Grid Impact Aware TSO-DSO Market Models for Flexibility Procurement: Coordination, Pricing Efficiency, and Information Sharing

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***Abstract*—This paper proposes five market models for the procurement of flexibility by transmission (TSO) and distribution system operators (DSOs), based on several TSO-DSO coordination schemes, including a disjoint distribution, disjoint transmission, common, fragmented, and multi-level market. The properties of these models are then analyzed. In particular, we prove that the common market is more efficient than the other market models. Then, different methods are proposed to adequately price TSO/DSO interface flows, when procuring cross-grid flexibility. We show that when interface flows are optimally priced, the fragmented and multi-level market solutions converge to those of the common market. To prevent the need for any network information sharing in the coordination schemes, decomposition methods based on bi-level programming and the alternating direction method of multipliers (ADMM) are proposed. A developed case study, considering an interconnected transmission-distribution system, corroborates the greater efficiency of the common market, the effect of adequate interface pricing on reducing procurement costs, and the capability of the decomposition methods to reach optimal market solutions with limited information exchange.**

***Index Terms*—TSO-DSO coordination, flexibility markets, optimization, power system economics, electricity markets.**

NOMENCLATURE

Indexes:

T/m	Transmission/distribution systems.
n, i, j	Nodes.
k	Flexibility service providers (FSPs).
e	Edges of the polygonal inner-approximation.

Sets:

$\mathcal{N}^D$	Distribution systems.
$\mathcal{N}^T/\mathcal{N}^m$	Transmission/distribution system nodes.
$\mathcal{L}^T/\mathcal{L}^m$	Transmission/distribution system lines.
$\mathcal{U}(n)/\mathcal{D}(n)$	Upward/downward FSPs connected to node n.
$\mathcal{K}(n)$	Descendent nodes of node n.
$\mathcal{E}$	Edges of the polygonal inner-approximation.

Parameters:

$A(n)$	Ancestor node of node n.
a_n^T/a_n^m	Anticipated base injection at transmission/distribution node n.
b_n^T/b_n^m	Anticipated base load at transmission/distribution node n.
$F_{ij}^{T,\max}/F_{ij}^{m,\max}$	Maximum thermal limits of transmission/distribution line $\{i, j\}$.
$\mathcal{G}_{(i,j),n}$	Generation shift factors.
$S_{A(n),n}^{m,\max}$	Maximum apparent power flow of line $\{A(n), n\}$.
$v_n^{m,\min}/v_n^{m,\max}$	Lower/upper limits of magnitude squared of the voltage at node n.
$r_{A(n),n}^m/x_{A(n),n}^m$	Resistance/reactance of line $\{A(n), n\}$.
$\gamma_e, \sigma_e, \delta_e$	Parameters defining the polygon of the inner-approximation.
$q_n^{m,\min}/q_n^{m,\max}$	Lower/upper limits of reactive power injection or offtake of node n.
$I_m^{p,\min}/I_m^{p,\max}$	Lower/upper active power transfer limit to distribution system m.
$I_m^{q,\min}/I_m^{q,\max}$	Lower/upper reactive power transfer limit to distribution system m.
$c_{k,n}^u/c_{k,n}^d$	Upward/downward bid price of FSP k.
$u_{k,n}^{T,\max}/d_{k,n}^{T,\max}$	Maximum offered quantity by upward/downward bid of FSP k.

Variables:

p_n^T/p_n^m	Net real power injection at transmission/distribution node n.
q_n^m	Net reactive power injection at node n.
F_{ij}^T/F_{ij}^m	Real power flow over transmission/distribution line $\{i, j\}$.
I_m^p/I_m^q	Active/reactive power transfer to distribution system m.
$Q_{A(n),n}^m$	Reactive power flow over line $\{A(n), n\}$.
v_n^m	Magnitude squared of the voltage at node n.
$u_{k,n}^T/u_{k,n}^m$	Dispatch level of transmission/distribution upward offer k.
$d_{k,n}^T/d_{k,n}^m$	Dispatch level of transmission/distribution downward offer k.
λ	Nodal or interface prices.

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This work is supported by the European Union Horizon 2020 research and innovation program under grant agreement No 824414 – CoordiNet Project.

I. INTRODUCTION

The increasing penetration of distributed generation in the distribution grid coupled with a growing electrification and digitalization at the consumer space are further endowing consumers with an unprecedented flexibility in their consumption and generation patterns. This consumer-level flexibility, dubbed distributed flexibility, along with flexibility available from medium and high voltage level assets, are essential to enable further integration of variable renewable energy resources into the grid. Indeed, this flexibility can be leveraged by transmission system operators (TSOs) and distribution system operators (DSOs) to meet their grid needs, including balancing services and congestion management, among others. To this end, the development of market mechanisms for the procurement of flexibility has been increasingly recommended in policy measures [1], and has been the center of development in the scientific literature [2]–[7] and in various international demonstration projects [8], [9].

Given that this flexibility can be used by different system operators (SOs), a need for TSO-DSO coordination naturally arises for the procurement of flexibility and the development of flexibility market mechanisms to 1) enable an efficient and coordinated procurement of flexibility from different voltage levels to meet the needs of the different SOs (also considering joint procurement when possible), 2) structure the level of access to flexibility assets by different SOs, and 3) ensure that a procured and activated flexibility by one SO does not lead to grid operational issues not only in its own grid but also in other interconnected grids.

Along these lines, conceptual aspects of TSO-DSO coordination (such as in [8], [10]) and initial formulations (such as in [3], [4]) have been proposed in the literature, which provide initial stepping stones towards the development of TSO-DSO flexibility procurement mechanisms. However, for introducing optimal designs of TSO-DSO flexibility markets, that are cognizant of the aforementioned TSO-DSO coordination needs, these initial efforts must be further developed using rigorous mathematical analyses and a detailed analytical and experimental assessment. Moreover, although some game-theoretic approaches to solve the TSO-DSO coordination problem (such as [2] and [11]) have been proposed, some key questions remain largely unexplored in the literature with respect to: 1) the efficiency of different coordination schemes going beyond common markets (e.g, comparing common markets to sequential/hierarchical markets), 2) the suitable pricing of interface power flows between different system operators, to adequately value the sharing of flexibility resources between different grids, and 3) the challenge of network information sharing between SOs (and absence thereof) to guarantee network operation security.

In order to close these gaps, this paper aims at introducing market models (rooted in sound TSO-DSO coordination), which enable devising a rigorous assessment of different possible TSO-DSO market structures, identifying possible challenges (practical and methodological), and promising solutions. More specifically, the contributions of this paper are summarized as follows:

1) Five TSO-DSO market models are developed, for the procurement of balancing and congestion management services including a) a disjoint transmission-level market; b) a disjoint distribution-level market; c) a common market; d) a fragmented market; and e) a multi-level market. The properties of each market model are analyzed while proving that the common market model is more efficient than the other proposed markets;
2) Several possible solutions are developed and analyzed for pricing TSO-DSO interface flows, which can adequately price caused imbalances in sequential markets (i.e. in the fragmented and multi-level market). We then prove that when the interface flow is priced optimally, the sequential market results are guaranteed to converge to the most efficient common market;
3) To account for possible network information sharing limitations between SOs, we propose a decentralization method based on the alternating direction method of multipliers (ADMM) to solve the market clearing problems of each of the markets, without the need for sharing any sensitive network information. In the proposed method, only interface flows and prices are shared among the SOs.

This current work provides a direct contribution to TSO-DSO flexibility market implementation (in the different demonstration campaigns in Greece, Spain, and Sweden) as part of the H2020 European project *CoordiNet* [8]. Indeed, the introduced models and analyses in this paper provide direct insights into flexibility market efficiency, TSO-DSO coordination structures, accounting for network constraints in market clearing models, enabling TSO-DSO coordination through adequate interface pricing, and limiting the need for TSO-DSO network information sharing, all of which have typically been key challenges in practical implementations.

The proposed models and solutions are tested on an interconnected transmission-distribution test system (based on adapted versions of the IEEE 14-bus transmission system interconnected to the Matpower 18-bus, 69-bus, and 141-bus distribution systems). The obtained results corroborates the analytical findings. For example, the numerical results show i) the highest efficiency of the common market, ii) the reduction of the total procurement cost by at least 25% in the fragmented and multi-level markets through adequately pricing the interface flows, and iii) the optimal iterative clearing of the different markets, using the proposed decomposition methods, while exchanging limited information.

The rest of this paper is organized as follows. In Section II, the TSO-DSO flexibility market models are presented and analyzed, while the pricing of the interface flows is addressed in Section III. The decentralized methods for clearing the different markets are described in Section IV. A numerical case study is provided in Section V, while Section VI concludes the paper.

II. Systems and Markets Models

We consider a network composed by a transmission system, operated by a TSO, connected to multiple distribution systems, each operated by a DSO. The meshed transmission system is denoted by a graph $G^T(\mathcal{N}^T, \mathcal{L}^T)$, where $\mathcal{N}^T$ is the set of nodes and $\mathcal{L}^T$ is the set of lines. A subset $\mathcal{N}^D \subseteq \mathcal{N}^T$ represents the TSO nodes that are connected to each distribution system. Each distribution system $m \in \mathcal{N}^D$ is also described by a graph $G^m(\mathcal{N}^m, \mathcal{L}^m)$. For ease of notation, we refer to this distribution system as DSO–m. As the distribution systems are considered to be radial, we define $A(n)$ as the ancestor node of $n \in \mathcal{N}^m$ and $\mathcal{K}(n)$ as the set of descendant nodes of $n \in \mathcal{N}^m$. The interface node of DSO–m with the transmission system, i.e., the node connecting distribution system m to a transmission system node in $\mathcal{N}^D \subseteq \mathcal{N}^T$, is denoted by n_0^m (this node also represents the root node of DSO–m).

The following notation is used to denote different parameters and variables within the transmission and distribution systems: 1) p_n^T/p_n^m denote the net real power injection at nodes $n \in \mathcal{N}^T/n \in \mathcal{N}^m$; 2) a_n^T/a_n^m and b_n^T/b_n^m denote, respectively, the vectors of anticipated base injection and load (i.e., prior to any flexibility activation) at all transmission/distribution systems nodes; 3) F_{ij}^T/F_{ij}^m denote the real power flow over line $\{i,j\} \in \mathcal{L}^T/\{i,j\} \in \mathcal{L}^m$; 4) $F_{ij}^{T,\max}/F_{ij}^{m,\max}$ are the maximum thermal limits of those lines; and 5) I_m^p and I_m^q denote the active and reactive power transfer to the distribution system DSO–m from transmission node $m \in \mathcal{N}^D$ (i.e. the flows between $m \in \mathcal{N}^D$ and n_0^m).

