Pacific Northwest NATIONAL LABORATORY

A comparative study of dark matter flow & hydrodynamic turbulence and its applications

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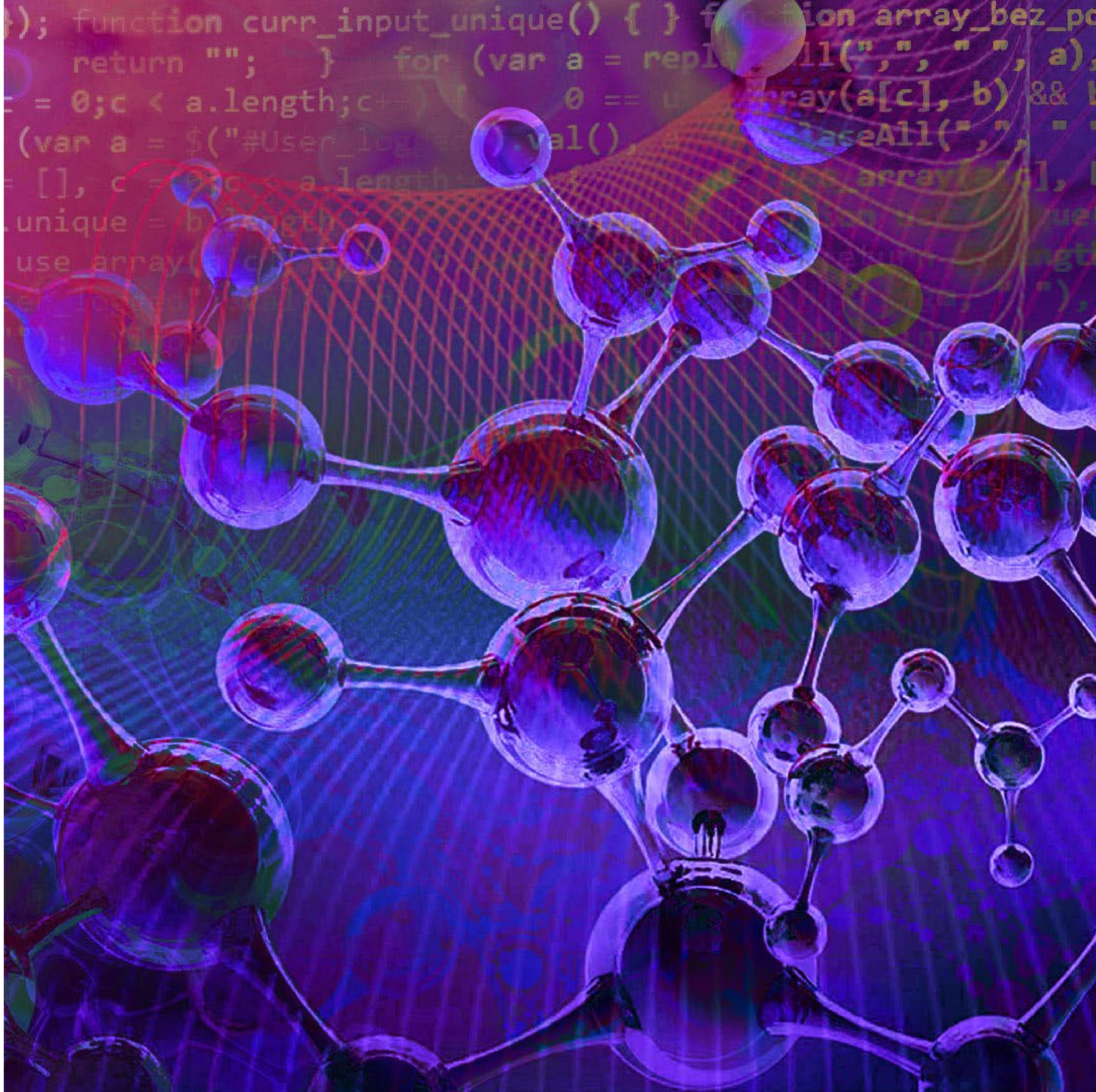


Preface

Dark matter, if exists, accounts for five times as much as ordinary baryonic matter. Therefore, dark matter flow might possess the widest presence in our universe. The other form of flow, hydrodynamic turbulence in air and water, is without doubt the most familiar flow in our daily life. During the pandemic, we have found time to think about and put together a systematic comparison for the connections and differences between two types of flow, both of which are typical non-equilibrium systems.

The goal of this presentation is to leverage this comparison for a better understanding of the nature of dark matter and its flow behavior on all scales. Science should be open. All comments are welcome.

Thank you!

Data repository and relevant publications

Structural (halo-based) approach:

0.	**Data https://dx.doi.org/10.5281/zenodo.6541230**
1.	Inverse mass cascade in dark matter flow and effects on halo mass functions https://doi.org/10.48550/arXiv.2109.09985
2.	Inverse mass cascade in dark matter flow and effects on halo deformation, energy, size, and density profiles https://doi.org/10.48550/arXiv.2109.12244
3.	Inverse energy cascade in self-gravitating collisionless dark matter flow and effects of halo shape https://doi.org/10.48550/arXiv.2110.13885
4.	The mean flow, velocity dispersion, energy transfer and evolution of rotating and growing dark matter halos https://doi.org/10.48550/arXiv.2201.12665
5.	Two-body collapse model for gravitational collapse of dark matter and generalized stable clustering hypothesis for pairwise velocity https://doi.org/10.48550/arXiv.2110.05784
6.	Evolution of energy, momentum, and spin parameter in dark matter flow and integral constants of motion https://doi.org/10.48550/arXiv.2202.04054
7.	The maximum entropy distributions of velocity, speed, and energy from statistical mechanics of dark matter flow https://doi.org/10.48550/arXiv.2110.03126
8.	Halo mass functions from maximum entropy distributions in collisionless dark matter flow https://doi.org/10.48550/arXiv.2110.09676

Statistics (correlation-based) approach:

0.	**Data https://dx.doi.org/10.5281/zenodo.6569898**
1.	The statistical theory of dark matter flow for velocity, density, and potential fields https://doi.org/10.48550/arXiv.2202.00910
2.	The statistical theory of dark matter flow and high order kinematic and dynamic relations for velocity and density correlations https://doi.org/10.48550/arXiv.2202.02991
3.	The scale and redshift variation of density and velocity distributions in dark matter flow and two-thirds law for pairwise velocity https://doi.org/10.48550/arXiv.2202.06515
4.	Dark matter particle mass and properties from two-thirds law and energy cascade in dark matter flow https://doi.org/10.48550/arXiv.2202.07240
5.	The origin of MOND acceleration and deep-MOND from acceleration fluctuation and energy cascade in dark matter flow https://doi.org/10.48550/arXiv.2203.05606
6.	The baryonic-to-halo mass relation from mass and energy cascade in dark matter flow https://doi.org/10.48550/arXiv.2203.06899

Applications of dark matter flow

The baryonic-to-halo mass relation from mass and energy cascade in dark matter flow

Xu Z., 2022, arXiv:2203.06899v1 [astro-ph.GA]

https://doi.org/10.48550/arXiv.2203.06899

Introduction

- The existence of dark matter (DM) is supported by numerous astronomical observations:
  - Flat rotation curves of spiral galaxies
  - Motion of galaxies in galaxy clusters
  - Gravitational lensing, Bullet clusters, CMB ……
- Though the nature of dark matter is still unclear, dark matter is believed to be cold (non-relativistic), collisionless, dissipationless, non-baryonic, barely interacting with baryons except through gravity, and sufficiently smooth with a fluid-like behavior.
- Total galaxy baryonic mass = stellar mass + cold gas.
- Stellar-to-halo mass relation (SHMR)
  - halo abundance matching approach
- Baryonic-to-halo mass relation (BHMR)

- Baryonic Tully and Fisher relation (BTFR):

$$v_f^4 = G m_b a_0$$ ← observed baryonic mass

- Halo mass m_h can be related to the halo virial radius r_h through constant density ratio Δ_c

$$m_h = \frac{4}{3}\pi (r_h)^3 \Delta_c \bar{\rho}_0 (a)$$

- The BHMR (m_b and m_h) can be obtained only if the relation between v_f and r_h is known.
- The BHMR from the mass and energy cascade of dark matter flow?
- What is the average mass fraction of baryons in all halos?
- What is the fraction of total baryons residing in all galaxies?

