Optimal Control of a Grid-connected Service Area for Plug-in Electric Vehicles Fast Charging under uncertain Power Demand

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Abstract— **In this paper we consider the problem of controlling a service area hosting stations for the provisioning of the electric vehicles fast charging service, having the support of an electric energy storage system and local power production from renewables. Key aspects motivating the work are the hard temporal constraint imposed by drivers requiring the fast charging service and the impact high aggregated power withdrawal has on the economic viability of the investment for the service area operator; consequently key control requirements include a congestion level driven tracking of the charging power demand and the flattening of power flow at the point of connection of the service area to the electricity grid, while keeping stable ESS operation. These opposing control objectives, together with the uncertain nature of the power demand and production, brings to the formulation of a stochastic model predictive control problem, based on a continuous/finite-time optimal control problem, for which the explicit form of solution is determined. Simulations are presented to validate the proposed approach.**

I. INTRODUCTION

In recent years the plug-in electric vehicle (PEV) fast charging has become the subject of huge investments for operators of the charging infrastructure and car manufacturers, with the trend of enabling the service at continuously increasing power levels [1][2]. Among the motivations, the increasing EVs travel range (and then the battery capacity) and the need of overcoming structural and logistic barriers for charging at home, that still make drivers skeptical about the transition to electromobility [3][4]. A key aspects for operators investing in the fast charging infrastructure is its economic viability, as the connection of charging stations (CSs) to the grid is subject to some costs for their installation and operation, that heavily depends on the nominal power requested at point of connection (POC). As a key requirement for the fast charging service is the charging time, the application of the well known smart charging concept [5] has to be restricted to situations of power congestion in the area hosting the CSs, and requires the support of other systems able to flatten the aggregated CSs power withdrawal at the POC. In this regard the energy storage system (ESS) technology can provide the needed additional degree of freedom, in consideration of its almost technical maturity and the potential adoption of second life batteries for containment of capital expenditure [6]. Consequently, the combination of CSs, an ESS and potentially local power generation from renewables in a microgrid, in the following referred to as "service area", is at the basis of fast charging viability for service area operators. In this paper we propose a control algorithm for PEVs charging power and ESS charging/discharging process in the service area according to the following objectives:

1) to flatten the power flowing at the POC, as avoiding power peaks allows to lower the connection fees payed by the operator of the service area to the grid, costs items which are partially transferred to the driver through the cost of the service;
2) to keep the state of charge (SOC) of the ESS as close as possible to a desired reference value (usually, half of the full charge), to ensure the ESS has at any point in time a sufficient reserve of energy to charge or discharge, respectively in case the power generation outbalances the charging power and in case of congestion;
3) to maintain the aggregated charging power of PEVs in the service area as close as possible to a certain reference signal represented by the power demand; the tracking is required to be stringent in case of low congestion, so as to serve the drivers in the minimum time, and more relaxed in case of congestion, so as to mitigate the ESS control effort for flattening the power flow at POC.

As these control requirements work in opposition each other, and in consideration of the fact that the future evolutions of the charging power demand and power generation from renewables are in principle uncertain, the problem is here modeled as an optimal control problem subject to uncertainty and time-dependent weights, addressed using the machinery of model predictive control (MPC) and calculus of variation based finite-time optimal control.

Optimal control is a popular control technique for problems related to resource management, see, e.g., [7], [8]. Numerous control problems have been formulated in literature, concerning microgrids equipped with controllable loads, especially PEVs, storage devices and local generation. The works found in the literature mainly differ for the specific requirements of their problems, the methodologies used and the modelling problems due to specific mathematical issues. Among the many applications, [9] addressed the issue of minimizing the voltage deviations in microgrids hosting

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distributed generation, while [10] proposes microgrid reconfiguration as a way to reduce operational costs and enhance security. In [11], an application to the problem of smart home energy management in a microgrid environment is presented. Several control methodologies, such as sliding mode control [12], MPC [13], [14], [15], [16] [17] [18], machine learning [19] were studied for many applications. Also the Pontryagin minimum principle (PMP) and the calculus of variations have been successfully applied in several problems related to optimal power control, in particular concerning hybrid vehicles [20], [21], [22] and microgrids [23], [24]. One of the main issues that make difficult to apply such control techniques is the impossibility, in some cases, to derive a closed-loop or a causal solution. To address this issue, some simplifications on the costate calculation from the necessary optimality conditions are proposed in the literature. In [20], the authors proposes that the costate of the system can be assumed constant for their hybrid vehicle control problem, with a resulting simplification on the optimal solution derivation. In [22], the authors combine PMP and MPC methodologies for the management of a PEV bus. Similar approach is used in [23], in this case to compute the optimal energy flow in a microgrid equipped with ESSs and Renewable Energy Sources, with the goal of minimizing the control effort for the ESS and the deviation of the SOC from a reference value; the costate has been considered constant also in [23]. Finally, some recent contributions also apply the PMP theory to the problem of optimal fast charging of Li-ion batteries, see [25] in particular.

