Some Mappings in N-Neutrosophic Supra Topological Spaces

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Abstract: The main aim of this paper defines some N-neutrosophic supra topological continuous mappings and some N-neutrosophic supra topological open mappings by weak neutrosophic supra topological open sets and their different properties are discussed. The relation between these N-neutrosophic supra continuous mappings are established with suitable examples.

Keywords: N-neutrosophic continuous mapping, S^*-N-neutrosophic k-open mapping, S^*-N-neutrosophic k-continuous mapping, N-supra neutrosophic k-open mapping, N-supra neutrosophic k-continuous mapping.

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1. Introduction

Zadeh [26] introduced the concept of fuzzy set theory by studying each element its membership values. The fuzzy topological space is a topological space defined on fuzzy sets, initiated by Chang [7]. Fuzzy supra topological spaces and their supra continuous mappings were defined by Abd El-Monsef and Ramadan [2]. Jayaparthasarathy [11] derived some contradicting examples of the statements of Abd El-Monsef and Ramadan [2] in fuzzy supra topological spaces. In 1986, Atanassov [4] introduced an intuitionistic fuzzy set as a generalization of the fuzzy set. Dogan Coker [9] extended the concept of fuzzy topological spaces into intuitionistic fuzzy topological spaces. The concept of intuitionistic fuzzy supra topological space was initiated by Turnal [19]. Florentin Smarandache [24] was the first one to develop the neutrosophic set theory, which is the generalization of Atanassov's intuitionistic fuzzy set theory. Recently many researchers [1, 6, 10, 25] developed the applications of neutrosophic sets in various fields such as artificial intelligence, biology, control systems, data analysis, economics, medical diagnosis, probability, etc. Salama et al. [22] defined the neutrosophic crisp sets and neutrosophic topological space.

In 1963, Levine [16] introduced semi-open sets and semi-continuous functions in classical topological spaces. Njastad [20] derived a classical topology using the α-open sets. Mashhour et al. [17] investigated the properties of pre-open sets. Andrijevic [3] established the behavior of β-open sets in classical topology. Mashhour et al. [18] introduced the concept of supra topological spaces by removing one topological condition and they further defined the supra semi-open set and supra semi-continuous function. Devi et al. [8] introduced the properties of α-open sets and α-continuous functions in supra topological spaces. Supra topological pre-open sets and their continuous functions are defined by Sayed [23]. Saeid Jafari et al. [21] investigated the properties of supra β-open sets and their continuity. In 2016, Lellis Thivagar et al. [13, 14, 27] originated the N-topological space with its own open sets. Apart from this, Lellis Thivagar et al. [15] introduced N-neutrosophic topological spaces with several properties.

Motivation of the work: The neutrosophic supra topological space is a new space developed by Jayaparthasarathy et al. [11]. In this area, some neutrosophic supra topological open sets, and their continuous mappings are defined. Arockia Dasan et al. [5], and Jayaparthasarathy et al. [12] further extended these neutrosophic supra topological spaces to N-neutrosophic supra topological spaces. In N-neutrosophic supra topological spaces, some weak open sets with some operators are only defined so far. Hence the motivation of this paper extends to define different properties of continuous and open mappings by using N-neutrosophic supra topological open sets as well as its weak open sets.

Organization of the paper: Section 2 of this paper presents some basic preliminaries of neutrosophic fuzzy sets and N-neutrosophic supra topological spaces. Section 3 introduces continuous mappings and open mappings using N-neutrosophic supra topological open sets. In section 4, we define some weak forms of continuous and weak open mappings in N-neutrosophic supra topological spaces, and the last section states summary and some of the future work in the conclusion and future work of this paper.

2 Preliminary

In this section, we discuss the basic definitions and properties of N-neutrosophic supra topological spaces which are useful in the sequel.

Definition 2.1 [24] Let X be a non-empty set. A neutrosophic set A having the form $A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$, where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x) \in]^-0, 1^+[$ represent the degree of membership (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$) and the degree of non-membership (namely $\gamma_A(x)$) respectively of each $x \in X$ to the set A such that $^-0 \leq \mu_A(x) + \sigma_A(x) + \gamma_A(x) \leq 3^+$ for all $x \in X$. For X, $N(X)$ denotes the collection of all neutrosophic sets of X.

Definition 2.2 [24] The following statements are true for neutrosophic sets A and B on X:

1. $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \leq \sigma_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$ if and only if $A \subseteq B$
2. $A \subseteq B$ and $B \subseteq A$ if and only if $A = B$.
3. $A \cap B = \{(x, min\{\mu_A(x), \mu_B(x)\}, min\{\sigma_A(x), \sigma_B(x)\}, max\{\gamma_A(x), \gamma_B(x)\}) : x \in X\}$.
4. $A \cup B = \{(x, max\{\mu_A(x), \mu_B(x)\}, max\{\sigma_A(x), \sigma_B(x)\}, min\{\gamma_A(x), \gamma_B(x)\}) : x \in X\}$.

More generally, the intersection and the union of a collection of neutrosophic sets $\{A_i\}_{i\in\Lambda}$, are defined by $\cap_{i\in\Lambda} A_i = \{(x, inf_{i\in\Lambda}\{\mu_{A_i}(x)\}, inf_{i\in\Lambda}\{\sigma_{A_i}(x)\}, sup_{i\in\Lambda}\{\gamma_{A_i}(x)\}) : x \in X\}$ and $\cup_{i\in\Lambda} A_i = \{(x, sup_{i\in\Lambda}\{\mu_{A_i}(x)\}, sup_{i\in\Lambda}\sigma_{A_i}\{(x)\}, inf_{i\in\Lambda}\{\gamma_{A_i}(x)\}) : x \in X\}$.

Definition 2.3 [11] Let A, B be two neutrosophic sets of X, then the difference of A and B is a neutrosophic set on X, defined as $A \setminus B = \{(x, |\mu_A(x) - \mu_B(x)|, |\sigma_A(x) - \sigma_B(x)|, 1 - |\gamma_A(x) - \gamma_B(x)|) : x \in X\}$. Clearly $X^c = X \setminus X = (x, 0, 0, 1) = \emptyset$ and $\emptyset^c = X \setminus \emptyset = (x, 1, 1, 0) = X$.

Notation 2.4 [11] Let X be a non-empty set. We consider the neutrosophic empty set as $\emptyset = \{(x, 0, 0, 1) : x \in X\}$ and the neutrosophic whole set as $X = \{(x, 1, 1, 0) : x \in X\}$.

Definition 2.5 [12] Let X be a non-empty set, $\tau_{n_1}{}^*, \tau_{n_2}{}^*, \ldots, \tau_{n_N}{}^*$ be N arbitrary neutrosophic supra topologies defined on X. Then the collection $N\tau_n{}^* = \{S \in N(X) : S = \cup_{i=1}^N A_i, \ A_i \in \tau_{n_i}{}^*\}$ is said to be a N-neutrosophic supra topology if it satisfies the following axioms:

1. $X, \emptyset \in N\tau_n{}^*$
2. $\cup_{i=1}^{\infty} S_i \in N\tau_n{}^*$ for all $S_i \in N\tau_n{}^*$

Then the N-neutrosophic supra topological space is the non-empty set X together with the collection $N\tau_n{}^*$, denoted by $(X, N\tau_n{}^*)$ and its elements are known as $N\tau_n{}^*$-open sets on X. A neutrosophic subset A of X is said to be $N\tau_n{}^*$-closed on X if $X \setminus A$ is $N\tau_n{}^*$-open on X. The set of all $N\tau_n{}^*$-open sets on X and the set of all $N\tau_n{}^*$-closed sets on X are respectively denoted by $N\tau_n{}^*O(X)$ and $N\tau_n{}^*C(X)$.

Definition 2.6 [12] Let $(X, N\tau_n{}^*)$ be a N-neutrosophic supra topological space and A be a neutrosophic set of X. Then

1. The $N\tau_n{}^*$-interior of A is defined by $int_{N\tau_n{}^*}(A) = \cup\{G : G \subseteq A$ and G is $N\tau_n{}^*$-open$\}$.
2. The $N\tau_n{}^*$-closure of A is defined by $cl_{N\tau_n{}^*}(A) = \cap\{F : A \subseteq F$ and F is $N\tau_n{}^*$-closed$\}$.

Definition 2.7 [5] A neutrosophic set A of a N-neutrosophic supra topological space $(X, N\tau_n{}^*)$ is called

1. N-neutrosophic supra α-open set if $A \subseteq int_{N\tau_n^*}(cl_{N\tau_n^*}(int_{N\tau_n^*}(A)))$.
2. N-neutrosophic supra semi-open set if $A \subseteq cl_{N\tau_n^*}(int_{N\tau_n^*}(A))$.
3. N -neutrosophic supra pre-open set if $A \subseteq int_{N\tau_n^*}(cl_{N\tau_n^*}(A))$.
4. N -neutrosophic supra β-open set if $A \subseteq cl_{N\tau_n^*}(int_{N\tau_n^*}(cl_{N\tau_n^*}(A)))$.
5. N -neutrosophic supra regular-open if $A = int_{N\tau_n^*}(cl_{N\tau_n^*}(A))$.

