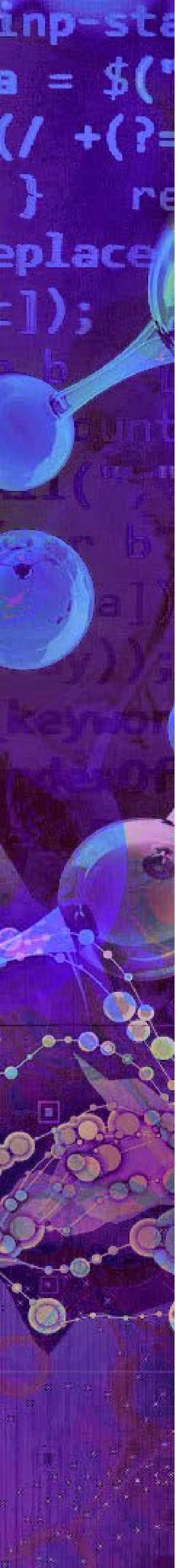

A comparative study of dark matter flow & hydrodynamic turbulence and its applications

May 2022

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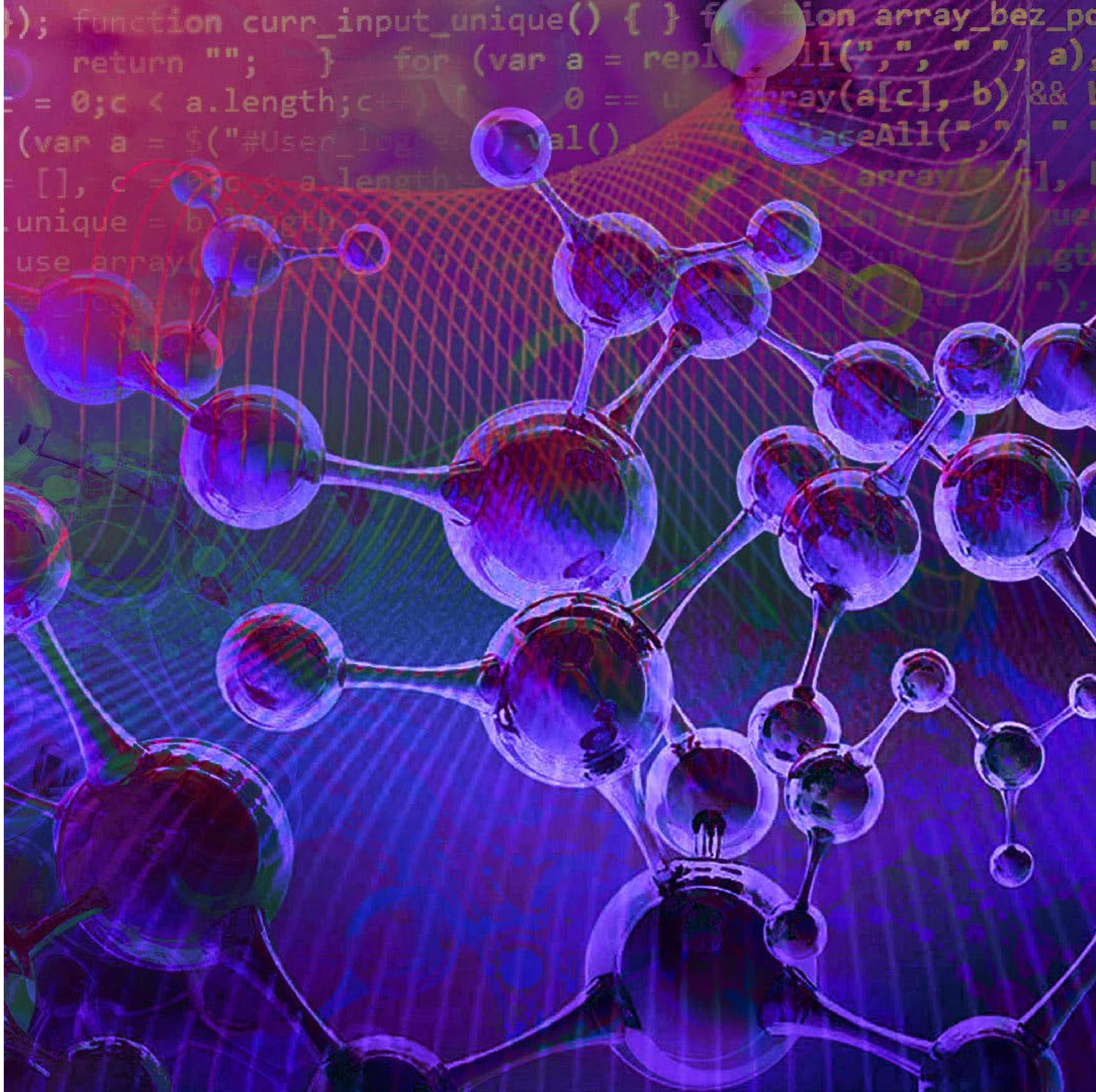


Preface

Dark matter, if exists, accounts for five times as much as ordinary baryonic matter. Therefore, dark matter flow might possess the widest presence in our universe. The other form of flow, hydrodynamic turbulence in air and water, is without doubt the most familiar flow in our daily life. During the pandemic, we have found time to think about and put together a systematic comparison for the connections and differences between two types of flow, both of which are typical non-equilibrium systems.

The goal of this presentation is to leverage this comparison for a better understanding of the nature of dark matter and its flow behavior on all scales. Science should be open. All comments are welcome.

Thank you!

Data repository and relevant publications

Structural (halo-based) approach:

0.	**Data https://dx.doi.org/10.5281/zenodo.6541230**
1.	Inverse mass cascade in dark matter flow and effects on halo mass functions https://doi.org/10.48550/arXiv.2109.09985
2.	Inverse mass cascade in dark matter flow and effects on halo deformation, energy, size, and density profiles https://doi.org/10.48550/arXiv.2109.12244
3.	Inverse energy cascade in self-gravitating collisionless dark matter flow and effects of halo shape https://doi.org/10.48550/arXiv.2110.13885
4.	The mean flow, velocity dispersion, energy transfer and evolution of rotating and growing dark matter halos https://doi.org/10.48550/arXiv.2201.12665
5.	Two-body collapse model for gravitational collapse of dark matter and generalized stable clustering hypothesis for pairwise velocity https://doi.org/10.48550/arXiv.2110.05784
6.	Evolution of energy, momentum, and spin parameter in dark matter flow and integral constants of motion https://doi.org/10.48550/arXiv.2202.04054
7.	The maximum entropy distributions of velocity, speed, and energy from statistical mechanics of dark matter flow https://doi.org/10.48550/arXiv.2110.03126
8.	Halo mass functions from maximum entropy distributions in collisionless dark matter flow https://doi.org/10.48550/arXiv.2110.09676

Statistics (correlation-based) approach:

0.	**Data https://dx.doi.org/10.5281/zenodo.6569898**
1.	The statistical theory of dark matter flow for velocity, density, and potential fields https://doi.org/10.48550/arXiv.2202.00910
2.	The statistical theory of dark matter flow and high order kinematic and dynamic relations for velocity and density correlations https://doi.org/10.48550/arXiv.2202.02991
3.	The scale and redshift variation of density and velocity distributions in dark matter flow and two-thirds law for pairwise velocity https://doi.org/10.48550/arXiv.2202.06515
4.	Dark matter particle mass and properties from two-thirds law and energy cascade in dark matter flow https://doi.org/10.48550/arXiv.2202.07240
5.	The origin of MOND acceleration and deep-MOND from acceleration fluctuation and energy cascade in dark matter flow https://doi.org/10.48550/arXiv.2203.05606
6.	The baryonic-to-halo mass relation from mass and energy cascade in dark matter flow https://doi.org/10.48550/arXiv.2203.06899

Applications of dark matter flow

Pacific Northwest
NATIONAL LABORATORY

The origin of MOND acceleration from mass and energy cascade in dark matter flow

Xu Z., 2022, arXiv:2203.05606v1 [astro-ph.CO]
https://doi.org/10.48550/arXiv.2203.05606

Introduction

- The existence of dark matter (DM) is supported by numerous astronomical observations:
  - Flat rotation curves of spiral galaxies
  - Motion of galaxies in galaxy clusters
  - Gravitational lensing
  - Bullet clusters, CMB ……
- Though the nature of dark matter is still unclear, dark matter is believed to be cold (non-relativistic), collisionless, dissipationless, non-baryonic, barely interacting with baryonic matter except through gravity, and sufficiently smooth with a fluid-like behavior.
- However, no conclusive signals have been detected in searches for dark matter particles.
- Alternative theory of dark matter: Modified Newtonian Dynamics (MOND)

- Empirical Tully and Fisher relation:

$v_f \propto M^{1/4}$ ⬅ observed baryonic mass

- MOND (Milgrom) is a popular empirical model to reproduce flat rotation curve without invoking dark matter hypothesis.

