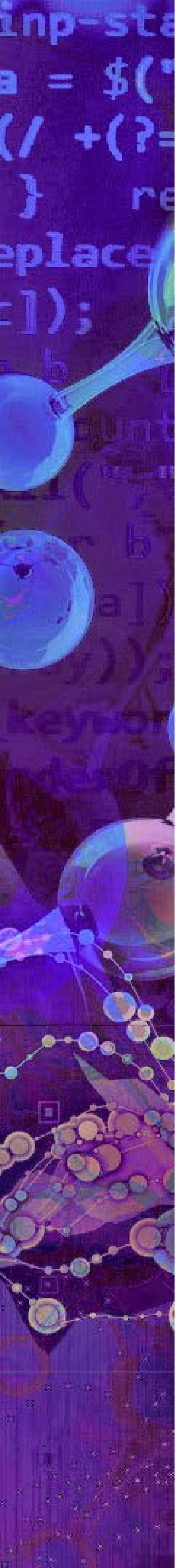

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A comparative study of dark matter flow & hydrodynamic turbulence and its applications

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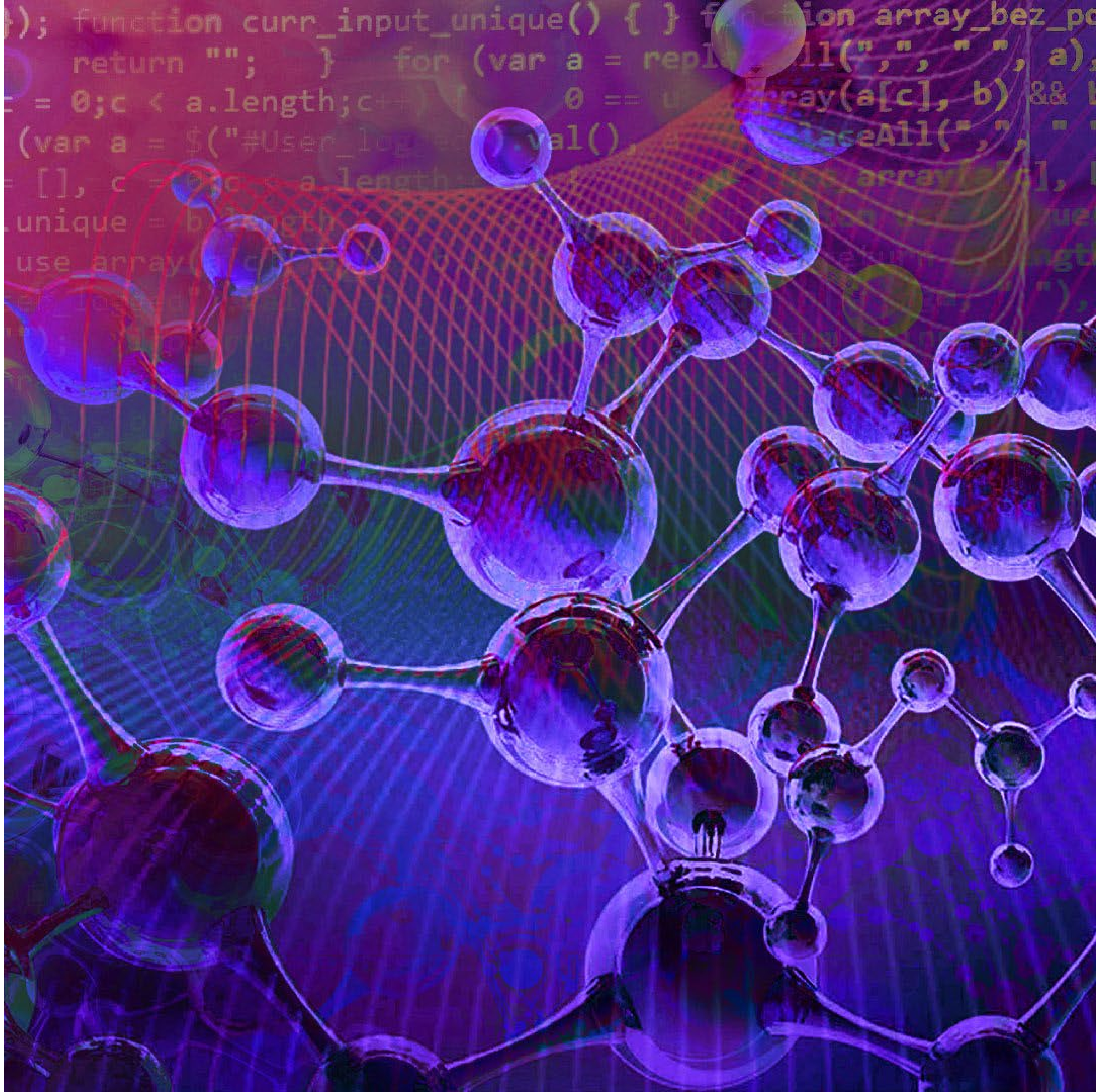

Preface

Dark matter, if exists, accounts for five times as much as ordinary baryonic matter. Therefore, dark matter flow might possess the widest presence in our universe. The other form of flow, hydrodynamic turbulence in air and water, is without doubt the most familiar flow in our daily life. During the pandemic, we have found time to think about and put together a systematic comparison for the connections and differences between two types of flow, both of which are typical non-equilibrium systems.

