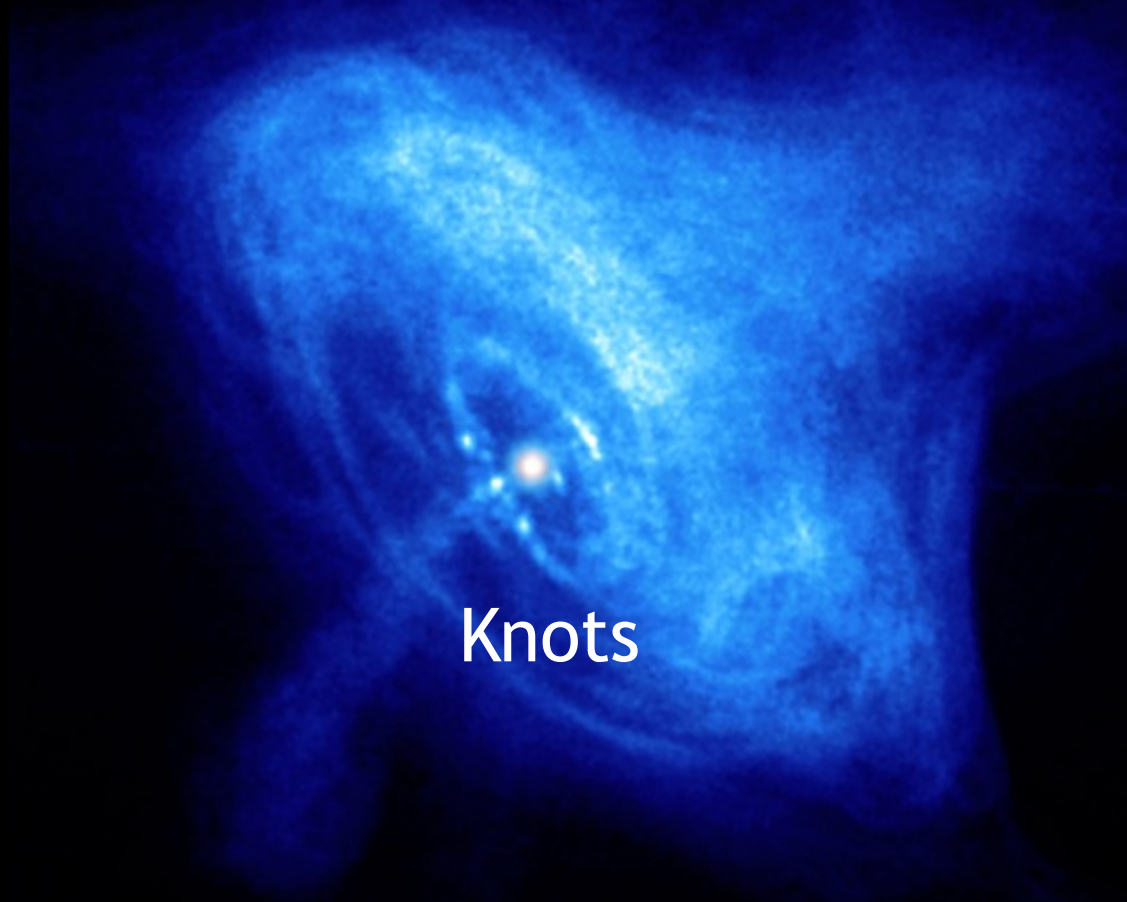
Jet-torus morphology of inner nebula





# THE JET PUZZLE

Crab Nebula (Weisskopf+ 00)



Knots



Arcs

Vela Nebula (Pavlov+ 01)

JET FROM  
 $R < R_{TS}$

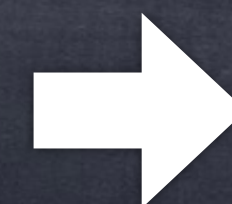
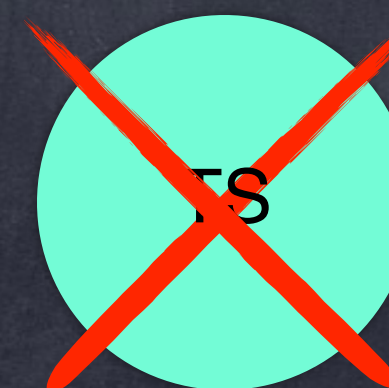


BUT NO MAGNETIC  
COLLIMATION  
IN  
RELATIVISTIC FLOWS  
[Lyubarsky & Eichler 01]

$$\Gamma \gg 1 \rightarrow \rho \vec{E} + \vec{J} \times \vec{B} \sim 0$$

$$F \propto \sin^2(\theta)$$

[Bogovalov & Khangoulia 02, Lyubarsky 02]

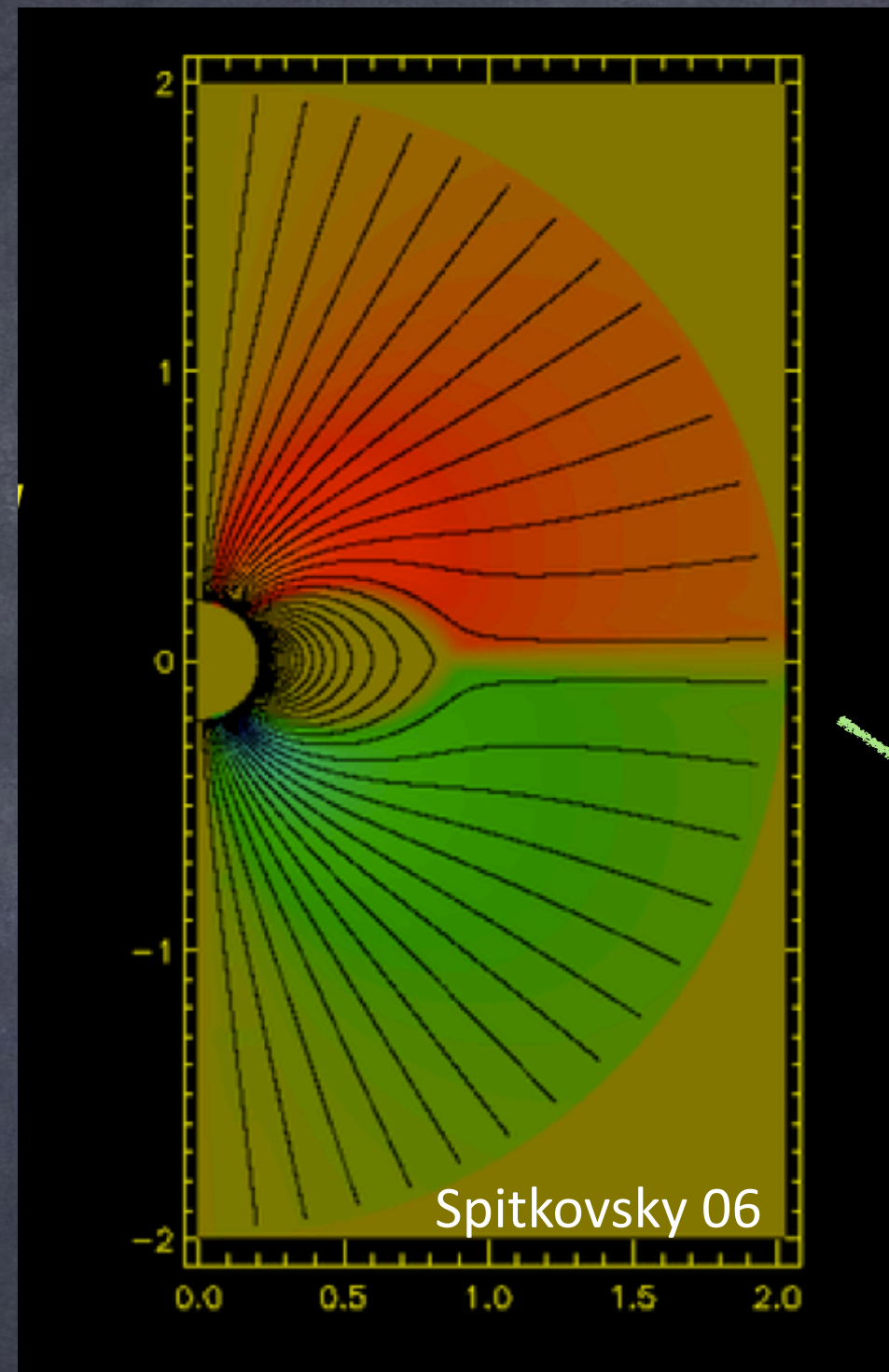


$$z_{TS} \ll R_{TS}$$





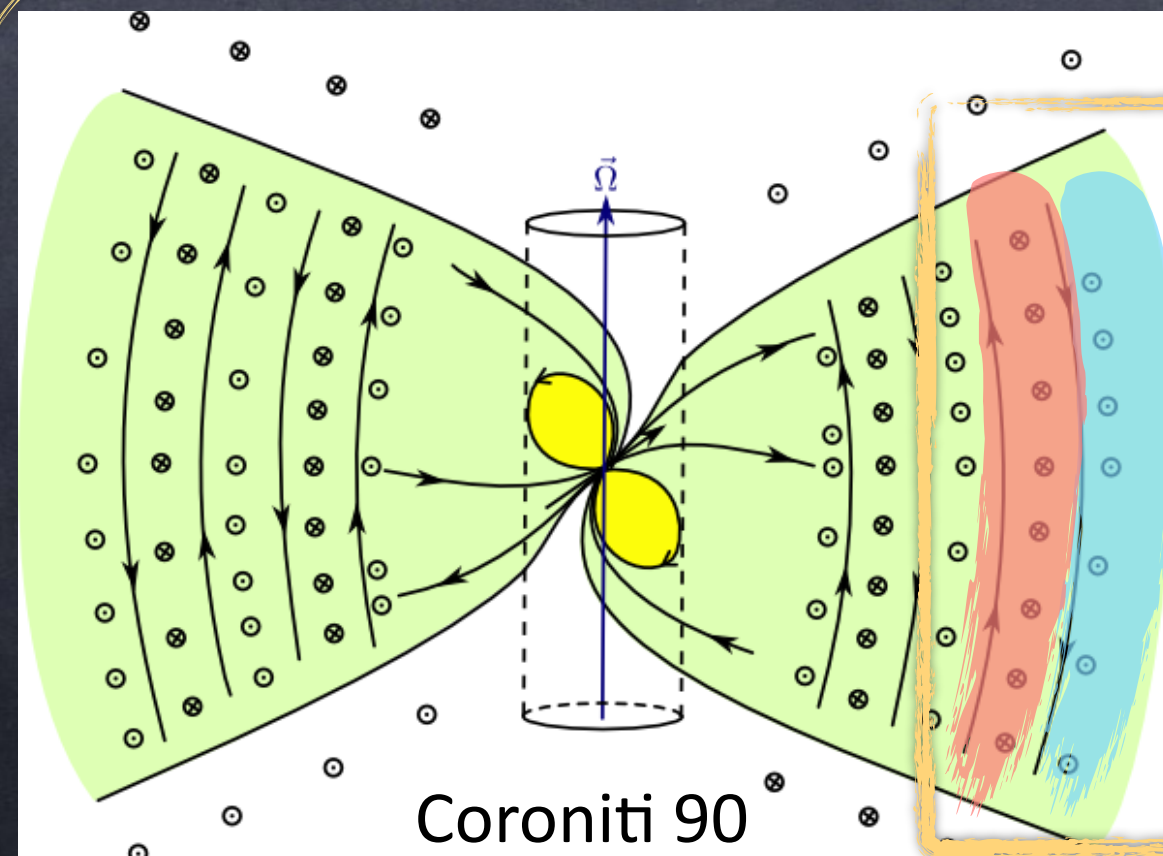
# THE ANISOTROPIC PULSAR WIND



Komissarov & Lyubarsky 03, 04; Del Zanna+ 04, 06; Bogovalov+ 05  
Camus+ 09; Volpi+ 08; Olmi+ 14, 15

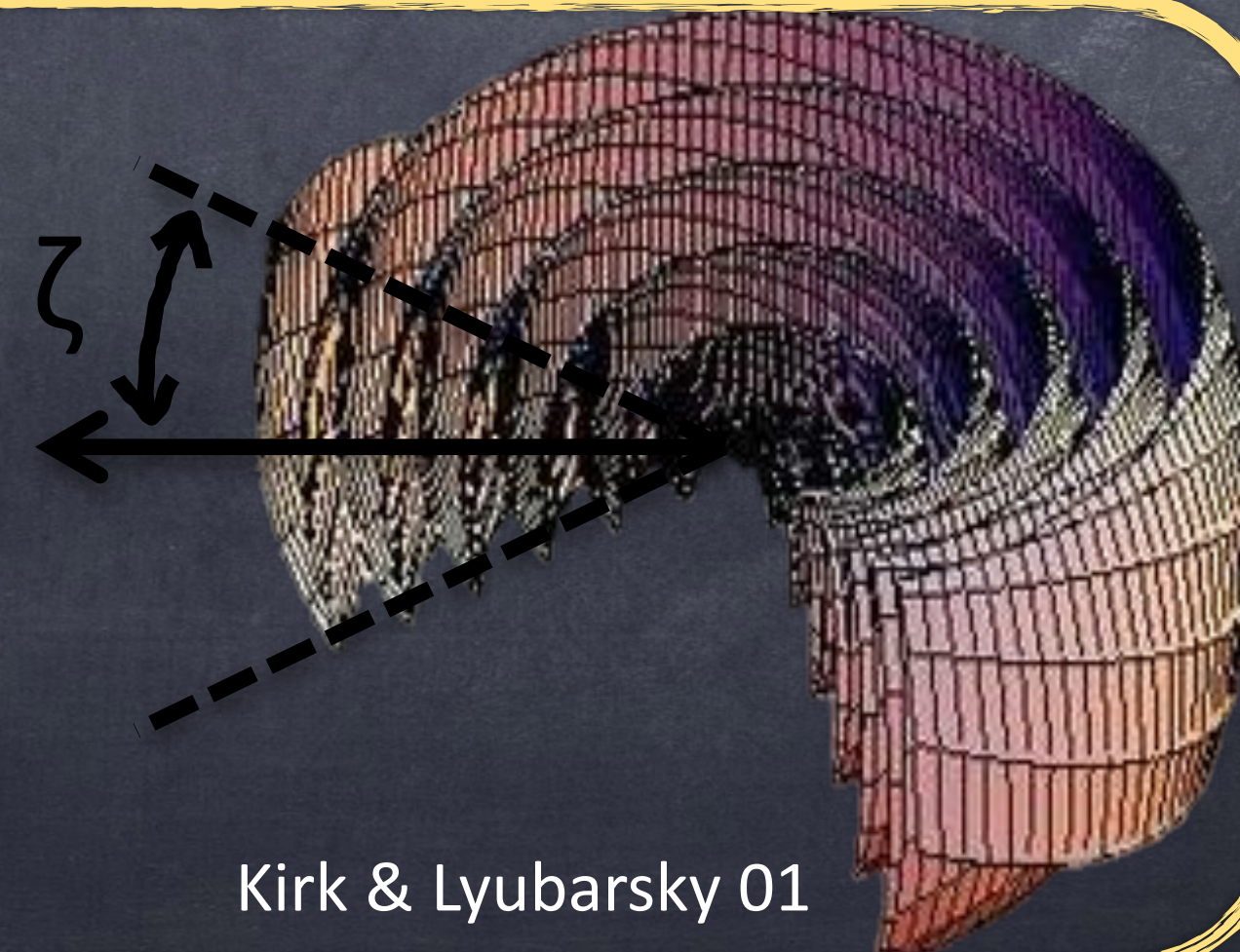
$$F(\theta) \propto \sin^2(\theta)$$

$$B(\theta) \propto \sqrt{\sigma} \sin \theta G(\theta)$$



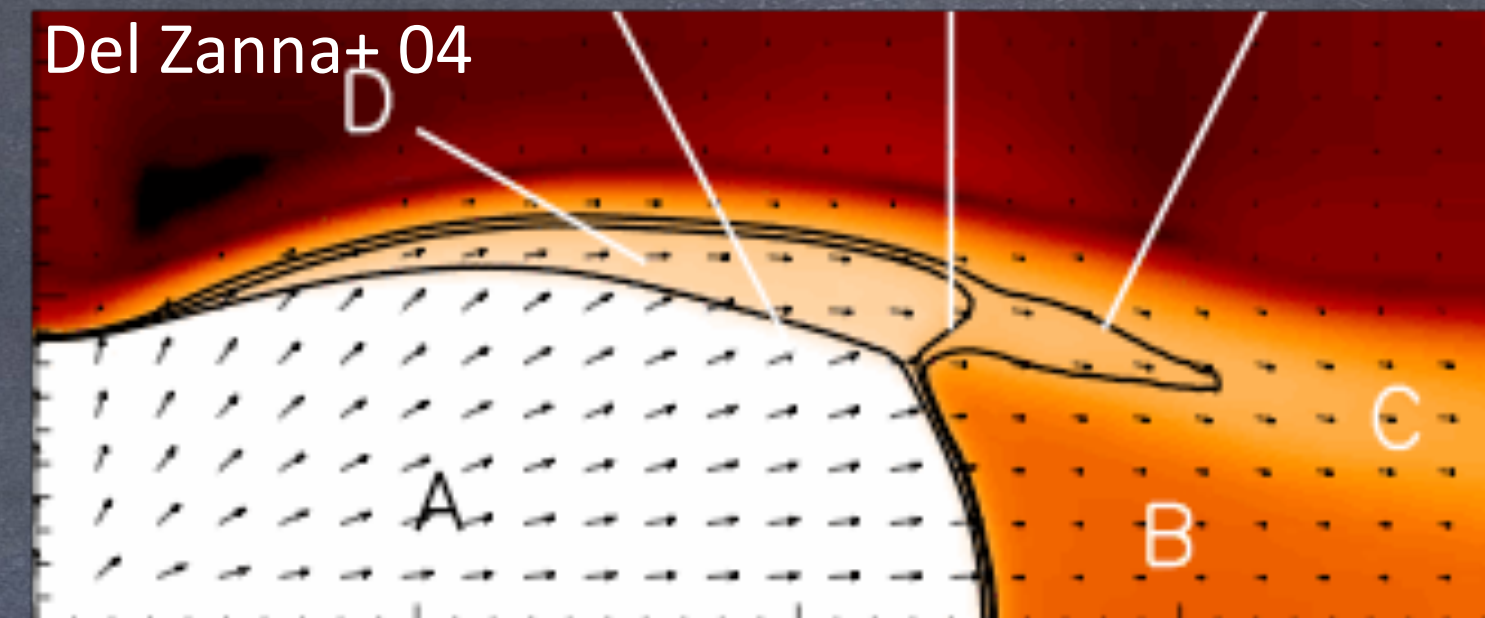
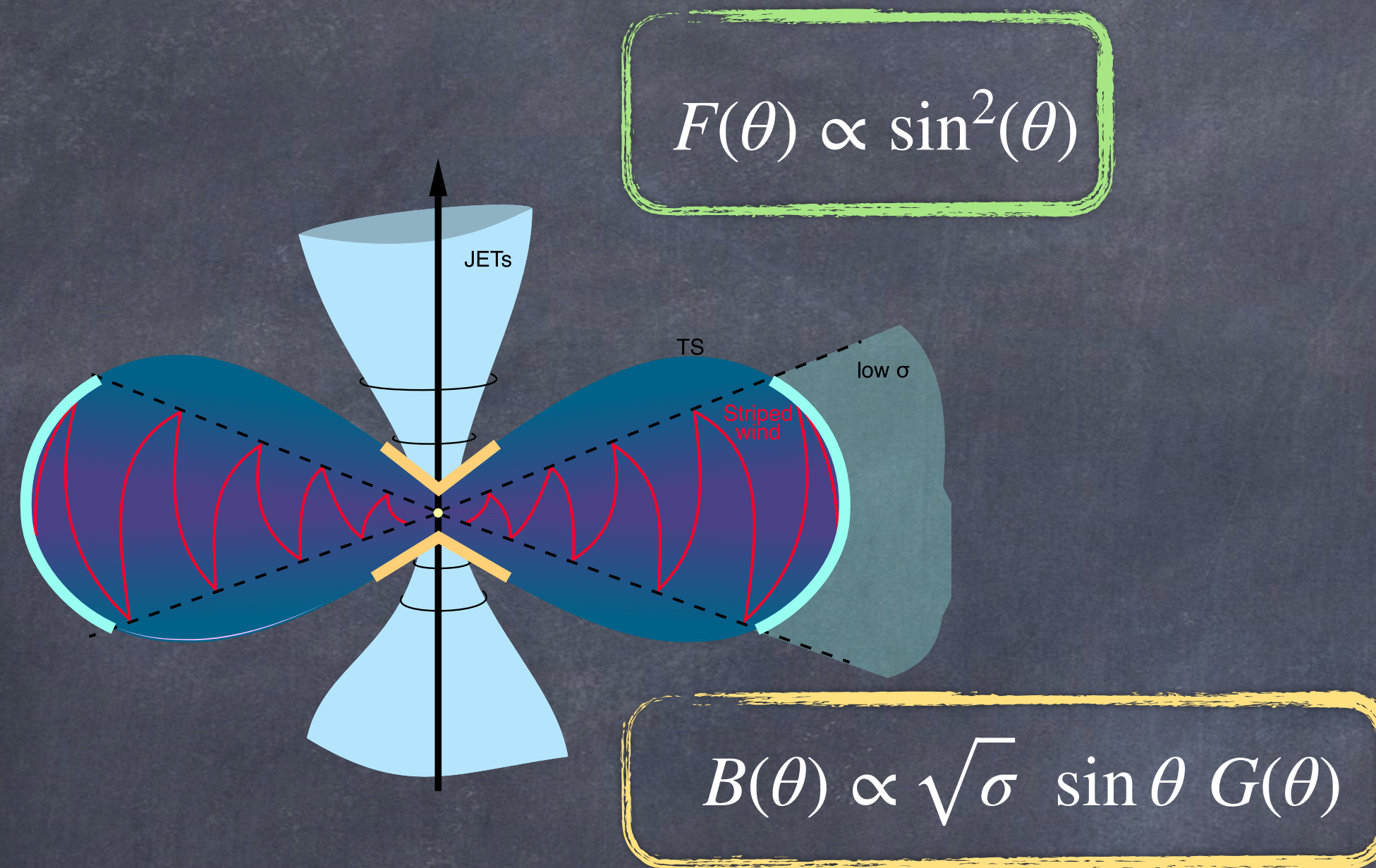
alternating  
stripes of  
opposite B  
polarities

dissipation in  
current sheet

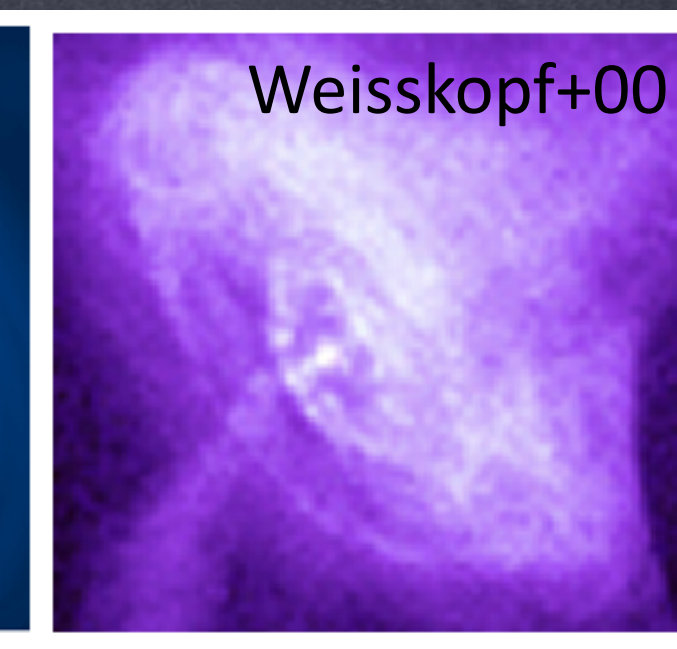
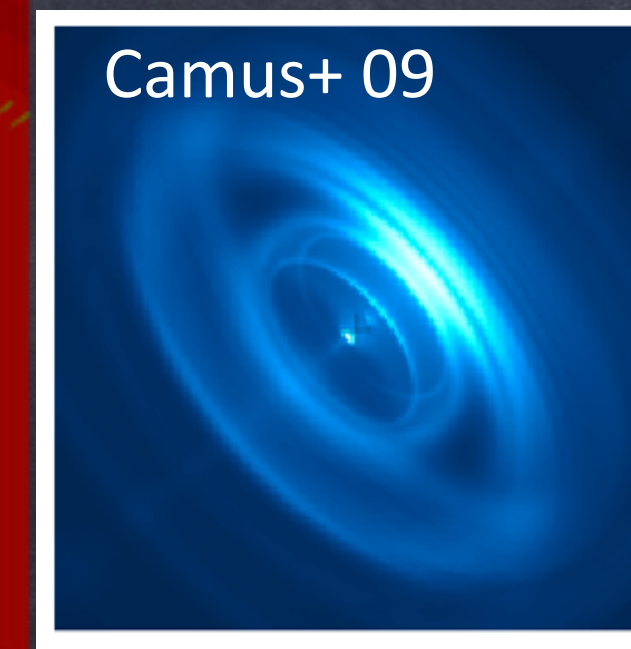
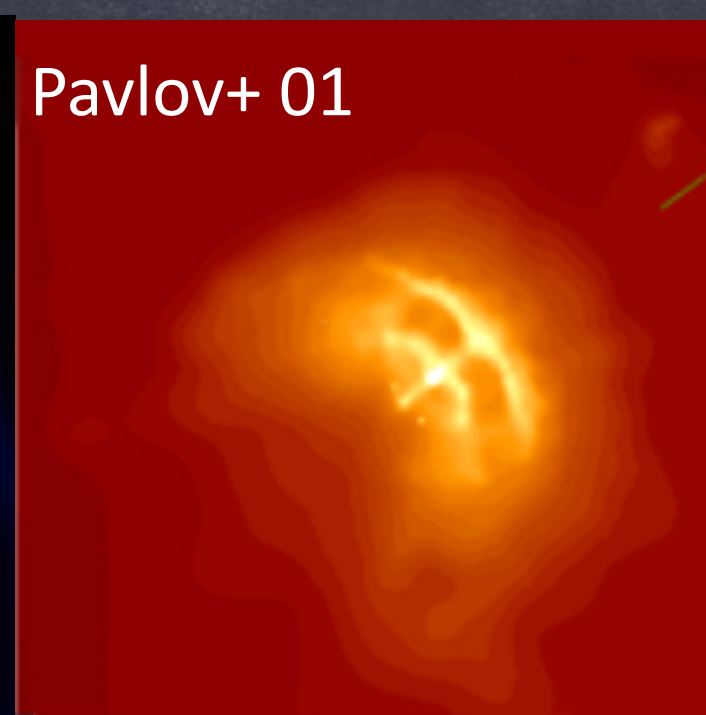