The transmission system is represented by the DC power flow model using generation shift factors ($\mathcal{G}_{(i,j),n}$), which captures the change in the active power flow over line $\{i,j\} \in \mathcal{L}^T$ due to a change in injection or offtake at node $n \in \mathcal{N}^T$. On the other hand, each distribution system DSO-m is described using the linearized power flow model proposed in [12], to account for reactive power injections/flows and voltages within each distribution systems, while keeping the representation linear. The additional parameters and variables used within each distribution system DSO-m are: 1) $Q_{A(n),n}^m$ is the reactive power flow over the line connecting nodes $A(n)$ and n for all $n \in \mathcal{N}^m$; 2) $S_{A(n),n}^{m,\max}$ is the maximum apparent power flow of line $\{A(n),n\} \in \mathcal{L}^m$; 3) v_n^m is the magnitude squared of the voltage at node $n \in \mathcal{N}^m$, with upper and lower limits $v_n^{m,\max}$ and $v_n^{m,\min}$, respectively; and 4) $r_{A(n),n}^m$ and $x_{A(n),n}^m$ are, respectively, the resistance and reactance of line $\{A(n),n\} \in \mathcal{L}^m$.

We note that, for ease of notation, this representation considers radial systems to be distribution systems and meshed systems to be transmission systems, but this is not to be interpreted as a restrictive condition as such, since meshed systems can also represent distribution systems. We note that considering different types of systems (interconnected meshed and radial systems) in the formulations serves to provide a wider view on the TSO-DSO coordination problem. Changing the distribution (radial) systems to meshed would not impact the nature of the market models presented next. Hence, this action can be readily accommodated as part of the presented TSO-DSO coordinated models.

In the next sections, we introduce the five TSO-DSO market models, which are summarized in Table I[1]. In Table I, direct sharing of resources means that an SO can directly purchase bids submitted from resources not connected within its own grid. As a result, there is a need to add the constraints of the different networks in its market clearing problem. On the other hand, indirect sharing of resources indicates that one system operator can indirectly benefit from its connection with the grid of another SO, by modifying the interface flow to meet its needs when clearing its market, without directly clearing/purchasing bids submitted from other SOs' networks, nor considering their network constraints.

We first present the Disjoint-Transmission and Disjoint-Distribution models, as they constitute the the building blocks for introducing the other three market models. Although the disjoint markets are separated, we represent them by one line in Table I to indicate that these markets can be solved independently (e.g. in parallel) when the multiple interconnected systems have congestion management/balancing needs.

[1]Figures representing the market models are available in [13].

A. Disjoint Transmission-Level Market

In this market model, the anticipated imbalance and/or line congestion at the transmission system is solved by the TSO using resources available only in its own grid. These resources are defined by upward/downward offers to the market from flexibility service providers (FSPs) operating on the transmission level. Both types of offers can be provided by the increase/decrease of generation or load. At each node $n \in \mathcal{N}^T$, we consider a set $\mathcal{U}(n)$ of FSPs offering upward flexibility, and a set $\mathcal{D}(n)$ of FSPs providing downward flexibility. We denote $u_{k,n}^T$ as the variable representing the dispatch level of upward offer $k \in \mathcal{U}(n)$, and $d_{k,n}^T$ as the variable for the downward offer dispatch $k \in \mathcal{D}(n)$, for all nodes $n \in \mathcal{N}^T$. Additionally, $c_{k,n}^u$ and $c_{k,n}^d$ represent the bid prices (cost of flexibility provision) of the submitted upward and downward offers, respectively. Finally, $u_{k,n}^{T,\max}$ and $d_{k,n}^{T,\max}$ are the maximum offered quantities by each bid. The objective of the TSO is to resolve the anticipated balancing and congestion issues in its grid at minimum cost. Thus, the disjoint transmission-level market clearing is described as follows:

$$\min_{\boldsymbol{u},\boldsymbol{d}} \sum_{n \in \mathcal{N}^T} \left(\sum_{k \in \mathcal{U}(n)} c_{k,n}^{u,T} u_{k,n}^T - \sum_{k \in \mathcal{D}(n)} c_{k,n}^{d,T} d_{k,n}^T \right) \tag{1a}$$

Subject to:

$$p_n^T = a_n^T - b_n^T + \sum_{k \in \mathcal{U}(n)} u_{k,n}^T - \sum_{k \in \mathcal{D}(n)} d_{k,n}^T, \ \forall n \in \mathcal{N}^T \setminus \mathcal{N}^D, \tag{1b}$$

$$p_n^T = a_n^T - b_n^T + \sum_{k \in \mathcal{U}(n)} u_{k,n}^T - \sum_{k \in \mathcal{D}(n)} d_{k,n}^T - I_n^p : (\lambda_n^T), \ \forall n \in \mathcal{N}^D, \tag{1c}$$

$$F_{ij}^T = \sum_{n \in \mathcal{N}^T} p_n^T \mathcal{G}_{(i,j),n}, \ \forall \{i,j\} \in \mathcal{L}^T, \tag{1d}$$

$$\sum_{n \in \mathcal{N}^T} p_n^T = 0, \tag{1e}$$

$$-F_{ij}^{T,\max} \le F_{ij}^T \le F_{ij}^{T,\max}, \forall \{i,j\} \in \mathcal{L}^T, \tag{1f}$$

$$0 \le u_{k,n}^T \le u_{k,n}^{T,\max}, \ \forall n \in \mathcal{N}^T, \ \forall k \in \mathcal{U}(n), \tag{1g}$$

$$0 \le d_{k,n}^T \le d_{k,n}^{T,\max}, \ \forall n \in \mathcal{N}^T, \ \forall k \in \mathcal{D}(n). \tag{1h}$$

Equations (1b) and (1c) calculate the net injection at nodes $n \in \mathcal{N}^T \setminus \mathcal{N}^D$ and interface nodes $n \in \mathcal{N}^D$, respectively; (1d) consists of the power flow equations over all the transmission lines, determined using sensitivity factors ($\mathcal{G}_{(i,j),n}$); (1e) is the power balancing equation; (1f) represents the line flow limits, while (1g) and (1h) capture the bid limits. Finally, I_n^p is considered to be a constant, as the market is disjoint, i.e., the TSO must procure its flexibility needs solely using resources connected to the transmission system, thus no sharing of resources is permitted (neither direct nor indirect).

TABLE I: Summary of the TSO-DSO Market Models

Market Model	Market Stages	Market Clearing	Sharing of Resources Connected at Transmission-level with DSO	Sharing of Resources Connected at Distribution-level with TSO	Sharing of Network Information
Disjoint (T&D)	N/A	Independent	No sharing	No sharing	No
Common	1	Joint	Complete sharing via common order book	Complete sharing via common order book	Yes
Fragmented	2	Sequential	Indirect sharing (DSO can change the interface flow)	No sharing	No
Multi-level	2	Sequential	Indirect sharing (DSO can change the interface flow)	Direct sharing (TSO can access – in Stage 2 – remaining, unpurchased bids from the distribution level, i.e., Stage 1)	Distribution network constraints inclusion in the TSO's market clearing, i.e., Stage 2

B. Disjoint Distribution-Level Market

In this market model, each DSO–m procures local resources to solve their anticipated congestion issues. No sharing of resources is permitted, thus only resources connected to their distribution system can be cleared. The offers are described similarly to the transmission system offers, but with superscript m instead of T to assert their distribution system location. Thus, the market clearing problem of a disjoint distribution level market of DSO-m is formulated as follows:

$$\min_{\boldsymbol{u},\boldsymbol{d},\boldsymbol{q}} \sum_{n\in\mathcal{N}^m}\left(\sum_{k\in\mathcal{U}(n)} c^{u,m}_{k,n}u^m_{k,n} - \sum_{k\in\mathcal{D}(n)} c^{d,m}_{k,n}d^m_{k,n}\right) \tag{2a}$$

Subject to:

$$p^m_n = a^m_n - b^m_n + \sum_{k\in\mathcal{U}(n)} u^m_{k,n} - \sum_{k\in\mathcal{D}(n)} d^m_{k,n}, \forall n\in\mathcal{N}^m \tag{2b}$$

$$p^m_n + F^m_{A(n)n} - \sum_{i\in\mathcal{K}(n)} F^m_{ni} = 0,\ \forall n\in\mathcal{N}^m\setminus n^m_0, \tag{2c}$$

$$q^m_n + Q^m_{A(n)n} - \sum_{i\in\mathcal{K}(n)} Q^m_{ni} = 0\ \forall n\in\mathcal{N}^m\setminus n^m_0, \tag{2d}$$

$$I^p_m - \sum_{i\in\mathcal{K}(n)} F^m_{ni} = 0 :\ (\lambda^D_n),\ \text{for } n = n^m_0, \tag{2e}$$

$$I^q_m - \sum_{i\in\mathcal{K}(n)} Q^m_{ni} = 0,\ \text{for } n = n^m_0, \tag{2f}$$

$$v^m_n = v^m_{A(n)} - 2r^m_{A(n)n}F^m_{A(n)n} - 2x^m_{A(n)n}Q^m_{A(n)n},\ \forall n\in\mathcal{N}^m\setminus n^m_0, \tag{2g}$$

$$\gamma_e F^m_{A(n)n} + \sigma_e Q^m_{A(n)n} + \delta_e S^{m,\max}_{A(n)n} \le 0, \forall e\in\mathcal{E}, \{A(n),n\}\in\mathcal{L}^m \tag{2h}$$

$$v^{m,\min}_n \le v^m_n \le v^{m,\max}_n, \forall n\in\mathcal{N}^m, \tag{2i}$$

$$q^{m,\min}_n \le q^m_n \le q^{m,\max}_n, \forall n\in\mathcal{N}^m, \tag{2j}$$

$$I^{q,\min}_m \le I^q_m \le I^{q,\max}_m, \tag{2k}$$

$$0 \le u^m_{k,n} \le u^{m,\max}_{k,n}, \forall n\in\mathcal{N}^m, \forall k\in\mathcal{U}(n), \tag{2l}$$

$$0 \le d^m_{k,n} \le d^{m,\max}_{k,n}, \forall n\in\mathcal{N}^m, \forall k\in\mathcal{D}(n). \tag{2m}$$

Equation (2b) calculates the net power injection at node $n\in\mathcal{N}^m$ considering the activated flexibility; (2c)-(2g) represent the linearized power flow equations in radial networks (considering the *LinDistFlow* model) [5], [12]; (2h) is a linearization of the complex flow limit constraint [5], [14]; (2i) and (2j) capture the limits of nodal voltage magnitudes and reactive power injections in order to meet operational limits; (2k) limits the reactive power transfer with the transmission grid; (2l) and (2m) reflect the limits of the submitted bids. Similarly to the disjoint transmission model, the interface flow I^p_m is kept constant, so that congestion management in each distribution system must be resolved only using resources connected within its own grid. Hence, no indirect sharing of resources can take place.

