

1 **Article**

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4 **Single-bit full adder and logic gate based on synthetic**

5 **antiferromagnetic bilayer skyrmions**

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Abstract

30 Skyrmion-based devices are promising candidates for non-volatile memory and low-delay time
31 computation. Many skyrmion-based devices execute operation by controlling skyrmion
32 trajectory, which can be impeded by the skyrmion Hall effect. Here, the design of skyrmion-
33 based arithmetic device built on synthetic antiferromagnetic (SyAF) structures are presented,
34 where the structure can greatly suppress skyrmion Hall effect. In this study, the operations of
35 skyrmion-based half adder, full adder and XOR logic gate are executed by introducing
36 geometric notches and tilted edges, which can annihilate or diverge skyrmion. Performance of
37 these skyrmion-based devices are evaluated, where the delay time and energy-delay product of
38 the single-bit full adder are 1.95 ns and 2.50×10^{-22} Js, which are only 12% and 79% of the
39 previously proposed skyrmion-based adder. This improvement is significant in the construction
40 of ripple-carry adder and ripple-carry adder-subtractor. Therefore, our skyrmion-based SyAF
41 arithmetic device are promising candidate to develop high-speed spintronic devices.

42 **Keywords** skyrmion; full adder; synthetic antiferromagnet; low delay time electronics;
43 micromagnets

44

1 Introduction

46 A magnetic skyrmion is a stable topologically non-trivial configuration that possess particle-
47 like properties [1-3]. There are many advantages that make skyrmion a promising technology
48 for beyond-CMOS computing device and future data storage devices, including its small size,
49 stable properties and easy current driveability [4-6]. In 2009, the existence of magnetic
50 skyrmion has been demonstrated experimentally in bulk MnSi exhibiting the Dzyaloshinskii–
51 Moriya interaction (DMI) [7-9]. Inspired by the experimental realizations of skyrmion, many
52 skyrmion-based logic gates and computational devices have been proposed [10-21]. However,
53 skyrmion-based arithmetic devices are relatively less discussed.

54 Conventional arithmetic devices are usually constructed using single-bit full adder [22,
55 23]. With appropriate matching of single-bit full adders and logic gates, basic arithmetic
56 operations such as addition, subtraction and multiplication can be realized in electronic circuit.
57 Therefore, the speed and energy consumption of arithmetic devices strongly depend on the
58 efficiency of the single-bit full adder [22-23]. In CMOS circuit, the single-bit adder is usually
59 constructed by the matching of multiple logic gates. With similar approach, Chauwin *et. al.*
60 group proposes the design of an efficient single-bit full adder by matching skyrmion-based logic
61 gates [10]. Alternatively, Song *et. al.* group demonstrates that the performance of single-bit full
62 adder can be further improved by performing logical operations concurrently by taking
63 advantages of the skyrmion annihilation properties [13]. However, both designs of single-bit
64 full adder rely heavily on precise control of skyrmion Hall effect, which can be challenging in
65 operation. As demonstrated in recent studies, the skyrmion Hall angle is sensitive to the
66 fluctuation of applied current density [24-26] and temperature [27, 28], causing changes in the
67 skyrmion trajectory and unintentionally annihilation. On the other hand, studies also describe
68 the issues of stabilizing skyrmions below 10 nm in ferromagnetic multilayers and ferromagnetic
69 thin films [29, 30].

70 To resolve these problems, synthetic antiferromagnets (SyAF) have been introduced. The
71 SyAF bilayer system is a ferromagnet / heavy-metal spacer / ferromagnet trilayer structures,
72 where the two ferromagnetic (FM) layers are strictly exchange-coupled in an antiferromagnetic
73 (AFM) configuration [31]. By using the SyAF structure, the skyrmion Hall effect is greatly
74 suppressed [32], and skyrmions with the size below 10 nm are stabilized without applying
75 magnetic field [33-34]. These properties are important to stabilize skyrmion in high speed
76 skyrmion-based devices, avoiding accidental skyrmion annihilation and allowing more
77 compact devices to be fabricated. Therefore, it is important to perform study on designing a
78 single-bit full adder based on the SyAF structure. Indeed, other racetrack fabrication methods

79 based on local modification of magnetic properties [35-36] are also important for building
 80 skyrmion-based devices.

81 In this paper, we propose designs of skyrmion-based single-bit half adder, single-bit full
 82 adder and XOR logic gate on SyAF bilayer structure. These designs execute computation by
 83 controlling skyrmion trajectory and annihilation with simple geometry. Since the skyrmion Hall
 84 effect is greatly suppressed on SyAF, our proposed devices are less sensitive to electric current
 85 and thermal fluctuation. We also demonstrate that the delay time and energy-delay product
 86 (EDP) of the single-bit full adder can be reduce to 1.95 ns and 2.50×10^{-22} Js per each
 87 operation, which is only 12% and 79% of the previously proposed skyrmion adder. Our results
 88 provide guidelines for the design of quick response skyrmion-based logic gate and single-bit
 89 full adder device.