The goal of this presentation is to leverage this comparison for a better understanding of the nature of dark matter and its flow behavior on all scales. Science should be open. All comments are welcome.

Thank you!

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Data repository and relevant publications

Structural (halo-based) approach:

0.	Data https://dx.doi.org/10.5281/zenodo.6541230
1.	Inverse mass cascade in dark matter flow and effects on halo mass functions https://doi.org/10.48550/arXiv.2109.09985
2.	Inverse mass cascade in dark matter flow and effects on halo deformation, energy, size, and density profiles https://doi.org/10.48550/arXiv.2109.12244
3.	Inverse energy cascade in self-gravitating collisionless dark matter flow and effects of halo shape https://doi.org/10.48550/arXiv.2110.13885
4.	The mean flow, velocity dispersion, energy transfer and evolution of rotating and growing dark matter halos https://doi.org/10.48550/arXiv.2201.12665
5.	Two-body collapse model for gravitational collapse of dark matter and generalized stable clustering hypothesis for pairwise velocity https://doi.org/10.48550/arXiv.2110.05784
6.	Evolution of energy, momentum, and spin parameter in dark matter flow and integral constants of motion https://doi.org/10.48550/arXiv.2202.04054
7.	The maximum entropy distributions of velocity, speed, and energy from statistical mechanics of dark matter flow https://doi.org/10.48550/arXiv.2110.03126
8.	Halo mass functions from maximum entropy distributions in collisionless dark matter flow https://doi.org/10.48550/arXiv.2110.09676

Statistics (correlation-based) approach:

0.	Data https://dx.doi.org/10.5281/zenodo.6569898
1.	The statistical theory of dark matter flow for velocity, density, and potential fields https://doi.org/10.48550/arXiv.2202.00910
2.	The statistical theory of dark matter flow and high order kinematic and dynamic relations for velocity and density correlations https://doi.org/10.48550/arXiv.2202.02991
3.	The scale and redshift variation of density and velocity distributions in dark matter flow and two-thirds law for pairwise velocity https://doi.org/10.48550/arXiv.2202.06515
4.	Dark matter particle mass and properties from two-thirds law and energy cascade in dark matter flow https://doi.org/10.48550/arXiv.2202.07240
5.	The origin of MOND acceleration and deep-MOND from acceleration fluctuation and energy cascade in dark matter flow https://doi.org/10.48550/arXiv.2203.05606
6.	The baryonic-to-halo mass relation from mass and energy cascade in dark matter flow https://doi.org/10.48550/arXiv.2203.06899

Applications of dark matter flow

Dark matter particle mass and properties from two-thirds law and energy cascade in dark matter flow

Xu Z., 2022, arXiv:2202.07240v1 [astro-ph.CO]
https://doi.org/10.48550/arXiv.2202.07240

Introduction

- The existence of dark matter (DM) is supported by numerous astronomical observations:
  - Rotation curves of spiral galaxies
  - Motion of galaxies in galaxy clusters
  - Gravitational lensing
  - Bullet clusters
  - CMB ……
- Though the nature of dark matter is still unclear, dark matter is believed to be cold (non-relativistic), collisionless, dissipationless, non-baryonic, and barely interacting with baryonic matter except through gravity.
- Dark matter must be sufficiently smooth on large scales with a fluid-like behavior that is best described by self-gravitating collisionless flow dynamics (SG-CFD).
- It is often assumed to be a thermal relic, weakly interacting massive particles (WIMPs)
- However, no conclusive signals have been detected in searches for thermal WIMPs.
  - Direct detection by underground experiments
    - XENON
    - DarkSide
    - LUX, SuperCDM
  - Indirect Astronomical observations like high energy cosmic rays
    - Pierre Auger Observatory
  - Production by the accelerator such as LHC

The null results from the detection of standard WIMP particles suggest new perspectives maybe needed.

A classical "top-down" example in physics

What is the typical speed of electron?

At the scale of electron, we have three fundamental constants

Vacuum permittivity	$\varepsilon_0 \sim s^4 \cdot A^2 \cdot kg^{-1} \cdot m^{-3}$	Required by Coulomb force
Elementary charge	$e \sim A \cdot s$	
Planck constant	$\hbar \sim m^2 \cdot kg \cdot s^{-1}$	Required by quantum effect

Even if the detail of physics is unknown, we can use simple dimensional analysis to predict the electron speed:

Electron speed: $v_e \propto e^2 / \varepsilon_0 \hbar \sim m \cdot s^{-1}$

Goal: can we apply similar method (by identifying key constants) to find dark matter particle properties ??

If we know the physics:

$$m_e v_e r_e = \hbar$$ Heisenberg's uncertainty principle

$$\underbrace{\frac{e^2}{4\pi\varepsilon_0 r_e}}_{\text{Potential energy}} = \underbrace{m_e v_e^2}_{\text{Kinetic energy}}$$ Virial theorem

More accurate electron speed: $v_e = \dfrac{e^2}{4\pi\varepsilon_0\hbar}$

Sommerfeld's interpretation of the fine structure constant:

$$\alpha = \frac{v_e}{c} = \frac{e^2}{4\pi\varepsilon_0\hbar c} \approx \frac{1}{137}$$

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What we need for predicting DM particle mass?