The set of all N-neutrosophic supra α-open (resp. N-neutrosophic supra semi-open, N-neutrosophic supra pre-open, N-neutrosophic supra β-open, N-neutrosophic supra regular-open) sets of $(X, N\tau_n{}^*)$ is denoted by $N\tau_n{}^*\alpha O(X)$ (resp. $N\tau_n{}^*SO(X), N\tau_n{}^*PO(X)$, $N\tau_n{}^*\beta O(X)$) and $N\tau_n{}^*RO(X)$. The complement of set of all N-neutrosophic supra α-open (resp. N-neutrosophic supra semi-open, N-neutrosophic supra pre-open and N-neutrosophic supra β-open) sets of $(X, N\tau_n{}^*)$ is called N-neutrosophic supra α-closed (resp. N-neutrosophic supra semi-closed, N-neutrosophic supra pre-closed, N-neutrosophic supra β-closed and N-neutrosophic supra regular closed) sets, denoted by $N\tau_n{}^*\alpha C(X)$ (resp. $N\tau_n{}^*SC(X), N\tau_n{}^*PC(X), N\tau_n{}^*\beta C(X)$) and $N\tau_n{}^*RC(X)$. Hereafter N-neutrosophic supra k-open set (shortly $N\tau_n{}^*k$-open set) is can be any one of the following: $N\tau_n{}^*$-open set, $N\tau_n{}^*\alpha$-open set, $N\tau_n{}^*$semi-open set, $N\tau_n{}^*$pre-open set, $N\tau_n{}^*\beta$-open set and $N\tau_n{}^*r$- open set.

Definition 2.8 [5] Let $(X, N\tau_n{}^*)$ be a N-Neutrosophic supra topological space and A be a subset of X.

1. The $kN\tau_n{}^*$-interior of A, is defined by

$$kint_{N\tau_n^*}(A) = \cup \{G : G \subseteq A \text{ and } G \in N\tau_n{}^*kO(X)\}.$$

2. The $kN\tau_n{}^*$-closure of A, is defined by

$$kcl_{N\tau_n^*}(A) = \cap \{F : A \subseteq F \text{ and } F \in N\tau_n{}^*kC(X)\}.$$

Definition 2.9 [15] Let X be a non-empty set, then $\tau_{n_1}, \tau_{n_2}, \ldots, \tau_{n_N}$ be N-arbitrary neutrosophic-topologies defined on X, then the collection $N_n\tau = \{S \subseteq X : S = (\cup_{i=1}^{N} A_i) \cup (\cap_{i=1}^{N} B_i), A_i, B_i \in \tau_{n_i}\}$ is called N-neutrosophic topology if the following axioms are satisfied.

1. $\emptyset, X \in N_n\tau$.
2. $\cup_{i=1}^{\infty} S_i \in N_n\tau$ for all $S_i \in N_n\tau$
3. $\cap_{i=1}^{n} S_i \in N_n\tau$ for all $S_i \in N_n\tau$.

Then $(X, N_n\tau)$ is called N_n-topological space on X. The element of $N_n\tau$ are known as N_n-open sets on X and its complement is called N_n-closed set on X.

Definition 2.10 [15] Let $(X, N_n\tau)$ and $(Y, N_n\sigma)$ be N-neutrosophic topological spaces. A mapping $f: X \to Y$ is said to be N-neutrosophic continuous on X if the inverse image of every $N_n\sigma$ -open set in Y is $N_n\tau$ -open in X.

3. Some Mappings in N-neutrosophic Supra Topological Spaces

In this section, we introduce continuous mappings in N-neutrosophic supra topological spaces and discuss their different properties.

Definition 3.1 Let $(X, N\tau_n)$ and $(Y, N\sigma_n)$ be N-neutrosophic topological spaces, $N\tau_n{}^*$ and $N\sigma_n{}^*$ be associated N-neutrosophic supra topologies with respect to $N\tau_n$ and $N\sigma_n$. A mapping $f: X \to Y$ is said to be N-supra neutrosophic continuous on X if the inverse image of every $N\sigma_n{}^*$-open set in Y is $N\tau_n{}^*$-open in X. If $N = 1$, then f is a supra neutrosophic continuous on X [11].

Definition 3.2 Let $(X, N\tau_n)$ and $(Y, N\sigma_n)$ be N-neutrosophic topological spaces. $N\tau_n{}^*$and $N\sigma_n{}^*$ be associated N-neutrosophic supra topologies with respect to $N\tau_n$ and $N\sigma_n$. A mapping $f: X \to Y$ is said to be S^*-N-neutrosophic continuous if the inverse image of every N-neutrosophic open set in $(Y, N\sigma_n)$ is N-neutrosophic supra open in $(X, N\tau_n{}^*)$.If $N = 1$, then f is a S^*-neutrosophic continuous on X [11].

Lemma 3.3. i. Every N-neutrosophic continuous mapping is S^*-N-neutrosophic continuous, but the converse need not be true.

ii. Every N-supra neutrosophic continuous mapping is S^*-N-neutrosophic continuous, but the converse need not be true.

iii. N-supra neutrosophic continuous and N-neutrosophic continuous mappings are independent each other.

Proof. The proof follows from the definition the converse and the independency are shown in the following example.

Example 3.4.(i) For $N = 3$, let $X = \{a, b\}$ and $Y = \{x, y\}$ with the neutrosophic topologies are

$\tau_{n_1}O(X) = \{\emptyset, X, ((0.3, 0.4), (0.2, 0.5), (0.1, 0.6))\}$, $\tau_{n_2}O(X) = \{\emptyset, X, ((0.7, 0.2), (0.6, 0.1), (0.8, 0))\}$,

$\tau_{n_3}O(X) = \{\emptyset, X\}$ and $\sigma_{n_1}O(Y) = \{\emptyset, Y\}$, $\sigma_{n_2}O(Y) = \{\emptyset, Y, ((0.3, 0.4), (0.2, 0.5), (0.1, 0.6))\}$, $\sigma_{n_3}O(X) = \{\emptyset, Y, ((0, 0.2), (0, 0), (1, 0.6))\}$. Then $3\tau_n O(X) = \{\emptyset, X, ((0.3, 0.4), (0.2, 0.5), (0.1, 0.6))$, $((0.7, 0.2), (0.6, 0.1), (0.8, 0))$, $((0.7, 0.4), (0.6, 0.5), (0.1, 0)), ((0.3, 0.2), (0.2, 0.1), (0.8, 0.6))\}$ and $3\sigma_n O(Y) = \{\emptyset, Y, ((0.3, 0.4), (0.2, 0.5), (0.1, 0.6)), ((0, 0.2), (0, 0), (1, 0.6))\}$. Let $3\tau_n{}^* O(X) = \{\emptyset, X, ((0.3, 0.4), (0.2, 0.5), (0.1, 0.6)), ((0.7, 0.2), (0.6, 0.1), (0.8, 0)), ((0.7, 0.4), (0.6, 0.5), (0.1, 0)), ((0.3, 0.2), (0.2, 0.1), (0.8, 0.6)), ((0, 0.2), (0, 0), (1, 0.6)), ((0.7, 0.6), (0.8, 0.5), (0.9, 0.4)), ((1, 0.8), (1, 1), (0, 0.4)), ((0.7, 0.6), (0.8, 0.5), (0.1, 0.4)), ((0.7, 0.6), (0.8, 0.5), (0.8, 0)), ((0.7, 0.6), (0.8, 0.5), (0.1, 0)), ((0.7, 0.6), (0.8, 0.5), (0.8, 0.4)), ((1, 0.8), (1, 1), (0, 0))\}$ and$3\sigma_n{}^* O(Y) = 3\sigma_n O(Y)$ be the associated 2-neutrosophic supra topological space. Define $f: X \to Y$ by $f(a) = x$, $f(b) = y$. Clearly, f is S^*-3-neutrosophic continuous and 3-supra neutrosophic continuous mapping on X but it is not 3-neutrosophic continuous map.

(ii) For $N=1$, let $X=\{a,b\}$ and $Y=\{x,y\}$ with the neutrosophic topologies are $\tau_n O(X)=\{\emptyset, X, ((0.7,0.6),(0.8,0.5),(0.9,0.4)), ((0.3,0.8),(0.4,0.9),(0.2,1)), ((0.3,0.6),(0.4,0.5),(0.9,1)), ((0.7,0.8),(0.8,0.9),(0.2,0.4)), ((0,0.6),(0,0.5),(1,1))\}$ and $\sigma_n O(Y)=\{\emptyset, Y\}$. Let ${\tau_n}^* O(X)=\tau_n O(X)$ and ${\sigma_n}^* O(Y)=\{\emptyset, Y, ((0.7,0.6),(0.8,0.5),(0.9,0.4)), ((0,0.6),(0,0.5),(1,1)), ((0.3,0.2),(0.2,0.2),(1,0.4)), ((0.3,0.6),(0.2,0.5),(1,0.4))\}$ be the associated neutrosophic supra topological space. Define $f: X \to Y$ by $f(a)=x$, $f(b)=y$. Clearly, f is S^*-neutrosophic continuous and neutrosophic continuous mapping on X but it is not supra neutrosophic continuous.

