Digital Option Analytics

This article presents a pricing model for skewed European interest rate digital option. The traditional pricing model is under the Black-Scholes framework. The new skew-adjusted model replicates a digital option by a portfolio of vanilla call options, and/or zero-coupon bonds and/or floating rate notes (FRNs). The new model provides a better approach to pricing skewed European interest rate digital options.

There are four types of digital options and the responding matured payoffs are as follows:

$$\text{Digital Call (Cash or Nothing) Payoff} = I_{\{f_T \geq K\}};$$
$$\text{Digital Put (Cash or Nothing) Payoff} = I_{\{f_T \leq K\}};$$
$$\text{Digital Call (Asset or Nothing) Payoff} = I_{\{f_T \geq K\}} \cdot f_T;$$
$$\text{Digital Put (Asset or Nothing) Payoff} = I_{\{f_T \leq K\}} \cdot f_T,$$

where *I* is indicator function; T is the Maturity of option; $f\,t$ is the Underlying interest rate at time t; K is the Strike; σ(K) is the Volatility function in terms of K.

For all those types of options, close-form solutions of prices are provided. The initial prices at $t = 0$ of the four digital options can be given by

$$\text{dgtlCallCash}(0,\ f_0, \text{K}) = df(0,T)[\Phi(d_-) - \frac{\partial F}{\partial \sigma}\frac{\partial \sigma}{\partial K}];$$
$$\text{dgtlPutCash}(0,\ f_0, \text{K}) = df(0,T)[\Phi(-d_-) + \frac{\partial F}{\partial \sigma}\frac{\partial \sigma}{\partial K}];$$
$$\text{dgtlCallAsset}(0,\ f_0, \text{K}) = df(0,T)[f_0\Phi(d_+) - K\frac{\partial F}{\partial \sigma}\frac{\partial \sigma}{\partial K}];$$
$$\text{dgtlPutAsset}(0,\ f_0, \text{K}) = df(0,T)[f_0\Phi(-d_+) + K\frac{\partial F}{\partial \sigma}\frac{\partial \sigma}{\partial K}],$$

where *df(,)* is the discount function and

$$F(f_0, K, T, \sigma) = f_0\Phi(d_+) - K\Phi(d_-);$$

$$d_+ = \frac{\ln(f_0 / K) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}, \quad d_- = d_+ - \sigma\sqrt{T}.$$

One may see that a skew-adjusted digital option can be approximately evaluated by a portfolio of vanilla call options, and/or zero-coupon bonds and/or FRNs. There are three ways to use this model:

1. $\frac{\partial\sigma}{\partial K} \approx \frac{\sigma(K+\Delta K) - \sigma(K)}{\Delta K}$ -- Method 1;
2. $\frac{\partial\sigma}{\partial K} \approx \frac{\sigma(K+\Delta K) - \sigma(K-\Delta K)}{2\Delta K}$ -- Moded 2;
3. $\frac{\partial\sigma}{\partial K} \approx \frac{\sigma(K) - \sigma(K-\Delta K)}{\Delta K}$ -- Method 3.

we find that the results are very sensitive to the method used for discretizing $\partial\sigma/\partial K$. This is because the volatility surface is not so smooth. We suggest that method 2 should be used for the calculations.

Reference:
https://finpricing.com/lib/EqBarrier.html