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The Lattice Computing Paradigm for Modeling Intelligence in Cyber-Physical Systems

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Outline

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2. Mathematical Background (a brief summary)
3. Data and Knowledge Representation (INs, Tree Data Structures, Ontologies)
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1. Introduction

- The CybSPEED Action aimed at creating an international and inter-sectorial network of organizations in order to perform research advancing a novel framework for analysis, modelling, synthesis and implementation of Cyber-Physical Systems (CPSs) for pedagogical rehabilitation in special education.
- The pursued research was a combination of robotic technology design, human-robot interaction with humanoid and non-humanoid robots in education, brain-inspired robotic and agent design, systems science, computational intelligence (CI) and Lattice Computing (LC) algorithms.
- A training course on LC in Morocco.
- Beneficiary IHU (former EMaTTech) has pursued as well modeling Cyber-Physical Systems (CPSs) based on the LC paradigm.

- A Cyber-Physical System (CPS) is defined as a device with both sensing and reasoning capacities.
- CPSs need to be driven by suitable models preferably with a capacity to deal effectively with both numerical (inherently associated with the “physical” system component(s)) and nonnumerical data (inherently associated with the “cyber” system component(s)).
- A typical method for dealing with nonnumerical data is by transforming them to numerical ones using “ad-hoc” techniques thus risking a non-reversible performance deterioration.

- An innovation of beneficiary IHU has been the engagement of the Lattice Computing (LC) paradigm for modelling based on numerical and/or nonnumerical data *per se* without transforming the latter data to the former data.
- An ensuing advantage is that LC enables *computing with semantics*, that is sensible for human-robot interaction applications, instead of “number crunching”.
- ~1/3 of IHU’s total 32 publications, compiled in the context of CybSPEED, were compiled with partner 11 (i.e., UH2C, Morocco); ~1/3 of IHU’s were compiled with another four beneficiaries /partners, namely UPV/EHU, Theater Tsvete, IR-BAS and KyuTech, respectively; the remaining of IHU’s 32 publications involved exclusively IHU researchers.

2. Mathematical Background (page 1/2)

Mathematical Lattice Theory or Order Theory.

Given a lattice $(\mathrm{L},\sqsubseteq)$, a *valuation* is a real function $v: \mathrm{L} \rightarrow \mathrm{R}$ given by $v(x \sqcap y) + v(x \sqcup y) = v(x) + v(y)$, $x,y \in \mathrm{L}$. A valuation is called *monotone* if $x \sqsubseteq y \Rightarrow v(x) \leq v(y)$, and *positive* if $x \sqsubset y < v(x) < v(y)$. A positive valuation $v: \mathrm{L} \rightarrow \mathrm{R}$ in $(\mathrm{L},\sqsubseteq)$ defines a metric distance $d: \mathrm{L} \times \mathrm{L} \rightarrow \mathrm{R}_0^+$ given by $d(x,y) = v(x \sqcup y) - v(x \sqcap y)$. A parametric valuation function $v(.)$ introduces tunable nonlinearities.

- Let $(\mathrm{L},\sqsubseteq)$ be a lattice. An *inclusion measure* function $\sigma: \mathrm{L} \times \mathrm{L} \rightarrow [0,1]$ is defined by the two conditions:

1. $u \sqsubseteq w \Leftrightarrow \sigma(u,w) = 1$
2. $u \sqsubseteq w \Leftrightarrow \sigma(x,u) \leq \sigma(x,w)$

(1)

An inclusion measure $\sigma: \mathrm{L} \times \mathrm{L} \rightarrow [0,1]$ can be interpreted as a *fuzzy order relation*; hence, the notations $\sigma(u,w)$ and $\sigma(u \sqsubseteq w)$ are used interchangeably. Any use of an inclusion measure $\sigma(.,.)$ is called *Fuzzy Lattice Reasoning*, or *FLR* for short.

Mathematical background (page 2/2)

There are at least two different functions for defining an inclusion measure σ: L×L→[0,1], both are based on a positive valuation function $v: \mathrm{L} \rightarrow \mathrm{R}$ in (L,⊑), as follows.

