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Modelling exponential decay to predict half-life of radioactive substance

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Abstract: This paper presents exponential decay modelling to predict half-life of a radioactive substance that is the time required for half of the radioactive material to decompose.

Keywords: exponential decay, radioactive material, prediction of half-life

1. Introduction

One of the most important characteristics of radioactivity is that it decays exponentially. That is, the exponential decay of a radioactive substance represents a decrease in the quantity of the radioactive substance over time. In this paper, the study focuses on exponential decay modelling that is used to predict half-life of a radioactive material, that is, the time taken for half of the radioactive substance to decompose.

2. Exponential Decay Model

Exponential decay model is very useful mathematical model for simulating the dose concentration of a drug over time. Also, it simulates the minimum and maximum concentrations of a drug administered intravenously using by geometric series [1, 2].

A quantity V is said to be subject to exponential decay if the quantity decreases at a rate proportional to its value over time t. Symbolically, this can be expressed as follows:

$$\frac{dV(t)}{dt} \propto V(T)$$

That is, $\frac{dV(t)}{dt} = kV(T)$ where K is decay constant, $(K < 0)$.

By solving this differential equation, we obtain

$V(T) = V_0 e^{kt}$, which is called the Exponential Decay Model.

3. Modelling Radioactive Decay

Radioactive decay is a common example of exponential decay. Radioactive materials, and some other substances, decompose according to a formula for exponential decay.

That is, the amount of radioactive material (V) present at time (t) is given by the equation

$$V(T) = V_0 e^{kt}$$

A radioactive substance is often described in terms of its half-life, which is the time taken for half the material to decompose.

Let us find the half-life of radium-226. After 500 years, a sample of radium-226 has decayed to 80.4% of its original mass.

Let V be the mass of radium present at time t (t = 0 corresponds to 500 years ago). We want to know for what time t is $V = (1/2)V_0$. However, we do not even know what k is yet. Once we know what k is, we can set V in the formula for exponential decay to be equal to $(1/2)V_0$, and then solve for t.

First we must determine k. We are given that after 500 years, the amount present is 80.4% of its original mass. That is, when t=500, V=0.804 V_0. Substituting these values into the formula for exponential decay, we obtain:

$$0.804\, V_0 = V_0\, e^{k(500)}.$$

Dividing through by V_0 gives us

$$0.804 = e^{500k}$$

which is an exponential equation.

To solve this equation, we take natural logs (ln) of both sides.

$$\ln(0.804) = \ln(e^{500k})$$

We know that $\ln(e^{500k}) = 500k$ by the cancellation properties of ln and e. So the equation becomes

$$\ln(0.804) = 500k$$

and

$$k = (\ln 0.804)/500.$$

This is the exact solution; evaluate the natural log with a calculator to get the decimal approximation k = -0.000436 .

Since we now know k, we can write the formula for the amount of radium present at time t as

$$V = V_0\, e^{-0.000436\, t}.$$

Now, we can finally find the half-life. We set $V = 1/2\, V_0$ and solve for t.

$$(1/2)V_0 = V_0\, e^{-0.000436\, t}$$

Dividing through by V_0 again, we get:

$$1/2 = e^{-0.000436\,t}.$$

To solve for t, take natural logs:

$$\ln(1/2) = \ln[e^{-0.000436\,t}].$$

Then applying the cancellation property for logarithms yields

$$\ln(1/2) = -0.000436\,t$$

So

$$t = \ln(1/2)/(-0.000436)$$

or $t = 1590$. The half-life of radium-226 is approximately 1590 years.

4. Conclusion

The research study focuses on exponential decay modelling that is used to predict half-life of a radioactive material, that is, the time taken for half of the radioactive substance to decompose. The exponential decay model has been used to find the half-life of radium-226 in this paper.

References

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