$a_0 \approx 1.2\times 10^{-10}\, m/s^2$ Critical MOND acceleration

$F = ma$ $\quad a \gg a_0$ Newtonian

$F = m a^2/a_0 \propto a^2$ $\quad a \ll a_0$ Deep MOND

$$\frac{GMm}{r^2} = m\frac{\left(v_f^2/r\right)^2}{a_0} \Rightarrow v_f = \left(GMa_0\right)^{1/4}$$

- What is the origin of MOND acceleration?
- What is the origin of deep "MOND" behavior?
- Could MOND be an intrinsic property of dark matter flow?
- Instead of falsifying, MOND supports the existence of dark matter?

Hydrodynamic turbulence vs. dark matter flow

Key attributes of hydrodynamic turbulence:

- Disorganized, chaotic, random;
- Nonrepeatability (sensitivity to initial cond.);
- Multiscale in length and time scales;
- Intermittency in space and time;

- Dissipative and collisional
- No long-range interaction
- Velocity fluctuation
- Vortex as fundamental building block
- Maximum entropy distribution (Gaussian)
- Incompressible on all scales
  - Divergence-free $\nabla \cdot \mathbf{v} = 0$
  - Constant density
- Energy cascade from large to small length scales

Key attributes of dark matter flow:

- Disorganized, chaotic, random;
- Nonrepeatability;
- Multiscale in mass/length/time scales;
- Intermittency in space and time;

- Dissipationless and collisionless
- Long-range gravity
- Velocity & acceleration fluctuation (→ MOND acceleration)
- Halos as fundamental building block (→ Deep MOND)
- Maximum entropy distribution? (the X dist.)
- Flow behavior is scale-dependent
  - Small scale: constant divergence $\nabla \cdot \mathbf{v} = \theta$
  - Large scale: irrotational (curl-free) $\nabla \times \mathbf{v} = 0$
- Mass/energy cascade from small to large mass scales

Energy cascade in hydrodynamic turbulence

- There exist an inertial range with a scale-independent rate of energy cascade (ε does not depend on eddy size l) for eddy size $\eta < l < L$. η is a dissipative scale determined by viscosity ν and ε.

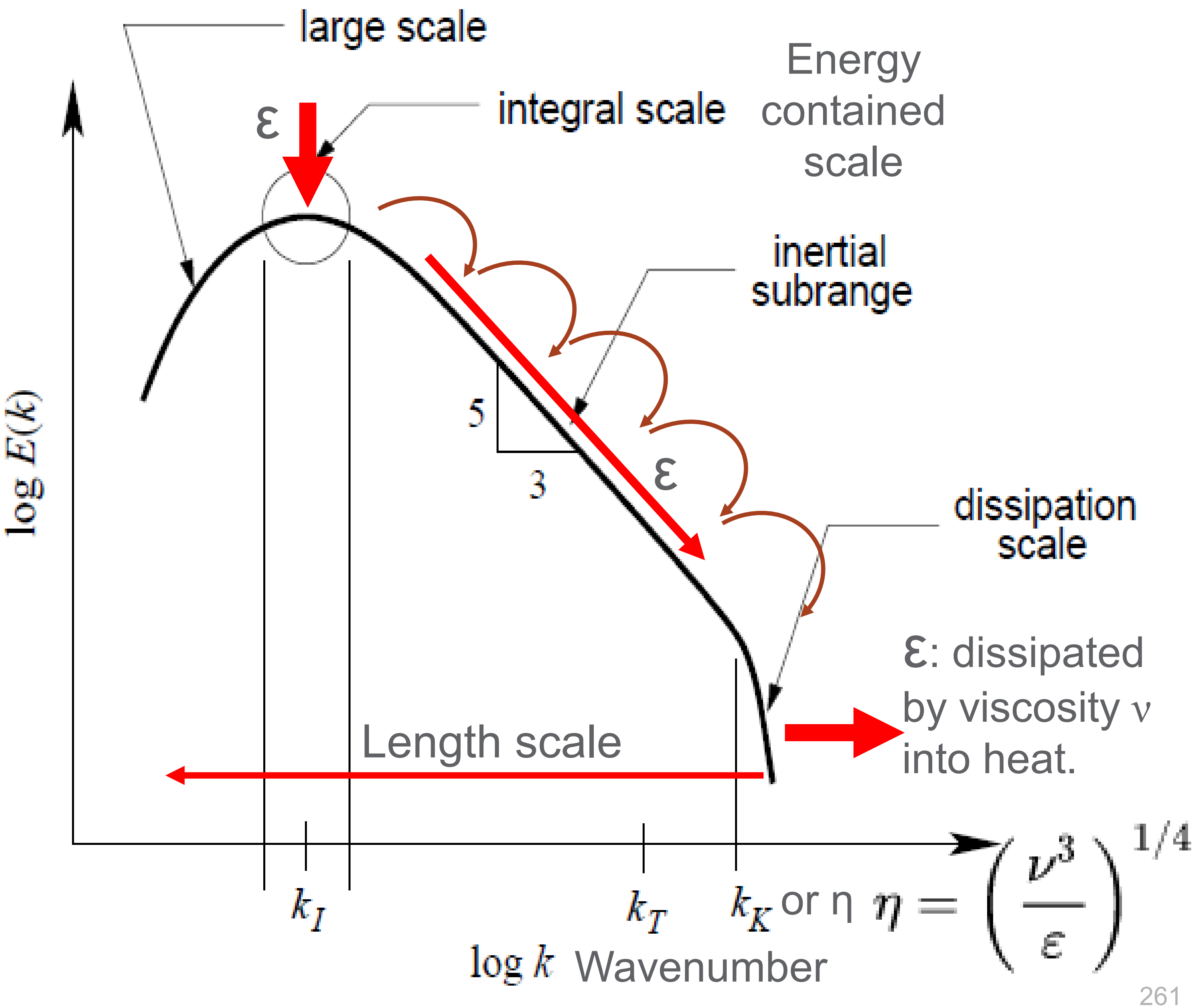
- In this range, inertial force is dominant over viscous force. For eddies with a characteristic velocity u and size l , the lifetime (turnaround time) of eddy is l/u. The rate ε can be computed as the kinetic energy passed per eddy lifetime.

