The Effect of Wall Compliance on the Stability of Jets and Wakes

Ryan Poole & **Matt Turner**

Department of Mathematics, University of Surrey, Guildford, UK, GU27 XH

Email: ryan.poole@surrey.ac.uk

Introduction

Compliant walls are surfaces which are able to bend, flex or compress, such as skin or sponge linings in audio studios. These can be modelled using compliant walls of 'Kramer' type [3], which consist of an elastic plate backed by springs, a viscous fluid substrate and a rigid plate, see Fig 1. The wall motion is governed by the equation $\left(m\frac{\partial^2}{\partial t^2} + d\frac{\partial}{\partial t} + B\frac{\partial^4}{\partial x^4} - T\frac{\partial^2}{\partial x^2} + K\right)\xi = p(h_2)$, where ξ is the displacement and p is the pressure.

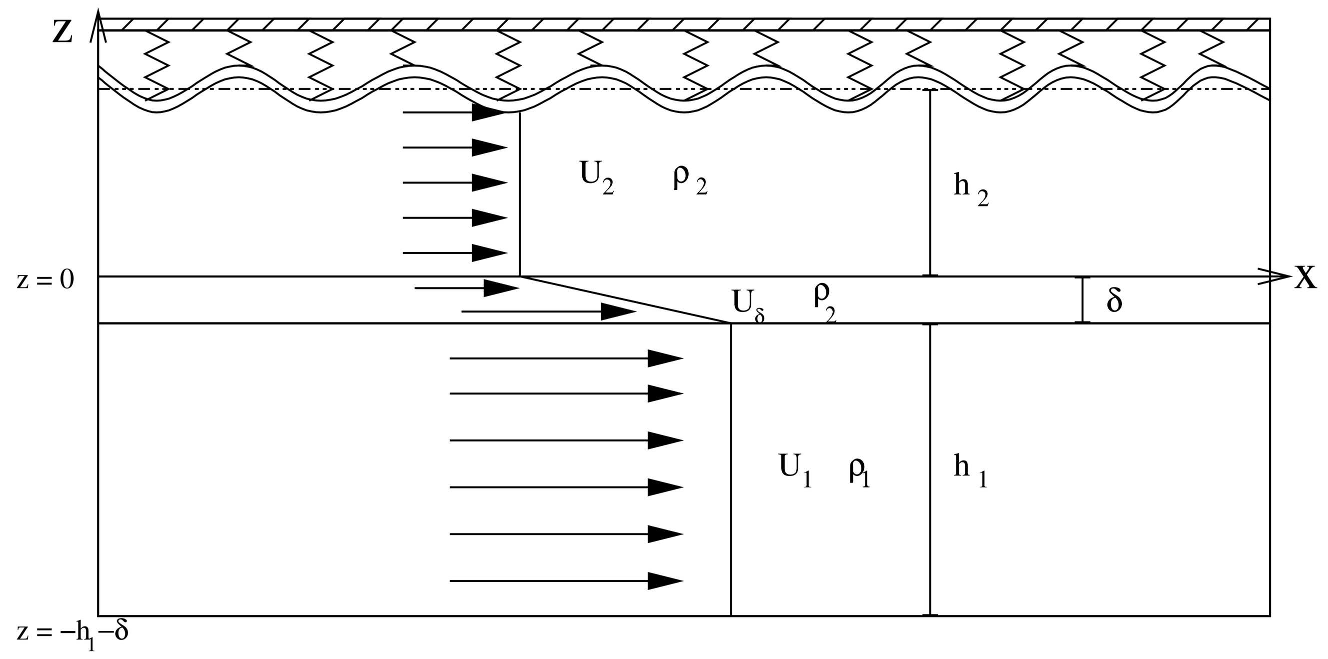

Fig 1: Schematic of the flow setup. Note that the flow is symmetric about $z = -h_1 - \delta$.

Here we consider three horizontal layers of inviscid, irrotational and incompressible fluid bounded by compliant walls. The outer layers have constant velocity U_2, density ρ_2 and width h_2. The inner flow has constant velocity U_1, density ρ_1 and width $2h_1$. We define h_1 as our characteristic length scale, $U_{ref} = (U_1 + U_2)/2$ as our velocity scale, and ρ_2 as our density scale.

Governing Equations: Dispersion Relations

The instability of the flow, pictured in Fig 1, is governed by the following dispersion relations [2] $\mathbb{D}^v \equiv A_1 D_+^v + B_1 D_-^v = 0$, and $\mathbb{D}^s \equiv A_1 D_-^s + B_1 D_+^s = 0$, where the superscript v and s represent the varicose (antisymmetric) and sinuous (symmetric) modes respectively. Here,

$$\frac{A_1}{B_1} = -e^{-2\alpha h}\left[\frac{Q + \alpha(1 - \Lambda - \omega/\alpha)^2}{Q - \alpha(1 - \Lambda - \omega/\alpha)^2}\right], \qquad (1)$$

where $Q = -m\omega^2 - \mathrm{i}d\omega + B\alpha^4 + T\alpha^2 + K$ is the wall admittance, and $\Lambda = (U_1 - U_2)/(U_1 + U_2)$, $S = \rho_1/\rho_2$ and $h = h_2/h_1$. α and ω are our spatial wave number and temporal frequencies respectively.