This paper reconsiders the work [26], where the deterministic fast charging control problem has been faced, under milder assumptions and extending the results in several directions. The distinctive aspects of this work are as follows. First, we formulate the control problem for service area fast charging as a stochastic finite-time optimal control problem, in which the charging power and the local power generation from renewables are assumed uncertain; we embed the problem in the framework of MPC to take advantage of the real time update of information about the plant to reduce the effect of uncertainty. Second, in order to properly meet the power tracking requirement, we introduce a cost term in the cost function which linearly depends on the level of power congestion in the service area; this choice, combined with the uncertain nature of the charging demand, is at the basis of the statistical properties of the charging demand which have to be known for the determination of the optimal control. Third, we provide the optimal control in an explicit form, which is useful in scenarios where the control is computed by local equipment characterized by low computational capabilities; the achieved control assumes, over the control window, the knowledge of the expected value and variance of the charging power demand and the expected value of local power generation in the service area, which are generally known by the service area operator in view of the availability of historical data. Finally, to the best of the authors' knowledge, this is one of the first works on the application of the theory of calculus of variation to the control of ESS and charging PEVs in a service area subject to the requirements presented above and also subject to uncertainty.

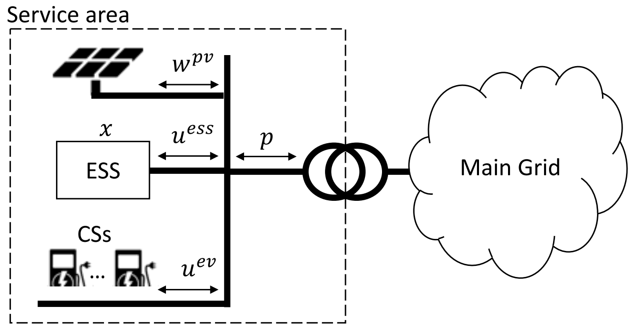

Fig. 1. Service area architecture considered in the paper.

The reminder of the paper is as follows. Section II presents the service area architecture considered in this work and the proposed open loop stochastic optimal control problem at the basis of the MPC framework. Section III presented the explicit MPC optimal control calculation. Section IV presents and analyzes the simulations in a simplified (even if realistic) charging scenario. Section V provides the conclusions and outlines possible future developments of the work.

II. PROBLEM FORMALIZATION

The architecture of the service area considered in this paper is presented in Fig. 1. In this setup, the ESS, the CSs and the power generation are directly connected to the POC. In what follows $p(t)$ denotes the power flowing at the POC at time t, $u^{ess}(t)$ the ESS charging power, $x(t)$ the relative ESS SOC with respect to a reference value (typically, 50% of the total charge), $u^{ev}(t)$ the actual cumulative power withdrawn by the PEVs, $\hat{u}^{ev}(t)$ the cumulative PEVs power demand, $u^{nom}(t)$ the nominal cumulative power that can be delivered by the CSs in the service area (i.e., $\hat{u}^{ev}(t) \leq u^{nom}(t)$, $\forall t$), $w^{pv}(t)$ the power generated in the area, assumed in the following to be provided by a photovoltaic plant. The ESS dynamics is given by $\dot{x}(t) = u^{ess}(t)$. Further, as consequence of the previously described electrical connection scheme, the obvious balance equation $p(t) = u^{ess}(t) + u^{ev}(t) - w^{pv}(t)$ holds at the POC.

In this work we propose to control the plant according to the well known MPC methodology principle [27]. Standard discrete-time MPC implementations are based on the periodic solution of an open loop optimal control problem, which is defined over a pre-defined future control window and leverages the knowledge, at the current time t_i, of the state and possibly the future evolution of other exogenous signals (in most cases forecasts). The open loop optimal control problem typically takes the form of an optimization problem that, depending on its nature, can be handled using proper solvers. The first sample in the computed optimal control sequence is actually applied to the plant for the entire sampling period before a new optimization takes place. In this work we take a slightly different perspective, motivated by the interest in finding a methodological solution that can be used in a scenario characterized by very low computational capabilities. We still assume to update the control on a periodic basis, but we formulate the open loop optimal control problem as a continuous-time optimal control problem subject to uncertainty and provide an explicit closed loop control law. Then, based on the control requirements introduced in Section I, the following open loop optimal control problem at the basis of the MPC framework is defined, in which $\mathbf{E}[\cdot]$ denotes the expected value operator.