Theorem 3.5. Let $(X, N{\tau_n}^*)$ and $(Y, N{\sigma_n}^*)$ be N-neutrosophic supra topological space. Then the following are equivalent:

i. A mapping $f:(X, N{\tau_n}^*) \to (Y, N{\sigma_n}^*)$ is N-supra neutrosophic continuous.
ii. The inverse image of every N-neutrosophic supra closed set in $(Y, N{\sigma_n}^*)$ is a N-neutrosophic supra closed set in $(X, N{\tau_n}^*)$.
iii. $cl_{N{\tau_n}^*}(f^{-1}(A)) \subseteq f^{-1}(cl_{N\sigma_n^*}(A)) \subseteq f^{-1}(cl_{N\sigma_n}(A))$ for every neutrosophic set A in Y.
iv. $f(cl_{N{\tau_n}^*}(B)) \subseteq cl_{N\sigma_n^*}(f(B) \subseteq cl_{N\sigma_n}(f(B))$ for every neutrosophic set B of X.
v. $f^{-1}(int_{N\sigma_n}(A)) \subseteq f^{-1}(int_{N{\sigma_n}^*}(A)) \subseteq int_{N{\tau_n}^*}(f^{-1}(A))$ for every neutrosophic subset A in Y.

Proof. $i \Rightarrow ii$: Assume that $f: X \to Y$ is N-supra neutrosophic continuous on X and let A be a $N{\sigma_n}^*$-closed set in Y. Then $Y-A$ is a $N{\sigma_n}^*$-open set in Y. Since f is N-supra neutrosophic continuous on X, then $f^{-1}(Y-A)$ is $N{\tau_n}^*$-open set in X. Then $X-f^{-1}(A)$ is $N{\tau_n}^*$-open set in X. Then $f^{-1}(A)$ is $N{\tau_n}^*$-closed set in X.
$ii \Rightarrow i$: Let A be $N{\sigma_n}^*$-open set in Y, then $Y-A$ is $N{\sigma_n}^*$-closed set in Y and by assumption, $f^{-1}(Y-A)=X-f^{-1}(A)$ is $N{\tau_n}^*$-closed in X. Thus $f^{-1}(A)$ is $N{\tau_n}^*$-open in X.
$ii \Rightarrow iii$: Since for each neutrosophic set A in Y, $cl_{N{\sigma_n}^*}(A)$ is a $N{\sigma_n}^*$-closed set in Y. Then $f^{-1}(cl_{N{\sigma_n}^*}(A))$ is $N{\tau_n}^*$ -closed in X. Thus $f^{-1}\left(cl_{N{\sigma_n}^*}(A)\right)=cl_{N{\tau_n}^*}(f^{-1}\left(cl_{N{\sigma_n}^*}(A)\right)) \supseteq cl_{N{\tau_n}^*}(f^{-1}(A))$ and implies $cl_{N{\tau_n}^*}(f^{-1}(A)) \subseteq f^{-1}(cl_{N\sigma_n^*}(A)) \subseteq f^{-1}(cl_{N\sigma_n}(A))$, since $cl_{N\sigma_n^*}(A) \subseteq cl_{N\sigma_n}(A)$.

$iii \Rightarrow iv$: Let B be the neutrosophic set in X, then

$f^{-1}(cl_{N\sigma_n}(f(B))) \supseteq f^{-1}(cl_{N{\sigma_n}^*}(f(B))) \supseteq cl_{N{\tau_n}^*}(f^{-1}(f(B))) \supseteq cl_{N{\tau_n}^*}(B)$ and so $(cl_{N{\tau_n}^*}(B)) \subseteq cl_{N\sigma_n^*}$

$(f(B) \subseteq cl_{N\sigma_n}(f(B))$.

$iv \Rightarrow ii$: Let A be $N{\sigma_n}^*$ -closed set in Y and $B=f^{-1}(A)$, then

$f\left(cl_{N{\tau_n}^*}(B)\right) \subseteq cl_{N{\sigma_n}^*}(f(B)) \subseteq cl_{N{\sigma_n}^*}(A)=A$ and $cl_{N{\tau_n}^*}(B) \subseteq f^{-1}\left(f\left(cl_{N{\tau_n}^*}(B)\right)\right) \subseteq f^{-1}(A)=B$.

Therefore $B=f^{-1}(A)$ is $N{\tau_n}^*$-closed in X.

$i \Rightarrow v$: Let A be a $N{\sigma_n}^*$ -open set in Y, then $f^{-1}(int_{N{\sigma_n}^*}(A))$ is $N{\tau_n}^*$ -open in X and $f^{-1}\left(int_{N{\sigma_n}^*}(A)\right)=int_{N{\tau_n}^*}(f^{-1}(int_{N{\sigma_n}^*}(A)) \subseteq int_{N{\tau_n}^*}(f^{-1}(A))$. Thus $f^{-1}(int_{N\sigma_n}(A)) \subseteq f^{-1}(int_{N{\sigma_n}^*}(A)) \subseteq int_{N{\tau_n}^*}(f^{-1}(A))$, since $int_{N\sigma_n^*}(A) \supseteq int_{N\sigma_n}(A)$.

$v \Rightarrow i$: Let A be $N\sigma_n{}^*$-open set in Y, then $f^{-1}(A) = f^{-1}(int_{N\sigma_n{}^*}(A)) \subseteq int_{N\tau_n{}^*}(f^{-1}(A))$ and so $f^{-1}(A)$ is $N\tau_n{}^*$-open in X.

Theorem 3.6. Let $(X, N\tau_n{}^*)$ and $(Y, N\sigma_n{}^*)$ be N-neutrosophic supra topological spaces. Then the following are equivalent:

i. A mapping $f : (X, N\tau_n{}^*) \to (Y, N\sigma_n{}^*)$ is S^*-N- neutrosophic continuous.
ii. The inverse image of every N-neutrosophic closed set in $(Y, N\sigma_n)$ is N-neutrosophic supra closed set in $(X, N\tau_n{}^*)$.
iii. $cl_{N\tau_n{}^*}(f^{-1}(A)) \subseteq f^{-1}(cl_{N\sigma_n}(A))$ for every neutrosophic set A in Y.
iv. $f(cl_{N\tau_n{}^*}(B)) \subseteq cl_{N\sigma_n}(f(B))$ for every neutrosophic set $B$of X.
v. $f^{-1}(int_{N\sigma_n}(A)) \subseteq int_{N\tau_n{}^*}(f^{-1}(A))$ for every neutrosophic subset A in Y.

Proof. The proof is similarly follows from the theorem 3.5.

Theorem 3.7. If $f : X \to Y$ and $g : Y \to Z$ are N-supra neutrosophic continuous mappings, then $g \circ f : X \to Z$ is N-supra neutrosophic continuous.

Proof. Let V be neutrosophic supra open set in Z then $g^{-1}(V)$ is neutrosophic supra open in Y and $f^{-1}(g^{-1}(V))$ is neutrosophic supra open in X, by hypothesis. Therefore $(g \circ f)^{-1}(V)$ is neutrosophic supra open in X and so $g \circ f$ is N-supra neutrosophic continuous.

Remark 3.8. The composition of two S^*-N-neutrosophic continuous mappings need not be S^*-N-neutrosophic continuous.

