How well do we know the scaling relations?

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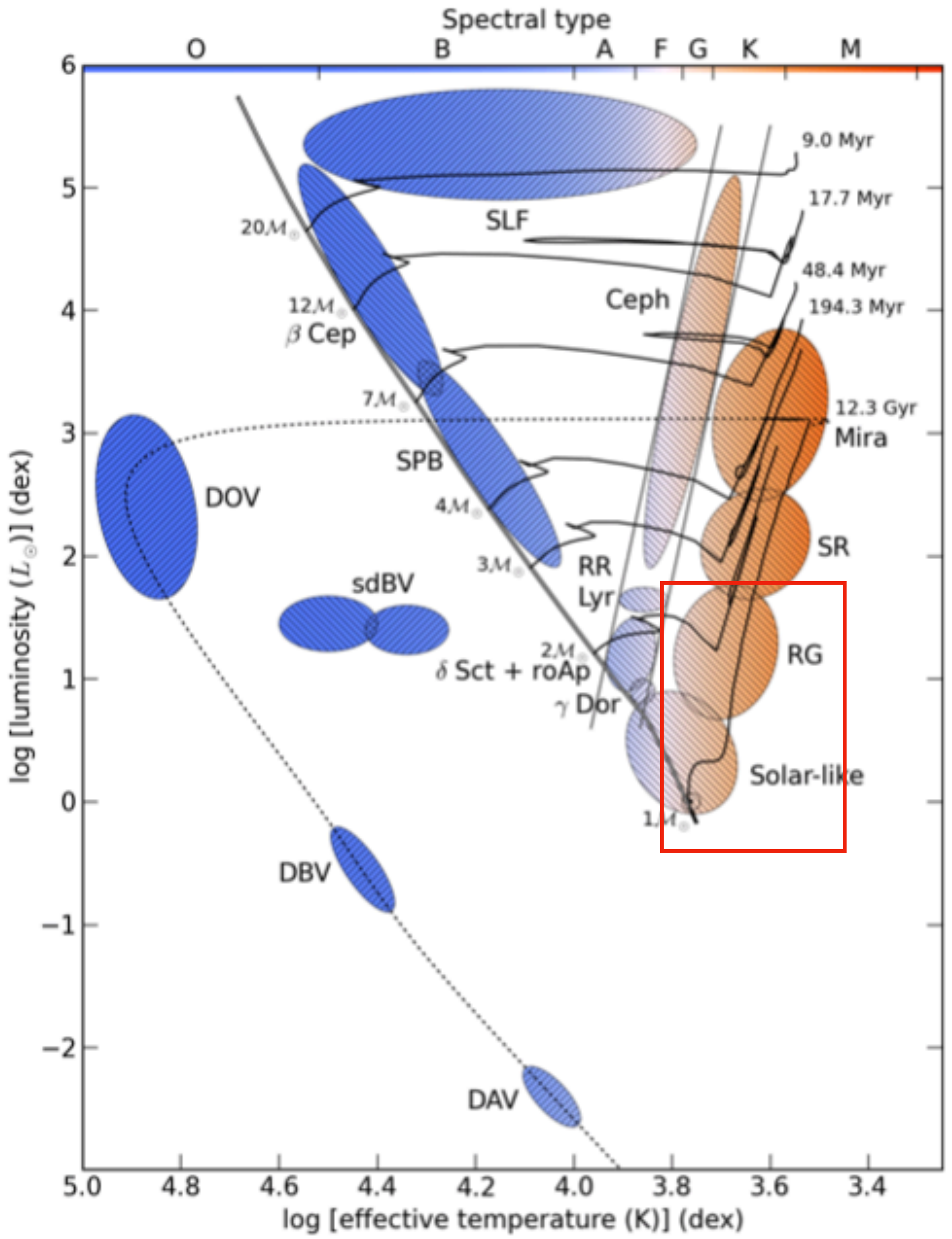
THE UNIVERSITY OF SYDNEY