$$\varepsilon \approx \frac{u^2}{(l/u)} \approx \frac{u^2}{l} u \;\Rightarrow\; u^3 \propto l$$

turnaround time

acceleration

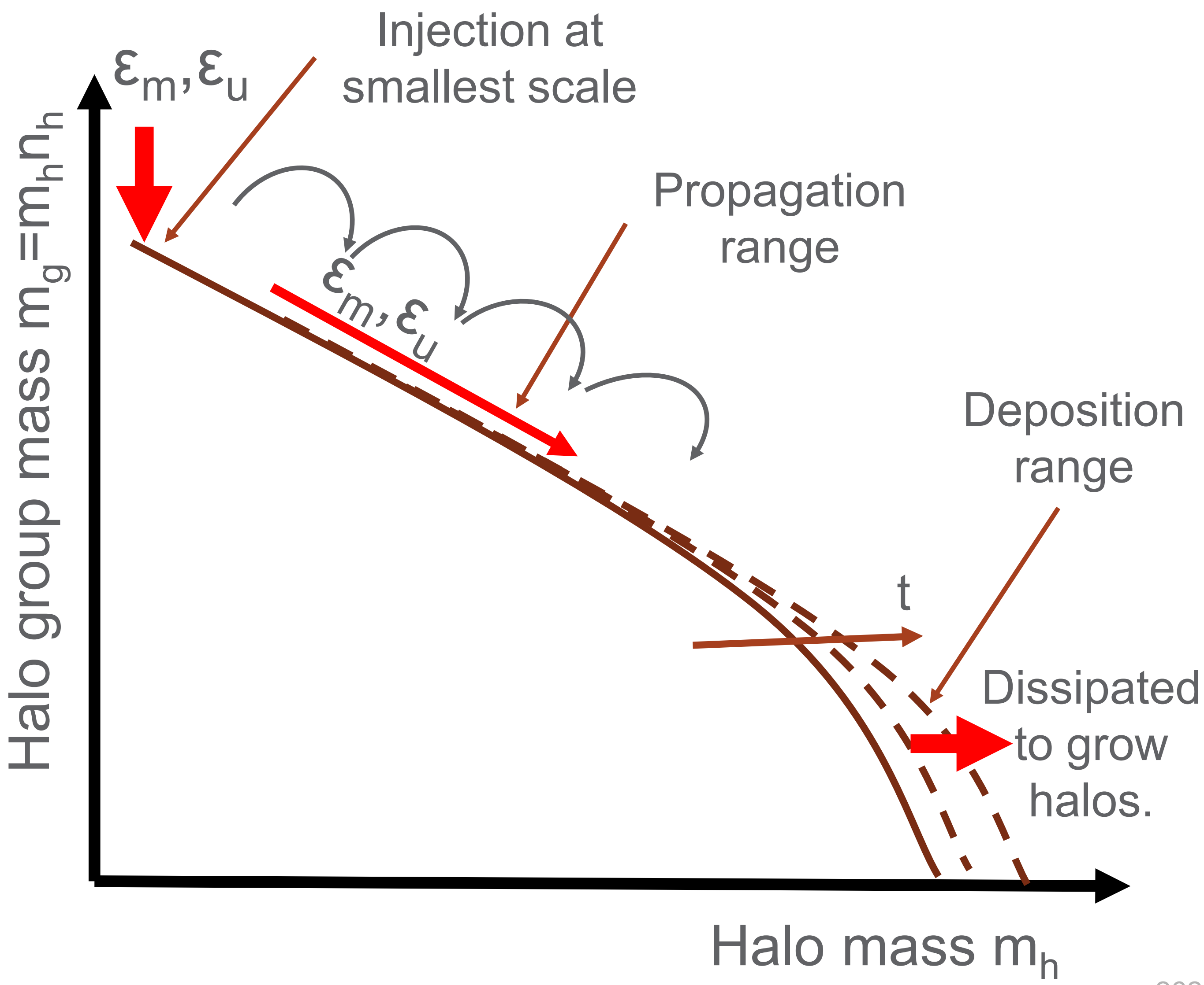
Big whirls have little whirls, That feed on their velocity;
And little whirls have lesser whirls, And so on to viscosity.



Mass/Energy cascade in dark matter flow (SG-CFD)

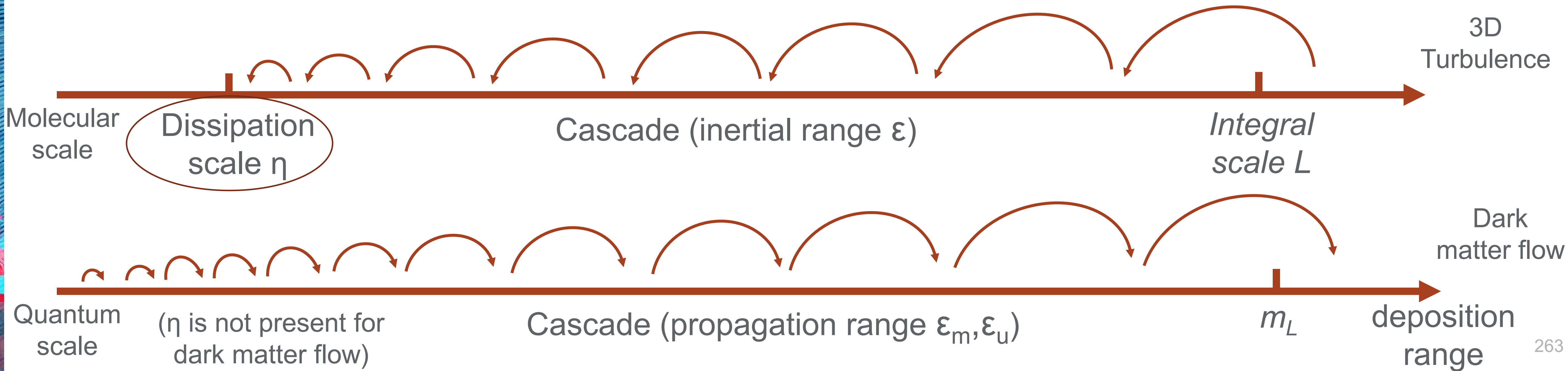
- Collisionless nature and long-range interaction.
- Long-range gravity requires a broad spectrum of halos to be formed to maximize system entropy. No halo structure for short-range forces.
- A continuous cascade of mass/energy from smaller to larger mass scales with a scale-independent rate of mass transfer ε_m and ε_u in a certain range of mass scales (propagation range).
- The mass/energy cascade is an intermediate statistically steady state for non-equilibrium systems to continuously maximize system entropy.
- The maximum entropy distribution of dark matter flow (the X distribution).

Little halos have big halos, That feed on their mass;
And big halos have greater halos, And so on to growth.



Mass/Energy cascade in dark matter flow (SG-CFD)

- Collisionless, no dissipation range in SG-CFD.
- The smallest length scale of inertial range is not limited by viscosity.
- This enable us to extend the scale-independent ε_u down to the smallest scale, where quantum effects become important
- Dark matter flow exhibits scale-dependent flow behaviors for peculiar velocity, i.e. a constant divergence flow on small scales and an irrotational flow on large scales.
- The constant divergence flow shares the same even order kinematic relations with those of incompressible (divergence free) flow. This hints to similar scaling laws holds for dark matter.



Constant (time and scale independent) rate of energy cascade

Power-law time evolution for energy in terms of rate of energy cascade ε_u:

$$K_p = -\varepsilon_{\mathbf{u}} t$$

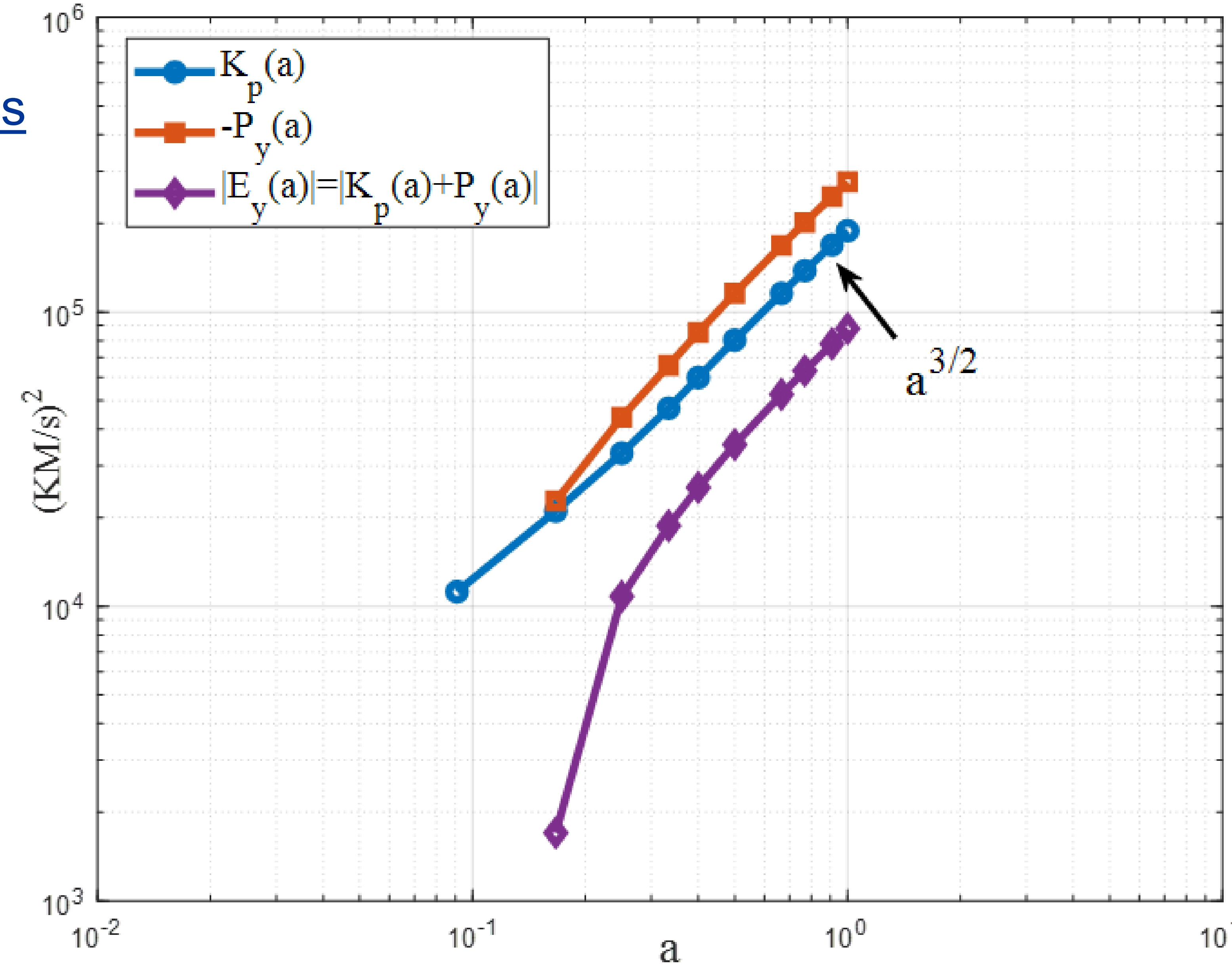
Power-law for Peculiar kinetic energy

$$P_y = \frac{7}{5}\varepsilon_{\mathbf{u}} t$$

Power-law for potential energy

$$\varepsilon_u = -\frac{K_p}{t} = -\frac{3}{2}\frac{u_0^2}{t_0} \approx -4.6\times10^{-7}\frac{m^2}{s^3}$$

Also see detail analysis for inverse kinetic energy cascade.



The time variation of specific kinetic and potential energies from *N*-body simulation.

The maximum entropy distribution in dark matter flow

In dark matter flow, the maximum entropy distribution of velocity can be derived as the X distribution:

$$X(v)=\frac{1}{2\alpha v_0}\frac{e^{-\sqrt{\alpha^2+(v/v_0)^2}}}{K_1(\alpha)}$$

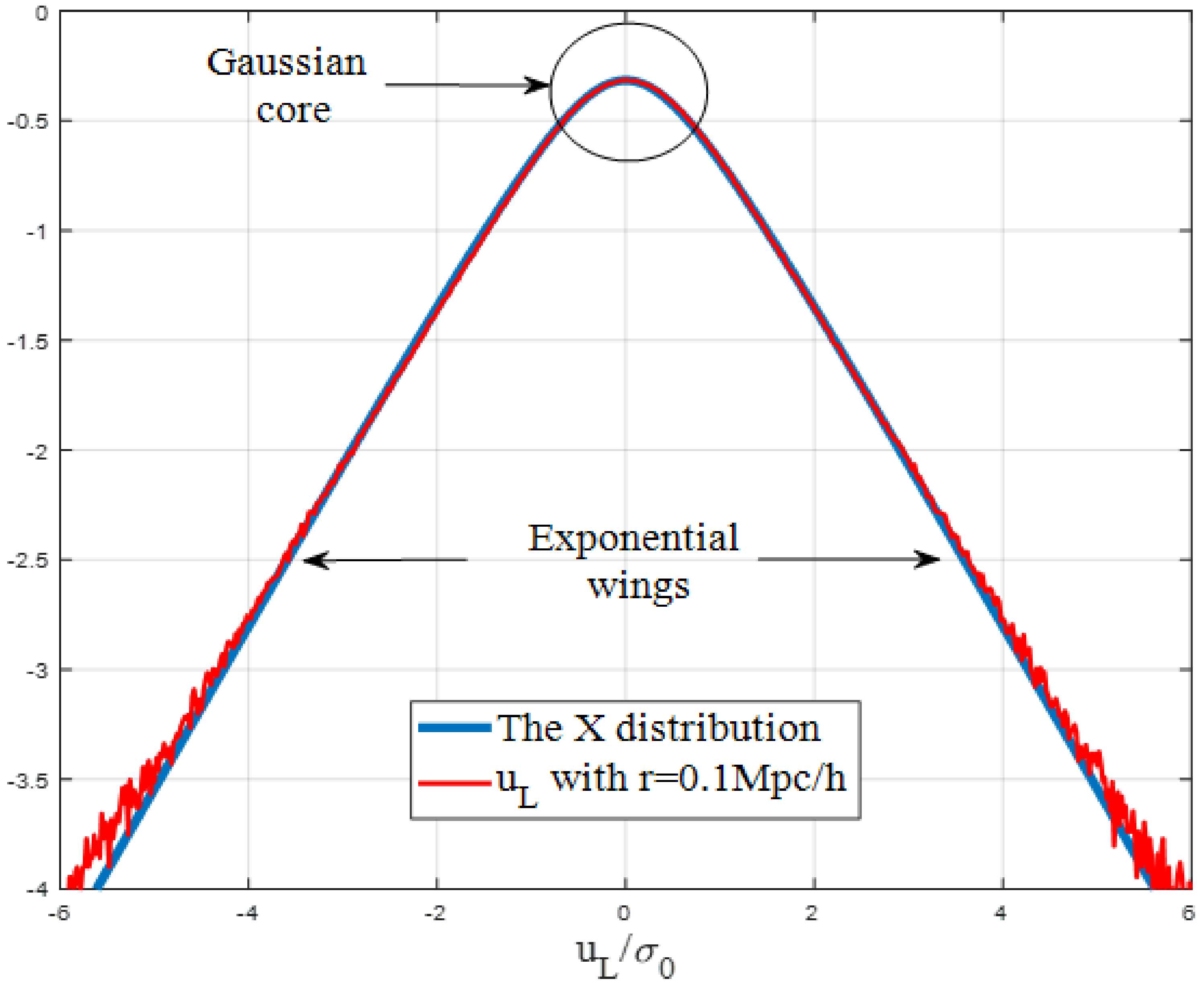
α: shape parameter;
v_0: velocity scale;

The relation between particle energy and velocity can be obtained from X distribution:

Energy per particle with a speed of v:

$$\varepsilon(v)=-\frac{X(v)v}{\partial X/\partial v}\left(\frac{3}{2}+\frac{3}{n}\right)$$

$$\varepsilon(v)=\frac{3}{2}\left(1+\frac{2}{n}\right)v_0^2\sqrt{\alpha^2+\left(\frac{v}{v_0}\right)^2}$$



The *X* distribution with a unit variance compared with the velocity distribution from *N*-body simulation