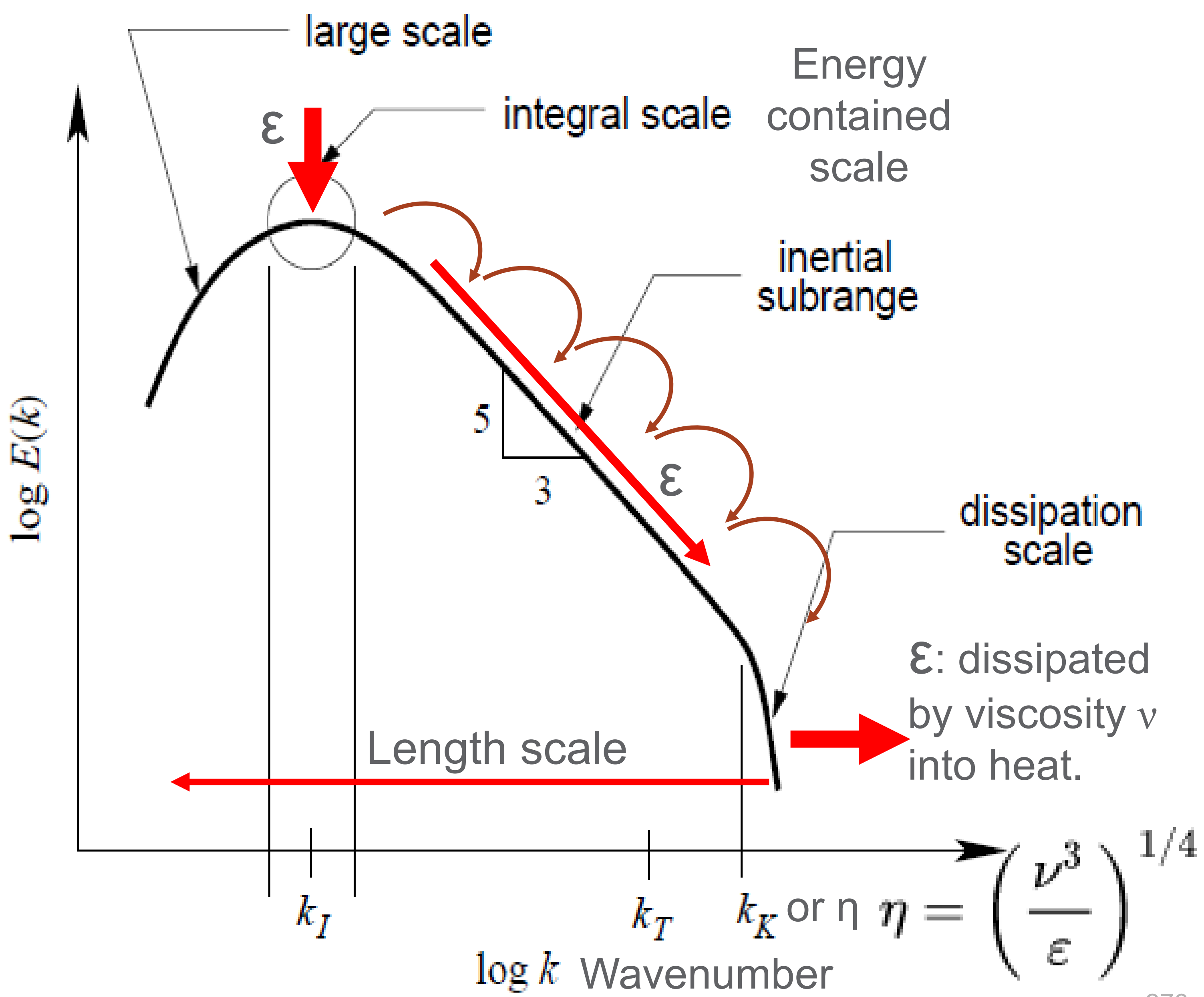
Energy cascade in hydrodynamic turbulence

- There exist an inertial range with a scale-independent rate of energy cascade (ε does not depend on eddy size l) for eddy size $\eta < l < L$. η is a dissipative scale determined by viscosity ν and ε.
- In this range, inertial force is dominant over viscous force. For eddies with a characteristic velocity u and size l , the lifetime (turnaround time) of eddy is l/u. The rate ε can be computed as the kinetic energy passed per eddy lifetime.

$$\varepsilon \approx \frac{u^2}{(l/u)} \approx \frac{u^2}{l} u \Rightarrow u^3 \propto l$$

turnaround time — acceleration

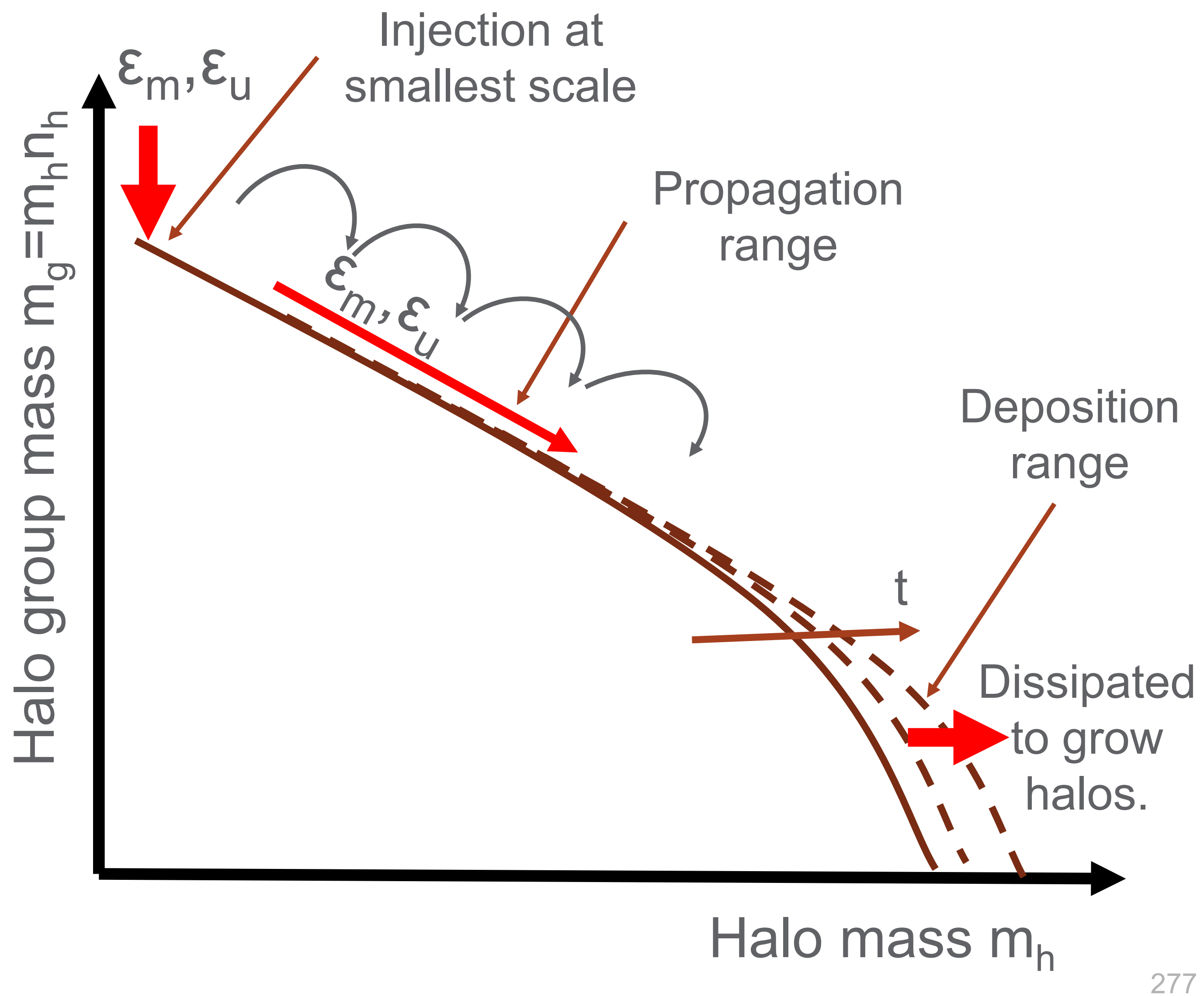
Big whirls have little whirls, That feed on their velocity;
And little whirls have lesser whirls, And so on to viscosity.