C. Common Market Model

The concept behind the common market is to reflect a setting in which the TSO and DSOs can jointly procure flexibility from the same pool of resources (i.e., from a common order book) to acquire their flexibility needs while meeting all grids' operational constraints. Hence, the flexibility resources are available to all SOs and the market is jointly cleared by, e.g., the TSO or a market operator, to optimally meet all the balancing and congestion management needs subject to the constraints of all participating grids. As a result, this market formulation joins the disjoint transmission and distribution markets as follows:

$$\min_{\boldsymbol{u},\boldsymbol{d},\boldsymbol{q}}\Bigg[\sum_{n\in\mathcal{N}^T}\left(\sum_{k\in\mathcal{U}(n)} c^{u,T}_{k,n}u^T_{k,n} - \sum_{k\in\mathcal{D}(n)} c^{d,T}_{k,n}d^T_{k,n}\right) + \sum_{n\in\mathcal{N}^m}\left(\sum_{k\in\mathcal{U}(n)} c^{u,m}_{k,n}u^m_{k,n} - \sum_{k\in\mathcal{D}(n)} c^{d,m}_{k,n}d^m_{k,n}\right)\Bigg], \tag{3a}$$

Subject to:

$$\text{(1b)–(1h), (2b)–(2m)}\ \forall m\in\mathcal{N}^D, \tag{3b}$$

$$\text{and}\quad I^{p,\min}_m \le I^p_m \le I^{p,\max}_m\ \forall m\in\mathcal{N}^D. \tag{3c}$$

The objective function (3a) equals the sum of the disjoint objective functions (1a) for the TSO, and (2a) for all DSOs. Moreover, all operational constraints and bid limits from the disjoint markets are considered. However, the interface flow I^p_m is no longer a constant, and an additional constraint to represent the interface line limit is added as (3c). This enables the interaction between the SOs to jointly procure flexibility from a common order book.

D. Fragmented Market Model

The proposed fragmented market model is a sequential market coordination scheme that follows two stages, in which system operators have direct access only to flexibility resources connected to their own systems. This coordination scheme enables DSOs to meet in the first market stage their flexibility needs – e.g., congestion management – while being able to induce limited imbalances that are later rectified in the next stage of the fragmented market by the TSO. This is referred to as an indirect sharing of resources (as this constitutes an implicit access of DSOs to flexibility available at the transmission system. Hence, in the first stage, the local DSO-level markets are run, and I^p_m in each DSO–m market can be modified from its base value (constrained by specified limits). Then, the TSO runs a disjoint central market to resolve its original balancing and congestion needs while accounting for new imbalances that were introduced by the re-dispatch in the first stage. Hence, in this second stage of the market, I_m^p is considered to be a constant updated based on the outcomes of the local market in the first stage, which means that the TSO does not have – neither direct nor indirect – access to distribution grids' flexibility resources. As such, the fragmented market model can be formulated as follows:

First stage – Distribution system DSO–m level market (to be run for each $m \in \mathcal{N}^D$):

$$\min_{\boldsymbol{u},\boldsymbol{d},\boldsymbol{q}} \sum_{n\in\mathcal{N}^m}\left(\sum_{k\in\mathcal{U}(n)} c_{k,n}^{u,m}u_{k,n}^m - \sum_{k\in\mathcal{D}(n)} c_{k,n}^{d,m}d_{k,n}^m\right), \tag{4a}$$

Subject to: (2b)–(2m), (4b)

$$\text{and} \quad I_m^{p,\min} \le I_m^p \le I_m^{p,\max}. \tag{4c}$$

Second stage – Transmission level market:

$$\min_{\boldsymbol{u},\boldsymbol{d}} \sum_{n\in\mathcal{N}^T}\left(\sum_{k\in\mathcal{U}(n)} c_{k,n}^{u,T}u_{k,n}^T - \sum_{k\in\mathcal{D}(n)} c_{k,n}^{d,T}d_{k,n}^T\right), \tag{5a}$$

Subject to: (1b)–(1h), (5b)

$$I_m^{p^*} = \sum_{j\in\mathcal{N}^m}\left(a_j^m - b_j^m + \sum_{k\in\mathcal{U}(j)} u_{k,j}^{m^*} - \sum_{k\in\mathcal{D}(j)} d_{k,j}^{m^*}\right) \forall m\in\mathcal{N}^D. \tag{5c}$$

where $I_m^{p^*}$ for each $m \in \mathcal{N}^D$, replaces I_n^p in (1c), and is the resulting value of the interface flow from the first stage of the fragmented market. $I_m^{p^*}$ is calculated as shown in (5c), where the starred quantities are the optimal decision variables from the first stage of the fragmented market.

As shown in the fragmented market formulation, each SO uses flexibility resources available in its own grid and, hence, has to account for the operational limits of its own network only (not requiring any network information sharing).

E. Multi-Level Market Model

The multi-level market is proposed to extend the concept of the fragmented market to allow TSOs to access flexibility bids submitted from resources connected to the distribution systems. As such, two levels of markets are also organized. First, local markets equal to the first stage of the fragmented markets are arranged, and each DSO can purchase resources available within its distribution system for congestion management, while indirectly using resources from the TSO through the interface flow. In the second stage, non-cleared bids of the first stage are forwarded to the TSO, which clears its market using resources connected to all systems. Here, differently from the fragmented market, the remaining resources located in the distribution systems are directly accessible by the TSO. Thus, the market clearing on the transmission level must take into consideration the constraints of the distribution systems so as not to violate the distribution system constraints. As such, the multilevel market model can be formulated as follows:

First stage – Distribution system DSO–m level market (to be run for each $m \in \mathcal{N}^D$):

$$\min_{\boldsymbol{u},\boldsymbol{d},\boldsymbol{q}} \sum_{n\in\mathcal{N}^m}\left(\sum_{k\in\mathcal{U}(n)} c_{k,n}^{u,m}u_{k,n}^m - \sum_{k\in\mathcal{D}(n)} c_{k,n}^{d,m}d_{k,n}^m\right), \tag{6a}$$

Subject to: (2b)–(2m), (6b)

$$\text{and} \quad I_m^{p,\min} \le I_m^p \le I_m^{p,\max}. \tag{6c}$$

Second stage – Transmission level market:

$$\min_{\boldsymbol{u},\boldsymbol{d},\boldsymbol{q}} \Bigg[\sum_{n\in\mathcal{N}^T}\left(\sum_{k\in\mathcal{U}(n)} c_{k,n}^{u,T}u_{k,n}^T - \sum_{k\in\mathcal{D}(n)} c_{k,n}^{d,T}d_{k,n}^T\right) + \sum_{n\in\mathcal{N}^m}\left(\sum_{k\in\mathcal{U}(n)} c_{k,n}^{u,m}u_{k,n}^m - \sum_{k\in\mathcal{D}(n)} c_{k,n}^{d,m}d_{k,n}^m\right)\Bigg], \tag{7a}$$

Subject to: (1b)–(1h), (2c)–(2k) $\forall m \in \mathcal{N}^D$, (7b)

$$I_m^{p,\min} \le I_m^p \le I_m^{p,\max} : (\mu_m^{\min}, \mu_m^{\max}),\ \forall m\in\mathcal{N}^D. \tag{7c}$$

$$p_n^m = a_n^m - b_n^m + \sum_{k\in\mathcal{U}(n)} (u_{k,n}^m + u_{k,n}^{m^*}) - \sum_{k\in\mathcal{D}(n)} (d_{k,n}^m + d_{k,n}^{m^*}),\quad \forall m\in\mathcal{N}^D, \forall n\in\mathcal{N}^m, \tag{7d}$$

$$0 \leqslant u_{k,n}^m \leqslant u_{k,n}^{m,\max} - u_{k,n}^{m^*},\ \forall m\in\mathcal{N}^D, \forall n\in\mathcal{N}^m, \forall k\in\mathcal{U}(n), \tag{7e}$$

$$0 \leqslant d_{k,n}^m \leqslant d_{k,n}^{m,\max} - d_{k,n}^{m^*},\ \forall m\in\mathcal{N}^D, \forall n\in\mathcal{N}^m, \forall k\in\mathcal{D}(n), \tag{7f}$$

where $u_{k,n}^{m^*}$ and $d_{k,n}^{m^*}$ are optimal values from the DSO–m level.

F. Efficiency of the Coordination Schemes

As the common market pools all resources together and clears the market jointly, it is expected to lead to the highest possible efficiency (i.e., meet the collective needs of all SOs at the minimum possible costs). This is indeed the case, as we prove next. In this regard, we first prove in Proposition 1 that the common market is more economically efficient than the disjoint market models. The proof that the common is also more efficient than the fragmented and multi-level markets is given in Corollary III.2, as it requires the introduction of additional results before readily deriving the proof.