Absolute Instability (AI): $\alpha, \omega \in \mathbb{C}$

Now we have a relationship between α and ω, we can carry out a range of analyses. If $\alpha \in \mathbb{C}, \omega \in \mathbb{R}$, we can carry out a spatial instability analysis. If we let $\alpha \in \mathbb{R}$ and $\omega \in \mathbb{C}$, we can carry out a temporal instability analysis. But what would we get if we let both $\alpha, \omega \in \mathbb{C}$? A spatio-temporal analysis?

AI is observed in flows where a disturbance grows in both space and time. Mathematically, we find AIs by looking for solutions of $\mathbb{D}^{(v,s)}(\alpha_0, \omega_0) = 0$, and $\mathbb{D}_\alpha^{(v,s)}(\alpha_0, \omega_0) = 0$. Only a few solutions (α, ω) represent AI, however. We call these Pinch Points. An example of a regular saddle and a pinch point is seen in Fig 2.

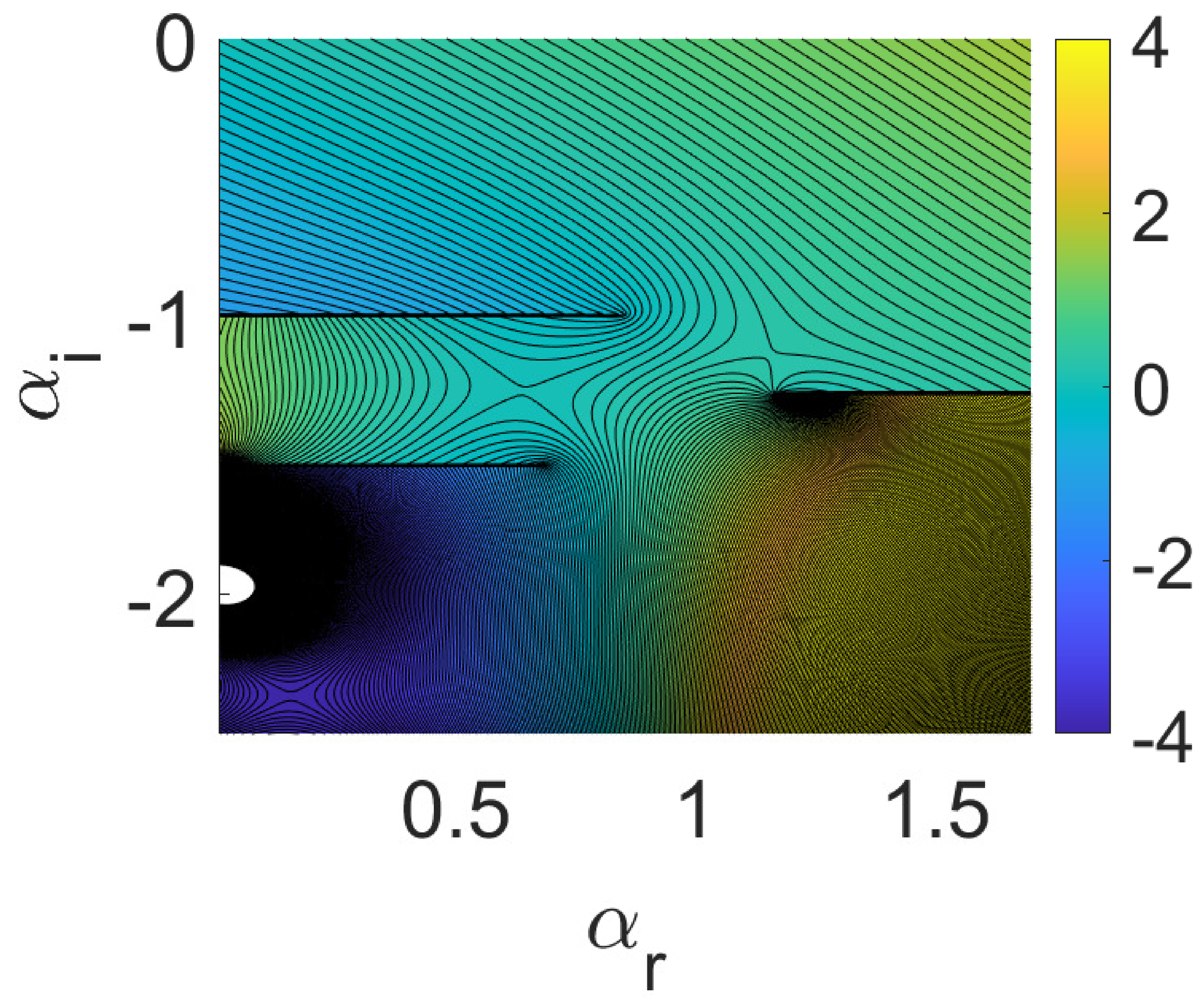

Fig 2: Contours of constant ω_i plotted in the complex α-plane for the varicose dispersion relation, with $(\Lambda, S, h) = (1, 1, 2)$ and wall parameters $(B, K, m) = (2, 10, 0.1)$, with $d = T = 0$. The right saddle is the only pinch point here.

