Constant Maturity Swap Model

A constant maturity swap (CMS) is an interest rate swap where floating rate equals the swap rate for a swap with a certain life (CMS tenor). For example, the floating payments on a CMS swap might be made every six months at a rate equal to the five-year swap rate (CMS tenor = 5 year). For convexity and timing value calculation for CMS rates, Hull-White formula with correlation coefficient, between CMS rate and forward rate, set at 0.7 is used.

We assume the future CMS swap rates f_t has a lognormal distribution and follows geometric Brownian motion as follows

$$df = \mu f dt + \sigma f dz$$

where μ is the mean and the variable z follows a Winner process.

- f_0 = forward swap rate at time 1 with swap tenor T_τ observed today
- f_t = forward swap rate at time t
- $P(f_t)$ = single leg swap payment discounted value at time t
- R_0 = current forward rate over $[T_1, T_2]$ calculated directly from the LIBOR curve.
- $\Sigma(f_0, T_1, \tau)$ = volatility extraction from LIBOR volatility surface with f_0, T_1 and swap tenor τ
- $\sigma = \Sigma(f_0, T_1, \tau)$
- $\sigma_R = \Sigma(R_0, T_1, dt)$, where $dt = T_2 - T_1$.
- ρ = correlation between LIBOR index and CMS index. It is a user input parameter.

This swap has same payment structure as in the floating leg of CMS swap and its value is derived from the forward swap rate *t f* as the internal rate of return.

$$P(f_t) = \sum_{i=1}^{n} \frac{C_i}{\prod_{j=1}^{i}(1+f_t \cdot \Delta t_j)} + \frac{1}{\prod_{j=1}^{n}(1+f_t \cdot \Delta t_j)}$$

where C the unit payment amount at time

There are two types of CMS swaps – regular and arrear CMS swap.

1. Regular CMS swap

The swap rate is calculated at the beginning of payment period. Both convexity and timing adjustment are required.

$$r_R^C(T_1, T_2, \tau) = E\left[f_{T_1}\right] = f_0 - \frac{f_0^2\left(e^{\sigma^2 T_1} - 1\right)}{2}\frac{P''(f_0)}{P'(f_0)} - \frac{f_0 \sigma \rho \sigma_{R_0} R_0 (T_2 - T_1) T_1}{1 + (T_2 - T_1) R_0}$$

2. Arrear CMS swap

The swap rate is calculated at the end of payment period. Only convexity adjustment is required since the swap rate is calculated and paid at the same time.

$$r_A^C(T_1, T_2, \tau) = E\left[f_{T_2}\right] = f_0 - \frac{f_0^2\left(e^{\sigma^2 T_2} - 1\right)}{2}\frac{P''(f_0)}{P'(f_0)}$$

Sensitivity of CMS swap pricing to LIBOR curve is available in Partial Differential Hedge (PDH) report. These sensitivities are Two-Way Delta and defined as

$$\text{LIBOR Curve Risk}\left(\alpha_i\right) = \frac{V\left(\alpha_i + x\right) - V\left(\alpha_i - x\right)}{2}$$

Reference:

https://finpricing.com/lib/EqBarrier.html