What is the mass of dark matter particles?

At the scale of DM particle, Assumptions:

- Only gravity is present without any other known interactions involved;
- DM particles still exhibit the wave-particle duality on the quantum level;

Then we have at least two fundamental constants:

Gravitational constant	$G \sim m^3 \cdot s^{-2} \cdot kg^{-1}$ $\longleftarrow$	Required by Newtonian gravity
Planck constant	$\hbar \sim m^2 \cdot kg \cdot s^{-1}$ $\longleftarrow$	Required by quantum effect

Dimensional analysis points out:

- No matter how you combine two constants, you cannot get mass;
- These two constants are not sufficient to solve problem;

Then what is the other constant besides these two?

This additional constant might come from the properties of dark matter flow.

Energy cascade in hydrodynamic turbulence

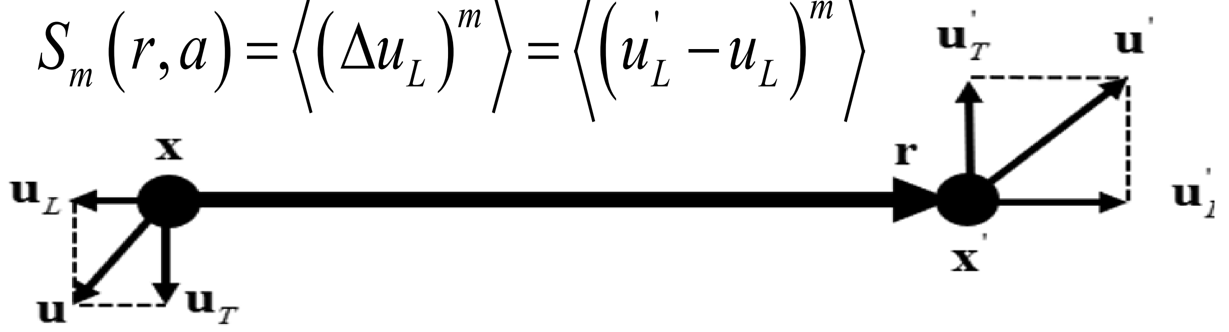
- There exist an inertial range with a scale-independent rate of energy cascade (ε does not depend on eddy size *l*) for eddy size η< *l* <*L*. η is a dissipative scale determined by viscosity ν and ε.

- In inertial range, inertial force is dominant over viscous force. A general scaling for velocity structure functions $S_m(r)$ for pairwise velocity Δu_L can be identified:

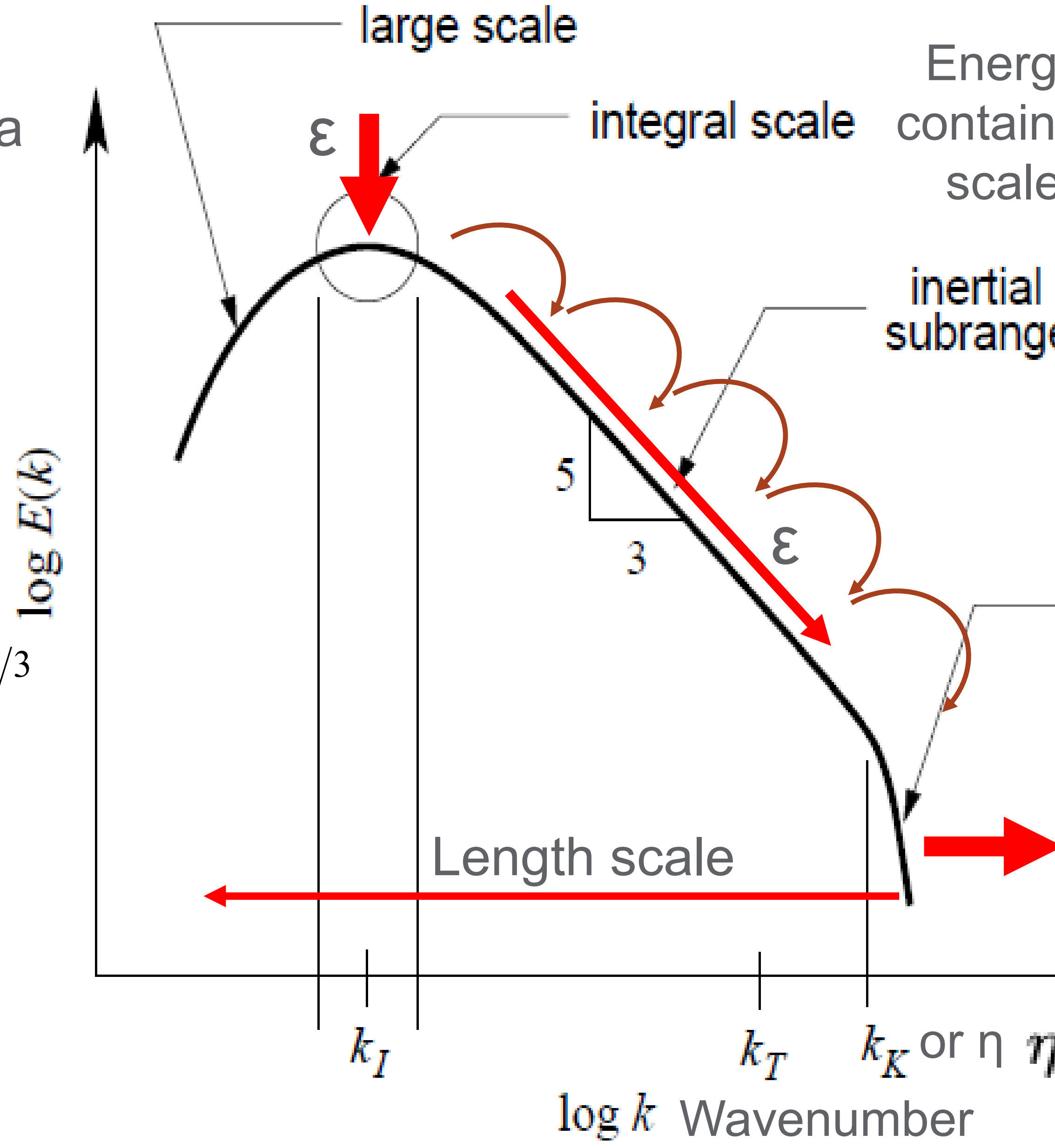
$$S_m(r) \propto (\varepsilon_u)^{m/3} r^{m/3} \xrightarrow{m=2} S_2 \propto (-\varepsilon_u)^{2/3} r^{2/3}$$