Problem 1. *(Fast charging optimal control problem in service area under uncertain charging demand and PV power production). Given an initial time t_i and a final time t_i+MT of problem definition (where T is the MPC time step and M a positive integer), given uncertain $\{\hat{u}^{ev}(t), t \in [t_i, t_i+MT]\}$ and $\{w^{pv}(t), t \in [t_i, t_i+MT]\}$ find*

$$\min_u \left\{ J(u) = \mathbf{E}\Big[S(x(t_i+MT)) + \\ + \int_{t_i}^{t_i+MT} L\big(x(t), u^{ess}(t), u^{ev}(t), t\big) dt \Big] \right\}, \tag{1}$$

with

$$S(x(t_i+MT)) = \frac{1}{2} s x(t_i+MT)^2 \tag{2}$$

$$L\big(x(t), u^{ess}(t), u^{ev}(t), t\big) = \\ = \frac{1}{2}\big[qx(t)^2 + rp(t)^2 + c(\hat{u}^{ev}(t))\big(u^{ev}(t) - \hat{u}^{ev}(t)\big)^2\big] \tag{3}$$

in which

$$c(\hat{u}^{ev}(t)) = c^{nom}\big(1 + u^{nom}(t) - \hat{u}^{ev}(t)\big), \tag{4}$$

subject to

$$x(t_i) = x_i, \tag{5}$$

$$\dot{x}(t) = u^{ess}(t), \quad \forall t \in [t_i, t_i+MT], \tag{6}$$

$$p(t) = u^{ess}(t) + u^{ev}(t) - w^{pv}(t), \quad \forall t \in [t_i, t_i+MT], \tag{7}$$

where s, q, r, c^{nom} are positive real numbers.

Once the optimal solution $u(t)^* = col(u^{ess}(t)^*, u^{ev}(t)^*)$ of Problem 1 is found, the control actuated on the plant is simply $u(t)^*|_{[t_i, t_i+T)}$. At time t_i+T the control is computed again moving the control window one step forward, based on the new available information.

It is straightforward for the reader to map the three terms appearing in the Lagrangian function (3) to the three control requirements stated in Section 1. The box constraints on $p(t)$, $u^{ess}(t)$ and $x(t)$ are not considered here; though several works in literature (e.g., [28], [29]) show how such kind of constraints can be handled in the context of a deterministic formulation of the addressed problem, the solution of an extended version of Problem 1 to the box constrained case is left to future works, for which the results reported in this paper are instrumental.

The following remarks are at the basis of the proposed contribution of this work and the development which follows.

Remark 1. As the charging power demand $\hat{u}^{ev}(t)$ and the power production $w^{pv}(t)$ are uncertain, the cost function $J(u)$ is expressed in terms of an expected value. This will bring, in the following section, to the identification of the set of statistical quantities which have to be known for the determination of the optimal control. For the same reason, and bearing in mind (7), the optimal trajectory of the power flowing at the POC can be given in terms of its expected value only.

Remark 2. The weight $c(\hat{u}^{ev}(t))$ appearing in the third term of (3) depends on the difference between the nominal charging power demand u^{nom}, namely the maximum power that can be delivered by the CSs in the service area, and the actual demand $\hat{u}^{ev}$. This choice is motivated by the idea to let the demand add an additional degree of freedom to the control of the area which progressively increases with the level of congestion; the parameter c^{nom} represents the weight in the case of maximum congestion. As the actual demand is assumed time dependent and uncertain, this weight will have a significant impact on the structure of the optimal control.

III. The optimal control

Problem 1 can be solved by using standard techniques from calculus of variations, provided that proper assumptions are made regarding the uncertain variable $\hat{u}^{ev}$. As a preliminary step, notice that, using the linearity property of the expected value and the fact that the ESS dynamics is not directly affected by uncertainty, the cost function assumes the form

$$J(u) = S(x(t_i+MT)) + \int_{t_i}^{t_i+MT} \tilde{L}\big(x(t), u^{ess}(t), u^{ev}(t), t\big) dt \tag{8}$$

in which

$$\tilde{L}\big(x(t), u^{ess}(t), u^{ev}(t), t\big) = \frac{1}{2}\big[qx(t)^2 + r\mathbf{E}[p(t)^2] + \\ + \mathbf{E}[c^{nom}\big(1 + u^{nom}(t) - \hat{u}^{ev}(t)\big)\big(u^{ev}(t) - \hat{u}^{ev}(t)\big)^2]\big]. \tag{9}$$