Mass/Energy cascade in dark matter flow (SG-CFD)

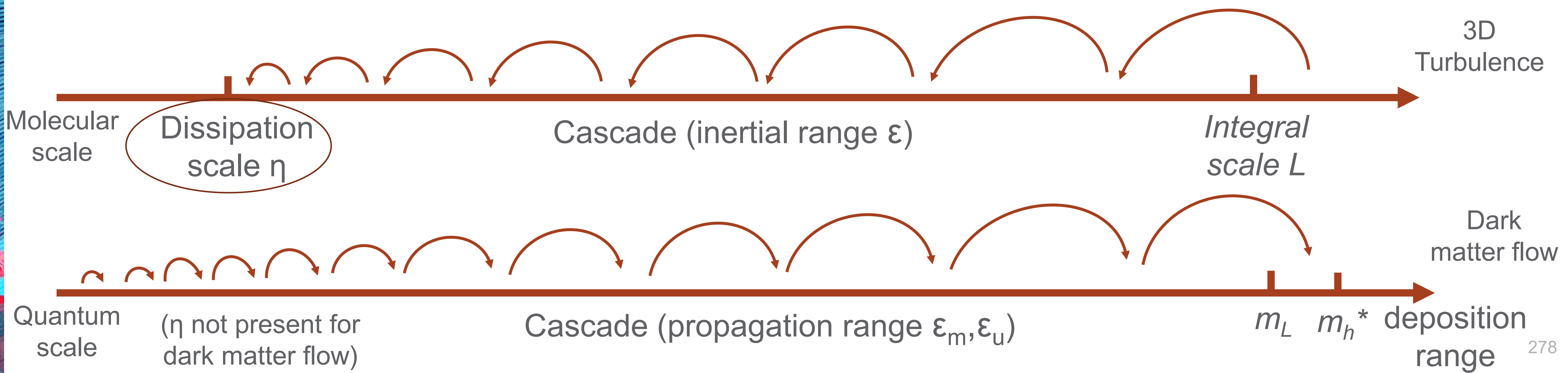
- Collisionless nature and long-range interaction.
- Long-range gravity requires a broad spectrum of halos to be formed to maximize system entropy. No halo structure for short-range forces.
- A continuous cascade of mass/energy from smaller to larger mass scales with a scale-independent rate of mass transfer ε_m and ε_u in a certain range of mass scales (propagation range).
- The mass/energy cascade is an intermediate statistically steady state for non-equilibrium systems to continuously maximize system entropy.
- The maximum entropy distribution of dark matter flow (the X distribution).

Little halos have big halos, That feed on their mass;
And big halos have greater halos, And so on to growth.



Mass/Energy cascade in dark matter flow (SG-CFD)

- Collisionless, no dissipation range in SG-CFD.
- The smallest length scale of inertial range is not limited by viscosity.
- This enable us to extend the scale-independent ε_u down to the smallest scale, where quantum effects become important
- Dark matter flow exhibits scale-dependent flow behaviors for peculiar velocity, i.e. a constant divergence flow on small scales and an irrotational flow on large scales.
- The constant divergence flow shares the same even order kinematic relations with those of incompressible (divergence free) flow. This hints to similar scaling laws holds for dark matter.



Constant (time and scale independent) rate of energy cascade

Power-law time evolution for energy in terms of rate of energy cascade ε_u:

$$K_p = -\varepsilon_{\mathbf{u}} t$$

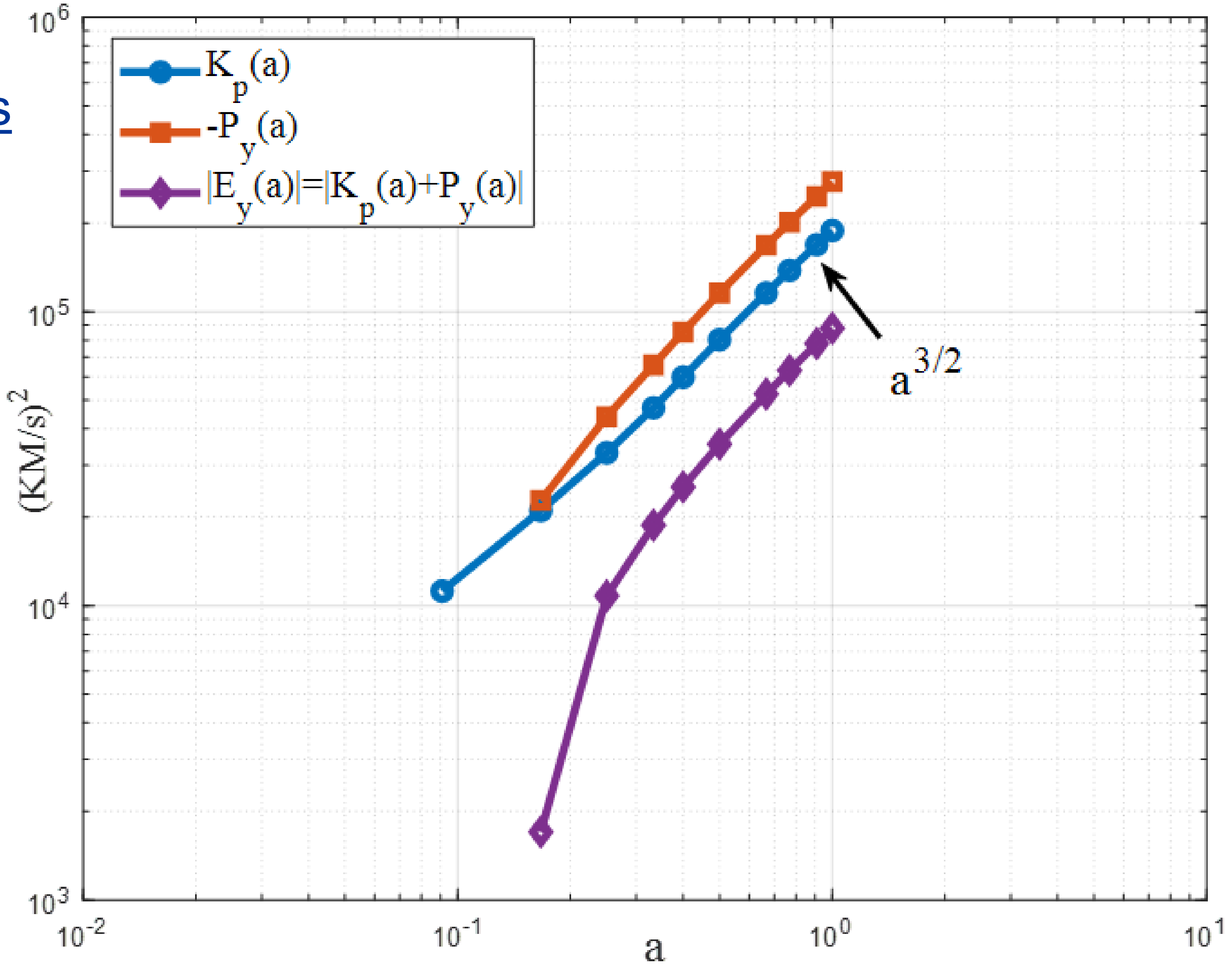
Power-law for Peculiar kinetic energy

$$P_y = \frac{7}{5}\varepsilon_{\mathbf{u}} t$$

Power-law for potential energy

$$\varepsilon_u = -\frac{K_p}{t} = -\frac{3}{2}\frac{u_0^2}{t_0} \approx -4.6\times10^{-7}\frac{m^2}{s^3}$$

Also see detail analysis for inverse kinetic energy cascade.