Example 3.9. For $N = 2$, let $X = \{a, b\}$, $Y = \{u, v\}$ and $Z = \{x, y\}$ with the neutrosophic topologies are $\tau_{n_1} = \{\emptyset, X\}$, $\tau_{n_2} = \{\emptyset, X, ((0.5, 0.5), (0.5, 0.5), (0.5, 0.5))\}$, $\sigma_{n_1} = \{\emptyset, Y\}$, $\sigma_{n_2} = \{\emptyset, Y, ((0.6, 0.6), (0.6, 0.6), (0.6, 0.6))\}$, $\eta_{n_1} = \{\emptyset, Z, ((0.3, 0.3), (0.3, 0.3), (0.3, 0.3))\}$ and $\eta_{n_2} = \{\emptyset, Z\}$ with the 2 -neutrosophic topologies are $2\tau_n O(X) = \{\emptyset, X, ((0.5, 0.5), (0.5, 0.5), (0.5, 0.5))\}$, $2\sigma_n O(Y) = \{\emptyset, Y, ((0.6, 0.6), (0.6, 0.6), (0.6, 0.6))\}$ and $2\eta_n O(Z) = \{\emptyset, Z, ((0.3, 0.3), (0.3, 0.3), (0.3, 0.3))\}$. Let $2\tau_n{}^* O(X) = \{\emptyset, X, ((0.5, 0.5), (0.5, 0.5), (0.5, 0.5)), ((0.6, 0.6), (0.6, 0.6), (0.6, 0.6)), ((0.4, 0.4), (0.4, 0.4), (0.4, 0.4))\}$ and $2\sigma_n{}^* O(Y) = \{\emptyset, Y, ((0.6, 0.6), (0.6, 0.6), (0.6, 0.6)), ((0.3, 0.3), (0.3, 0.3), (0.3, 0.3)), ((0.7, 0.7), (0.7, 0.7), (0.7, 0.7))\}$ be the associated 2-neutrosophic supra topologies with respect to $2\tau_n$ and $2\sigma_n$. Then the mapping $f : X \to Y$ and $g : Y \to Z$ are defined respectively by $f(a) = u, f(b) = v, g(u) = x, g(v) = y$ are S^* - 2 -neutrosophic continuous. But $g \circ f$ is not S^*-2-neutrosophic continuous.

Theorem 3.10. If $f : X \to Y$ is S^*-N-neutrosophic continuous and $g : Y \to Z$ is N-neutrosophic continuous, then $g \circ f : X \to Z$ is S^*-N-neutrosophic continuous.

Proof. Let V be N-neutrosophic open set in Z, then by hypothesis, $g^{-1}(V)$ is N-neutrosophic open in Y and $f^{-1}(g^{-1}(V))$ is N-neutrosophic supra open in X implies $(g \circ f)^{-1}(V)$ is N-neutrosophic supra open in X. Therefore $g \circ f$ is S^*-N-neutrosophic continuous.

Theorem 3.11. If $f : X \to Y$ is N -supra neutrosophic continuous and $g : Y \to Z$ is S^*-N-neutrosophic continuous, then $g \circ f : X \to Z$ is S^*-N-neutrosophic continuous.

Proof. Let V be N-neutrosophic open set in Z then $g^{-1}(V)$ is N-neutrosophic supra open in Y and $f^{-1}(g^{-1}(V))$ is N-neutrosophic supra open in X implies $(g \circ f)^{-1}(V)$ is N-neutrosophic supra open in X. Therefore $g \circ f$ is S^*-N-neutrosophic continuous.

Definition 3.12. Let $(X, N\tau_n)$ and $(Y, N\sigma_n)$ be N -neutrosophic topological spaces. $N\tau_n{}^*$ and $N\sigma_n{}^*$ be associated N-neutrosophic supra topologies with respect to $N\tau_n$ and $N\sigma_n$. A mapping $f : X \to Y$ is said to be N-supra neutrosophic open if the image of every $N\tau_n{}^*$-open set in X is a $N\sigma_n{}^*$-open set in Y.

Definition 3.13. Let $(X, N\tau_n)$ and $(Y, N\sigma_n)$ be N-neutrosophic topological spaces. $N\tau_n{}^*$ and $N\sigma_n{}^*$ be associated N-neutrosophic supra topologies with respect to $N\tau_n$ and $N\sigma_n$. A mapping $f : X \to Y$ is said to be S^*-N-neutrosophic open if the image of every neutrosophic $N\tau_n$-open set in X is $N\sigma_n{}^*$-open set in Y.

Lemma 3.14. Every N-supra neutrosophic open mapping is S^*-N-neutrosophic open, but the converse need not be true.

Proof. The proof follows from the definitions, the converse part is shown in the following example.

Example 3.15. Consider the example 3.4 (ii), define $g : Y \to X$ by $g(x) = a$, $g(y) = b$. Clearly, g is S^*-N neutrosophic open map but it is not supra neutrosophic open map.

Theorem 3.16. Let $f : X \to Y$ be a N-supra neutrosophic open mapping. Then for each neutrosophic subset A of X,

i. $f(int_{N\tau_n{}^*}(A)) \subseteq int_{N\sigma_n{}^*}(f(A)$.
ii. $f(cl_{N\tau_n{}^*}(A)) \supseteq cl_{N\sigma_n{}^*}(f(A))$.

Proof.

i. Since $int_{N\tau_n{}^*}(A) \subseteq A$, then $f\left(int_{N\tau_n{}^*}(A)\right) \subseteq f(A)$ and $int_{N\sigma_n{}^*}\left(f\left(int_{N\tau_n{}^*}(A)\right)\right) \subseteq int_{N\sigma_n{}^*}(f(A))$. Since $int_{N\tau_n{}^*}(A)$ is $N\tau_n{}^*$-open in X, then $f(int_{N\tau_n{}^*}(A))$ is $N\tau_n{}^*$-open in Y. Therefore $int_{N\sigma_n{}^*}(f(int_{N\tau_n{}^*}(A))) = f(int_{N\tau_n{}^*}(A))$. Hence $f(int_{N\tau_n{}^*}(A)) \subseteq int_{N\sigma_n{}^*}(f(A))$.
ii. Since $cl_{N\tau_n{}^*}(A) \supseteq A$, then $f\left(cl_{N\tau_n{}^*}(A)\right) \supseteq f(A)$ and $cl_{N\sigma_n{}^*}\left(f\left(cl_{N\tau_n{}^*}(A)\right)\right) \supseteq cl_{N\sigma_n{}^*}(f(A))$. Since $cl_{N\tau_n{}^*}(A)$ is $N\tau_n{}^*$-closed set in X, then $X - cl_{N\tau_n{}^*}(A)$ is $N\tau_n{}^*$-open set in X and $f(X - cl_{N\tau_n{}^*}(A))$ is $N\sigma_n{}^*$-open in Y. That is $Y - f(cl_{N\tau_n{}^*}(A))$ is $N\sigma_n{}^*$-open in Y implies that $f(cl_{N\tau_n{}^*}(A))$ is $N\sigma_n{}^*$-closed in Y and so $cl_{N\sigma_n{}^*}(f(cl_{N\tau_n{}^*}(A))) = f(cl_{N\tau_n{}^*}(A))$. Therefore, $f(cl_{N\tau_n{}^*}(A)) \supseteq cl_{N\sigma_n{}^*}(f(A))$.

Theorem 3.17. Let $f : X \to Y$ be a S^*-N-neutrosophic open mapping. Then for each neutrosophic subset A of X,

i. $f(int_{N\tau_n}(A)) \subseteq int_{N\sigma_n{}^*}(f(A)$.
ii. $f(cl_{N\tau_n}(A)) \supseteq cl_{N\sigma_n{}^*}(f(A))$.

Proof. The proof follows directly from theorem 3.16.

4 Some Weak Mappings in $N\tau_n{}^*$-Topological Space

In this section, we introduce some weak forms of continuous functions in N-neutrosophic supra topological spaces and investigate the relationship between them. Throughout the section, N-neutrosophic supra k-open set (shortly $N\tau_n{}^*k$-open set) is can be any one of the following: $N\tau_n{}^*$-open set, $N\tau_n{}^*\alpha$-open set, $N\tau_n{}^*$semi-open set, $N\tau_n{}^*$pre-open set, $N\tau_n{}^*\beta$-open set, and $N\tau_n{}^*r$- open set.

Definition 4.1. Let $(X, N\tau_n)$ and $(Y, N\sigma_n)$ be N-neutrosophic topological spaces. $N\tau_n{}^*$ and $N\sigma_n{}^*$ be associated N-neutrosophic supra topologies with respect to $N\tau_n$ and $N\sigma_n$. A mapping $f : X \to Y$ is said to be N-supra neutrosophic k-continuous ((shortly $N\tau_n{}^*k$-continuous) is can be any one of the following: $N\tau_n{}^*\alpha$-continuous, $N\tau_n{}^*$ semi continuous, $N\tau_n{}^*$ pre continuous, $N\tau_n{}^*\beta$-continuous and $N\tau_n{}^*r$- continuous) on X if the inverse image of every $N\sigma_n{}^*$-open set in Y is a $N\tau_n{}^*k$-open in X.

Definition 4.2. Let $(X, N\tau_n)$ and $(Y, N\sigma_n)$ be N-neutrosophic topological spaces. $N\tau_n{}^*$ and $N\sigma_n{}^*$ be associated N-neutrosophic supra topologies with respect to $N\tau_n$ and $N\sigma_n$. A mapping $f : X \to Y$ is said to be S^*-N-neutrosophic k-continuous (is can be any one of the following: S^*-N-neutrosophic α-continuous, S^*-N-neutrosophic semi continuous, S^*-N-neutrosophic pre continuous, S^*-N-neutrosophic β-continuous and S^*-N-neutrosophic r-continuous) if the inverse image of every N-neutrosophic open set in $(Y, N\sigma_n)$ is N-neutrosophic supra k-open set in $(X, N\tau_n{}^*)$.