# 2D MHD NUMERICAL MODELING: RINGS AND TORII



A: ULTRARELATIVISTIC WIND  
B: SUBSONIC OUTFLOW  
C: SUPERSONIC FUNNEL

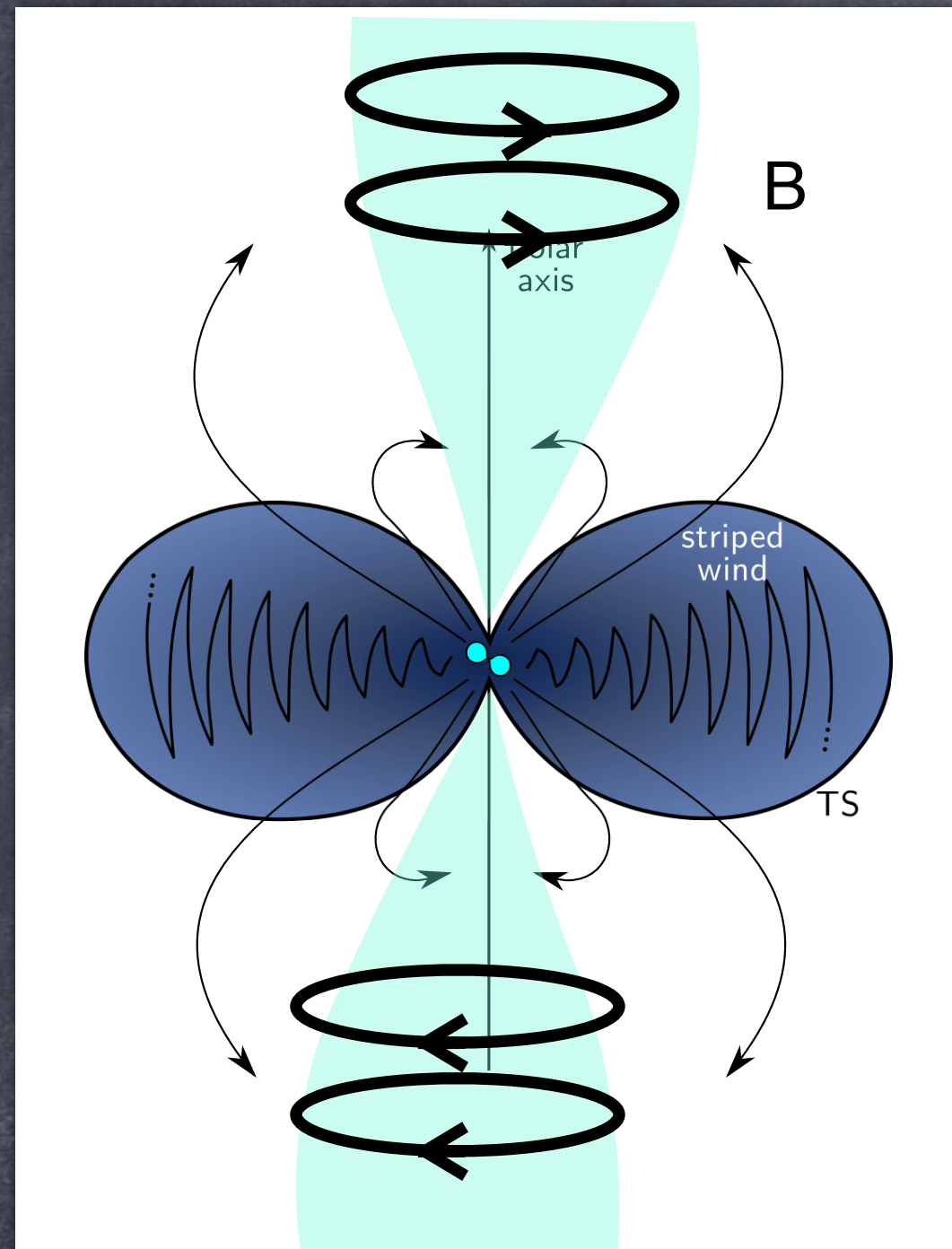




# CONSTRAINING THE PULSAR MULTIPLICITY

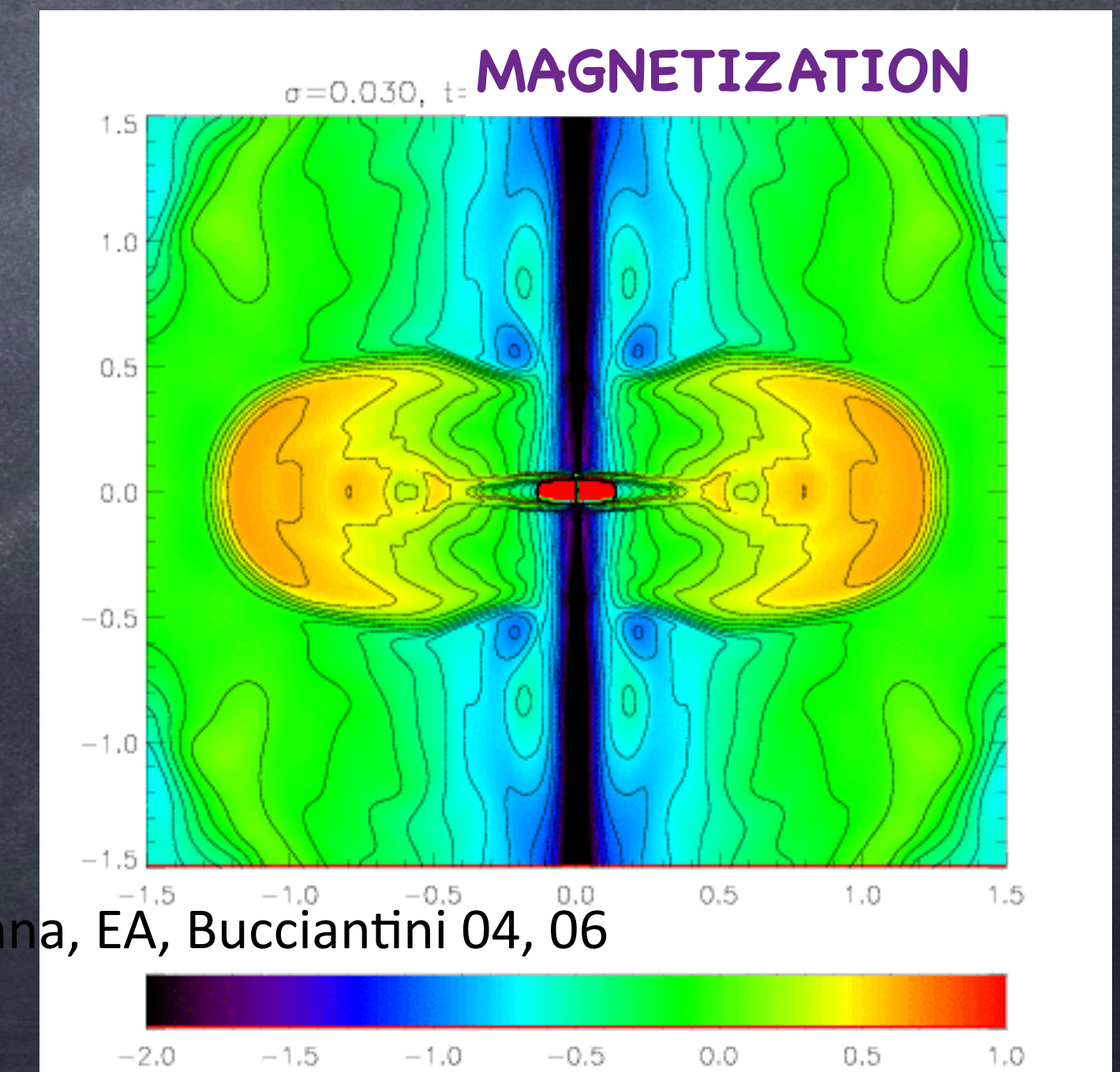
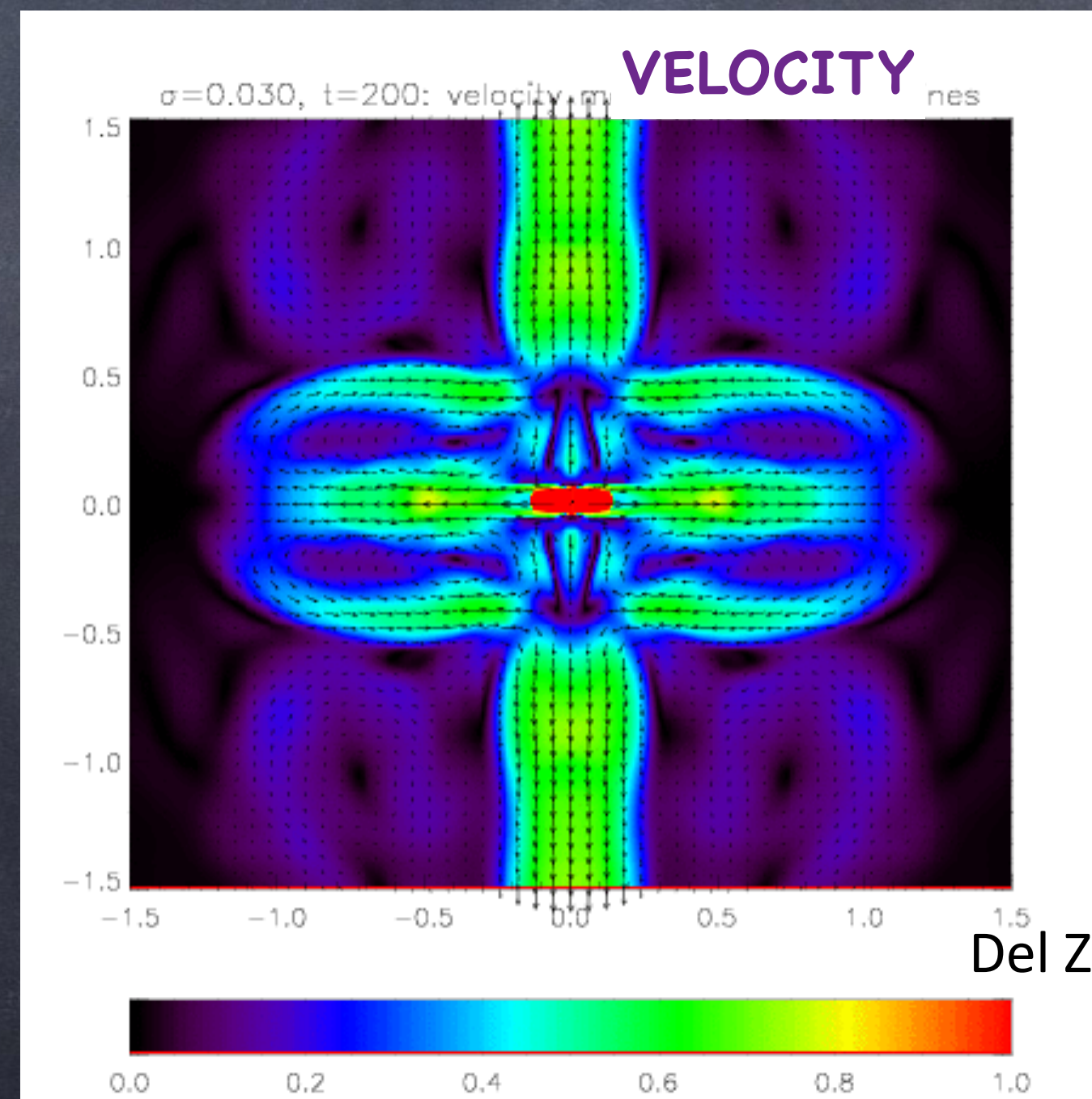


# 2D MHD NUMERICAL MODELING: JETS



EQUIPARTITION  
NEEDED FOR  
JET FORMATION

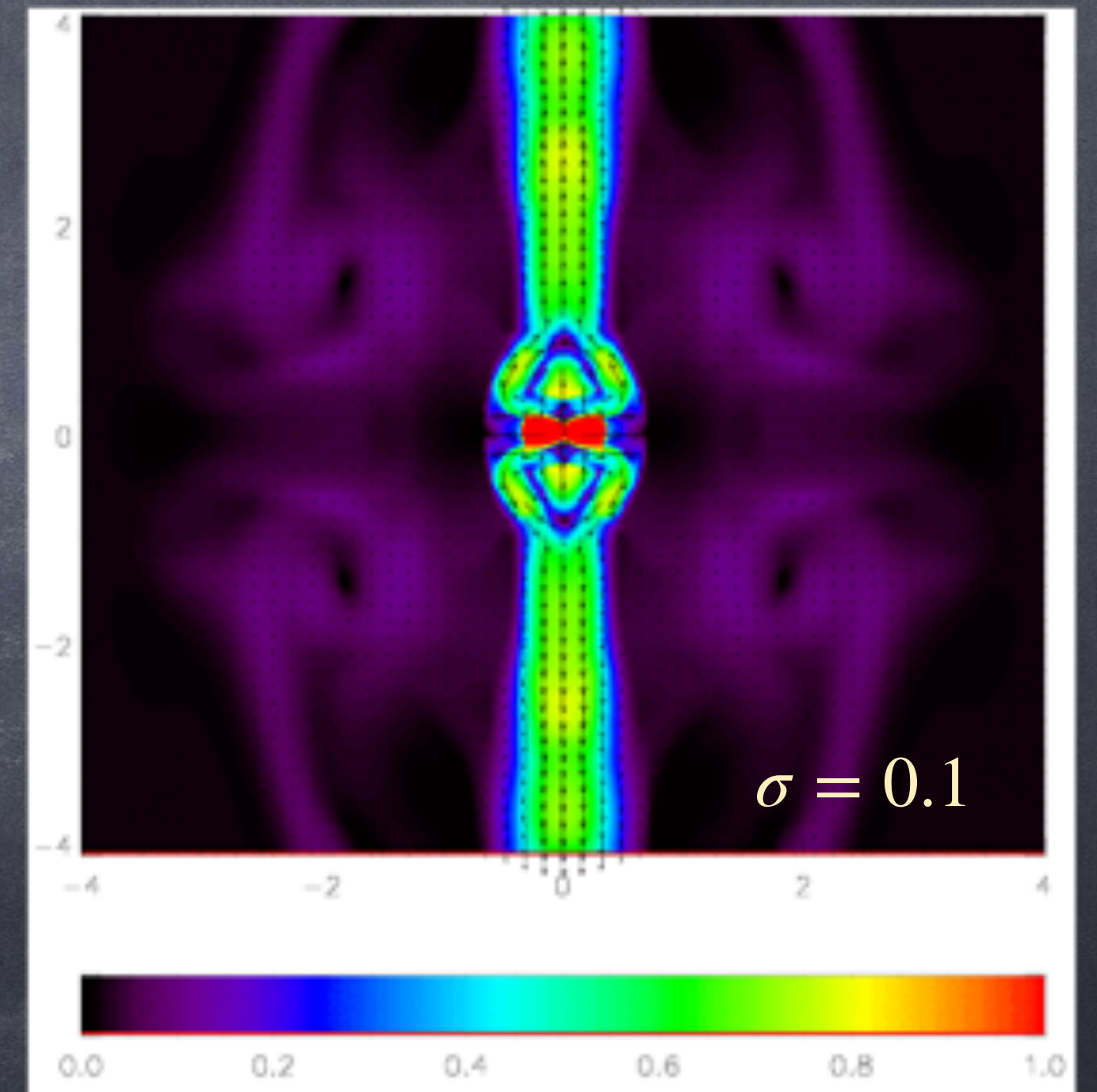
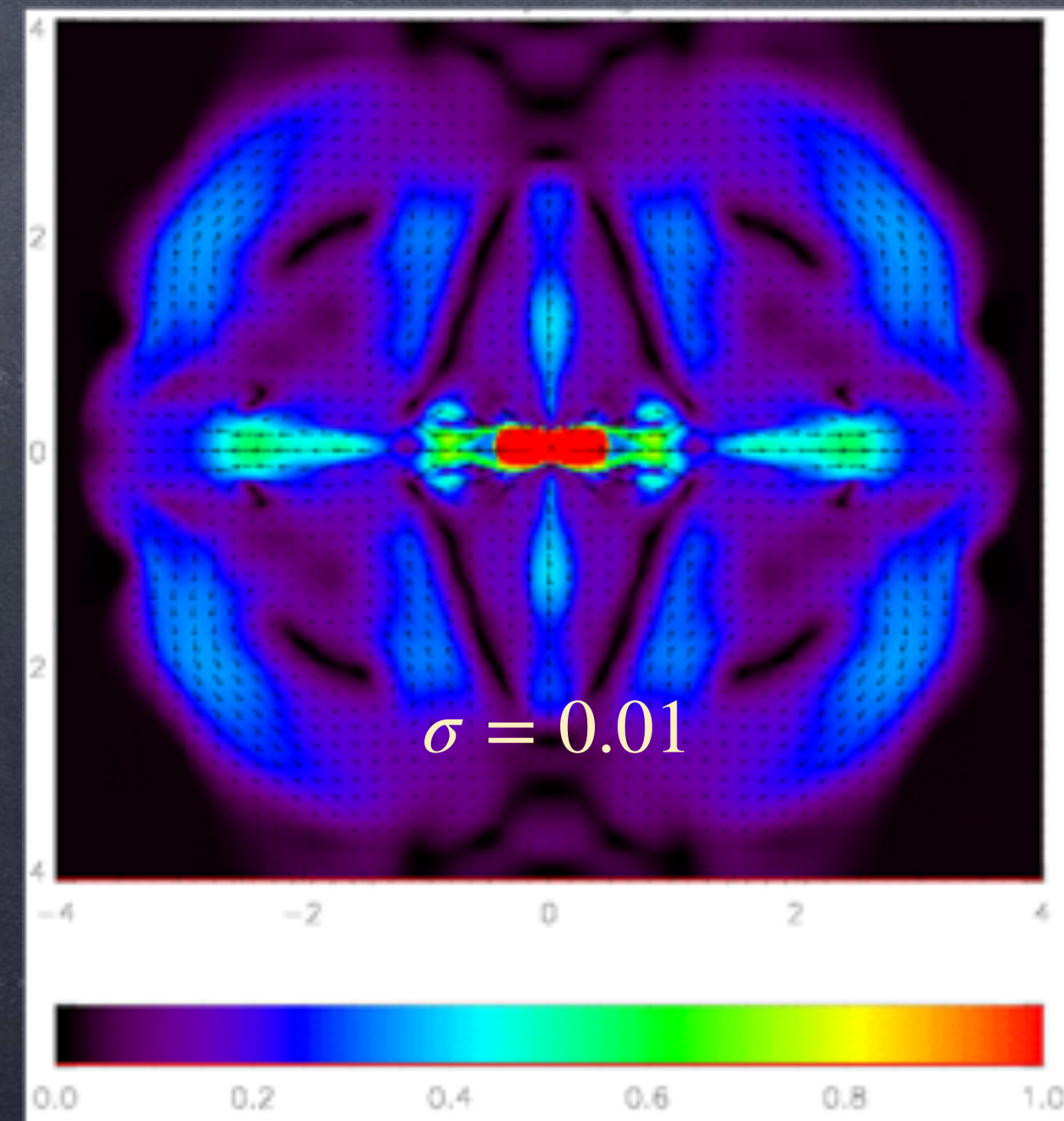
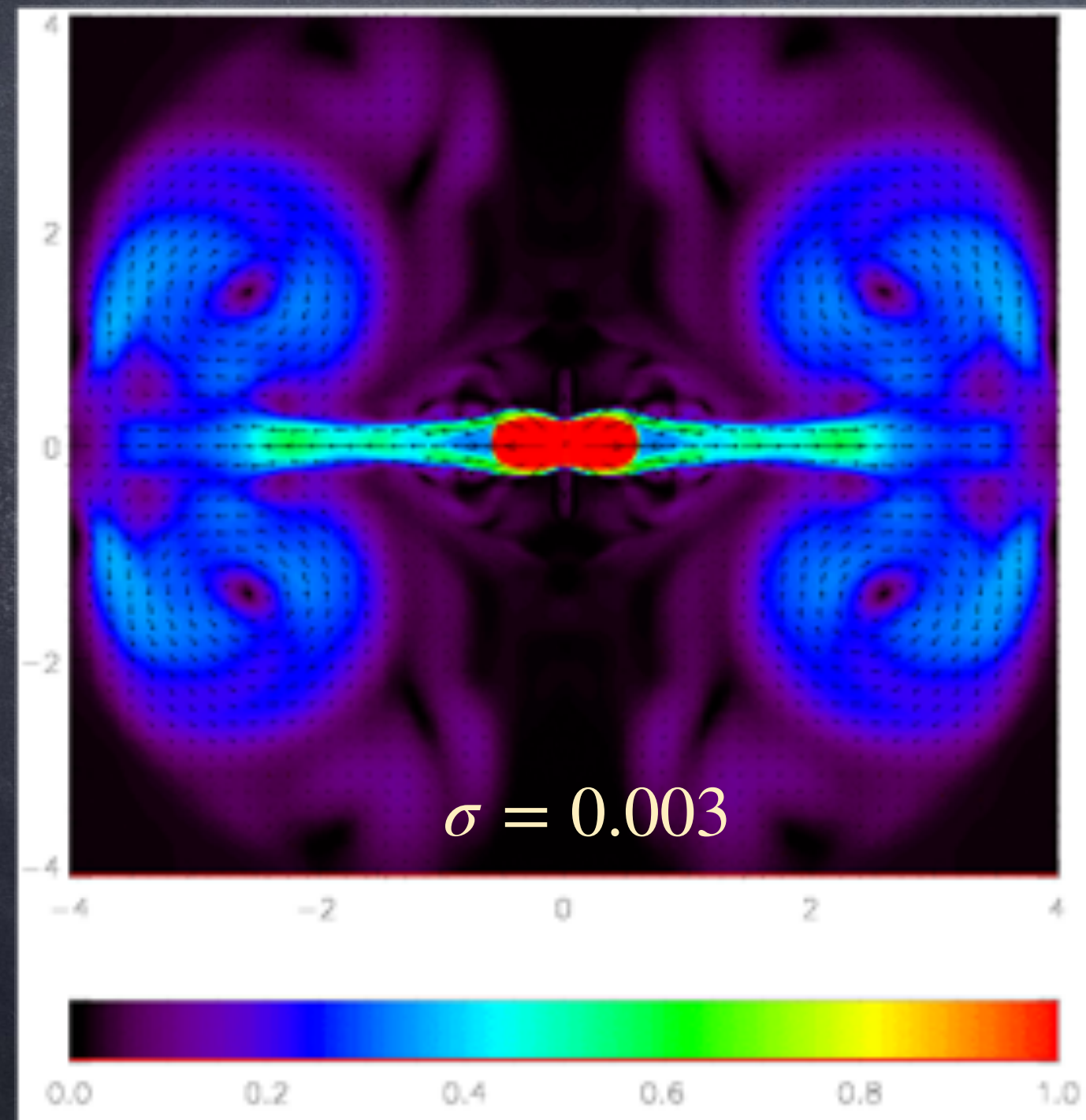
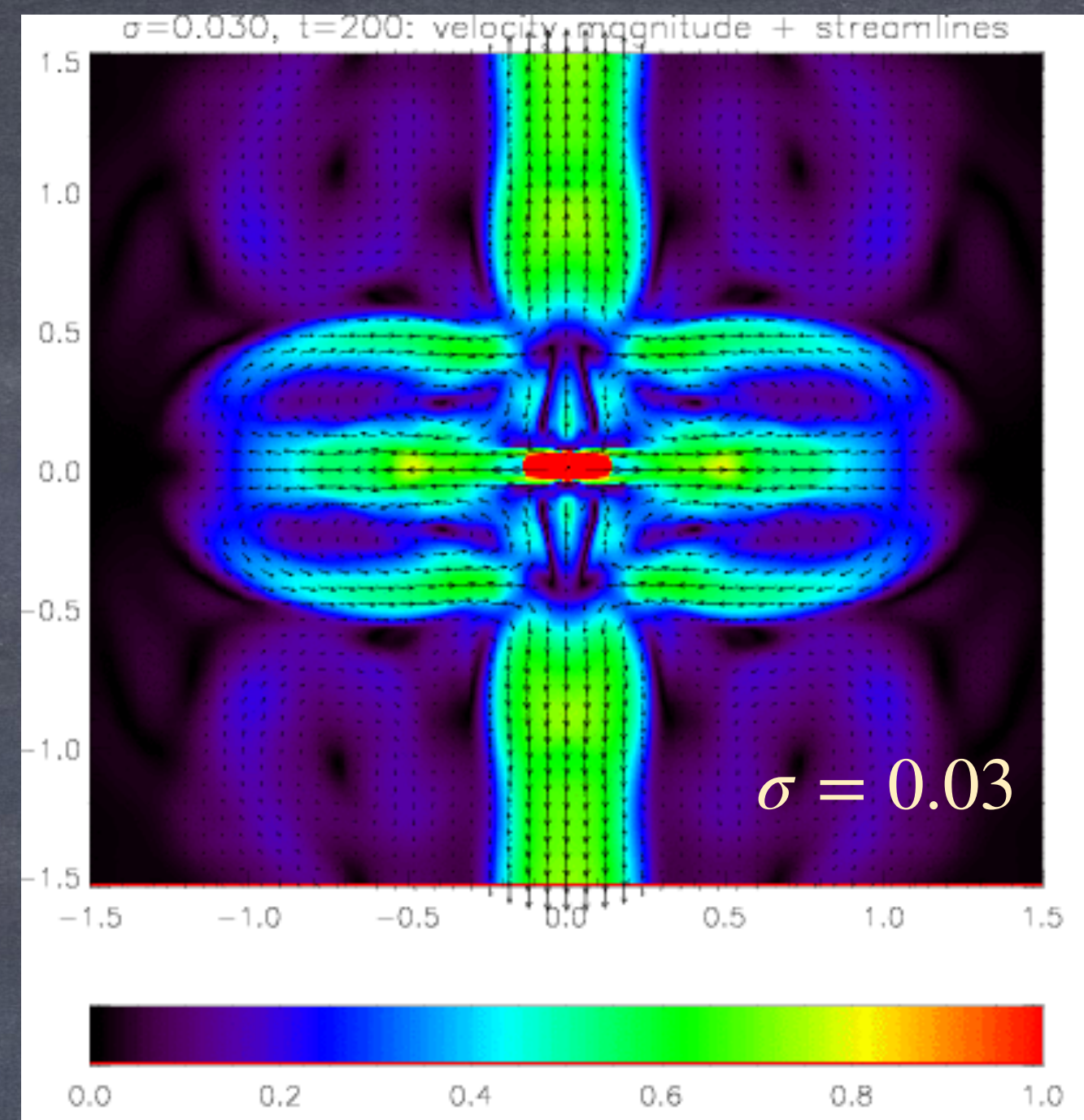
IN 2D JETS REQUIRE  $\sigma > 0.03$



Del Zanna, EA, Bucciantini 04, 06

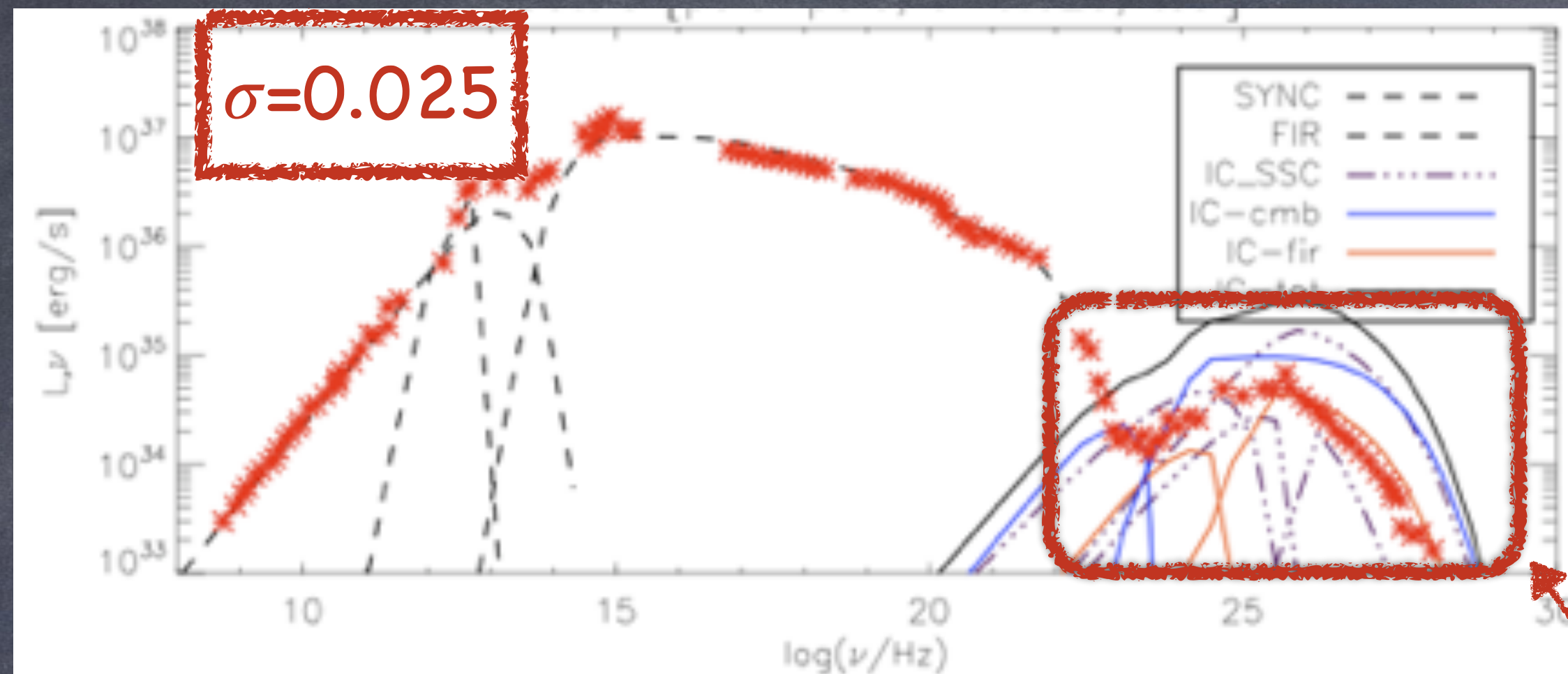


# MAKING A JET

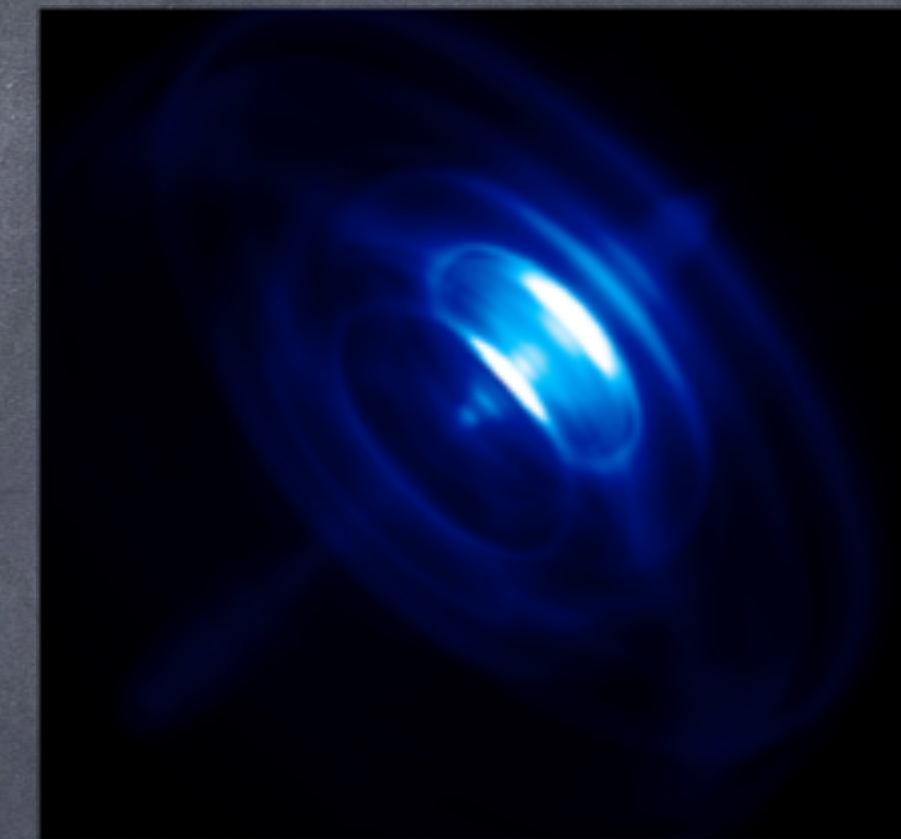




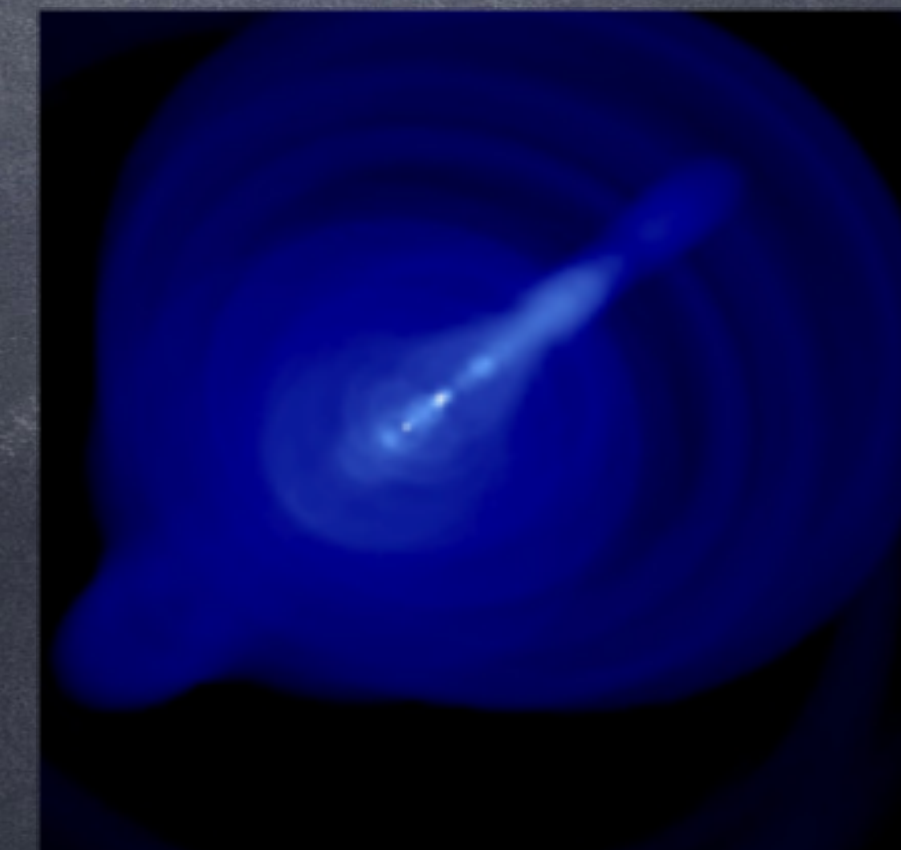
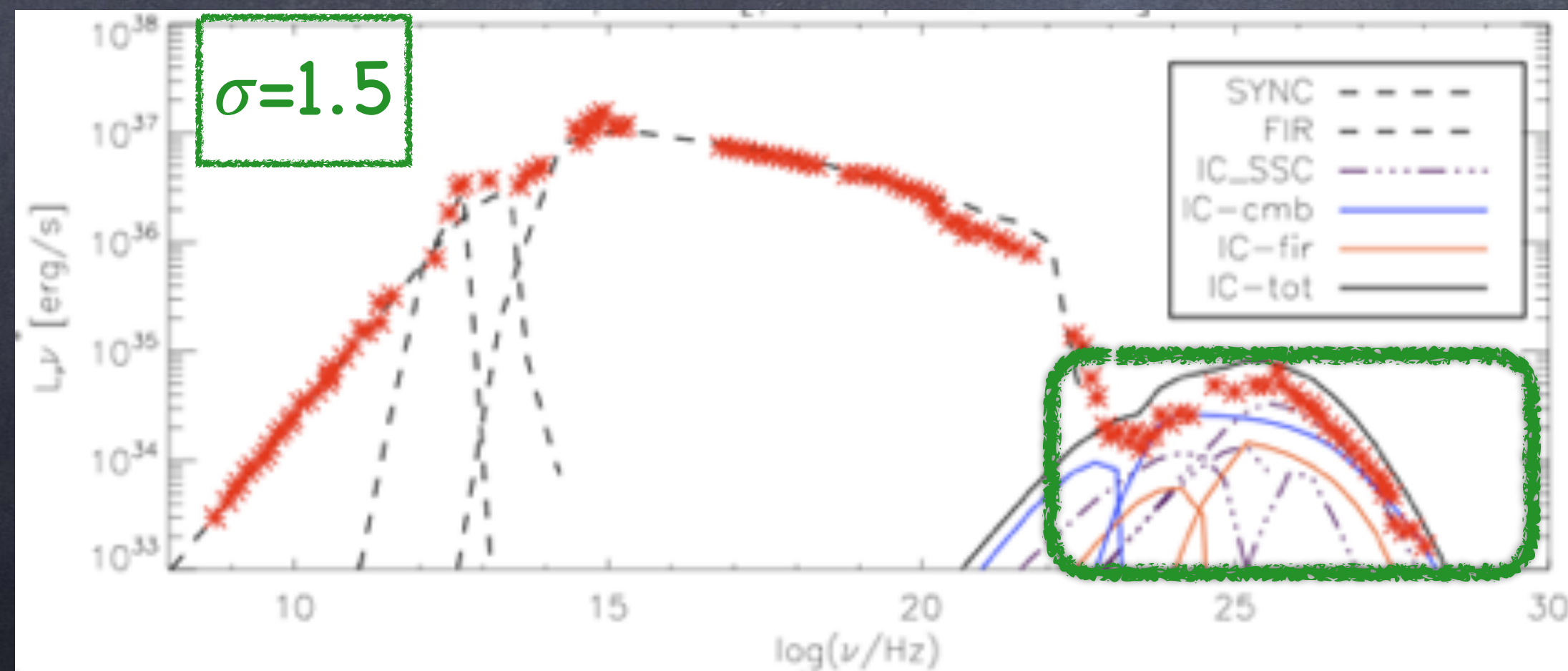
# BEHIND PRETTY PICTURES



