



CHIZIQLI ALGEBRAIK TENGLAMALAR SISTEMASINI YECHISHNING "ITERASIYA" USULI

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ABSTRACT

Ushbu maqolada bugungi kundagi globallashuv jarayonlarida yoshlarimiz internet tarmoqlaridan to'g'ri foydalanish, information xurujlardan qanday ximoyalanish mumkinligi ularning salbiy oqibatlari, axborot xurujlarining turlari ularga qarshi kurashish usullari keltirilgan.

Quyidagi chiziqli algebraik tenglamalar sistemasini qaraymiz:

[illegible]

Agar barcha bu sistemada barcha a_{ii} lar noldan farqli bo'lsa, u holda uni quyidagi keltirilgan ko'rinishga o'tamiz:

$$\begin{cases} x_1 = \alpha_{11}x_1 + \alpha_{12}x_2 + \dots + \alpha_{1n}x_n + \beta_1 \\ x_2 = \alpha_{21}x_1 + \alpha_{22}x_2 + \dots + \alpha_{2n}x_n + \beta_2 \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ x_n = \alpha_{n1}x_1 + \alpha_{n2}x_2 + \dots + \alpha_{nn}x_n + \beta_n \end{cases} \quad (2)$$

Bu yerda $\beta_i = \frac{b_i}{a_{ii}}$ ($i = 1, 2, \dots, n$),

$$\alpha_{ij} = \begin{cases} 0, & i = j; \\ -\frac{a_{ij}}{a_{ii}}, & i \neq j. \end{cases}$$

Quyidagi belgilashlarni kiritamiz:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad \boldsymbol{\alpha} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \dots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \dots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{n1} & \alpha_{n2} & \dots & \alpha_{nn} \end{bmatrix}, \quad \boldsymbol{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix}$$

va (2) sistemani quyidagi ko`rinishda
yozib olamiz:

$$x = \alpha x + \beta \text{ .} \quad (3)$$

Iterasiyaning ketma-ket yaqinlashuvlarini quyidagi tarzda olamiz. Nolinchi yaqinlashuv $x^{(0)}$ sivatida β vektorni tanlaymiz. Uni (3) tenglikning o'ng tomoniga qo'yib, $x^{(1)}$ ni hosil qilamiz va hakoza bu jarayonni davom ettirib yaqinlashuvlarning quyidagi vektor ketma-ketligiga ega bo'lamiz:

$$\begin{aligned} \mathbf{x}^{(1)} &= \alpha \mathbf{x}^{(0)} + \beta - \text{birinchi yaqinlashuv,} \\ \mathbf{x}^{(2)} &= \alpha \mathbf{x}^{(1)} + \beta - \text{ikkinchi} \\ \text{yaqinlashuv,} \end{aligned} \quad (4)$$

$$x^{(k+1)} = \alpha x^{(k)} + \beta \quad - \quad (k+1) \text{ chi}$$

yaqinlashuv,

Agar $k \rightarrow \infty$ da $\mathbf{x}^{(k)}$ vektorlar ketma-ketligining $\boldsymbol{\theta}$ limiti mavjud bo'lsa, u holda $\mathbf{x}^{(k+1)} = \boldsymbol{\alpha} \mathbf{x}^{(k)} + \boldsymbol{\beta}$ tenglikning har ikki tomonida $k \rightarrow \infty$ da limitga o'tib,

$$\theta = \alpha\theta + \beta$$

munosabatni hosil qilamiz, bundan esa, θ (3) tenglamani echimi ekanligi kelib chiqadi.



Iterasiyaning tenglama yechimiga yaqinlashishining yetarli sharti quyidagi teoremda keltirilgan:

Teorema. Agar α matrisaning ixtiyoriy bir normasi birdan kichik: $\|\alpha\| < 1$ bo'lsa, u holda (4) iterasiya yaqinlashuvchi bo'lib, uning limiti (3) tenglamaning echimi bo'ladi.