Lemma 4.3. Every N-supra neutrosophic k-continuous mapping is S^*-N-neutrosophic k-continuous, but the converse need not be true.

Proof. The proof follows from the definition; the converse part is shown in the following example.

Example 4.4. Consider the example 3.4(i), f is S^*-3-neutrosophic k-continuous and 3-supra neutrosophic k-continuous mapping on X.

Theorem 4.5. Let $(X, N\tau_n{}^*)$ and $(Y, N\sigma_n{}^*)$ be N-neutrosophic supra topological space. Then the following are equivalent:

i. A mapping $f : (X, N\tau_n{}^*) \to (Y, N\sigma_n{}^*)$ is N-supra neutrosophic k-continuous.
ii. The inverse image of every N-neutrosophic supra closed set in $(Y, N\sigma_n{}^*)$ is a N-neutrosophic supra k-closed set in $(X, N\tau_n{}^*)$.
iii. $kcl_{N\tau_n{}^*}(f^{-1}(A)) \subseteq f^{-1}(kcl_{N\sigma_n{}^*}(A))$ for every neutrosophic set A of Y.
iv. $f(kcl_{N\tau_n{}^*}(B)) \subseteq kcl_{N\sigma_n{}^*}(f(B))$ for every neutrosophic set B of X.
v. $f^{-1}(kint_{N\sigma_n{}^*}(A)) \subseteq kint_{N\tau_n{}^*}(f^{-1}(A))$ for every neutrosophic subset A of Y.

Proof. The proof can be similarly derive as that of theorem 3.5

Theorem 4.6. Let $(X, N\tau_n{}^*)$ and $(Y, N\sigma_n{}^*)$ be N-neutrosophic supra topological space. Then the following are equivalent:

i. A mapping $f : (X, N\tau_n{}^*) \to (Y, N\sigma_n{}^*)$ is S^*-N- neutrosophic k-continuous.
ii. The inverse image of every N-neutrosophic closed set in $(Y, N\sigma_n)$ is a N-neutrosophic supra k-closed set in $(X, N\tau_n{}^*)$.
iii. $kcl_{N\tau_n{}^*}(f^{-1}(A)) \subseteq f^{-1}(kcl_{N\sigma_n}(A))$ for every neutrosophic set A of Y.
iv. $f(kcl_{N\tau_n{}^*}(B)) \subseteq kcl_{N\sigma_n}(f(B))$ for every neutrosophic set B of X.
v. $f^{-1}(kint_{N\sigma_n{}^*}(A)) \subseteq kint_{N\tau_n{}^*}(f^{-1}(A))$ for every neutrosophic subset $A$of Y.

Proof. The proof is straightforward from theorem 3.5.

Theorem 4.7. The following statements are true for the mapping $f : (X, N\tau_n{}^*) \to (Y, N\sigma_n{}^*)$:

i. Every N-supra neutrosophic r-continuous is N-supra neutrosophic continuous.
ii. Every N-supra neutrosophic continuous is N-supra neutrosophic α-continuous.
iii. Every N-supra neutrosophic α-continuous is N-supra neutrosophic semi-continuous.
iv. Every N-supra neutrosophic α-continuous is N-supra neutrosophic pre-continuous.
v. Every N-supra neutrosophic semi-continuous is N-supra neutrosophic β-continuous.
vi. Every N-supra neutrosophic pre-continuous is N-supra neutrosophic β-continuous.

Proof. The proof follows directly from the fact that theorem 4.2 of [12] and theorem 14 of [5].

The converse of the above theorem need not be true as shown in the following example.

Example 4.8. (i) For $N = 4$. Let $X = \{a, b\}$ and $Y = \{x, y\}$ with the neutrosophic topologies are $\tau_{n_1}O(X) = \{\emptyset, X, ((0.6, 0.1), (0.7, 0.2), (0.8, 0))\}, \tau_{n_2}O(X) = \{\emptyset, X, ((0.6, 0.3), (0.5, 0.4), (0.3, 0.5))\}$, $\tau_{n_3}O(X) = \{\emptyset, X, ((0.6, 0.3), (0.7, 0.4), (0.3, 0))\}$, $\tau_{n_4}O(X) = \{\emptyset, X\}$ and $\sigma_{n_1}O(Y) = \{\emptyset, Y, ((0.6, 0.3), (0.7, 0.4), (0.3, 0))\}$, $\sigma_{n_2}O(Y) = \{\emptyset, Y, ((0.3, 0.1), (0.7, 0.2), (0.8, 0))\}$, $\sigma_3 O(Y) = \{\emptyset, Y\}$, and $\sigma_{n_4}O(Y) = \{\emptyset, Y, ((0.6, 0.3), (0.5, 0.4), (0.3, 0.5))\}$. Then $4\tau_n O(X) = \{\emptyset, X, ((0.6, 0.1), (0.7, 0.2), (0.8, 0)), ((0.6, 0.3), (0.5, 0.4), (0.3, 0.5)), ((0.6, 0.3), (0.7, 0.4), (0.3, 0)), ((0.6, 0.1), (0.5, 0.2), (0.8, 0.5)), ((0.6, 0.3), (0.7, 0.4), (0.3, 0))\}$ and $4\sigma_n O(Y) = \{\emptyset, Y, ((0.6, 0.1), (0.7, 0.2), (0.8, 0)), ((0.6, 0.3), (0.5, 0.4), (0.3, 0.5)), ((0.6, 0.3), (0.7, 0.4), (0.3, 0)), ((0.6, 0.1), (0.5, 0.2), (0.8, 0.5))\}$. Let $4\tau_n{}^*O(X) = 4\tau_n O(X)$, $4\sigma_n{}^*O(Y) = 4\sigma_n O(Y)$ be the associated 4-neutrosophic supra topological space. Define $f : X \to Y$ by $f(a) = x$, $f(b) = y$. Clearly, f is 4-supra neutrosophic continuous mapping on X but it is not 4-supra neutrosophic r-continuous mapping on X.

(ii) For $N = 2$, let $X = \{a, b\}$ and $Y = \{x, y\}$ with the neutrosophic topologies are $\tau_{n_1} = \{\emptyset, X, ((0.3, 0.4), (0.3, 0.4), (0.4, 0.5))\}, \tau_{n_2} = \{\emptyset, X, ((0.4, 0.4), (0.4, 0.4), (0.4, 0.4))\}$ and $\sigma_{n_1} = \{\emptyset, Y\}, \sigma_{n_2} = \{\emptyset, Y, ((0.4, 0.6), (0.4, 0.6), (0.3, 0.4))\}$. Then $2\tau_n O(X) = \{\emptyset, X, ((0.3, 0.4), (0.3, 0.4), (0.4, 0.5)), ((0.4, 0.4), (0.4, 0.4), (0.4, 0.4))\}$ and $2\sigma_n O(Y) = \{\emptyset, Y, ((0.4, 0.6), (0.4, 0.6), (0.3, 0.4)\}$. Let $2\tau_n{}^*O(X) = \{\emptyset, X, ((0.3, 0.4), (0.3, 0.4), (0.4, 0.5)), ((0.4, 0.2), (0.4, 0.2), (0.5, 0.4)), ((0.4, 0.4), (0.4, 0.4), (0.4, 0.4))\}$ and $2\sigma_n{}^*O(Y) = \{\emptyset, Y, ((0.4, 0.6), (0.4, 0.6), (0.3, 0.4)), ((0.4, 0.4), (0.4, 0.4), (0.4, 0.4))\}$ be the associated 2-neutrosophic supra topological space. Define $f : X \to Y$ by $f(a) = x$ and $f(b) = y$. Therefore, f is 2-supra neutrosophic α-continuous, 2-supra neutrosophic semi-continuous, 2-supra neutrosophic pre-continuous and 2-supra neutrosophic β-continuous on X but not 2-supra neutrosophic continuous.

(iii) For $N = 2$, let $X = \{a, b\}$ and $Y = \{x, y\}$. Consider $\tau_{n_1}O(X) = \{\emptyset, X, ((0.3, 0.5), (0.3, 0.5), (0.4, 0.5))\}, \tau_{n_2}O(X) = \{\emptyset, X, ((0.4, 0.3), (0.4, 0.3), (0.5, 0.6))\}$ and $\sigma_{n_1}O(Y) = \{\emptyset, Y\}$ and $\sigma_{n_2}O(Y) = \{\emptyset, Y, ((0.4, 0.4), (0.4, 0.4), (0.5, 0.4))\}$. Then $2\tau_n O(X) = \{\emptyset, X, ((0.3, 0.5), (0.3, 0.5), (0.4, 0.5)), ((0.4, 0.3), (0.4, 0.3), (0.5, 0.6)), ((0.4, 0.5), (0.4, 0.5), (0.4, 0.5)), ((0.3, 0.3), (0.3, 0.3), (0.5, 0.5))\}$ and $2\sigma_n O(Y) = \{\emptyset, Y, ((0.4, 0.4), ((0.4, 0.4), (0.5, 0.4))\}$. Let $2\tau_n{}^*O(X) = 2\tau_n O(X)$ and $2\sigma_n{}^*O(Y) = 2\sigma_n O(Y)$ be the associated 2-neutrosophic supra topological space. Define $f : X \to Y$ by $f(a) = x$ and $f(b) = y$. Then f is $2\tau_n{}^*$-supra neutrosophic semi-continuous and 2-supra neutrosophic β-continuous but it is not 2-supra neutrosophic α-continuous and not 2-supra neutrosophic pre-continuous.