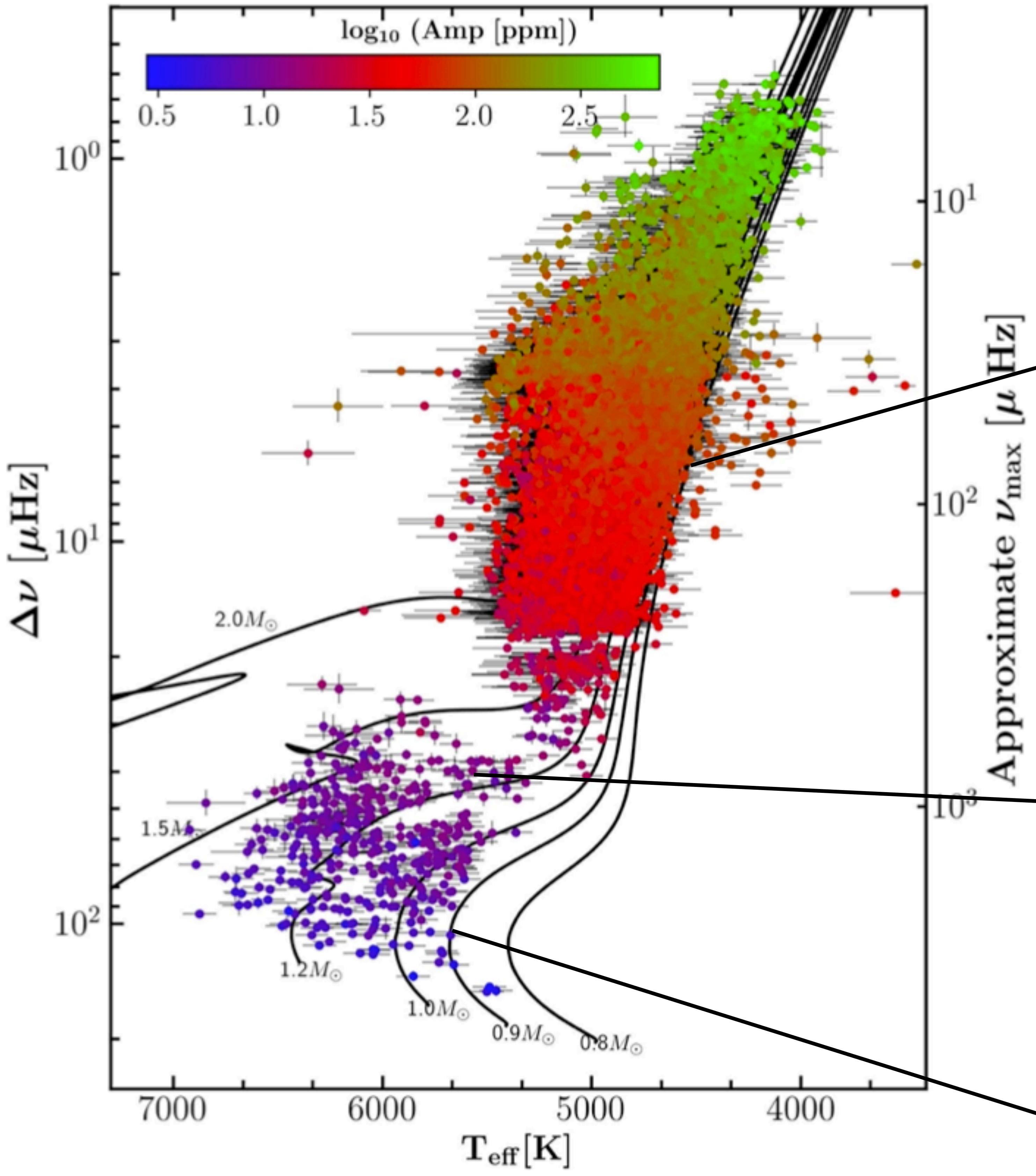
STELLAR ASTROPHYSICS CENTRE

13/Jun/2022 @MW-Gaia Workshop

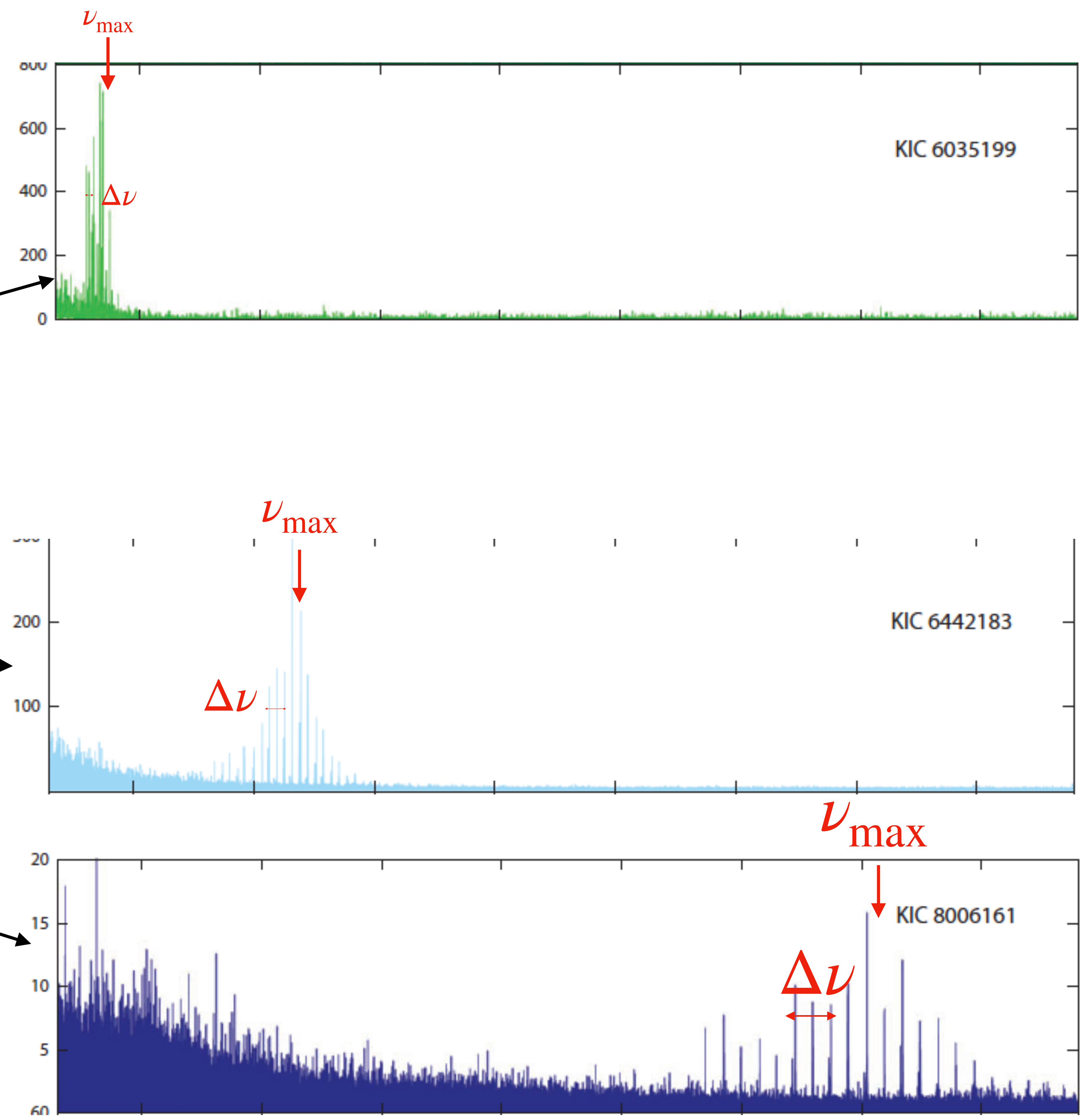
Solar-like oscillations



Pápics et al. (2013)
Aerts et al. (2021)



Yu J. et al. (2018)



Chaplin et al. (2014)

Scaling relations

- Ulrich 1986, Brown et al. 1991, Kjeldsen & Bedding 1995

$$\frac{\Delta\nu}{\Delta\nu_\odot} \approx \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{R}{R_\odot}\right)^{-3/2}$$

$$\frac{\nu_{\rm max}}{\nu_{\rm max,\odot}} \approx \left(\frac{M}{M_\odot}\right)^{1} \left(\frac{R}{R_\odot}\right)^{-2} \left(\frac{T_{\rm eff}}{T_{\rm eff,\odot}}\right)^{-1/2}$$

- Stello et al. 2008, Kallinger et al. 2010

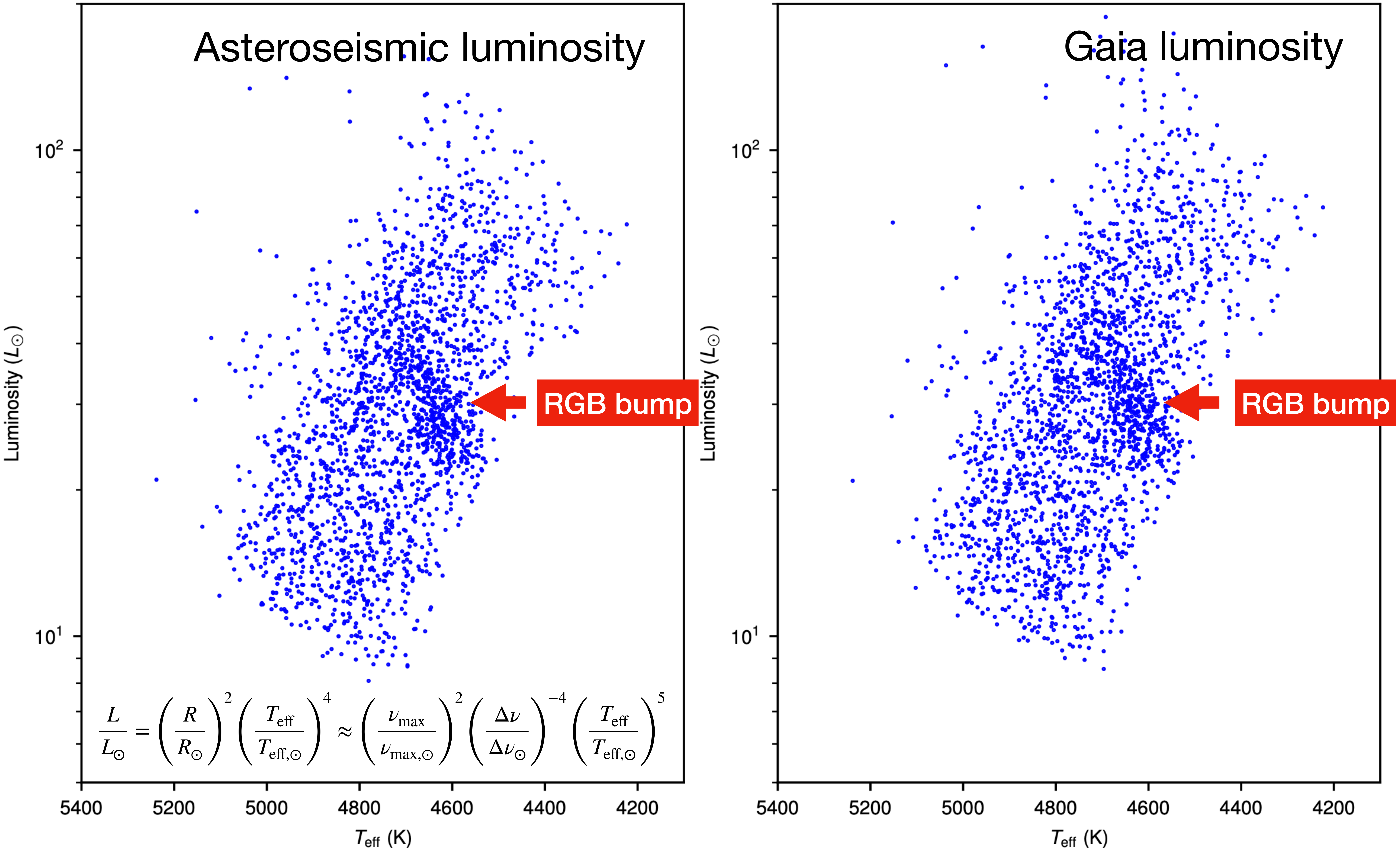
$$\frac{M}{M_\odot} \approx \left(\frac{\nu_{\rm max}}{\nu_{\rm max,\odot}}\right)^{3} \left(\frac{\Delta\nu}{\Delta\nu_\odot}\right)^{-4} \left(\frac{T_{\rm eff}}{T_{\rm eff,\odot}}\right)^{-3/2}$$

$$\frac{R}{R_\odot} \approx \left(\frac{\nu_{\rm max}}{\nu_{\rm max,\odot}}\right)^{1} \left(\frac{\Delta\nu}{\Delta\nu_\odot}\right)^{-2} \left(\frac{T_{\rm eff}}{T_{\rm eff,\odot}}\right)^{1/2}$$

