IJESRT: 11(5), May, 2022 ISSN: 2277-9655

# International Journal of Engineering Sciences & Research Technology

**(A Peer Reviewed Online Journal)**
**Impact Factor: 5.164**

**IJESRT**

IJESRT
INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

# REVAN SOMBOR INDICES AND THEIR EXPONENTIALS FOR CERTAIN NANOTUBES

**V. R. Kulli**
Department of Mathematics, Gulbarga University, Gulbarga 585 106, India

**DOI**: 10.5281/zenodo.6623554

## ABSTRACT
In this paper, we introduce the modified Revan Sombor index, Revan Sombor exponential and modified Revan Sombor exponential of a graph. We compute the Revan Sombor index, modified Revan Sombor index and their corresponding exponentials of certain nanotubes.

**Keywords:** *molecular structure, Revan Sombor index, modified Revan Sombor index, nanotube.*
***Mathematics Subject Classification:*** *05C05, 05C07, 05C92.*

## 1. INTRODUCTION
Let $G$ be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex $u$ is the number of vertices adjacent to $u$. Let $\Delta$ and $\delta$ denote, respectively, the maximum and minimum degree among the vertices of the graph $G$. The edge connecting the vertices $u$ and $v$ will be denoted by $uv$.

One of the main directions of recent research in chemical graph theory is the study and application of graph-based molecular structural descriptors, usually referred to as "topological indices" [1]. An important group of such descriptors are the vertex-degree-based (VDB) topological indices, whose general form is

$$TI = TI(G) = \sum_{uv\in E(G)} \Phi(d_G(u), d_G(v))$$

where $\Phi(x, y)$ is a pertinently chosen function satisfying the condition $\Phi(x, y) = \Phi(y, x)$. Some of the simplest, oldest, and most detailed studied VDB indices are the first and second Zagreb index

$$M_1 = M_1(G) = \sum_{uv\in E(G)} \left[d_G(u) + d_G(v)\right] \quad , \quad M_2 = M_2(G) = \sum_{uv\in E(G)} d_G(u)d_G(v)$$

Another, recently introduced group of VDB indices [2, 3] are the Sombor and Nirmala indices

$$SO = SO(G) = \sum_{uv\in E(G)} \sqrt{d_G(u)^2 + d_G(v)^2}$$

$$N(G) = \sum_{uv\in E(G)} \sqrt{d_G(u) + d_G(v)}$$

as well as the reverse Sombor index [4]

$$SO_{rev} = SO_{rev}(G) = \sum_{uv\in E(G)} \sqrt{\left[\Delta - d_G(u) + 1\right]^2 + \left[\Delta - d_G(v) + 1\right]^2} \; .$$

Recently, some Sombor and Nirmala indices were studied in [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22].

Denote by $r_G(u)$ the Revan vertex degree of a vertex $u$ in $G$, defined as $r_G(u) = \Delta + \delta - d_G(u)$. In 2017 [23], Kulli conceived a class of Revan-type indices, defined in analogy to the Zagreb indices as

$$R_1(G) = \sum_{uv\in E(G)} \left[r_G(u) + r_G(v)\right],$$

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

$$R_2(G)=\sum_{uv\in E(G)} r_G(u)r_G(v).$$

In [24], Kulli et al. introduced the Revan Sombor index of a graph and defined it as,

$$RSO(G)=\sum_{uv\in E(G)}\sqrt{r_G(u)^2+r_G(v)^2}=\sum_{uv\in E(G)}\sqrt{[\Delta+\delta-d_G(u)]^2+[\Delta+\delta-d_G(\mathrm{v})]^2}.$$

The Revan Sombor exponential of a graph $G$ is defined as

$$RSO(G,x)=\sum_{uv\in E(G)} x^{\sqrt{r_G(u)^2+r_G(v)^2}}.$$

We introduce the modified Revan Sombor index of a graph $G$ and it is defined as

$$^{m}RSO(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{r_G(u)^2+r_G(v)^2}}.$$

We define the modified Revan Sombor exponential of a graph $G$ as

$$^{m}RSO(G,x)=\sum_{uv\in E(G)} x^{\frac{1}{\sqrt{r_G(u)^2+r_G(v)^2}}}.$$

Recently, some Revan indices were studied in [25, 26].