Volpi+ 08, Olmi+14



$$B_{sim} \approx 10^{-5} \text{ G}$$



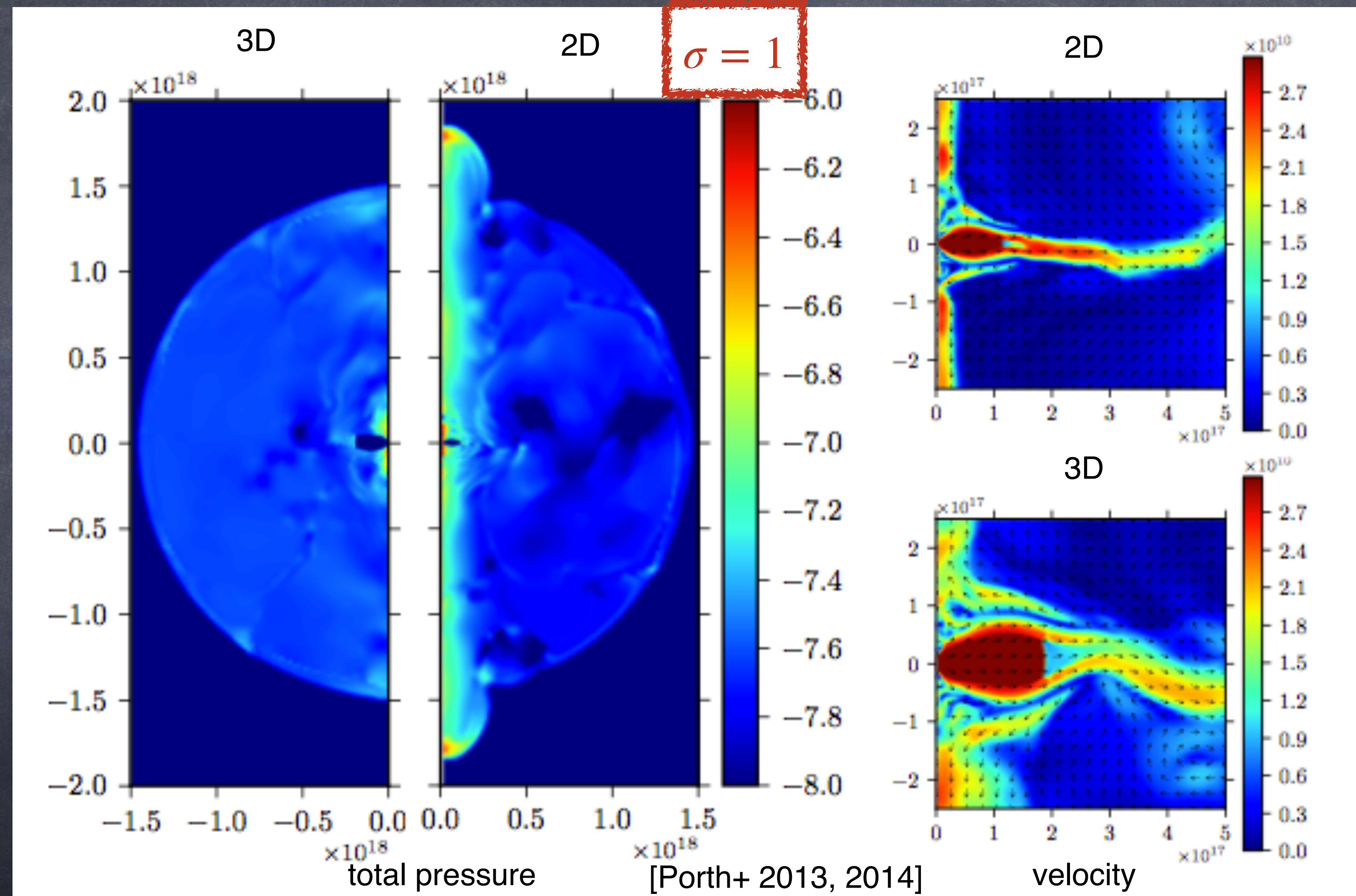
$$B_{obs} \approx 10^{-4} \text{ G}$$



# 3D RMHD SIMULATIONS

GLOBAL DYNAMICS DIFFERENT

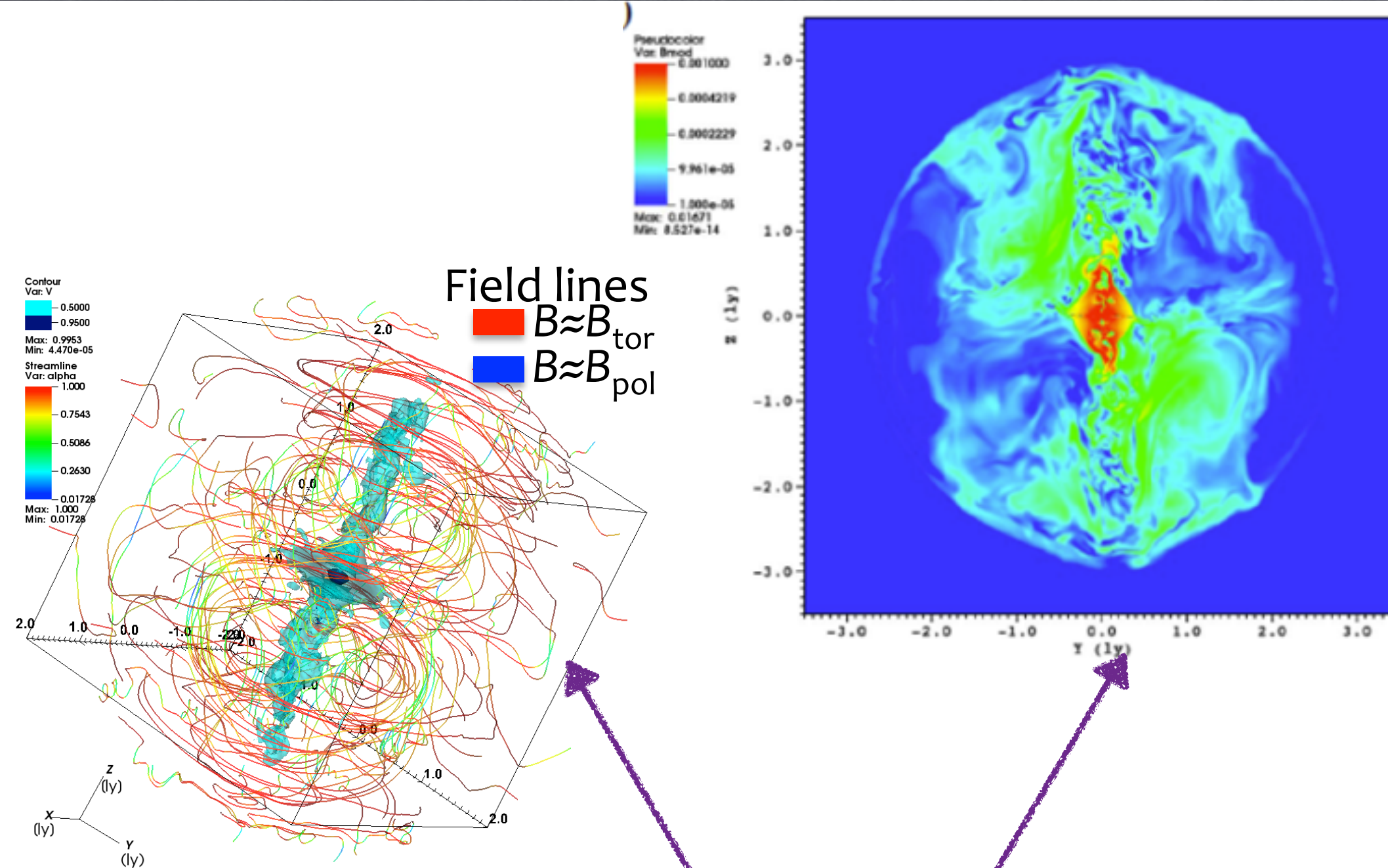
INNER DYNAMICS SIMILAR



EARLY SUGGESTION (Begelman 98): KINKS REDUCE HOOP STRESS WITH LITTLE DISSIPATION

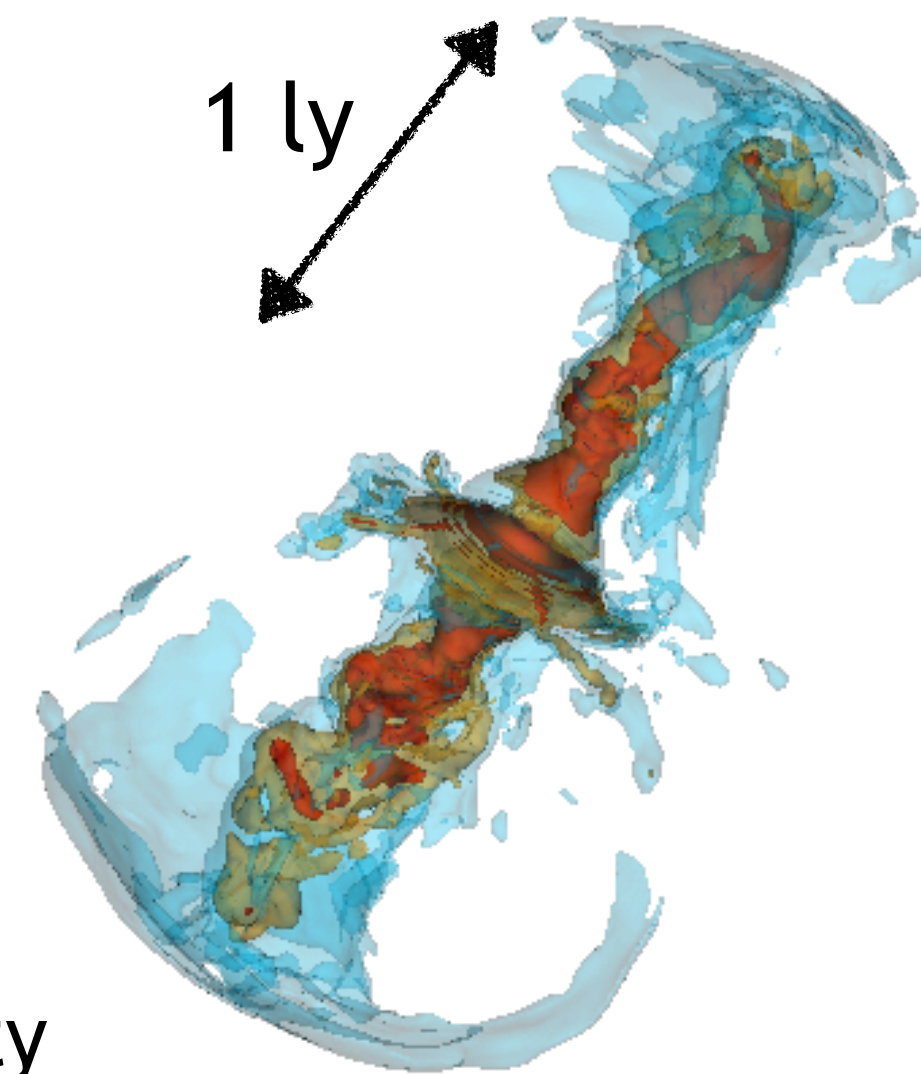


# LONGER 3D RMHD SIMULATIONS

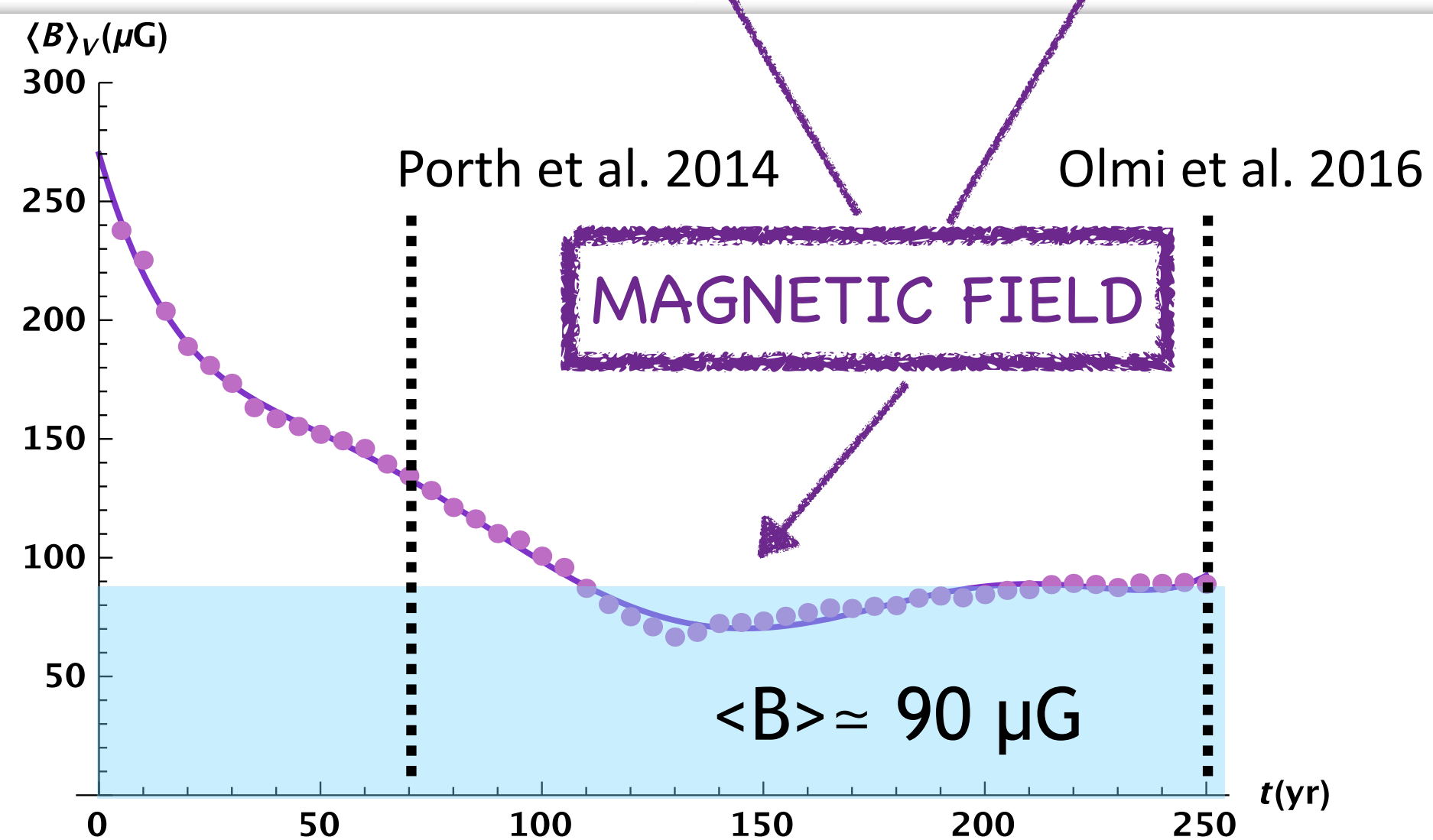


SELF SIMILAR PHASE  
FULLY REACHED

0.25c  
0.5c  
0.7c



$$\sigma = 1.5$$

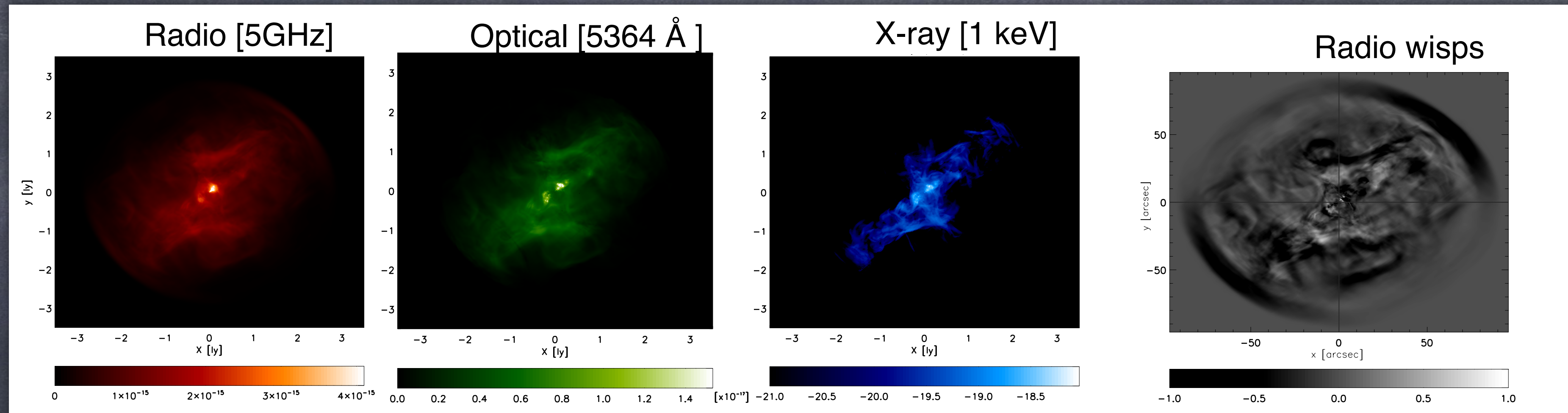




# ALL IS SOLVED?

- ✓ SHRINKAGE AND WISPS VARIABILITY OK
- NO BRIGHT X-RAY TORUS

Olmi+ 16



## ...NOT YET...

AVERAGE FIELD STILL  
TOO LOW



- ARTIFICIAL STEEPENING OF X-RAY PARTICLE SPECTRUM STILL NEEDED
- IC SPECTRUM STILL OVERESTIMATED

**EVEN HIGHER  $\sigma$  NEEDED ON AVERAGE**

IS THIS GOOD?



# PARTICLE ACCELERATION MECHANISMS: SUMMARY OF REQUIREMENTS

$$\dot{E} = \kappa \dot{N}_{GJ} m_e \Gamma c^2 \left( 1 + \frac{m_i}{\kappa m_e} + \sigma \right)$$

$$\sigma = \frac{B^2}{4\pi n_{\pm} m_e c^2 \Gamma^2}$$

FERMI MECHANISM

MAGNETIZATION:  
REQUIRES LOW

HOWEVER  
SEE  
VARIANTS

DRIVEN MAGNETIC RECONNECTION

MAGNETIZATION:  
REQUIRES HIGH

PLASMA MULTIPLICITY:  
REQUIRES HIGH

ION CYCLOTRON ABSORPTION  
IN  
ION DOPED PLASMA

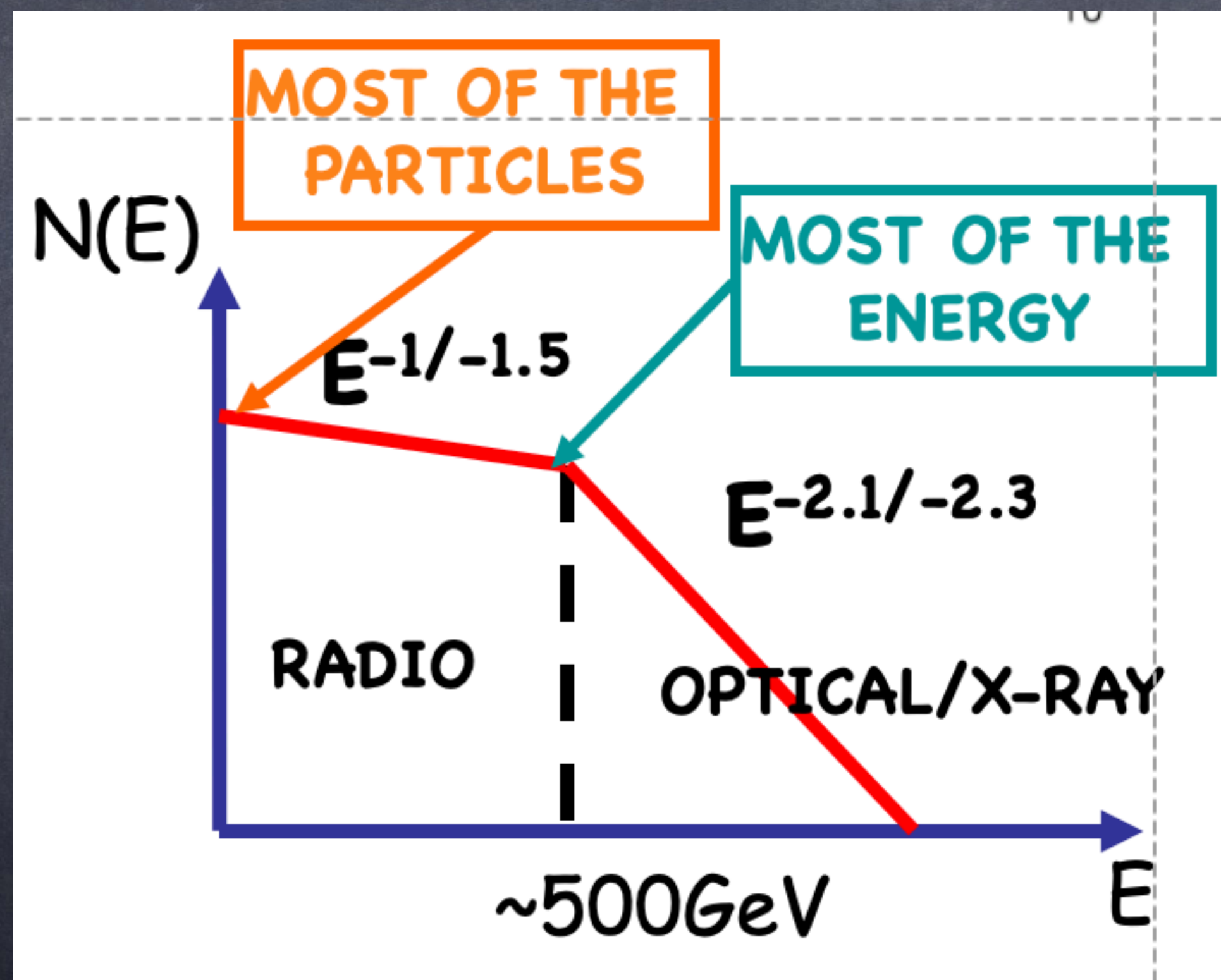
PLASMA MULTIPLICITY:  
REQUIRES LOW



# CONSTRAINING THE PULSAR MULTIPLICITY



# $\kappa$ IS CONSTRAINED BY RADIO EMITTING PARTICLES

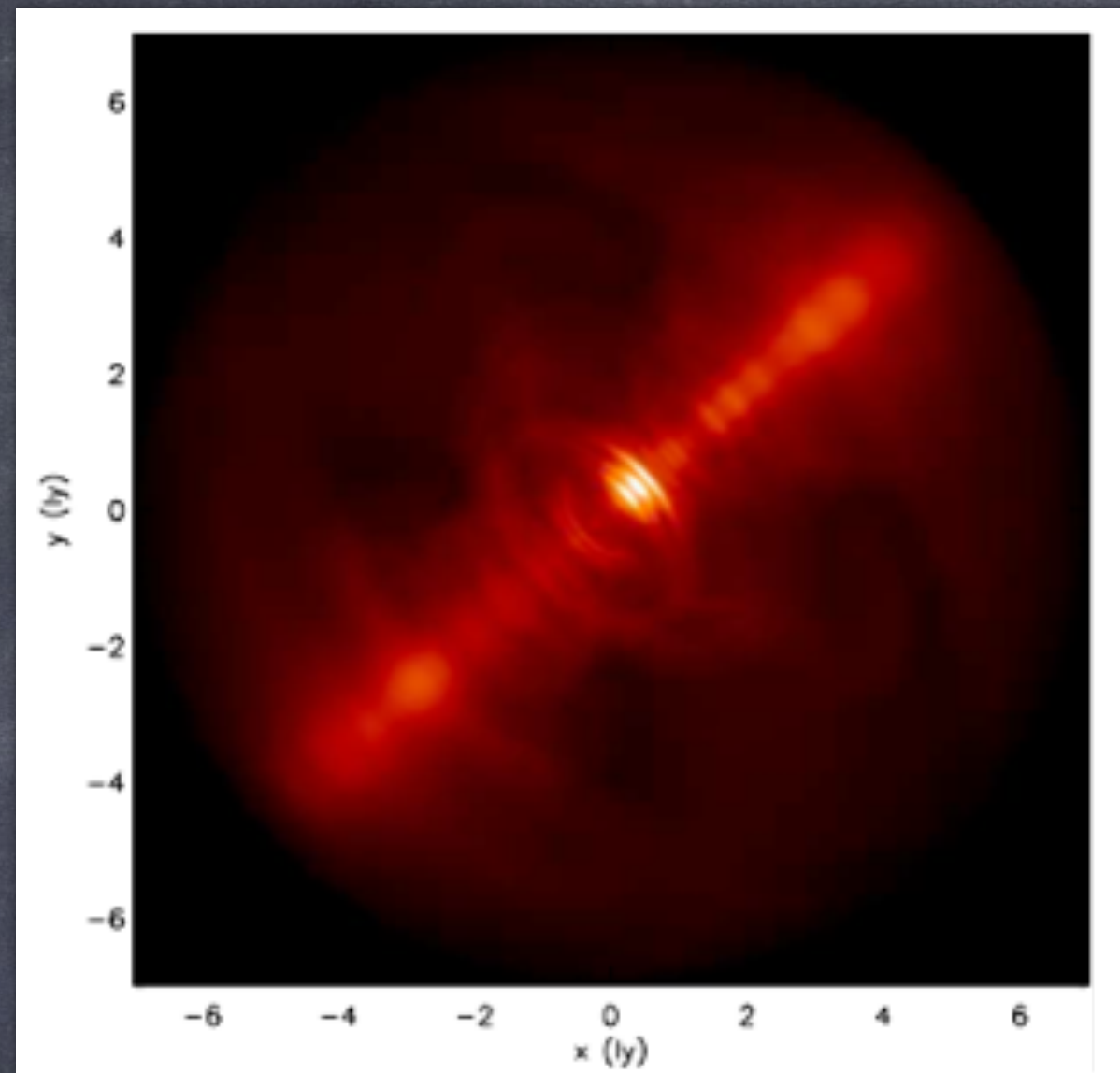


RADIO EMITTING PARTICLES  
HAVE LONG LIFETIMES:  
DO NOT NEED TO BE PART OF  
THE FLOW

IF PART OF THE FLOW  
 $\kappa \approx 10^6$      $\Gamma \approx 10^4$   
 OTHERWISE  
 $\kappa \approx 10^3-10^4$      $\Gamma \approx 10^6-10^7$



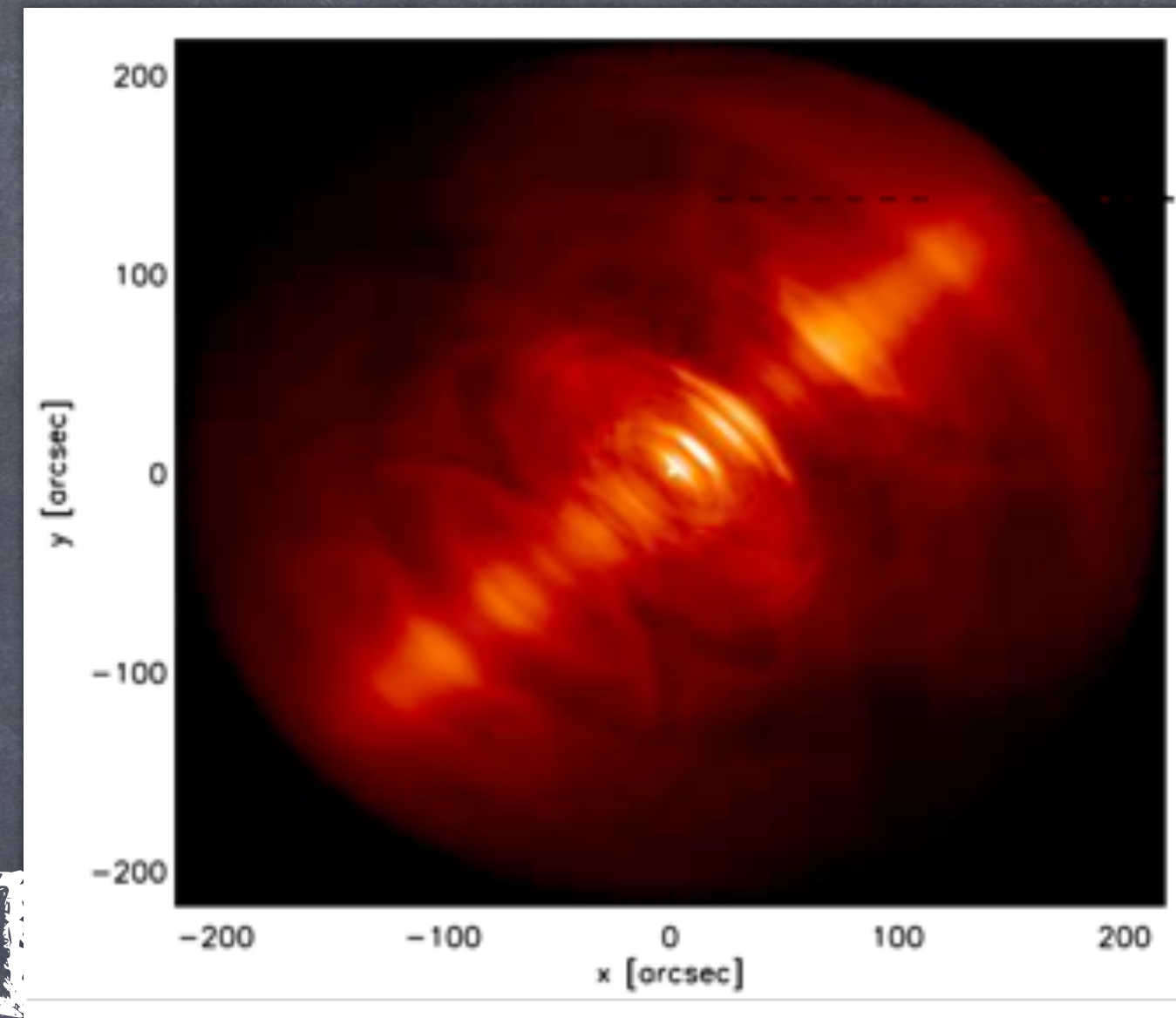
# RADIO EMISSION



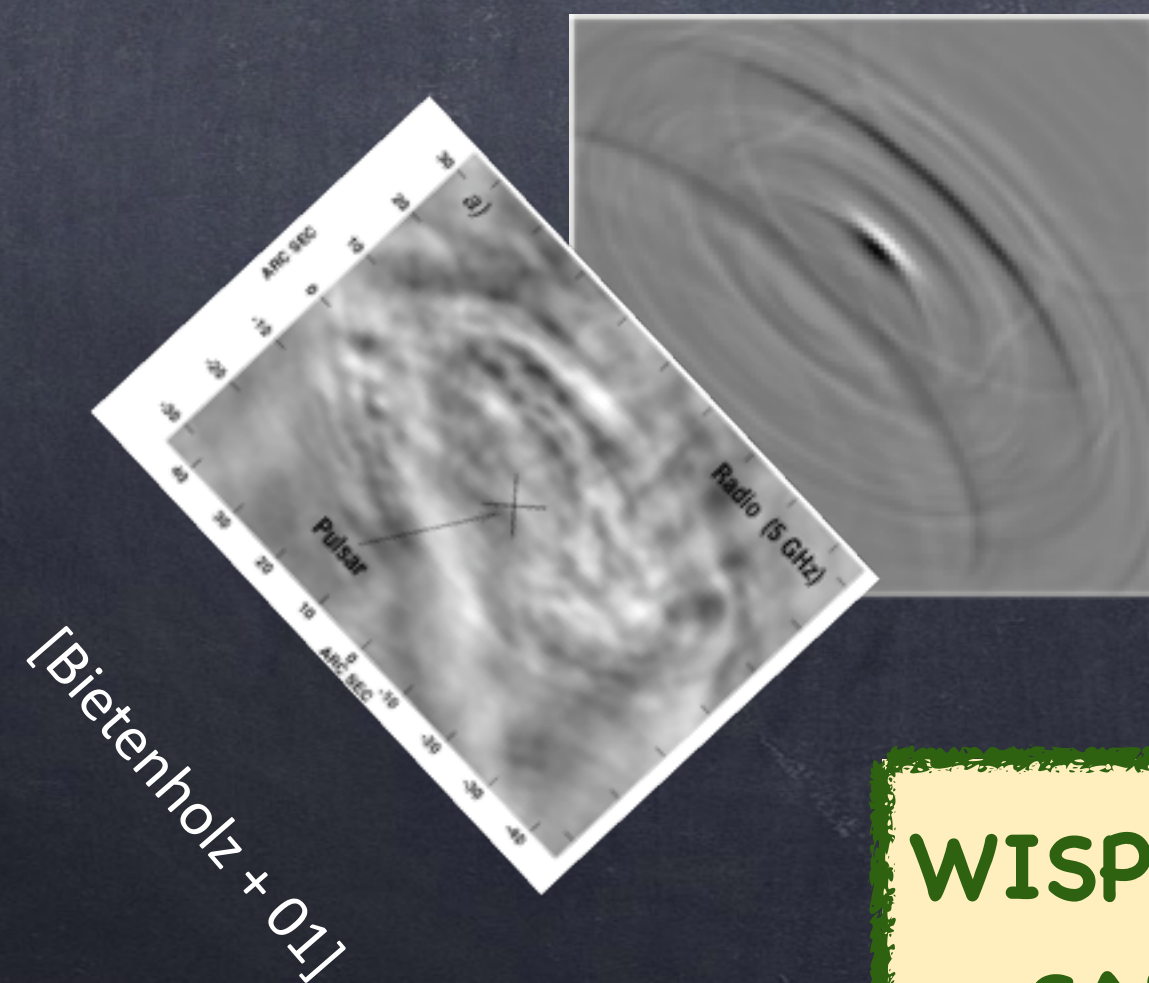
SHOCK ACCELERATION  
& ADVECTION

EMISSION MAPS  
CANNOT DISTINGUISH

UNIFORM INJECTION



Olmi + 2014

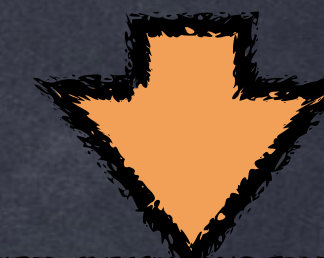


[Bietenholz + 01]