Tedious but simple calculations show that the Hamiltonian of the system is

$$\begin{aligned}\mathcal{H} :=\ & \tilde{L}\big(x(t), u^{ess}(t), u^{ev}(t), t\big) + \lambda(t) u^{ess}(t) = \\ & = \frac{1}{2} q x(t)^2 + \\ & + \frac{1}{2} r \Big(u^{ess}(t)^2 + u^{ev}(t)^2 + \mathbf{E}[w^{pv}(t)^2] + \\ & + 2u^{ess}(t)u^{ev}(t) - 2u^{ess}(t)\mathbf{E}[w^{pv}(t)] + \\ & - 2u^{ev}(t)\mathbf{E}[w^{pv}(t)] \Big) + \\ & + \frac{1}{2} c^{nom} \Big(u^{ev}(t)^2 + u^{nom}(t)u^{ev}(t)^2 - \mathbf{E}[\hat{u}^{ev}(t)^3] + \\ & + \mathbf{E}[\hat{u}^{ev}(t)^2] + u^{nom}(t)\mathbf{E}[\hat{u}^{ev}(t)^2] + 2u^{ev}(t)\mathbf{E}[\hat{u}^{ev}(t)^2] + \\ & + 2u^{ev}(t)\mathbf{E}[\hat{u}^{ev}(t)] + 2u^{nom}(t)u^{ev}(t)\mathbf{E}[\hat{u}^{ev}(t)] + \\ & + u^{ev}(t)^2\mathbf{E}[\hat{u}^{ev}(t)] \Big) + \lambda u^{ess}(t).\end{aligned} \tag{10}$$

Problem 1 is convex, and the resulting sufficient optimality conditions are

$$\dot{\lambda}(t) = -\frac{\partial \mathcal{H}}{\partial x(t)} = -qx(t), \tag{11}$$

$$\begin{aligned} &\frac{\partial \mathcal{H}}{\partial u^{ess}(t)} = 0 \\ &\Leftrightarrow ru^{ess}(t) + ru^{ev}(t) - r\mathbf{E}[w^{pv}(t)] + \lambda(t) = 0 \\ &\Leftrightarrow u^{ess}(t) = -\frac{1}{r}\lambda(t) - u^{ev}(t) + \mathbf{E}[w^{pv}(t)], \end{aligned} \tag{12}$$

$$\begin{aligned} &\frac{\partial \mathcal{H}}{\partial u^{ev}(t)} = 0 \\ &\Leftrightarrow r\Big(u^{ev}(t) + u^{ess}(t) - \mathbf{E}[w^{pv}(t)]\Big) + \\ &+ c^{nom}\Big(u^{ev}(t) + u^{nom}(t)u^{ev}(t) + \mathbf{E}[\hat{u}^{ev}(t)^2] + \\ &+ \mathbf{E}[\hat{u}^{ev}(t)] - u^{nom}(t)\mathbf{E}[\hat{u}^{ev}(t)] - u^{ev}(t)\mathbf{E}[\hat{u}^{ev}(t)]\Big) = 0 \\ &\Leftrightarrow u^{ev}(t) = \frac{1}{c(t)}\lambda(t) + \\ &- \frac{1}{c(t)}c^{nom}\Big(\mathbf{E}[\hat{u}^{ev}(t)^2] - (1+u^{nom})\mathbf{E}[\hat{u}^{ev}(t)]\Big), \end{aligned} \tag{13}$$

$$\lambda(t_i + MT) = \frac{\partial S}{\partial x(t_i + MT)} = sx(t_i + MT). \tag{14}$$

where $c(t) = c^{nom}(1 + u^{nom}(t) - \mathbf{E}[\hat{u}^{ev}(t)])$ and where the last step in (13) is achieved by substituting $u^{ess}(t)$ as in (12). Equation (11) is the costate equation, (12) and (13) the equations of the control, and (14) the transversality condition.

By plugging (13) into (12), we rewrite the latter as

$$\begin{aligned} u^{ess}(t) = &-\frac{1}{r'(t)}\lambda(t) + \frac{c^{nom}}{c(t)}\Big(\mathbf{E}[\hat{u}^{ev}(t)^2] + \\ &- \big(1 + u^{nom}(t)\big)\mathbf{E}[\hat{u}^{ev}(t)]\Big) + \mathbf{E}[w^{pv}(t)], \end{aligned} \tag{15}$$

in which

$$r'(t) := \left[\frac{1}{r} + \frac{1}{c(t)}\right]^{-1}. \tag{16}$$

It is possible to notice that (15) is a generalization of equation (12) in [26], having dropped the assumption of exact knowledge of $w^{pv}(t)$ and $\hat{u}^{ev}(t)$.

After plugging (12) into the state dynamics, the state and the costate dynamics define the following two-point boundary value problem

$$\begin{bmatrix}\dot{x}(t)\\ \dot{\lambda}(t)\end{bmatrix} = \underbrace{\begin{bmatrix}0 & -\frac{1}{r'(t)}\\ -q & 0\end{bmatrix}}_{:=A(t)}\begin{bmatrix}x(t)\\ \lambda(t)\end{bmatrix} + \underbrace{\begin{bmatrix}-1\\ 0\end{bmatrix}}_{:=B}w(t) \tag{17}$$

with boundary conditions $x(t_i) = x_i$ and $\lambda(t_f) = sx(t_f)$, and where we have defined

$$\begin{aligned} w(t) := &-\frac{c^{nom}}{c(t)}\big(\mathbf{E}[\hat{u}^{ev}(t)^2] + \\ &- \big(1 + u^{nom}(t)\big)\mathbf{E}[\hat{u}^{ev}(t)]\big) - \mathbf{E}[w^{pv}(t)]. \end{aligned} \tag{18}$$