In this paper, the Revan Sombor indices of $HC_5C_7[p,q]$ nanotubes, $SC_5C_7[p,q]$ nanotubes, $KTUC_4C_8[p,q]$ nanotubes, $KTUC_4C_8[p,q]$ nanotubes are computed. For nanotubes see [12] and references cited therein.

## 2. RESULTS FOR $HC_5C_7[p,q]$ NANOTUBES

We consider $HC_5C_7[p,q]$ nanotubes in which $p$ is the number of heptagones in the first row and $q$ rows of pentagones repreated alternately. The 2-$D$ lattice ofnanotube $HC_5C_7[8,4]$ is shown in Figure 1.

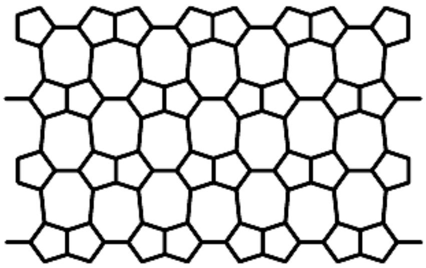

**Figure 1. 2-$D$ lattice of $HC_5C_7[8,4]$ nanotube**

Let $G$ be the graph of $HC_5C_7[p,q]$ nanotubes. We see that the vertices of $G$ are either of degree 2 or 3. Therefore $\Delta(G) = 3$ and $\delta(G) = 2$. By algebraic method, we obtain that $G$ has $4pq$ vertices and $6pq - p$ edges. In $G$, there are two types of edges based on the degree of end vertices of each edge as follows:

$E_{23}= \{uv\in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_{23}| = 4p$.
$E_{33} = \{uv\in E(G)\mid d_G(u) = d_G(v) = 3\}$, $|E_{33}| = 6pq - 5p$.

We have $r_G(u) =\Delta(G) + \delta(G) - d_G(u) = 5 - d_G(u)$.

Thus there are two types of Revan edges based on the degree of end Revan vertices of each Revan edge as given in Table 1.

**Table 1: Revan edge partition of $HC_5C_7[p, q]$**

| $r_G(u), r_G(v) \setminus uv\in E(G)$ | (3, 2) | (2, 2) |
|---|---|---|
| Number of edges | $4p$ | $6pq - 5p$ |

In the following theorem, we compute the exact formulas of $RSO(HC_5C_7[p,q])$, $RSO(HC_5C_7[p,q], x)$ for $HC_5C_7[p,q]$ nanotubes.

**Theorem 1.**Let $G$ be the graph of a nanotube $HC_5C_7[p, q]$.Then

(i) $RSO\left(HC_5C_7[p,q]\right)=12\sqrt{2}pq+\left(4\sqrt{13}-10\sqrt{2}\right)p.$

(ii) $RSO\left(HC_5C_7[p,q],x\right)=4px^{\sqrt{13}}+\left(6pq-5p\right)x^{2\sqrt{2}}.$

**Proof:** From definitions and by using Table 1, we deduce

(i) $$RSO\left(HC_5C_7[p,q]\right)=\sum_{uv\in E(G)}\sqrt{r_G(u)^2+r_G(v)^2}=(3^2+2^2)^{\frac{1}{2}}4p+(2^2+2^2)^{\frac{1}{2}}(6pq-5p)$$
$$=12\sqrt{2}pq+\left(4\sqrt{13}-10\sqrt{2}\right)p.$$

(ii) $$RSO\left(HC_5C_7[p,q],x\right)=\sum_{uv\in E(G)}x^{\sqrt{r_G(u)^2+r_G(v)^2}}=4px^{(3^2+2^2)^{\frac{1}{2}}}+\left(6pq-5p\right)x^{(2^2+2^2)^{\frac{1}{2}}}$$
$$=4px^{\sqrt{13}}+\left(6pq-5p\right)x^{2\sqrt{2}}.$$

In the following theorem, we compute the exact formulas of ${}^{m}RSO\left(HC_5C_7[p,q]\right)$ ${}^{m}RSO\left(HC_5C_7[p,q],x\right)$ for $HC_5C_7[p,q]$ nanotubes.