Proposition 1. *The common market is guaranteed to return a lower or equal procurement cost and, hence, is more (economically) efficient than the disjoint markets.*

Proof. As the common market model is linear, it can readily be presented as a standard compact linear program in line with the work in [11], as follows:

$$\text{(LP)} \qquad \min_{\boldsymbol{x},\boldsymbol{z}} \ \Phi_0(\boldsymbol{x}_0) + \sum_{m\in\mathcal{N}^D} \Phi_m(\boldsymbol{x}_m), \tag{8a}$$

$$s.t. \ \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{z} \le \boldsymbol{d}, \tag{8b}$$

$$\boldsymbol{x}_0 \in \mathcal{X}_0, \tag{8c}$$

$$\boldsymbol{x}_m \in \mathcal{X}_m, \forall m\in\mathcal{N}^D, \tag{8d}$$

$$\boldsymbol{z} \in \mathcal{Z}, \tag{8e}$$

where $\boldsymbol{x}_0$ is the vector of the TSO's decision variables, $\boldsymbol{x}_m$ is DSO m's vector of decision variables, $\boldsymbol{z}$ is a vector of dependent variables, and matrices $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{d}$ capture the parameters in equations (1b)–(1e), (2b)–(2h). When TSO and DSOs cooperate in a single market, Problem (8) can be interpreted as a characteristic function game $G \triangleq \left(\mathcal{N}^D \cup \{\text{TSO}\}, v\right)$, as we have previously shown in [11], with characteristic function defined for any coalition $C \subseteq \mathcal{N}^D \cup \{\text{TSO}\}$ as follows:

$$v(C) = \sum_{i \in C} \Phi_i(\boldsymbol{x}_i^*) \ , \tag{9}$$

where $(\boldsymbol{x}_i^*)_{i \in C}$ is the optimum of the optimization problem:

$$\min_{(\boldsymbol{x}_i)_{i\in C}, (\boldsymbol{z}_i)_{i \in C}} \quad \sum_{i \in C} \Phi_i(\boldsymbol{x}_i), \tag{10a}$$

$$s.t. \quad \boldsymbol{Ax} + \boldsymbol{Bz} \leq \boldsymbol{d}, \tag{10b}$$

$$\boldsymbol{x}_i \in \mathcal{X}_i, \boldsymbol{z}_i \in \mathcal{Z}_i, \forall m \in C, \tag{10c}$$

This characteristic function game G, can be proven to be concave (as shown in [11]). This result, extended to the current problem, implies that the total procurement cost of the common market is lower or equal to the sum of the procurement costs on the disjoint markets (due to the proven concavity). This implies that the common market cannot be less efficient than the disjoint transmission and distribution markets. □

Proposition 1 highlights the greater efficiency of the common market as compared to the disjoint markets. To further extend this comparison to the sequential markets (fragmented and multi-level), we first investigate how the interface flows can be priced within these markets, and the consequences thereof, in the next section.

III. Optimal Pricing of Interface Flows

In the sequential markets described in Section II, i.e. fragmented and multi-level markets, a market clearing in the distribution-level stage (i.e. First Stage) can lead to “unpriced” imbalances for the TSO. As the distribution systems clear their markets first, they can procure excessive downward flexibility in their systems to reduce their total cost (even if no longer needed to resolve congestions), which will generate an upward need at the transmission level to keep the system balance. The “unpriced” imbalance must be settled by the second stage of the coordination schemes, harming the total welfare of the procurement process.

To prevent the DSOs from procuring excessive flexibility (just for revenue derivation) in the first stages of the sequential markets, without a grid need for it, we develop three methods. In the first, DSOs are prevented from changing the interface flow in the first stage of the fragmented and multi-level markets. Therefore, the variable I_m^p is treated as a constant in equations (4c) and (6c). This method is the most conservative approach as it limits the exploitation of flexibility resources. For example, this method renders the fragmented market model equal to the disjoint market models.

In the second method, the interface flow is priced at the midpoint between the most expensive downward flexibility bid and the least expensive upward flexibility bid of each distribution system. As a result, the interface flow becomes more expensive than all downward flexibility bids, and the DSOs will no longer benefit from purchasing such flexibility unless there is a grid need for it (e.g. to alleviate congestion) in the first stage of the sequential market schemes. The bids from the DSOs are used to define the midpoint price due to two reasons: 1) to prevent unnecessary purchasing of downward bids by the DSOs, and 2) each DSO does not need to access submitted bids from other systems to compute the price. This pricing method derives from applications in day-ahead and balancing markets at the transmission levels, namely, in the single price imbalance settlement mechanisms [15].

It is important to note that choosing any interface price in between the least expensive upward bid and the most expensive downward bid would lead to the same result. The goal of providing the midpoint method is to propose a simple empirical method which can lead to highly satisfactory results. Therefore, if the complexity involved in applying the optimal pricing (as discussed next) or in exchanging information (as discussed in Section IV) is blocking the potential implementation of optimal interface flow pricing, than the use of simple methods such as the midpoint can constitute a good practical alternative.

In the third method, the interface flow is priced optimally based on a virtual run of the common market, to capture the real optimal value to the system from providing flexibility through the connection points. As demonstrated in Proposition 2, the optimal price can be derived from the power flow equations at the interface between the systems. This can be done by running the common market in problem (11), calculating the dual variables of equation (11h) (λ_m), and adding the term $\lambda_m I_m^p$ to the objective function of the DSOs in the first stages of the fragmented (4a) and multi-level (6a) markets. The addition of this optimal price will make the solution of the sequential markets equal to the solution of the common market, in terms of total procurement cost, as also shown in Proposition 2 and Corollary III.1. As the virtual run of the common market implies the sharing of network information of all system operators, we propose in Section IV a method for obtaining those in a distributed way, hence, avoiding the need for network information sharing.

Proposition 2. *If the interface flows are optimally priced, the result of the fragmented market is equal to the common market.*

Proof. Consider two different interface flows $\overline{z}_m$ and $\tilde{z}_m$, where the first is considered from the perspective of the TSO – for all its interface nodes $m \in \mathcal{N}^D$ – and the second from the perspective of the DSOs, for all DSOs $m \in \mathcal{N}^D$. As $\overline{z}_m = \tilde{z}_m$ for the connections between the TSO node m to the corresponding DSO–m, the common market model can be written as a compact linear program (LP) considering the duplicated interface flows as follows:

$$\text{(LP)} \quad \min_{\boldsymbol{x}, \tilde{\boldsymbol{z}}, \overline{\boldsymbol{z}}} \ \Phi_0(\boldsymbol{x}_0) + \sum_{m \in \mathcal{N}^D} \Phi_m(\boldsymbol{x}_m), \tag{11a}$$

$$s.t. \quad \boldsymbol{A}_0\boldsymbol{x}_0 + \boldsymbol{B}_0\overline{\boldsymbol{z}} = 0, \tag{11b}$$

$$\boldsymbol{A}_n\boldsymbol{x}_m + \boldsymbol{B}_n\tilde{z}_m = 0, \forall m \in \mathcal{N}^D, \tag{11c}$$

$$\boldsymbol{x}_0 \in \mathcal{X}_0, \tag{11d}$$

$$\boldsymbol{x}_m \in \mathcal{X}_m, \forall m \in \mathcal{N}^D, \tag{11e}$$

$$\overline{\boldsymbol{z}} \in \mathcal{Z}, \tag{11f}$$

$$\tilde{\boldsymbol{z}} \in \mathcal{Z}, \tag{11g}$$

$$\tilde{z}_m - \overline{z}_m = 0 : (\lambda_m), \forall m \in \mathcal{N}^D, \tag{11h}$$

in which $\boldsymbol{x}_0$ are the decision variables of the TSO, and $\boldsymbol{x}_m$ are the decision variables of the DSO–m. Equation (11b) captures (1c), and (11c) captures (2e). $\mathcal{X}_0$ contains all other constraints of the transmission system, while $\mathcal{X}_m$ represents all other constraints of distribution system m. $\mathcal{Z}$ is the limit of interface flows in equation (3c). $\Phi_0(\boldsymbol{x}_0)$ equals (1a), and $\Phi_n(\boldsymbol{x}_m)$ equals (2a). Moreover, λ_m are the dual variables of equations (11h).

We define the decentralized problems of each system operator considering the interface price and the new variables for interface flows. In the case of distribution system DSO-m:

$$\min_{\boldsymbol{x}_m,\tilde{z}_m} \quad \Phi_n(\boldsymbol{x}_m) + \lambda_m \tilde{z}_m, \tag{12a}$$
$$s.t. \quad \boldsymbol{A}_n \boldsymbol{x}_m + \boldsymbol{B}_m \tilde{z}_m = 0, \tag{12b}$$
$$\boldsymbol{x}_m \in \mathcal{X}_m, \tag{12c}$$
$$\tilde{z}_m \in \mathcal{Z}_m. \tag{12d}$$

For the TSO:

$$\min_{\boldsymbol{x}_0,\overline{\boldsymbol{z}}} \; \Phi_0(\boldsymbol{x}_0) - \sum_{m\in\mathcal{N}^D} \lambda_m \overline{z}_m, \tag{13a}$$
$$s.t. \quad \boldsymbol{A}_0 \boldsymbol{x}_0 + \boldsymbol{B}_0 \overline{\boldsymbol{z}} = 0, \tag{13b}$$
$$\boldsymbol{x}_0 \in \mathcal{X}_0, \tag{13c}$$
$$\overline{\boldsymbol{z}} \in \mathcal{Z}. \tag{13d}$$

The above two problems clear at $\tilde{z}_m - \overline{z}_m = 0$ for all $m \in \mathcal{N}^D$. We show next that the primal/dual solution of (11) is also a solution of those two problems.

Consider $\boldsymbol{x}_0^*$, $\boldsymbol{x}_m^*$, $\overline{z}^*$, $\tilde{z}^*$ as an optimal solution of problem (11), and let λ_m^* be obtained by the optimal dual variables of the constraints (11h). This optimal solution clearly respects condition $\tilde{z}_m - \overline{z}_m = 0$, as it solves the rewritten common market problem. We have now to prove that this optimal solution is also optimal for problems (12), for all $m \in \mathcal{N}^D$, and for (13). The Lagrangian dual problem of (11) is:

$$\max_{\boldsymbol{\lambda}} \left\{ \min_{\boldsymbol{x},\overline{\boldsymbol{z}},\tilde{\boldsymbol{z}}} \Phi_0(\boldsymbol{x}_0) + \sum_{m\in\mathcal{N}^D} \Phi_m(\boldsymbol{x}_m) + \sum_{m\in\mathcal{N}^D} \lambda_n(\tilde{z}_m - \overline{z}_m) \text{ s.t. (11b)–(11g)} \right\}$$
$$= \max_{\boldsymbol{\lambda}} \left\{ \left\{ \min_{\boldsymbol{x}_0,\overline{\boldsymbol{z}}} \Phi_0(\boldsymbol{x}_0) - \sum_{m\in\mathcal{N}^D} \lambda_m \overline{z}_m \text{ s.t. (11b), (11d), (11f)} \right\} + \sum_{m\in\mathcal{N}^D} \left\{ \min_{\boldsymbol{x}_m,\tilde{z}_m} \Phi_m(\boldsymbol{x}_m) + \lambda_m \tilde{z}_m \text{ s.t. (11c), (11e), (11g)} \right\} \right\}. \tag{14}$$

The equality in (14) holds since the first inner problem is separable for each market participant (TSO and DSOs), i.e., no constraints or variables are shared as the interface flow was duplicated. For a fixed vector $\boldsymbol{\lambda}$, the inner problems of the right-hand side are equal to problems (12) for all $m \in \mathcal{N}^D$, and (13), written for each SO, which must be solved by the optimal $\boldsymbol{x}_0^*$, $\boldsymbol{x}_m^*$, $\overline{z}^*$, $\tilde{z}^*$ to obtain the desired result.