Eslatma. $n \times m$ o'lchamli matrisalar chiziqli fazosida ixtiyoriy

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{bmatrix}$$

matrisa uchun quyidagi uch hil norma aniqlangan:

$$\|A\|_m = \max_{1 \leq i \leq n} \left\{ \sum_{j=1}^m |a_{ij}| \right\} \quad (m\text{-norma});$$

$$\|A\|_l = \max_{1 \leq j \leq m} \left\{ \sum_{i=1}^n |a_{ij}| \right\} \quad (l\text{-norma});$$

$$\|A\|_k = \sqrt{\sum_{i=1}^n \sum_{j=1}^m a_{ij}^2} \quad (k\text{-norma}).$$

(3) tenglamani bunday iterasion usul bilan taqribiy yechishda k - qadamdagi hatolik quyidagi tengsizlik bilan baholanadi

$$\|\theta - x^{(k)}\| \leq \frac{\|\alpha\|^{k+1}}{1 - \|\alpha\|} \|\beta\|. \quad (5)$$

Bu tengsizlikdan tenglama yechimini berilgan ε aniqlikda topish uchun zaruriy bo'lgan iterasiyalar soni k ni baholash mumkin.

$x^{(k)}$ yechimning haqiqiy yechim θ dan farqi berilgan ε dan ortmasligi uchun quyidagi:

$$\|\theta - x^{(k)}\| \leq \frac{\|\alpha\|}{1 - \|\alpha\|} \|x^{(k)} - x^{(k-1)}\| \leq$$

$$\frac{\|\alpha\|^{k+1}}{1 - \|\alpha\|} \|\beta\| \leq \varepsilon \quad (6)$$

tengsizlik o'rinni bo'lishi kerak.

(6) tengsizlikdan haqiqiy yechim θ ning ε aniqlikdagi taqribiy qiymati

sifatida olinadiga $x^{(k)}$ ni hisoblash uchun qulay tanlash sharti kelib chiqadi:

$$\|x^{(k)} - x^{(k-1)}\| \leq \frac{1 - \|\alpha\|}{\|\alpha\|} \varepsilon \quad (7).$$

Misol. Quyidagi taenglamalar sistemasini iterasiya usuli bilan $\varepsilon = 10^{-2}$ aniqlikda yeching.

$$\begin{cases} 5x_1 - x_2 + 2x_3 = 8 \\ x_1 + 4x_2 - x_3 = -4 \\ x_1 + x_2 + 4x_3 = 4 \end{cases}$$

Bu sistemani (2) ko'rinishga keltiramiz:

$$\begin{cases} x_1 = 0 \cdot x_1 + 0,2x_2 - 0,4x_3 + 1,6 \\ x_2 = -0,25x_1 + 0 \cdot x_2 + 0,25x_3 - 1 \\ x_3 = -0,25x_1 - 0,25x_2 + 0 \cdot x_3 + 1 \end{cases}$$

Iterasiya ketma-ketligini yozamiz:

$$\begin{cases} x_1^{(k+1)} = 0 \cdot x_1^{(k)} + 0,2x_2^{(k)} - 0,4x_3^{(k)} + 1,6 \\ x_2^{(k+1)} = -0,25x_1^{(k)} + 0 \cdot x_2^{(k)} + 0,25x_3^{(k)} - 1 \\ x_3^{(k+1)} = -0,25x_1^{(k)} - 0,25x_2^{(k)} + 0 \cdot x_3^{(k)} + 1 \end{cases} \quad (8)$$

Sistemaning α matrisasi

$$\alpha = \begin{bmatrix} 0 & 0,2 & -0,4 \\ -0,25 & 0 & 0,25 \\ -0,25 & -0,25 & 0 \end{bmatrix}$$

uchun iterasiya yaqinlashishining etarli sharti bajariladi, chunki:

$$\|\alpha\|_m = \max_{1 \leq i \leq 3} \left\{ \sum_{j=1}^3 |\alpha_{ij}| \right\} = \max\{0,6; 0,5; 0,5\} = 0,6 < 1.$$