- $sigma - meet$: $\sigma_{\sqcap}(x,u) = v(x \sqcap u)/v(x)$
- $sigma - join$: $\sigma_{\sqcup}(x,u) = v(u)/v(x \sqcup u)$

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- $sigma - join$: $\sigma_{\sqcup}(x,u) = v(u)/v(x \sqcup u)$

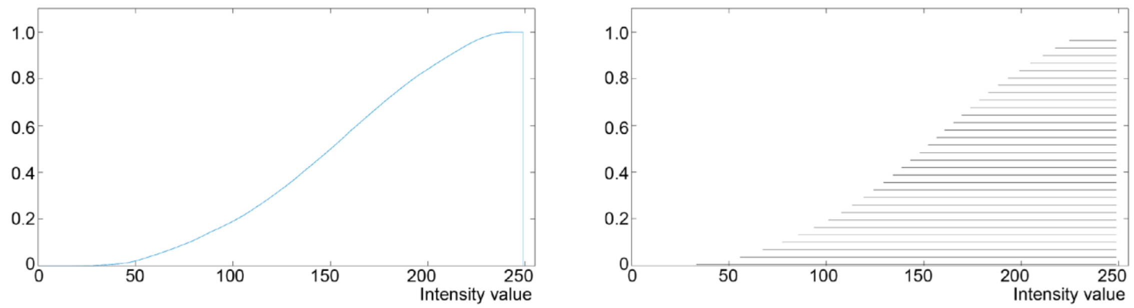
3. Data and Knowledge Representation

3.1 Intervals' Numbers (INs)

Two equivalent representations of an Intervals' Number (IN).

(a) The **membership-function-representation** of an IN is amenable to interpretations; e.g. it may represent a distribution function.

(b) The **interval-representation** of an IN is amenable to useful algebraic operations.



(a) (b)

The aforementioned equivalence stems from an established fuzzy-set-theory mathematical result, namely the “resolution identity theorem”, which specifies that a fuzzy set can (equivalently) be represented either by its *membership function* or by its α-*cuts*.

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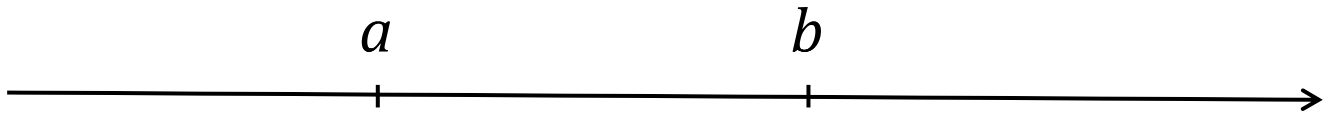
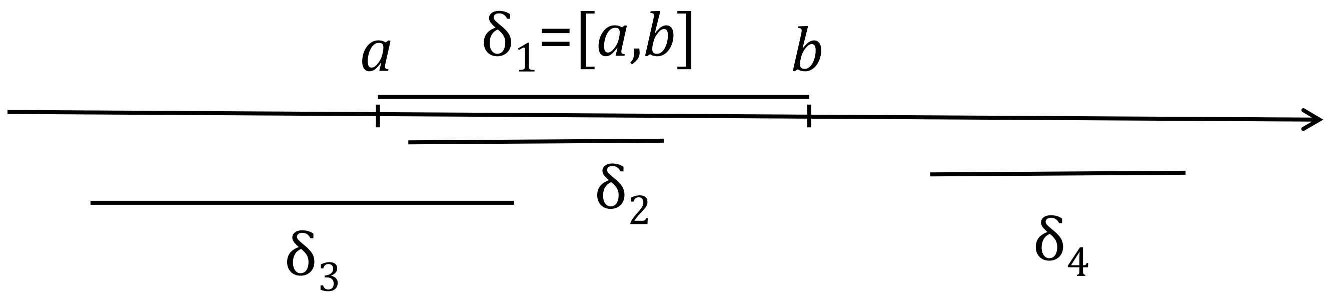
Analysis in the context of **mathematical Lattice Theory**, or **Order Theory**: A hierarchy of mathematical lattices stemming from R