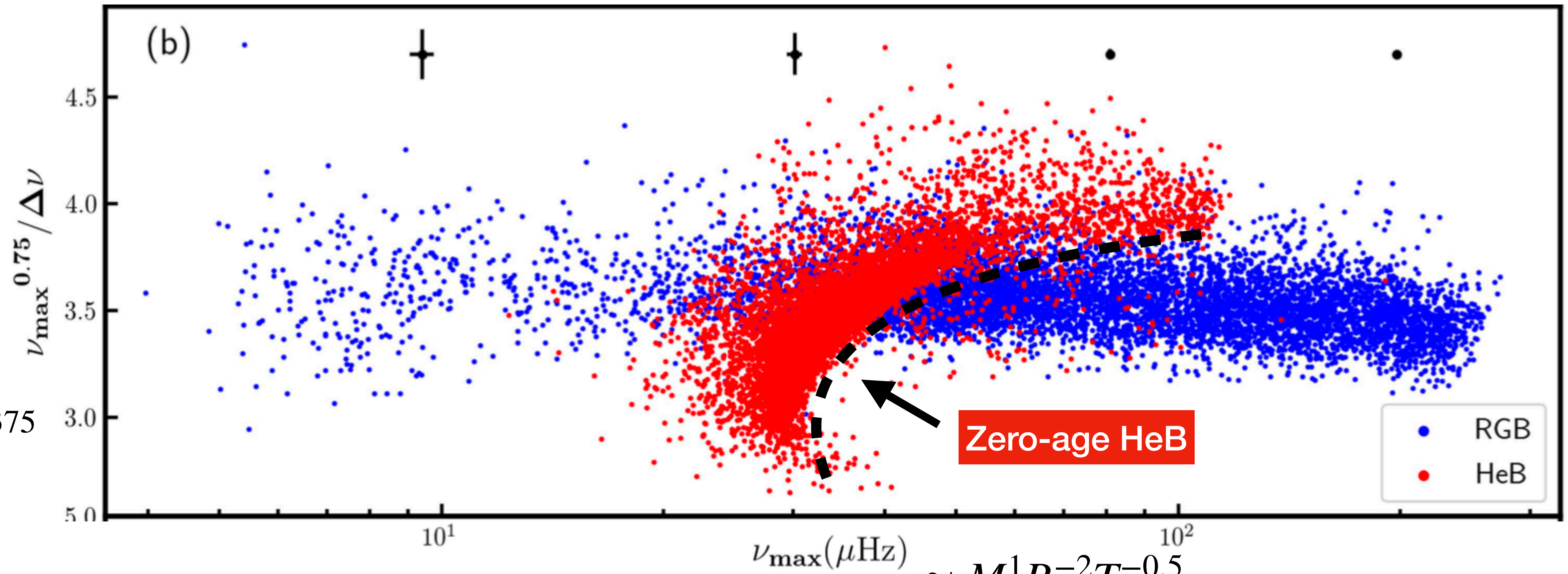
Exoplanet host

Galactic archeology

Kepler red-giant-branch stars



Kepler red-clump stars

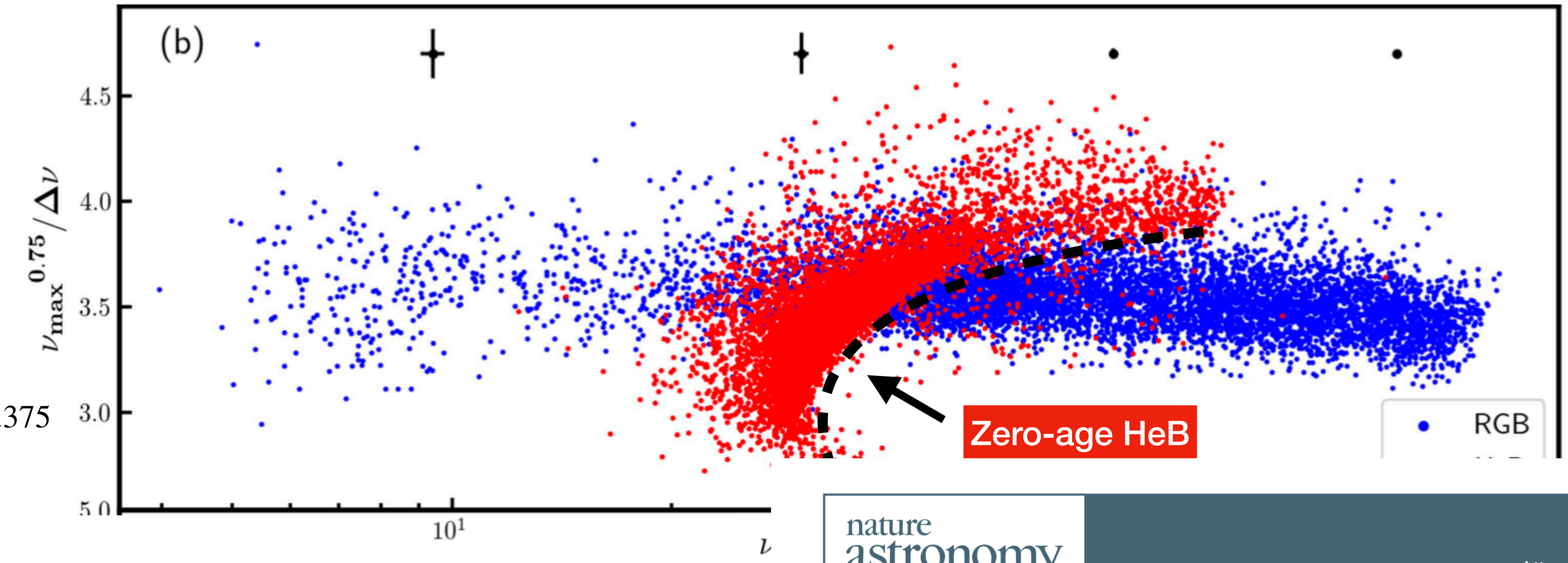


$\approx M^{0.25} T_{\mathrm{eff}}^{-0.375}$

$\approx M^{1} R^{-2} T_{\mathrm{eff}}^{-0.5}$

Yu J. et al. (2018)

Kepler red-clump stars



$\approx M^{0.25} T_{\mathrm{eff}}^{-0.375}$

nature astronomy

LETTERS

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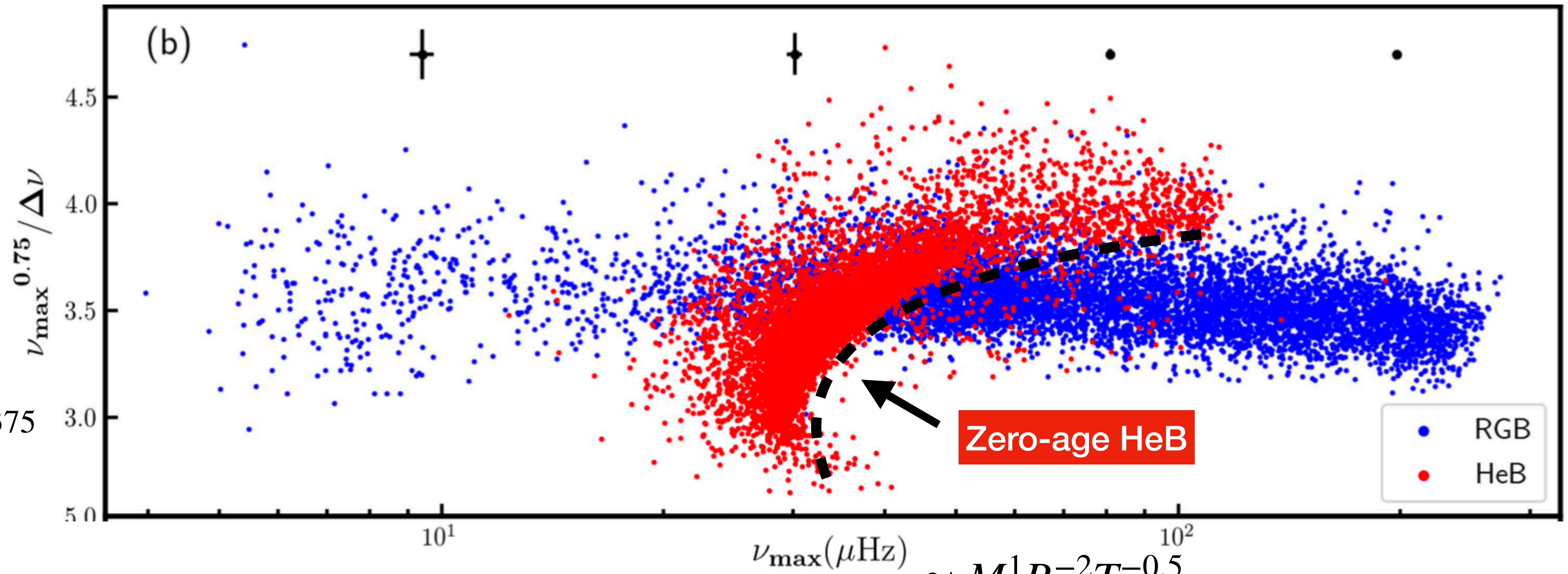
Discovery of post-mass-transfer helium-burning red giants using asteroseismology

Yaguang Li[1,2], Timothy R. Bedding[1,2], Simon J. Murphy[1,2], Dennis Stello[2,3], Yifan Chen[1], Daniel Huber[4], Meridith Joyce[5], Dion Marks[1], Xianfei Zhang[6], Shaolan Bi[6], Isabel L. Colman[7], Michael R. Hayden[1], Daniel R. Hey[1,2], Gang Li[8], Benjamin T. Montet[3,9], Sanjib Sharma[1] and Yaqian Wu[10]

Kepler red-clump stars



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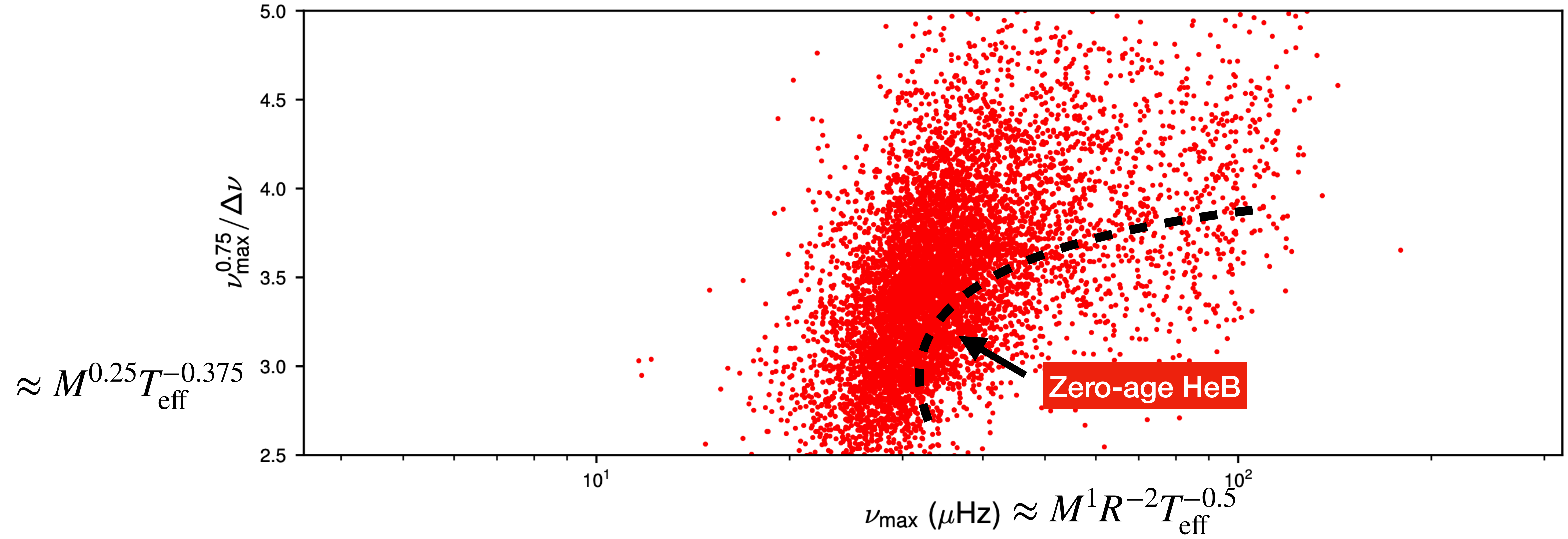
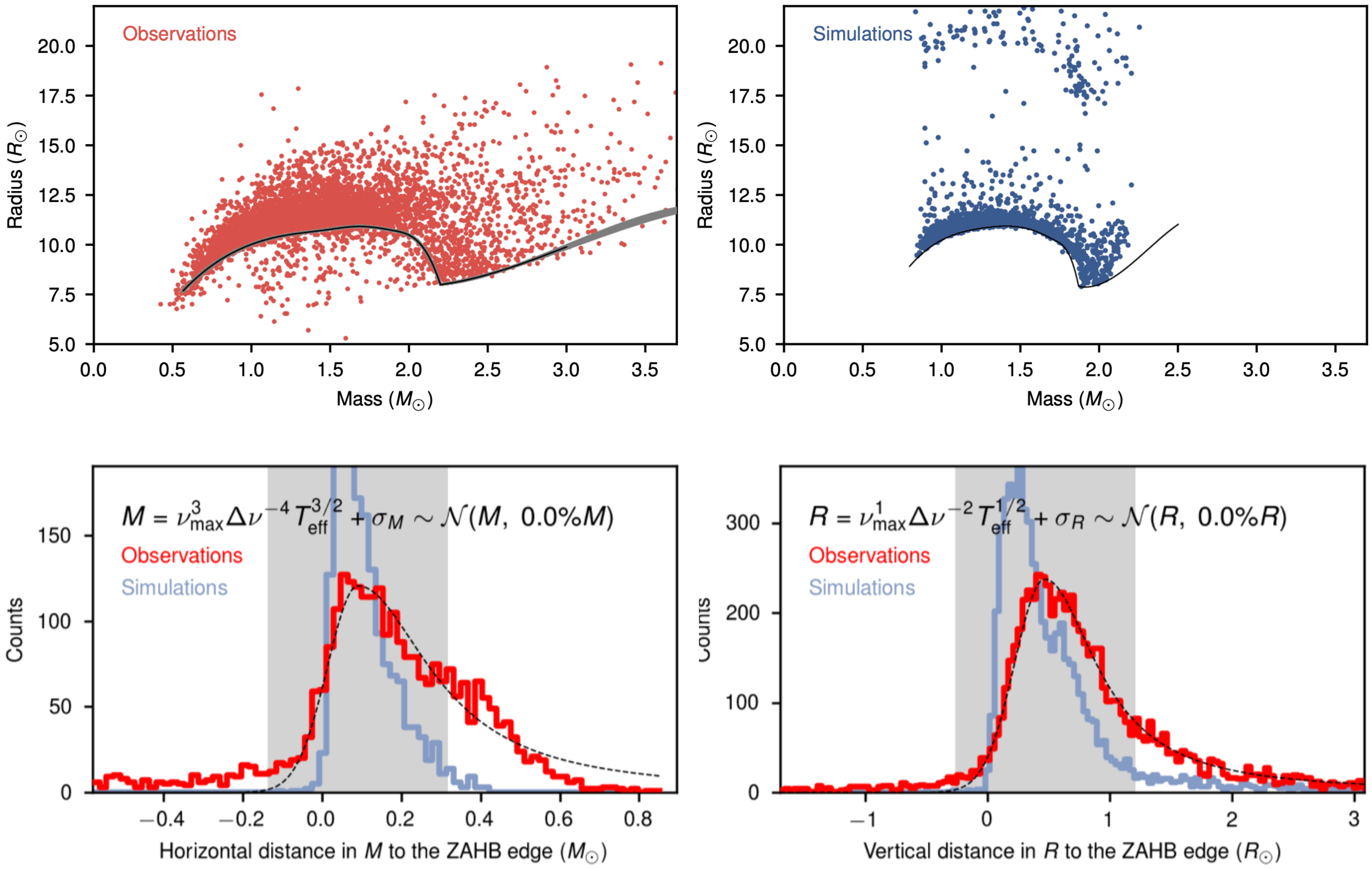


