



PROOF OF SOME TRIGONOMETRIC IDENTITIES

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KEYWORDS

identity, function,
trigonometric function,
equation, system of equations

ABSTRACT

As is known, it is an important factor in the development of the independent thinking of the student, the ability to express his point of view, compare and find a common opinion. Mathematical proof plays an important role in the development of such mathematical abilities of students. One of them is trigonometric identities. Below we come up with some of the methods of proving some trigonometric identities.

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BA'ZI TRIGONOMETRIK AYNİYATLARNI ISBOTLASH

KALIT SO'ZLAR:

ayniyat, funksiya,
trigonometrik funksiya,
tenglama, tenglamalar
sistemi

ANNOTATSIYA

Ma'lumki o'quvchining mustaqil fikirlashi, o'z nuqtayi-nazarini ifoda etishi, taqqoslash va umumiy fikrni topish qobiliyatini rivojlantirish muhim omil hisoblanadi. O'quvchilarning bunday matematik qobiliyatlarini rivojlantirishda matematik isbotlashlar muhim rol o'ynaydi. Shulardan biri trigonometrik ayniyatlardir. Quyida biz ba'zi trigonometrik ayniyatlarning isbotlash usullaridan ba'zilarini keltirib o'tamiz.

Zamonaviy ta'limning eng muhim vazifalaridan biri shaxs tarbiyasida yangicha yondashuvni tashkil etishdir. Bu jarayonlarda o'quvchi va talabalarning ijodiy faollik, mustaqil fikrlash orqali bilimlarini turli masalalarni yechishga qo'llay olish ko'nikmalari rivojlanadi. Xususan, matematikaga iqtidorli o'quvchi va talabalarda matematik qobiliyatlarni va izlanuvchanlikni tarbiyalash muhimdir [1-18]. Bunda turli mazmundagi qiyinlik darajasi yuqoriroq matematik masalalarni yechishda mavjud bilimlardan foydalana bilish ulardagi ijodkorlikni rivojlantiradi [19-34]. Ushbu maqolada trigonometrik ayniyatlar bilan bog'liq ba'zi masalalar o'rganiladi. Bu kabi masalalarni yechishda odatda quyidagi usullardan foydalaniladi:

- a) berilgan ayniyatning chap qismidagi ifodada aynan shakl almashtirish orqali o'ng qismini keltirib chiqarish;
- b) berilgan ayniyatning chap va o'ng qismlaridagi ifodalarning ayirmasi aynan nolga tengligini ko'rsatish
- c) berilgan ayniyatning chap va o'ng qismlarida aynan shakl almashtirishlar orqali ularning tengligini ko'rsatish

Quyida bu usullarning qo'llanilishiga oid ba'zi misollarni ko'rsatib o'tamiz.

1-misol. Agar $\alpha > 0$, $\beta > 0$, $\gamma > 0$ bo'lib, $\alpha + \beta + \gamma = \frac{\pi}{2}$ bo'lsa,

$tg\alpha \cdot tg\beta + tg\beta \cdot tg\gamma + tg\gamma \cdot tg\alpha = 1$ ayniyatni isbotlang.

Isbot. Ayniyatning chap qismidagi ifodadan o'ng qismidagi ifodani keltirib chiqaramiz:

$$\begin{aligned} tg\alpha \cdot tg\beta + tg\beta \cdot tg\gamma + tg\gamma \cdot tg\alpha &= tg\alpha \cdot tg\beta + tg\gamma(tg\alpha + tg\beta) = \\ &= tg\alpha \cdot tg\beta + tg\left(\frac{\pi}{2} - (\alpha + \beta)\right)(tg\alpha + tg\beta) = tg\alpha \cdot tg\beta + ctg(\alpha + \beta)(tg\alpha + tg\beta) = \\ &= tg\alpha \cdot tg\beta + \frac{1 - tg\alpha \cdot tg\beta}{tg\alpha + tg\beta}(tg\alpha + tg\beta) = 1. \end{aligned}$$

2-misol.

Ayniyatni

isbotlang:

$$\frac{1}{2} \left(\frac{1 - \cos 2\alpha}{\sec^2 \alpha - 1} + \frac{1 + \cos 2\alpha}{\operatorname{cosec}^2 \alpha - 1} \right) + ctg 2\alpha + \cos 2\alpha + \sin 2\alpha = \frac{2\sqrt{2} \sin\left(\frac{\pi}{4} + 2\alpha\right) \cos^2\left(\frac{\pi}{4} - \alpha\right)}{\sin 2\alpha}. \quad (1)$$