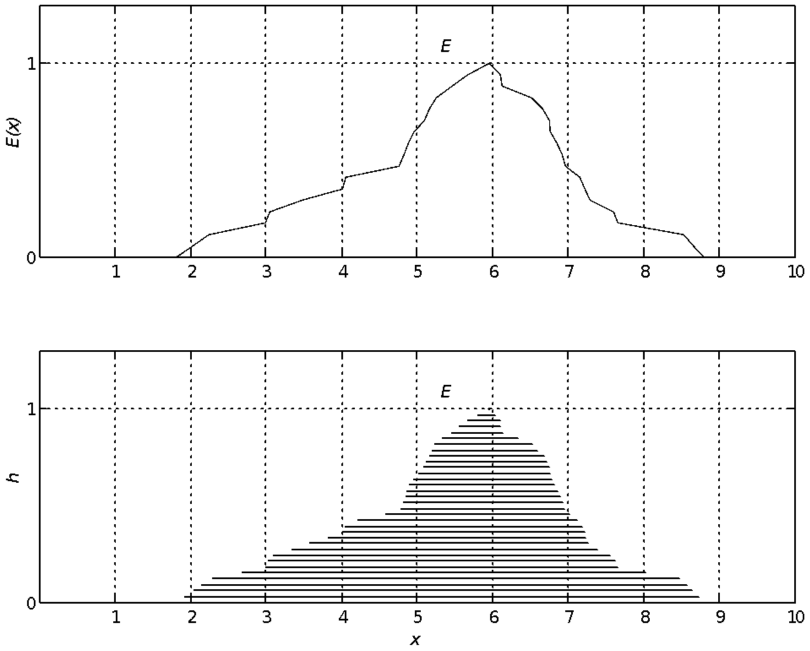
a) Level-0; the lattice (R;≤) of real numbers



b) Level-1; the lattice (I_1;⊆) of intervals



c) Level-2; the lattice (F_1;≼) of Intervals' Numbers (INs)



The lattice (F_1;≼) is a *metric cone*; therefore, ARMA models can be developed.

Useful mathematical instruments include:

A metric distance function D_1: $F_1 \times F_1 \to R_0^+$ defined as

$$D_1(F,G) = \int_0^1 d_1(F_h, G_h)dh, \text{ where}$$

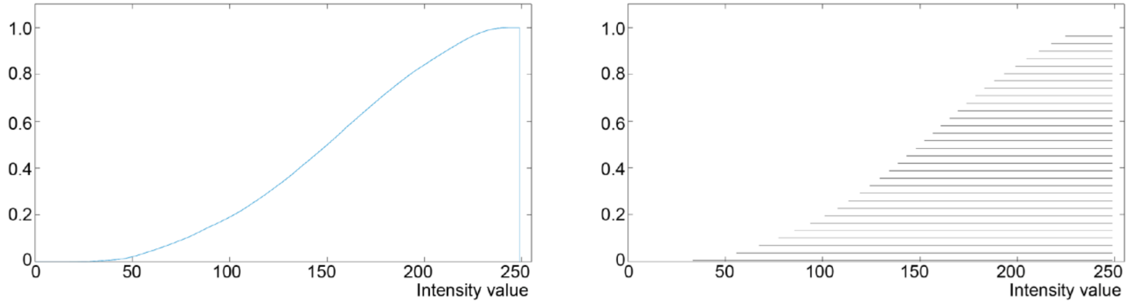
d_1 is a metric distance function d_1: $I_1 \times I_1 \to R_0^+$ defined as

$$d_1([a,b],[c,d]) = v(\theta(a \wedge c)) - v(\theta(a \vee c)) + v(b \vee d) - v(b \wedge d), \text{ where}$$

function v: $R \to [0,+\infty)$ is strictly increasing, whereas function $\theta: R \to R$ is strictly decreasing.