Two-thirds law

$$S_m(r,a) = \langle (\Delta u_L)^m \rangle = \langle (u_L' - u_L)^m \rangle$$



Big whirls have little whirls, That feed on their velocity;
And little whirls have lesser whirls, And so on to viscosity.

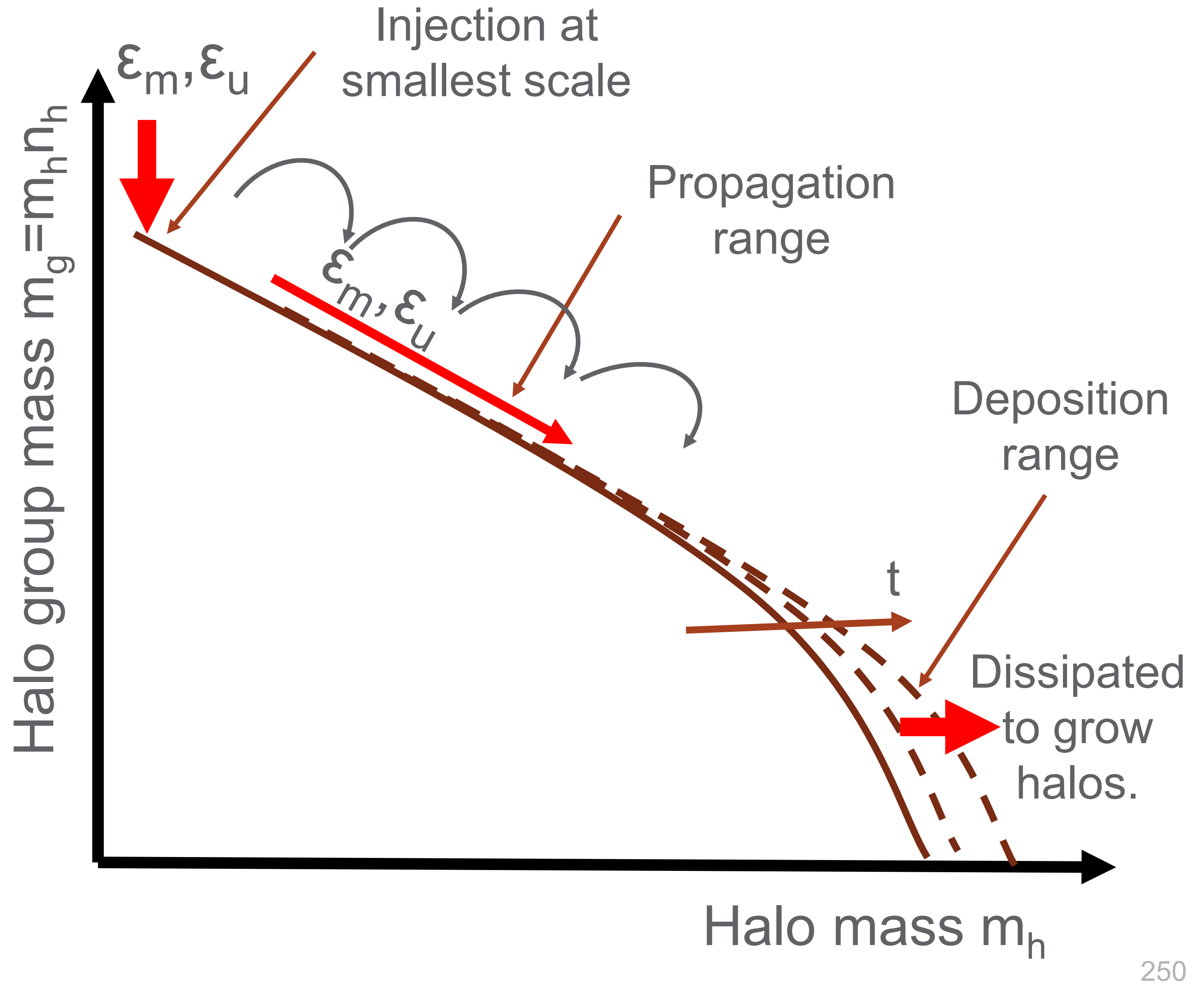


$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}$$

Mass/Energy cascade in dark matter flow (SG-CFD)

