# Quadratic Average Enhanced Curve Bootstrapping Algorithm

Quadratic Average Enhanced Curve Bootstrapping Algorithm (QAECBA) is intended to replace the existing Enhanced Curve Bootstrapping Algorithm (ECBA) and algorithm. The new approach achieves the ”smoothness” of both spot and forward zero-curves, and produces the instantaneous forward rate curve directly.

Partial Differential Hedge (PDH) analysis is used to compare different curve construction algorithms. Generally speaking, the PDH analysis consists in calculating the sensitivity of a given portfolio to a change in the yield or price of the instruments from which the zero-curve (discount curve, forecast curve) used to price a portfolio is bootstrapped.

Because the bootstrapping algorithms differ, we expect different sensitivities per instrument, but the portfolio’s total delta sensitivities should be close. Also, the sensitivities per instrument should have the same sign, implying hedge numbers having the same sign (positive for long positions, negative for short positions).

Let *Ii*, i = 1, ..., n be the instruments used to bootstrap a zero-curve C . We can think of *Ii* as a function of its yield (or price) which we denote by yi (of course, there are other characteristics, such as maturity, that determine an instrument, but they will not be reflected in our notation). Consequently, our zero curve is a (vector-valued) function of all yi, i = 1, ..., n, but via Ii, i = 1, ..., n (and the bootstrapping algorithm used, but we do not carry it in our notation):

$$\mathscr{C} := \mathscr{C}(I_1(y_1), ..., I_i(y_i), ..., I_n(y_n)).$$

Let Δy be a small positive quantity (for example, 0.0001 or 0.001). For each j from 1 to n, we shift (up, down) the yield yj of the j'th instrument by Δy. Consequently, the zero-curve changes. We denote the new values by:

$$\mathscr{C}_j^+ := \mathscr{C}(I_1(y_1), ..., I_{j-1}(y_{j-1}), I_j(y_j + \Delta y), I_{j+1}(y_{j+1}), ..., I_n(y_n)),$$
$$\mathscr{C}_j^- := \mathscr{C}(I_1(y_1), ..., I_{j-1}(y_{j-1}), I_j(y_j - \Delta y), I_{j+1}(y_{j+1}), ..., I_n(y_n)).$$

Let S be a portfolio of interest-rate derivatives (for example, swaps or swaptions). We think of it as a linear combination of its underlying derivatives (the coefficients representing positions, positive for long, negative for short).

In order to price this portfolio we use the above zero-curve (discount curve). This means that the net present value S is a function of C (that is, of yi, i = 1, ..., n, via the bootstrapping instrument definitions and the bootstrapping algorithm used). We denote the net present value by NPV. Corresponding to the three curve instances described above, we have three present values of S:

$$NPV(S, \mathscr{C}),\ NPV(S, \mathscr{C}_j^+),\ NPV(S, \mathscr{C}_j^-).$$

We can introduce now the sensitivities computed in the PDH analysis, delta (up, down, two-way) and gamma, per bootstrapping instrument (for each j = 1, ..., n):

$$\begin{aligned}
\Delta_j^+ &:= NPV(S, \mathscr{C}_j^+) - NPV(S, \mathscr{C}), \\
\Delta_j^- &:= NPV(S, \mathscr{C}) - NPV(S, \mathscr{C}_j^-), \\
\Delta_j &:= (NPV(S, \mathscr{C}_j^+) - NPV(S, \mathscr{C}_j^-))/2, \\
\Gamma_j &:= NPV(S, \mathscr{C}_j^+) + NPV(S, \mathscr{C}_j^-) - 2 \cdot NPV(S, \mathscr{C})
\end{aligned}$$

The hedge number for a bootstrapping instrument, say Ij , is representing the position we should take in Ij to offset the change in the portfolio's value. More precisely, if we denote by Hj the hedge number, and by NPV (Ij ,C )) the net present value of the bootstrapping instrument (priced itself under different instances of the newly bootstrapped curve), then the following equation must be satisfied:

$$NPV(S, \mathscr{C}_j^+) - NPV(S, \mathscr{C}) + H_j \cdot (NPV(I_j, \mathscr{C}_j^+) - NPV(I_j, \mathscr{C})) = 0.$$

Note that the numerator is Δ+j, and, consequently, dependent on the bootstrapping algorithm used (the denominator has a weaker dependency on the bootstrapping algorithm used as the second term is not affected by the algorithm change - for example, if Ij is a par swap, then its NPV must be 0 no matter which algorithm we are using).

Once we fix the portfolio S (including the pricing models for our derivatives) and the instruments used to bootstrap the curve C (and all its attributes except bootstrapping algorithm), the sensitivities above are dependent on the bootstrapping algorithm and Δy only.

Reference:
https://finpricing.com/lib/EqSpread.html