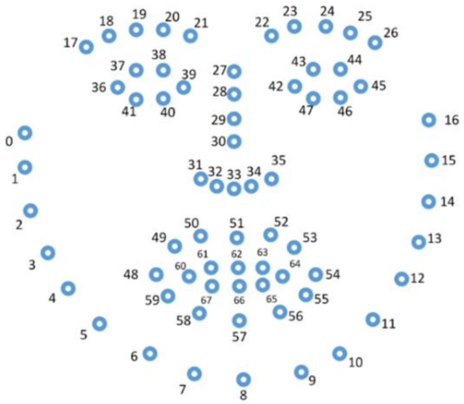
The main advantage of an IN:

All-order data statistics can be represented, using a “small” number of L intervals. Therefore, no *ad hoc* feature extraction is required; only an optimal estimation of the undelying positive valuation function is required.

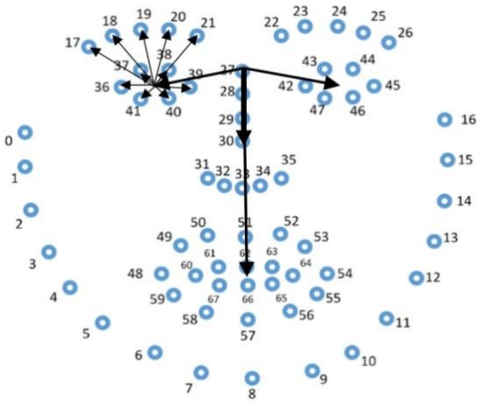


(a) (b)

3.2 Tree Data Structures



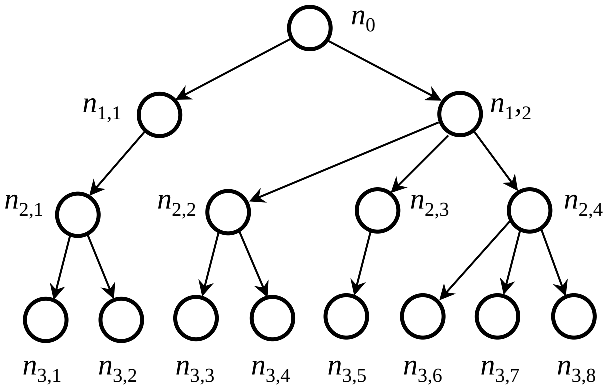
(a)



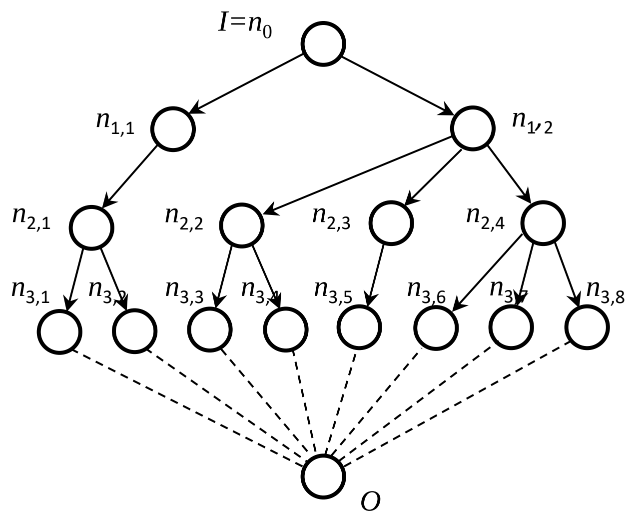
(b)

(a) 68 facial landmark points on a human face. (b) The *unit vector* defined along the nose. The first three *primary vectors* are also shown to the centers of the eyes and mouth as well as *secondary vectors* from the left eye center.