**Theorem 2.**Let $G$ be the graph of a nanotube $HC_5C_7[p, q]$.Then

(i) ${}^{m}RSO\left(HC_5C_7[p,q]\right)=12\sqrt{2}pq+\left(4\sqrt{13}-10\sqrt{2}\right)p.$

(ii) ${}^{m}RSO\left(HC_5C_7[p,q],x\right)=4px^{\sqrt{13}}+\left(6pq-5p\right)x^{2\sqrt{2}}.$

**Proof:** From definitions and by using Table 1, we deduce

(i) $${}^{m}RSO\left(HC_5C_7[p,q]\right)=\sum_{uv\in E(G)}\frac{1}{\sqrt{r_G(u)^2+r_G(v)^2}}=\frac{4p}{\sqrt{3^2+2^2}}+\frac{(6pq-5p)}{\sqrt{2^2+2^2}}$$
$$=\frac{3pq}{\sqrt{2}}+\left(\frac{4}{\sqrt{13}}-\frac{5}{2\sqrt{2}}\right)p.$$

(ii) $${}^{m}RSO\left(HC_5C_7[p,q],x\right)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{r_G(u)^2+r_G(v)^2}}}=4px^{\frac{1}{\sqrt{3^2+2^2}}}+\left(6pq-5p\right)x^{\frac{1}{\sqrt{2^2+2^2}}}$$
$$=4px^{\frac{1}{\sqrt{13}}}+\left(6pq-5p\right)x^{\frac{1}{2\sqrt{2}}}.$$

## 3. RESULTS FOR $SC_5C_7[p,q]$ NANOTUBES

In this section, we focus on the family of nanotubes, denoted by $SC_5C_7[p,q]$, in which $p$ is the number of heptagons in the first row and $q$ rows of vertices and edges are repeated alternately. The 2-D lattice of nanotube $SC_5C_7[p,q]$ is presented in Figure 2.

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

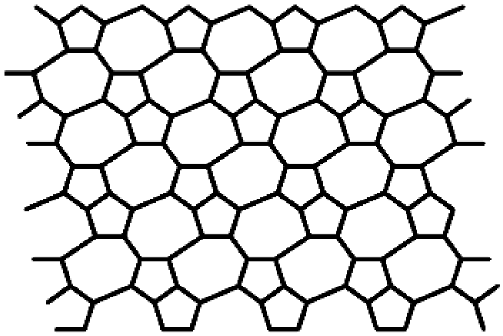

**Figure 2: 2-*D* lattice of nanotube $SC_5C_7[p,q]$**

Let $G$ be the graph of $SC_5C_7[p,q]$. We see that the vertices of $G$ are either of degree 2 or 3. Therefore $\Delta(G) = 3$ and $\delta(G) = 2$. By calculation, we obtain that $G$ has $4pq$ vertices and $6pq - p$ edges. Also by calculation, we get that $G$ has three types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = q$.
$E_2 = \{uv\in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_2| = 6q$.
$E_2 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_3| = 6pq - p - 7q$.

We have $r_G(u) = \Delta(G) + \delta(G) - d_G(u) = 5 - d_G(u)$.

Thus there are three types of Revan edges based on the degree of end Revan vertices of each Revan edge as given in Table 2.

**Table 2: Revan edge partition of $SC_5C_7[p, q]$**

| $r_G(u), r_G(v) \setminus uv\in E(G)$ | (3, 3) | (3, 2) | (2, 2) |
|---|---|---|---|
| Number of edges | $q$ | $6q$ | $6pq - p - 7q$ |

In the following theorem, we compute the exact formulas of $RSO(SC_5C_7[p,q])$, $RSO(SC_5C_7[p,q], x)$ for $SC_5C_7[p,q]$ nanotubes.

**Theorem 3.**Let $G$ be the graph of a nanotube $SC_5C_7[p, q]$.Then

(i) $RSO\left(SC_5C_7\left[p,q\right]\right) = 12\sqrt{2}pq - 2\sqrt{2}p + \left(6\sqrt{13} - 11\sqrt{2}\right)q.$

(ii) $RSO\left(SC_5C_7\left[p,q\right],x\right) = qx^{3\sqrt{2}} + 6qx^{\sqrt{13}} + \left(6pq - p - 7q\right)x^{2\sqrt{2}}.$

**Proof:** From definitions and by using Table 2, we derive

(i) $RSO\left(SC_5C_7\left[p,q\right]\right) = \sum_{uv\in E(G)} \sqrt{r_G(u)^2 + r_G(v)^2}$

$$= \left(3^2 + 3^2\right)^{\frac{1}{2}} q + \left(3^2 + 2^2\right)^{\frac{1}{2}} 6q + \left(2^2 + 2^2\right)^{\frac{1}{2}} \left(6pq - p - 7q\right)$$
$$= 12\sqrt{2}pq - 2\sqrt{2}p + \left(6\sqrt{13} - 11\sqrt{2}\right)q.$$