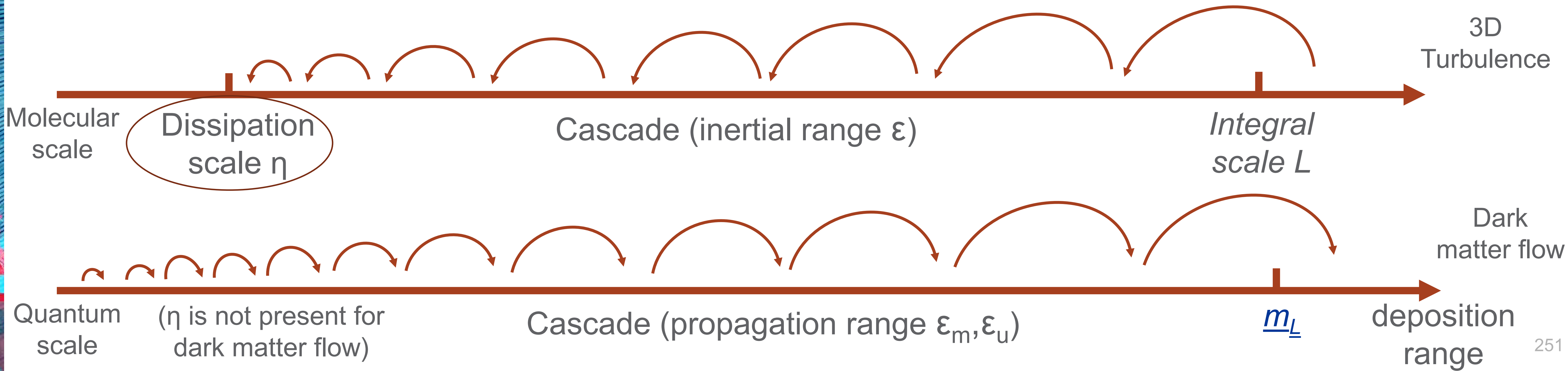
- Collisionless nature and long-range interaction.
- Long-range gravity requires a broad spectrum of halos to be formed to maximize system entropy.
- A continuous cascade of mass/energy from smaller to larger mass scales with a scale-independent rate of mass transfer ε_m and ε_u in a certain range of mass scales (propagation range).
- The mass/energy cascade is an intermediate statistically steady state for non-equilibrium systems to continuously maximize system entropy.
- The maximum entropy distribution of dark matter flow.

Little halos have big halos, That feed on their mass;
And big halos have greater halos, And so on to growth.



Mass/Energy cascade in dark matter flow (SG-CFD)

- Collisionless, no dissipation range in SG-CFD.
- The smallest length scale of inertial range is not limited by viscosity.
- This enable us to extend the scale-independent ε_u down to the smallest scale, where quantum effects become important
- Dark matter flow exhibits scale-dependent flow behaviors for peculiar velocity, i.e. a constant divergence flow on small scales and an irrotational flow on large scales.
- The constant divergence flow shares the same even order kinematic relations with those of incompressible (divergence free) flow. This hints to similar scaling laws holds for dark matter.



Constant (time and scale independent) rate of energy cascade

[Power-law time evolution for energy in terms of rate of energy cascade] ε_u:

$$K_p = -\varepsilon_{\mathbf{u}} t$$

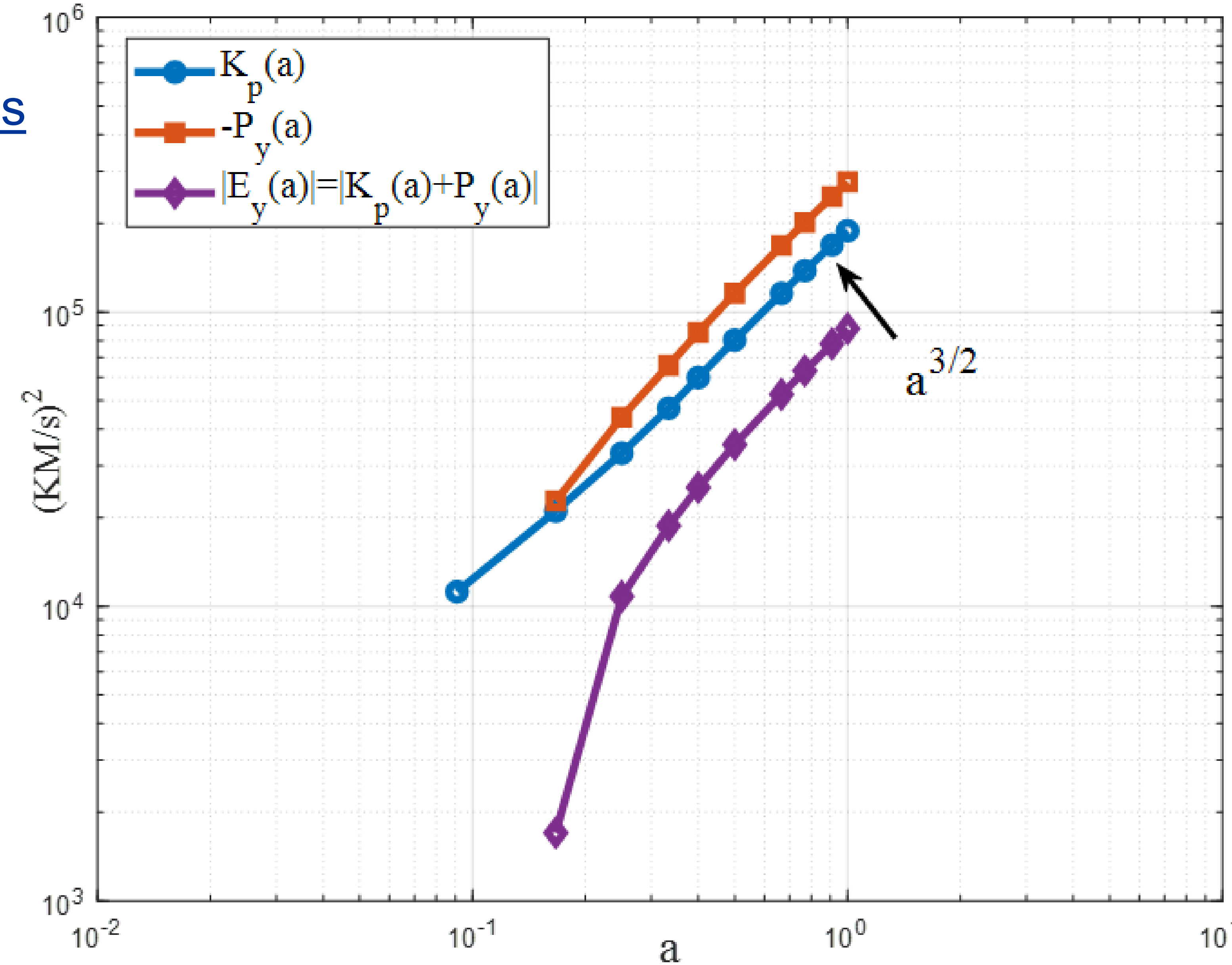
Power-law for Peculiar kinetic energy

$$P_y = \frac{7}{5}\varepsilon_{\mathbf{u}} t$$

Power-law for potential energy

$$\varepsilon_u = -\frac{K_p}{t} = -\frac{3}{2}\frac{u_0^2}{t_0} \approx -4.6\times 10^{-7}\frac{m^2}{s^3}$$

[Also see detail analysis for inverse kinetic energy cascade.]



The time variation of specific kinetic and potential energies from *N*-body simulation.

The two-thirds law on small scales

Odd order moment (generalized stable clustering hypothesis):

$$S_{2n+1}^{lp}(r) = (2n+1) S_1^{lp}(r) S_{2n}^{lp}(r) \propto r^1$$

Even order (two-thirds law):

$$S_{2n}^{lp}(r) - 2^n u^{2n} K_{2n}(\Delta u_L, 0) = \beta_{2n}^*(r/r_s)^{2/3} \propto r^{2/3}$$

Second order (two-thirds law):

$$S_2^{lp}(r) - 2u^2 = S_{2r}^{lp} = \beta_2^*(r/r_s)^{2/3} \propto r^{2/3}$$

Introduce a velocity scale:

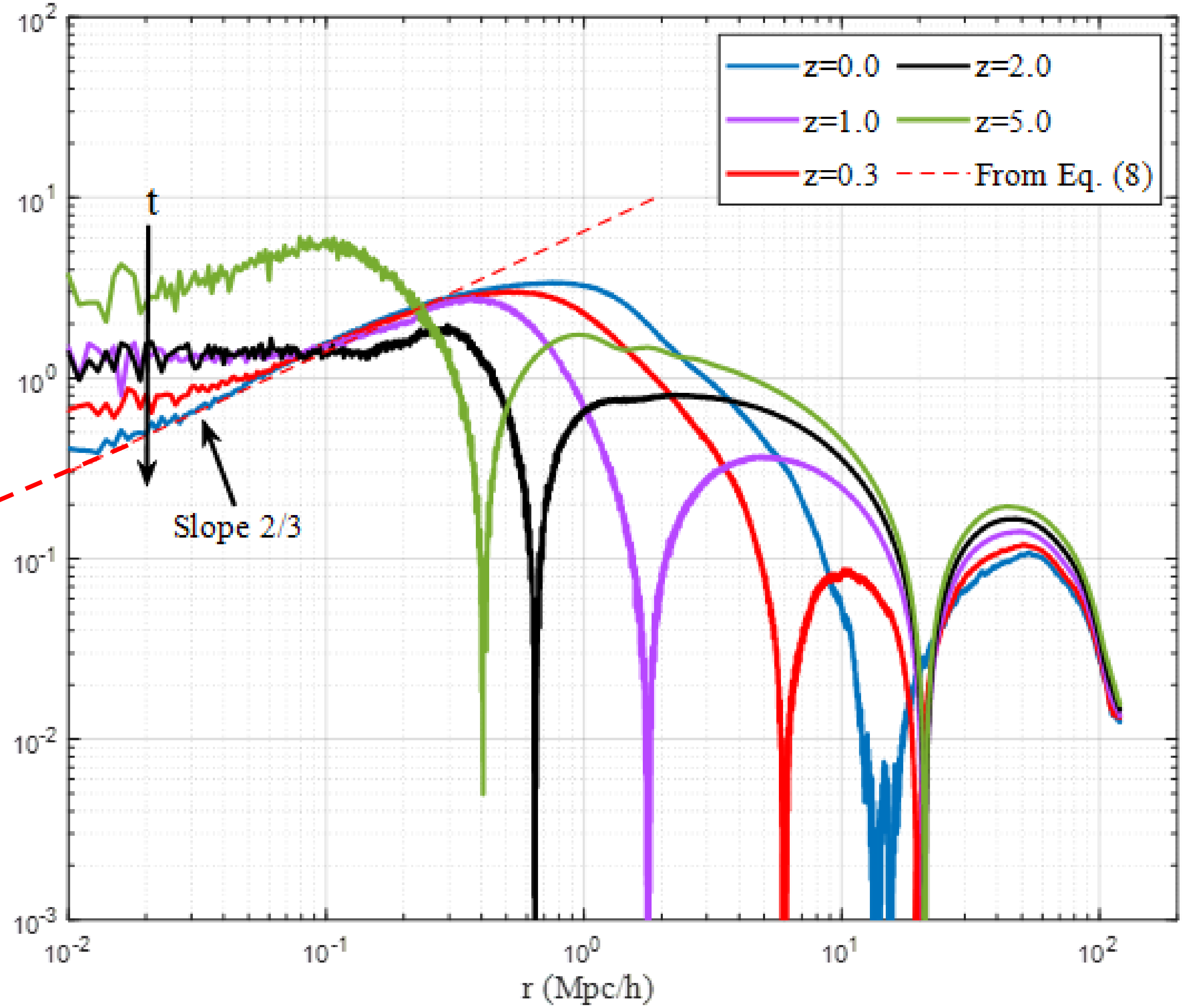
$$v_l^2 = S_{2r}^{lp}(r) \Big/ \left(2^{2/3} \beta_2^* a^{3/2}\right)$$