3.2 Tree Data Structures (cont.)



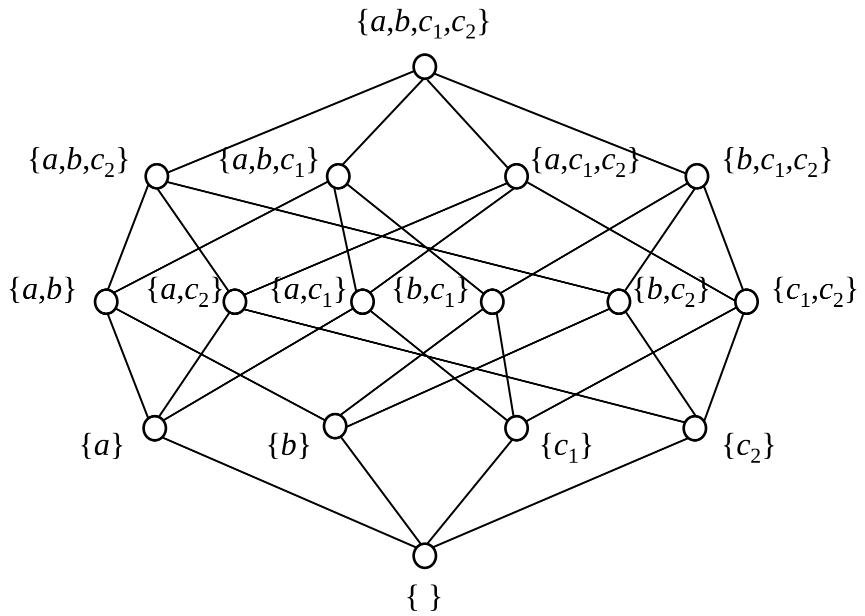
(a)



(b)

(a) A tree data structure example. (b) A lattice results in by inserting an additional level including the least lattice element O. The tree root corresponds to the greatest lattice element I.

3.3 Ontologies



A Boolean lattice ontology of the four inequality constraints *a*: $TSS_{min} < TSS < TSS_{MAX}$, *b*: $TA_{min} < TA < TA_{MAX}$, c_1: $pH < pH_{MAX}$ and c_2: $pH_{min} < pH$, regarding grape maturity indices TSS, TA and pH.

4. Computational Experiments and Results

Special attention has been given to novel CPS models based on logic/reasoning.

From a computational point of view the LC models are promising for CPS applications because they

(a) can deal with disparate types of data (including both numerical data for the “physical” system component(s) and non-numerical data for the “cyber” system component(s)).

(b) can compute with semantics.

(c) can rigorously deal with ambiguity.

(d) can naturally engage logic and reasoning.

(e) can process data fast.

4. Computational Experiments and Results (cont.)

Logic-based LC models as well as other LC models, such as IN regressor models, implementable in software on CPSs, have been both developed (e.g. see publications in the Appendices).

5. Discussion and Conclusion

- ❑ Mathematical lattice-ordered data representations, including INs, tree data structures and ontologies, have been considered in applications of interest to the CybSPEED project as explained in section 3.

- ❑Our proposed LC models drew the attention of extraordinary research, lately:

• F. Braga and H. S. Pinto, “Composing Music Inspired by Sculpture: A Cross-Domain Mapping and Genetic Algorithm Approach”, *Entropy*, vol. 24, no. 4, 468, 2022. https://doi.org/10.3390/e24040468

- ❑New projects have been accessed.

Acknowledgement

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http://humain-lab.cs.ihu.gr/index.php/portfolio-item/cybspeed/?lang=en

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Appendix A

V. G. Kaburlasos (Guest Editor), Special Issue on "Lattice Computing: A Mathematical Modelling Paradigm for Cyber-Physical System Applications", *Mathematics*, vol. 10, no. 2, 271, 2022. https://www.mdpi.com/journal/mathematics/special_issues/Lattice_Computing (8 papers).