(ii) $RSO\left(SC_5C_7\left[p,q\right],x\right) = \sum_{uv\in E(G)} x^{\sqrt{r_G(u)^2 + r_G(v)^2}} = qx^{\left(3^2+3^2\right)^{\frac{1}{2}}} + 6qx^{\left(3^2+2^2\right)^{\frac{1}{2}}} + \left(6pq - p - 7p\right)x^{\left(2^2+2^2\right)^{\frac{1}{2}}}$

$$= qx^{3\sqrt{2}} + 6qx^{\sqrt{13}} + \left(6pq - p - 7q\right)x^{2\sqrt{2}}.$$

In the following theorem, we compute the exact formulas of ${}^{m}RSO\left(SC_5C_7\left[p,q\right]\right)$ ${}^{m}RSO\left(SC_5C_7\left[p,q\right],x\right)$ for $SC_5C_7[p,q]$ nanotubes.

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

**Theorem 4.** Let $G$ be the graph of a nanotube $SC_5C_7[p, q]$.Then

(i) $^{m}RSO\left(SC_5C_7\left[p,q\right]\right)=12\sqrt{2}pq+\left(4\sqrt{13}-10\sqrt{2}\right)p.$

(ii) $^{m}RSO\left(SC_5C_7\left[p,q\right],x\right)=4px^{\sqrt{13}}+\left(6pq-5p\right)x^{2\sqrt{2}}.$

**Proof:** From definitions and by using Table 2, we deduce

(i)
$$^{m}RSO\left(SC_5C_7\left[p,q\right]\right)=\sum_{uv\in E(G)}\frac{1}{\sqrt{r_G(u)^2+r_G(v)^2}}$$
$$=\frac{q}{\sqrt{3^2+3^2}}+\frac{6q}{\sqrt{3^2+2^2}}+\frac{(6pq-p-7q)}{\sqrt{2^2+2^2}}$$
$$=\frac{3pq}{\sqrt{2}}-\frac{p}{2\sqrt{2}}+\left(\frac{1}{3\sqrt{2}}+\frac{6}{\sqrt{13}}-\frac{7}{2\sqrt{2}}\right)q.$$

(ii)
$$^{m}RSO\left(SC_5C_7\left[p,q\right],x\right)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{r_G(u)^2+r_G(v)^2}}}$$
$$=qx^{\frac{1}{\sqrt{3^2+3^2}}}+6qx^{\frac{1}{\sqrt{3^2+2^2}}}+\left(6pq-p-7q\right)x^{\frac{1}{\sqrt{2^2+2^2}}}$$
$$=qx^{\frac{1}{3\sqrt{2}}}+6qx^{\frac{1}{\sqrt{13}}}+\left(6pq-p-7q\right)x^{\frac{1}{2\sqrt{2}}}.$$

## 4. RESULTS FOR KTUC$_4$C$_8$[p,q] NANOTUBES

In this section, we focus on the graph structure of a family of $TUC_4C_8(S)$ nanotubes. The 2-$D$ lattice of $TUC_4C_8(S)$ is denoted by $KTUC_4C_8[p,q]$, where $q$ is the number of columns and $q$ is the number of rows. The graph of $KTUC_4C_8[p,q]$ is shown in Figure 3.

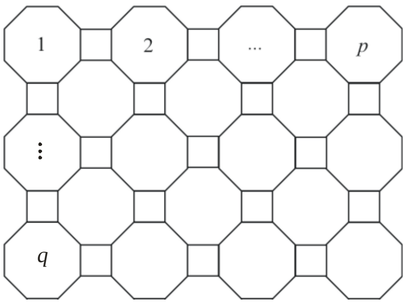


**Figure 3: The graph of $KTUC_4C_8[p,q]$ nanotube**

Let $G$ be the graph of a nanotube $KTUC_4C_8[p,q]$. We see that the vertices of $G$ are either of degree 2 or 3. Therefore $\Delta(G) = 3$ and $\delta(G) = 2$. By calculation, we obtain that $G$ has $12pq - 2p - 2q$ edges. The graph $G$ has three types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = 2p + 2q+ 4$.