Scatter both $\nu_{\rm max}$ and $\Delta\nu$ by 10%

Kepler red-clump stars



The intrinsic scatter

- We used features in the red giant population to test the intrinsic scatter of the scaling relations (Li, Yaguang et al. 2021).
  - 0.5% ($\Delta\nu$), 1.1% ($\nu_{\rm max}$), 1.7% (M), and 0.4% (R)
- The random uncertainties from measuring $\Delta\nu$ and $\nu_{\rm max}$ dominate the scatter that blurs the features.

Are there any ways to improve the measurements?

$\Delta\nu$

Huber et al. (2010, the SYD pipeline)

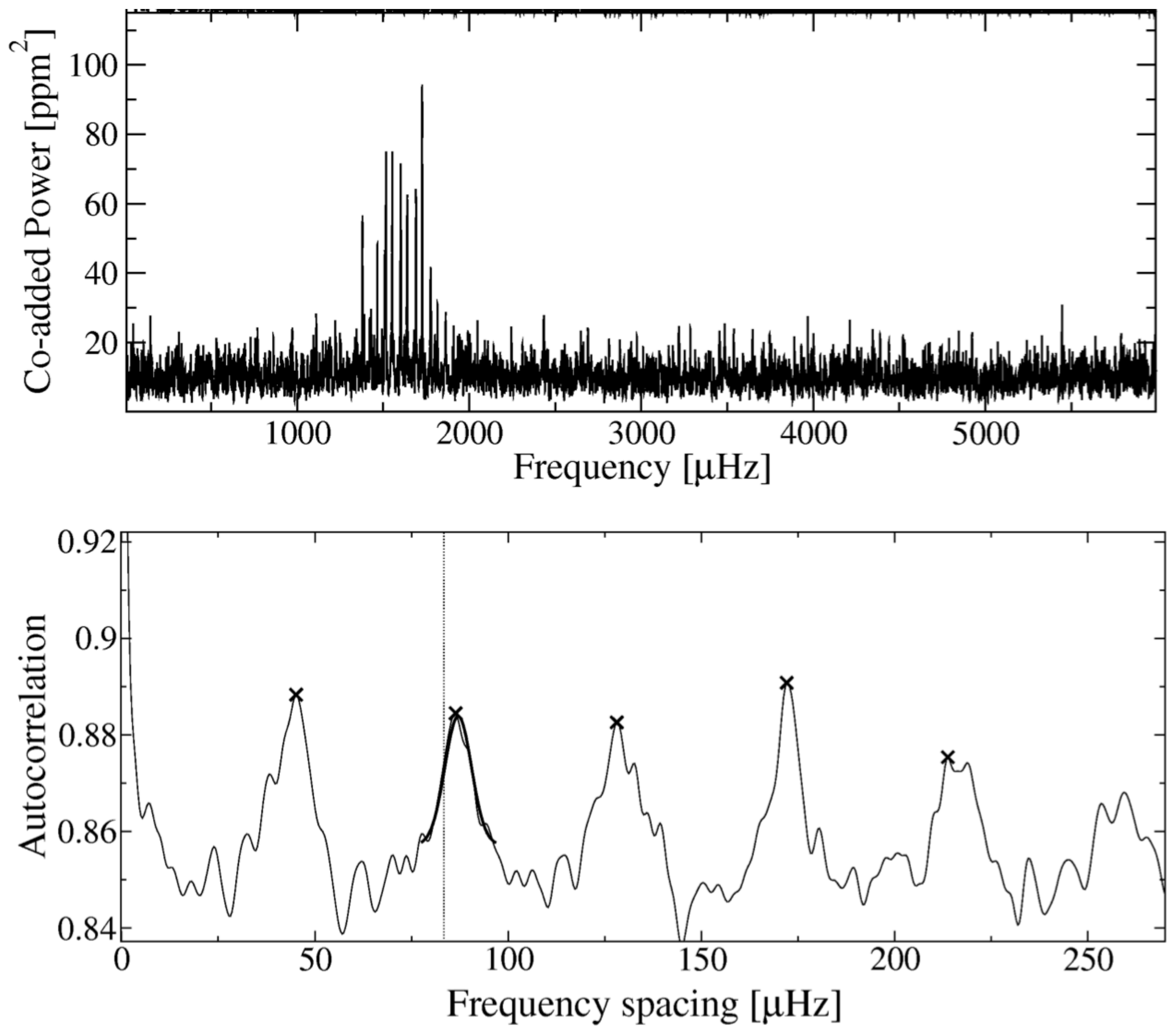


Figure 5: ACF of the background-corrected co-added power spectrum of a simulated Kepler star. Black crosses mark the five highest peaks. The grey line is a fit to the peak among the five which is closest to $\Delta\nu_{\rm exp}$ (vertical dotted line).

Kallinger et al. (2012)