The time variation of specific kinetic and potential energies from *N*-body simulation.

Dimensional analysis for critical mass scales

The smallest mass scale (dark matter particle mass)

At the smallest scale, three fundamental constants:

Gravitational constant	$G = 6.67 \times 10^{-11}\, m^3/(kg \cdot s^2)$
Rate of energy cascade	$\varepsilon_u = -4.6 \times 10^{-7}\, m^2/s^3$
Planck constant	$\hbar = 1.05 \times 10^{-34}\, kg \cdot m^2/s$

Simple dimensional analysis predicts:

Mass scale:
$$m_X \propto \left(-\varepsilon_u \hbar^5 / G^4\right)^{\frac{1}{9}} \approx 8.7 \times 10^{-16}\, kg = 0.5 GeV$$

Length scale: $l_X \propto \left(-G\hbar/\varepsilon_u\right)^{\frac{1}{3}}$

Time scale: $t_X \propto \left(G^2\hbar^2/\varepsilon_u^5\right)^{\frac{1}{9}}$

The largest mass scale (critical halo mass)

Three fundamental constants:

Gravitational constant	$G = 6.67 \times 10^{-11}\, m^3/(kg \cdot s^2)$
Rate of energy cascade	$\varepsilon_u = -4.6 \times 10^{-7}\, m^2/s^3$
Velocity dispersion or Hubble constant H	$u_0 \equiv u(a=1) = 354.61\, km/s$

Simple dimensional analysis predicts:

Mass scale: $m_L \propto -u_0^5/(G\varepsilon_u) \approx 9.14 \times 10^{13} M_\odot$

Length scale: $l_L \propto -u_0^3/\varepsilon_u \approx 3.14 Mpc$

Time scale: $t_L \propto u_0^2/\varepsilon_u \approx 8.7 \times 10^9\, yr$

The baryonic-to-halo mass ratio from energy cascade

Baryonic Tully-Fisher relation (BTFR):

$$v_f^4 = Gm_b a_0$$

Halo mass and halo size relation:

$$m_h = \frac{4}{3}\pi r_h^3 \Delta_c \bar{\rho}_0 a^{-3}$$

Baryonic Tully-Fisher relation (BTFR):

$$v_f^4 = Gm_b a_0$$

Halo mass and halo size relation:

$$m_h = \frac{4}{3}\pi r_h^3 \Delta_c \bar{\rho}_0 a^{-3}$$

Rate of energy cascade

$$\varepsilon_u = -\beta_f \frac{u^2}{r_h/v_f} a^q$$

Small halos $<m_L$: Baryonic mass in equilibrium with DM, i.e. same kinetic energy u^2

$$v_{cir} = \frac{4}{9}\sqrt{\frac{\Delta_c}{2}}\beta_f v_f a^q \propto (m_h)^{1/3} a^{-1/2}$$

$$r_h = \frac{4}{9}\beta_f v_f H^{-1} a^q \propto (m_h)^{1/3} a^1$$

$$v_f = \frac{9}{4\beta_f}\left(\frac{2}{\Delta_c}\right)^{\frac{1}{3}} (Gm_h H)^{1/3} a^{-q} \propto (m_h)^{1/3} a^0$$

$$\varepsilon_u = -\alpha_f \frac{v_f^2}{r_h/v_f} a^p$$

Turnaround time

Large halos $>m_L$: Baryonic mass and DM are two miscible phases sharing same rate of cascade.

$$v_{cir} = \frac{4}{9}\sqrt{\frac{\Delta_c}{2}}\alpha_f \frac{v_f^3}{u^2} a^p \propto (m_h)^{1/3} a^{-1/2}$$

$$r_h = \frac{4}{9}\alpha_f \frac{v_f^3}{Hu^2} a^p \propto (m_h)^{1/3} a^1$$

$$v_f = \left(\frac{3}{2\sqrt{\alpha_f}}\right)^{\frac{2}{3}} \left(\frac{2}{\Delta_c}\right)^{\frac{1}{9}} (Gm_h H)^{1/9} u^{2/3} a^{-p/3} \propto (m_h)^{1/9} a^{(1-p)/3}$$

Critical scales and Baryonic-Halo-Mass Ratio

Critical rotation speed: $v_{fc} = ua^{(q-p)/2}\sqrt{\beta_f/\alpha_f}$

Critical circular speed: $v_{cc} = \frac{4}{9}\sqrt{\frac{\Delta_c}{2}}\sqrt{\frac{\beta_f^3}{\alpha_f}ua^{(3q-p)/2}}$

Critical halo size: $r_{hc} = \frac{4}{9}a^{(3q-p)/2}uH^{-1}\beta_f\sqrt{\beta_f/\alpha_f}$

Critical halo mass: $m_{hc} = \frac{16}{81}\left(\frac{\beta_f^3}{\alpha_f}\right)^{3/2}\left(\frac{\Delta_c}{2}\right)\left(\frac{u^5}{G\varepsilon_u}\right)a^{\frac{3}{2}(3q-p)}$

Critical baryonic mass: $m_{bc} = \frac{2}{\Delta_c}\left(\frac{\beta_f}{\alpha_f}\right)^2\left(\frac{u^5}{G\varepsilon_u}\right)a^{2(q-p)}$

Mass scale m_L

The baryonic mass in small halos:

$$m_b = \left(M_{c1}\right)^{-1/3}\left(m_h\right)^{4/3} \qquad M_{c1}(a) = \left(\frac{2}{3}\right)^{16}\left(\beta_f a^q\right)^{12}\left(\frac{\Delta_c}{2}\right)^7\left(\frac{u^5}{G\varepsilon_u}\right)$$

The baryonic mass in large halos:

$$m_b = \left(M_{c2}\right)^{5/9}\left(m_h\right)^{4/9} \qquad M_{c2}(a) = \left(\frac{2}{3}\right)^{-\frac{16}{5}}\left(\alpha_f a^p\right)^{-\frac{12}{5}}\left(\frac{2}{\Delta_c}\right)^{\frac{13}{5}}\left(\frac{u^5}{G\varepsilon_u}\right)$$

The baryonic-halo-mass ratio in critical halos:

$$A(z) \equiv \frac{m_{bc}}{m_{hc}} = \left(\frac{M_{c2}}{M_{c1}}\right)^{5/24} = \frac{81\left(2/\Delta_c\right)^2}{16\left(\alpha_f\right)^{1/2}\left(\beta_f\right)^{5/2}}a^{-(5q+p)/2}$$