$$(-\varepsilon_u) = \frac{2v_l^2}{r} v_l = \frac{2v_l^2}{r/v_l}$$

Acceleration

Turnaround time

Extend all the way to the smallest scale



Variation of normalized reduced longitudinal structure function and two-thirds law

Postulating dark matter particle mass and properties

At the smallest scale, we have three fundamental constants:

Gravitational constant	$G = 6.67\times10^{-11}\, m^3/\left(kg\cdot s^2\right)$	Required by Newtonian gravity
Rate of energy cascade	$\varepsilon_u = -4.6\times10^{-7}\, m^2/s^3$	Required by dark matter flow
Planck constant	$\hbar = 1.05\times10^{-34}\, kg\cdot m^2/s$	Required by quantum effect

If we know the physics:

$m_X v_X \cdot l_X/2 = \hbar$	Heisenberg's uncertainty principle
$a_X \cdot v_X = -\varepsilon_u$	Energy cascade (two-thirds law)
$Gm_X/l_X^2 = a_X$	Acceleration
$Gm_X/l_X = 2v_X^2$	Virial theorem

Even if the detail of physics is unknown, we can use simple dimensional analysis to predict :

Mass scale: $m_X \propto \left(-\varepsilon_u \hbar^5/G^4\right)^{\frac{1}{9}}$

Length scale: $l_X \propto \left(-G\hbar/\varepsilon_u\right)^{\frac{1}{3}}$

Time scale: $t_X \propto \left(G^2\hbar^2/\varepsilon_u^5\right)^{\frac{1}{9}}$

$$m_X = \left(-256\varepsilon_u \hbar^5/G^4\right)^{\frac{1}{9}} = 1.62\times10^{-15}\,kg = 0.90\times10^{12}\,GeV$$

$$l_X = \left(-2G\hbar/\varepsilon_u\right)^{\frac{1}{3}} = 3.12\times10^{-13}\,m$$

$$t_X = l_X/v_X = \left(-32G^2\hbar^2/\varepsilon_u^5\right)^{\frac{1}{9}} = 7.51\times10^{-7}\,s$$

Velocity scale: $v_X = \left(\varepsilon_u^2 \hbar G/4\right)^{\frac{1}{9}} = 4.16\times10^{-7}\,m/s$

Acceleration scale: $a_X = \left(-4\varepsilon_u^7/(\hbar G)\right)^{\frac{1}{9}} = 1.11\,m/s^2$

Postulating dark matter particle mass and properties

Density scale: $m_X / l_X^3 \approx 5.33\times10^{22}\ kg/m^3$ ⟷ Nuclear density: $10^{17}\ kg/m^3$

Power scale (Joule/s):

$$\mu_X = m_X a_X \cdot v_X = F_X \cdot v_X = -m_X \varepsilon_u = \left(-\frac{256\varepsilon_u^{10}\hbar^5}{G^4}\right)^{\frac{1}{9}} = 7.44\times10^{-22}\ kg\cdot m^2/s^3 = 0.0046\,eV/s$$

Energy scale: $\mu_X t_X/4 = \hbar/t_X = \frac{1}{2}m_X v_X^2 = 0.87\times10^{-9}\,eV$

Rydberg energy of 13.6 eV for the ionization energy of the hydrogen atom

Particle lifetime: $\tau_X = \frac{m_X c^2}{\mu_X} = -\frac{c^2}{\varepsilon_u} = 2\times10^{23}\,s = 6.2\times10^{15}\,yr \approx \frac{\hbar e^{1/\alpha_X}}{m_X c^2}$