Notice that (19) is a time varying system, because $r'(t)$ depends on $\mathbf{E}[\hat{u}^{ev}(t)]$. Its explicit solution can be however computed in the present case provided that a proper assumption on $\mathbf{E}[\hat{u}^{ev}(t)]$ is made. Specifically we assume that $\mathbf{E}[\hat{u}^{ev}(t)]$ is piece-wise constant over the same time periods characterizing the MPC framework. This assumption is reasonable, as in practice this signal may be built starting from historical sampled metering data. As a consequence of this assumption the time-dependent dynamical matrix of system (19) is piece-wise constant. Based on this assumption, the following lemma summarizes the main result of the previous work [26], which is instrumental to the determination of the dynamics of system (19) and, consequently, the solution of Problem 1 and the optimal control of the MPC scheme.

Lemma 1.1. *Consider a system of the form*

$$\begin{bmatrix}\dot{x}(t)\\ \dot{\lambda}(t)\end{bmatrix} = A(t)\begin{bmatrix}x(t)\\ \lambda(t)\end{bmatrix} + Bw(t) \tag{19}$$

defined over the time interval $[t_i, t_f]$, in which $x(t) \in \mathbb{R}$, $\lambda(t) \in \mathbb{R}$, $w(t) \in \mathbb{R}$, $A(t) \in \mathbb{R}^{2\times 2}$, $B \in \mathbb{R}^2$, and assume that $x(t) = x_i$, $\lambda(t_f) = sx(t_f)$ for a given positive real number s. Consider the time instants $t_i = t_1 < t_2 < ... < t_N = t_f$, with $N \geq 2$, such that, for any $n \in \{1, ..., N-1\}$, the matrix $A(t)$ is constant over the time interval $[t_n, t_{n+1})$. Then the state of the system at the generic time t in the interval $[t_n, t_{n+1})$ is given by

$$\begin{bmatrix}x(t)\\ \lambda(t)\end{bmatrix} = e^{A_n(t-t_n)}\begin{bmatrix}x(t_n)\\ \lambda(t_n)\end{bmatrix} + \int_{t_n}^{t} e^{A_n(t-\tau)}Bw(\tau)d\tau, \tag{20}$$

in which

$$\begin{aligned} \begin{bmatrix}x(t_n)\\ \lambda(t_n)\end{bmatrix} = &\left(\prod_{k=1}^{n-1} e^{A_{n-k}\Delta t_{n-k}}\right)\begin{bmatrix}x(t_1)\\ \lambda(t_1)\end{bmatrix} + \\ &+ \sum_{h=1}^{n-1}\left[\prod_{j=1}^{n-1-h} e^{A_{n-j}\Delta t_{n-j}}\right] \\ &\left[\int_{t_h}^{t_{h+1}} e^{A_h(t_{h+1}-\tau)}Bw(\tau)d\tau\right], \end{aligned} \tag{21}$$

where A_n denotes the matrix A evaluated over the time interval $[t_n, t_{n+1})$ and $\Delta t_j := t_{j+1} - t_j$. The initial condition $\lambda(t_i)$ is given by

$$\lambda(t_i) = s_1 x(t_i) + \sum_{h=1}^{N-1}\left[\prod_{j=1}^{h-1} d_j\right]\int_{t_h}^{t_{h+1}} H_h(t_{h+1}-\tau)w(\tau)d\tau, \tag{22}$$

in which

$$\begin{aligned} H_h(t_{h+1}-\tau) &= \frac{\Phi_{h,21}(t_{h+1}-\tau) - s_{h+1}\Phi_{h,11}(t_{h+1}-\tau)}{\Phi_{h,22}(\Delta t_h) - s_{h+1}\Phi_{h,12}(\Delta t_h)} \\ d_j &= \frac{1}{\Phi_{j,22}(\Delta t_j) - s_{j+1}\Phi_{j,12}(\Delta t_j)} \\ s_h &= -\frac{\Phi_{h,21}(\Delta t_h) - s_{h+1}\Phi_{h,11}(\Delta t_h)}{\Phi_{h,22}(\Delta t_h) - s_{h+1}\Phi_{h,12}(\Delta t_h)} \\ s_N &= s \quad h = 1, ..., N-1 \quad j = 1, ..., N-2. \end{aligned} \tag{23}$$

where $\Phi_{h,ij}$ denotes the (i,j) entry of the state transition matrix $e^{A_h t}$.