$E_2 = \{uv\in E(G) \mid d_G(u) =2, d_G(v) = 3\}$, $|E_2| = 4p + 4q- 8$.

$E_3 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_3| = 12pq - 8p - 8q + 4$.

We have $r_G(u) =\Delta(G) + \delta(G) - d_G(u) = 5 - d_G(u)$.

Thus there are three types of Revan edges based on the degree of end Revan vertices of each Revan edge as given in Table 3.

***Table 3: Revan edge partition of $KTUC_4C_8[p, q]$***

| $r_G(u), r_G(v)$/ $uv \in E(G)$ | (3, 3) | (3, 2) | (2, 2) |
|---|---|---|---|
| Number of edges | $2p+2q+4$ | $4p+4q-8$ | $12pq-8p-8q+4$ |

In the following theorem, we compute the exact formulas of $RSO(KTUC_4C_8[p,q])$, $RSO(KTUC_4C_8[p,q], x)$ for $KTUC_4C_8[p,q]$ nanotubes.

**Theorem 5.** Let $G$ be the graph of a nanotube $KTUC_4C_8[p,q]$. Then

(i) $$RSO(KTUC_4C_8[p,q]) = 24\sqrt{2}pq + \left(4\sqrt{13} - 14\sqrt{2}\right)p + \left(4\sqrt{13} - 10\sqrt{2}\right)q + 4\sqrt{2} - 8\sqrt{13}.$$

(ii) $$RSO(KTUC_4C_8[p,q], x) = (2p+2q+4)x^{3\sqrt{2}} + (4p+4q-8)x^{\sqrt{13}} + (12pq-8p-8q+4)x^{2\sqrt{2}}.$$

**Proof:** From definitions and by using Table 3, we obtain

(i) $$RSO(KTUC_4C_8[p,q]) = \sum_{uv\in E(G)} \sqrt{r_G(u)^2 + r_G(v)^2}$$

$$= (3^2+3^2)^{\frac{1}{2}}(2p+2q+4) + (3^2+2^2)^{\frac{1}{2}}(4p+4q-8) + (2^2+2^2)^{\frac{1}{2}}(12pq-8p-8q-4)$$
$$= 24\sqrt{2}pq + \left(4\sqrt{13} - 14\sqrt{2}\right)p + \left(4\sqrt{13} - 10\sqrt{2}\right)q + 4\sqrt{2} - 8\sqrt{13}.$$

(ii) $$RSO(KTUC_4C_8[p,q], x) = \sum_{uv\in E(G)} x^{\sqrt{r_G(u)^2 + r_G(v)^2}}$$

$$= (2p+2q+4)x^{(3^2+3^2)^{\frac{1}{2}}} + (4p+4q-8)x^{(3^2+2^2)^{\frac{1}{2}}} + (12pq-8p-8q+4)x^{(2^2+2^2)^{\frac{1}{2}}}.$$
$$= (2p+2q+4)x^{3\sqrt{2}} + (4p+4q-8)x^{\sqrt{13}} + (12pq-8p-8q+4)x^{2\sqrt{2}}.$$

In the following theorem, we compute the exact formulas of ${}^{m}RSO(KTUC_4C_8[p,q])$, ${}^{m}RSO(KTUC_4C_8[p,q], x)$ for $KTUC_4C_8[p,q]$ nanotubes.

**Theorem 6.** Let $G$ be the graph of a nanotube $KTUC_4C_8[p,q]$. Then

(i) $${}^{m}RSO(KTUC_4C_8[p,q]) = \frac{6pq}{\sqrt{2}} + \left(\frac{2}{3\sqrt{2}} + \frac{4}{\sqrt{13}} - \frac{4}{\sqrt{2}}\right)p + \left(\frac{2}{3\sqrt{2}} + \frac{4}{\sqrt{13}} - \frac{4}{\sqrt{2}}\right)q + \left(\frac{4}{3\sqrt{2}} - \frac{8}{\sqrt{13}} - \frac{2}{\sqrt{2}}\right).$$

(ii) $${}^{m}RSO(KTUC_4C_8[p,q], x) = (2p+2q+4)x^{\frac{1}{3\sqrt{2}}} + (4p+4q-8)x^{\frac{1}{\sqrt{13}}} + (12pq-8p-8q+4)x^{\frac{1}{2\sqrt{2}}}.$$