We need to verify that the optimal solution solves the inner minimization problems in (14). The strong duality for linear programs guarantees that $\boldsymbol{\lambda}^*$ solves the left-hand side of (14), and:

$$\Phi_0(\boldsymbol{x}_0^*) + \sum_{m\in\mathcal{N}^D} \Phi_m(\boldsymbol{x}_m^*) = \max_{\boldsymbol{\lambda}} \left\{ \min_{\boldsymbol{x},\overline{\boldsymbol{z}},\tilde{\boldsymbol{z}}} \Phi_0(\boldsymbol{x}_0) + \sum_{m\in\mathcal{N}^D} \Phi_m(\boldsymbol{x}_m) + \sum_{m\in\mathcal{N}^D} \lambda_m(\tilde{z}_m - \overline{z}_m) \text{ s.t. (11b)–(11g)} \right\}. \tag{15}$$

Using (11h) multiplied by $\boldsymbol{\lambda}$, and the fact that $\boldsymbol{\lambda}^*$ solves (14), equation (15) can be rewritten as:

$$\Phi_0(\boldsymbol{x}_0^*) + \sum_{m\in\mathcal{N}^D} \Phi_m(\boldsymbol{x}_m^*) + \sum_{m\in\mathcal{N}^D} \lambda_m^*(\tilde{z}_m - \overline{z}_m) = \left\{ \min_{\boldsymbol{x}_0,\overline{\boldsymbol{z}}} \Phi_0(\boldsymbol{x}_0) - \sum_{m\in\mathcal{N}^D} \lambda_n^* \overline{z}_m \text{ s.t. (11b), (11d), (11f)} \right\} + \sum_{m\in\mathcal{N}^D} \left\{ \min_{\boldsymbol{x}_m,\tilde{z}_m} \Phi_m(\boldsymbol{x}_m) + \lambda_m^* \tilde{z}_m \text{ s.t. (11c), (11e), (11g)} \right\}. \tag{16}$$

Since $\boldsymbol{x}_m^*$ and $\tilde{z}_m^*$ satisfy constraints (11c), (11e), (11g) for all $m \in \mathcal{N}^D$, and $\boldsymbol{x}_0^*$ and $\overline{\boldsymbol{z}}^*$ satisfy constraints (11b), (11d), (11f) – they are feasible for each system operator problem in the right-hand side of (16) – they must be optimal solutions for these problems as well. If not, equation (16) would not hold, contradicting strong duality for linear programs. This proves that an optimal solution of problem (11) is also optimal for problems (12) for all $m \in \mathcal{N}^D$ and (13).

Problem (12) equals the first stage of the fragmented market with optimal penalty factor. In the second stage, $\overline{\boldsymbol{z}}$ is fixed by the result of the first stage. Therefore, the second stage of the fragmented is equal to problem (13) with $\overline{\boldsymbol{z}}$ fixed and equal to $\tilde{\boldsymbol{z}}$, guaranteeing the optimal solution described in this proof. □

Proposition 2 proves that if the interface flow is optimally priced, the fragmented market results would converge to the common market. This result is further extended to the multi-level market as shown in Corollary III.1.

Corollary III.1. *If the interface flows are optimally priced, the result of the multi-level is equal to the common market.*

Proof. The first stage of the multi-level market considering the interface flow pricing equals problem (12). As the solution of this problem is optimal when λ_m is defined by the dual of equation (11h), the second stage of the multi-level reduces to problem (13), as no other resources at distribution level are competitive (given the interface flow limits). □

The results of Proposition 2 and Corollary III.1 clearly highlight the importance of adequately pricing the interface flows. Indeed, even though the fragmented market does not allow access for the TSO to distribution-level bids, and that the multi-level market provides priority access to the DSO, when the interface flow price captures the real (optimal) value of power exchange between the SOs (through the dual prices of these interface flows), the results of each sequential market would make use of this interface flow optimally, converging to the common market, which is the most efficient market as shown next.

Corollary III.2. *If $\boldsymbol{\lambda}$ is not optimally defined (equal to the dual of equation (11h)), the common market will always return an optimal solution more (economically) efficient or equal the optimal solution of the fragmented or multi-level markets.*

Proof. The right-hand side of the strong duality in equation (15) is separable

$$\Phi_0(\boldsymbol{x}_0^*) + \sum_{n\in\mathcal{N}^D} \Phi_n(\boldsymbol{x}_n^*)$$
$$= \max_{\boldsymbol{\lambda}} \left\{ \left\{ \min_{\boldsymbol{x}_0,\overline{\boldsymbol{z}}} \Phi_0(\boldsymbol{x}_0) - \sum_{n\in\mathcal{N}^D} \lambda_n \overline{z}_n \text{ s.t. (11b), (11d), (11f)} \right\} \right.$$

$$+ \sum_{n \in \mathcal{N}^D} \left\{ \min_{\boldsymbol{x}_n, \tilde{z}_n} \Phi_n(\boldsymbol{x}_n) + \lambda_n \tilde{z}_n \text{ s.t. } (11c), (11e), (11g) \right\} \Bigg\}, \quad (17)$$

and its value is minimized by the optimal price vector $\boldsymbol{\lambda}$. If any other interface price vector is chosen, the left-hand side of equation (17) would be less than or equal to its right-hand side. This means that the solution of the fragmented would be worse than the solution of the common market if a non-optimal price vector is chosen. The multi-level solution would also be worse, given that its first stage equals the first stage of the fragmented market. Finally, if the price vector $\boldsymbol{\lambda} = \boldsymbol{0}$ (no interface flow price) is not an optimal solution of the right-hand side of (17), the original fragmented and multi-level markets (without interface price) will return a solution worse or at most equal the common market solution. □

The derivation of Proposition 2 and the results that ensued, imply that without access to flexibility resources from the distribution system (bid sharing) and without the need for any network information sharing (i.e., the TSO problem does not need to consider network limitations from the distribution system), the fragmented market can achieve the same efficiency as the most efficient common market, which is a striking result. However, we note that this is only achieved if the interface flow is priced optimally, while the optimal price of the interface flow is considered to be obtained from a virtual run of the common market, which requires the sharing of network information of all the systems. Hence, the sharing of information is intrinsically embedded in the virtual run of the common market, which may not be always possible, specially because the fragmented market is originally set up to prevent such network information sharing. To deal with the information sharing limitation while reaching an optimal pricing of interface flows, we propose adequate distributed mechanisms next.

IV. Network Information Sharing Limitation

Among the proposed market models, two of them involve information sharing between the participating system operators (SOs). First, the common market is a joint procurement process in which the network information of all SOs must be provided to the market operator (or the entity responsible for the market clearing). Second, in the multi-level market, the DSOs must provide their network information to the TSO (or to a third-party market operator) in the second stage so its market clearing takes into account the operational limits of the impacted grids when procuring flexibility in from the distribution levels. As this may face practical obstacles, we propose alternatives to allow for the safe clearing of those markets with limited need of information sharing.

Related to this information sharing challenge, a recent research stream in the literature on energy markets has aimed at proposing fully distributed and privacy-preserving algorithms to compute a market equilibrium. In this body of literature, proposed algorithms that are classified as guaranteeing minimum information exchange rely on the sole release of price signals [16], [17]. This is a significant departure from classical centralized optimization paradigms, which would require that the system operators have precise information about all local network constraints.

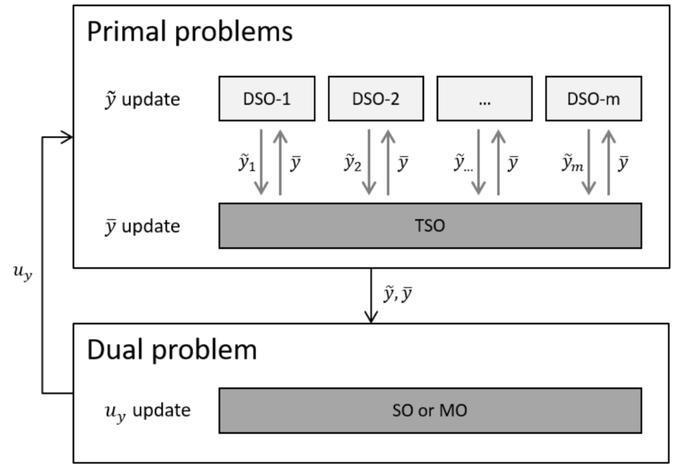

Fig. 1: The application of ADMM to decompose the common and multi-level markets. Primal problems are solved by each system operator (SO) while the dual problem can be solved by one SO or a market operator (MO).

To achieve the minimum information sharing goal and enhance the privacy-preserving aspect of the multi-level and common market models, we apply the alternating direction method of multipliers (ADMM), which is able to solve complex optimization problems by breaking them in smaller problems that are easier to solve. In energy markets, the method has been widely applied to decompose problems in which multiple stakeholders have conflicting interests, e.g. prosumers and SOs [18], investors in capacity markets [19], electricity and natural gas networks [20], producers and consumers in nodal pricing markets [21], among others.

In our case, TSO and DSOs seek to procure flexibility to resolve their congestion management and balancing needs at a minimal cost. Therefore, the goal behind applying ADMM is to decompose the TSO-DSO joint problems (the common and multi-level markets) into one problem per SO by relaxing the coupling constraints, and solve them in a distributed manner while exchanging just dual variables and fine-tuning the interface flows. As a result, the algorithm is privacy-preserving in the sense that it is no longer necessary to share the entire network information, but only dual variables of coupling constraints while regulating the interface flows to converge to a global variable.