Based on the above mentioned result, it is straightforward to determine the solution of Problem 1. In particular, the optimal state and costate trajectories are determined by (20) after substitution of (21) and the initial costate (22), in which the time interval Δt_h is equal to the MPC time step T, $N-1=M$. and $\Phi_{h,ij}$ is the (i,j) entry of the transition matrix

$$e^{A_h(t-t_h)} := \begin{bmatrix} \Phi_{h,11}(t-t_h) & \Phi_{h,12}(t-t_h) \\ \Phi_{h,21}(t-t_h) & \Phi_{h,22}(t-t_h) \end{bmatrix} =$$
$$= \begin{bmatrix} \cosh\left(\sqrt{\frac{q}{r'_h}}(t-t_h)\right) & -\frac{1}{\sqrt{qr'_h}}\sinh\left(\sqrt{\frac{q}{r'_h}}(t-t_h)\right) \\ -\sqrt{qr'_h}\sinh\left(\sqrt{\frac{q}{r'_h}}(t-t_h)\right) & \cosh\left(\sqrt{\frac{q}{r'_h}}(t-t_h)\right) \end{bmatrix}. \tag{24}$$

Consequently, plugging (20) into (13) and (15) provides the optimal solution of Problem 1. Finally, the optimal trajectory of the expected value of the power flowing at the POC is found evaluating the expected value of both sides of (7), In the light of the above, the MPC optimal control is given by

$$\begin{aligned} u^{ess}(t)^* = \frac{1}{r'(t)}\Bigg[& -\Phi_{i,21}(t-t_i)x(t_i) - \Phi_{i,22}(t-t_i)\lambda(t_i) + \\ & + \int_{t_i}^{t} \Phi_{i,21}(t-t_i)w(\tau)d\tau\Bigg] + \mathbf{E}[w^{pv}(t)] + \\ & + \frac{c^{nom}}{c(t)}\Big[\mathbf{E}[\hat{u}^{ev}(t)^2] - (1+u^{nom}(t))\mathbf{E}[\hat{u}^{ev}(t)]\Big], \end{aligned} \tag{25}$$

$$\begin{aligned} u^{ev}(t)^* = \frac{1}{c(t)}\Bigg\{ & \Phi_{i,21}(t-t_i)x(t_i) + \Phi_{i,22}(t-t_i)\lambda(t_i) + \\ & - \int_{t_i}^{t} \Phi_{i,21}(t-t_i)w(\tau)d\tau] + \\ & - c^{nom}\Big[\mathbf{E}[\hat{u}^{ev}(t)^2] - (1+u^{nom}(t))\mathbf{E}[\hat{u}^{ev}(t)]\Big]\Bigg\}, \end{aligned} \tag{26}$$

where

$$\begin{aligned} \Phi_{i,21}(t-t_i) &= -\sqrt{qr'_i}\sinh\left(\sqrt{\frac{q}{r'_i}}(t-t_i)\right) \\ \Phi_{i,22}(t-t_i) &= \cosh\left(\sqrt{\frac{q}{r'_i}}(t-t_i)\right), \end{aligned} \tag{27}$$

which holds $\forall t \in [t_i, t_i+T)$[1].

Remark 3. The optimal control depends on $\mathbf{E}[\hat{u}^{ev}(t)]$, $\mathbf{E}[\hat{u}^{ev}(t)^2]$ (or equivalently on the variance $\sigma[\hat{u}^{ev}(t)]$) and $\mathbf{E}[w^{pv}(t)]$. Specifically the need of knowing the expected value and variance of the charging demand comes from the chosen lagrangian function, and more in particular from the weight (4), which linearly depends on $u^{nom}(t) - \hat{u}^{ev}(t)$. It is straightforward to see that higher grade dependence of the form $(u^{nom}(t) - \hat{u}^{ev}(t))^m$ (for a given positive integer m), motivated for example by potential better control tuning, would require the knowledge of $\mathbf{E}[\hat{u}^{ev}(t)^{m+1}]$.

[1]With little abuse of notation, in (25)(26)(27) the subscript i has been used to indicate that $\Phi_{i,21}$, $\Phi_{i,22}$ and r'_i refer to time period $[t_i, t_i+T)$.

TABLE I
PARAMETERS OF THE ALGORITHM

q	**r**	**s**	$\mathbf{c^{nom}}$	$\mathbf{u^{nom}(t)}$	**T**	**MT**
1	2	10	1	550	5m	4h