**Proof:** From definitions and by using Table 3, we obtain

(i) $${}^{m}RSO(KTUC_4C_8[p,q]) = \sum_{uv\in E(G)} \frac{1}{\sqrt{r_G(u)^2 + r_G(v)^2}}$$

$$= \frac{(2p+2q+4)}{\sqrt{3^2+3^2}} + \frac{(4p+4q-8)}{\sqrt{3^2+2^2}} + \frac{(12pq-8p-8q-4)}{\sqrt{2^2+2^2}}$$
$$= \frac{6pq}{\sqrt{2}} + \left(\frac{2}{3\sqrt{2}} + \frac{4}{\sqrt{13}} - \frac{4}{\sqrt{2}}\right)p + \left(\frac{2}{3\sqrt{2}} + \frac{4}{\sqrt{13}} - \frac{4}{\sqrt{2}}\right)q + \left(\frac{4}{3\sqrt{2}} - \frac{8}{\sqrt{13}} - \frac{2}{\sqrt{2}}\right).$$

(ii) $$^{m}RSO(KTUC_4C_8[p,q],x)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{r_G(u)^2+r_G(v)^2}}}$$

$$=(2p+2q+4)x^{\frac{1}{\sqrt{3^2+3^2}}}+(4p+4q-8)x^{\frac{1}{\sqrt{3^2+2^2}}}+(12pq-8p-8q+4)x^{\frac{1}{\sqrt{2^2+2^2}}}.$$

$$=(2p+2q+4)x^{\frac{1}{3\sqrt{2}}}+(4p+4q-8)x^{\frac{1}{\sqrt{13}}}+(12pq-8p-8q+4)x^{\frac{1}{2\sqrt{2}}}.$$

## 5. RESULTS FOR $GTUC_4C_8[p,q]$ NANOTUBES

In this section, we focus on the graph structure of family of $TUC_4C_8(S)$ nanotubes. The 2-dimensional lattice of $TUC_4C_8(S)$ is denoted by $G=GTUC_4C_8[p,q]$ where $p$is the number of columns and $q$is the number of rows. The graph of$GTUC_4C_8[p,q]$ is depicted in Figure 4.

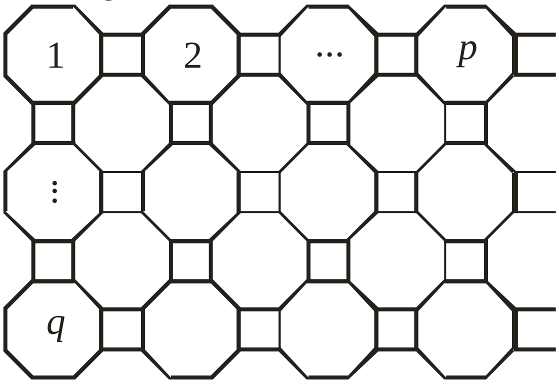


**Figure 4: The graph of $GTUC_4C_8[p,q]$ nanotube**

Let $G$ be the molecular graph of $GTUC_4C_8[p,q]$ nanotube. We see that the vertices of $G$ are either of degree 2 or 3. Therefore $\Delta(G) = 3$ and $\delta(G) = 2$. By calculation, we obtain that $G$ has $12pq - 2p$ edges. Also by calculation, we obtain that $G$ has three types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = 2p$.

$E_2 = \{uv\in E(G) \mid d_G(u) =2, d_G(v) = 3\}$, $|E_2| = 4p$.

$E_3 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_3| = 12pq - 8p$.

We have $r_G(u) =\Delta(G) + \delta(G) - d_G(u) = 5 - d_G(u)$.

Thus there are three types of Revan edges based on the degree of end Revan vertices of each Revan edge as given in Table 4.

**Table 4: Revan edge partition of $G$**

| $r_G(u), r_G(u) \setminus uv \in E(G$ | (3, 3) | (3, 2) | (2, 2) |
|---|---|---|---|
| Number of edges | $2p$ | $4p$ | $12pq - 8p$ |

In the following theorem, we compute the exact formulas of $RSO(GTUC_4C_8[p,q])$, $RSO(GTUC_4C_8[p,q], x)$ for $GTUC_4C_8[p,q]$ nanotubes.