1. V. G. Kaburlasos, C. Lytridis, E. Vrochidou, C. Bazinas, G. A. Papakostas, A. Lekova, O. Bouattane, M. Youssfi, T. Hashimoto, "Granule-based-classifier (GbC): a lattice computing scheme applied on tree data structures," *Mathematics*, vol. 9, no. 22, 2889, 2021.
2. N. S. T. Hirata, G. A. Papakostas, "On Machine-Learning Morphological Image Operators," *Mathematics*, vol. 9, no. 16, 1854, 2021.

Appendix A (*cont.*)

3. F. J. Valverde-Albacete, C. Peláez-Moreno, "Four-Fold Formal Concept Analysis Based on Complete Idempotent Semifields," *Mathematics*, vol. 9, no. 2, 173, 2021.
4. F. J. Valverde-Albacete, C. Peláez-Moreno, "The Singular Value Decomposition over Completed Idempotent Semifields," *Mathematics*, vol. 8, no. 9, 1577, 2020.
5. G. X. Ritter, G. Urcid, L.-D. Lara-Rodríguez, "Similarity Measures for Learning in Lattice Based Biomimetic Neural Networks," *Mathematics*, vol. 8, no. 9, 1439, 2020.
6. M. E. Valle, "Reduced Dilation-Erosion Perceptron for Binary Classification," *Mathematics*, vol. 8, no. 4, 512, 2020.
7. C. Lytridis, A. Lekova, C. Bazinas, M. Manios, V. G. Kaburlasos, "WINkNN: Windowed Intervals' Number kNN Classifier for Efficient Time-Series Applications," *Mathematics*, vol. 8, no. 3, 413, 2020.
8. J.-B. Liu, M. Munir, Q. Munir, A. R. Nizami, "Some Metrical Properties of Lattice Graphs of Finite Groups." *Mathematics*, vol. 7, no. 5, 398, 2019.

Appendix B

All LC algorithms published during CybSPEED by IHU (former EMaTTEch) co-authors – The first 10 with an Acknowledgement to CybSPEED .

1. [A06] G.A. Papakostas, V.G. Kaburlasos, “Modeling in cyber-physical systems by lattice computing techniques: the case of image watermarking based on intervals’ numbers”, *Proceedings of the World Congress on Computational Intelligence (WCCI) 2018, FUZZ-IEEE Program*, Rio de Janeiro, Brazil, 8-13 July 2018, pp. 491-496.
2. [A13] V. Kaburlasos, E. Vrochidou, “Social robots for pedagogical rehabilitation: trends and novel modeling principles”. In: *Cyber-Physical Systems for Social Applications*, M. Dimitrova & H. Wagatsuma (Eds.), pp. 1-21, 2019. IGI Global: Pennsylvania, USA. ISBN13: 9781522578796, DOI: 10.4018/978-1-5225-7879-6.

Appendix B (*cont.*)

3. [A16] C. Lytridis, A. Lekova, C. Bazinas, M. Manios, V.G. Kaburlasos, “WINkNN: Windowed Intervals’ Number kNN classifier for efficient time-series applications”, *Mathematics*, vol. 8, no. 3, 413, 2020. https://www.mdpi.com/2227-7390/8/3/413
4. [A19] V.G. Kaburlasos, “The Lattice Computing (LC) paradigm”. In: Francisco J. Valverde-Albacete, Martin Trnecka (Eds.), *Proceedings of the 15th International Conference on Concept Lattices and their Applications (CLA 2020)*, Tallinn, Estonia, 29 June - 1 July 2020, pp. 1-8. Tallinn University of Technology, Estonia, ISBN: 978-9949-83-557-7. http://ceur-ws.org/Vol-2668/
5. [A21] V. G. Kaburlasos, C. Lytridis, C. Bazinas, S. Chatzistamatis, K. Sotiropoulou, A. Najoua, M. Youssfi, O. Bouattane, “Head pose estimation using lattice computing techniques”, *28th International Conference on Software, Telecommunications and Computer Networks (SoftCOM 2020)*, Hvar, Croatia, 17-19 September 2020.