The resulting process is shown in Figure 1, in which $\tilde{y}$ are the DSO variables to be exchanged, $\overline{y}$ are the TSO variables to be exchanged, and u_y are the scaled dual variables of the coupling constraints. We note that the variables in the vectors $\tilde{y}, \overline{y}$, and u_y depend on the market model, i.e. common or multi-level.

A. Common Market Clearing with Limited Information Sharing

In order to solve the common market with limited information sharing, we apply the alternating direction method of multipliers (ADMM) to problem (11). In this version of the common market, only equation (11h) couples the problems of the TSO and DSOs. Thus, an augmented Lagrangian can be written as:

$$\mathcal{L}_{\rho}^{\text{CM}}(\boldsymbol{x},\overline{\boldsymbol{z}},\tilde{\boldsymbol{z}},\boldsymbol{u}) = \Phi_0(\boldsymbol{x}_0) + \sum_{m\in\mathcal{N}^D} \Phi_m(\boldsymbol{x}_m) + \frac{\rho}{2}\left\|\tilde{\boldsymbol{z}} - \overline{\boldsymbol{z}} + \boldsymbol{u}\right\|_2^2 - \frac{\rho}{2}\left\|\boldsymbol{u}\right\|_2^2. \quad (18)$$

In which $u_m = \frac{1}{\rho}\lambda_m$ are the scaled dual variable [22], and ρ is a penalty parameter of the ADMM. As the problems become separable and the objective functions $\Phi_0(.)$ and $\Phi_m(.)$ are linear, thus convex, the ADMM can be applied [22]. In each iteration $k+1$ of the method, the SOs update their local decision variables and the interface flow by minimizing the augmented Lagrangian function considering the decision variables and interface flows of the other SOs fixed and equal to the result of iteration k (or $k+1$, depending on the order). Each $m \in \mathcal{N}^D$ solves problem (19) to obtain an updated value of the interface flow $\tilde{z}_m^{k+1}$ while accounting for the feasibility space of the other variables in $\boldsymbol{x}_m$. Notice that those problems can be solved in parallel, as the DSOs do not share constraints or variables among themselves.

$$\tilde{z}_m^{k+1} := \underset{\tilde{z}_m\in\mathcal{Z}_m}{\text{argmin}}\, \Phi_m(\boldsymbol{x}_m) + \frac{\rho}{2}\left\|\tilde{z}_m - \overline{z}_m^k + u_m^k\right\|_2^2 - \frac{\rho}{2}\left\|u_m^k\right\|_2^2$$
$$\text{s.t. } \boldsymbol{A}_m\boldsymbol{x}_m + \boldsymbol{B}_m\tilde{z}_m = 0,\ \boldsymbol{x}_m \in \mathcal{X}_m. \quad (19)$$

Then, the TSO solves problem (20) to obtain updated values of the interface flows $\overline{\boldsymbol{z}}^{k+1}$ while accounting for the feasibility space of the other variables in $\boldsymbol{x}_0$. Finally, the scaled dual variables are updated according to (21).

$$\overline{\boldsymbol{z}}^{k+1} := \underset{\overline{\boldsymbol{z}}\in\mathcal{Z}}{\text{argmin}}\, \Phi_0(\boldsymbol{x}_0) + \frac{\rho}{2}\left\|\tilde{\boldsymbol{z}}^{k+1} - \overline{\boldsymbol{z}} + \boldsymbol{u}^k\right\|_2^2 - \frac{\rho}{2}\left\|\boldsymbol{u}^k\right\|_2^2$$
$$\text{s.t. } \boldsymbol{A}_0\boldsymbol{x}_0 + \boldsymbol{B}_0\overline{\boldsymbol{z}} = 0,\ \boldsymbol{x}_0 \in \mathcal{X}_0 \quad (20)$$
$$u_m^{k+1} := u_m^k + \tilde{z}_m^{k+1} - \overline{z}_m^{k+1},\ \forall m \in \mathcal{N}^D. \quad (21)$$

The ADMM is guaranteed to converge when $\Phi_0(.)$ and $\Phi_n(.)$ for all $m \in \mathcal{N}^D$ are closed, proper, and convex (which is the case of the linear objective functions in our models), and the variables form a compact and convex set (which is the case of all our decision variables and interface flows as they are constrained to the linear equations defining the sets $\mathcal{X}_0$, $\mathcal{X}_m$ and $\mathcal{Z}$)—see [22].

Finally, the ADMM decomposition generates local problems, which can be solved by each system operator, without sharing network information: only the interface flows and prices (scaled dual variable) are shared among the SOs. One should note that each SO solves a fragmented market with penalty factor plus an extra term accounting for the difference between the duplicated interface flows, which converges to zero—see the unscaled version of the augmented Lagrangian in (18). Moreover, at the end of the process, the interface prices are defined. Therefore, applying the ADMM is also a method to solve optimally the fragmented market with interface flow penalty (third method presented in section III) without knowing the interface price beforehand.

B. Multi-Level Clearing with Limited Information Sharing

To solve the multi-level market with limited information sharing while pricing the interface flow of its first stage, a bi-level optimization is applied. For simplicity of presentation of the bi-level program, we consider that the DSOs solve the first stage together. This will not impact the results of the decentralization process as the DSOs do not share any information. Variables related to the distribution systems are represented by $\boldsymbol{x}_m^u$ when regarding the first stage of the multi-level, and by $\boldsymbol{x}_m^l$ for the second stage. The interface flows are also indexed for each stage. The upper level problem is the first stage of the multi-level market, since the distribution systems move first. Then, the lower level problem is the second stage of the multi-level, which is the decision problem of the TSO. We consider that the TSO prices the interface flow of the first stage according to its interface node prices, given by (1c) (λ_m^T for all $m \in \mathcal{N}^D$). These prices represent the cost the TSO bears due to changes in the interface flow. Therefore, the bi-level program (BLP) is defined as:

$$\text{(BLP)} \quad \min_{\boldsymbol{x}_m^u,\boldsymbol{z}^u} \sum_{m\in\mathcal{N}^D}\left[\Phi_m(\boldsymbol{x}_m^u) + \lambda_m^T z_m^u\right], \quad (22a)$$
$$s.t.\ H_m^u(\boldsymbol{x}_m^u,\boldsymbol{z}_m^u) = 0\ \ \forall m\in\mathcal{N}^D, \quad (22b)$$
$$G_m^u(\boldsymbol{x}_m^u,\boldsymbol{z}_m^u) \le 0\ \ \forall m\in\mathcal{N}^D, \quad (22c)$$
$$\boldsymbol{\lambda}^T \in \underset{\boldsymbol{\lambda}^T}{\text{argmin}}\left\{\Phi_0(\boldsymbol{x}_0) + \sum_{m\in\mathcal{N}^D}\Phi_m(\boldsymbol{x}_m^l)\right. \quad (22d)$$
$$s.t.\ \boldsymbol{A}_0\boldsymbol{x}_0 + \boldsymbol{B}_0\boldsymbol{z}^l = 0 : (\boldsymbol{\lambda}^T), \quad (22e)$$
$$H_0(\boldsymbol{x}_0) = 0 : (\boldsymbol{\beta}_0), \quad (22f)$$
$$G_0(\boldsymbol{x}_0) \le 0 : (\boldsymbol{\alpha}_0), \quad (22g)$$
$$\boldsymbol{A}_m(\boldsymbol{x}_m^l + \boldsymbol{x}_m^u) + \boldsymbol{B}_m z_m^l = 0 :\ (\lambda_m^D),\ \forall m\in\mathcal{N}^D \quad (22h)$$
$$H_m^l(\boldsymbol{x}_m^l,\boldsymbol{x}_m^u) = 0 : (\boldsymbol{\beta}_m), \forall m\in\mathcal{N}^D, \quad (22i)$$
$$G_m^l(\boldsymbol{x}_m^l,\boldsymbol{x}_m^u) \le 0 : (\boldsymbol{\alpha}_m), \forall m\in\mathcal{N}^D, \quad (22j)$$
$$\left. F_m^l(z_m^l) \le 0 :\ (\boldsymbol{\mu}_m),\ \forall m\in\mathcal{N}^D\right\} \quad (22k)$$

in which $H_m^u(\cdot)$ represents the equality constraints of multi-level first stage (2b)–(2g); $G_m^u(\cdot)$ represents the inequality constraints of first stage (2h)–(2m),(6c); equation (22e) equals (1c); $H_0(\cdot)$ are the equality constraints of the transmission system in second stage (1b),(1d),(1e); $G_0(\cdot)$ are the inequality constraints of the transmission system in second stage (1f)–(1h); equation (22h) equals (2e); $H_m^l(\cdot)$ represents equality constraints of distribution systems in second stage (2c),(2d),(2f),(2g),(7d); $G_m^l(\cdot)$ are inequality constraints of distribution systems in second stage (2h)–(2k),(7e),(7f); and (22k) represents equation (7c). Duals of the second stage constraints are indicated in parenthesis ($\boldsymbol{\beta}$, $\boldsymbol{\alpha}$, $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$).