**Theorem 7.**Let $G$ be the graph of a nanotube $GTUC_4C_8[p,q]$. Then

(i) $$RSO(GTUC_4C_8[p,q])=24\sqrt{2}pq+\left(4\sqrt{13}-10\sqrt{2}\right)p.$$

(ii) $$RSO(GTUC_4C_8[p,q],x)==2px^{3\sqrt{2}}+4px^{\sqrt{13}}+(12pq-8p)x^{2\sqrt{2}}.$$

**Proof:** From definitions and by using Table 4, we obtain

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

(i) $RSO(GTUC_4C_8[p,q]) = \sum_{uv\in E(G)} \sqrt{r_G(u)^2 + r_G(v)^2}$

$$= (3^2+3^2)^{\frac{1}{2}}\, 2p + (3^2+2^2)^{\frac{1}{2}}\, 4p + (2^2+2^2)^{\frac{1}{2}}\, (12pq-8p)$$
$$= 24\sqrt{2}pq + \left(4\sqrt{13} - 10\sqrt{2}\right)p.$$

(ii) $RSO(GTUC_4C_8[p,q],x) = \sum_{uv\in E(G)} x^{\sqrt{r_G(u)^2 + r_G(v)^2}}$

$$= 2px^{(3^2+3^2)^{\frac{1}{2}}} + 4px^{(3^2+2^2)^{\frac{1}{2}}} + (12pq-8p)\,x^{(2^2+2^2)^{\frac{1}{2}}}.$$
$$= 2px^{3\sqrt{2}} + 4px^{\sqrt{13}} + (12pq-8p)\,x^{2\sqrt{2}}.$$

In the following theorem, we compute the exact formulas of ${}^{m}RSO(GTUC_4C_8[p,q])$, ${}^{m}RSO(GTUC_4C_8[p,q],x)$ for $GTUC_4C_8[p,q]$ nanotubes.

**Theorem 8.** Let $G$ be the graph of a nanotube $GTUC_4C_8[p,q]$. Then

(i) ${}^{m}RSO(GTUC_4C_8[p,q]) = \frac{6pq}{\sqrt{2}} + \left(\frac{2}{3\sqrt{2}} + \frac{4}{\sqrt{13}} - \frac{4}{\sqrt{2}}\right)p.$

(ii) ${}^{m}RSO(GTUC_4C_8[p,q],x) = 2px^{\frac{1}{3\sqrt{2}}} + 4px^{\frac{1}{\sqrt{13}}} + (12pq-8p)\,x^{\frac{1}{2\sqrt{2}}}.$

**Proof:** From definitions and by using Table 4, we obtain

(i) ${}^{m}RSO(GTUC_4C_8[p,q]) = \sum_{uv\in E(G)} \frac{1}{\sqrt{r_G(u)^2 + r_G(v)^2}}$

$$= \frac{2p}{\sqrt{3^2+3^2}} + \frac{4p}{\sqrt{3^2+2^2}} + \frac{(12pq-8p)}{\sqrt{2^2+2^2}}$$
$$= \frac{6pq}{\sqrt{2}} + \left(\frac{2}{3\sqrt{2}} + \frac{4}{\sqrt{13}} - \frac{4}{\sqrt{2}}\right)p.$$

(ii) ${}^{m}RSO(GTUC_4C_8[p,q],x) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{r_G(u)^2 + r_G(v)^2}}}$

$$= 2px^{\frac{1}{\sqrt{3^2+3^2}}} + 4px^{\frac{1}{\sqrt{3^2+2^2}}} + (12pq-8p)\,x^{\frac{1}{\sqrt{2^2+2^2}}}$$
$$= 2px^{\frac{1}{3\sqrt{2}}} + 4px^{\frac{1}{\sqrt{13}}} + (12pq-8p)\,x^{\frac{1}{2\sqrt{2}}}.$$

## 6. CONCLUSION

In this study, we have introduced the modified Revan Sombor index of a graph. The Revan Sombor and modified Revan Sombor indices and their exponentials of certain naotubes have been computed. In Medical Science, Chemical, Medical, biological pharmaceutical properties of molecular structure are essential for drug design. Their properties can be studied by the topological index calculation. .

**Acknowledgement**
This research is supported by IGTRC No. BNT/IGTRC/ 2022:2207:107 International Graph Theory Research Center, Banhatti 587311, India.