Isbot. Ayniyatni isbotlash uchun (1) ning chap va o'ng qismlari ayirmasi nolga tengligini ko'rsatamiz

$$\begin{aligned}
 & \frac{1}{2} \left(\frac{1 - \cos 2\alpha}{\sec^2 \alpha - 1} + \frac{1 + \cos 2\alpha}{\operatorname{cosec}^2 \alpha - 1} \right) + \operatorname{ctg} 2\alpha + \cos 2\alpha + \sin 2\alpha - \frac{2\sqrt{2} \sin\left(\frac{\pi}{4} + 2\alpha\right) \cos^2\left(\frac{\pi}{4} - \alpha\right)}{\sin 2\alpha} = \\
 & = \frac{1}{2} \left[\frac{2 \sin^2 \alpha}{(1 + \operatorname{tg}^2 \alpha) - 1} + \frac{2 \cos^2 \alpha}{(1 + \operatorname{ctg}^2 \alpha) - 1} \right] + \operatorname{ctg} 2\alpha + \cos 2\alpha + \sin 2\alpha - \frac{\sqrt{2} \sin\left(\frac{\pi}{4} + 2\alpha\right) 2 \cos^2\left(\frac{\pi}{4} - \alpha\right)}{\sin 2\alpha} = \\
 & = \left(\frac{\sin^2 \alpha}{\operatorname{tg}^2 \alpha} + \frac{\cos^2 \alpha}{\operatorname{ctg}^2 \alpha} \right) + \operatorname{ctg} 2\alpha + \cos 2\alpha + \sin 2\alpha - \frac{\sqrt{2} \left(\sin \frac{\pi}{4} \cos 2\alpha + \cos \frac{\pi}{4} \sin 2\alpha \right) \left[1 + \cos\left(\frac{\pi}{2} - 2\alpha\right) \right]}{\sin 2\alpha} = \\
 & = (\cos^2 \alpha + \sin^2 \alpha) + \operatorname{ctg} 2\alpha + \cos 2\alpha + \sin 2\alpha - \frac{\sqrt{2} \left(\frac{\sqrt{2}}{2} \cos 2\alpha + \frac{\sqrt{2}}{2} \sin 2\alpha \right) (1 + \sin 2\alpha)}{\sin 2\alpha} = \\
 & = 1 + \frac{\cos 2\alpha}{\sin 2\alpha} + \cos 2\alpha + \sin 2\alpha - \frac{(\cos 2\alpha + \sin 2\alpha)(1 + \sin 2\alpha)}{\sin 2\alpha} = \\
 & = \frac{\cos 2\alpha + \sin 2\alpha}{\sin 2\alpha} + \cos 2\alpha + \sin 2\alpha - \frac{(\cos 2\alpha + \sin 2\alpha)(1 + \sin 2\alpha)}{\sin 2\alpha} = \\
 & = \frac{\cos 2\alpha + \sin 2\alpha + \sin 2\alpha(\cos 2\alpha + \sin 2\alpha)}{\sin 2\alpha} - \frac{(\cos 2\alpha + \sin 2\alpha)(1 + \sin 2\alpha)}{\sin 2\alpha} = \\
 & = \frac{(\cos 2\alpha + \sin 2\alpha)(1 + \sin 2\alpha)}{\sin 2\alpha} - \frac{(\cos 2\alpha + \sin 2\alpha)(1 + \sin 2\alpha)}{\sin 2\alpha} = 0
 \end{aligned}$$

Ayniyat isbotlandi.

3-misol. Agar $\operatorname{tg} \gamma = \frac{\sin \alpha \sin \beta}{\cos \alpha + \cos \beta}$ (2) bo'lib, bunda

$$\begin{cases} 2\pi k < \gamma < \frac{\pi}{2} + 2\pi k, \\ 2\pi m < \gamma < \frac{\pi}{2} + 2\pi m, \\ 2\pi n < \gamma < \frac{\pi}{2} + 2\pi n, \end{cases}$$

(3) bo'lsa, $\operatorname{tg} \frac{\gamma}{2} = \operatorname{tg} \frac{\alpha}{2} \operatorname{tg} \frac{\beta}{2}$ (4) bo'lishini isbotlang.

Isbot. (2) tenglikning ikkala qismini kvadratga ko'tarib, ikkala qismiga ham 1 ni qo'shamiz

$$1 + \operatorname{tg}^2 \gamma = 1 + \frac{\sin^2 \alpha \sin^2 \beta}{(\cos \alpha + \cos \beta)^2}$$

va bundan

$$\frac{1}{\cos^2 \gamma} = \frac{\cos^2 \alpha + 2 \cos \alpha \cos \beta + \cos^2 \beta + \sin^2 \alpha \sin^2 \beta}{(\cos \alpha + \cos \beta)^2},$$

$$\cos^2 \gamma = \frac{(\cos \alpha + \cos \beta)^2}{\cos^2 \alpha + 2 \cos \alpha \cos \beta + \cos^2 \beta + (1 - \cos^2 \alpha)(1 - \cos^2 \beta)},$$

$$\cos^2 \gamma = \frac{(\cos \alpha + \cos \beta)^2}{(1 + \cos \alpha \cos \beta)^2},$$

yoki

$$|\cos \gamma| = \frac{|\cos \alpha + \cos \beta|}{|1 + \cos \alpha \cos \beta|},$$