Appendix B (*cont.*)

6. [A22] V. G. Kaburlasos, C. Lytridis, C. Bazinas, G. A. Papakostas, A. Naji, M. Hicham Zaggaf, K. Mansouri, M. Qbadou, M. Mestari, “Structured human-head pose representation for estimation using fuzzy lattice reasoning (FLR)”, *The Fourth International Conference on Intelligent Computing in Data Sciences (ICDS 2020)*, Fez, Morocco, 21-23 October 2020.
7. [A25] E. Vrochidou, C. Lytridis, C. Bazinas, G.A. Papakostas, H. Wagatsuma, V.G. Kaburlasos, “Brain signals classification based on fuzzy lattice reasoning”, *Mathematics*, vol. 9, no. 9, 1063, 2021. https://doi.org/10.3390/math9091063
8. [A26] N. S. T. Hirata, G. A. Papakostas, “On Machine-Learning Morphological Image Operators”, *Mathematics*, vol. 9, no. 16, p. 1854, Aug. 2021. https://doi.org/10.3390/math9161854

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Appendix B (*cont.*)

9. [A28] V. G. Kaburlasos, C. Lytridis, E. Vrochidou, C. Bazinas, G. A. Papakostas, A. Lekova, O. Bouattane, M. Youssfi, T. Hashimoto, “Granule-based-classifier (GbC): a lattice computing scheme applied on tree data structures”, *Mathematics*, vol. 9, no. 22, 2889, 2021. https://www.mdpi.com/2227-7390/9/22/2889
10. [A29] V. G. Kaburlasos (Guest Editor), Special Issue on “Lattice Computing: A Mathematical Modelling Paradigm for Cyber-Physical System Applications”, *Mathematics*, vol. 10, no. 2, 271, 2022. https://www.mdpi.com/2227-7390/10/2/271
11. Peter Sussner, George Papakostas (chairmen) Special Session on “Recent Advances in Lattice Computing”, World Congress on Computational Intelligence (WCCI) 2018, FUZZ-IEEE Program. 8-13 July 2018, Rio de Janeiro, Brazil.

Appendix B (*cont.*)

12. Peter Sussner, Vassilis Kaburlasos (chairmen) Special Session on “Recent Approaches Toward Lattice Computing”, 2022 North American Fuzzy Information Processing Society (NAFIPS 2022) Conference. 31 May -3 June 2022, Halifax, Nova Scotia, Canada.
13. V. G. Kaburlasos, E. Vrochidou, F. Panagiotopoulos, Ch. Aitsidis, A. Jaki, “Time series classification in cyber-physical system applications by intervals’ numbers techniques”, *Proceedings of the IEEE International Conference on Fuzzy Systems* (*FUZZ-IEEE* 2019), New Orleans, Louisiana, USA, 23-26 June 2019.
14. C. Bazinas, E. Vrochidou, C. Lytridis, V. G. Kaburlasos, “Time-series of distributions forecasting in agricultural applications: an intervals’ numbers approach”, *7th International Conference on Time Series and Forecasting* (*ITISE 2021*), Gran Canaria, Spain, 19-21 July 2021. In: *MDPI Engineering Proceedings* **2021**, 5 (1), 12; https://www.mdpi.com/2673-4591/5/1/12

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15. V. G. Kaburlasos, C. Bazinas, E. Vrochidou, E. Karapatzak, “Agricultural yield prediction by difference equations on data-induced cumulative possibility distributions”, *2022 North American Fuzzy Information Processing Society (NAFIPS 2022) Conference*, Halifax, Nova Scotia, Canada, 31 May - 3 June 2022.

16. V. G. Kaburlasos, C. Lytridis, G. Siavalas, T. Pachidis, S. Theocharis, “Fuzzy lattice reasoning (FLR) for decision-making on an ontology of constraints toward agricultural robot harvest”, *15th International FLINS (Fuzzy Logic and Intelligent Technologies in Nuclear Science) Conference (FLINS 2022) on Machine learning, Multi agent and Cyber physical systems*, Tianjin, China, 26-28 August 2022. (submitted)

Thank you

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Questions ?

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