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

## REFERENCES

[1] V.R.Kulli, Graph indices, in Hand Book of Research in Advanced Applications of Graph Theory In Modern Society, M.Pal, S.Samanta, A.Pal (eds.) IGI Global, USA (2020) 66-91.
[2] I.Gutman Geometric approach to degree based topological indices: Sombor indices, *MATCH Common. Math. Comput. Chem* 86 (2021) 11-16
[3] V.R.Kulli, Nirmala index, *International Journal of Mathematics Trends and Technology*, 67(3) (2021) 8-12.
[4] N.N.Swamy, T.Manohar, B.Sooryanarayana and I.Gutman, Reverse Sombor index, *Bulletin of International Mathematical Virtual Institute*, in press.
[5] V.R.Kulli, Neighborhood Sombor index of some nanostructures, *International Journal of Mathematics Trends and Technology*, 67(5) (2021) 101-108.
[6] S.Alikhani and N.Ghanbari, Sombor index of polymers, *MATCH Commun. Math. Comput. Chem.* 86 (2021).
[7] R.Cruz, I.Gutman and J.Rada, Sombor index of chemical graphs, *Appl. Math. Comput.* 399 (2021) 126018.
[8] H.Deng, Z.Tang and R.Wu, Molecular trees with extremal values of Sombor indices, *Int. J. Quantum Chem.*DOI: 10.1002/qua.26622.
[9] B.Horoldagva and C.Xu, On Sombor index of graphs, *MATCH Commun. Math. Comput. Chem.* 86 (2021).
[10] V.R.Kulli, Sombor indices of certain graph operators, *International Journal of Engineering Sciences and Research Technology*, 10(1) (2021) 127-134.
[11] V.R.Kulli, Multiplicative Sombor indices of certain nanotubes, *International Journal of Mathematical Archive*, 12(3) (2021) 1-5.
[12] V.R.Kulli, $\delta$-Sombor index and its exponential for certain nanotubes, *Annals of Pure and Applied Mathematics*, 23(1) (2021) 37-42.
[13] V.R.Kulli and I.Gutman, Computation of Sombor indices of certain networks, *SSRG International Journal of Applied Chemistry*, 8(1) (2021) 1-5.
[14] I.Redzepović, Chemical applicability of Sombor indices, *J. Serb. Chem. Soc.*(2021) https://doi.org/10.2298/JSC20:1215006R.
[15] T.Reti, T. Došlić and A. Ali, On the Sombor index of graphs, *Contributions of Mathematics*, 3 (2021) 11-18.
[16] V.R.Kulli, Computation of multiplicative Banhatti-Sombor indices of certain benzenoid systems, *International Journal of Mathematical Archive*, 12(4) (2021) 24-30.
[17] V.R.Kulli, On Banhatti-Sombor indices, *SSRG International Journal of Applied Chemistry*, 8(1) (2021) 21-25.
[18] V.R.Kulli, On second Banhatti-Sombor indices, *International Journal of Mathematical Archive*, 12(5) (2021) 11-16.
[19] J.Rada, L.M.Rodriguez and J.M.Sigarreta, General properties of Sombor indices, *Discrete Applied Mathematics*, 299 (2021) 87-97.
[20] V.R.Kulli and I.Gutman, On some mathematical properties of Nirmala index, *Annals of Pure and Applied Mathematics*, 23(2) (2021) 93-99.
[21] M.R.Nandargi and V.R.Kulli, The ($a$ ,$b$)-Nirmala index, *International Journal of Engineering Sciences and Research Technology*, 11(2) (2022) 37-42.
[22] I.Gutman and V.R.Kulli, Nirmala energy, *Open Journal of Discrete Applied Mathematics*, 4(2) (2021) 11-16.
[23] V.R. Kulli, Revan indices of oxide and honeycomb networks, *International Journal of Mathematics and its Applications*, 5(4-E) (2017) 663-667.
[24] V.R.Kulli and I.Gutman, Revan Sombor index, *Journal of Mathematics and Informatics*,22 (2022) 23-27.
[25] V.R. Kulli, On the product connectivity Revan index of certain nanotubes, *Journal of Computer and Mathematical Sciences*, 8(10) (2017) 562-567.

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

[26] V.R.Kulli, F-Revan index and R-Revan polynomial of some families of Benzenoid systems, *Journal of Global Research in mathematical Archives*, 5(11) (2018) 1-6.