The lower level in problem (22) is convex and regular, since all equations are linear, including the objective function. Moreover, Slater's condition applies to the problem: at least one of the inequalities imposing limits to dispatch and flows are non-bidding if variables' upper and lower bounds are not equal. Therefore, it can be replaced by its Karush-Kuhn-Tucker conditions [23], yielding a single-level reduction reformulation of the problem [24]:

$$\text{(SLR)} \quad \min_{\boldsymbol{x}_m^u,\boldsymbol{z}^u} \sum_{m\in\mathcal{N}^D}\left[\Phi_m(\boldsymbol{x}_m^u) + \lambda_m^T z_m^u\right], \quad (23a)$$
$$s.t.\ \text{(22b), (22c), (22e)–(22k)} \quad (23b)$$
$$\nabla_{\boldsymbol{x}^l,\boldsymbol{x}_0,\boldsymbol{z}} L(\boldsymbol{x}^u,\boldsymbol{x}^l,\boldsymbol{x}_0,\boldsymbol{z},\boldsymbol{\lambda},\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\mu}) = 0 \quad (23c)$$
$$\boldsymbol{\alpha}_0 G_0(\boldsymbol{x}_0,\boldsymbol{z}^l) = 0 \quad (23d)$$

$$\boldsymbol{\alpha}_m G_m^l(\boldsymbol{x}_m^l, \boldsymbol{x}_m^u, \boldsymbol{z}_m^l) = 0, \forall m \in \mathcal{N}^D \tag{23e}$$

$$\boldsymbol{\mu}_m F_m^l(z_m^l) = 0, \ \forall m \in \mathcal{N}^D \tag{23f}$$

$$\boldsymbol{\alpha}_0 \geq 0, \boldsymbol{\alpha}_m \geq 0, \boldsymbol{\mu}_m \geq 0, \forall m \in \mathcal{N}^D \tag{23g}$$

where $L(\boldsymbol{x}^u, \boldsymbol{x}^l, \boldsymbol{x}_0, \boldsymbol{z}^l, \boldsymbol{\lambda}^T, \boldsymbol{\alpha}, \boldsymbol{\beta})$ is the Lagrangian of the lower level problem. Similarly to the common market clearing with limited information sharing in Section IV-A, the SLR problem in (23) can be solved using ADMM by identifying the coupling constraints and building an augmented Lagrangian to decompose the problem. Again, the interface flow variable in the second stage couples the problems of distribution and transmission systems in the lower level problem, and it can be duplicated in an analogous way. However, this coupling also impacts other stationary constraints in equation (23c), and other variables must be also duplicated: the interface flow prices at TSO side in the second stage ($\boldsymbol{\lambda}^T$); the interface flow prices at DSO side in the second stage ($\boldsymbol{\lambda}^D$), which are the duals of equation (2e); and the duals of the interface flow limits in the second stage ($\boldsymbol{\mu}_m$). Considering these new variables and the inclusion of constraints to close their gaps, the scaled augmented Lagrangian of the problem can be written as:

$$\begin{aligned}&\mathcal{L}_\rho^{\text{ML}}(\boldsymbol{x}^u, \boldsymbol{x}^l, \boldsymbol{x}_0, \overline{\boldsymbol{z}}^l, \tilde{\boldsymbol{z}}^l, \overline{\boldsymbol{\lambda}}^T, \tilde{\boldsymbol{\lambda}}^T, \overline{\boldsymbol{\lambda}}^D, \tilde{\boldsymbol{\lambda}}^D, \overline{\boldsymbol{\mu}}, \tilde{\boldsymbol{\mu}}, \boldsymbol{u}) \\ &= \sum_{m \in \mathcal{N}^D} \left[\Phi_m(\boldsymbol{x}_m^u) + \tilde{\lambda}_m^T z_m^u\right] + \frac{\rho}{2}\left\|\tilde{\boldsymbol{z}}^l - \overline{\boldsymbol{z}}^l + \boldsymbol{u}_z\right\|_2^2 - \frac{\rho}{2}\left\|\boldsymbol{u}_z\right\|_2^2 \\ &+ \frac{\rho}{2}\left\|\tilde{\boldsymbol{\lambda}}^T - \overline{\boldsymbol{\lambda}}^T + \boldsymbol{u}_{\lambda^T}\right\|_2^2 - \frac{\rho}{2}\left\|\boldsymbol{u}_{\lambda^T}\right\|_2^2 + \frac{\rho}{2}\left\|\tilde{\boldsymbol{\lambda}}^D - \overline{\boldsymbol{\lambda}}^D + \boldsymbol{u}_{\lambda^D}\right\|_2^2 \\ &- \frac{\rho}{2}\left\|\boldsymbol{u}_{\lambda^m}\right\|_2^2 + \frac{\rho}{2}\left\|\tilde{\boldsymbol{\mu}} - \overline{\boldsymbol{\mu}} + \boldsymbol{u}_\mu\right\|_2^2 - \frac{\rho}{2}\left\|\boldsymbol{u}_\mu\right\|_2^2. \end{aligned} \tag{24}$$

Through the linearization of the objective function and the complementary slackness constraints in (23d)–(23f), ADMM could be applied to the separable problems[2]. Similarly to the explanation in last section, the SOs update their local variables and duplicated variables by minimizing equation (24) considering the decision variables of the other SOs fixed. Each DSO $m \in \mathcal{N}^D$ solves problem (25), which can be performed in parallel. Then, the TSO solves problem (26). Finally, the scaled dual variables are updated according to (27)–(30).

$$\begin{aligned}&\tilde{z}_m^{k+1}, \tilde{\lambda}_m^{T,k+1}, \tilde{\lambda}_m^{D,k+1}, \tilde{\mu}_m^{k+1} = \underset{\tilde{z}_m, \tilde{\lambda}_m^T, \tilde{\lambda}_m^D, \tilde{\mu}_m}{\text{argmin}} \Big[\Phi_m(\boldsymbol{x}_m^u) + \tilde{\lambda}^T z_m^u \\ &+ \frac{\rho}{2}\left\|\tilde{z}_m - \overline{z}_m^k + u_{z,m}^k\right\|_2^2 + \frac{\rho}{2}\left\|\tilde{\lambda}_m^T - \overline{\lambda}_m^{T,k} + u_{\lambda^T,m}^k\right\|_2^2 \\ &+ \frac{\rho}{2}\left\|\tilde{\lambda}_m^D - \overline{\lambda}_m^{D,k} + u_{\lambda^D,m}^k\right\|_2^2 + \frac{\rho}{2}\left\|\tilde{\mu}_m - \overline{\mu}_m^k + u_{\mu,m}^k\right\|_2^2\Big] \\ &\quad \text{s.t. (22b), (22c), (22h)–(22j), (23e),} \\ &\quad\quad \text{(23c), (22k), (23f), (23g) w.r.t. DSOs' variables} \end{aligned} \tag{25}$$

$$\begin{aligned}&\overline{z}_m^{k+1}, \overline{\lambda}_m^{T,k+1}, \overline{\lambda}_m^{D,k+1}, \overline{\mu}_m^{k+1} = \underset{\overline{z}_m, \overline{\lambda}_m^T, \overline{\lambda}_m^D, \overline{\mu}_m}{\text{argmin}} \Big[\frac{\rho}{2}\left\|\tilde{z}_m^{k+1} - \overline{z}_m + u_{z,m}^k\right\|_2^2 \\ &+ \frac{\rho}{2}\left\|\tilde{\lambda}_m^{T,k+1} - \overline{\lambda}_m^T + u_{\lambda^T,m}^k\right\|_2^2 + \frac{\rho}{2}\left\|\tilde{\lambda}_m^{D,k+1} - \overline{\lambda}_m^D + u_{\lambda^D,m}^k\right\|_2^2 \\ &+ \frac{\rho}{2}\left\|\tilde{\mu}_m^{k+1} - \overline{\mu}_m + u_{\mu,m}^k\right\|_2^2\Big] \\ &\quad \text{s.t. (22e)–(22g), (23d),} \\ &\quad\quad \text{(23c), (22k), (23f), (23g) w.r.t. TSOs' variables} \end{aligned} \tag{26}$$

[2]Note that when dealing with nonconvex possibly nonsmooth optimization problems, ADMM convergence is guaranteed under sufficient non-restricitive conditions [18], [25].

TABLE II: Flexibility procurement costs (in €) for the different systems, and total costs, in the different market models.

System	Disjoint	Common	Fragmented	Multi-level
DN_18	36.970	51.967	21.973	21.973
DN_69	27.429	45.734	16.021	16.021
DN_141	12.496	45.098	-1.867	-1.867
TN	215.932	77.910	355.757	351.811
Total	292.828	220.709	391.884	387.938

$$u_{z,m}^{k+1} := u_{z,m}^k + \tilde{z}_m^{k+1} - \overline{z}_m^{k+1}, \ \forall m \in \mathcal{N}^D \tag{27}$$

$$u_{\lambda^T,m}^{k+1} := u_{\lambda^T,m}^k + \tilde{\lambda}_m^{T,k+1} - \overline{\lambda}_m^{T,k+1}, \ \forall m \in \mathcal{N}^D \tag{28}$$

$$u_{\lambda^D,m}^{k+1} := u_{\lambda^D,m}^k + \tilde{\lambda}_m^{D,k+1} - \overline{\lambda}_m^{D,k+1}, \ \forall m \in \mathcal{N}^D \tag{29}$$

$$u_{\mu,m}^{k+1} := u_{\mu,m}^k + \tilde{\mu}_m^{k+1} - \overline{\mu}_m^{k+1}, \ \forall m \in \mathcal{N}^D \tag{30}$$

V. Numerical Results and Analyses

The case study is constructed as an interconnected system consisting of the IEEE 14-bus (TN) transmission network connected to three distribution networks: the Matpower [26] 18-bus (DN_18), 69-bus (DN_69), and 141-bus (DN_141) systems. Base demand profiles are added to all buses to show anticipated imbalance in the transmission system. Moreover, to create anticipated congestion in the system, the capacity limits of the lines are adapted. Also, each distribution system is connected to the transmission system through one line, which is limited to a 1 MW capacity. Flexibility bids are randomly generated in the different nodes. For downward flexibility bids, the prices are drawn from a uniform distribution in the range [10,15] €/MWh, and for upward flexibility bids, they are drawn from the range [45,50] €/MWh. The bids' maximum quantities are generated according to the base demand or supply of the node from which they are connected. A minimum value for the quantity is imposed as 0.01 MW[3].

We first investigate the economic efficiency of the coordination schemes proposed in Section II, i.e. disjoint, common, fragmented, and multi-level markets, without pricing the interface flows. The resulting total cost for each market model applied to the case study is presented in Table II, which are calculated using equations (1a) plus (2a) for the disjoint; (3a) for the common; (4a) plus (5a) for the fragmented; and (6a) plus (7a) for the multi-level.

As demonstrated in Proposition 1 and Corollary III.2, the common market is the cheapest/most efficient market model. The fact that the disjoint market is more efficient than the fragmented and multi-level models would suggest that having uncoordinated markets solved in parallel could be better than coordinating transmission and distribution systems using the sequential coordination schemes. However, the result is rather explained by the "unpriced" imbalances generated by the distribution systems: in Table II, the cost for the DNs is lower in fragmented and multi-level markets than in the other two, demonstrating that those systems dispatched unnecessary downward flexibility locally for a profit (up to the interface line limit), specially DN_141, and increased the imbalance in the transmission system, leading to higher costs on the transmission level.