(3) shartlardan $\cos \gamma > 0$, $\cos \alpha > 0$, $\cos \beta > 0$, ekanligi va bundan

$$\cos \gamma = \frac{\cos \alpha + \cos \beta}{1 + \cos \alpha \cos \beta}, \quad (4)$$

bo'lishi kelib chiqadi. Proporsiyani xossasiga ko'ra

$$\frac{1 - \cos \gamma}{1 + \cos \gamma} = \frac{1 + \cos \alpha \cos \beta - (\cos \alpha + \cos \beta)}{1 + \cos \alpha \cos \beta + (\cos \alpha + \cos \beta)},$$

$$\frac{1 - \cos \gamma}{1 + \cos \gamma} = \frac{(1 - \cos \beta) - \cos \alpha(1 - \cos \beta)}{(1 + \cos \beta) + \cos \alpha(1 + \cos \beta)},$$

yoki

$$\frac{1 - \cos \gamma}{1 + \cos \gamma} = \frac{(1 - \cos \beta)(1 - \cos \alpha)}{(1 + \cos \beta)(1 + \cos \alpha)}, \quad (5)$$

$1 - \cos x = 2 \sin^2 \frac{x}{2}$, $1 + \cos x = 2 \cos^2 \frac{x}{2}$, formulalarga ko'ra (5) tenglikni quyidagicha

yo'zish mumkin: $tg^2 \frac{\gamma}{2} = tg^2 \frac{\beta}{2} tg^2 \frac{\alpha}{2}$ bundan $\left| tg \frac{\gamma}{2} \right| = \left| tg \frac{\beta}{2} \right| \left| tg \frac{\alpha}{2} \right|$

bo'lishi kelib chiqadi. (3) shartlarga ko'ra $tg \frac{\gamma}{2} > 0$, $tg \frac{\beta}{2} > 0$, $tg \frac{\alpha}{2} > 0$,

Demak,

$$tg \frac{\gamma}{2} = tg \frac{\beta}{2} tg \frac{\alpha}{2} \quad \text{tenglikka ega bo'lamiz.}$$

4-misol. Agar $\begin{cases} \cos 2\alpha + \cos \alpha = \frac{m}{a} \\ \sin 2\alpha + \sin \alpha = \frac{n}{b} \end{cases}$ (6) bo'lsa,

$$\frac{2m}{a} = \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right) \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} - 3 \right). \quad (7) \text{ bo'lishini isbotlang.}$$

Isbot. (6) sistemani quyidagicha yozamiz:

$$\begin{cases} 2 \cos \frac{3\alpha}{2} \cos \frac{\alpha}{2} = \frac{m}{a} \\ 2 \sin \frac{3\alpha}{2} \cos \frac{\alpha}{2} = \frac{n}{b} \end{cases} \quad (8)$$

Oxirgi sistemadagi tengliklarni kvadratga ko'tarib qo'shamiz

$$4 \cos^2 \frac{\alpha}{2} = \frac{m^2}{a^2} + \frac{n^2}{b^2} \quad (9)$$

(8) sistemaning birinchi tenglamasida $\cos \frac{3\alpha}{2}$ uchun kosinusning uchlangan

argument formulasini qo'llab

$$2 \left(4 \cos^3 \frac{\alpha}{2} - 3 \cos \frac{\alpha}{2} \right) \cos \frac{\alpha}{2} = \frac{m}{a},$$

yoki

$$2 \cos^2 \frac{\alpha}{2} \left(4 \cos^2 \frac{\alpha}{2} - 3 \right) = \frac{m}{a},$$

Bu ifodani (9) dan foydalanib quyidagicha yozish mumkin:

$$\frac{1}{2} \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right) \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} - 3 \right) = \frac{m}{a}$$

yoki

$$\left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right) \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} - 3 \right) = \frac{2m}{a}$$

isbotlanishi talab etilgan tenglikka kelamiz.