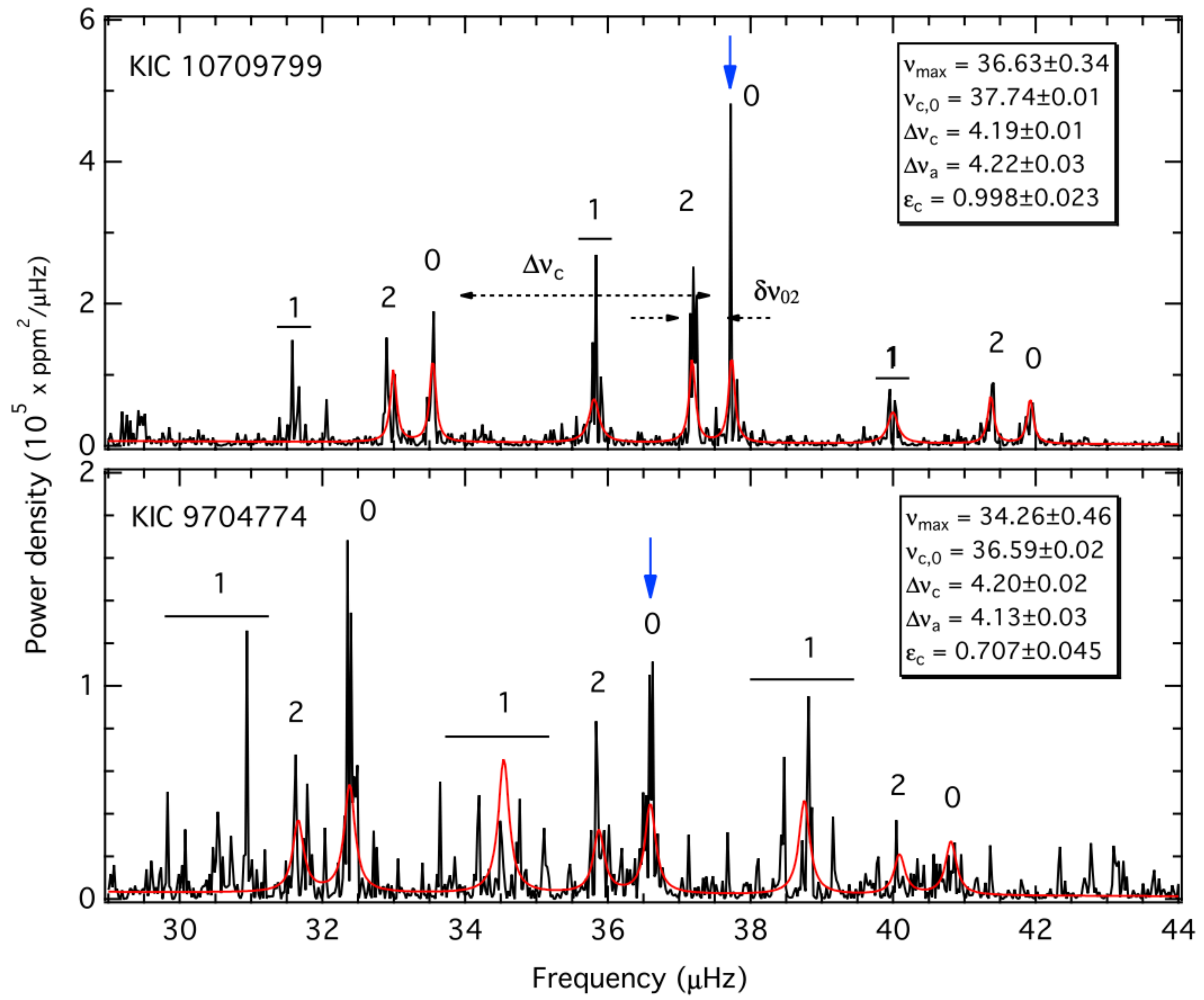
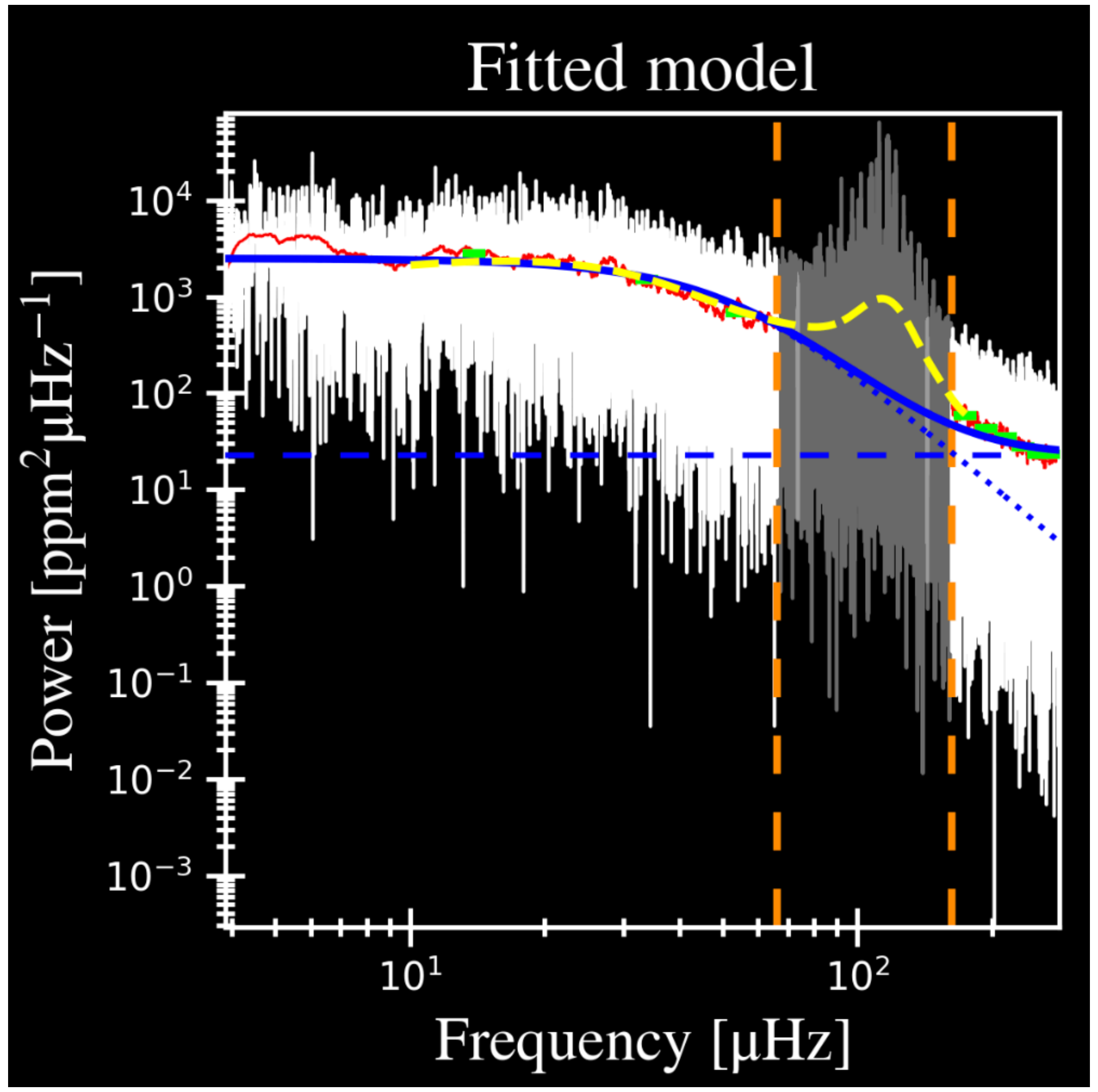


Fig. 3. Power density spectra of an RGB (*top*) and RC (*bottom*) star with almost identical $\Delta\nu_c$ but with the central radial modes ($\nu_{c,0}$, indicated by blue arrows) at significantly different frequencies (resulting in different ϵ_c). The red lines indicate the best fit model to determine the p-mode spacings, $\Delta\nu_c$ and $\delta\nu_{02}$. Numbers give the mode degree.

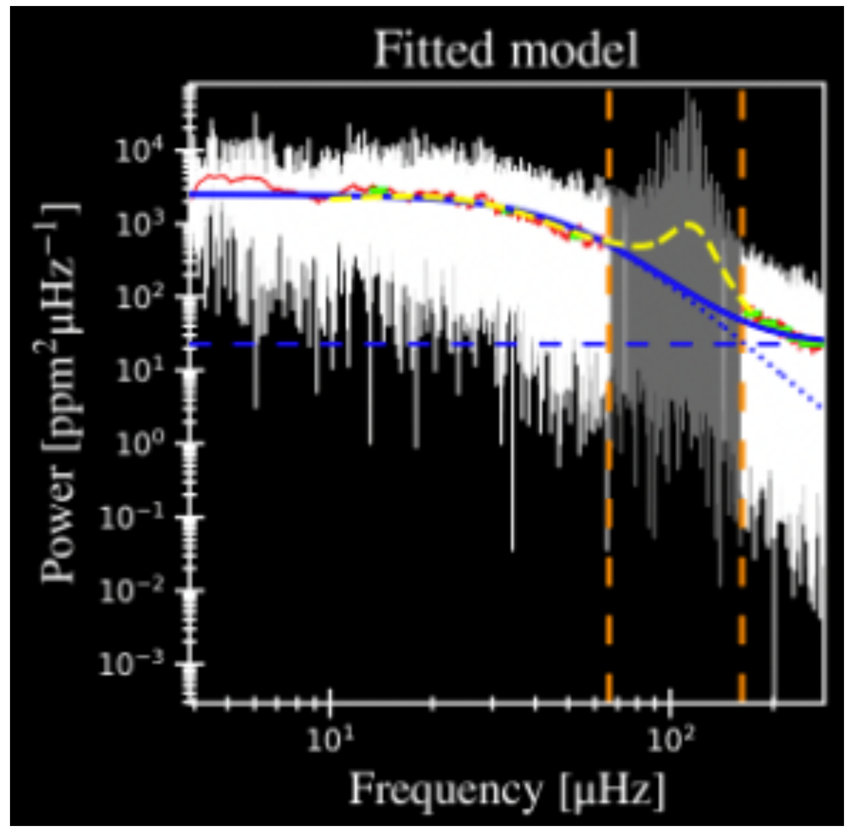
ν_{max}

Huber et al. (2010, the SYD pipeline)

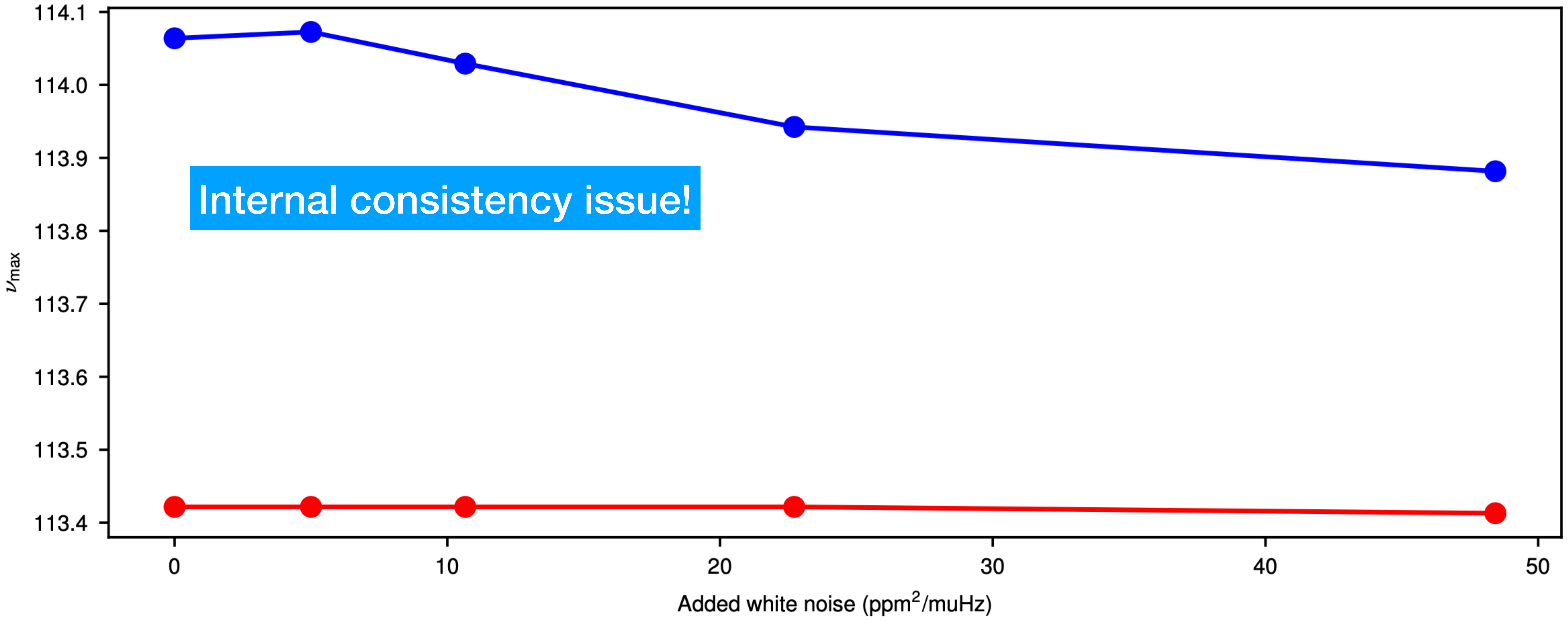


- ν_{max} from fitted power spectra (a Gaussian profile + background)
- ν_{max} from smoothed power spectra w/ background subtraction
- ν_{max} from smoothed power spectra w/o background subtraction