Particle energy in dark matter flow

$$X(v)=\frac{1}{2\alpha v_0}\frac{e^{-\sqrt{\alpha^2+(v/v_0)^2}}}{K_1(\alpha)}\qquad \varepsilon(v)=-\frac{X(v)v}{\partial X/\partial v}\left(\frac{3}{2}+\frac{3}{n}\right)$$

Particle energy:

$$\varepsilon(v)=\frac{3}{2}\left(1+\frac{2}{n}\right)v_0^2\sqrt{\alpha^2+\left(\frac{v}{v_0}\right)^2}$$

Gaussian core for $|v|\ll v_0$

$$\varepsilon(v)\approx\frac{3}{2}\left(1+\frac{2}{n}\right)\left(\alpha v_0^2+\frac{v^2}{2\alpha}\right)\propto v^2$$

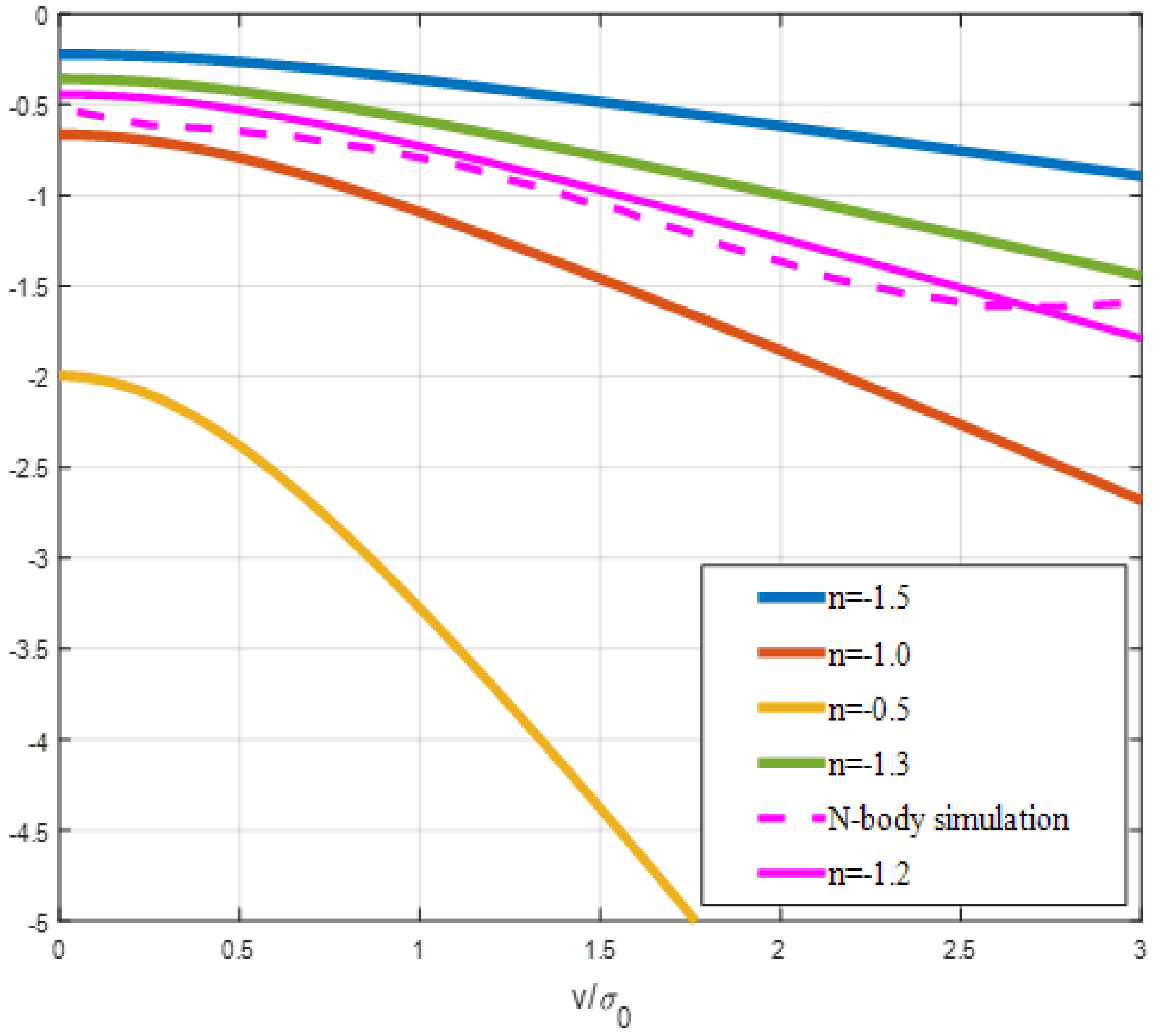
Inner halo, Newtonian behavior

Exponential wings for $|v|\gg v_0$

$$\varepsilon(v)\approx\frac{3}{2}\left(1+\frac{2}{n}\right)v_0 v\propto v$$

Outer region of halo, non-Newtonian behavior

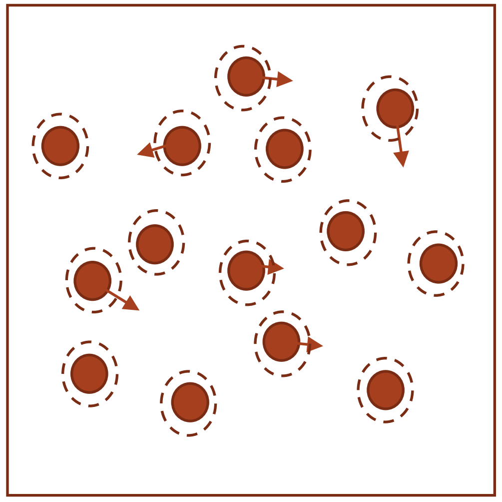
External field effects and MOND??



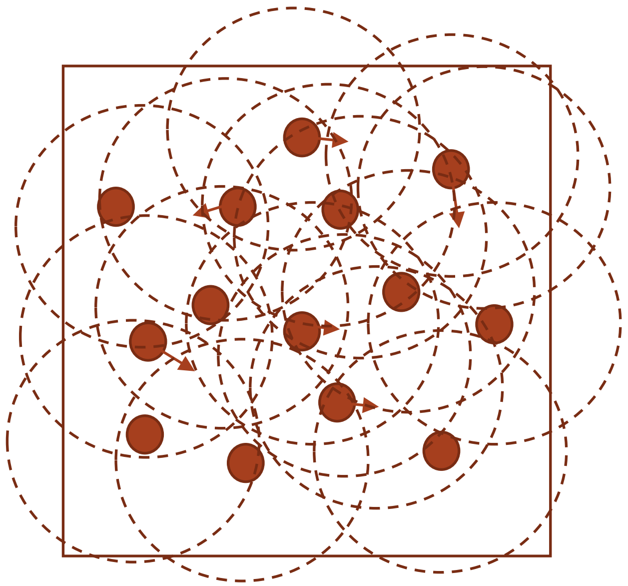
Comparison with N-body simulation

Acceleration fluctuation in dark matter flow

- In kinetic theory of gases, molecules undergo random elastic collisions with a short-range of interaction. Only velocity fluctuation and no fluctuation of acceleration.
- The long-range gravity in dark matter flow leads to fluctuations in acceleration, in addition to the fluctuation in velocity.
- This unique feature hints to the potential generalization of standard Brownian/Langevin dynamics to include acceleration fluctuation in dark matter flow.
- Critical MOND acceleration can be related to the fluctuation of acceleration.