WISPS ARE THE  
SAME TOO



RADIO PARTICLES DO NOT  
NEED TO BE CURRENTLY  
ACCELERATED AT THE SHOCK



$\kappa \sim 10^3 - 10^4$  IS VIABLE

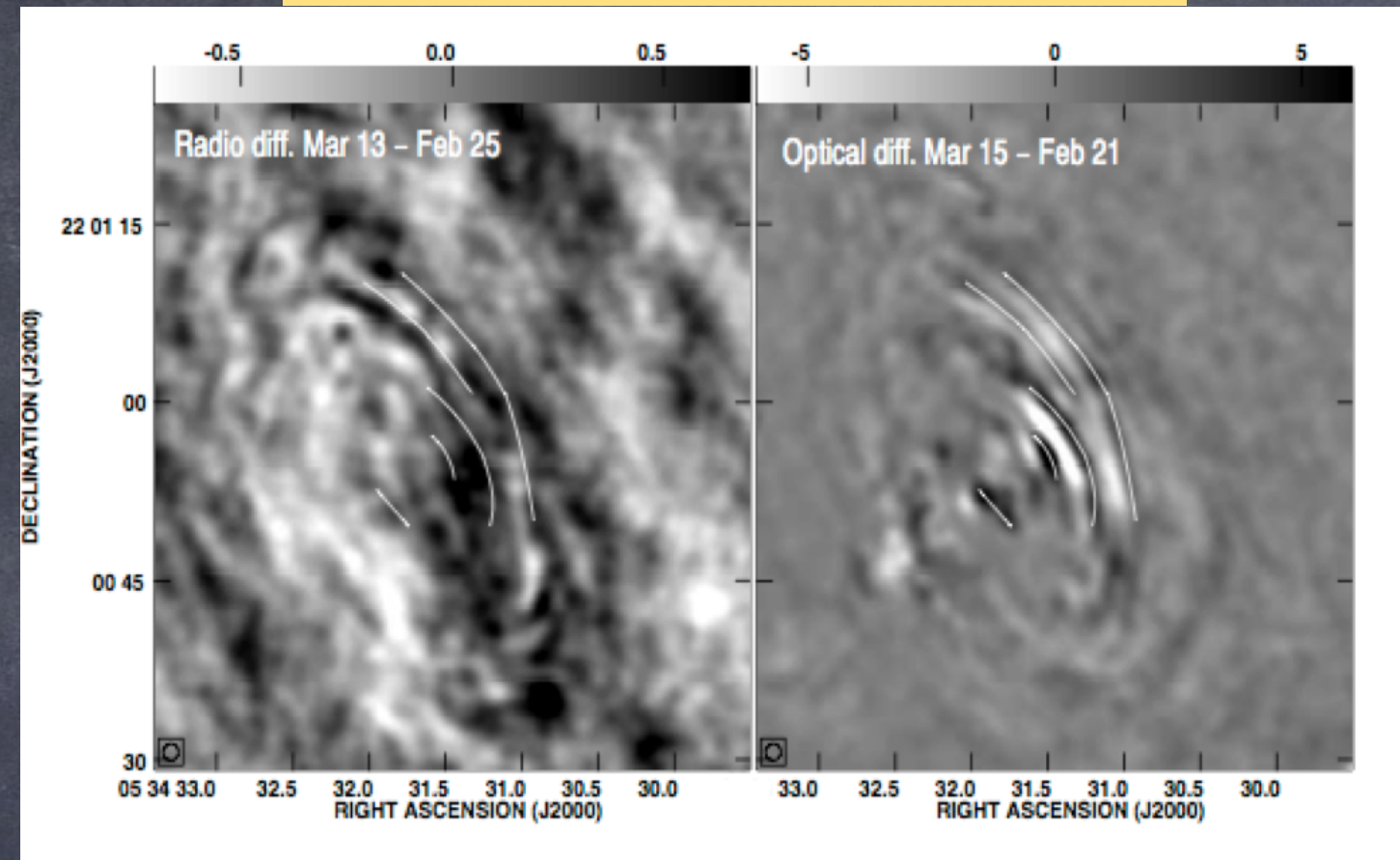


CONSTRAINING THE  
LOCATIONS OF  
PARTICLE  
ACCELERATION

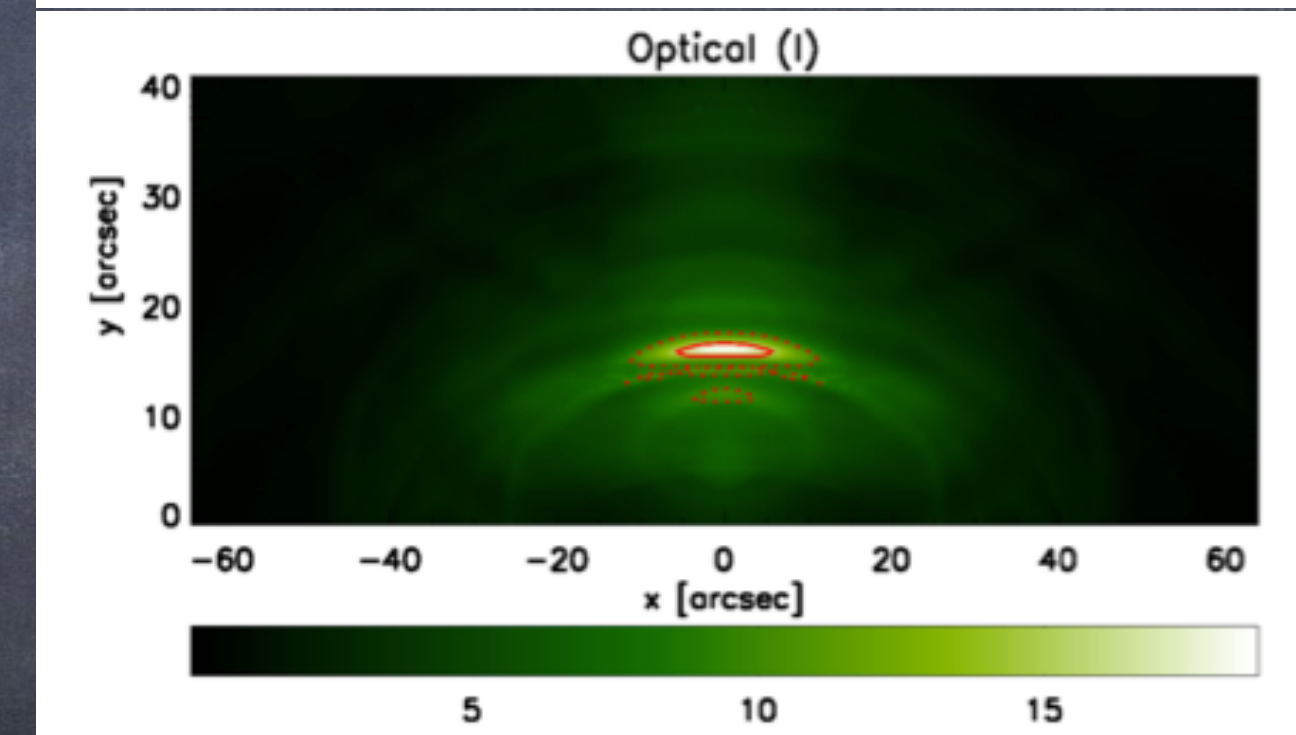
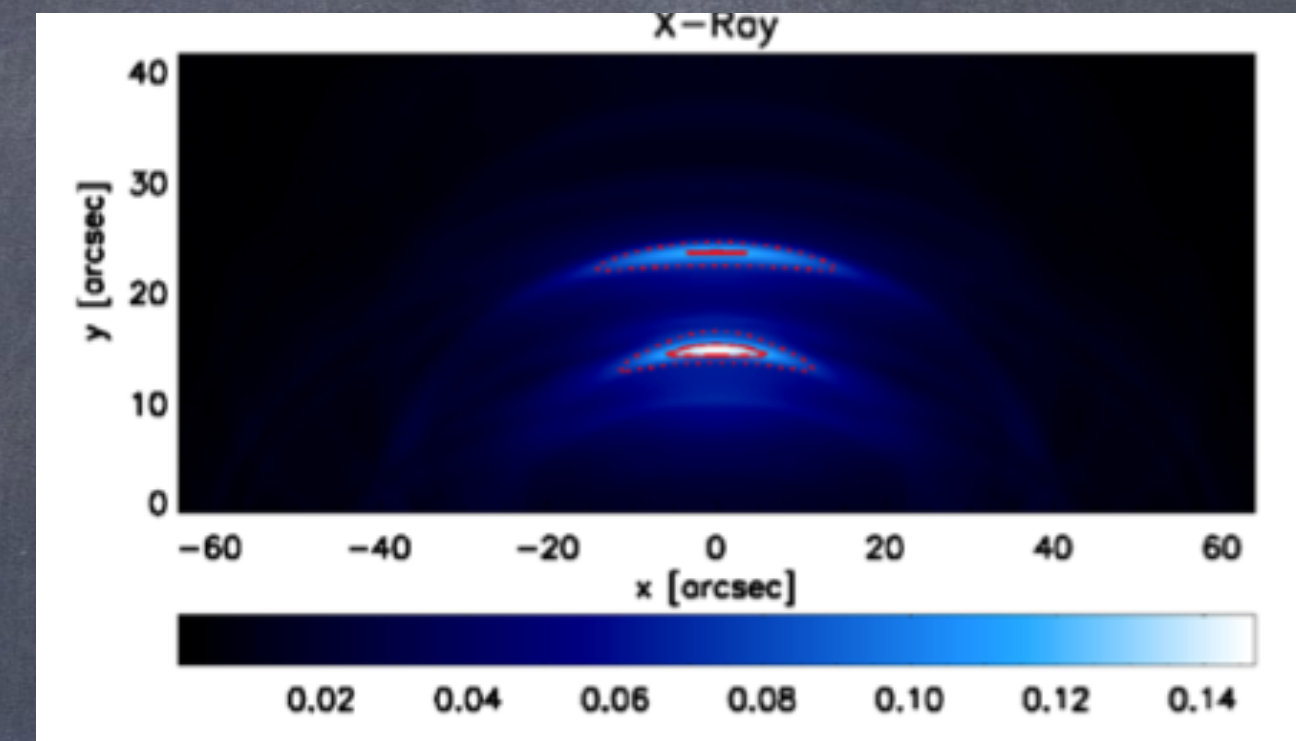


# VARIABILITY IN THE INNER NEBULA

## RADIO VS OPTICAL WISPS

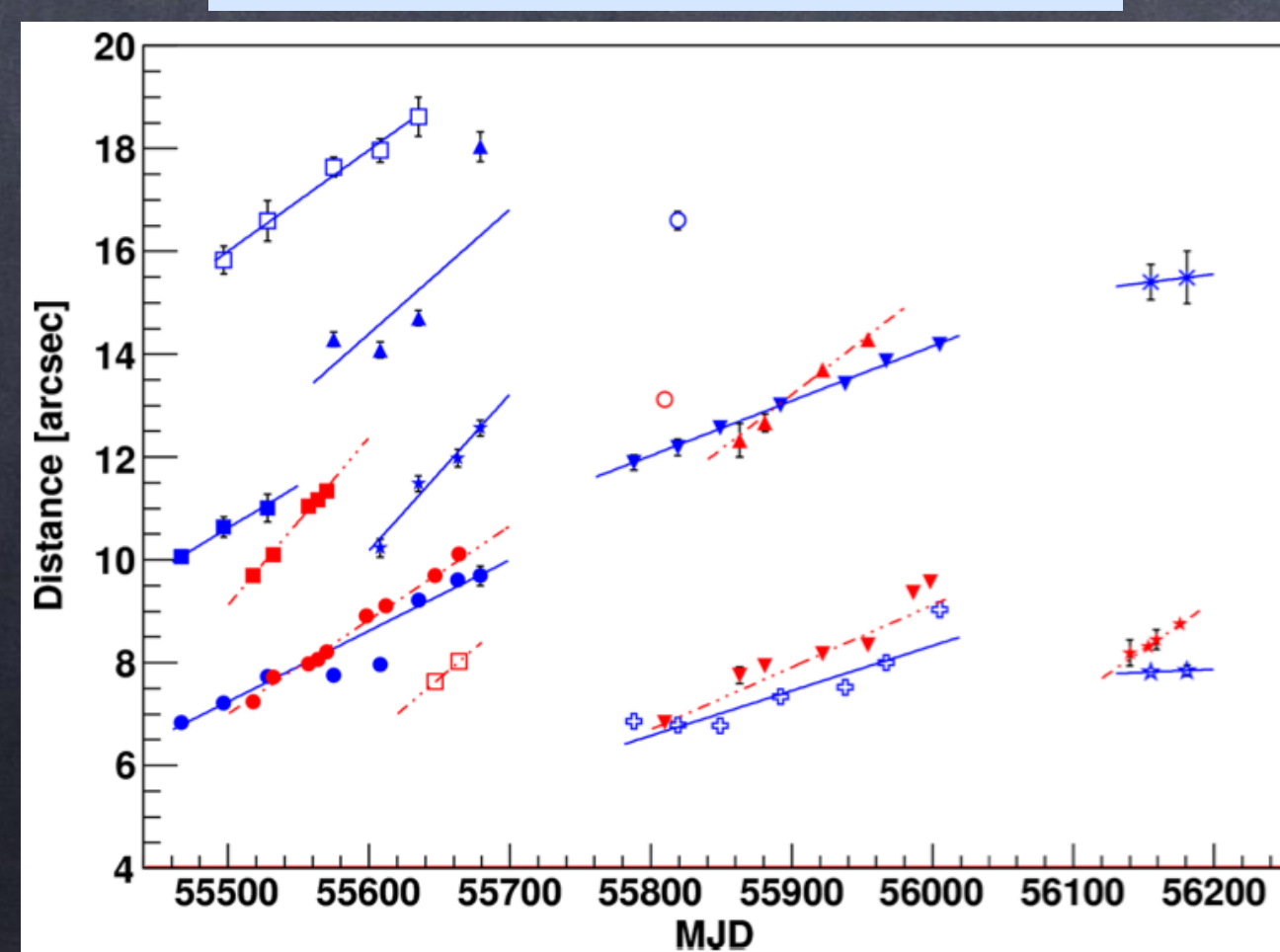


[Bietenholz + 2004]



[Olmi et al. 2015]

## X-RAY VS OPTICAL WISPS

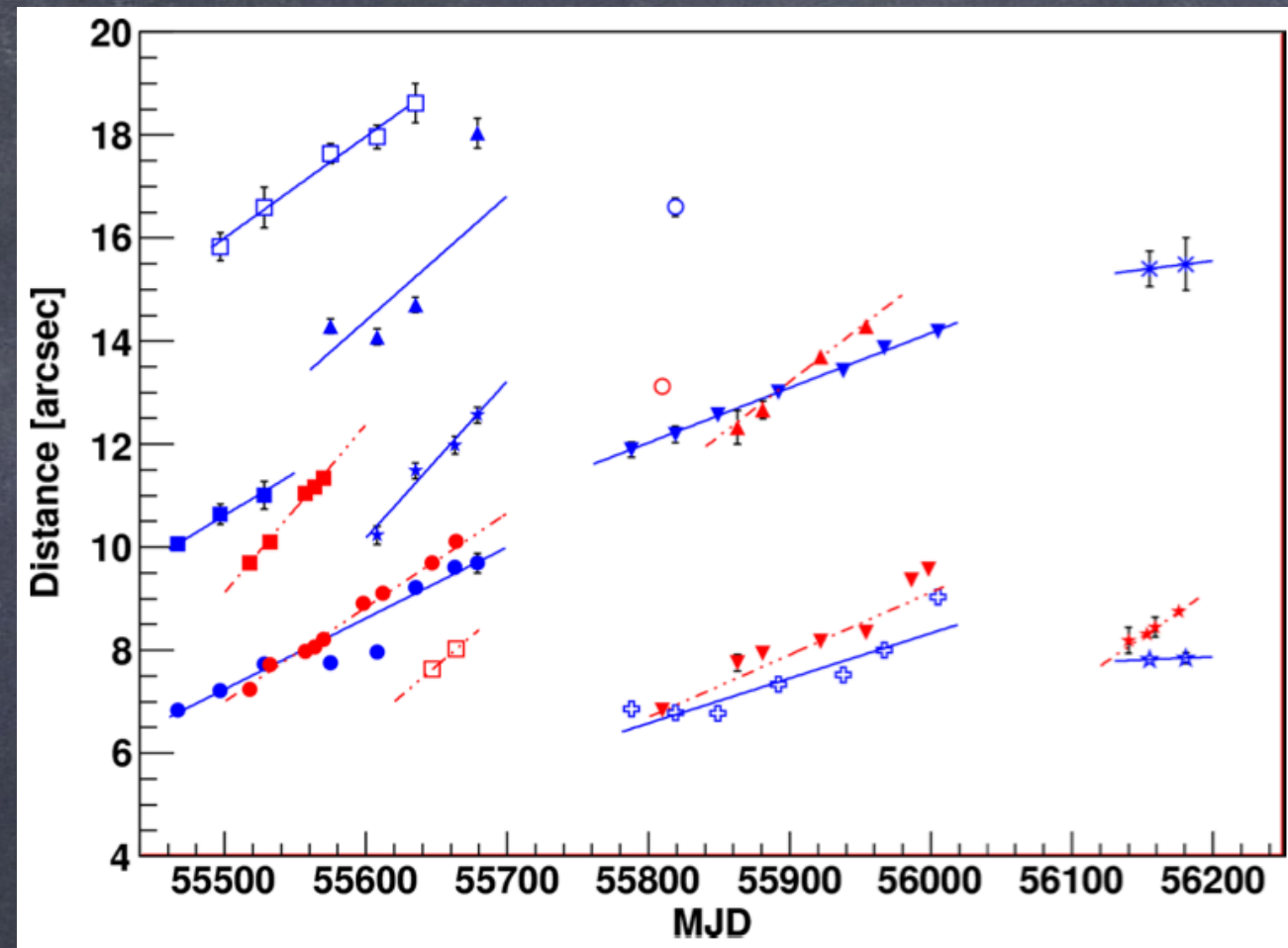


[Schweizer et al. 2013]



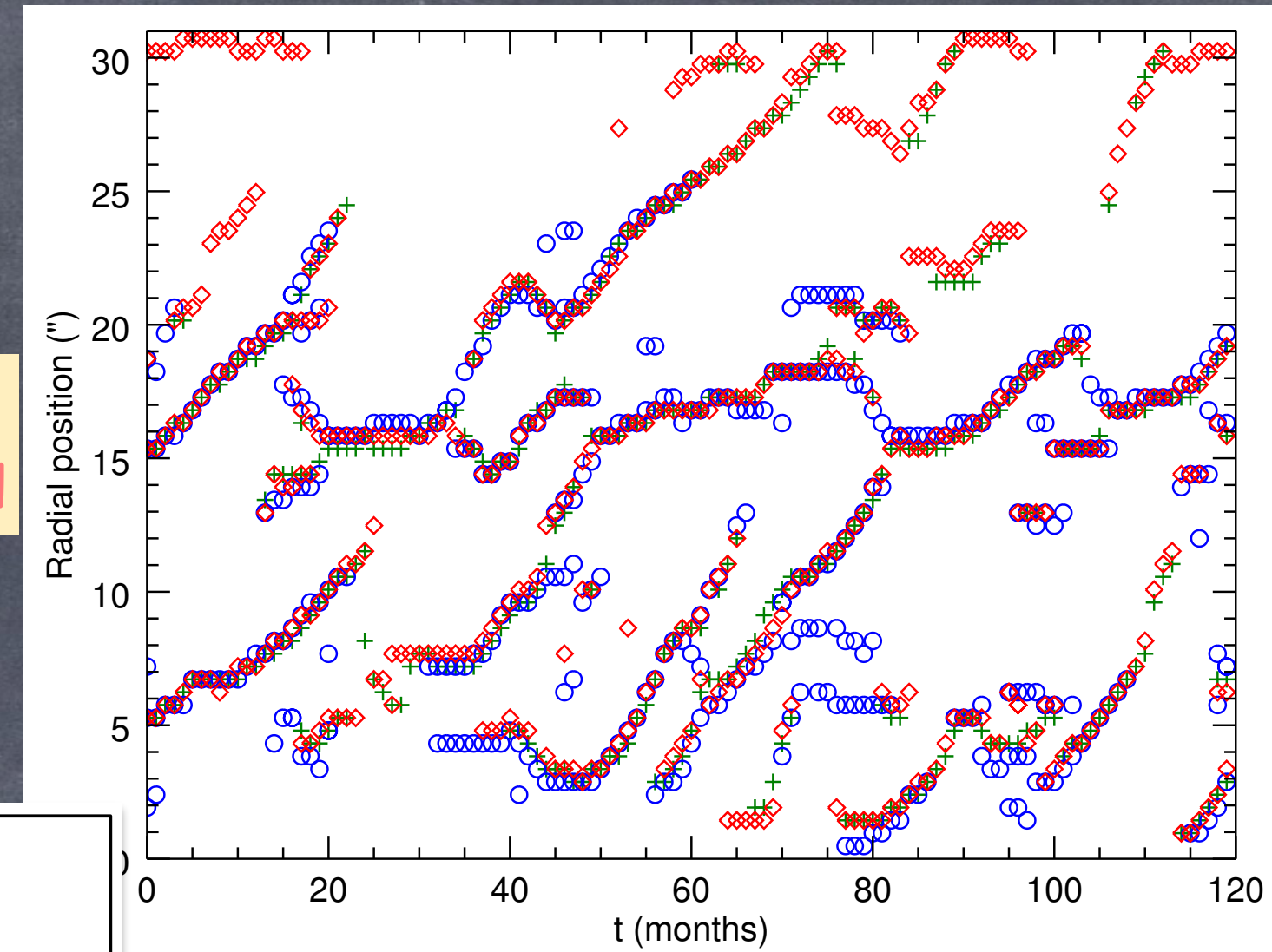
# HINTS ON LOCATIONS OF PARTICLE ACCELERATION

X-RAY VS OPTICAL WISPS



[Schweizer et al. 2013]

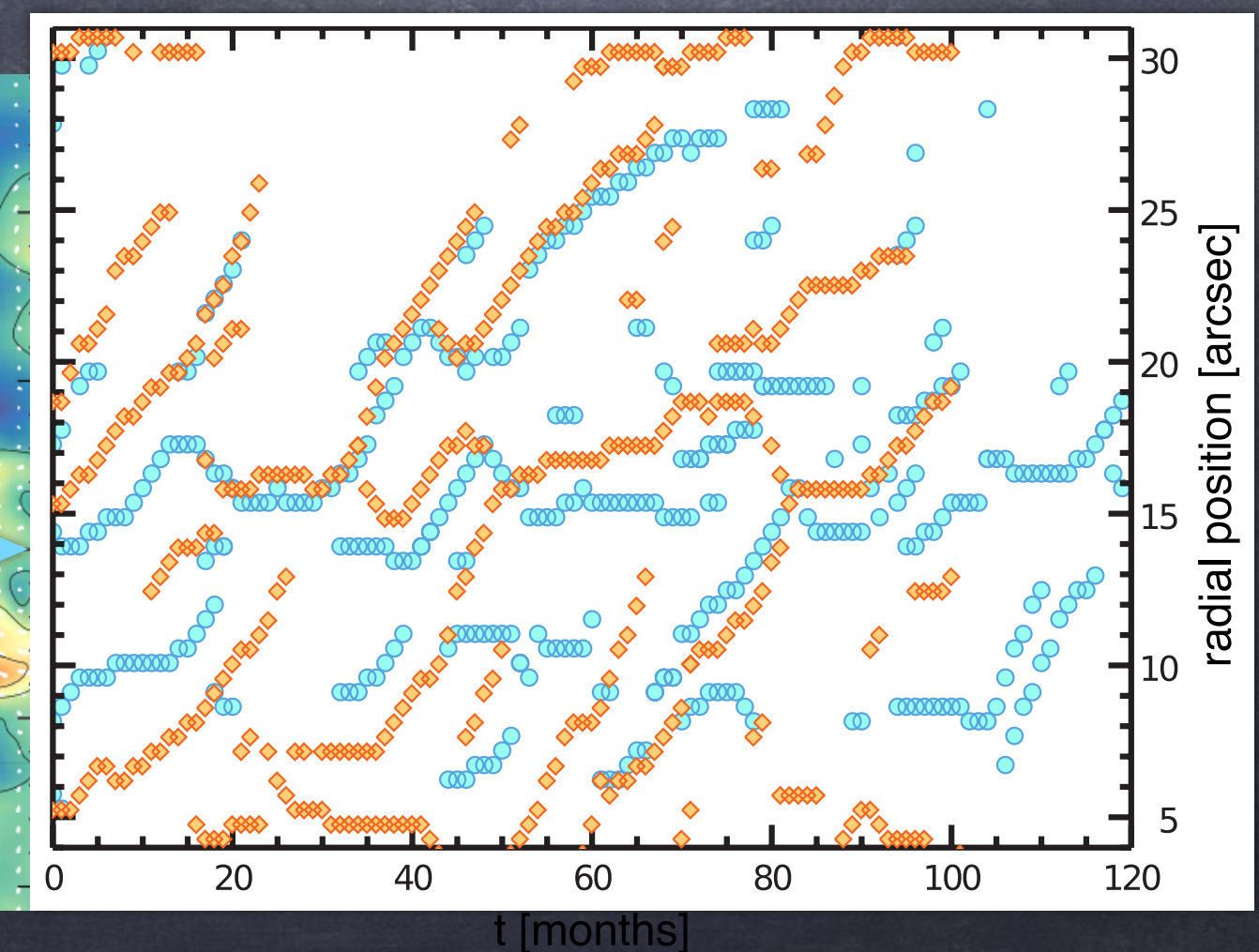
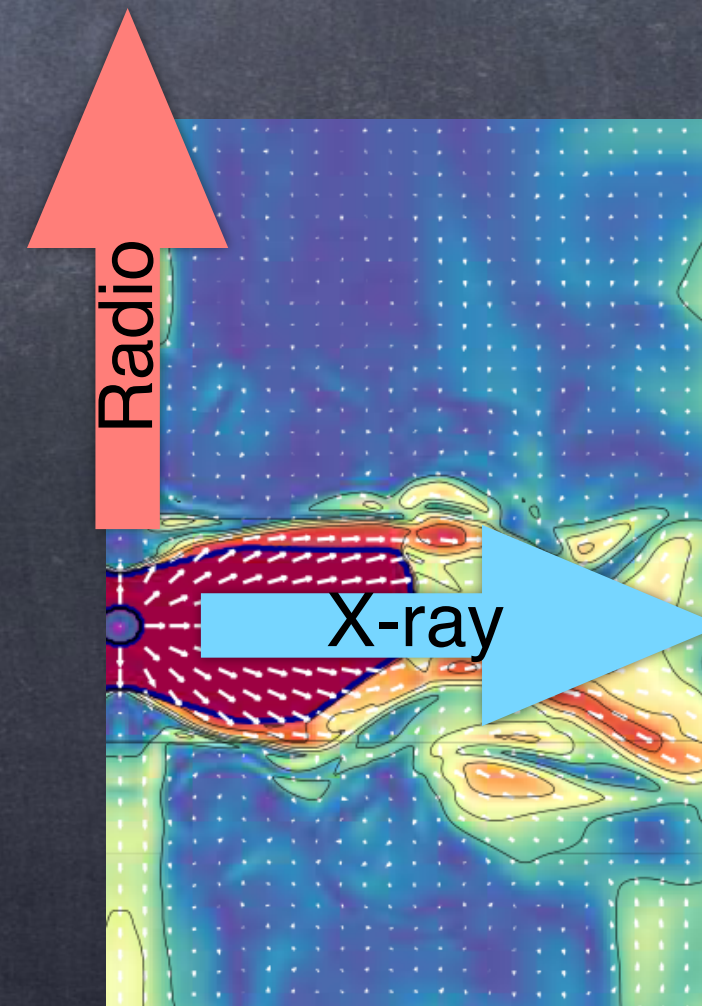
ISOTROPIC  
ACCELERATION



[Olmi et al. 2015]