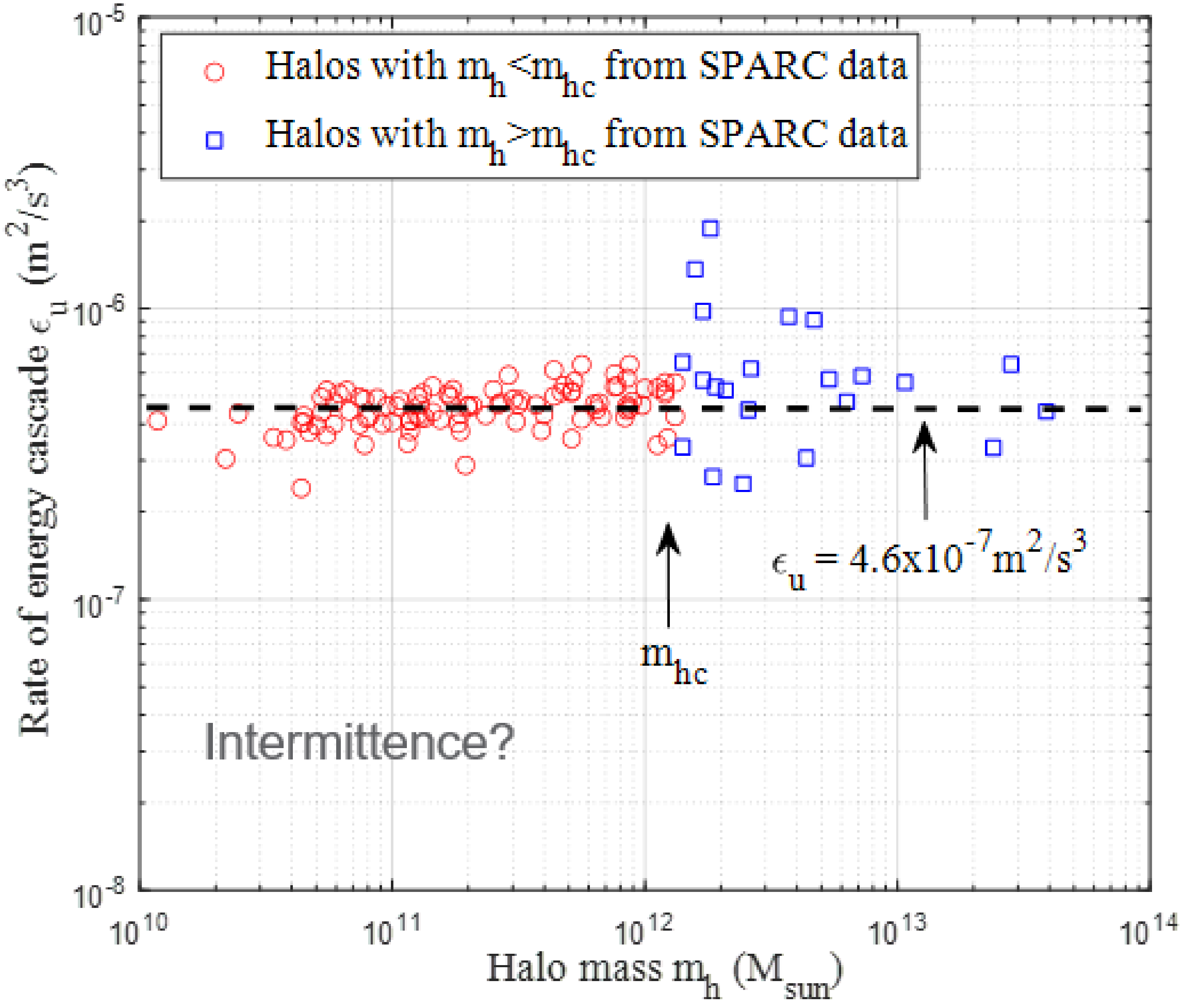
$$A(z=0) \approx 0.076$$

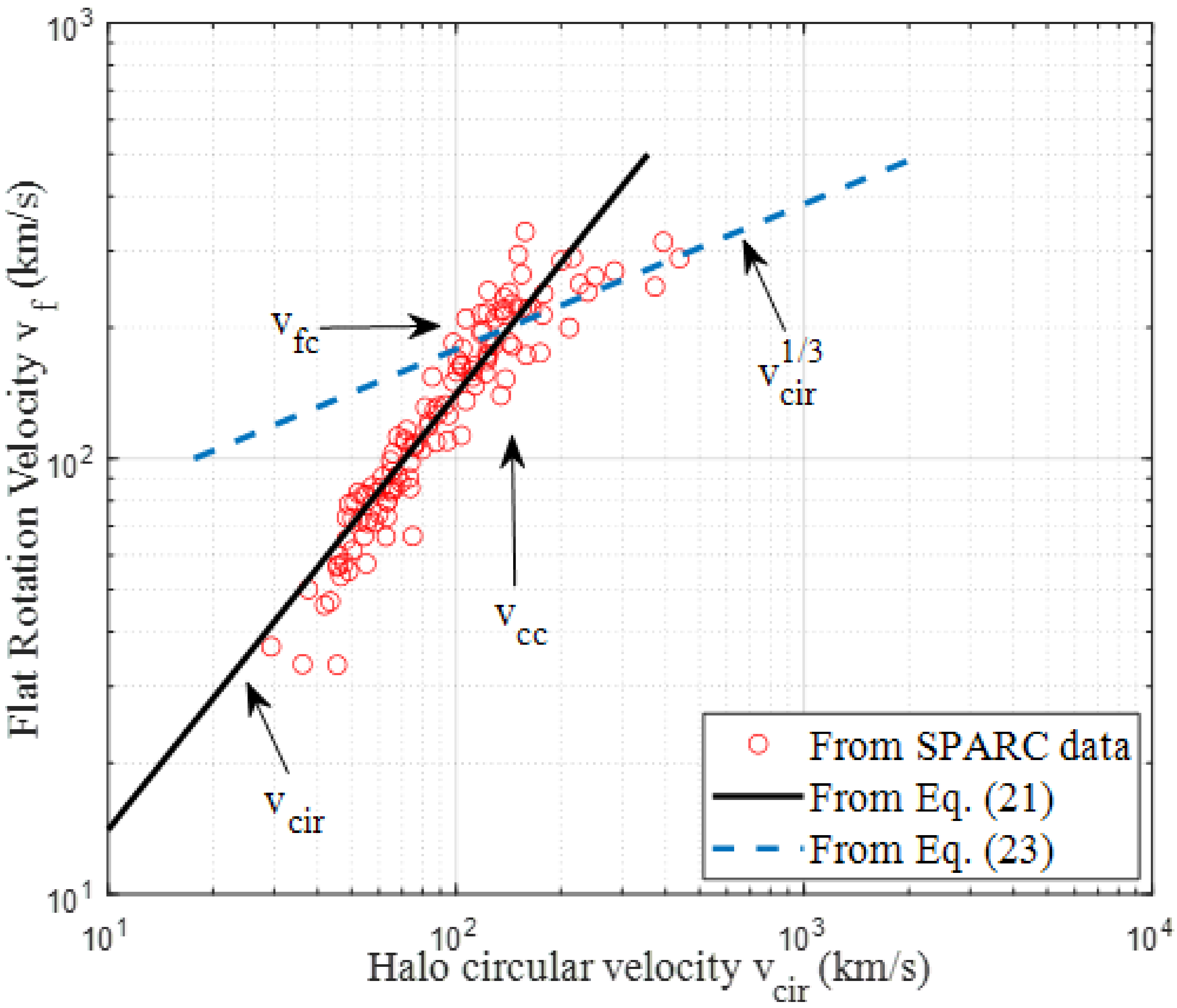
Relevant parameters for baryonic-to-halo mass ratio