(iv) For $N = 2$, let $X = \{a, b\}$ and $Y = \{x, y\}$. Consider $\tau_{n_1}O(X) = \{\emptyset, X, ((0.3, 0.5), (0.3, 0.5), (0.4, 0.5))\}, \tau_{n_2}O(X) = \{\emptyset, X, ((0.4, 0.5), (0.4, 0.5), (0.4, 0.2))\}$. $\sigma_{n_1}O(Y) = \{\emptyset, Y, ((0.4, 0.5), (0.4, 0.5), (0.5, 0.4))\}, \sigma_{n_2}O(Y) = \{\emptyset, Y\}$. Then $2\tau_n O(X) = \{\emptyset, X, ((0.3, 0.5), (0.3, 0.5), (0.4, 0.5)), ((0.4, 0.5), (0.4, 0.5), (0.4, 0.2))\}$, and $2\sigma_n O(Y) = \{\emptyset, Y, ((0.4, 0.5), (0.4, 0.5), (0.5, 0.4))\}$. Let $2\tau_n{}^*O(X) = \{\emptyset, X, ((0.3, 0.5), (0.3, 0.5), (0.4, 0.5)), ((0.4, 0.3), (0.4, 0.3), (0.5, 0.2)), ((0.4, 0.5), (0.4, 0.5), (0.4, 0.2))\}$ and $2\sigma_n{}^*O(Y) = \{\emptyset, Y, ((0.4, 0.5), (0.4, 0.5), (0.5, 0.4)), ((0.4, 0.5), (0.4, 0.5), (0.4, 0.2))\}$ be the associated 2-neutrosophic supra topological space. Define $f : X \to Y$ by $f(a) = x$ and $f(b) = y$. Then f is 2-supra neutrosophic pre-continuous and 2-supra neutrosophic β-continuous but it is not 2-supra neutrosophic α-continuous and not 2-supra neutrosophic semi-continuous.

Theorem 4.9. The following statements are true for the mapping $f : (X, N\tau_n{}^*) \to (Y, N\sigma_n{}^*)$:

i. Every S^*-N-neutrosophic r-continuous is S^*-N-neutrosophic continuous.

ii. Every S^*-N-neutrosophic continuous is S^*-N-neutrosophic α-continuous.
iii. Every S^*-N-neutrosophic α-continuous is S^*-N-neutrosophic semi-continuous.
iv. Every S^*-N-neutrosophic α-continuous is S^*-N-neutrosophic pre-continuous.
v. Every S^*-N-neutrosophic semi-continuous is S^*-N-neutrosophic β-continuous.
vi. Every S^*-N-neutrosophic pre-continuous is S^*-N-neutrosophic β-continuous.

Proof. The proof follows directly from the fact that theorem 4.2 of [12] and theorem 14 of [5].

The converse of the above theorem need not be true as shown in the following example.

Example 4.10. Consider the example 4.8(i) f is S^*-4-neutrosophic continuous mapping on X but it is not S^*4-neutrosophic r-continuous mapping on X.

Consider the example 4.8.(ii) , f is S^*-2-neutrosophic α-continuous, S^*-2-neutrosophic semi-continuous, S^*-2-neutrosophic pre-continuous and S^*-2-neutrosophic β-continuous on X but not S^*-2-neutrosophic continuous.

Consider the example 4.8.(iii), f is S^*-2-neutrosophic semi-continuous and S^*-2-neutrosophic β-continuous but it is not S^*-2-neutrosophic α-continuous and not S^*-2-neutrosophic pre-continuous.

Consider the example 4.8.(iv), Then f is S^*-2-neutrosophic pre-continuous and S^*-2-neutrosophic β-continuous but it is not S^*-2-neutrosophic α-continuous and not S^*-2-neutrosophic semi-continuous.

Theorem 4.11.A function $f: X \to Y$ is N-supra neutrosophic α-continuous on X if and only if N-supra neutrosophic semi-continuous and N-supra neutrosophic pre-continuous.

Proof. The proof can be derive from the fact of theorem 4.6 of [12].

Theorem 4.12.A function $f: X \to Y$ is S^*-N-neutrosophic α-continuous on X if and only if S^*-N-neutrosophic semi-continuous and S^*-N-neutrosophic pre-continuous.

Proof. The proof of the theorem is directly following from theorem 4.6 of [12].

Theorem 4.13. If $f: X \to Y$ and $g: Y \to Z$ are N-supra neutrosophic k-continuous mappings, then $g \circ f: X \to Z$ is N-supra neutrosophic k-continuous.

Proof. Let V be a N-neutrosophic supra k-open set in Z, then $g^{-1}(V)$ is neutrosophic supra k-open in Y and $f^{-1}(g^{-1}(V))$ is neutrosophic supra k-open in X implies $(g \circ f)^{-1}(V)$ is neutrosophic supra k-open in X. Therefore $g \circ f$ is N-supra neutrosophic k-continuous.

Remark 4.14. The composition of two S^*-N-neutrosophic k-continuous mappings need not be S^*-N-neutrosophic k-continuous.

Example 4.15. For $N = 2$, let $X = \{a, b\}$, $Y = \{u, v\}$ and $Z = \{x, y\}$ with the neutrosophic topologies are $\tau_{n_1} = \{\emptyset, X, ((0.3, 0.7), (0.4, 0.8), (0.5, 0.9))\}$, $\tau_{n_2} = \{\emptyset, X, ((0.3, 0.8), (0.5, 0.8), (0.5, 0.8))\}$, $\sigma_{n_1} = \{\emptyset, Y\}$, $\sigma_{n_2} = \{\emptyset, Y, ((0.3, 0.7), (0.4, 0.8), (0.5, 0.9))\}$, $\eta_{n_1} = \{\emptyset, Z, ((0.7, 0.3), (0.6, 0.2), (0.5, 0.1))\}$

and $\eta_{n_2} = \{\emptyset, Z\}$ with the 2 -neutrosophic topologies are

$2\tau_n O(X) = \{\emptyset, X, ((0.3, 0.7), (0.4, 0.8), (0.5, 0.9)), ((0.3, 0.8), (0.5, 0.8), (0.5, 0.8))\}$, $2\sigma_n O(Y) =$

$\{\emptyset, Y, ((0.3, 0.7), (0.4, 0.8), (0.5, 0.9))\}$ and $2\eta_n O(Z) = \{\emptyset, Z, ((0.7, 0.3), (0.6, 0.2), (0.5, 0.1))\}$. Let

$2\tau_n{}^* O(X) = \{\emptyset, X, ((0.5, 0.8), (0.5, 0.8), (0.5, 0.8)), ((0.3, 0.7), (0.4, 0.8), (0.5, 0.9)), ((0.3, 0.8), (0.5, 0.8),$

$(0.5, 0.8))\}$ and $2\sigma_n{}^*O(Y) = \{\emptyset, Y, ((0.3, 0.7), (0.4, 0.8), (0.5, 0.9)), ((0.7, 0.3), (0.6, 0.2), (0.5, 0.1)), ((0.7, 0.7), (0.6, 0.8), (0.5, 0.1))\}$ be the associated 2-neutrosophic supra topologies with respect to $2\tau_n$ and $2\sigma_n$. Then the mapping $f: X \to Y$ and $g: Y \to Z$ are defined respectively by $f(a) = u, f(b) = v, g(u) = x, g(v) = y$ are S^* - 2 -neutrosophic α -continuous S^* - 2 -neutrosophic semi-continuous S^* - 2 -neutrosophic pre-continuous, S^*-2-neutrosophic β-continuous. But $g \circ f$ is not S^*-2-neutrosophic k-continuous. Consider the example 3.9. f and g is S^*-2-neutrosophic r-continuous. But $g \circ f$ is not S^*-2-neutrosophic r-continuous.

Theorem 4.16. If $f: X \to Y$ be S^* - N -neutrosophic k -continuous and $g: Y \to Z$ is N-neutrosophic continuous, then $g \circ f: X \to Z$ is S^*-N-neutrosophic k-continuous.

Proof. Let V be a N-neutrosophic open set in Z, then $g^{-1}(V)$ is N-neutrosophic open in Y and $f^{-1}(g^{-1}(V))$ is N-neutrosophic supra k-open in X implies $(g \circ f)^{-1}(V)$ is N-neutrosophic supra k-open in X. Therefore $g \circ f$ is S^*-N-neutrosophic k-continuous.