ν_{max}



- ν_{max} from smoothed power spectra w/ background subtraction



- ν_{max} from smoothed power spectra w/o background subtraction

Improving measurements

- Determine $\Delta\nu$ from mode frequencies
- Determine ν_{max} from power spectra by not fitting the background, but “assuming” it

Scaling relations

The systematic offset

- Ulrich 1986, Brown et al. 1991, Kjeldsen & Bedding 1995

$$\frac{\Delta\nu}{\Delta\nu_\odot} = f_{\Delta\nu}\left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{R}{R_\odot}\right)^{-3/2}$$

$$\frac{\nu_{\mathrm{max}}}{\nu_{\mathrm{max},\odot}} = f_{\nu_{\mathrm{max}}}\left(\frac{M}{M_\odot}\right)^{1}\left(\frac{R}{R_\odot}\right)^{-2}\left(\frac{T_{\mathrm{eff}}}{T_{\mathrm{eff},\odot}}\right)^{-1/2}$$

- Stello et al. 2008, Kallinger et al. 2010

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$$\frac{R}{R_\odot} = \left(\frac{\nu_{\mathrm{max}}}{f_{\nu_{\mathrm{max}}}\nu_{\mathrm{max},\odot}}\right)^{1}\left(\frac{\Delta\nu}{f_{\Delta\nu}\Delta\nu_\odot}\right)^{-2}\left(\frac{T_{\mathrm{eff}}}{T_{\mathrm{eff},\odot}}\right)^{1/2}$$

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- Benchmark from other fundamental data
  - Binaries (Gaulme et al. 2016; Brogaard et al. 2018; Benbakoura et al. 2021; …)
  - Astrometric radii (Silva Aguirre et al. 2012; Huber et al. 2017; Sahlholdt & Silva Aguirre 2018; Hall et al. 2019; Khan et al. 2019; Zinn et al. 2019; …)
  - Stellar clusters, Interferometric radii, …

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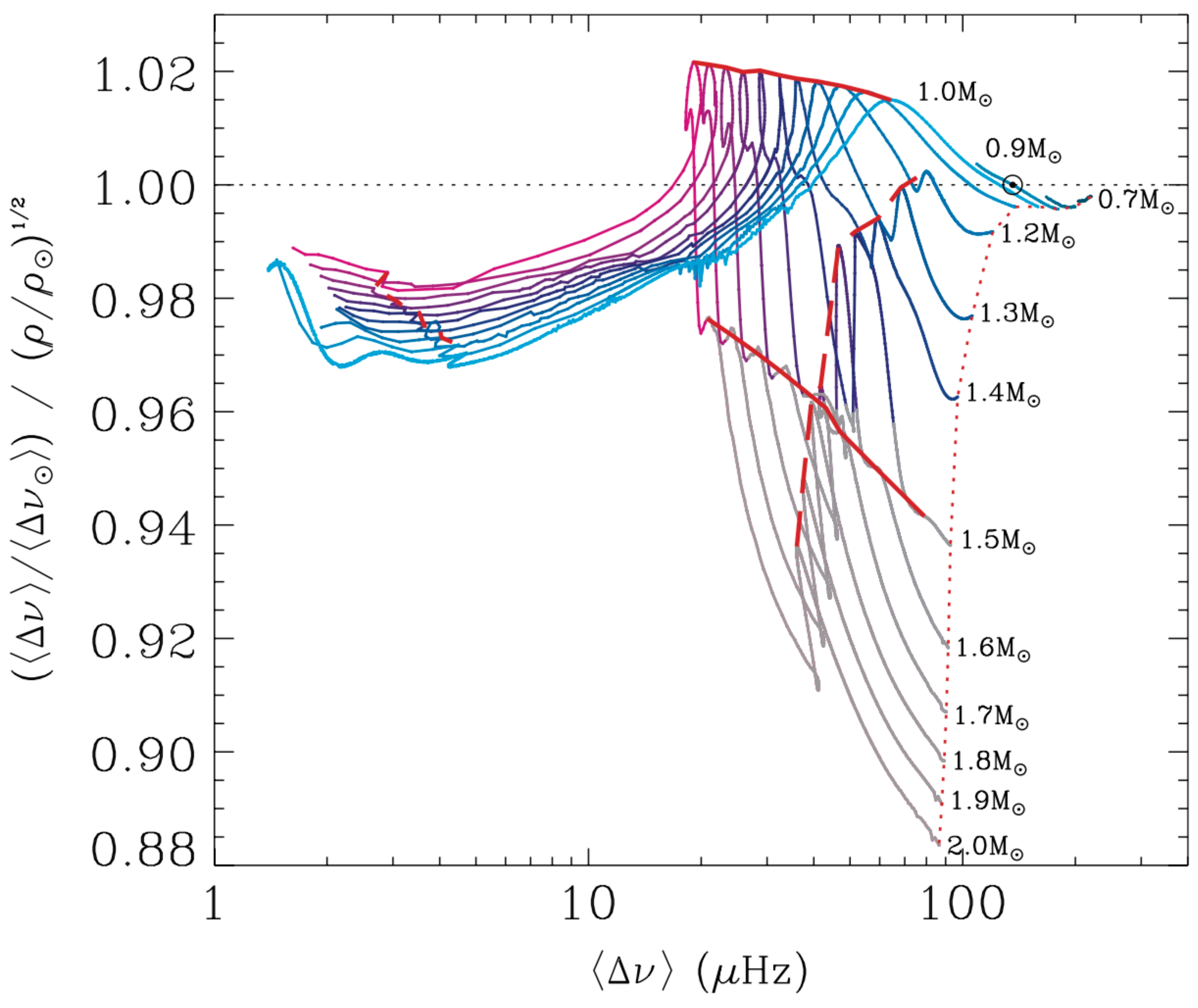
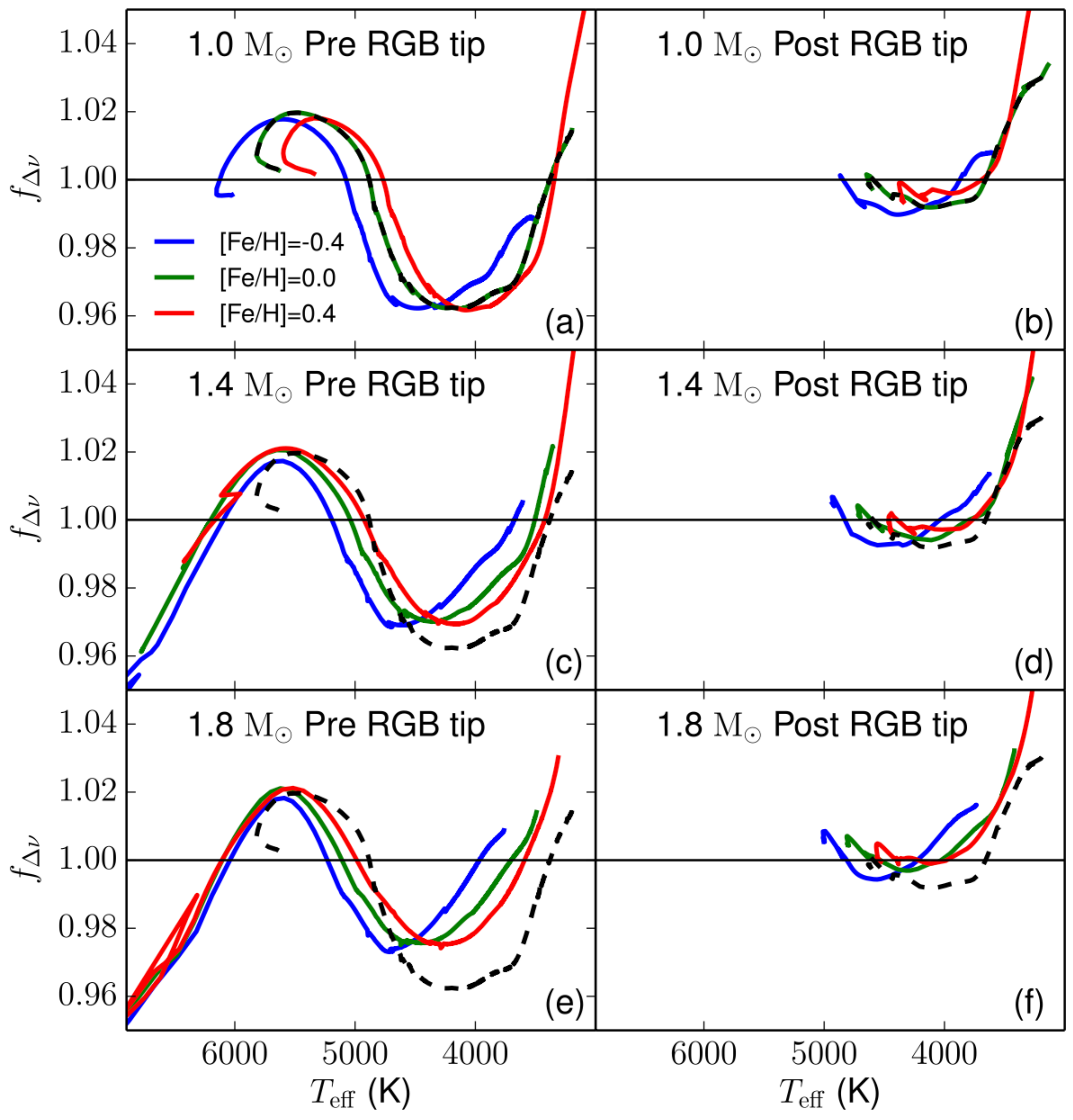
- Corrections from stellar models
  - $f_{\Delta\nu}$: $f_{\Delta\nu}(M, t, [\mathrm{M/H}]) = \Delta\nu/\sqrt{\rho}$ (White et al. 2011; Guggenberger et al. 2016; Sharma et al. 2016; Rodrigues et al. 2017; Serenelli et al. 2017; Pinsonneault et al. 2018; …)

$$f_{\Delta\nu}(M, t, [\mathrm{M/H}]) = \Delta\nu/\sqrt{\rho}$$



White et al. (2011)