[3]The full data set including the network models and order books is available at https://doi.org/10.5281/zenodo.5734914

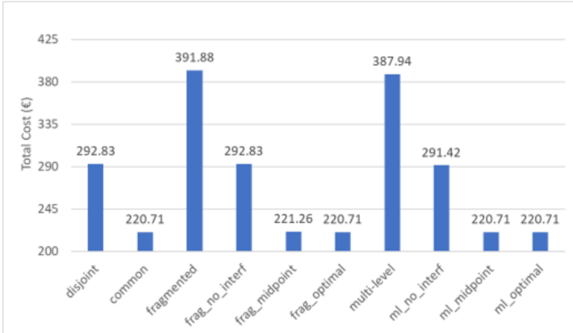

Fig. 2: Total flexibility procurement cost (in €) of the market models, including the three methods of interface flow pricing.

TABLE III: Total interface flow penalty – i.e., monetary amount – payed (positive) or received (negative) by the different systems, in the sequential markets when the midpoint or optimal pricing methods of the interface flow is applied (in €).

System	frag_midpoint	frag_optimal	ml_midpoint	ml_optimal
DN_18	30.005	46.071	30.005	46.071
DN_69	24.871	45.829	29.955	45.829
DN_141	12.413	45.975	30.005	45.975
TN	-67.289	-137.876	-89.965	-137.876

To address this aspect, the interface flows of the first stages of the fragmented and multi-level markets are priced following the three methods described in section III: 1) the distribution systems are not allowed to change the interface flow (frag_no_interf and ml_no_interf); 2) the price is defined by the midpoint between the most expensive downward flexibility and the least expensive upward flexibility (frag_midpoint and ml_midpoint); 3) the price is optimally defined by a virtual run of the common market in problem (11) (frag_optimal and ml_optimal). The sequential markets are run considering the different interface flow prices, for the same case study, and results are shown in Fig. 2. For ease of comparison, the results in Table II are repeated in the plot.

We note that the interface flow penalties are applied to each system operator's objective function in the fragmented and multi-level market, and they represent the monetary amount that the DSO would pay to (or receive from) the TSO, for every unit change in the interface power flow. When summing up the objective functions of all system operators, the penalties are thus cancelled (their summation is equal to zero). In Table III, we present the interface penalty payed (positive) or received (negative) by each system operator in each market model for each interface pricing method. As shown in the table, under the optimal method, results of the fragmented (frag_optimal) and multi-level (ml_optimal) are equal, following the mathematical derivations. We do not show results for Method 1, as in this method modification to the interface flow is not permitted, resulting in no interface penalty to be payed/received by any SO.

As shown in the plot, the total flexibility procurement cost of the sequential markets is significantly reduced when their first stages are prevented from changing the interface flows: from €391.88 to €292.83 in the case of the fragmented, and from €387.94 to €291.42 in the case of the multi-level. When distribution systems cannot modify the interface flow, the congestion management in the distribution systems is performed while keeping those systems balanced, preventing them from purchasing downward flexibility unless there is a local grid need for it. However, this method is a highly restrictive solution in which the benefit of coordination is significantly reduced. For example, the solutions of frag_no_interf and ml_no_interf are close to the disjoint solution (for the case of fragmented, the solution is equal to the disjoint, as discussed in Section III).

In the case of method 2, results are close to (for the fragmented market) or equal to (for the multi-level) the common market. Moreover, as shown in Table III, the DSOs ought to pay the TSO up to €30.01 to compensate for the further imbalances generated in the transmission system due to the result of their markets in Stage 1. Therefore, our numerical results show that this practical and easy to implement approach is able to prevent unnecessary dispatch of resources at distribution systems, through adequately pricing interface flows, while guaranteeing TSO-DSO coordination and the sharing of flexibility resources. However, as the interface flow price is calculated according to the submitted bids to the market, market monitoring is essential to ensure no strategic behavior is preformed by the FSPs to manipulate the interface flow prices.

As method 3 prices the interface flow optimally, the total cost of frag_optimal and ml_optimal are equal to the result of the common market, following the results in Proposition 2 and Corollary III.1. In addition, as shown in Table III, the DSOs ought to pay even more to the transmission system operator (up to €46.07) to compensate for the further imbalances caused in the transmission system. Although the result is promising, with optimality guaranteed even in the coordination scheme where the SOs can independently run their markets without sharing information, i.e. the fragmented, this method implies a virtual run of the common market to define the optimal price, which in practice may not be possible. This drawback is addressed through the proposed decentralized techniques, which are able to define the optimal interface prices while solving the problems with limited information sharing.

In this respect, we apply the proposed ADMM to solve the common market, as described in Section IV-A, and the multi-level market, as described in Section IV-B, with limited information sharing. In Table IV, we show the results of the final iteration of the ADMM applied to the common market, in terms of interface flows and prices. As can be seen, the duplicated variables of each DSO problem and of the TSO ($\tilde{z}_m$ and $\overline{z}_m$) converge to the same value (-1.0 MW for all transmission-distribution connections), which is equal to the result of the common market. Moreover, the dual variables $\lambda_m = \rho \times u_m$ also converge to an interface flow price close to the optimal one: the maximal difference is 0.09%.

In Fig. 3, convergence results of the ADMM are shown. The dashed lines represent the result of the objective function of the DSOs' problem (19) (DN_18, DN_69, DN_141), and

TABLE IV: Final interface flows and prices of ADMM compared to optimal results from the common market.

System	Interface Flow (MW)			Interface Price (€/MWh)	
	$\tilde{z}_m$	$\overline{z}_m$	Optimal	$\rho \times u_m$	Optimal
DN_18	-1.00	-1.00	-1.00	46.03	46.07
DN_69	-1.00	-1.00	-1.00	45.79	45.83
DN_141	-1.00	-1.00	-1.00	45.96	45.98

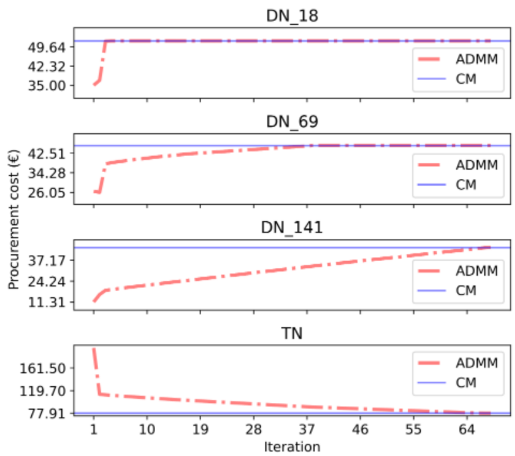

Fig. 3: Convergence of the procurement cost for each SO when applying ADMM to solve the common market.

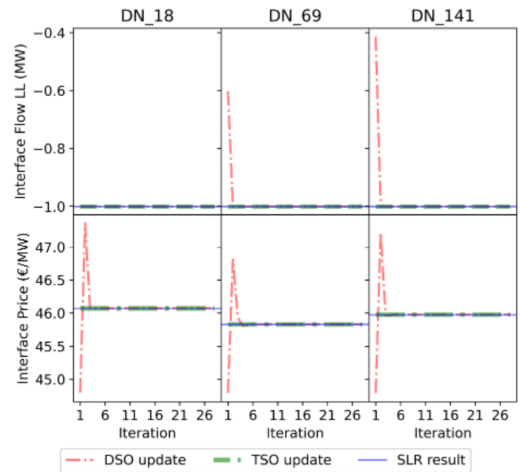

Fig. 4: Convergence of interface flows and prices per interconnection when applying ADMM to solve the multi-level market.

the objective function of the TSO's problem (20) (TN). The procurement costs of flexibility of all system operators converge to the common market result in Table II (also indicated by the horizontal line in the plots). The convergence required 68 iterations, where only the local values of the interface flows and prices are exchanged between the SOs.

In the case of the multi-level market decentralization, the proposed bi-level program reduced to a single level (SLR) returns the same result as the common market. Even though the proposition uses the price imposed by the TSO nodes to penalize DSO's utilization of interface flow ($\boldsymbol{\lambda}^T$ instead of the optimal $\boldsymbol{\lambda}$ given in Proposition 2), the bi-level model reduced (SLR) and decomposed (ADMM) reach the same optimal solution as the common market for the case study.

In Figure 4, we show the ADMM convergence in terms of lower level interface flows ($\tilde{z}^l$ from the side of DSOs and $\overline{z}^l$ from the side of the TSO), and of the interface prices ($\tilde{\boldsymbol{\lambda}}^T$ and $\overline{\boldsymbol{\lambda}}^T$). As can be seen, after 5 iterations, the values converge to the solution of the SLR, which are equal to the solution of the common market. The other 23 iterations represent the fine-tuning of the other duplicated variables to reach complete convergence. This quick convergence has a great practical advantage as this first limits the clearing time (enabling the practical implementation in a time-restrictive market clearing environment) while reducing the communication needs between the different SOs.

It is worth noticing that no network data (e.g. systems' topology and parameters) need to be exchanged between SOs in both decomposition methods. Moreover, the decomposition of the proposed bi-level model for the multi-level market is able to solve this coordination scheme optimally, without the need of a common market run to define the interface prices.

As a final remark, the presented numerical results are specific to our numerical case analysis. However, they serve to highlight and corroborate the mathematical derivations and results presented in the paper: 1) the common market model is the most efficient model, as developed in Proposition 1 and Corollary III.2; 2) if the interface flows are optimally priced, the solutions of the fragmented and multi-level markets are as efficient as the solution of the common market, as proven in Proposition 2 and Corollary III.1; and 3) the common and multi-level markets can be optimally solved using the distributive algorithm (i.e., ADMM) while returning the optimal interface flow prices, as shown in Section IV. Those analytical derivations and results are case-agnostic. Hence, any other numerical case would return the same conclusions aforementioned, which means that those results are generalizable, even though the numerical values obtained are specific to the case study.

VI. Conclusion

In this paper, we have proposed five TSO-DSO coordination market models (disjoint transmission, disjoint distribution, common, fragmented, and multi-level) for the procurement of flexibility from different voltage levels, while incorporating all the needed grid constraints. After proving that the common market is the most economically efficient, we have proposed three methods to accommodate the interface flows between SOs in sequential markets (fragmented and multi-level) to avoid exploitation of market advantages through unpriced imbalances. We have then shown that the optimally-defined interface flow prices render the multi-level and fragmented markets as efficient as the common market. In addition, decomposition models based on ADMM were proposed to solve the common and multi-level markets without requiring any network information sharing between the SOs. The models and analytical results were further tested using an elaborate case study, which has corroborated the derived results and provided direct insights for practical implementations. The work has generated key recommendations and insights for efficient TSO-DSO flexibility markets in a European context as part of the H2020 CoordiNet project.

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