Table 2. Parameters for deriving baryonic-to-halo mass ratio

Δ_c	200	p	$7/4$	M_{c1}	$3.01\times10^{15}a^{-9/4}M_{sun}$
ε_u	$4.6\times10^{-7}\,m^2/s^3$	q	$-1/2$	M_{c2}	$1.29\times10^{10}a^{-9/20}M_{sun}$
H_0	$1.62\times10^{-18}\,1/s$	α_f	0.5	m_{hc}	$1.33\times10^{12}a^{-9/8}M_{sun}$
u_0	$354.61\,km/s$	β_f	0.16	m_{bc}	$1.01\times10^{11}a^{-3/4}M_{sun}$
$a_0(z=0)$	$1.2\times10^{-10}\,m/s^2$	m	4	$A(z)$	$0.0761a^{3/8}$
η_0	0.76	q_0	0.556	m_h^*	$4\times10^{13}a^{3/2}M_{sun}$ [27]

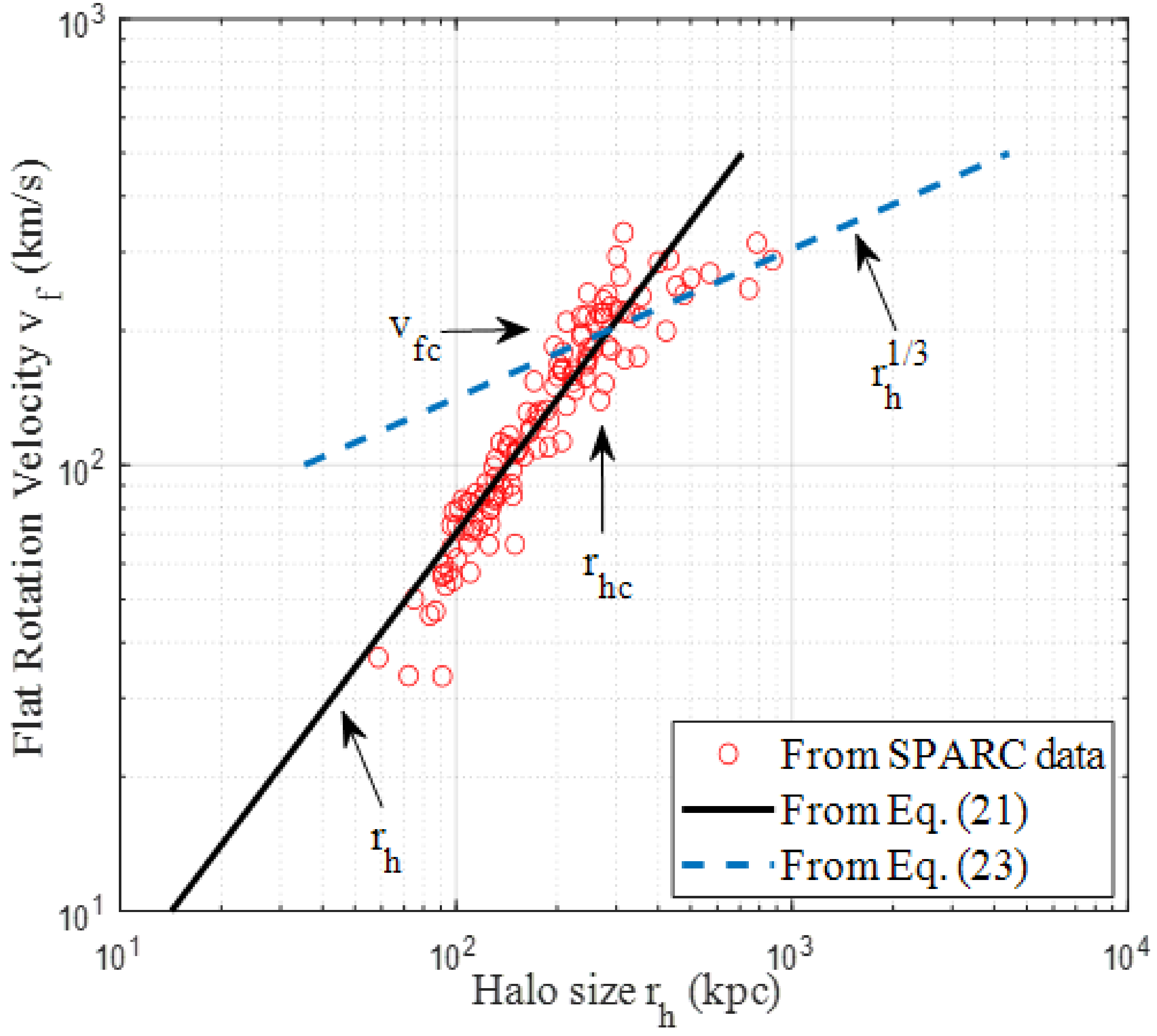
SPARC (Spitzer Photometry & Accurate Rotation Curves) data and model

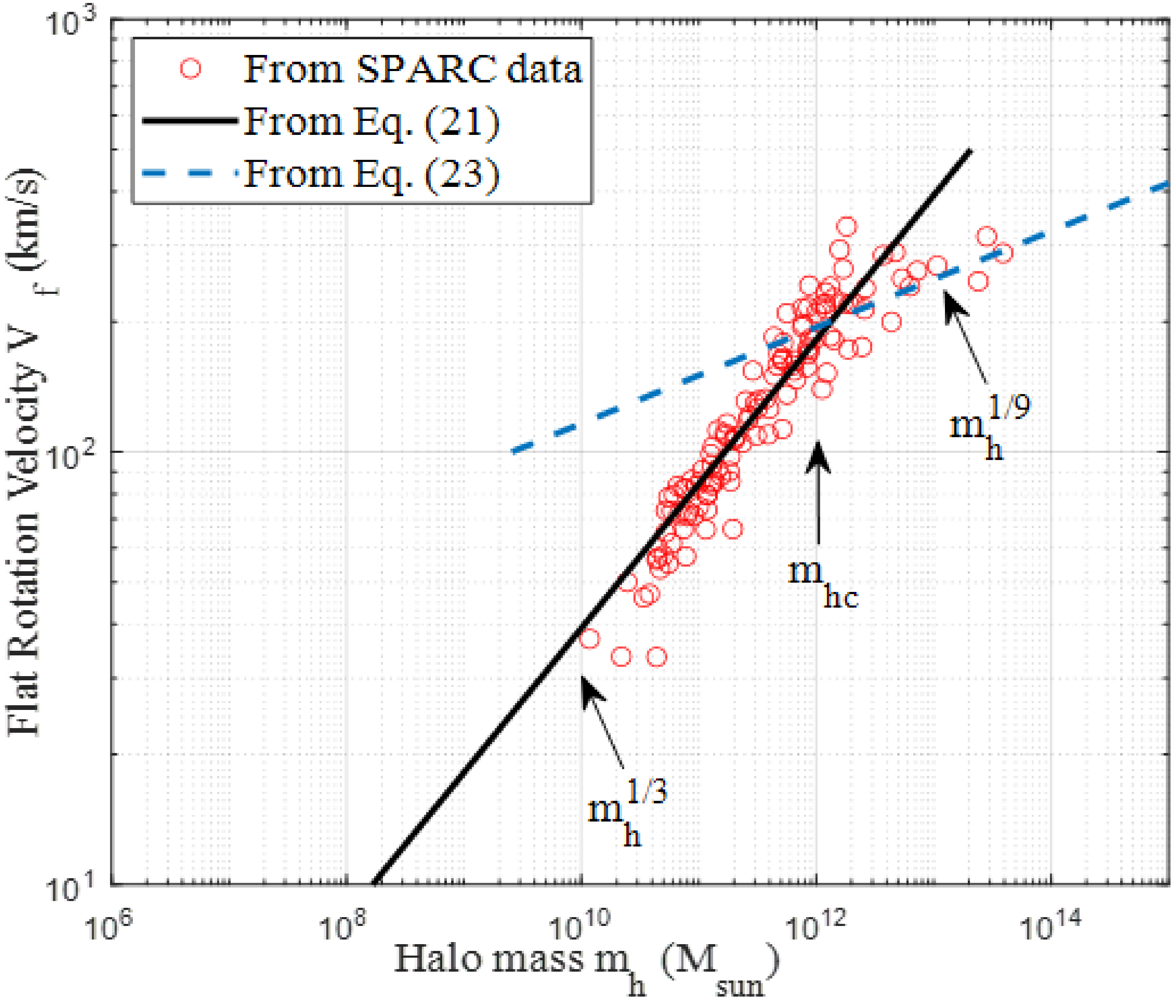




Halos have different rate of energy cascade with an average around ε_u (spatial intermittence in dark matter flow?)