X-RAY EMITTERS  
FROM EQUATOR  
LOWER ENERGY ANYWHERE





# IMPLICATIONS ON ACCELERATION MECHANISMS

NEBULAR DYNAMICS AND  
HIGH ENERGY EMISSION  
PROPERTIES

$$\sigma \gtrsim 1$$

TOO LARGE FOR  
FERMI ACCELERATION  
BUT TURBULENCE  
MIGHT HELP

MODELLING OF  
RADIO EMISSION

$$\kappa \approx \text{few} \times 10^3$$

AND

$$\Gamma > \text{few} \times 10^6$$

VIABLE

ION CYCLOTRON  
VIABLE

MODELLING OF  
MULTIFREQUENCY  
VARIABILITY OF  
INNER NEBULA

ACCELERATION OF  
LOW AND HIGH  
ENERGY PARTICLES IN  
DIFFERENT REGIONS

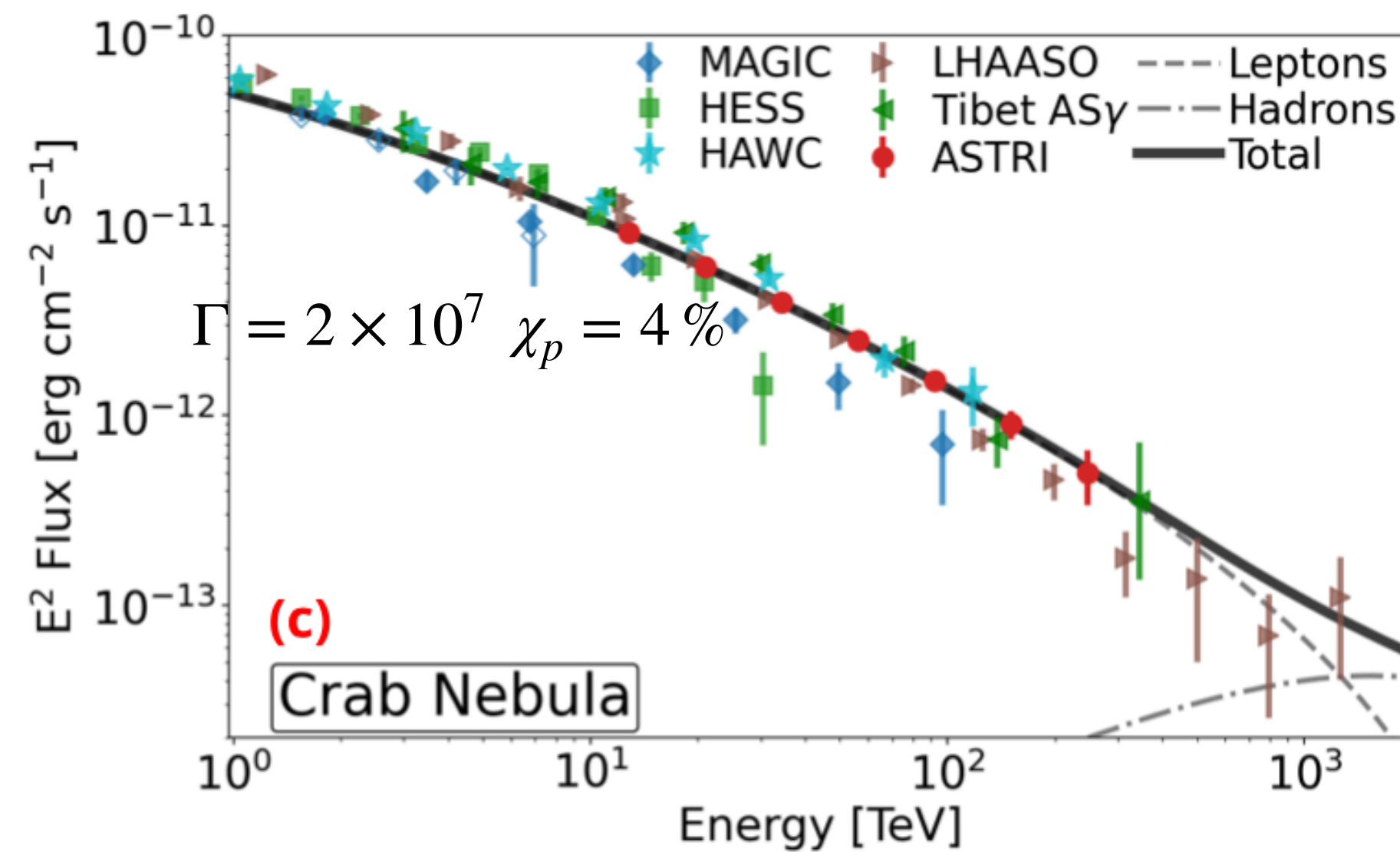
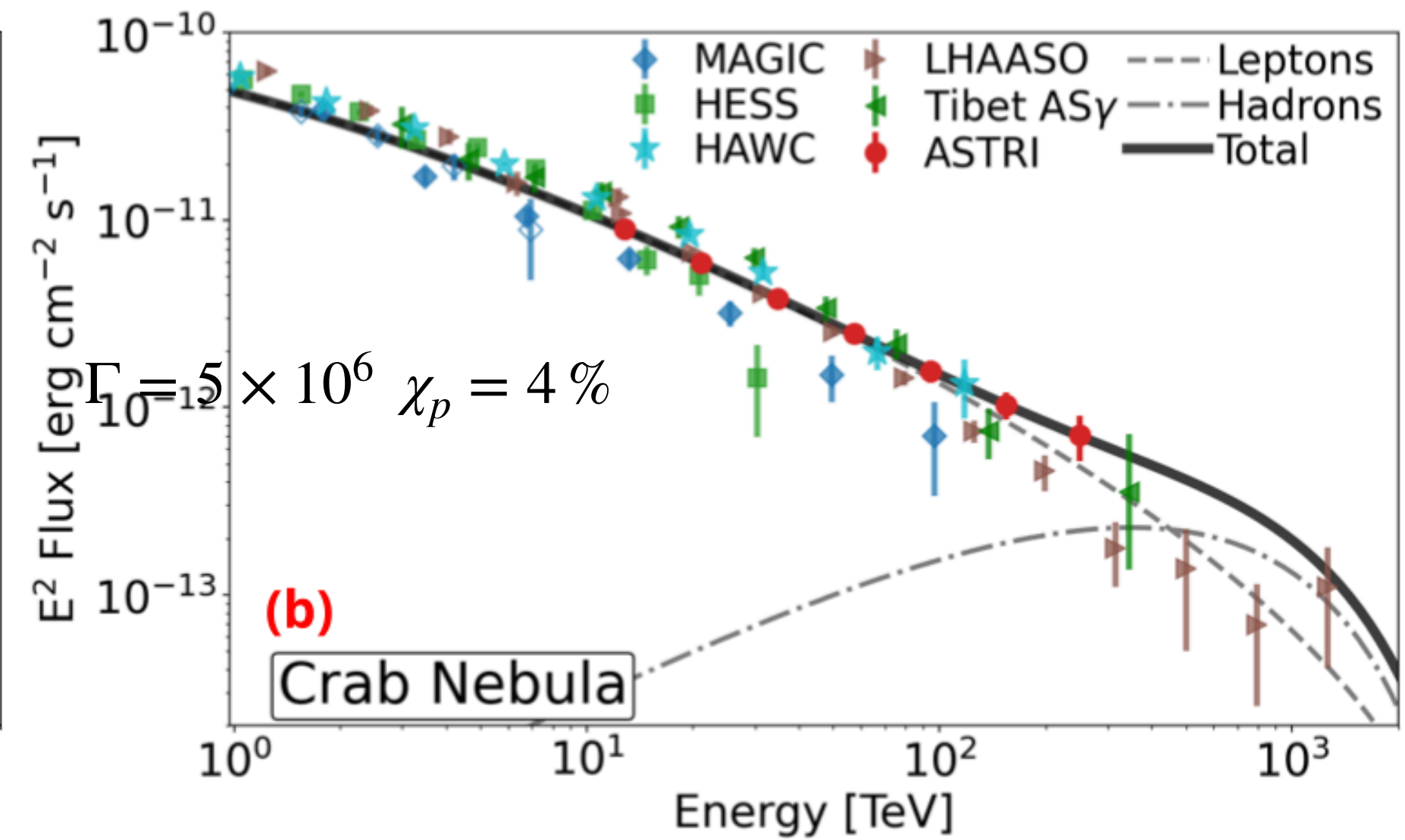
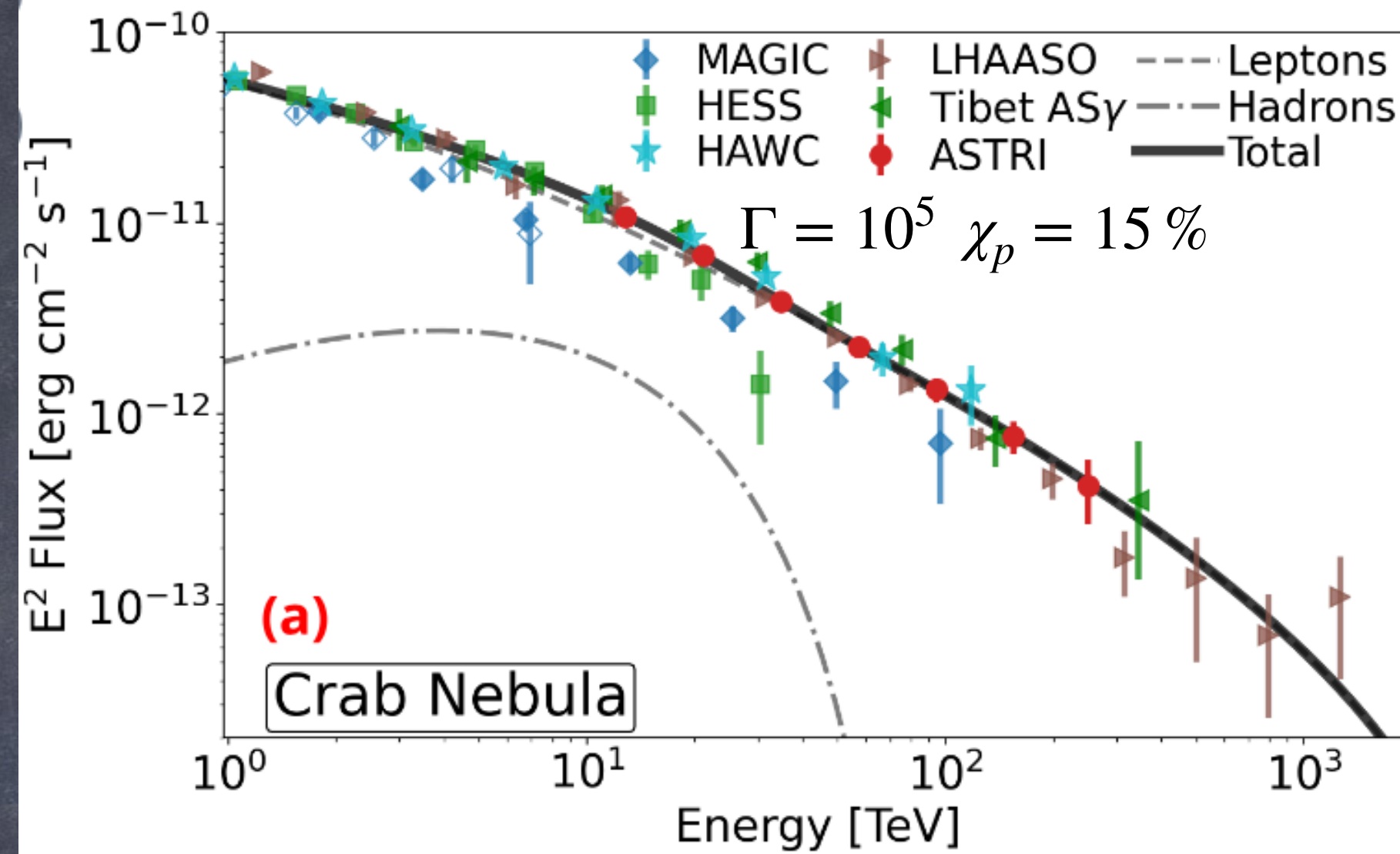
LOW ENERGY FROM  
TURBULENT  
ACCELERATION IN  
THE NEBULA?



PWNe AT EHE  
GAMMA-RAYS



# HADRONS IN CRAB?



Fiori, EA + in prep.

$$Q_p(E) \propto \delta(E - m_p c^2 \Gamma)$$

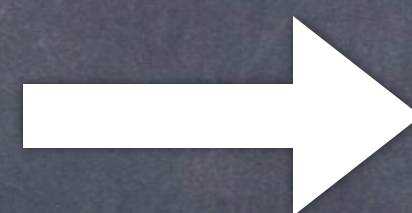
(EA & Arons 06; EA, Guetta, Blasi 03)



# MAXIMUM ENERGY IN A PWN

IN YOUNG ENERGETIC SYSTEMS ACCELERATION IS LOSS LIMITED

$$t_{acc} = \frac{E}{e\xi_E Bc} < t_{loss} = \frac{6\pi(mc^2)^2}{\sigma_T c B^2 E}$$



$$E_{max} \approx 6 \text{ PeV } \xi_E^{1/2} B_{-4}^{1/2}$$

STRICT LIMIT FROM THE PSR POTENTIAL DROP  $\Phi_{PSR} = \sqrt{\dot{E}/c}$

$$E_{max,abs} = e\xi_E B_{TS} R_{TS}$$

$$\frac{B_{TS}^2}{4\pi} = \xi_B \frac{\dot{E}}{4\pi R_{TS}^2 c}$$



$$E_{max,abs} = e\xi_E \xi_B^{1/2} \sqrt{\dot{E}/c} \approx 1.8 \text{ PeV } \xi_E \xi_B^{1/2} \dot{E}_{36}^{1/2}$$



# LEPTONIC OR HADRONIC PEVATRONS?

## 12 SOURCES DETECTED BY LHAASO ABOVE 100 TeV

**Table 1 | UHE  $\gamma$ -ray sources**

Source name	RA (°)	dec. (°)	Significance above 100 TeV ( $\times\sigma$ )	$E_{\text{max}}$ (PeV)	Flux at 100 TeV (CU)
LHAASO J0534+2202	83.55	22.05	17.8	$0.88 \pm 0.11$	1.00(0.14)
LHAASO J1825-1326	276.45	-13.45	16.4	$0.42 \pm 0.16$	3.57(0.52)
LHAASO J1839-0545	279.95	-5.75	7.7	$0.21 \pm 0.05$	0.70(0.18)
LHAASO J1843-0338	280.75	-3.65	8.5	$0.26 - 0.10^{+0.16}$	0.73(0.17)
LHAASO J1849-0003	282.35	-0.05	10.4	$0.35 \pm 0.07$	0.74(0.15)
LHAASO J1908+0621	287.05	6.35	17.2	$0.44 \pm 0.05$	1.36(0.18)
LHAASO J1929+1745	292.25	17.75	7.4	$0.71 - 0.07^{+0.16}$	0.38(0.09)
LHAASO J1956+2845	299.05	28.75	7.4	$0.42 \pm 0.03$	0.41(0.09)
LHAASO J2018+3651	304.75	36.85	10.4	$0.27 \pm 0.02$	0.50(0.10)
LHAASO J2032+4102	308.05	41.05	10.5	$1.42 \pm 0.13$	0.54(0.10)
LHAASO J2108+5157	317.15	51.95	8.3	$0.43 \pm 0.05$	0.38(0.09)
LHAASO J2226+6057	336.75	60.95	13.6	$0.57 \pm 0.19$	1.05(0.16)

Cao+ 2021

PeV PROTONS OR ELECTRONS?

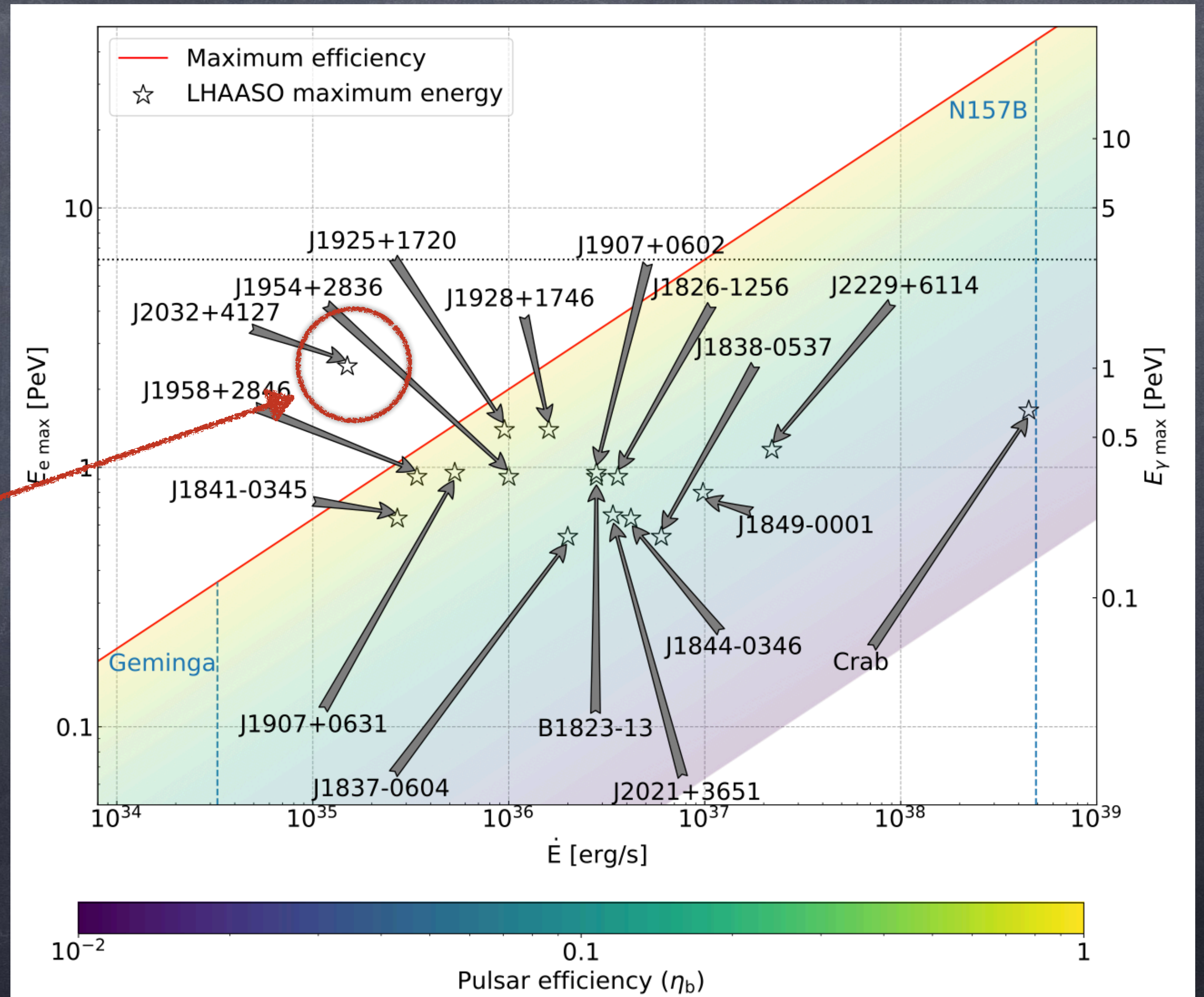
ALL SOURCES HAVE A PSR IN THE FIELD BUT....



# LHAASO PEVATRONS AND PWNe

MAXIMUM  
ELECTRON ENERGY AS A FUNCTION  
OF PSR POTENTIAL DROP  
AND LHAASO SOURCES

CYGNUS



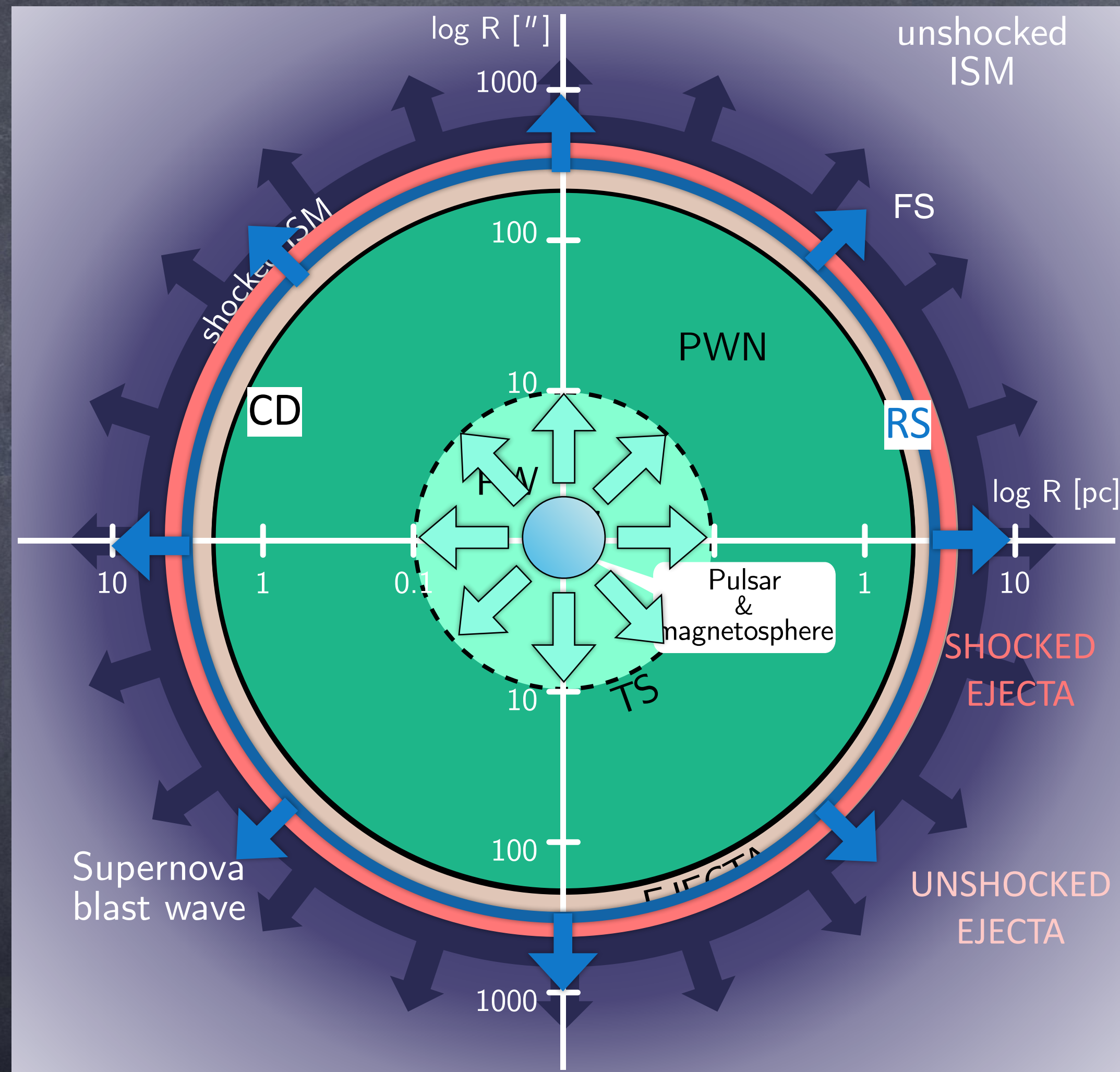
Lopez-Coto + 2022



EVOLVED PWINE



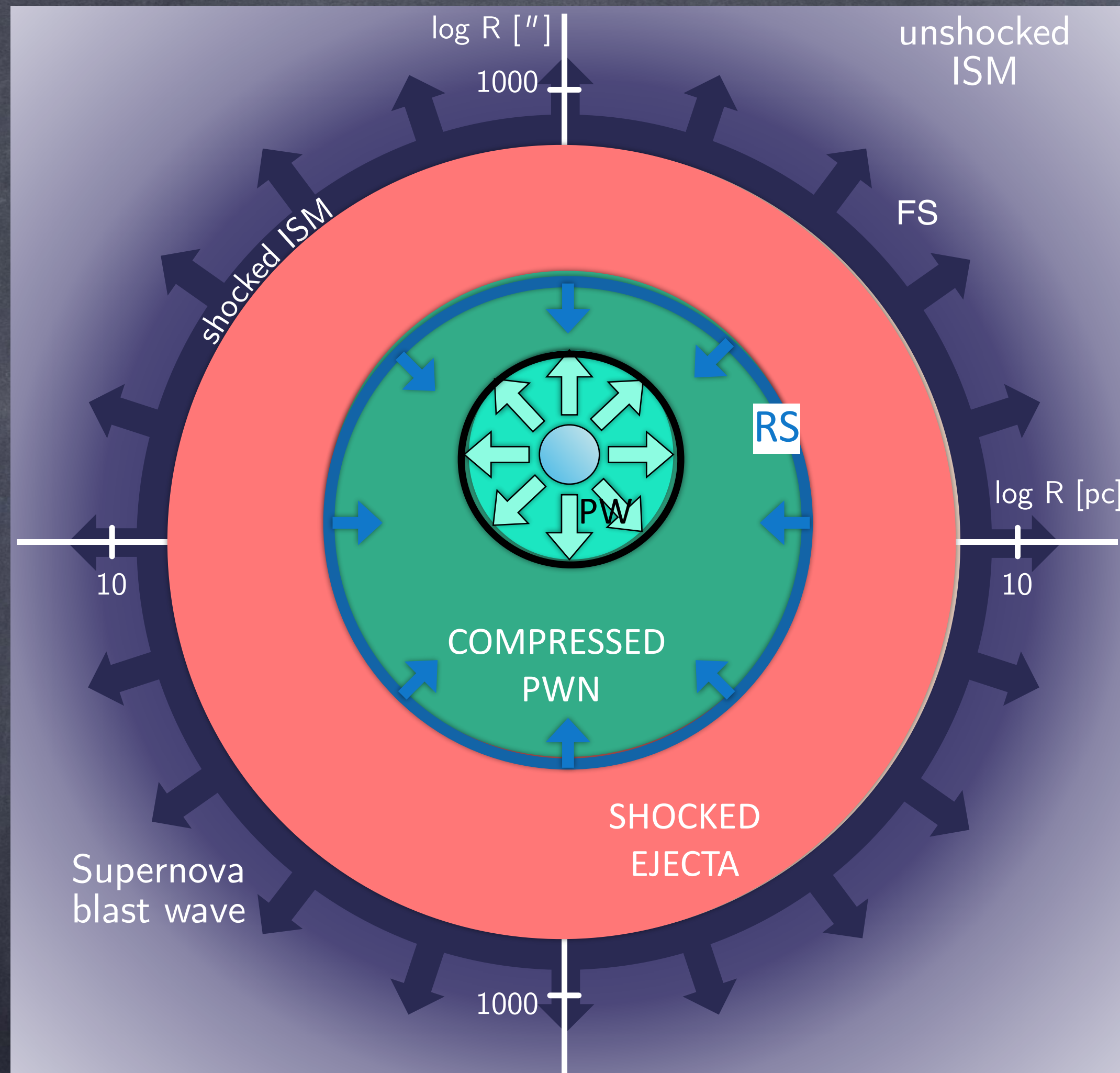
# BASIC PICTURE FOR YOUNG SYSTEMS



Adapted from Kennel & Coroniti 1984  
[Del Zanna & Olmi 2017]