Short range: molecule acceleration vanishes



Long range: nonvanishing and fluctuating acceleration

Acceleration distributions in dark matter flow

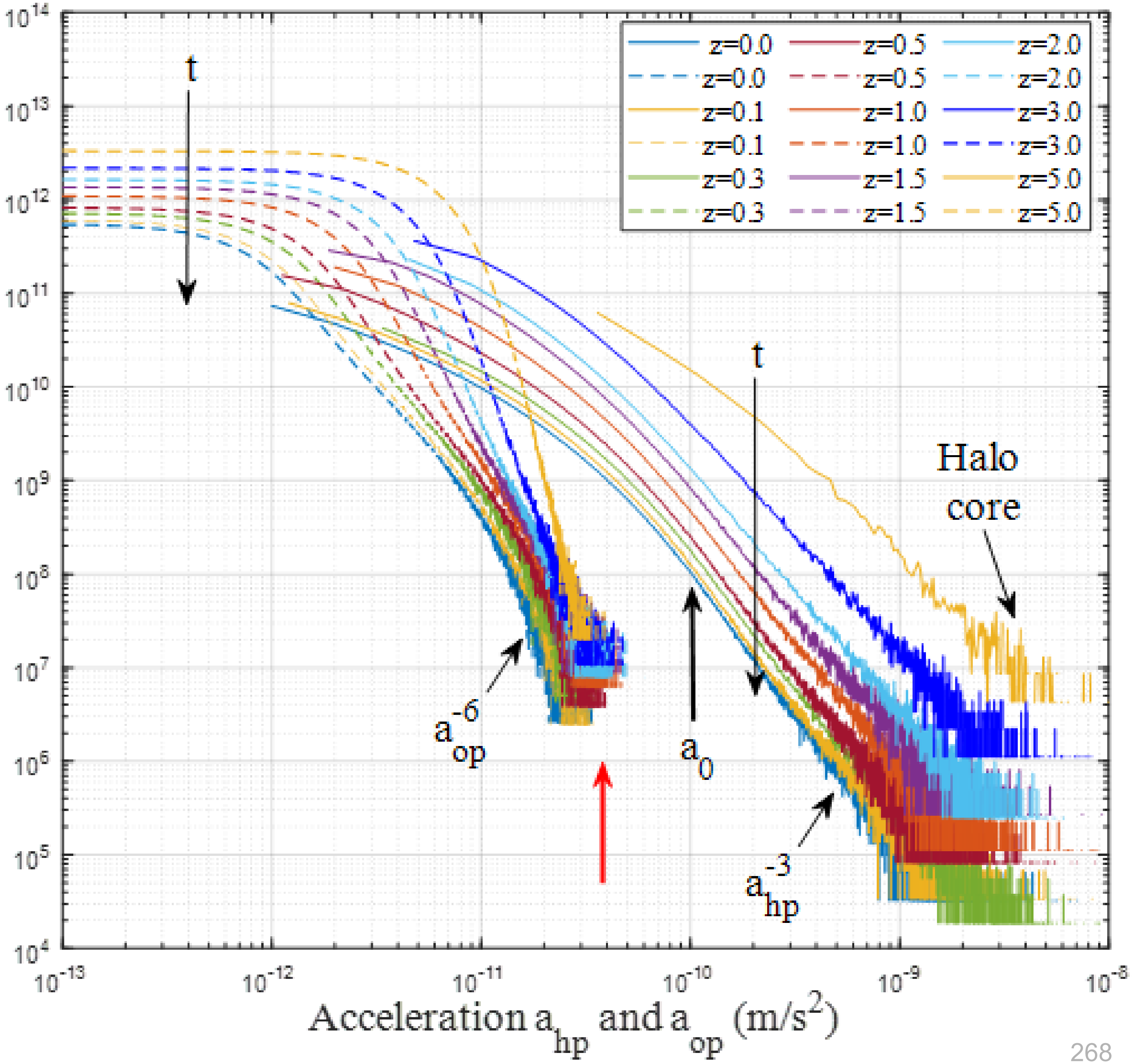
Fluctuation leads to distributions of acceleration

Proper acceleration for particle i:

$$\mathbf{a}_p = \frac{Gm_p}{a^2}\sum_{j\neq i}^{N}\frac{\mathbf{x}_i - \mathbf{x}_j}{\left|\mathbf{x}_i - \mathbf{x}_j\right|^3}$$

Halo-based non-projection approach for acceleration distributions:

- Halo particle acceleration: a_{hp}
- Out-of-halo particles acceleration: a_{op} (Gaussian)
- Acceleration decreases with time
- A long tail $\sim a_{hp}^{-3}$ in halo core region
- MOND acceleration a_0 is right in the middle
- Analytical models of acceleration distribution? (future work)



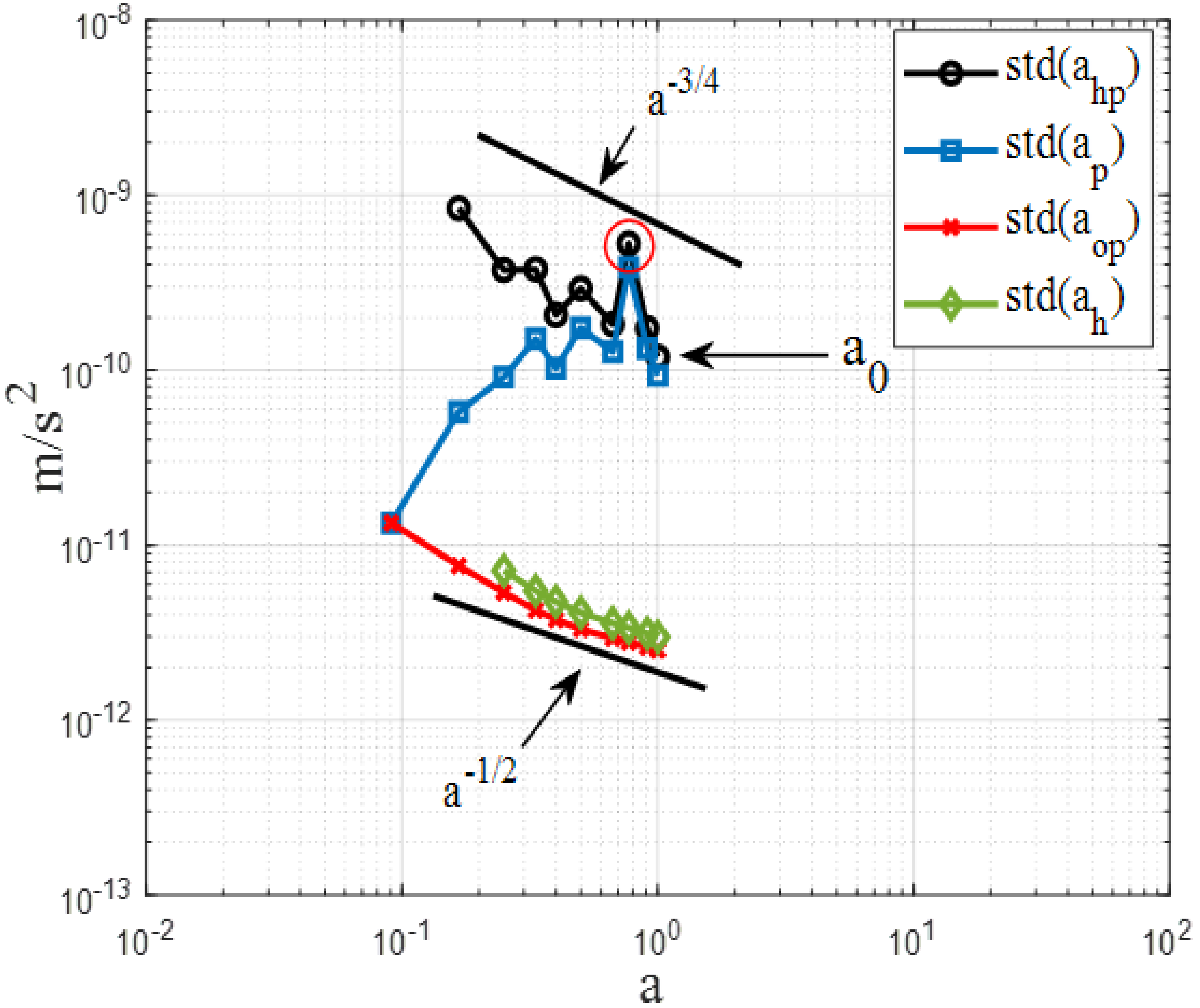
The variation of acceleration with redshift