SPARC data and model





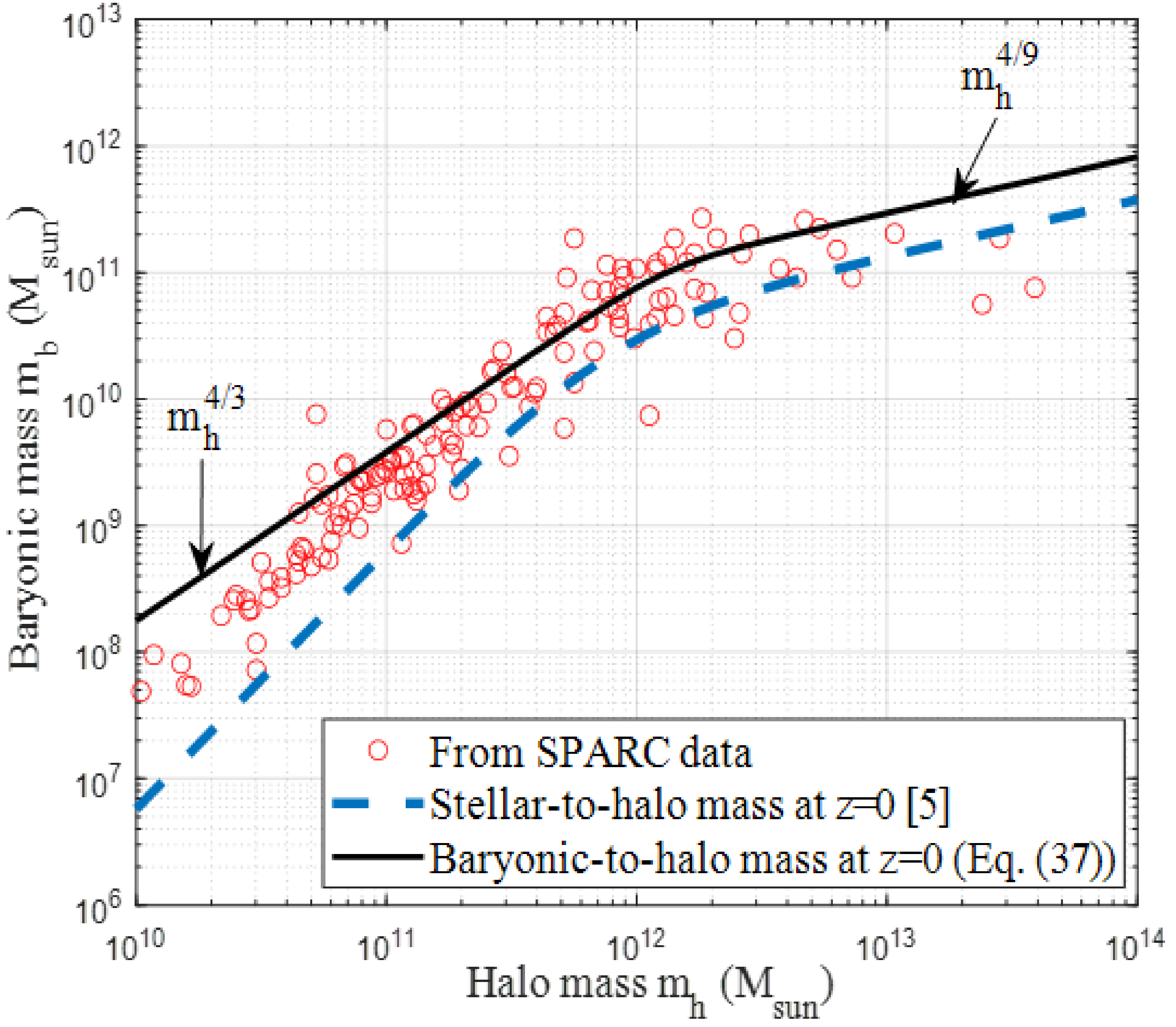
SPARC data and model

Baryonic mass in small halos: $m_b = (M_{c1})^{-1/3} (m_h)^{4/3}$

Baryonic mass in large halos: $m_b = (M_{c2})^{5/9} (m_h)^{4/9}$

Model incorporate two limits:

$$\frac{m_b}{m_h} = 2^{\frac{1}{m}} A(z) \left[\left(\frac{m_h}{m_{hc}(z)} \right)^{-\frac{m}{3}} + \left(\frac{m_h}{m_{hc}(z)} \right)^{\frac{5m}{9}} \right]^{-\frac{1}{m}}$$



- Dash line: the stellar-to-halo mass ratio obtained from halo abundance matching approach (require to match the stellar mass function)

- The scaling 4/9 law for both SHMR and BHMR

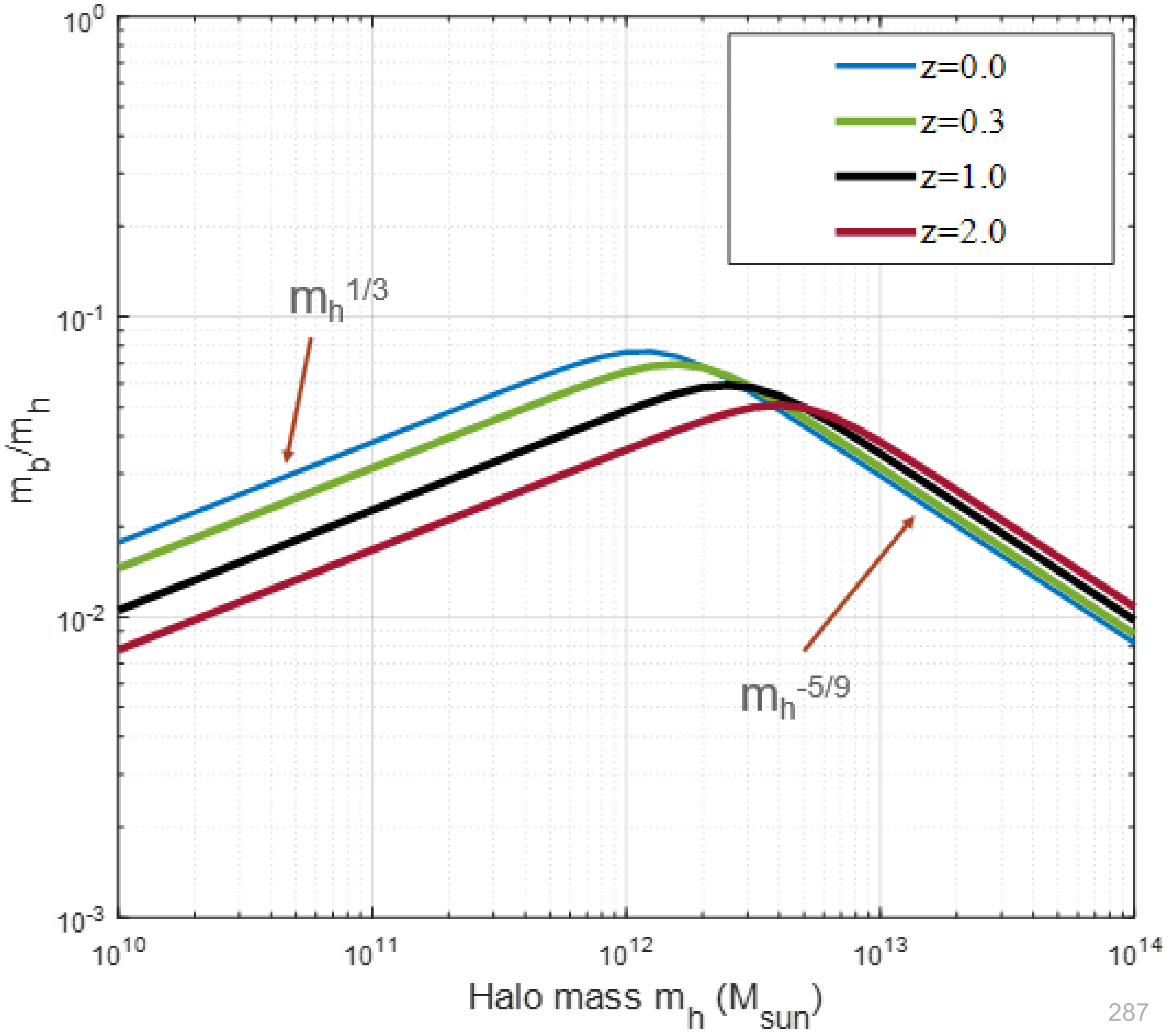
Redshift variation of baryonic-to-halo mass ratio