# PWN EVOLUTION



SNR EXPANSION

SLOWS DOWN

+

LARGE FRACTION OF  
ALL THE PULSARS

BORN WITH

HIGH KICK VELOCITY



COMPRESSED PWN  
OFFSET PW

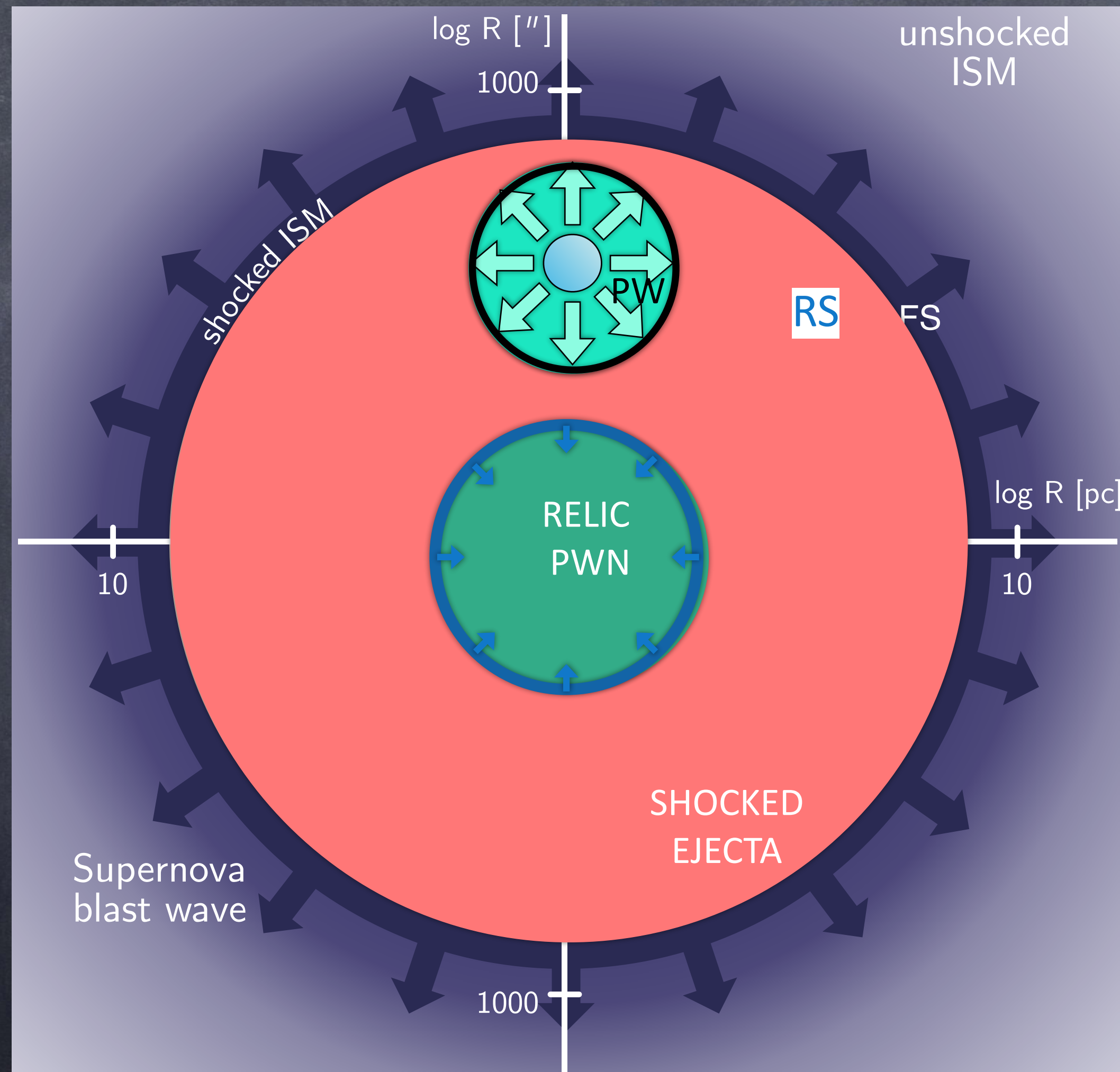


REVERBERATION PHASE

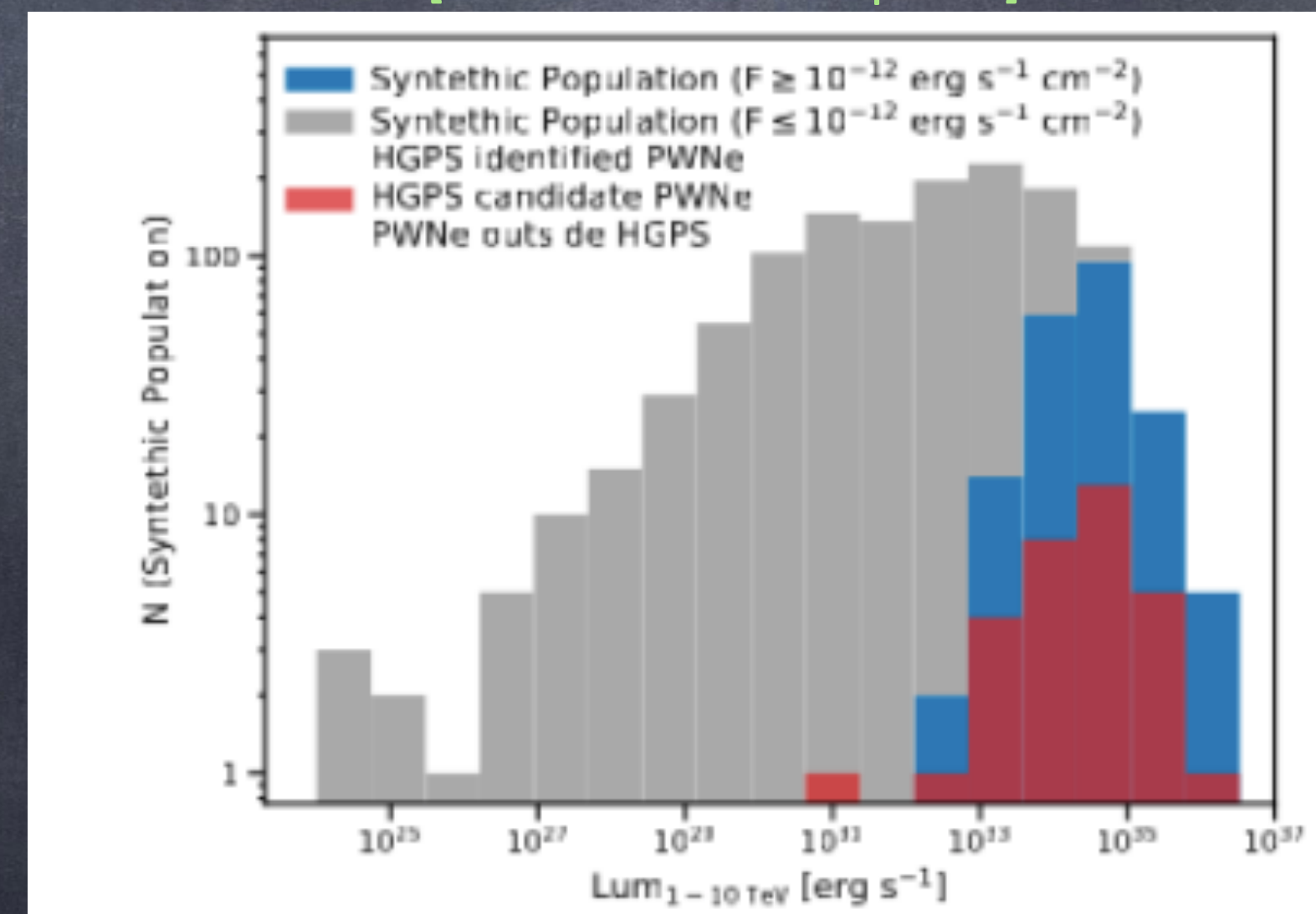


# RELIC NEBULAE

PSR MAY CROSS RS DURING COMPRESSION  
AND LEAVE A RELIC

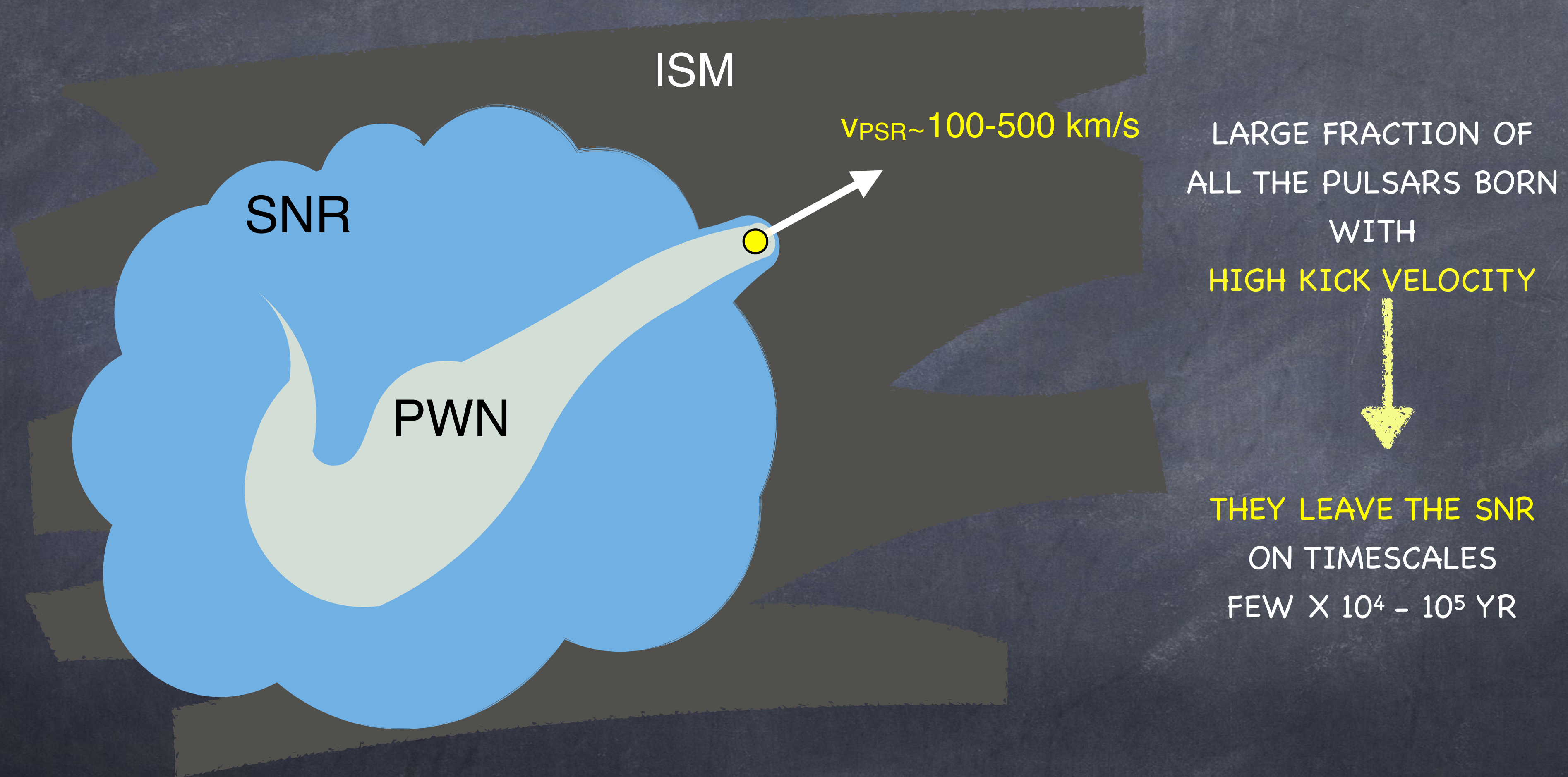


EVENTUALLY  
MOST GAMMA-RAY BRIGHT  
X-RAY DIM PWNe  
[Fiori+ MNRAS in press]



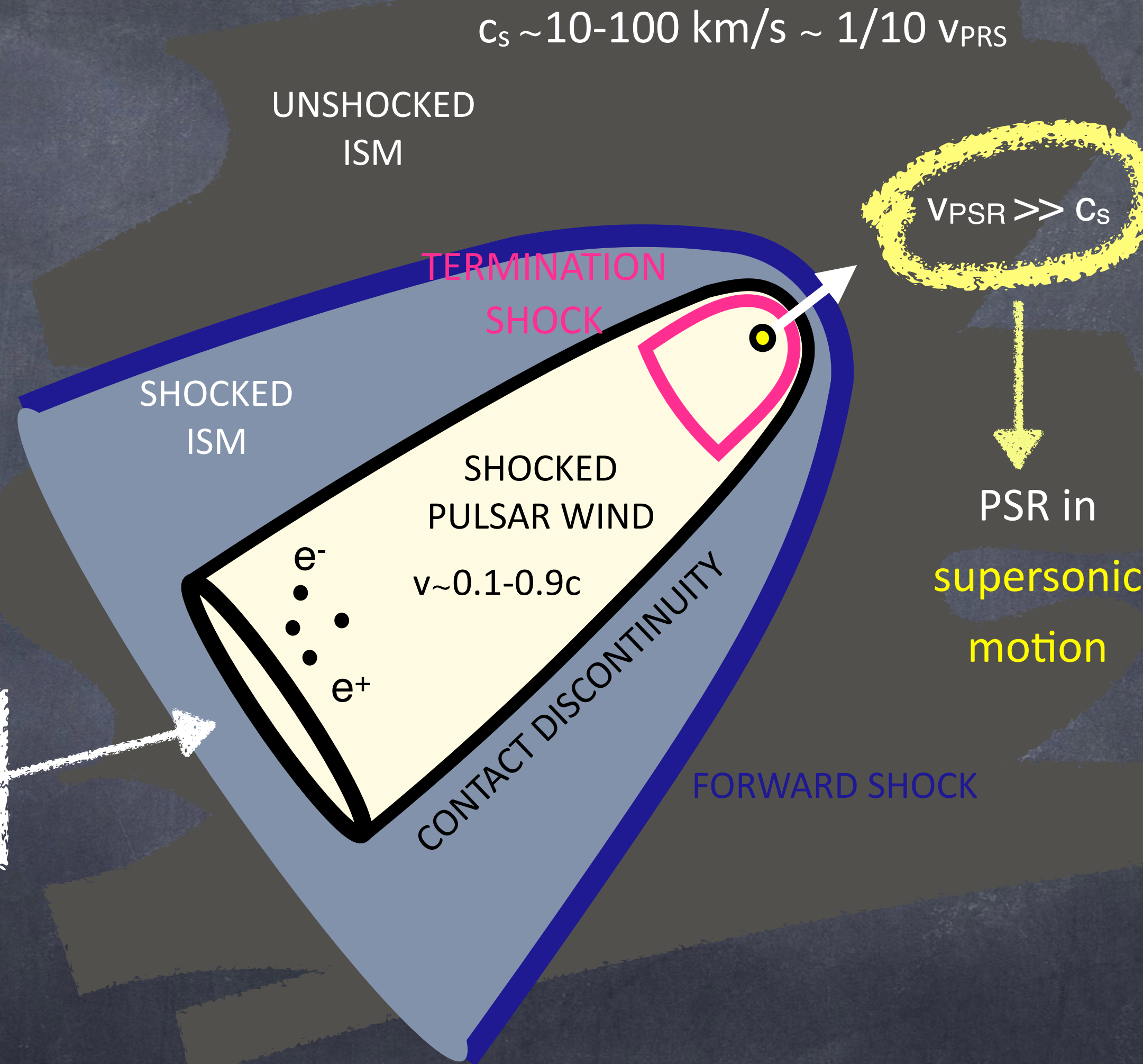
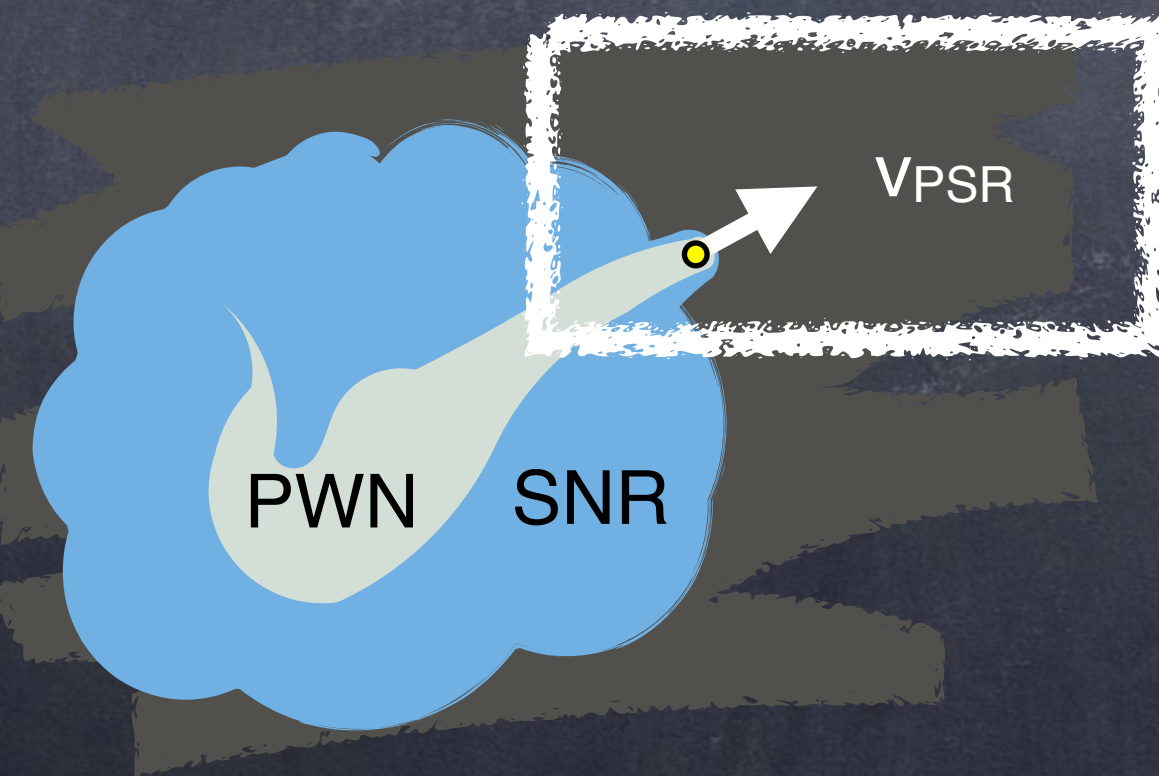
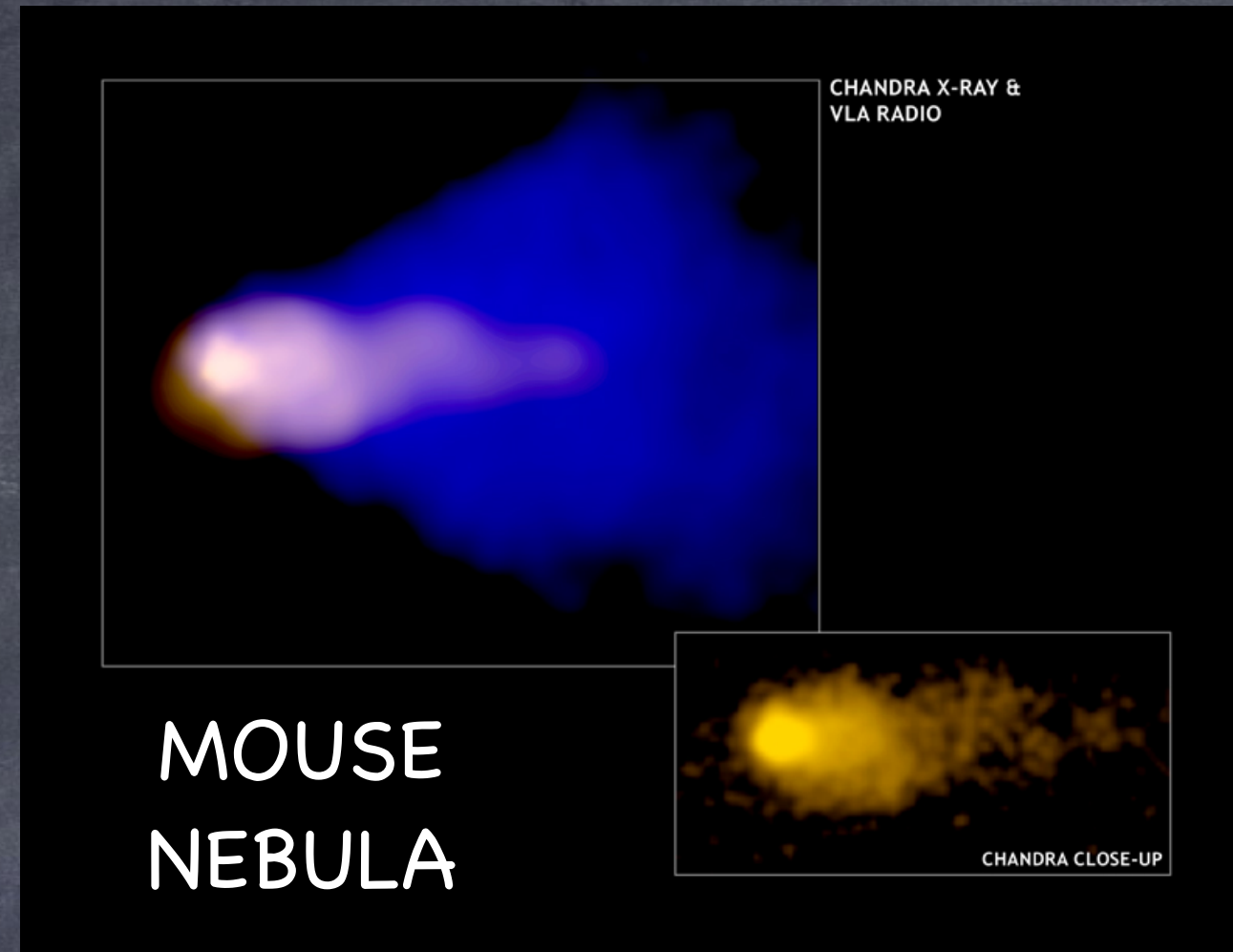


# EVOLVED PWNe



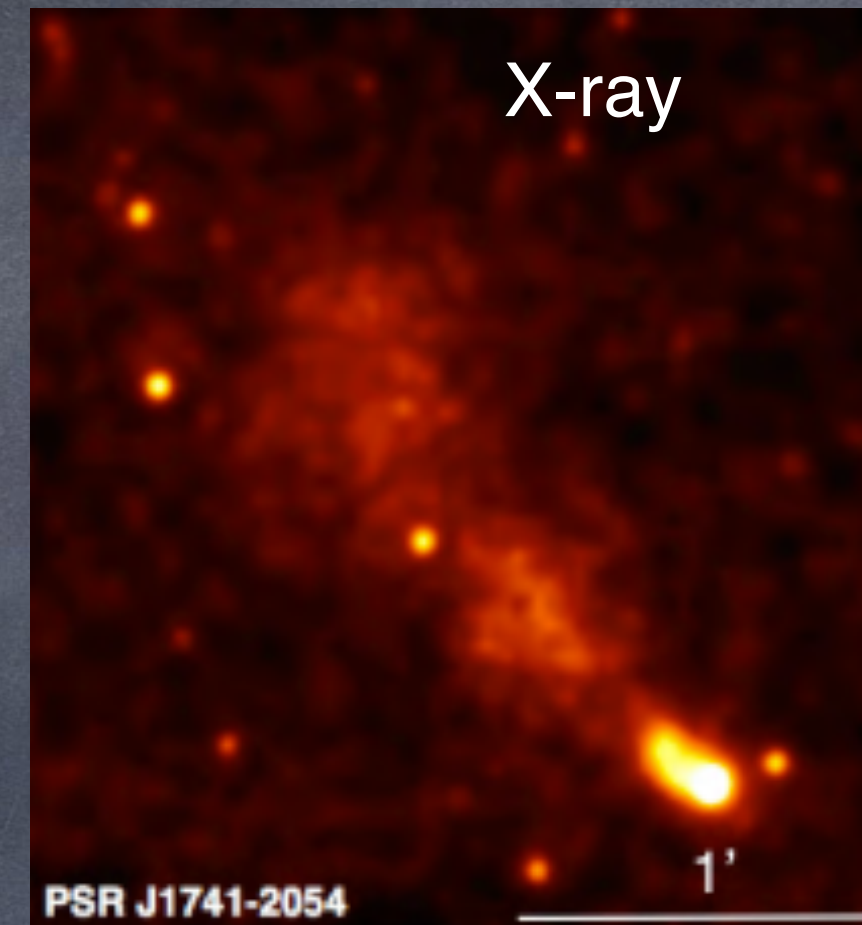
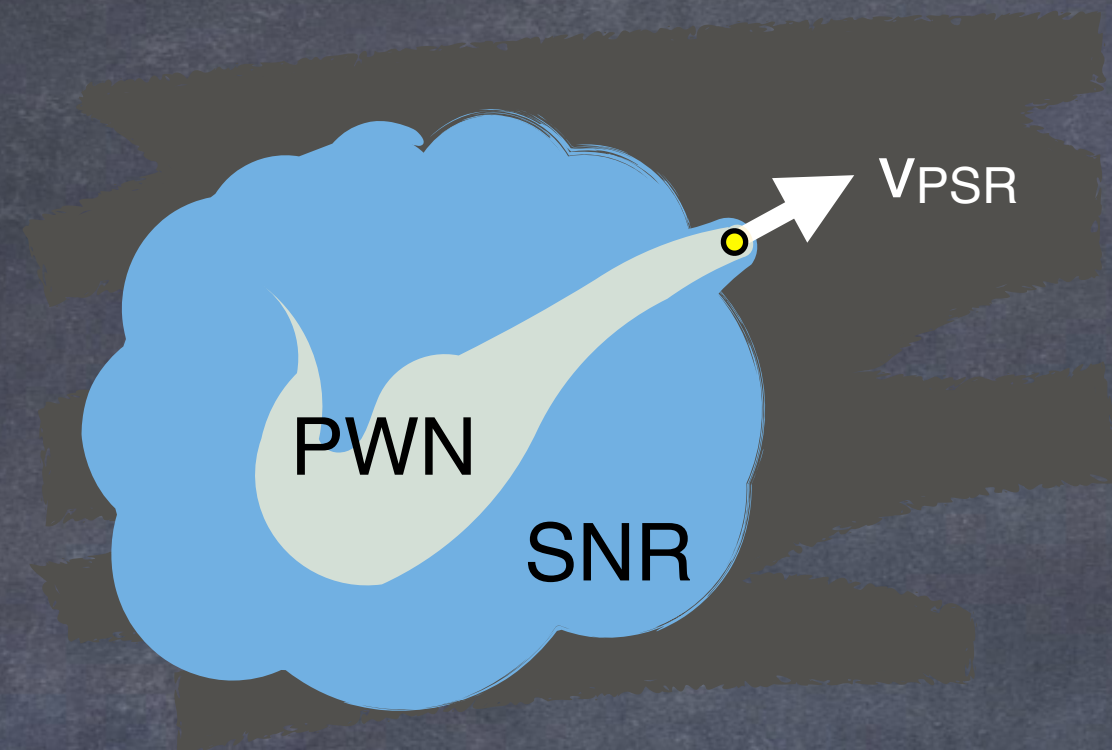


# BOW SHOCK NEBULAE



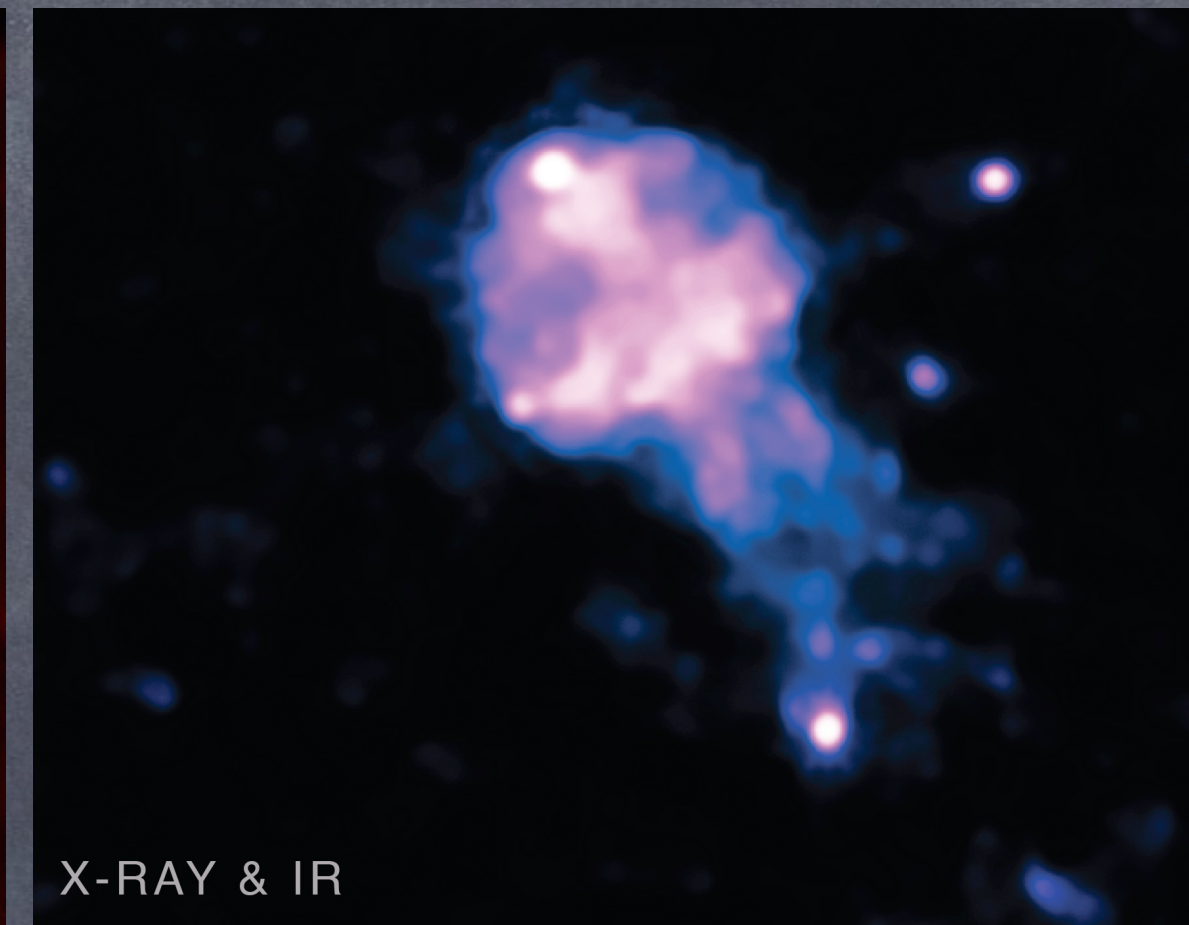


# OBSERVATIONS: COMETARY NEBULAE



**PSR J1741-2054**

[Kargaltsev et al. 2016]



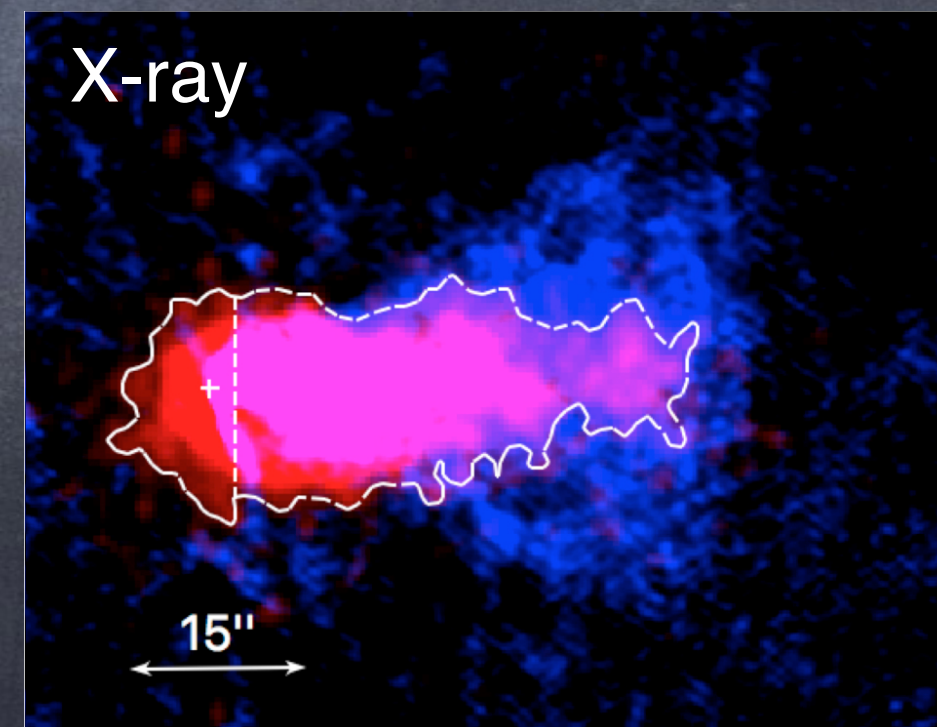
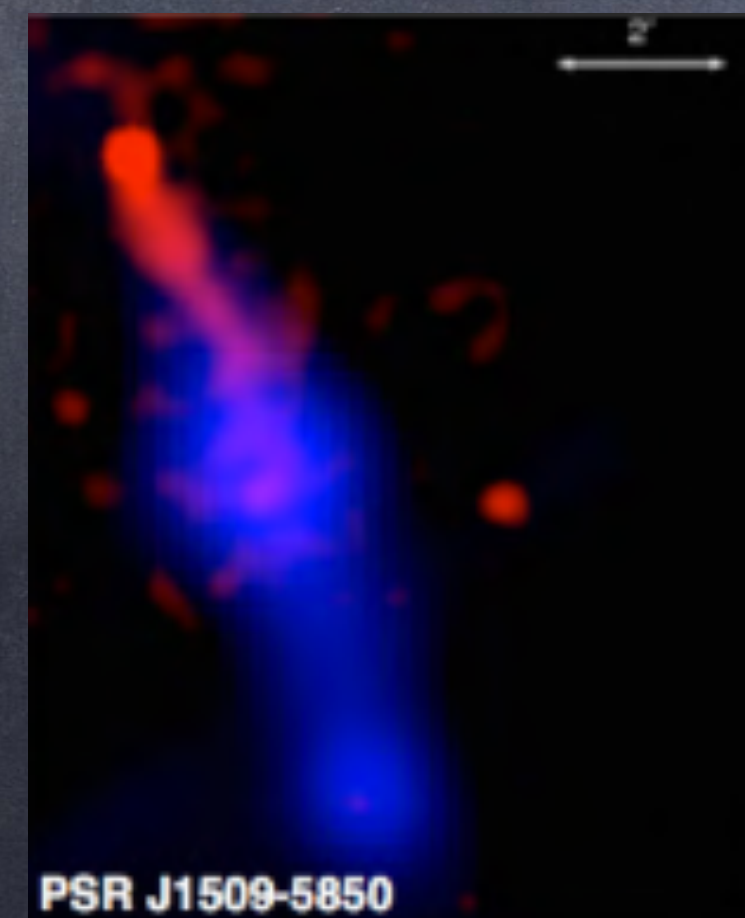
**B0355+54**

[Emre et al. 2005]



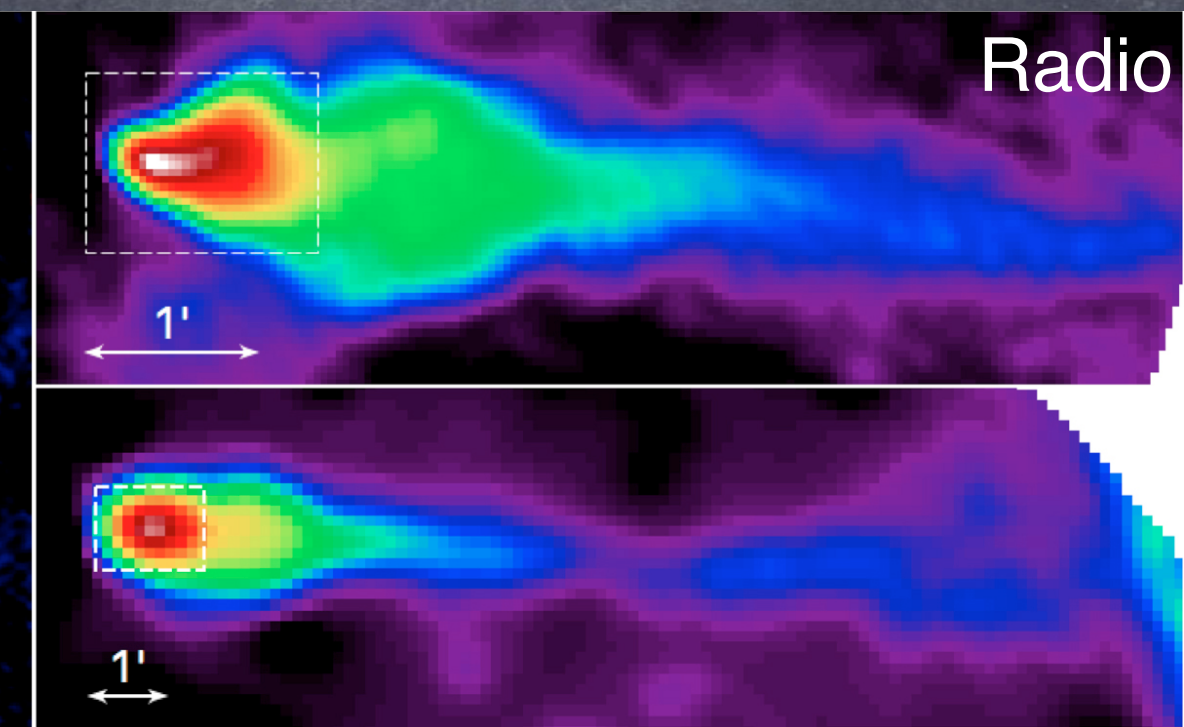
**PSR J1509-5850**

[Hui & Becker 2007, Klinger et al. 2016]



**Mouse PWN**

[Yusef-Zadeh & Bally 1987, Yusef-Zadeh & Gaensler 2005, Klinger et al. 2018]