Halo-based non-projection approach

Root-mean-square accelerations:

- Acceleration of all particles: a_p
- Halo particle acceleration: $a_{hp} \sim a^{-3/4}$
- Out-of-halo particles acceleration: $a_{op} \sim a^{-1/2}$
- Halo acceleration: $a_h \sim a^{-1/2}$

- All typical accelerations decrease with time
- The only exception a_{hp} at z=0.3 requires further confirmation
- Halos and out-of-halo particles have similar accelerations that are much smaller due to greater distance
- At z=0, the typical acceleration of halo particles matches the critical MOND acceleration



The variation of typical (root-mean-square) accelerations with scale factor a

The variation of acceleration with halo size

Acceleration decomposition:
(similar to velocity decomposition)

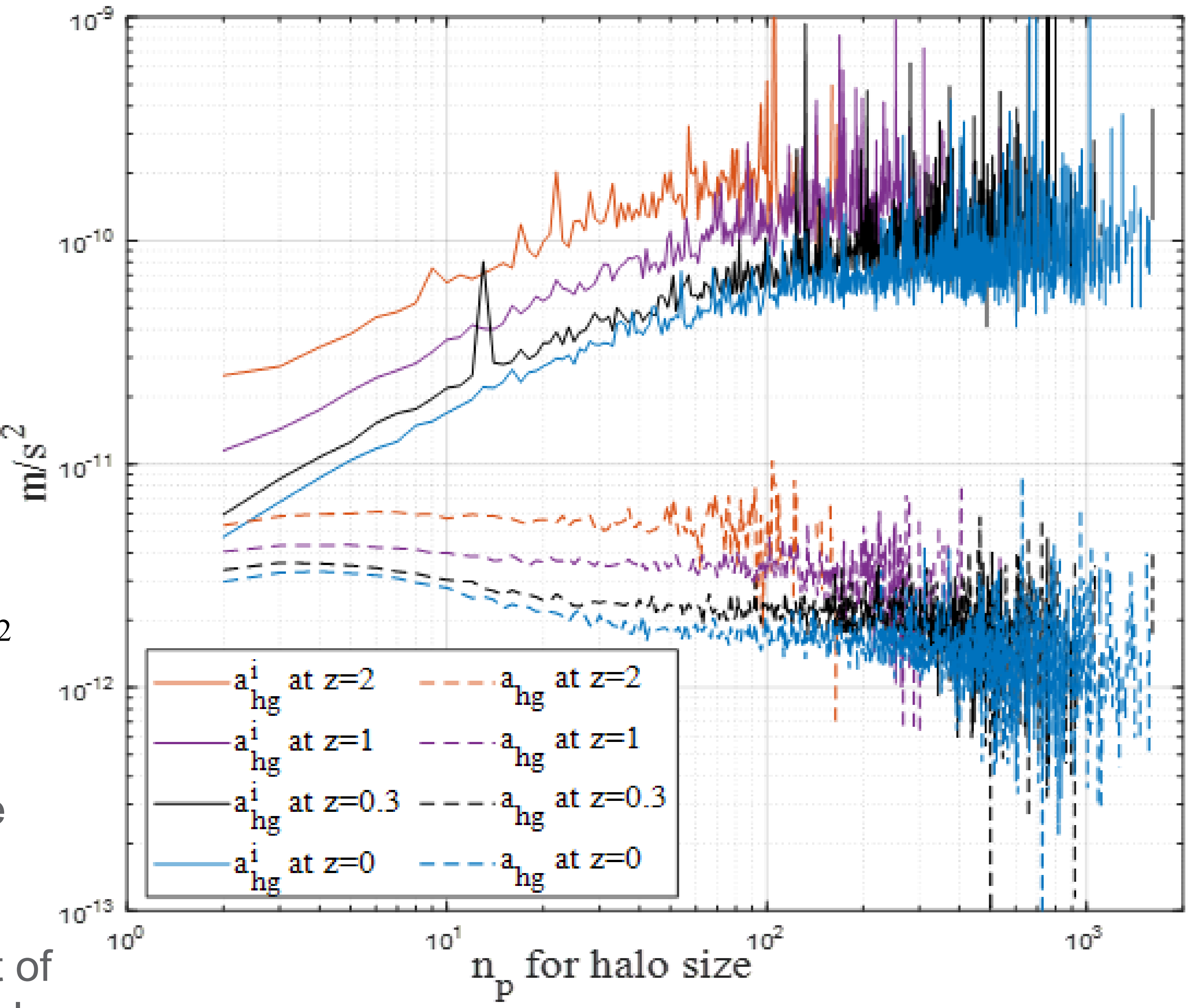
Intra-halo acceleration: $\mathbf{a}_{hp}^{\mathrm{i}} = \mathbf{a}_{hp} - \langle \mathbf{a}_{hp} \rangle_h = \mathbf{a}_{hp} - \mathbf{a}_h$

Halo acceleration (inter-halo): $\mathbf{a}_h = \langle \mathbf{a}_{hp} \rangle_h = \frac{1}{n_p} \sum_{k=1}^{n_p} \mathbf{a}_{hp}$

Group average intra-halo | Inter-halo acceleration

$a_h^{\mathrm{i}} = \left\langle \left| \mathbf{a}_{hp}^{\mathrm{i}} \right|^2 \right\rangle_h^{1/2}$ $\quad a_{hg}^{\mathrm{i}} = \left\langle a_h^{\mathrm{i}} \right\rangle_g$ $\quad a_{hg} = \left\langle \left| \mathbf{a}_h \right|^2 \right\rangle_g^{1/2}$

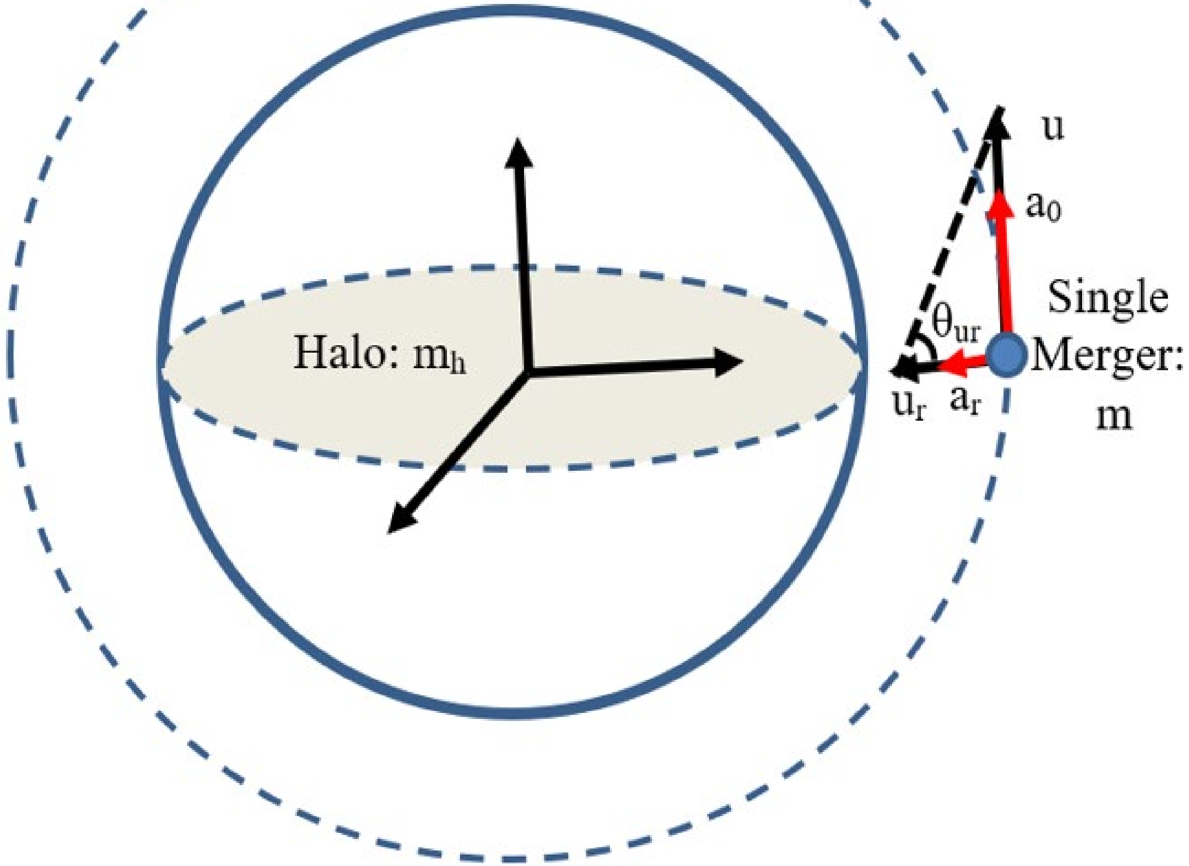
- Acceleration in halos increases with halo size and reaches about for large halos.
- Acceleration of halos is relatively independent of halo size, much smaller than acceleration in halos.