Models for baryonic-to-halo mass ratio:

$$\frac{m_b}{m_h} = 2^{\frac{1}{m}} A(z) \left[\left(\frac{m_h}{m_{hc}(z)} \right)^{-\frac{m}{3}} + \left(\frac{m_h}{m_{hc}(z)} \right)^{\frac{5m}{9}} \right]^{-\frac{1}{m}}$$

m is a parameter to adjust the transition;

- There exist a maximum BHMR ~0.076 at critical halo mass $m_{hc}=1.33\times10^{12}$ M_{sun}
- The critical halo mass decreases with time
- The maximum BHMR increases with time



Redshift evolution of baryonic-halo-mass relation

Overall cosmic baryonic-to-DM mass ratio (including both halos and out-of-halo) is ~18.8% in ΛCDM model:

Baryonic-to-DM mass ratio in out-of-halos

Baryonic-to-halo mass ratio in all halos

$$A_{boh}(z)=\frac{0.188-A_{dh}(z)A_{bh}(z)}{1-A_{dh}(z)}$$

Fraction of DM mass in halos

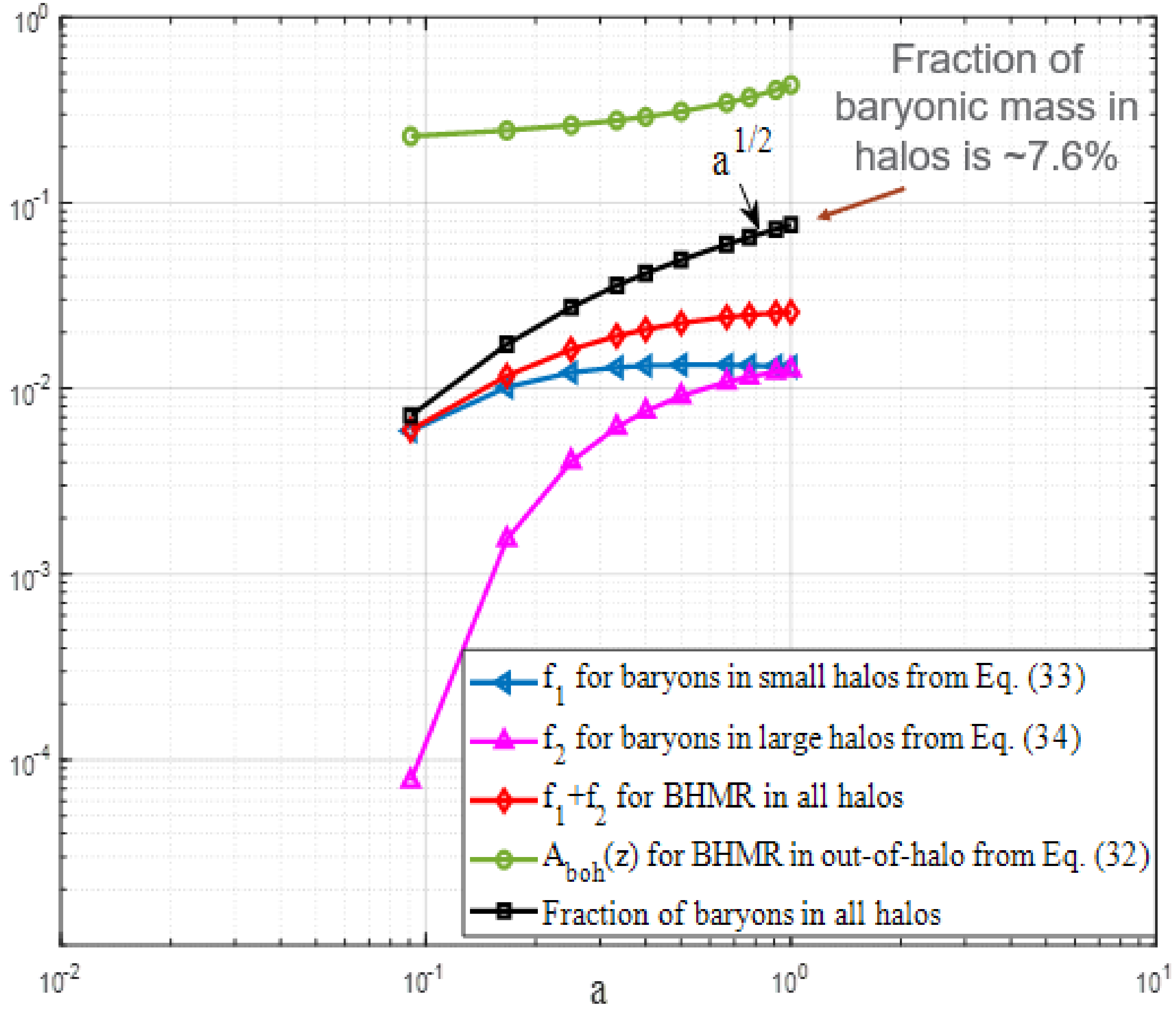
Use double-λ mass function to compute:

$$f_1=\int_0^{\nu_c} f_{D\lambda}(\nu)\left(M_{c1}\right)^{-1/3}\left(\nu^{3/2}m_h^*\right)^{1/3}d\nu$$

The baryonic-to-halo mass ratio in small halos

$$f_2=\int_{\nu_c}^{\infty} f_{D\lambda}(\nu)\left(M_{c2}\right)^{5/9}\left(\nu^{3/2}m_h^*\right)^{-5/9}d\nu$$

The baryonic-to-halo mass ratio in large halos



Redshift evolution of BHMR

Summary and keywords

Halo mass function	Mass/energy cascade	Tully-Fisher relation
Modified Newtonian Dynamics	Stellar-to-halo mass relation	Baryonic-to-halo mass relation

- Review direct energy cascade from large to small scales in hydrodynamic turbulence
- Reveal inverse mass and energy cascade that is unique for dark matter flow
- Present a fundamental theory for baryonic-to-halo mass ratio based on the mass/energy cascade in dark matter flow (agrees well with SPARC data)
- Predict a maximum baryonic-to-halo mass ratio ~0.076 for halos with a critical mass (agrees with SPARC data) and an average ratio ~0.024 for all halos
- Predict two distinct regimes for small and large halos, respectively, with critical halo mass and size explicitly derived (agrees with observations of stellar-to-halo mass ratio).
- Predict the fraction of total baryons in all galaxies is ~7.6% and that fraction increases with time (agrees very well with astronomical surveys including optical Sloan Digital Sky Survey and HIPASS). Most baryons (~92.4%) are not in galaxies.