Sharma et al. (2016)

$$f_{\Delta\nu}(M, t, [\mathrm{M/H}]) = \Delta\nu/\sqrt{\rho}$$

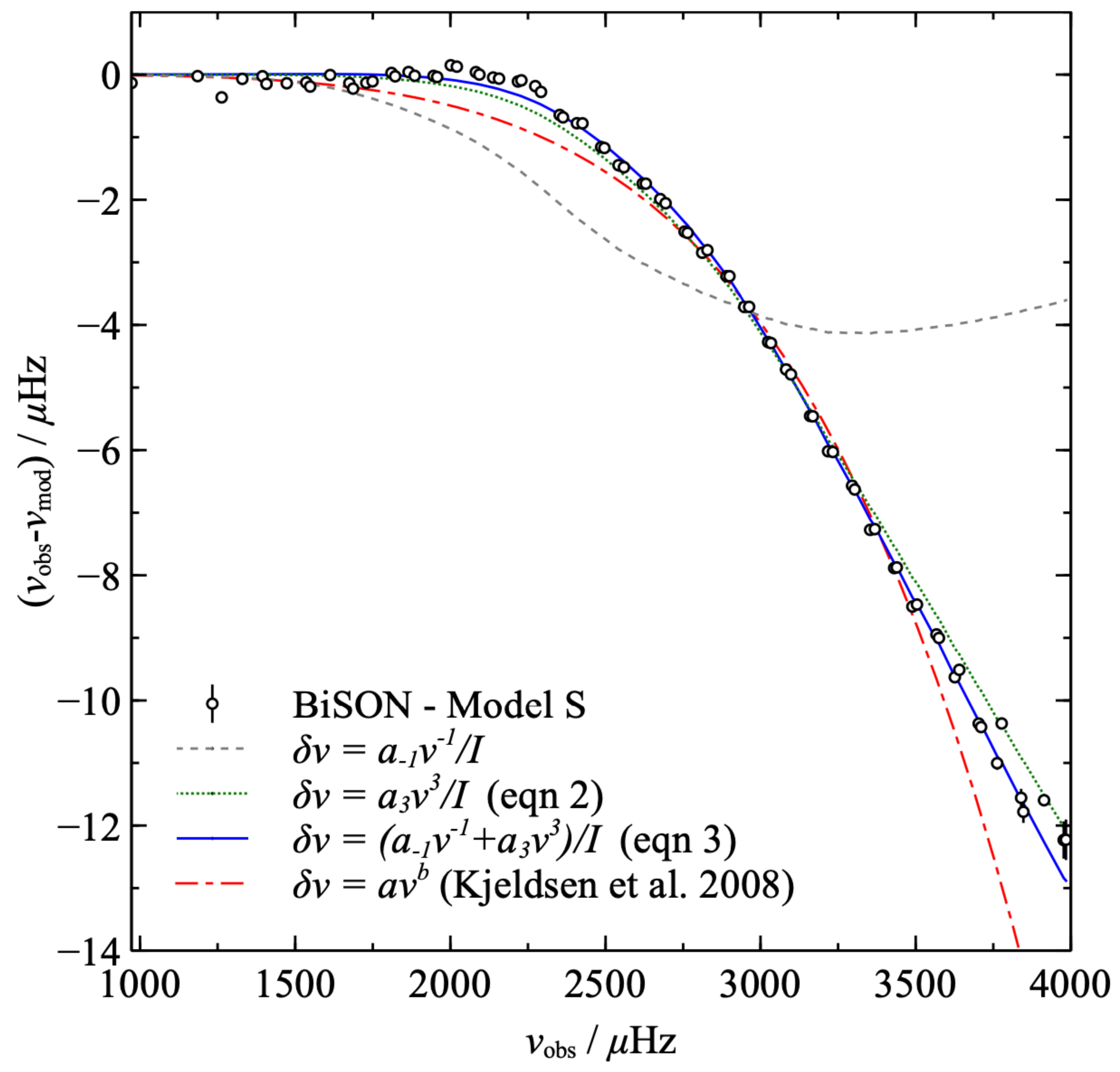


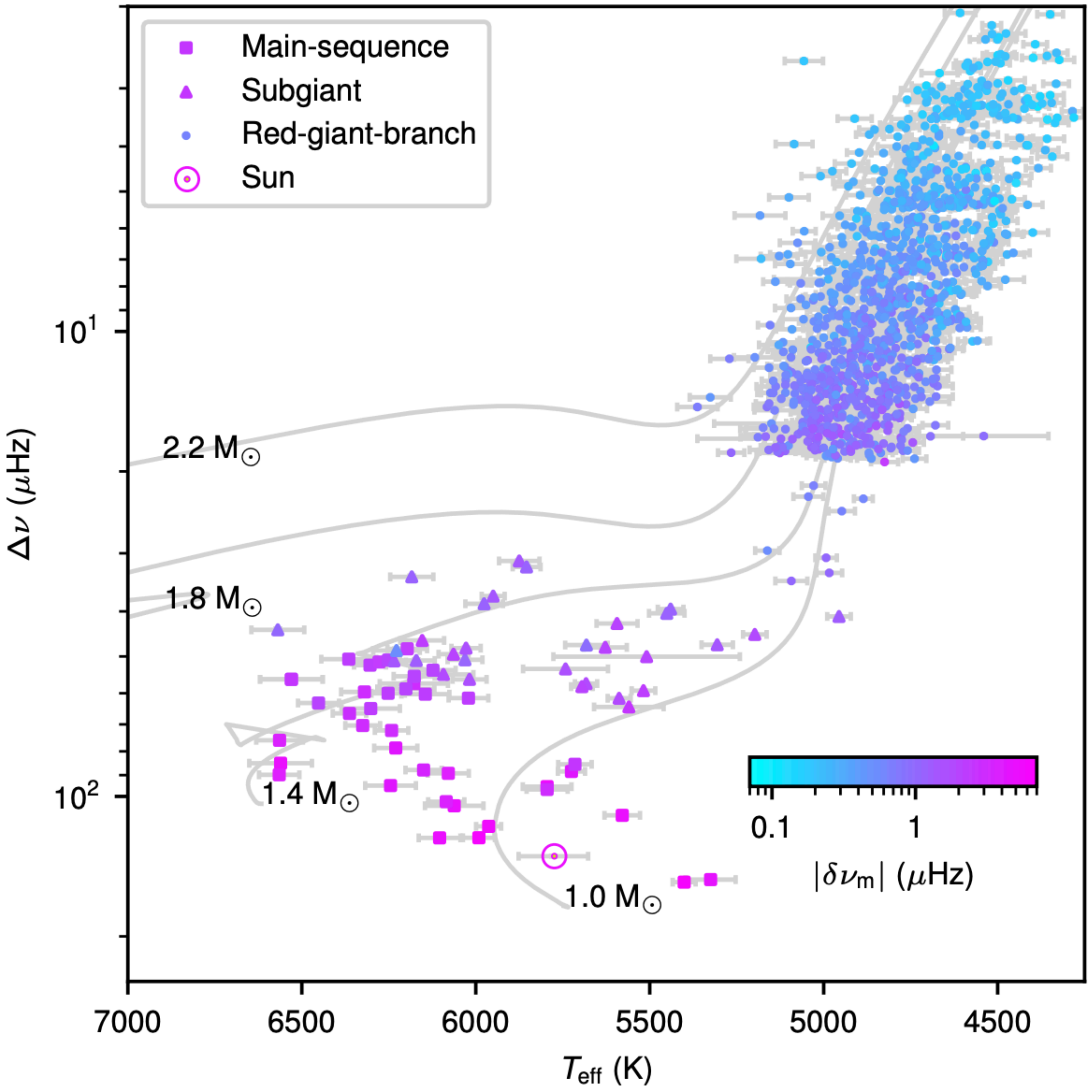
Fig. 1. Frequency differences between a standard solar model (Model S, Christensen-Dalsgaard et al. 1996) and observations of low-degree modes ($\ell \leq 3$) by BiSON. The lines show fits made using an inverse term (dashed), cubic term (dotted) or both terms (solid). The fit with the inverse term is quite poor, with the cubic term much better and with both terms somewhat better still. The dot-dashed lines show a power law, fit to nine radial orders about $\nu_{\max} = 3090\,\mu$Hz, as is used in the frequency correction proposed by Kjeldsen et al. (2008).