In order to determine whether a saddle is a pinch point or not, we use Briggs' Criterion [1], which involves locating the originating plane of the hills that make up a saddle point. If one hill originates in the upper half plane, and the other in the lower half, we have a pinch point, otherwise we have a saddle. If this pinch point has a $\omega_i > 0$ we have an AI.

Results & Conclusions

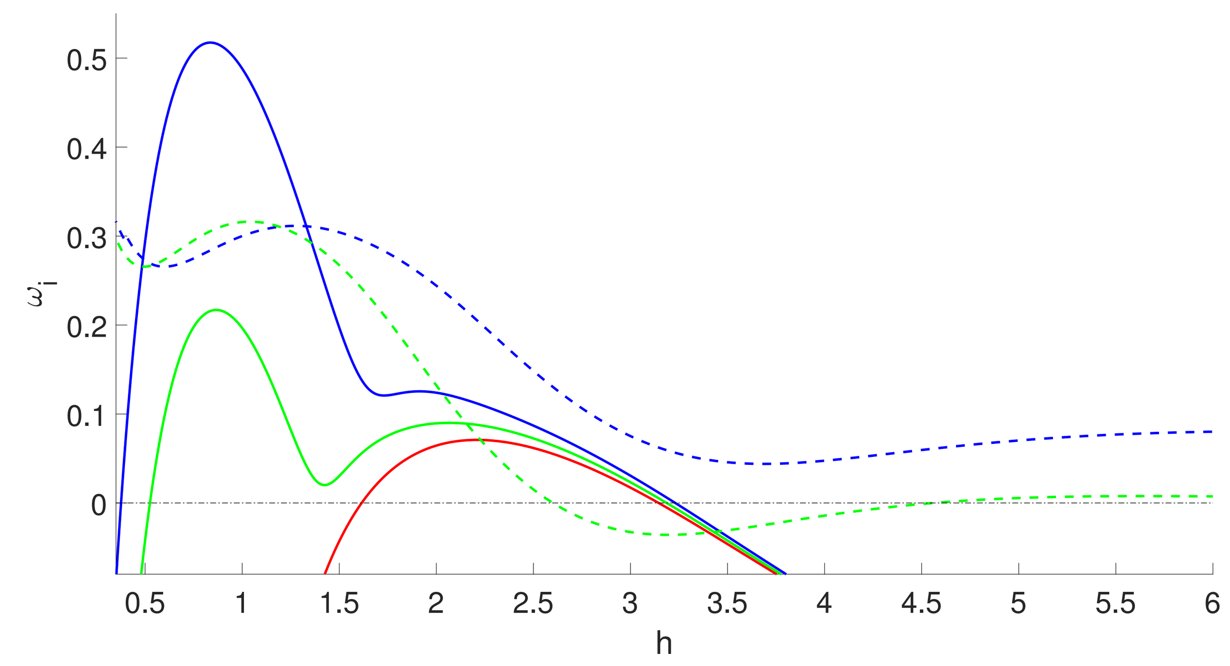

Fig 4: Plot of the AI saddle growth rates ω_i vs. h, for three different wall configurations, with fixed $K = 10, m = 0.1, d = T = 0$, and we vary $B = 0.1, 1, 2$ for red, green and blue curves respectively

An in-depth AI analysis tracks how the growth rate of a given pinch point varies as we vary a wall parameters. We have shown that there are cases in which, if we have AI in the large or small limit of Q, and a convectively unstable flow in the other limit, the walls have a drastic effect on the stability of the flow.

We have also shown that wall modes exist and can dominate the flow, even if there is no AI in the large Q limit, see Fig 3. This shows growth rates in a flow bounded by three different wall configurations. The solid lines represent the same flow based mode in each of the different setups. The dashed lines represent a wall based mode, which, in the case of the blue set, maintains and AI for a greater range of h than in the large Q limit.

To conclude, we have shown that compliant walls can enhance existing AI modes, as well as induce AI where there was only convective instability before. This gives us a new and interesting method into controlling a flows instability, though we need to be sure that we do not induce more instability in the walls themselves.

References

[1] R. J. Briggs. Electron-stream interaction with plasmas. MIT, 1964.

[2] M. P. Juniper. The full impulse response of two-dimensional jet/wake flows and implications for confinement. J. Fluid Mech., 590:163, 2007.

[3] M. O. Kramer. Boundary-layer stabilization by distributed damping. Journal of the Aerospace Sciences, 27(1):69–69, 1960.