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FRACTIONAL DERIVATIVE MODEL TO CHARACTERIZE THE TRANSVERSAL VISCOELASTIC BEHAVIOR OF *GUADUA ANGUSTIFOLIA*

Luis Felipe Zuluaga, Universidad del Valle, Colombia, luis.zuluaga@correounivalle.edu.co
Daniel Gómez, Universidad del Valle, Colombia, daniel.gomez@correounivalle.edu.co
José Jaime García, Universidad del Valle, Colombia, jose.garcia@correounivalle.edu.co

ABSTRACT

Stress-relaxation tests on *Guadua angustifolia* rings were conducted to determine the behavior under sustained transverse loading. A generalized Maxwell's model with three parameters showed good accuracy in fitting the experimental data. However, a significant dispersion of model parameters was observed. Thus, the Scott-Blair fractional derivative model with two parameters was formulated to fit the experimental data. Model fittings showed excellent approximations to experimental data ($\bar{R}^2$ = 0.996) with fewer parameters than a traditional model. Fractional models appear to be a good alternative to theoretically analyze the viscoelasticity of bamboo.

KEYWORDS

Bamboo viscoelasticity; Bamboo transverse properties; Fractional calculus; Scott-Blair model.

INTRODUCTION

Guadua angustifolia (GA) is a species of bamboo native to America, with a special interest in countries like Colombia because of its excellent mechanical properties. This material is used in construction given its high axial strength, similar to low carbon steel, and fast growth compared to wood (Paudel, 2008). However, its usage is limited to temporary or traditional construction, partially due to the complexity of bamboo joints (Jayanetti & Follett, 2008). Some connections require perforations, cuts, and mortar infill, creating brittle failure mechanisms, and making them relatively tedious for production. Another option that creates ductile mechanisms of failure (Moran et al., 2018) consists of using steel clamps around the bamboo culm, eliminating perforations in the material and making it easier to implement on a large scale. This connection applies a transversal load to the culm. Research has shown that bamboo exhibits viscoelastic properties (Amada & Lakes, 1997; Habibi et al., 2016), meaning the mechanical response changes over time.

Numerous traditional models have been developed to represent the viscoelastic response of bamboo (Wei et al., 2020). These models are represented by springs and dashpots arranged in series and parallel. A widely used model is the generalized Maxwell model, where the number of parameters can be adjusted depending on the complexity of the material response. A more recent field of study is fractional calculus (Mainardi, 2010), where the main interest is the determination of fractional order derivatives and integrals. A rheological model based on the fractional derivative opens the possibility for advanced models, to better represent complex viscoelastic responses with fewer parameters compared to traditional models (Schmidt & Gaul, 2002). Then, our objective was to assess the capability of a Scott-Blair fractional model with two parameters to represent the transversal viscoelastic response of GA.

MATERIALS AND METHODS

Material and experimental set-up

The material used for testing was obtained by cutting *Guadua*'s culms in the transverse direction. These elements have a shape similar to a ring, with a nominal length of 20 mm. The other dimensions of the rings are presented in Table 1.

Table 1: Dimensions of tested rings

n	Diameter (mm)		Thickness (mm)		Length (mm)	
27	mean	COV (%)	mean	COV (%)	mean	COV (%)
	116.01	2.27	10.97	15.85	20.25	5.32

To obtain the viscoelastic response in the transverse direction of *Guadua*, the edge bearing test was selected (ISO, 2019). The test consisted of imposing a vertical displacement on the rings and maintaining it for a period of time. A displacement of 1 mm was selected since it is small enough to stay within the linear response. The test duration was 10 minutes, obtaining the short-time response of the material. Displacement was measured with an LVDT, and force response with a load cell. Tests were performed under ambient conditions to simulate everyday conditions where the material would be used.

Figure 1: Experimental setup

Theoretical formulation

Based on viscoelasticity theory, a hereditary integral is needed to obtain stress in terms of strain. This integral consists of a convolution between the relaxation modulus and the strain derivative, described by the equation,

$$\sigma(t) = \int_0^t E(t-\tau)\frac{d\varepsilon}{d\tau}d\tau \qquad \text{Eq. 1}$$

where σ is the stress, t is the time, E is the relaxation modulus, τ is a dummy variable, and ε is the strain.

For the generalized Maxwell model, the relaxation modulus is as follows (Gutierrez-Lemini, 2014).

$$E(t) = E_0 + \sum_{i=1}^{N} E_i e^{-t/\tau_i} \qquad \text{Eq. 2}$$

where E_0 is the equilibrium or long-term modulus, and E_i and τ_i are the elastic modulus and relaxation time of the branch *i*, respectively.

The number of parameters required for the model depends on the complexity of the material response. For this study a generalized Maxwell model with one branch, also known as Standard Linear Solid (SLS), was selected (Figure 2).

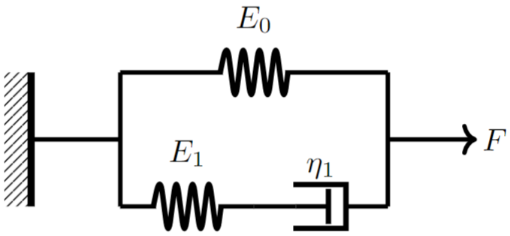


Figure 2: Sketch of a generalized Maxwell model with one branch

Fractional calculus allows calculating derivatives of non-integer order. Common elements, like springs and dashpots, used in traditional viscoelastic models represent a relationship between stress and an integer order derivative of strain, e.g., 0 for the spring, and 1 for the dashpot. In contrast with fractional calculus, a fractional-order element is obtained, representing either a spring, a dashpot, or a combination spring-dashpot, depending on the derivative order. The Scott-Blair model is the simplest fractional order element, characterized by only two parameters. The relaxation modulus of this element is (Mainardi, 2010),

$$E(t) = E_0 \frac{t^{-\alpha}}{\Gamma(1-\alpha)} \qquad \text{Eq. 3}$$

where Γ is the Gamma function and α is the derivative order.