Theorem 4.17. If $f: X \to Y$ is N -supra neutrosophic k -continuous and $g: Y \to Z$ is S^* - N -neutrosophic k -continuous (or N -neutrosophic continuous), then $g \circ f: X \to Z$ is S^*-N-neutrosophic k-continuous.

Proof. Let V be a N-neutrosophic open set in Z. Since g is S^*-N-neutrosophic k-continuous, then $g^{-1}(V)$ is N-neutrosophic supra k-open in Y. Since f is N-supra neutrosophic k-continuous, then $f^{-1}(g^{-1}(V))$ is N-neutrosophic supra k-open in X implies $(g \circ f)^{-1}(V)$ is N-neutrosophic supra k-open in X. Therefore $g \circ f$ is S^*-N-neutrosophic k-continuous.

Definition 4.18. Let $(X, N\tau_n)$ and $(Y, N\sigma_n)$ be N-neutrosophic topological spaces. $N\tau_n{}^*$ and $N\sigma_n{}^*$ be associated N-neutrosophic supra topologies with respect to $N\tau_n$ and $N\sigma_n$. A mapping $f: X \to Y$ is said to be N-supra neutrosophic k-open on X if the image of every $N\tau_n{}^*$-open set in X is a $N\sigma_n{}^*k$-open in Y.

Definition 4.19. Let $(X, N\tau_n)$ and $(Y, N\sigma_n)$ be N-neutrosophic topological spaces. $N\tau_n{}^*$ and $N\sigma_n{}^*$ be associated N-neutrosophic supra topologies with respect to $N\tau_n$ and $N\sigma_n$. A mapping $f: X \to Y$ is said to be N-supra neutrosophic k-closed on X if the image of every $N\tau_n{}^*$-closed set in X is a $N\sigma_n{}^*k$-closed in Y.

Definition 4.20. Let $(X, N\tau_n)$ and $(Y, N\sigma_n)$ be N-neutrosophic topological spaces. $N\tau_n{}^*$ and $N\sigma_n{}^*$ be associated N-neutrosophic supra topologies with respect to $N\tau_n$ and $N\sigma_n$. A mapping $f: X \to Y$ is said to be S^*-N-neutrosophic k-open mapping on X if the image of every N-neutrosophic open set in $(X, N\tau_n)$ is N-neutrosophic supra k-open in $(Y, N\sigma_n{}^*)$.

Definition 4.21. Let $(X, N\tau_n)$ and $(Y, N\sigma_n)$ be N-neutrosophic topological spaces. $N\tau_n{}^*$ and $N\sigma_n{}^*$ be associated N-neutrosophic supra topologies with respect to $N\tau_n$ and $N\sigma_n$. A mapping

$f: X \to Y$ is said to be S^*-N-neutrosophic k-closed mapping on X if the image of every N-neutrosophic closed set in $(X, N\tau_n)$ is N-neutrosophic supra k-closed in $(Y, N\sigma_n{}^*)$.

Lemma 4.22. Every N-supra neutrosophic k-open mapping is S^*-N-neutrosophic k-open but the converse need not be true.

Proof. The proof is trivially true from the definition, the converse part is shown in the following example.

Example 4.23 Consider the example 3.15.(ii), g is S^*-2-neutrosophic k-open map on X but it is not 2-supra neutrosophic k-open map

Theorem 4.24. Let $f: X \to Y$ be a N- supra neutrosophic k-open mapping. Then for each neutrosophic subset A of X,

i. $f(kint_{N\tau_n{}^*}(A)) \subseteq kint_{N\sigma_n{}^*}(f(A))$.

ii. $f(kcl_{N\tau_n{}^*}(A)) \supseteq kcl_{N\sigma_n{}^*}(f(A))$.

Proof. The proof is similarly follows from theorem 3.16.

Theorem 4.25. Let $f: X \to Y$ be a S^*-N-neutrosophic open mapping. Then for each neutrosophic subset A of X,

i. $f(kint_{N\tau_n}(A)) \subseteq kint_{N\sigma_n{}^*}(f(A))$.

ii. $f\left(kcl_{N\tau_n}(A)\right) \supseteq kcl_{N\sigma_n{}^*}(f(A))$.

Proof. This proof is straightforward from theorem 3.17.

Theorem 4.26. Let $(X, N\tau_n)$ and $(Y, N\sigma_n)$ be N-neutrosophic topological spaces. $N\tau_n{}^*$ and $N\sigma_n{}^*$ be associated N-neutrosophic supra topologies with respect to $N\tau_n$ and $N\sigma_n$. Let A be then neutrosophic subset of X. Then

i. If $f: X \to Y$ is N-supra neutrosophic α-closed if and only if $cl_{N\sigma_n{}^*}(int_{N\sigma_n{}^*}(cl_{N\sigma_n{}^*}(f(A)))) \subseteq f(cl_{N\tau_n{}^*}(A))$.

ii. If $f: X \to Y$ is N-supra neutrosophic semi-closed if and only if $int_{N\sigma_n{}^*}(cl_{N\sigma_n{}^*}(f(A)) \subseteq f(cl_{N\tau_n{}^*}(A))$.

iii. If $f: X \to Y$ is N-supra neutrosophic pre-closed if and only if $cl_{N\sigma_n{}^*}(int_{N\sigma_n{}^*}(f(A))) \subseteq f(cl_{N\tau_n{}^*}(A))$.

iv. If $f: X \to Y$ is N-supra neutrosophic β-closed if and only if $int_{N\sigma_n{}^*}(cl_{N\sigma_n{}^*}(int_{N\sigma_n{}^*}(f(A)))) \subseteq f(cl_{N\tau_n{}^*}(A))$.

Proof. i. Assume that f be N-supra neutrosophic α-closed and $A \subseteq X$ and $cl_{N\tau_n{}^*}(A)$ is $N\tau_n{}^*$ -closed in X. Then $f(cl_{N\tau_n{}^*}(A))$ is $N\sigma_n{}^*\alpha$ -closed in Y and so, $f\left(cl_{N\tau_n{}^*}(A)\right) \supseteq cl_{N\sigma_n{}^*}\left(int_{N\sigma_n{}^*}\left(cl_{N\sigma_n{}^*}\left(f\left(cl_{N\tau_n{}^*}(A)\right)\right)\right)\right)$. Now $f(A) \subseteq f(cl_{N\tau_n{}^*}(A))$ then $cl_{N\sigma_n{}^*}(f(A)) \subseteq cl_{N\sigma_n{}^*}(f(cl_{N\tau_n{}^*}(A))$ and $cl_{N\sigma_n{}^*}(int_{N\sigma_n{}^*}\left(cl_{N\sigma_n{}^*}(f(A))\right) \subseteq cl_{N\sigma_n{}^*}\left(int_{N\sigma_n{}^*}\left(cl_{N\sigma_n{}^*}\left(f\left(cl_{N\tau_n{}^*}(A)\right)\right)\right)\right)$. Hence $cl_{N\sigma_n{}^*}(int_{N\sigma_n{}^*}(cl_{N\sigma_n{}^*}(f(A))) \subseteq f(cl_{N\tau_n{}^*}(A))$. Conversely, suppose the condition holds. Let F be any $N\tau_n{}^*$ -closed set in X. Then $cl_{N\tau_n{}^*}(F) = F$. By the condition, $cl_{N\sigma_n{}^*}(int_{N\sigma_n{}^*}(cl_{N\sigma_n{}^*}(f(F)))) \subseteq f(cl_{N\tau_n{}^*}(F)) = f(F)$ which gives $f(F)$ is $N\sigma_n{}^*\alpha$-closed in Y and so f is N-supra neutrosophic α-closed.

ii. Assume that f be N-supra neutrosophic semi-closed and $A \subseteq X$ and $cl_{N\tau_n{}^*}(A)$ is $N\tau_n{}^*$ -closed in X. Then $f(cl_{N\tau_n{}^*}(A))$ is $N\sigma_n{}^*$ -semi closed in Y and $f\left(cl_{N\tau_n{}^*}(A)\right) \supseteq int_{N\sigma_n{}^*}\left(cl_{N\sigma_n{}^*}\left(f\left(cl_{N\tau_n{}^*}(A)\right)\right)\right)$. Now $f(A) \subseteq f\left(cl_{N\tau_n{}^*}(A)\right)$ then $cl_{N\sigma_n{}^*}(f(A)) \subseteq cl_{N\sigma_n{}^*}(f(cl_{N\tau_n{}^*}(A))$ and $int_{N\sigma_n{}^*}(cl_{N\sigma_n{}^*}(f(A))) \subseteq int_{N\sigma_n{}^*}(cl_{N\sigma_n{}^*}(f(cl_{N\tau_n{}^*}(A))))$. Then