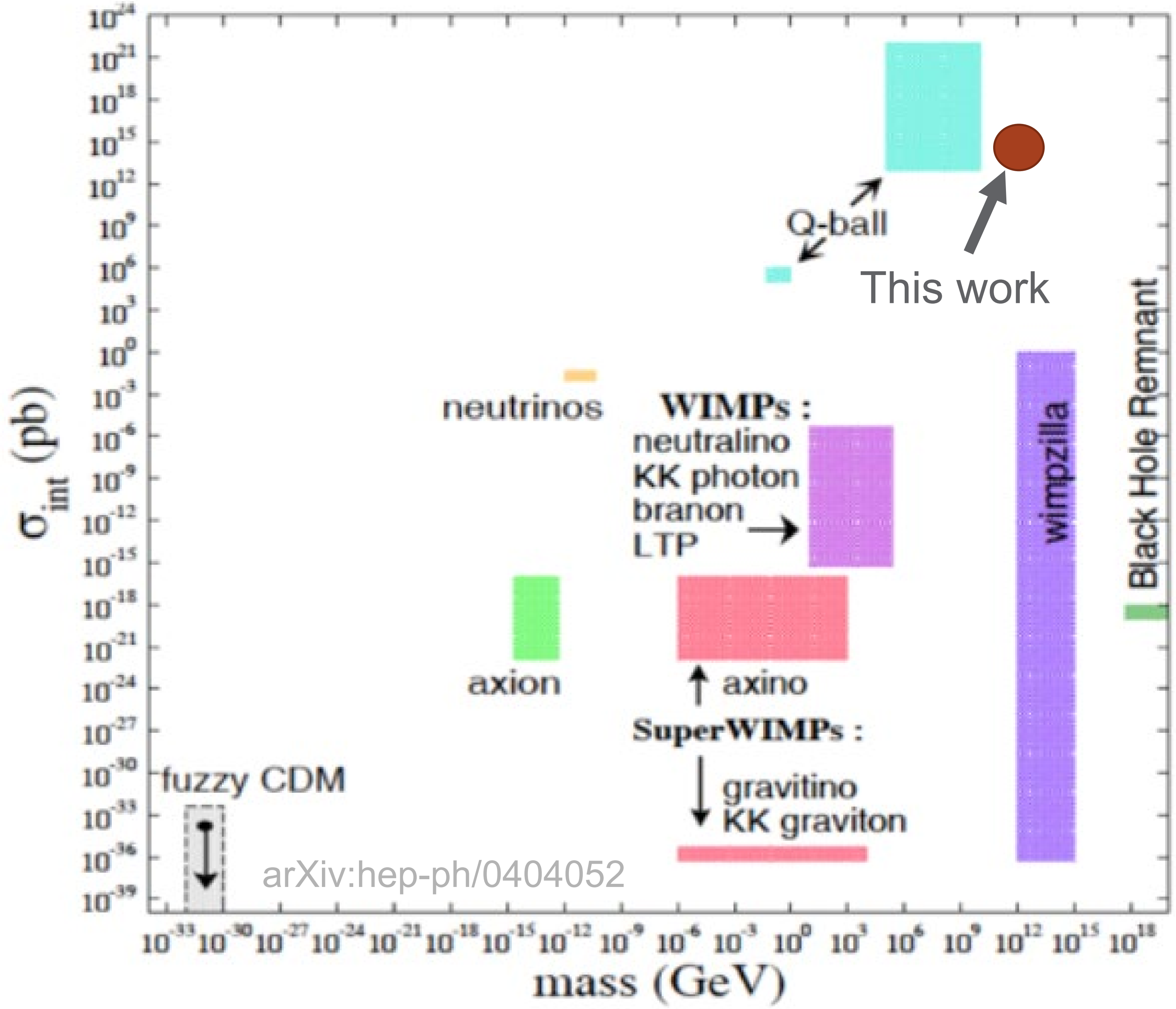
If instantons are responsible for the decay [1]:

$$\tau_X = \frac{\hbar e^{1/\alpha_X}}{m_X c^2} = 6.2\times10^{15}\,yr$$

➡ Fine structure constant: $\alpha_X \approx \frac{1}{136.85}$

[1] Anchordoqui, L.A., et al.,Astroparticle Physics, 2021. 132.

Where is our prediction?



Summary and keywords

Dark matter flow	Mass/energy cascade	Dark matter particle mass
Two-thirds law	Rate of energy cascade	Fine structure constant

- Establish connections between dark matter flow and hydrodynamic turbulence.
- Review direct energy cascade from large to small scales in hydrodynamic turbulence with the smallest length scale η determined by viscosity and the rate of cascade ε.
- Review the inverse energy cascade in dark matter flow from small to large mass scales with a constant rate of energy cascade.
- Two-thirds law for pairwise velocity dispersion on small scale r.
- The collisionless nature of dark matter flow enables us to extend constant rate of cascade and two-thirds law down to the smallest scale where quantum effects are dominant.
- Suggests a heavy dark matter scenario by combining rate of energy cascade, Planck constant, and gravitational constant to predict dark matter particles with a mass ~0.9x1012 GeV and a size ~3x10-13 m.