Factorial of Sum of Two nonnegative Integers is equal to Multiple of the Product of Factorial of the Two Nonnegative Integers

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Abstract: This paper presents a theorem in factorial functions with the sum of any two nonnegative integers that is equal to multiple of the product of factorial of the same two nonnegative integers.

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1. Introduction

Integers involving in factorial functions or factorials [1-8] are non-negative numbers. These have several applications in computing, science, and engineering.

Definition: Factorial of any non-negative integer n, denoted by $n!$, is defined as a product of all nonnegative integers less than or equal to n.

For example, 5! =1×2×3 ×4 ×5 = 120. Note that zero factorial is always one, that is, 0! =1.

2. A Theorem in Factorials

The theorem [1-8] states that the factorial of sum of any two nonnegative integers is equal to multiple of a product of factorials of the same two nonnegative integers.

Let N = {0, 2, 3, 4, . . ., } be a set of natural number including zero.

Theorem: For any two integers $m, n \geq 0, (m+n)! = \text{k} \times \text{m}! \times n!, (k \geq 0 \ \& \ k \in N)$.

$Proof. (m+n)! = \text{k} \times \text{m}! \times n!$ can be proved by mathematical induc$tion$.

$\text{Basis. Let m} = 2 \text{ and n} = 3.\ (2+3)! = 720 = 60 \times 2! \times 3! \text{ is obviously true.}$

Inductive hypothesis. Let us assume that it is true for $(m-b)$ and $(n-c)$,

$$\text{that is, } \big((m-b)+(n-c)\big)! = \text{h} \times (\text{m}-\text{b})! \times (n-c)!,$$

where $m \geq b \geq 0$ and $n \geq c \geq 0$ & $b, c \in N$.

Inductive Step. We must show that the hypothesis is true for $(\text{m}-\text{b}+\text{b})$ and $(n-c+c)$.
$\big((m-b+b)+(n-c+c)\big)! = h \times (m-b+b)! \times (n-c+c)!, (h \geq 0 \ \& \ h \in N)$.
By simplifying this result, we get $(m+n)! = \text{k} \times \text{m}! \times n!, \quad (h = k)$.
Hence, theorem is proved.

Corollary: For any k nonnegative integers $n_1, n_2, n_3, \cdots$ *and* n_k,
$(n_1 + n_2 + n_3 + \cdots + n_k)! = (a_1 \times a_2 \times a_3 \times \cdots \times a_{k-1}) \times n_1! \times n_2! \times n_3! \times \cdots \times n_k!$,

$$\text{that is, } \left(\sum_{i=1}^{k} n_i\right)! = A \prod_{i=1}^{k} n_i\,!,$$

where $\mathrm{A} = a_1 \times a_2 \times a_3 \times \cdots \times a_{k-1}$ and $A, a_1, a_2, a_3, \cdots, a_{k-1}$ are coefficients.

For instance,
If $n_1 = n_2 = n_3 = \cdots = n_k = n$. Then, $(n_1 + n_2 + n_3 + \cdots + n_k)! = (k \times n)!$.
If $n_1 = n_2 = n_3 = \cdots = n_k = 0$. Then, $(n_1 + n_2 + n_3 + \cdots + n_k)! = (k \times 0)! = 0! = 1$.
If $n_1 = n_2 = n_3 = \cdots = n_k = 1$. Then, $(n_1 + n_2 + n_3 + \cdots + n_k)! = (k \times 1)! = k!$.
If $n_1 = n_2 = n_3 = \cdots = n_k = 2$. Then, $(n_1 + n_2 + n_3 + \cdots + n_k)! = (k \times 2)! = (2k)!$.
If $n_1 = n_2 = n_3 = \cdots = n_k = k$. Then, $(n_1 + n_2 + n_3 + \cdots + n_k)! = (k \times k)! = k^2!$.

This novel idea can help to the researchers working in computational science, management, science, and engineering.

3. Conclusion

In this article, an innovative combinatorial technique and theorem are introduced and the theorem states that the factorial of sum of any k nonnegative integers is equal to multiple of the product of factorials of the k nonnegative integers. This methodological advance can enable the researchers working in computational science, management, science and engineering to solve the most real life problems and meet today's challenges [9].

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