Cross Currency Basis Swaption Model

A Cross-currency basis swap, which is also called FX basis swap, is a contract between two parties that one side receives a floating rate (plus a possible spread) of currency A and the other receives a floating rate (also plus a possible spread) of currency B.

Between the two currencies, suppose that currency A is more liquid than currency B. For example, USD is currency A and CAD is currency B. That is, in the currency A world, there is a market difference in supply and demand of currency B. Thus the traditional interest rate parity in the currency A world may not valid without any adjustment.

Specifically, let Q_t be the exchange rate at a time t which is expressed as number of units in currency A per unit of currency B, $Q(t; T)$ be the forward exchange rate at the time t matured at a time $T > t$, $\mathrm{df}^A(t; T)$ be the discounting factor in currency A and $\mathrm{df}^B(t; T)$ be the one in currency B. The traditional interest rate parity can be expressed as

$$Q(t,T) = Q_t \times \frac{\mathrm{df}^{\mathrm{B}}(t,T)}{\mathrm{df}^{\mathrm{A}}(t,T)} \ .$$

However, currency B, as a commodity in the currency A world, has extra convenience yield so that the relation (1) is valid provided that the discounting factor $\mathrm{df}^B(t; T)$ has to be adjusted, denoted by df_B($t; T$) or equivalently, the currency B zero curve has to be adjusted by a spread from the view point of currency A world.

Let us consider a FX basis swap formulated as follows. Let $T_0 < T_1 < \ldots < T_N$ be swap payment dates, dcf^A_i and dcf^B_i be day count fractions of the ith period $[T_{i-1}; T_i]$ on currencies A and B, respectively, where $i = 1; \ldots ; N$.

There is initial exchange at T_0 and final exchange at T_N of notional principals of N^A and N^B, respectively. At each payment date T_i, one party pays A-floating rate plus a spread x^A in currency A and the other pays B-floating rate plus a spread x^B in currency B. Let $t < T_0$ be a valuation time. Due to the reason mentioned above, the time t-value in currency A of the FX basis swap (paying A-floating and receiving B-floating) can be given by

$$\mathrm{PV}_t = (x^{\mathrm{B}} - \Lambda) \cdot \mathrm{pv01}_t^{\mathrm{B}} \cdot N^{\mathrm{B}} \cdot Q_t \; - \; x^{\mathrm{A}} \cdot \mathrm{pv01}_t^{\mathrm{A}} \cdot N^{\mathrm{A}} \; ,$$

where

$$\mathrm{pv01}_t^{\mathrm{A}} = \sum_{i=1}^{N} \mathrm{dcf}_i^{\mathrm{A}} \times \mathrm{df}^{\mathrm{A}}(t, T_i) \; , \qquad \mathrm{pv01}_t^{\mathrm{B}} = \sum_{i=1}^{N} \mathrm{dcf}_i^{\mathrm{B}} \times \tilde{\mathrm{df}}^{\mathrm{B}}(t, T_i) \; ,$$

and Λ is a market variable which is called balance spread (of currency B with respect to currency A). It also depends on the underlying swap term and observation time.

An Fx basis swaption is the option to enter an in-the-money underlying Fx basis swap. With the notation defined above and consider the same Fx basis swap, suppose that the option expiry is T_0. Then the matured payoff of the option can be written as

$$[\mathrm{PV}_{T_0}]^+ = \left[(x^{\mathrm{B}} - \Lambda_{T_0}) \cdot \mathrm{pv01}_{T_0}^{\mathrm{B}} \cdot N^{\mathrm{B}} \cdot Q_{T_0} \; - \; x^{\mathrm{A}} \cdot \mathrm{pv01}_{T_0}^{\mathrm{A}} \cdot N^{\mathrm{A}}\right]^+$$

Due to the lack of quantitative analysis of the balance spread process ¤ and correlations among balance spread, exchange rate, currency A risk-free rate and currency B risk-free rate, it should be noted that pricing Fx basis swaption is much more difficult than pricing regular interest rate swaption. Adopting some assumptions is necessary for simplifying the pricing problem. We introducing the following assumptions.

[1] Risk-free rates in currency A and currency B are deterministic. Both zero curves can be obtained from the market at any time;

[2] The adjusted short rate in curency B (from the view point of the currency A world) is deterministic and the adjusted zero curve can also be obtained from the market at any time;

[3] The Exchange rate follows a geometric Brownian motion,

[4] The balance spread ¤ follows a Brownian motion, i.e.,

$$\mathrm{d}\Lambda_t = \mu_\Lambda \mathrm{d}t + \sigma_\Lambda \mathrm{d}W_t^\Lambda \ ,$$

where μ_A and σ_A are constants;

[5] Processes *WQ* and *W*¤ are independent.

With those assumptions, then the European type and American type Fx basis swaption value at a time $t < T0$ can be approximated by using standard 3D tree in the exchange rate and the exponential of the balance spread.

Reference:
https://finpricing.com/lib/EqBarrier.html