$int_{N\sigma_n^*}\left(cl_{N\sigma_n^*}(f(A))\right) \subseteq f\left(cl_{N\tau_n^*}(A)\right)$. Conversely, suppose the condition holds. Let F be any $N\tau_n{}^*$ -closed set in X. Then $cl_{N\tau_n^*}(F) = F$. By the condition, $int_{N\sigma_n^*}(cl_{N\sigma_n^*}(f(F)) \subseteq f(cl_{N\tau_n^*}(A)) = f(F)$ which gives $f(F)$ is $N\sigma_n{}^*$-semi-closed in Y. So f is N-supra neutrosophic semi-closed.

iii. Assume that f be N -supra neutrosophic pre-closed and $A \subseteq X$. Then $cl_{N\tau_n^*}(A)$ is $N\tau_n{}^*$ -closed in X. Hence $f(cl_{N\tau_n^*}(A))$ is $N\sigma_n{}^*$ -pre closed in Y and so, $f(cl_{N\tau_n^*}(A)) \supseteq cl_{N\sigma_n^*}(int_{N\sigma_n^*}(f(cl_{N\tau_n^*}(A))))$. Now $f(A) \subseteq f(cl_{N\tau_n^*}(A))$. That is $int_{N\sigma_n^*}(f(A)) \subseteq int_{N\sigma_n^*}(f(cl_{N\tau_n^*}(A)))$. We have $cl_{N\sigma_n^*}\left(int_{N\sigma_n^*}(f(A))\right) \subseteq cl_{N\sigma_n^*}\left(int_{N\sigma_n^*}\left(f\left(cl_{N\tau_n^*}(A)\right)\right)\right)$. Hence $cl_{N\sigma_n^*}(int_{N\sigma_n^*}(f(A))) \subseteq f(cl_{N\tau_n^*}(A))$. Conversely, suppose the condition holds. Let F be any $N\tau_n{}^*$ -closed set in X. Then $cl_{N\tau_n^*}(F) = F$. By the condition, $cl_{N\sigma_n^*}(int_{N\sigma_n^*}(f(F)) \subseteq f(cl_{N\tau_n^*}(A)) = f(F)$ which gives $f(F)$ is $N\sigma_n{}^*$-pre-closed in Y. So f is N-supra neutrosophic supra pre-closed.

iv. Assume that f be N-supra neutrosophic β-closed and $A \subseteq X$. Then $cl_{N\tau_n^*}(A)$ is $N\tau_n{}^*$-closed in X. Hence $f(cl_{N\tau_n^*}(A))$ is $N\sigma_n{}^*\beta$ -closed in Y and so, $f\left(cl_{N\tau_n^*}(A)\right) \supseteq int_{N\sigma_n^*}\left(cl_{N\sigma_n^*}\left(int_{N\sigma_n^*}\left(f\left(cl_{N\tau_n^*}(A)\right)\right)\right)\right)$. Now $f(A) \subseteq f(cl_{N\tau_n^*}(A))$. That is $int_{N\sigma_n^*}(f(A)) \subseteq int_{N\sigma_n^*}(f(cl_{N\tau_n^*}(A)))$. We have $int_{N\sigma_n^*}(cl_{N\sigma_n^*}(int_{N\sigma_n^*}(f(A)))) \subseteq int_{N\sigma_n^*}(cl_{N\sigma_n^*}(int_{N\sigma_n^*}(f(cl_{N\tau_n^*}(A)))))$. Hence $int_{N\sigma_n^*}(cl_{N\sigma_n^*}(int_{N\sigma_n^*}(f(A)))) \subseteq f(cl_{N\tau_n^*}(A))$. Conversely, suppose the condition holds. Let F be any $N\tau_n{}^*$ -closed set in X. Then $cl_{N\tau_n^*}(F) = F$. By the condition, $int_{N\sigma_n^*}(cl_{N\sigma_n^*}(int_{N\sigma_n^*}(f(F)))) \subseteq f(cl_{N\tau_n^*}(A)) = f(F)$ which gives $f(F)$ is $N\sigma_n{}^*$-pre closed in Y. So f is N-supra neutrosophic β-closed mapping.

Theorem 4.27. Let $(X, N\tau_n)$ and $(Y, N\sigma_n)$ be N-neutrosophic topological spaces. $N\tau_n{}^*$ and $N\sigma_n{}^*$ be associated N-neutrosophic supra topologies with respect to $N\tau_n$ and $N\sigma_n$. Let A be then neutrosophic subset of X. Then

i. If $f : X \to Y$ is S^* - N -neutrosophic α -closed if and only $cl_{N\sigma_n^*}(int_{N\sigma_n^*}(cl_{N\sigma_n^*}(f(A)))) \subseteq f(cl_{N\tau_n}(A))$.

ii. If $f : X \to Y$ is S^* - N -neutrosophic semi-closed if and only if $int_{N\sigma_n^*}(cl_{N\sigma_n^*}(f(A)) \subseteq f(cl_{N\tau_n}(A)$.

iii. If $f : X \to Y$ is S^* - N -neutrosophic pre-closed if and only if $cl_{N\sigma_n^*}(int_{N\sigma_n^*}(f(A)) \subseteq f(cl_{N\tau_n}(A))$.

iv. If $f : X \to Y$ is S^* - N -neutrosophic β -closed if and only if $int_{N\sigma_n^*}(cl_{N\sigma_n^*}(int_{N\sigma_n^*}(f(A)))) \subseteq f(cl_{N\tau_n}(A))$.

Proof. This proof is similarly follows from theorem 4.26

Theorem 4.28. Let $(X, N\tau_n)$ and $(Y, N\sigma_n)$ be N-neutrosophic topological spaces. $N\tau_n{}^*$ and $N\sigma_n{}^*$ be associated N-neutrosophic supra topologies with respect to $N\tau_n$ and $N\sigma_n$. Let A be then neutrosophic subset of X. Then

i. If $f : X \to Y$ is N -supra neutrosophic α -open if and only if $f(int_{N\tau_n^*}(A)) \subseteq int_{N\sigma_n^*}(cl_{N\sigma_n^*}(int_{N\sigma_n^*}(f(A))))$.

ii. If $f : X \to Y$ is N -supra neutrosophic semi-open if and only if $f(int_{N\tau_n^*}(A)) \subseteq cl_{N\sigma_n^*}(int_{N\sigma_n^*}(f(A))$.

iii. If $f : X \to Y$ is N - supra neutrosophic pre-open if and only if $f\left(int_{N\tau_n^*}(A)\right) \subseteq int_{N\sigma_n^*}\left(cl_{N\sigma_n^*}(f(A))\right)$.

iv. If $f : X \to Y$ is N -supra neutrosophic β -open $f(int_{N\tau_n^*}(A)) \subseteq cl_{N\sigma_n^*}(int_{N\sigma_n^*}(cl_{N\sigma_n^*}(f(A))))$.

Proof. This proof follows from theorem 4.26.

Theorem 4.29. Let $(X, N\tau_n)$ and $(Y, N\sigma_n)$ be N-neutrosophic topological spaces. $N\tau_n{}^*$ and $N\sigma_n{}^*$ be associated N-neutrosophic supra topologies with respect to $N\tau_n$ and $N\sigma_n$. Let A be then neutrosophic subset of X. Then

i. $f: X \to Y$ is S^* - N -neutrosophic α -open if and only if $f(int_{N\tau_n}(A)) \subseteq int_{N\sigma_n^*}(cl_{N\sigma_n^*}(int_{N\sigma_n^*}(f(A))))$.

ii. $f: X \to Y$ is S^* - N -neutrosophic semi-open if and only if $f(int_{N\tau_n}(A)) \subseteq cl_{N\sigma_n^*}(int_{N\sigma_n^*}(f(A)))$.

iii. A mapping $f: X \to Y$ is S^* - N -neutrosophic pre-open if and only if $f(int_{N\tau_n}(A)) \subseteq int_{N\sigma_n^*}(cl_{N\sigma_n^*}(f(A)))$.

iv. A mapping $f: X \to Y$ is S^* - N -neutrosophic β -open if and only if $f(int_{N\tau_n}(A)) \subseteq cl_{N\sigma_n^*}(int_{N\sigma_n^*}(cl_{N\sigma_n^*}(f(A))))$.

Proof. This proof is straightforward from theorem 4.26.

5 Conclusion and Future Work

Neutrosophic supra topological space is one of the new research areas to deal with the uncertainty concept and it is a generalized form of fuzzy supra topological spaces as well as intuitionistic fuzzy supra topological spaces. This paper theoretically introduced N-neutrosophic supra topological mappings with suitable examples. The properties and relationship between N-neutrosophic supra topological mappings are derived. We can construct the real-life application of these N-neutrosophic supra topological sets and mappings in the future and implement these concepts to other applicable research areas of topology such as Rough topology, Fuzzy topology, intuitionistic topology, Digital topology, and so on.

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