The original of MOND acceleration

Assume a_0 is the typical acceleration scale of fluctuation,
u is the typical velocity scale of fluctuation, θ_{ur} is the angle of incidence.

$$\cot(\theta_{ur}) = \frac{1}{3\pi}$$



The rate of energy cascade in terms of a_0, u and θ_{ur} :

$$\varepsilon_u = -a_r u_r = -a_0(a)\cot(\theta_{ur}) u(a)\cot(\theta_{ur})$$

$$a_0(a) = -\frac{\Delta_c}{2}\cdot\frac{\varepsilon_u}{u} = -(3\pi)^2\frac{\varepsilon_u}{u} = \frac{81}{4}\pi^2 H_0 \frac{u_0^2}{u} \propto a^{-3/4}$$

The rate of energy cascade:

$$\varepsilon_u \approx -\frac{3}{2}\frac{u^2}{t} = -\frac{3}{2}\frac{u_0^2}{t_0} = -\frac{9}{4}H_0 u_0^2 = -4.6\times10^{-7}\frac{m^2}{s^3}$$

$$a_0(a=1) \approx 200 H_0 u_0 \approx 1.2\times10^{-10}\, m/s^2$$

Energy cascade

Potential connection with dark energy??

$$\rho_{vac} = \frac{\Lambda c^2}{8\pi G} = \frac{3\pi}{2G}\left(\frac{(3\pi)^2\varepsilon_u}{u_0}\right)^2 = \frac{3\pi}{2}\frac{a_0^2 H_0}{GH} \propto \frac{a_0^2}{H}$$

$$a_0(z=0) \approx c\frac{(\Lambda/3)^{1/2}}{2\pi}$$

- Ideal gas pressure P ∝ temperature T ∝ velocity fluctuation
- DE density ∝ a_0^2 ∝ acceleration fluctuation (implies an entropic origin?)

The deep MOND behavior

- Fluctuation of acceleration introduces a scale of acceleration a_0;
- Deep MOND for particles with acceleration $a_p \ll a_0$.
- Consider a one-dimensional dark matter flow with a velocity scale v_0 and acceleration scale a_0

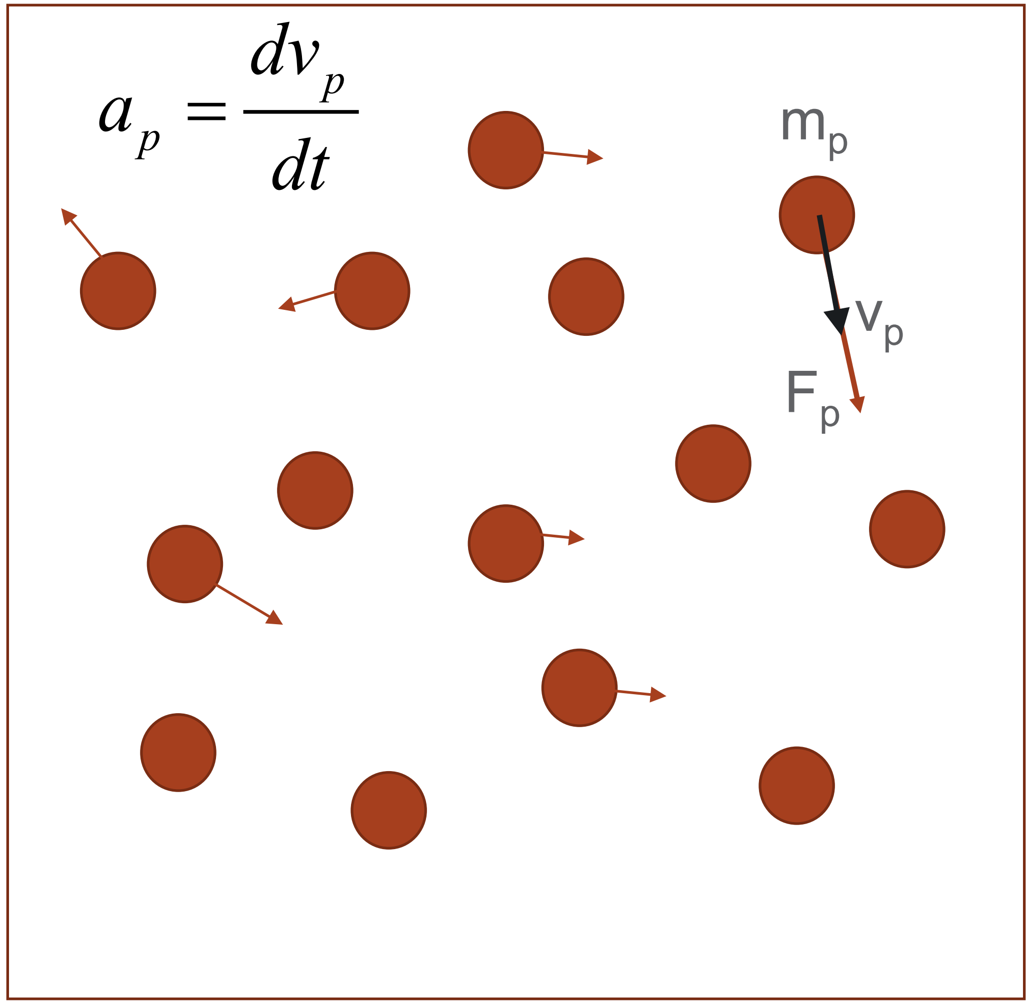
$$\frac{1}{2}\frac{dv_p^2}{dt} = v_p \frac{dv_p}{dt} = a_p v_p = a_0 v_0 = -\varepsilon_u$$

Constant rate of Energy cascade

$$\varepsilon_K(v) = v_0 v_p$$

Maximum entropy distribution: particle kinetic energy is proportional to velocity

$$F_p v_p = m_p \frac{d\varepsilon_K}{dt} \Rightarrow F_p = m_p \frac{v_0}{v_p} a_p = m_p \frac{a_p^2}{a_0} \propto a_p^2$$



Baryonic mass subject to external force F_p is suspended in and in equilibrium with dark matter flow

Summary and keywords

Modified Newtonian Dynamics	Constant rate of energy cascade	Maximum entropy distribution
Critical MOND acceleration	Mass/energy cascade	Deep MOND

- Direct energy cascade from large to small scales in hydrodynamic turbulence
- Inverse energy cascade in dark matter flow from small to large mass scales with a constant rate of cascade
- Long-range interaction of dark matter flow leads to a fluctuation in acceleration with a typical scale a_0
- The acceleration fluctuation in N-body simulation exactly matches the value of critical MOND acceleration
- The acceleration fluctuation in dark matter flow as the origin of MOND acceleration that can be related to the constant rate of energy flux.
- Suggest dark energy density might be also related to the acceleration fluctuation.
- Both Newtonian dynamics and “deep-MOND” behavior can be recovered based on the maximum entropy distribution and energy cascade in dark matter flow.