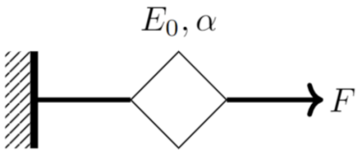


Figure 3: Sketch of Scott-Blair model

Model parameter fitting was done with the MATLAB function FMINCON. To measure the model quality to fit the experimental data, the determination coefficient R^2 was defined as,

$$R^2 = 1 - \frac{\sum(\sigma - \sigma_t)^2}{\sum(\sigma - \bar{\sigma})^2}, \qquad \text{Eq. 10}$$

where σ is the experimental stress, σ_t the theoretical stress, and $\bar{\sigma}$ is the mean stress. Additionally, the difference between the peak experimental and theoretical stresses was calculated as,

$$\%Peak\ error = \left|\frac{\sigma_{peak} - \sigma_{t\ peak}}{\sigma_{peak}}\right| * 100. \qquad \text{Eq. 11}$$

where σ_{peak} is the peak experimental stress and $\sigma_{t\ peak}$ the peak theoretical stress.

RESULTS

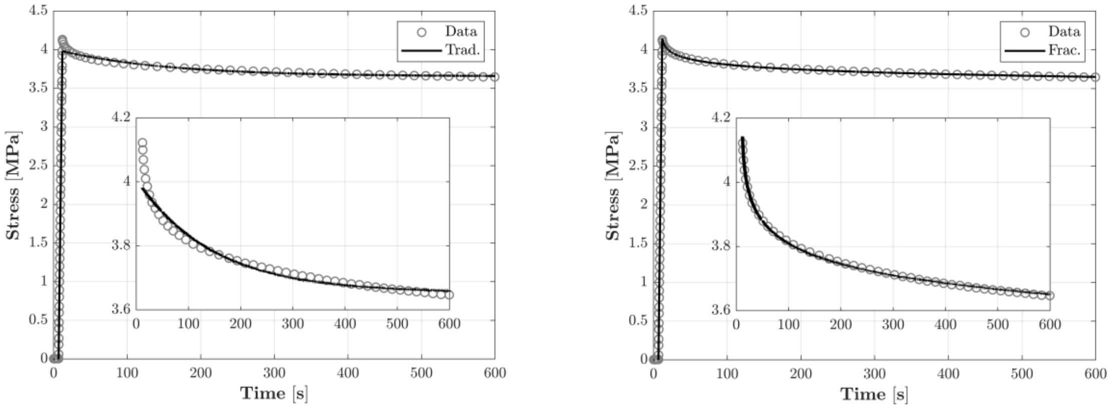
Both the traditional and the fractional models show a good capability to fit the experimental data given the high coefficient of determination and the relatively low error in estimating the peak load (Table 2). On the other hand, the simple fractional derivative model provided fitting results as good as those of the traditional model with fewer parameters.

Table 2. Model fitting results

n	Traditional model parameters						R^2		Peak error (%)	
	E_0 (MPa)		E_1 (MPa)		τ_1 (s)					
27	mean	COV (%)	mean	COV (%)	mean	COV (%)	mean	COV (%)	mean	COV (%)
	608.48	34.64	89.33	30.39	180.75	12.96	0.996	0.28	4.37	22.94

Fractional model parameters				R^2		Peak error (%)	
E_0 (MPa)		α					
mean	COV (%)	mean	COV (%)	mean	COV (%)	mean	COV (%)
771.71	28.9	0.038	33.31	0.996	0.31	1.92	63.06

The fractional model provided a better representation of the experimental curve in the first stage of relaxation and a closer representation of the peak load, since the error mean was 1.92% compared with 4.37% for the traditional model (Figure 4). In terms of computational time, both models spent similar amounts of time during the fitting process.



a) Traditional model fit a) Fractional model fit

Figure 4: Experimental data with traditional and fractional model fitting. Circles represent experimental data and solid lines represent viscoelastic model fittings.

DISCUSSION

The relaxed modulus E_0 for the fractional model was higher than that of the traditional model. Also, the coefficient of variation for this parameter was reduced for the advanced model compared to the traditional model. Still, a significant variation in parameters can be expected given that *Guadua* is a natural material with geometry and rigidity variations depending on variables such as the probe's location on the culm height and age.

The parameter α of the fractional model has a mean value closer to zero, meaning that the fractional element tends to represent the material more like an elastic material than a viscous one. To represent more complex curves with enriched fractional models, future fittings may take this parameter as a fixed value and adjust the others, using similar techniques as those when fitting with a generalized Maxwell's model, when the relaxation times are defined *a priori*. Overall, the fractional viscoelastic model could represent experimental data with an accuracy similar to the traditional model and even better when considering the peak error.

CONCLUSIONS

An advanced viscoelastic model was chosen to compare its advantages to the traditional one, considering this advanced model has fewer parameters than the traditional one. Results show an excellent approximation to experimental data, requiring fewer parameters and being capable of representing the whole stress history, including the zone close to the peak that the traditional model cannot fully cover. This advanced model can represent the transversal viscoelastic response of *Guadua angustifolia* with clear advantages over the traditional model proposed.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest associated with the work presented in this paper.

DATA AVAILABILITY

Data on which this paper is based is available from the authors upon reasonable request.

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