Ball & Gizon (2014)

Surface effect!

$$\delta\nu = \left(a_{-1}(\nu/\nu_{\mathrm{ac}})^{-1} + a_3(\nu/\nu_{\mathrm{ac}})^3\right)/\mathcal{I}$$

Prescribing the surface effect

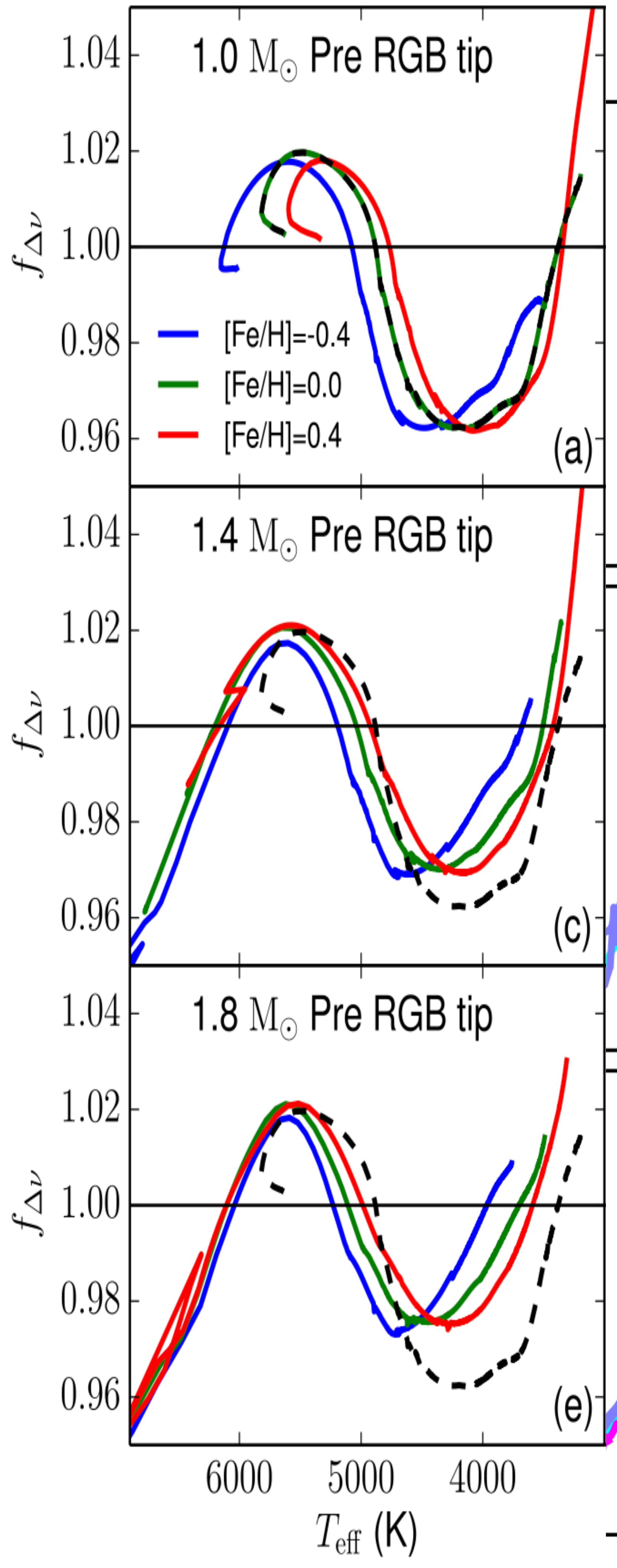


$$\delta\nu = \left(a_{-1}(\nu/\nu_\mathrm{ac})^{-1} + a_3(\nu/\nu_\mathrm{ac})^3\right)/\mathcal{I}$$

$$\delta\nu_m = a \cdot (g/g_\odot)^b \cdot (T_\mathrm{eff}/T_{\mathrm{eff},\odot})^c \cdot (d \cdot [\mathrm{M/H}] + 1)$$

Li, Yaguang et al. (to be submitted)

$$f_{\Delta\nu}(M, t, [\mathrm{M/H}]) = \Delta\nu/\sqrt{\rho}$$



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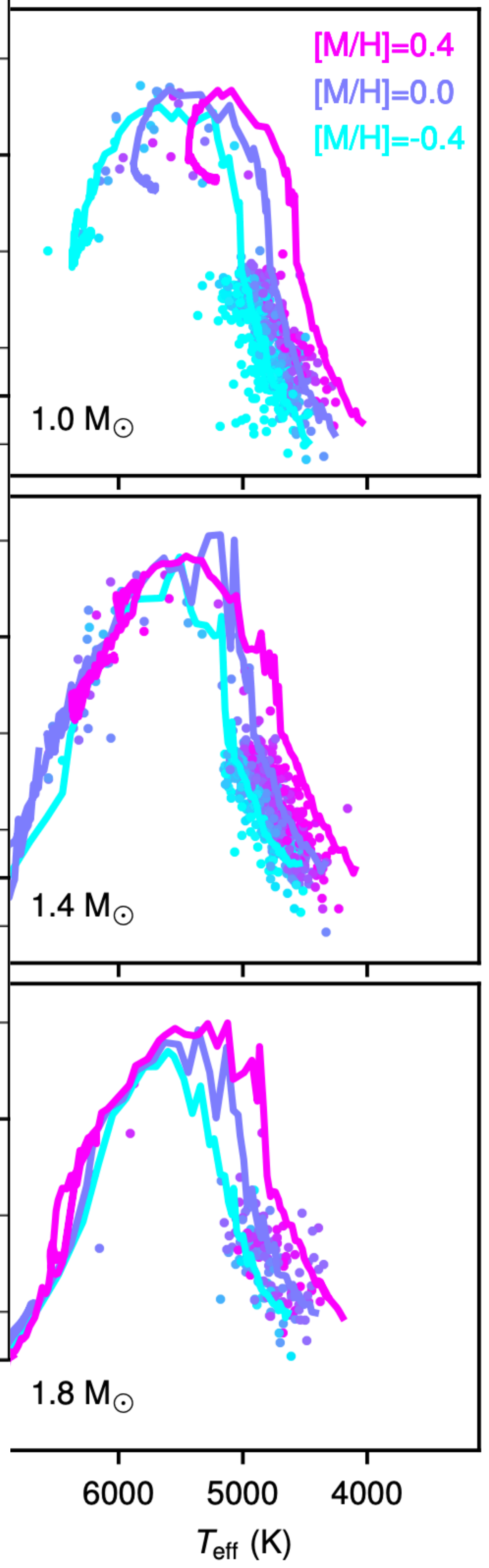

This work (with surface correction)

~2% decrease on the RGB!

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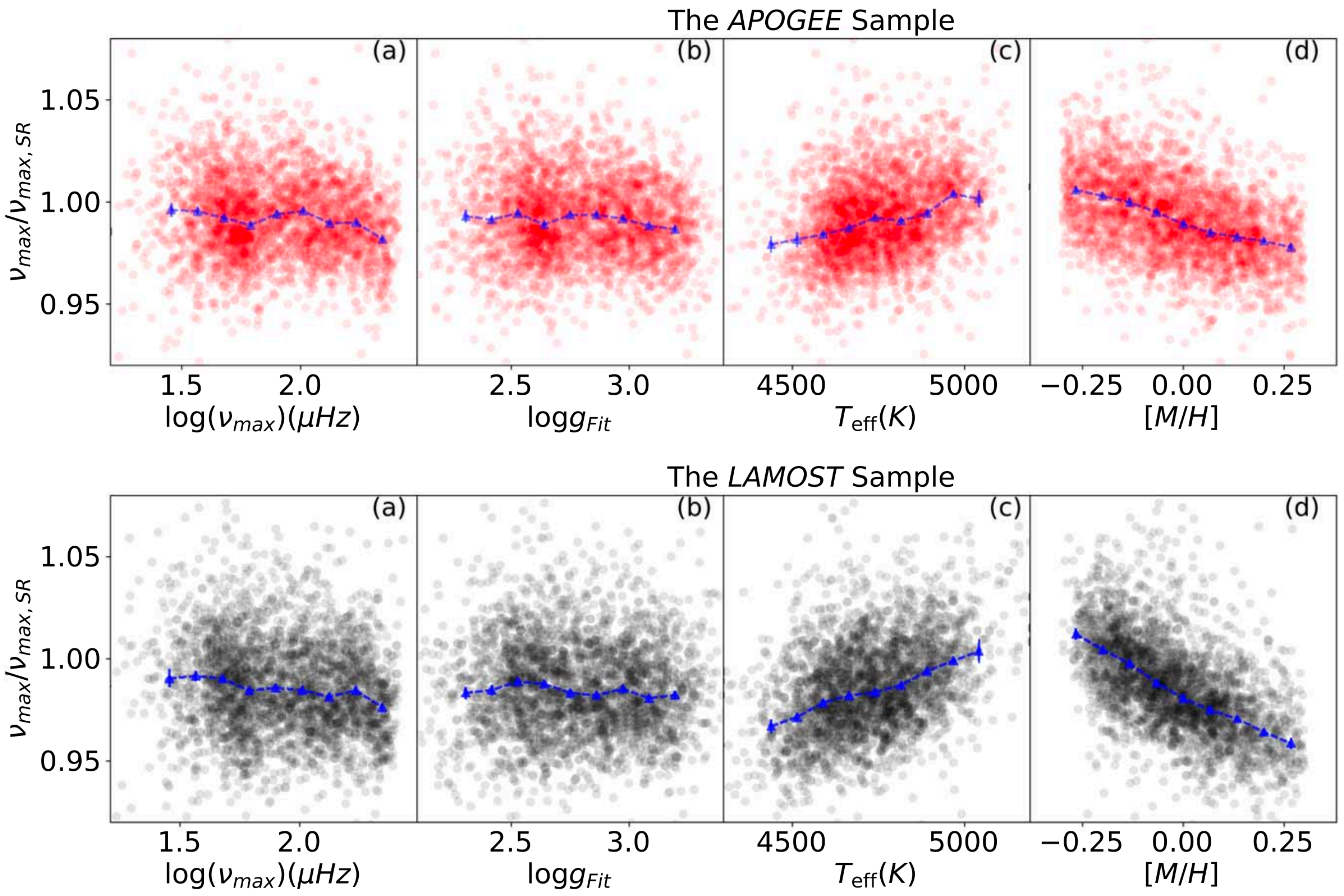
  - Surface corrections
  - $f_{\nu_{\max}}$: assume 1

Is there a way to test?

$$f_{\nu_{\max}} = \nu_{\max}\ (\text{true, observed})\ /\ \nu_{\max}\ (\text{modelled, scaling})$$

$T_{\rm eff}$, L, [M/H], ν_{nl}, …



Li, Tanda et al. (2022)

Summary

How well do we know about the scaling relations?

- The intrinsic scatter of the scaling relations is small: 0.5% ($\Delta\nu$), 1.1% ($\nu_{\max}$), 1.7% (M), and 0.4% (R).
- New techniques can be developed to improve the measurements of $\Delta\nu$ and $\nu_{\max}$ from data.
- Gaia benchmark data will be able to calibrate the scaling relations.
- Accounting for the surface effect decreases $f_{\Delta\nu}$ by 0-2%.
- The values for $f_{\nu_{\max}}$ are around 1, and the potential dependence on metallicity requires more tests.