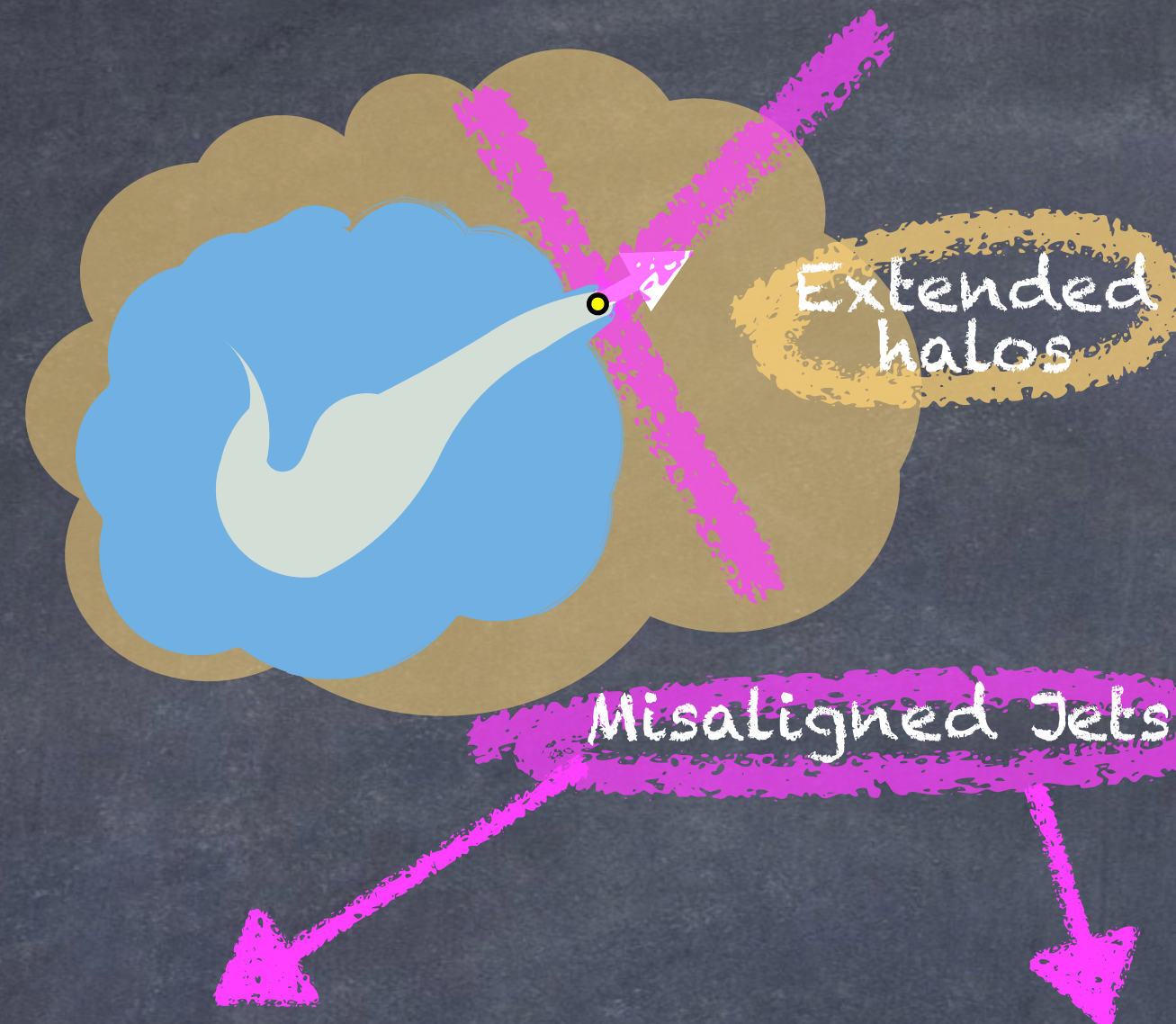




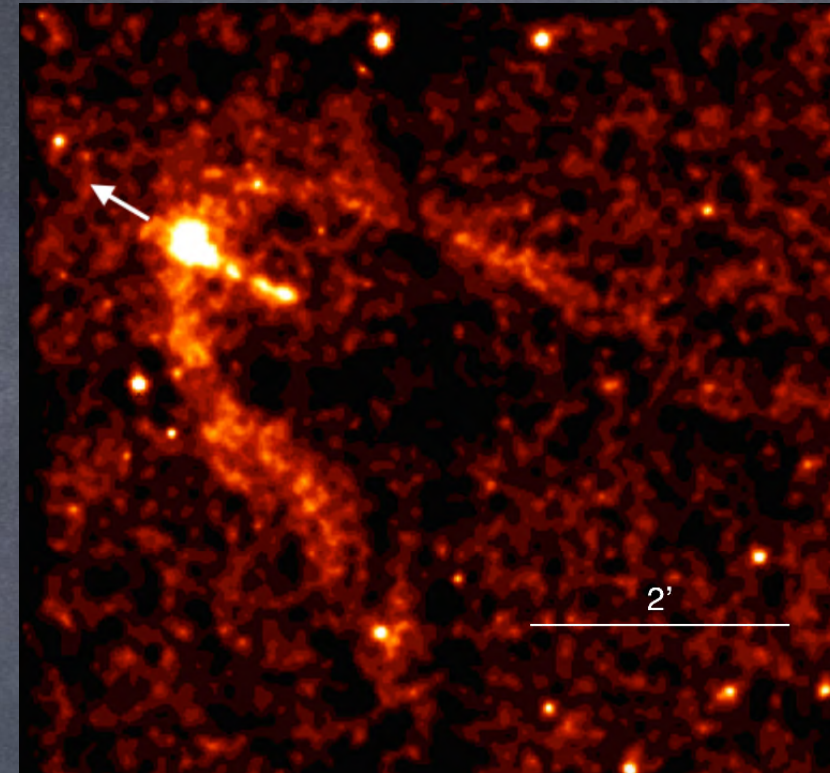
EVOLVED SYSTEMS  
AND  
PARTICLE ESCAPE



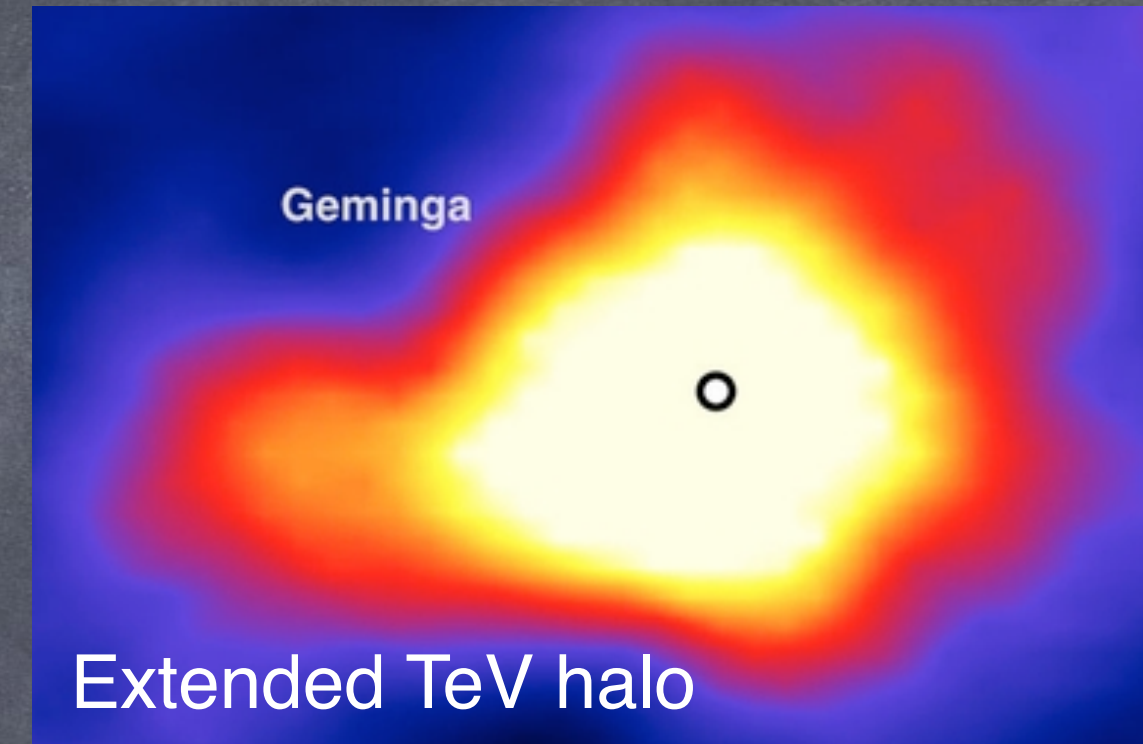
# OBSERVATIONS: JETS AND HALOES



X-ray



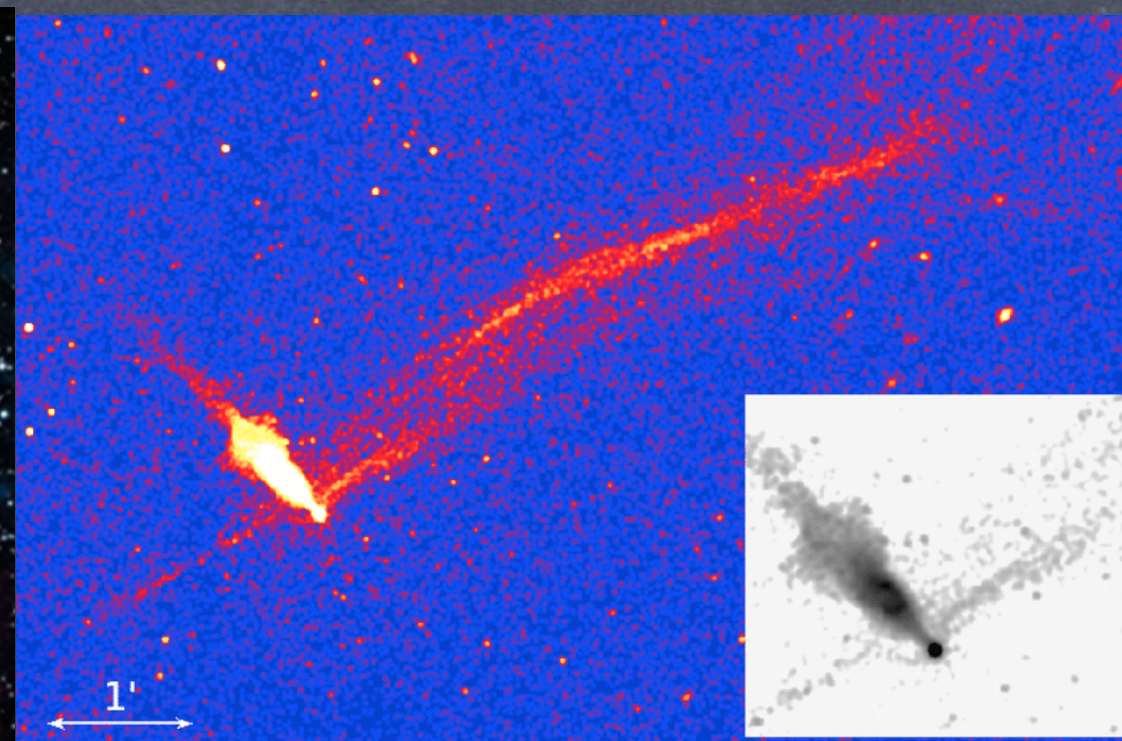
Geminga  
[Posselt+ 2017]



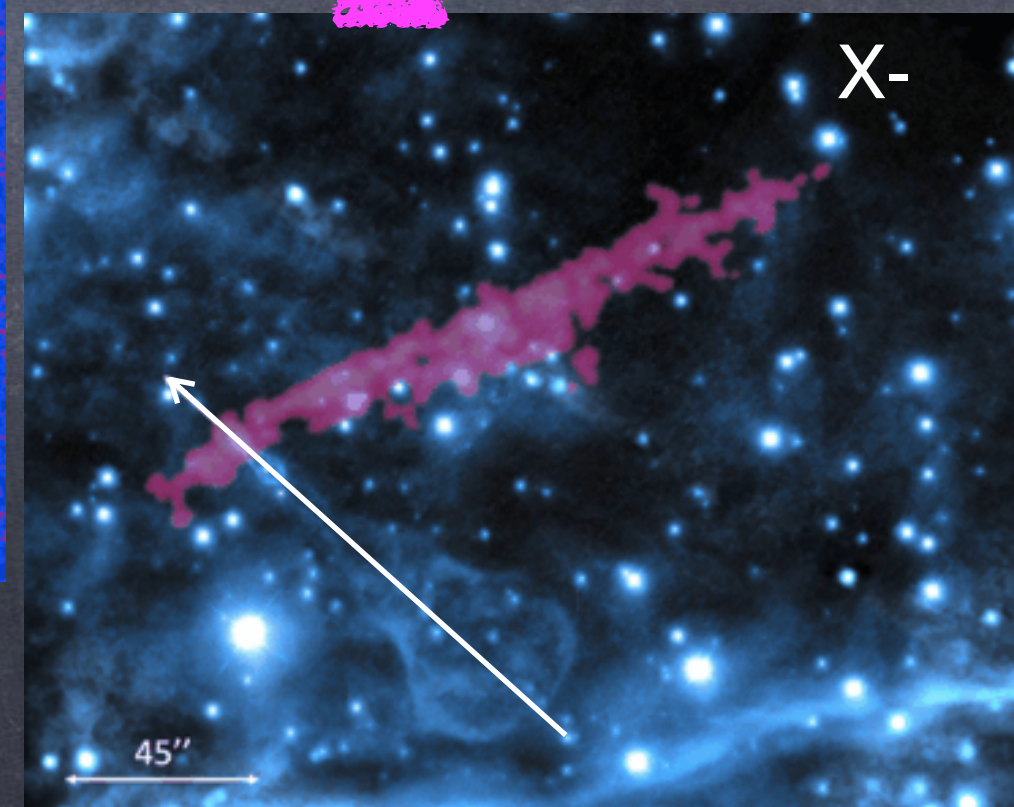
Extended TeV halo  
[Abeysekara+ 2017]



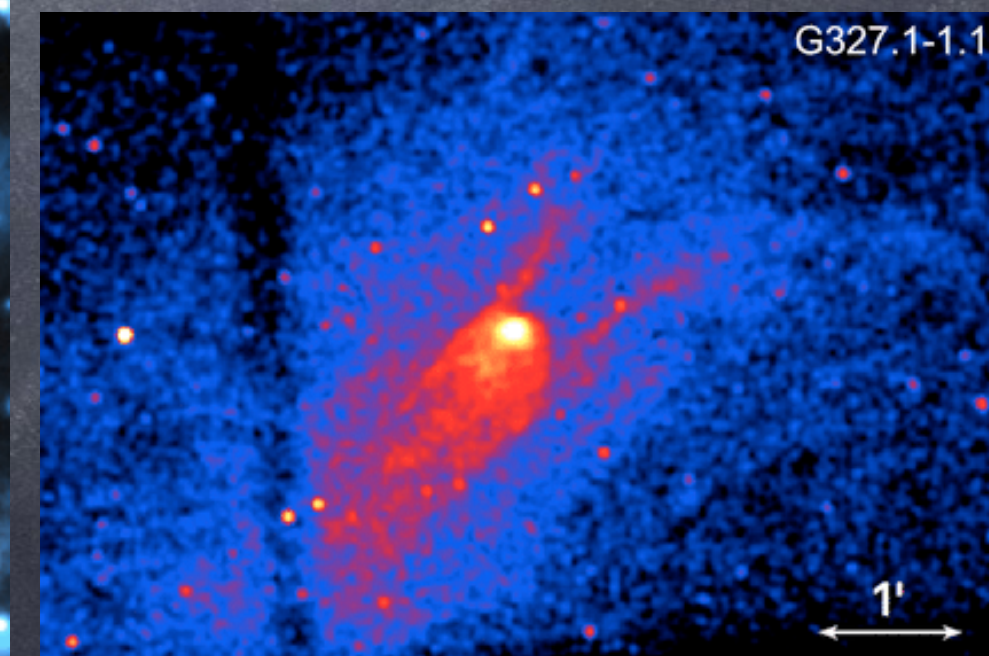
PSR J1509-5850  
[Klinger+ 2016]



Lighthouse nebula  
[Pavan+ 2016]



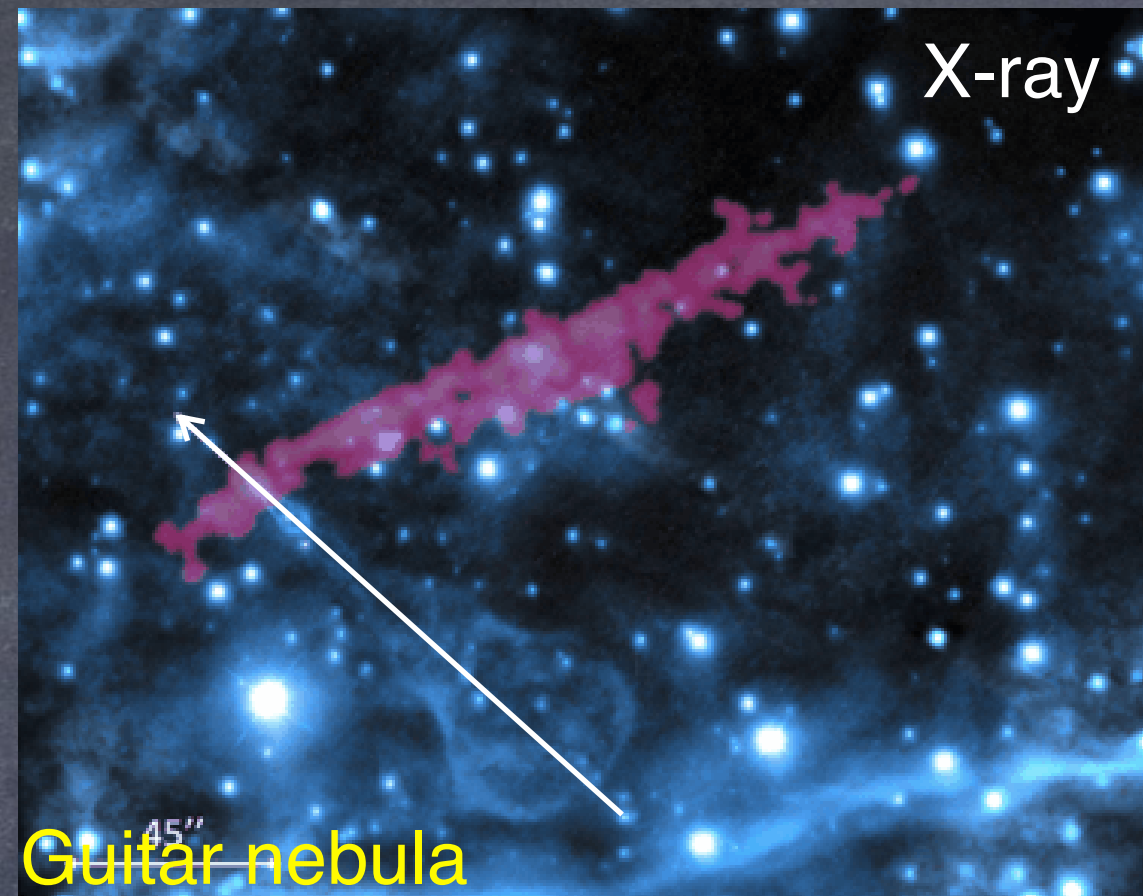
Guitar nebula  
[Cordes+ 1993, Wong+ 2003]



G327  
[Temim+ 2009]

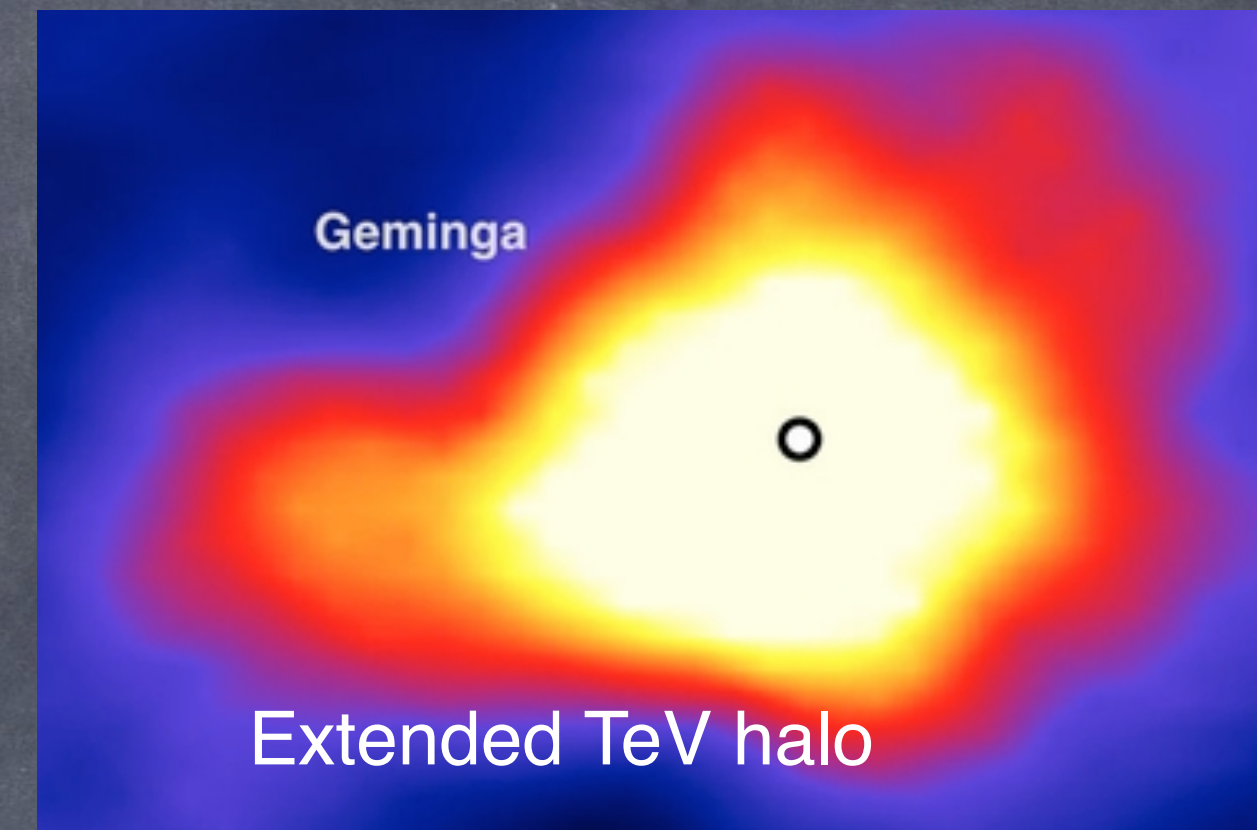


# INTERPRETATION: JETS AND HALOES



[Cordes+ 1993, Wong+ 2003]

JETS CONSISTENT WITH  
SYNCHROTRON EMISSION  
OF PARTICLES WITH  $E \approx e\Phi_{\text{PSR}}$   
IN A FEW  $\times 10\mu\text{G}$  MAGNETIC FIELD  
[Bandiera 2008]

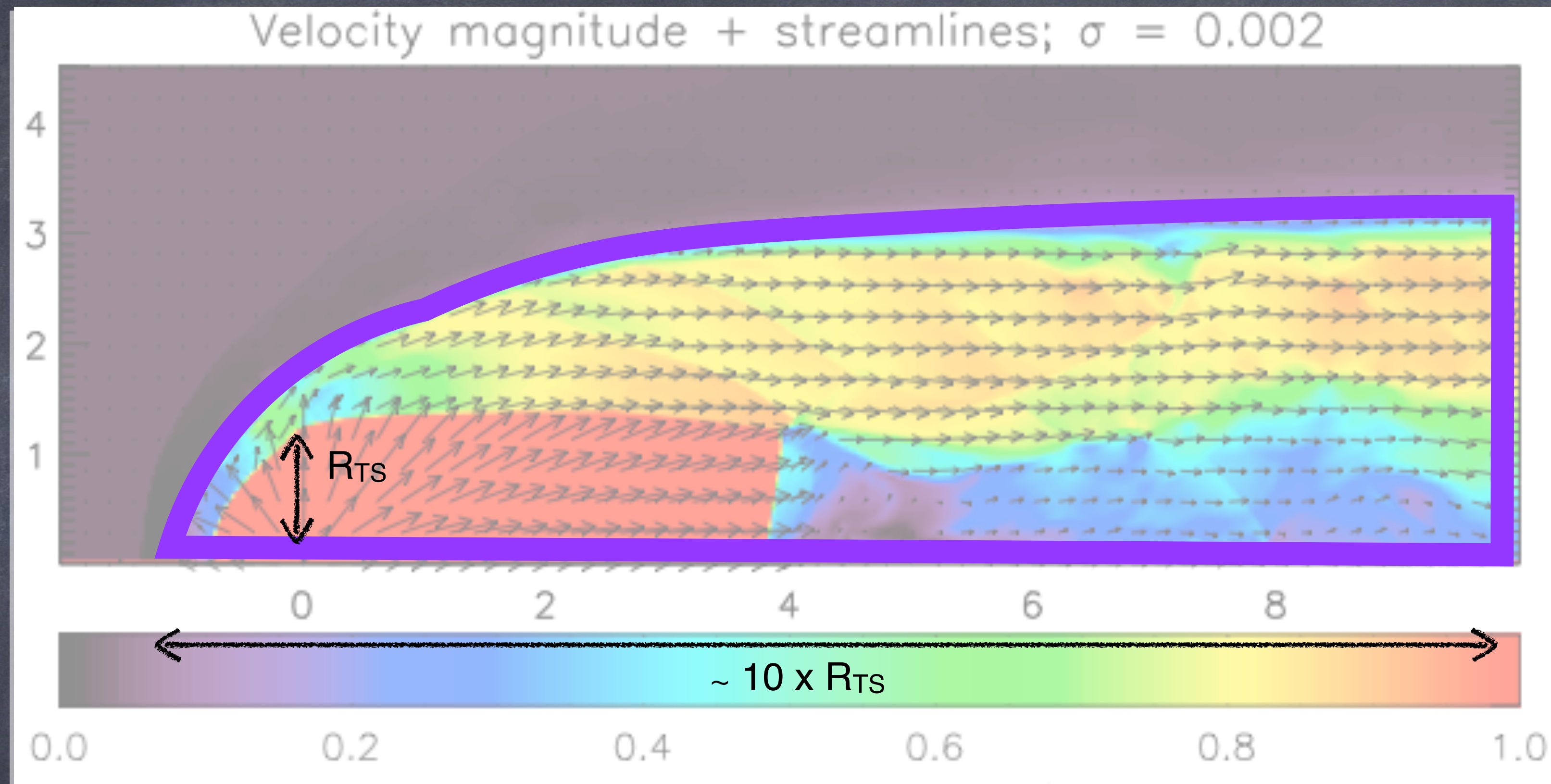


[Abeysekara+ 2017]

HALOS CONSISTENT WITH  
ICS EMISSION  
OF PARTICLES WITH  $E \approx e\Phi_{\text{PSR}}$   
IN A  $\approx \mu\text{G}$  MAGNETIC FIELD  
AND  $D \approx 10^{-2}D_{\text{gal}}$   
[Abeysekara+ 2017, Lopez-Coto & Giacinti 2018,  
Lopez-Coto + 2021]



# 2D RMHD MODELS OF BS PWNE



Bucciantini, EA, Del Zanna 2005

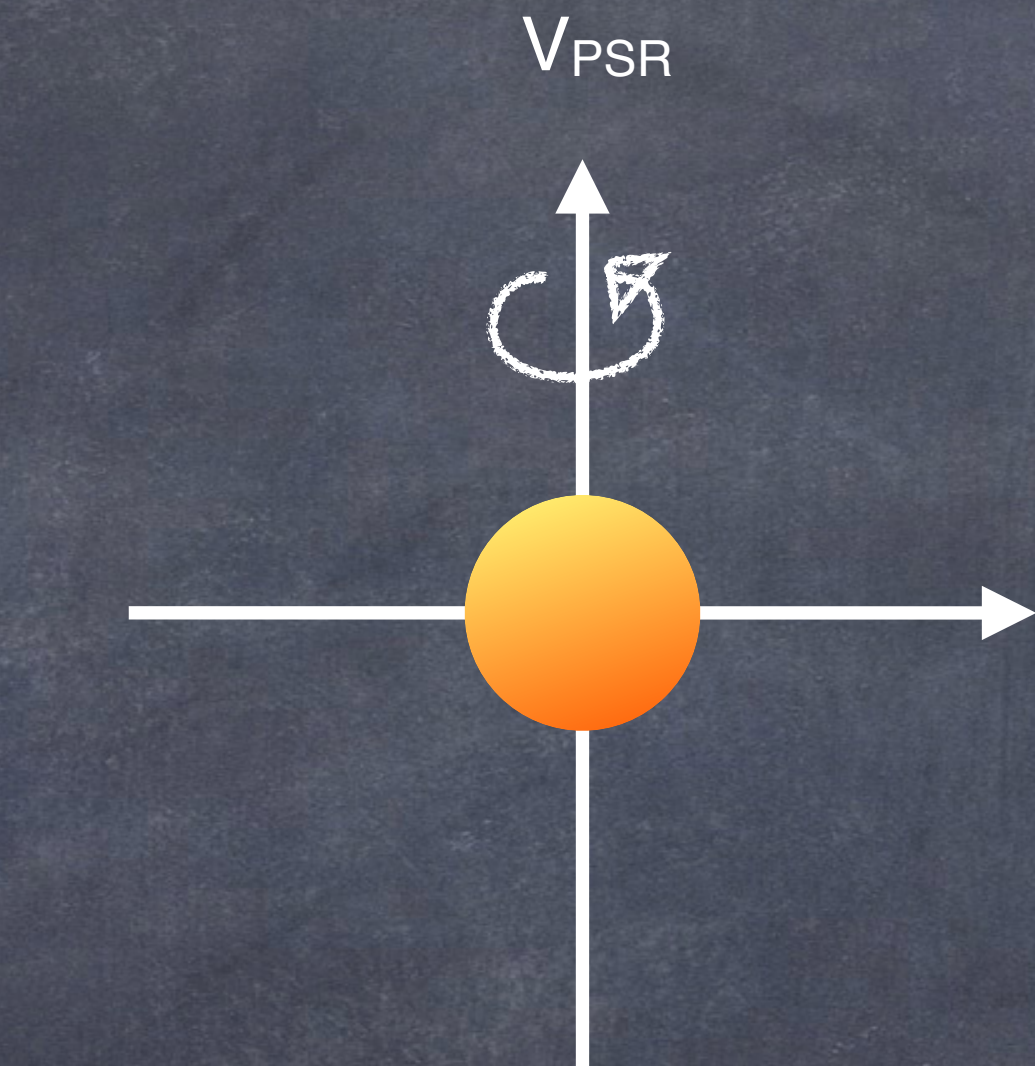
FORMATION OF BOW SHOCK, TS DEFORMATION, CYLINDRICAL TAIL WITH  
MILDLY RELATIVISTIC OUTFLOW IN THE TAIL