5-misol. Ayniyatni isbotlang: $8ctg24\alpha + 4tg12\alpha + 2tg6\alpha + tg3\alpha = ctg3\alpha$

Isbot.

$$\begin{aligned} 8ctg24\alpha + 4tg12\alpha + 2tg6\alpha + tg3\alpha &= \frac{8 \cos 24\alpha}{\sin 24\alpha} + \frac{4 \sin 12\alpha}{\cos 12\alpha} + \frac{2 \sin 6\alpha}{\cos 6\alpha} + \frac{\sin 3\alpha}{\cos 3\alpha} = \\ &= \frac{8 \cos 24\alpha}{8 \sin 3\alpha \cos 3\alpha \cos 6\alpha \cos 12\alpha} + \frac{4 \sin 12\alpha}{\cos 12\alpha} + \frac{2 \sin 6\alpha}{\cos 6\alpha} + \frac{\sin 3\alpha}{\cos 3\alpha} = \\ &= \frac{\cos 24\alpha + 4 \sin 12\alpha \sin 3\alpha \cos 3\alpha \cos 6\alpha + 2 \sin 6\alpha \sin 3\alpha \cos 3\alpha \cos 12\alpha + \sin^2 3\alpha \cos 6\alpha \cos 12\alpha}{\sin 3\alpha \cos 3\alpha \cos 6\alpha \cos 12\alpha} = \\ &= \frac{8 \left(\cos 24\alpha + \sin^2 12\alpha + \sin^2 6\alpha \cos 12\alpha + \sin^2 3\alpha \cos 6\alpha \cos 12\alpha \right)}{\sin 24\alpha} = \\ &= \frac{8 \left(\cos 24\alpha + \frac{1}{2} - \frac{1}{2} \cos 24\alpha + \cos 12\alpha \left(\frac{1}{2} - \frac{1}{2} \cos 12\alpha \right) + \left(\frac{1}{2} - \frac{1}{2} \cos 6\alpha \right) \cos 6\alpha \cos 12\alpha \right)}{\sin 24\alpha} = \\ &= \frac{8 \left(\frac{1}{2} + \frac{1}{2} \cos 24\alpha + \frac{1}{2} \cos 12\alpha - \frac{1}{2} \cos^2 12\alpha + \left(\frac{1}{2} \cos 6\alpha - \frac{1}{2} \cos^2 6\alpha \right) \cos 12\alpha \right)}{\sin 24\alpha} = \\ &= \frac{8 \left(\frac{1}{2} + \frac{1}{2} \cos 24\alpha + \frac{1}{2} \cos 12\alpha - \frac{1}{4} - \frac{1}{4} \cos 24\alpha + \frac{1}{2} \cos 6\alpha \cos 12\alpha - \frac{1}{4} (1 - \cos 12\alpha) \cos 12\alpha \right)}{\sin 24\alpha} = \end{aligned}$$

$$\begin{aligned}
 &= \frac{8\left(\frac{1}{8} + \frac{1}{8}\cos 24\alpha + \frac{1}{4}\cos 12\alpha + \frac{1}{4}\cos 18\alpha + \frac{1}{4}\cos 6\alpha\right)}{\sin 24\alpha} = \frac{1 + \cos 24\alpha + 2\cos 12\alpha + 2\cos 18\alpha + 2\cos 6\alpha}{\sin 24\alpha} = \\
 &= \frac{1 + \cos 24\alpha + \frac{2\sin 3\alpha \cos 12\alpha + 2\sin 3\alpha \cos 18\alpha + 2\sin 3\alpha \cos 6\alpha}{\sin 3\alpha}}{\sin 24\alpha} = \\
 &= \frac{1 + \cos 24\alpha + \frac{\sin 15\alpha - \sin 9\alpha + \sin 21\alpha - \sin 15\alpha + \sin 9\alpha - \sin 3\alpha}{\sin 3\alpha}}{\sin 24\alpha} = \frac{1 + \cos 24\alpha + \frac{\sin 21\alpha - \sin 3\alpha}{\sin 3\alpha}}{\sin 24\alpha} = \\
 &= \frac{\sin 3\alpha \cos 24\alpha + \sin 21\alpha}{\sin 3\alpha \sin 24\alpha} = \frac{\frac{1}{2}\sin 27\alpha - \frac{1}{2}\sin 21\alpha + \sin 21\alpha}{\sin 3\alpha \sin 24\alpha} = \frac{\frac{1}{2}\sin 27\alpha + \frac{1}{2}\sin 21\alpha}{\sin 3\alpha \sin 24\alpha} = \\
 &= \frac{\frac{1}{2}2\sin 24\alpha \cos 3\alpha}{\sin 3\alpha \sin 24\alpha} = \operatorname{ctg} 3\alpha
 \end{aligned}$$

Xulosa o'rnida shuni ta'kidlash muhimki trigonometrik ayniyatlarni isbotlashda yuqoridagi usullarni qo'llash o'quvchida ijodkorlokni, bilimlarning umumlashtirilishini ya'ni oldingi bilimlarni, formulalarni o'z o'rnida qo'llay olish kabi ko'nikmalarni talab qiladi[1-34].

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