Boshlang'ich yaqinlashish sifatida ozod hadlar ustunini olamiz:

$$x^{(0)} = \begin{bmatrix} 1,6 \\ -1 \\ 1 \end{bmatrix}.$$

Kerakli aniqlikka erishish uchun zarur bo'lgan iterasiyalar sonini

$$\frac{\|\alpha\|^{k+1}}{1 - \|\alpha\|} \|\beta\| \leq \varepsilon$$

tengsizlikdan aniqlaymiz:

$$k \geq \frac{-2 + \lg 0,25}{\lg 0,6} - 1 \approx 11.$$



Bu yerda $\|\alpha\|_m = 0,6$; $\|\beta\|_m = \|x^{(0)}\|_m = 1,6$ ekanligi hisobga olindi.

Endi ketma-ket uchinchi yaqinlashishgacha taqribiy yechimlarni hisoblaymiz:

$$x^{(1)} = \begin{bmatrix} 0 & 0,2 & -0,4 \\ -0,25 & 0 & 0,25 \\ -0,25 & -0,25 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1,6 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1,6 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{cases} x_1^{(1)} = 0,2(-1) - 0,4 + 1,6 = 1. \\ x_2^{(1)} = -0,25 \cdot 1,6 + 0,25 - 1 = -1,15 \\ x_3^{(1)} = -0,25 \cdot 1,6 - 0,25 \cdot (-1) + 1 = 0,85 \end{cases}$$

Demak, $x^{(1)} = \begin{bmatrix} 1 \\ -1,15 \\ 0,85 \end{bmatrix}$ bo'ladi.

Bundan foydalanib ikkinchi yaqinlashishni

topamiz:

$$x^{(2)} = \begin{bmatrix} 0 & 0,2 & -0,4 \\ -0,25 & 0 & 0,25 \\ -0,25 & -0,25 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1,15 \\ 0,85 \end{bmatrix} + \begin{bmatrix} 1,6 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{cases} x_1^{(2)} = 0,2(-1,15) - 0,4 \cdot 0,85 + 1,6 = 1,03 \\ x_2^{(2)} = -0,25 \cdot 1 + 0,25 \cdot 0,85 - 1 = -1,0375 \\ x_3^{(2)} = -0,25 \cdot 1 - 0,25 \cdot (-1,15) + 1 = 1,0375 \end{cases}$$

Demak, $x^{(2)} = \begin{bmatrix} 1,03 \\ -1,0375 \\ 1,0375 \end{bmatrix}$ bo'ladi.

Bundan foydalanib uchinchi yaqinlashishni topamiz:

$$x^{(3)} = \begin{bmatrix} 0 & 0,2 & -0,4 \\ -0,25 & 0 & 0,25 \\ -0,25 & -0,25 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1,03 \\ -1,0375 \\ 1,0375 \end{bmatrix} + \begin{bmatrix} 1,6 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{cases} x_1^{(3)} = 0,2(-1,0375) - 0,4 \cdot 1,0375 + 1,6 = 0,9775 \\ x_2^{(3)} = -0,25 \cdot 1,03 + 0,25 \cdot 1,0375 - 1 = -0,9981 \\ x_3^{(3)} = -0,25 \cdot 1,03 - 0,25 \cdot (-1,0375) + 1 = 1,0018 \end{cases}$$

Demak, iterasiyaning uchinchi yaqinlashishidan so'ng tenglamaning taqribiy yechimi: $x_1 = 0,9775$; $x_2 = -0,9981$; $x_3 = 1,0018$ bo'ladi. (izoh: tenglamaning haqiqiy yechimi: $x_1 = 1$; $x_2 = -1$; $x_3 = 1$).