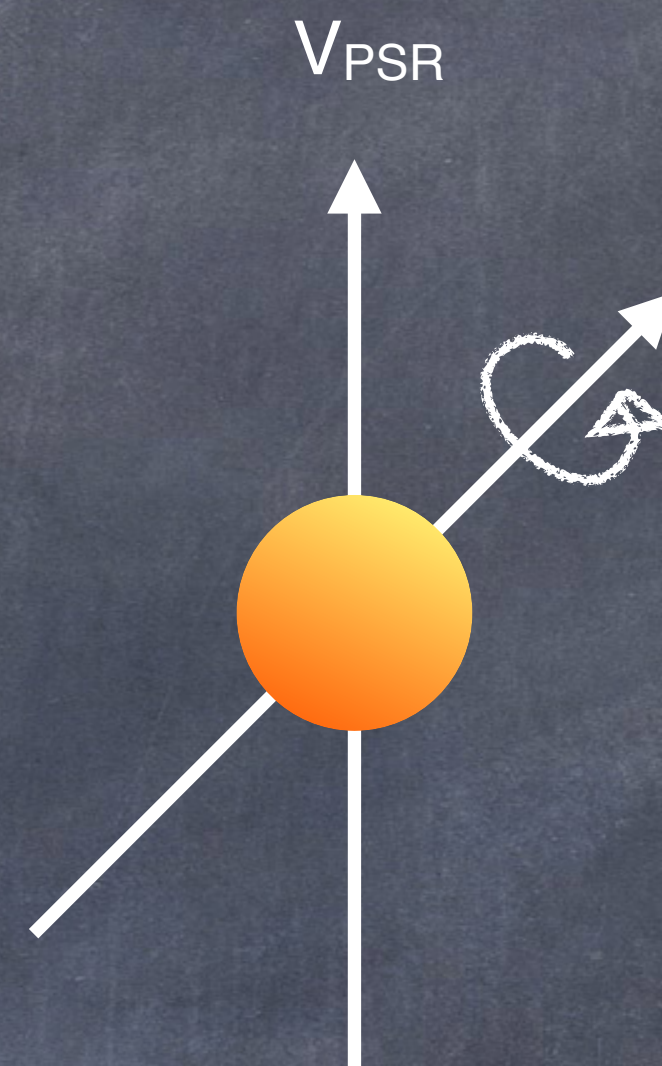
# 3D MHD MODELS OF BSPWNe

## PARAMETERS OF THE PULSAR WIND

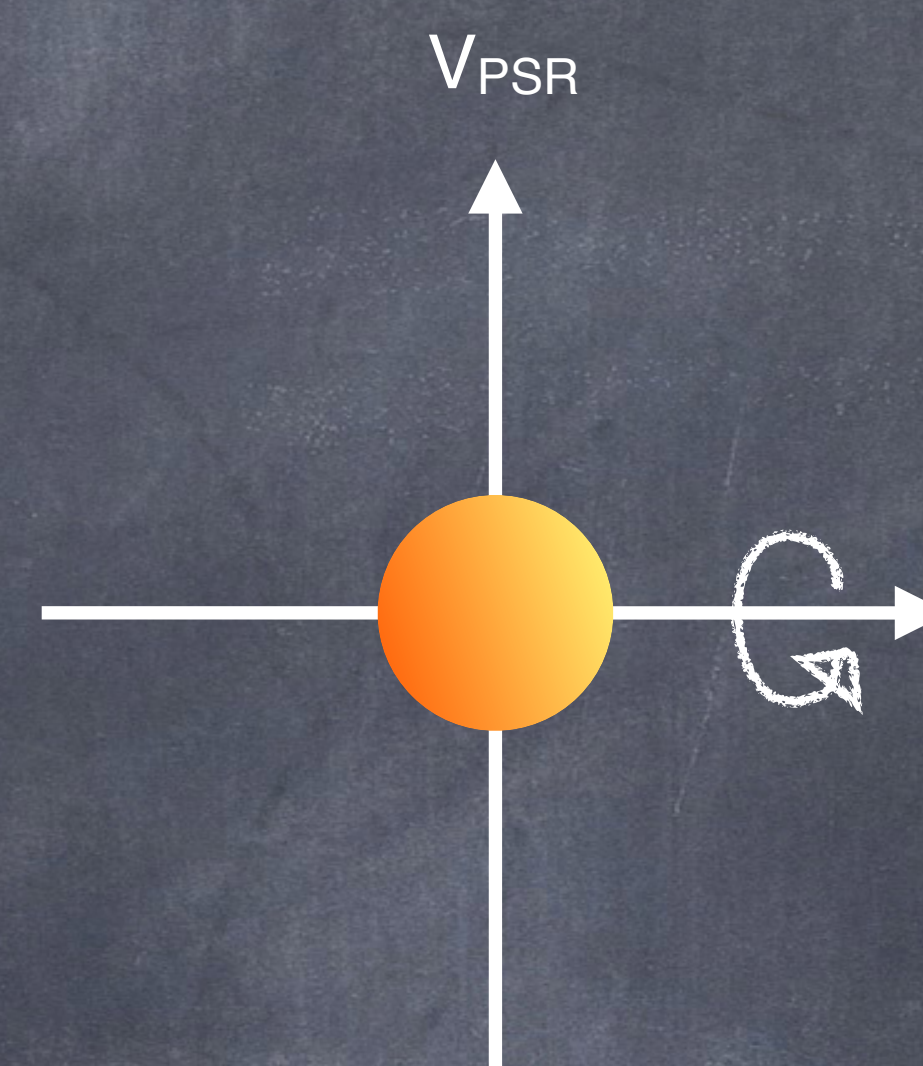
Inclination of the spin-axis and pulsar speed:



Spin-axis aligned with pulsar motion  $\Phi_M=0^\circ$



$\Phi_M=45^\circ$



$\Phi_M=90^\circ$

Anisotropy in the energy flux:  $F(\psi) \propto 1 + \alpha \sin^2 \psi$   
 $\psi$  colatitude from the spin-axis



ISOTROPIC case  
( $\alpha=0$ )



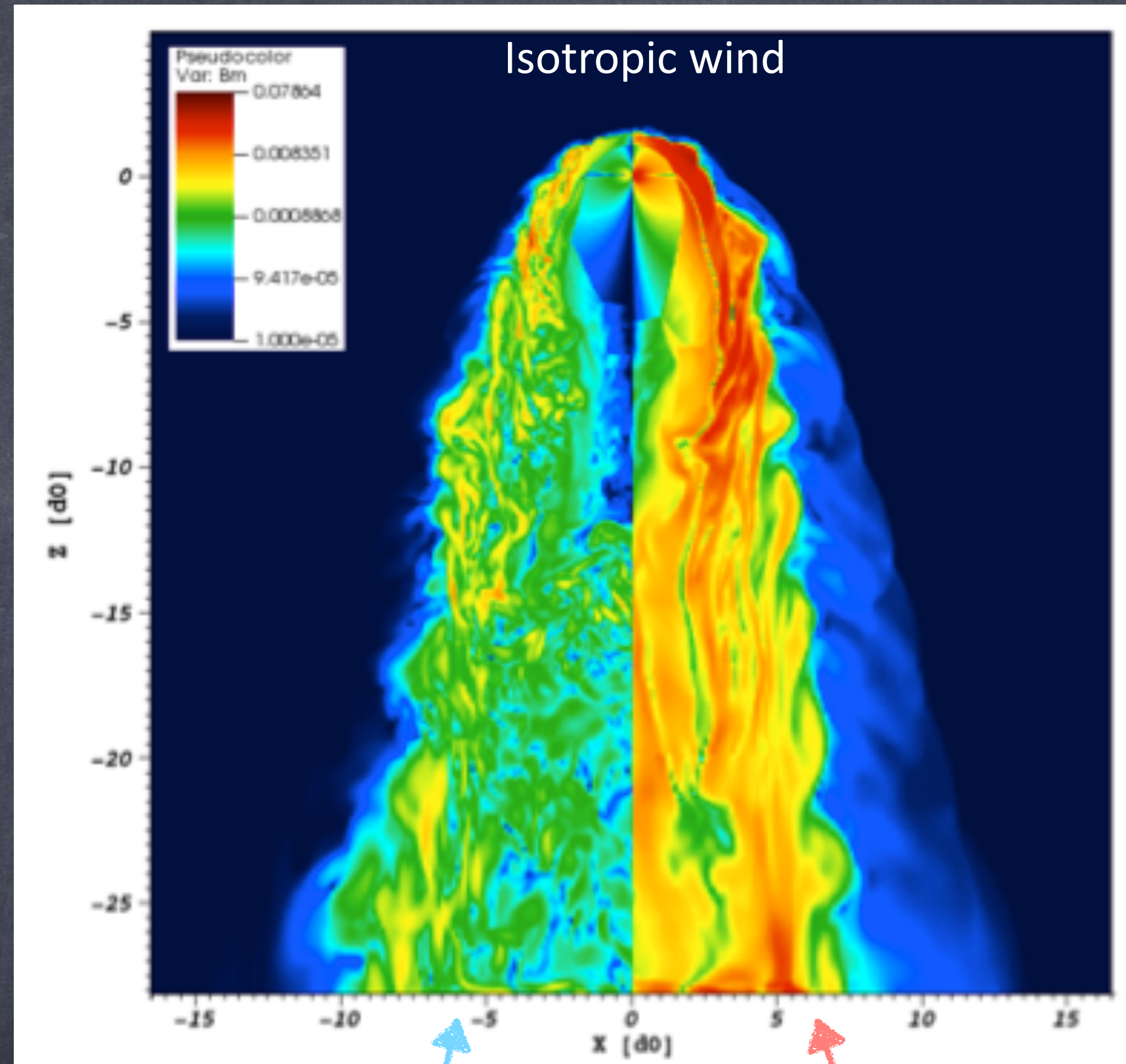
ANISOTROPIC case ( $\alpha \neq 0$ )

Wind magnetization:  $0.01 \lesssim \sigma \lesssim 1$



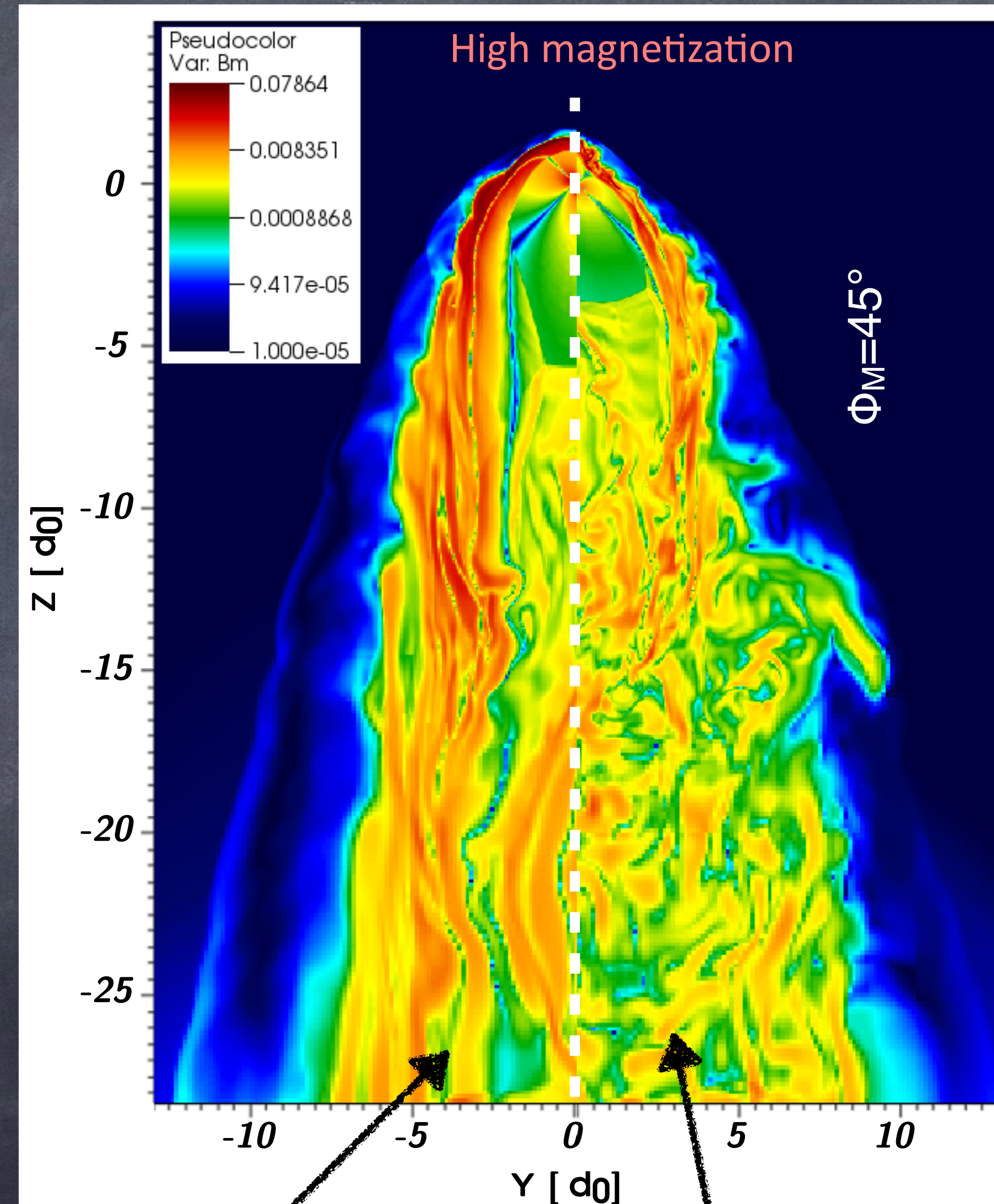
# 3D RMHD SIMULATIONS OF BSPWNe

[Olmi & Bucciantini 2019]



Low magnetization

High magnetization

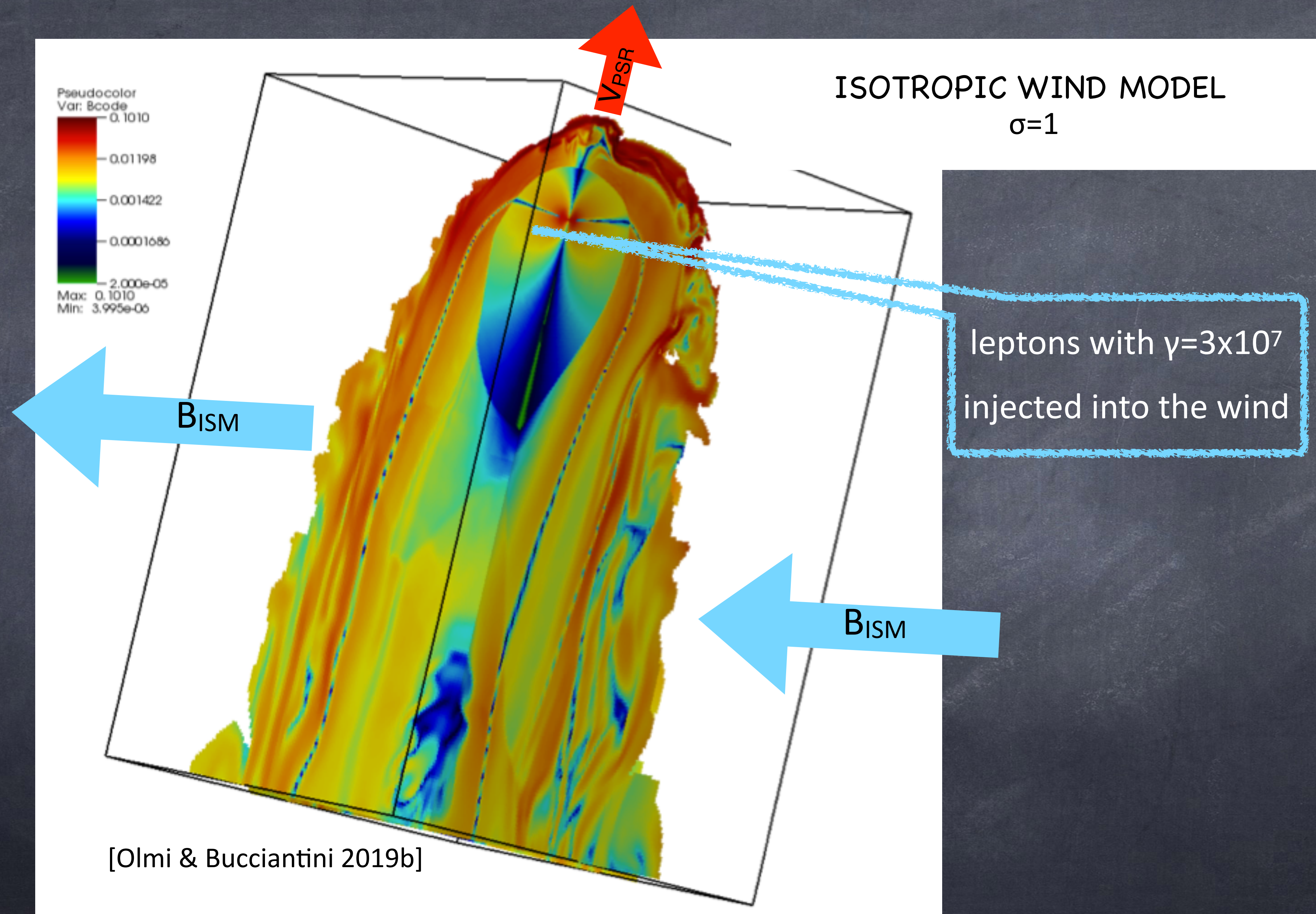


Isotropic wind

Anisotropic wind



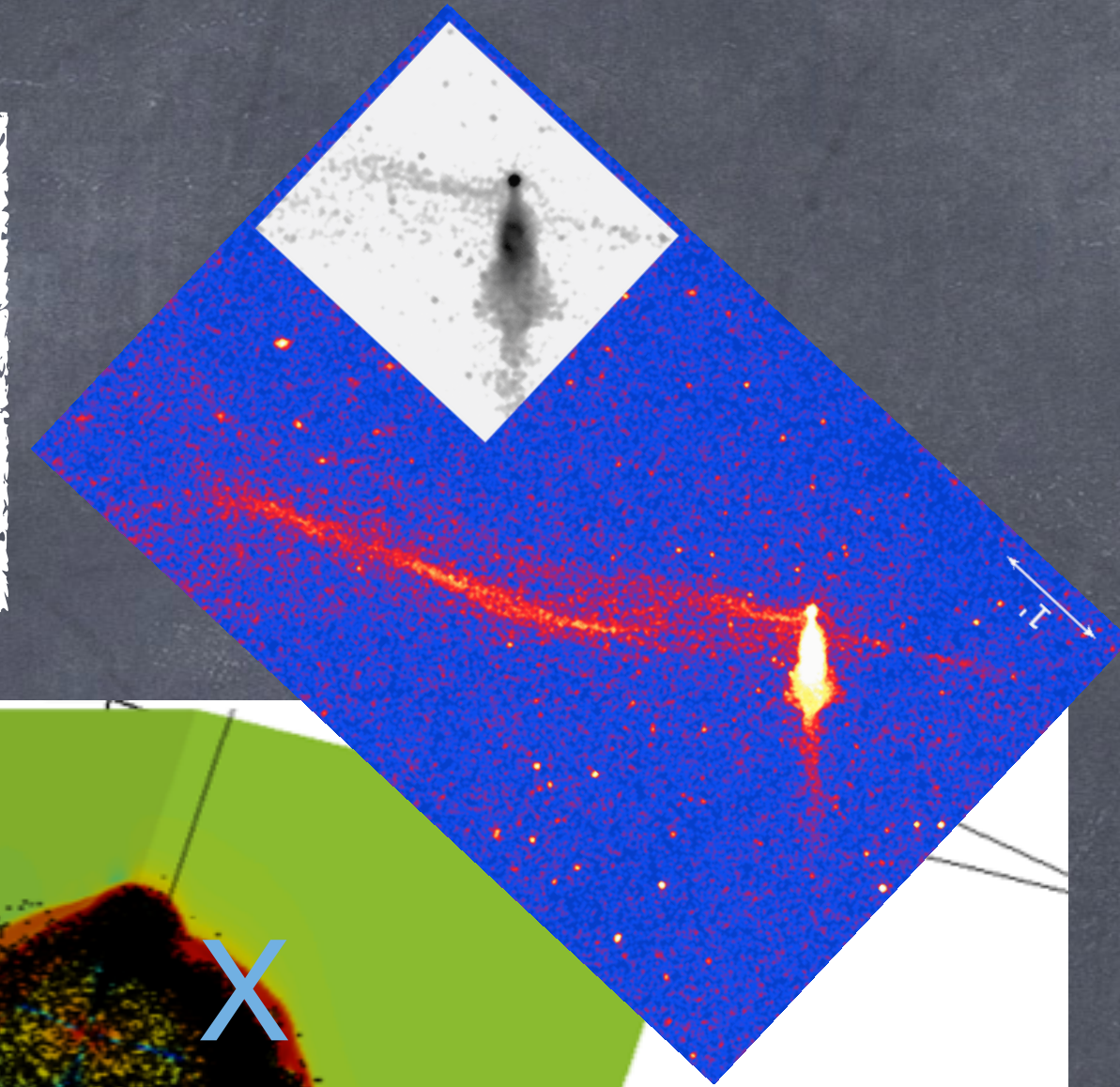
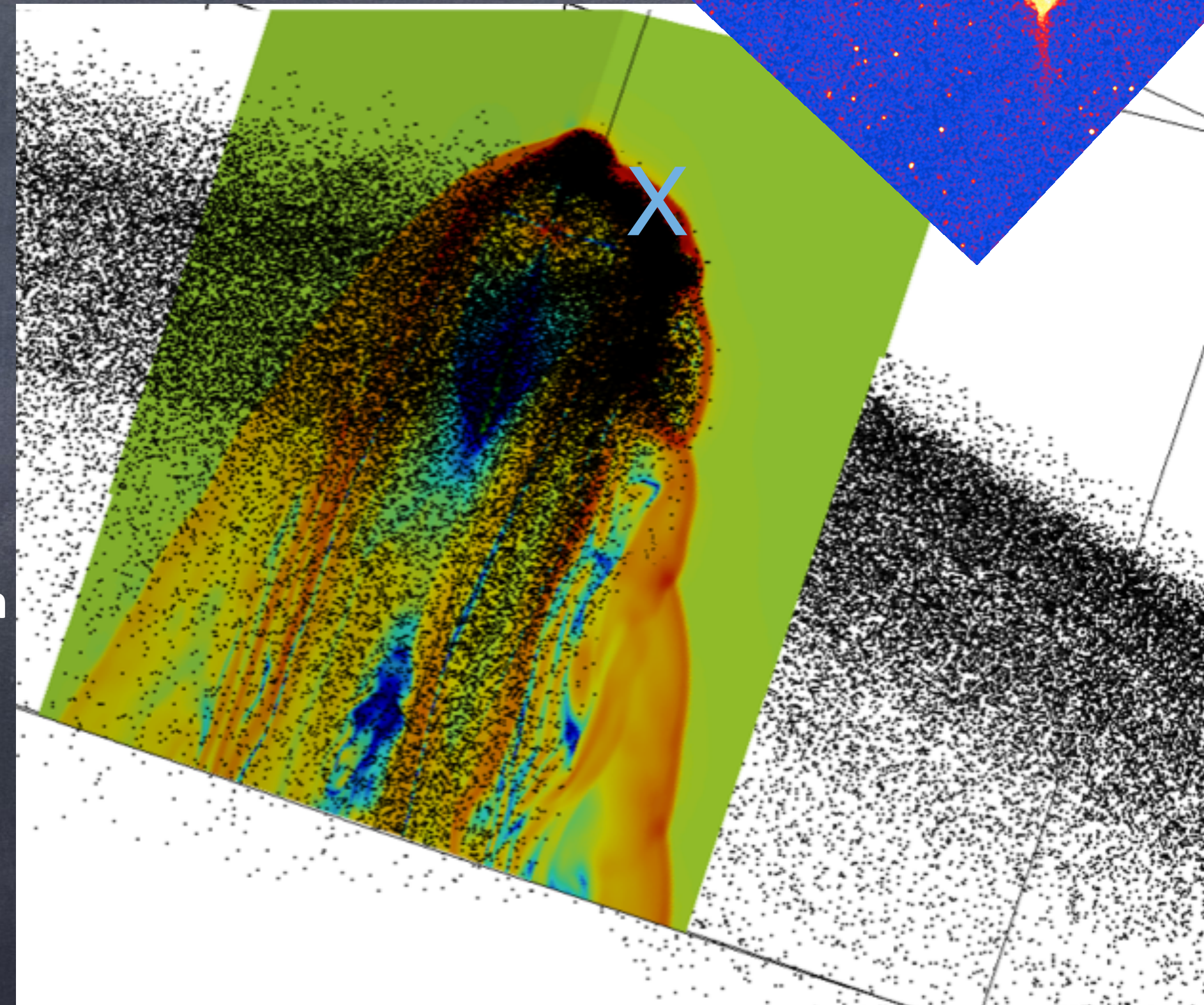
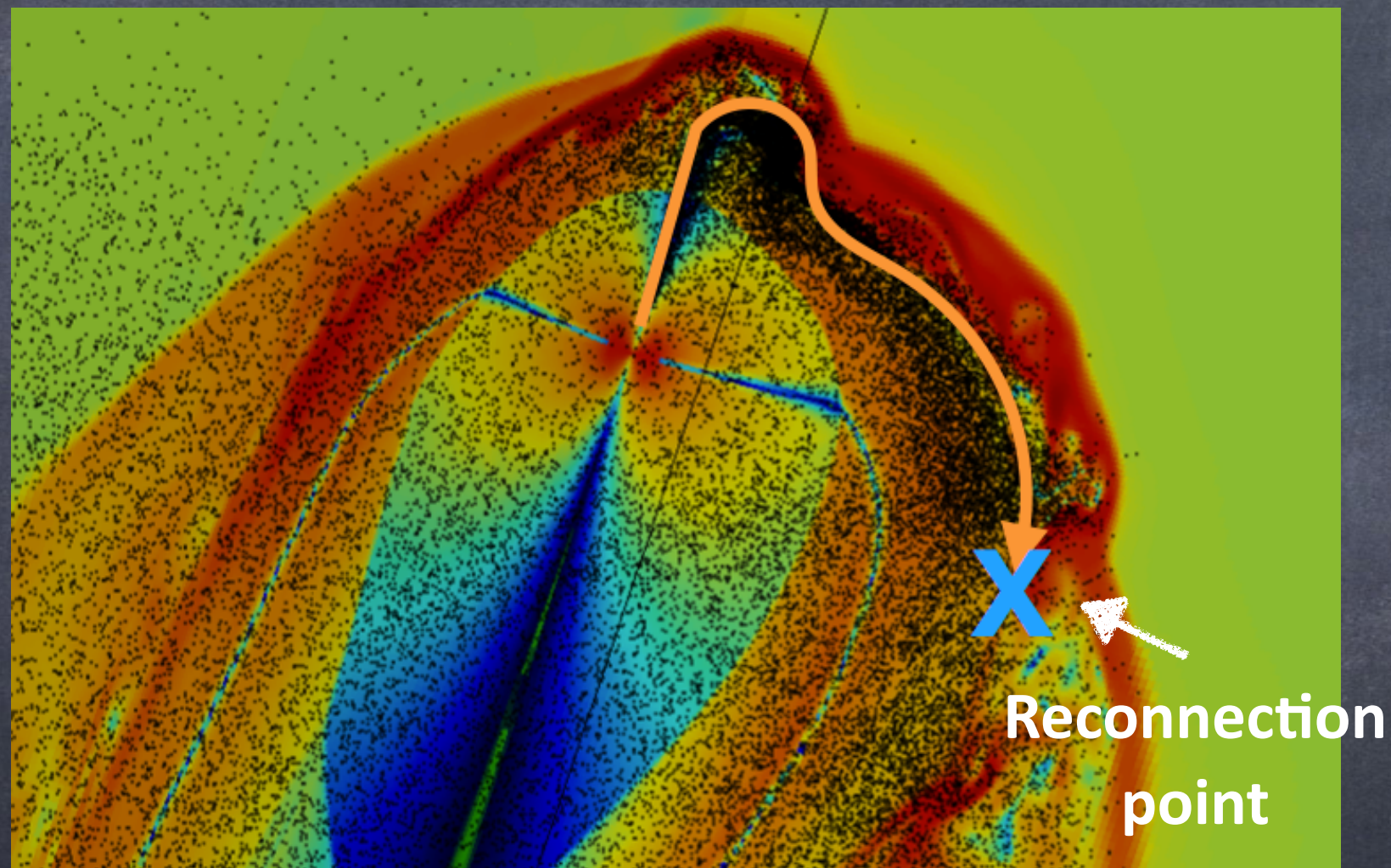
# PARTICLE ESCAPE FROM BSPWN





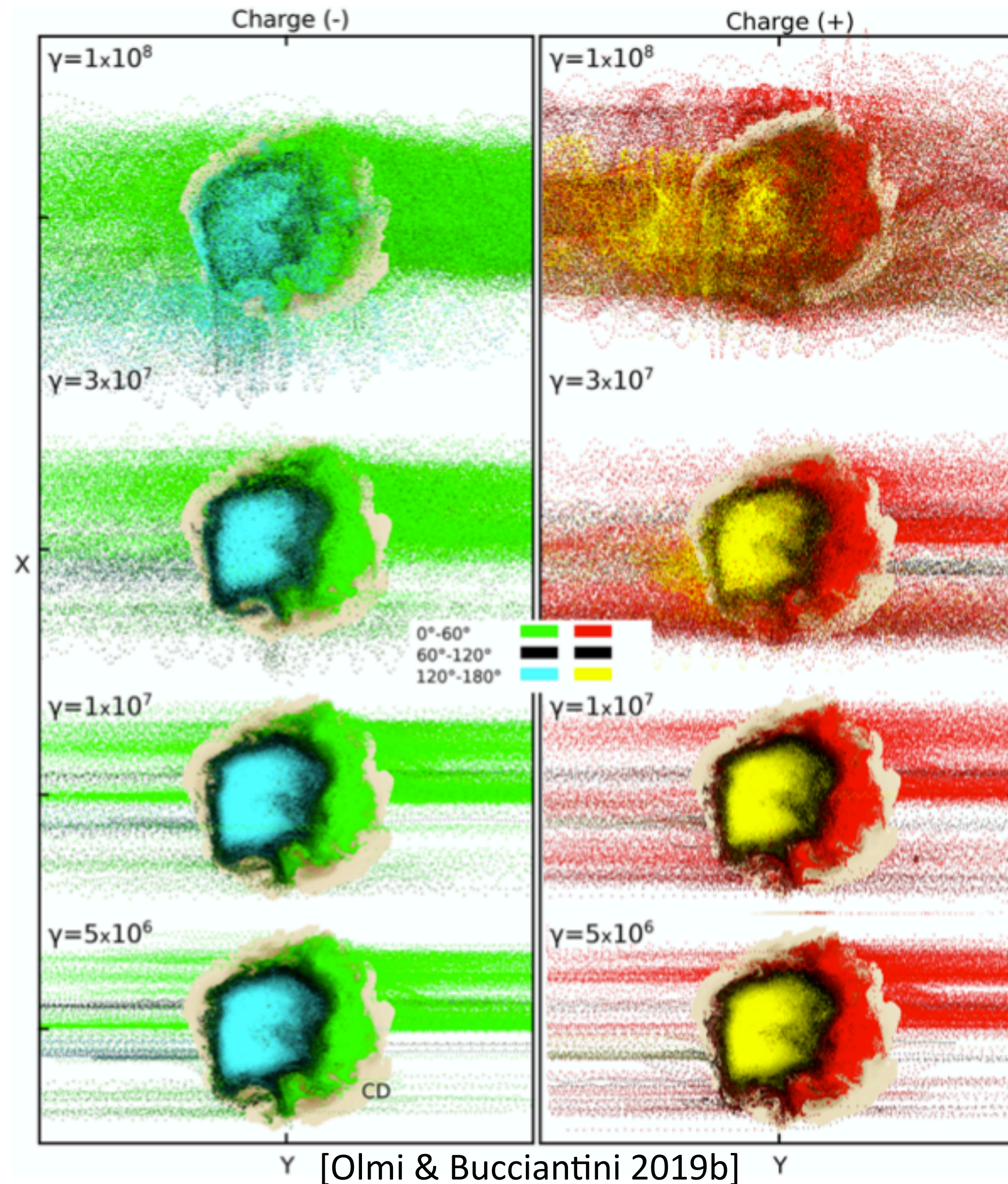
# PARTICLE ESCAPE FROM BOW SHOCK PWNe

HIGH ENERGY PARTICLES  
INJECTED CLOSE TO THE POLAR AXIS  
STREAM OUT FROM RECONNECTION POINT AND  
FORM JETS IN THE ISM B-FIELD





# ENERGY DEPENDENCE OF THE ESCAPE



## WITH INCREASING ENERGY:

- LARGER FRACTION OF PARTICLES
- MORE ISOTROPIC RELEASE

## AT GeV ENERGIES:

- ESCAPE EXPECTED ONLY FROM THE TAIL

## NOTICE THAT:

- ENERGY DEPENDENT ESCAPE PROBABILITY MAKES HALO SPECTRUM NON TRIVIAL
- ESCAPE IS CHARGE SEPARATED!
- IF LOW AMBIENT B BELL INSTABILITY POSSIBLE...



# SUMMARY

ENERGETIC PWNe CAN BE PEVATRONS, BOTH LEPTONIC AND HADRONIC IN PRINCIPLE, SOURCES OF MODIFIED TRANSPORT REGIONS, SOURCES OF CR LEPTONS

HOWEVER

WE STILL DO NOT UNDERSTAND HOW PARTICLES ARE ACCELERATED

NOR HOW PARTICLES ESCAPE....

LOTS OF INTERESTING WORK TO DO IN THE FIELD!!!!