IV. Numerical Simulations

The simulations have been run in Matlab R2021a and consider an operation period of the service area of 12 hours (from 6:00 to 18:00, i.e., the hours when the photovoltaic plant is operative). The parameters characterizing the optimal control problem are reported in table I. The parameters q, r and s have been chosen so to have good peek shaving for power $p(t)$ at POC while reaching as much as possible the reference value for the final state of ESS stored energy $x(t_i+MT)$. Moreover, the c^{nom} parameter has been chosen to have a small weight on the EV power tracking term of (9) in case of congestion. The maximum charging power for each PEV has been set to 50 kW and a total number of 10 charging points has been considered. Moreover, the expected value $\mathbf{E}[\hat{u}^{ev}(t)]$ and the actual value $\hat{u}^{ev}(t)$ (this latter was unknown a priori) considered for the simulations are represented in Figure 2. The parameter $u^{nom}(t)$ has been fixed to the constant value of 550 kW (i.e., 50 kW more than the maximum aggregated power of the PEVs) in order to have a smaller weight for the tracking term of (3) if $\mathbf{E}[\hat{u}^{ev}]$ is near to the maximum aggregated power of the PEVs (i.e., almost all the CSs occupied and charging at max power), while having a greater weight if few CSs are occupied and/or the PEVs are charging at low power (e.g., because of overheated batteries or almost-charged batteries). The simulations considered a photovoltaic power production from the PV plant as depicted in Figure 3, where the solid line represents the actual value of $w^{pv}(t)$, while the dotted line represents the expected value $\mathbf{E}[w^{pv}(t)]$. The algorithm runs at each time step t_i and is fed with the expected curves for the aggregated PEV demand $\mathbf{E}[\hat{u}^{ev}(t)]$ and the PV power production $\mathbf{E}[w^{pv}(t)]$ for the time window $[t_i, t_i+MT)$; as the power demand and production are know at t_i, it is reasonable to put $\mathbf{E}[\hat{u}^{ev}(t)] = \hat{u}^{ev}(t)$ and $\mathbf{E}[w^{pv}(t)] = w^{pv}(t)$ for all $t \in [t_i, t_i+T)$. Indeed, at each time step t_i, the actual values $\hat{u}^{ev}(t)$ and $w^{pv}(t)$ are assumed unknown for any time $t \geq t_i+T$, while are known only their expected values $\mathbf{E}[\hat{u}^{ev}(t)]$ and $\mathbf{E}[w^{pv}(t)]$ and the variance term $\mathbf{E}[\hat{u}^{ev}(t)^2]$. This means that at each time step the MPC algorithm takes the actual measures of $\hat{u}^{ev}$ and w^{pv}, while uses their estimated values for the rest of the time window to compute the optimal control signals as in (25) and (26). In correspondence of the actual measured values, the variance is considered 0.

Figure 4 shows the charging power reference $\hat{u}^{ev}$ as dashed line, while the actual power allocated to the PEVs u^{ev} is indicated as solid line. From this figure it is possible to notice that the actual power is almost equal to the reference

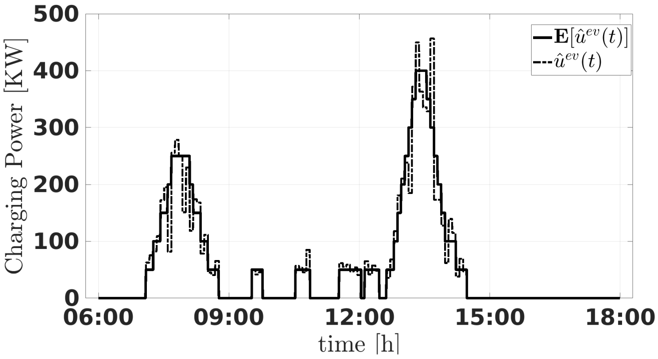

Fig. 2. Power demand from electric vehicles: $\mathbf{E}[\hat{u}^{ev}(t)]$ (continuous line), $\hat{u}^{ev}(t)$ (dashed line)

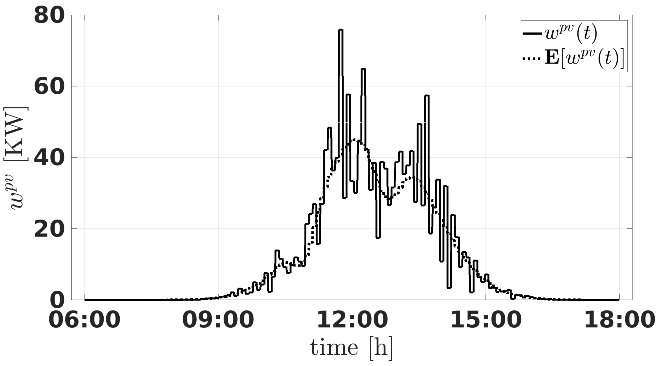

Fig. 3. Photovoltaic power production profile: $w^{pv}(t)$ (solid line), $\mathbf{E}[w^{pv}(t)]$ (dotted line)

one, except for the cases with high load, where there is a small mismatch between the reference and the actual power for the PEVs. Where the load of the PEVs is lower, the reference is tracked with high fidelity, due to the shape of the weight $c(\hat{u}^{ev})$. Based on the shape of this weight, it is also possible trade-off tracking performances of u^{ev} in favour of power shaving of power p at POC and/or of ESS power u^{ess}.

Figures 5 and 6 show respectively the ESS charging/discharging power and the ESS SOC evolution. As the simulation begins, the ESS pre-charges in prevision of the first load peak, that is the only visible to the algorithm (the time window for the MPC is set to 4 hours). After the

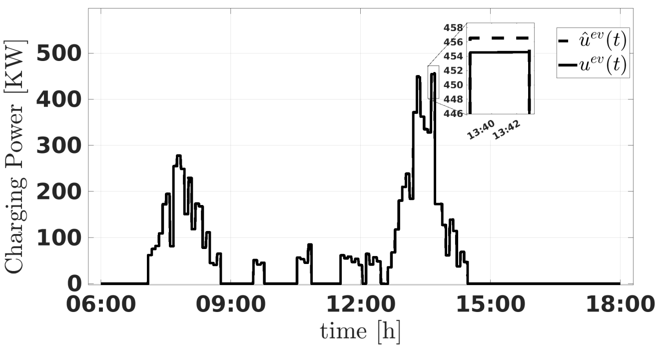

Fig. 4. $u^{ev}(t)$ (continuous line), $\hat{u}^{ev}(t)$ (dashed line)

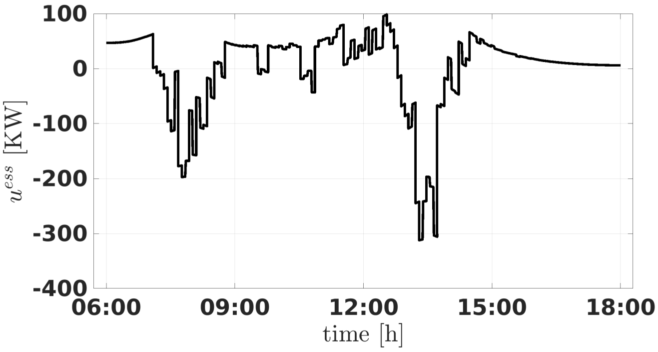

Fig. 5. ESS charging/discharging power.

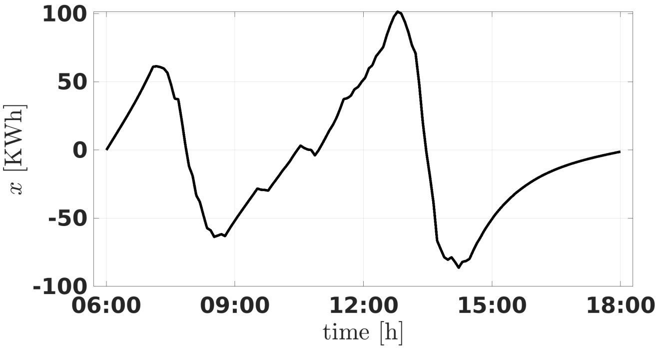

Fig. 6. ESS SOC.

first demand peak, it continues pre-charging (even more than before) in prevision of the second peak, also thanks to the peak of power production from the PV plant, that are both now visible to the MPC controller.

This results in a shaved power profile at POC. Indeed, even during the second charging peak, the power requested to the main grid is about 1/5 of the charging power, even if the PV plant contributes with only 20-60 kW, thanks to the effort of the ESS. Moreover, the control law for the ESS power is able to bring back the ESS state-of-charge to the reference value at the end of the simulation, so no additional power is required for the ESS to reach the reference SOC value.

The results presented above then show the effectiveness of

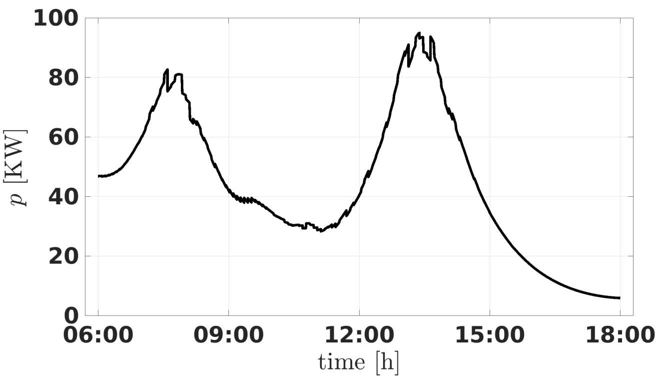

Fig. 7. Power flow at the connection with the grid.

the proposed control even with uncertainties on charging demand and solar power production, with the only assumptions of knowing the expected charging demand (and its variance) and the expected PV plant power production.

V. Conclusions and Future Works

This paper proposed an optimal control algorithm for controlling the charging of PEVs in a service area hosting an ESS with uncertainties on the power demand from the PEVs and on the power production of the PV plant. The goal of this work is to control the PEV charging process and the ESS power in order to serve the PEV users as fast as possible, while shaving the power profile at the POC with the grid (which results in lower operation costs of the charging area). Moreover, the optimal control is re-optimized at each time step in order to reduce the uncertainties and to perform better tracking performances and flatter power profile at POC with the main grid.

Future works will focus on the inclusion of physical constraints for the involved equipment (ESS charging power, POC, ESS state-of-charge, etc.), as well as of the PEVs' charging dynamics, in order to capture in a finer way the charging process. Also, an assessment of the economical benefits due to the flattening of the power profile at the POC will be carried out.

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