90 **2 Model and methods**

91 Fig. 1a shows the schematic diagram of the proposed device. Two heavy metal layers (HM)
 92 sandwiches the patterned trilayer SyAF structure. The patterned trilayer SyAF are shown in Fig.
 93 1b-d and are used as the skyrmion conduit. Magnetic tunnel junctions (MTJ) are installed on
 94 top of the SyAF/HM layer for skyrmion nucleation and detection. This MTJ-based skyrmion
 95 nucleation and detection circuit are adopted from previous skyrmion devices and are spice-
 96 compatible [17, 37]. The skyrmion in the patterned SyAF layers is driven by spin-orbit torque
 97 (SOT), which is induced by applying electric current to the HM layers to produce vertical spin
 98 currents to the SyAF structure. [38]. Thus, skyrmion moves in the electric current direction.

99 The operation of the proposed devices is designed based on the analysis of SOT driven
 100 skyrmion motion properties described by Thiele equation [13, 32]:

$$101 \quad \mathbf{G} \times \mathbf{v} - D\alpha\mathbf{v} + \mathbf{F}_{\text{st}} + \mathbf{F}_{\text{ext}} = \mathbf{0} \quad (1)$$

102 with the gyromagnetic vector $\mathbf{G} = \hat{z}G = \hat{z}(4\pi Q)M_s d/\gamma$, the topological charge Q , the
 103 gyromagnetic ratio γ and dissipation constant $D = -(\frac{16}{3})\pi M_s d/\gamma$. The first term is the

104 gyroscopic force related to the skyrmion Hall effect and the second term is the dissipative forces
 105 that oppose the skyrmion movement in the direction of the velocity \mathbf{v} . The third term $\mathbf{F}_{st} =$
 106 $\sigma\kappa j\hat{z} \times \hat{p}$ is spin torque force, with spin Hall angle dependent constant $\sigma = -\pi\hbar\theta_{SH}/2e$,
 107 characteristic length of skyrmion κ and $\hat{p} = -\hat{y}$ in our simulation. The fourth term \mathbf{F}_{ext} is an
 108 external force; including forces induce by the conduit's boundary, notches or tilted edge.

109 Skyrmions movement on the FM layer are determined by the balances of these four
 110 forces. Considering the skyrmion in the free space moving along the x-axis, i.e. $\mathbf{F}_{ext} = 0$:

$$111 \quad \mathbf{v}_x = \frac{\alpha D}{G^2 + \alpha^2 D^2} \mathbf{F}_{st} \quad \mathbf{v}_y = \frac{\mathbf{G}}{G^2 + \alpha^2 D^2} \mathbf{F}_{st} \quad (2)$$

112 Thus, the skyrmion transverse movement \mathbf{v}_y is related to G and its direction depends on the
 113 topological charge of the skyrmion Q. In SyAF, the skyrmions on top FM layer and bottom FM
 114 layer are strictly exchange-coupled in an antiferromagnetic (AFM) configuration. Hence, the
 115 skyrmions have opposite sign of Q and the transverse movement will cancel out each other. As
 116 a result, with balance of the three forces: gyroscopic force ($\mathbf{G} \times \mathbf{v}$), dissipative force ($D\alpha\mathbf{v}$),
 117 and spin torque force (\mathbf{F}_{st}), skyrmion moves in a straight line in free space.

118 When the skyrmion is close to the boundary of the conduits, changes of magnetization at
 119 the boundary induced repulsive force ($\mathbf{F}_{ext} = \mathbf{F}_b$) to the skyrmion. For titled edge, transverse
 120 force ($\mathbf{F}_b = F_b\hat{y}$) is extracted from the boundary, causing the skyrmions to move along the
 121 boundary. When the forces exceeds the threshold value, a skyrmion annihilates at the boundary
 122 [39, 40].

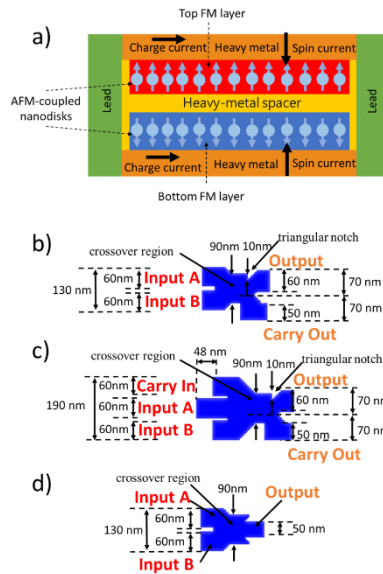
123 The adders and logic gate operation are demonstrated using micromagnetic simulations
 124 performed on the GPU-accelerated open-source micromagnetic simulator software package
 125 MuMax3 [41], which have been used in simulating SyAF system as reported in Ref. [42-45].
 126 In real experimental samples, a spacer layer is added to adjust the interlayer exchanges coupling
 127 between two ferromagnetic layers. In the simulation, we focus on the magnetic layers and
 128 ignore the physical thickness of the nonmagnetic spacer layer, as the spacer is nonmagnetic and

129 is usually much smaller than 1 nm in experiments [46]. However, we do consider the interlayer
 130 exchange coupling based on the RKKY-type exchange interaction. Therefore, in the simulation,
 131 we applied interlayer exchange coupling between two Ferromagnetic layer and and ignore the
 132 thickness of nonmagnetic spacer layer. The software simulates the magnetization dynamics by
 133 solving the Landau–Lifshitz–Gilbert (LLG) equation augmented with SOT [47-48].

$$134 \quad \frac{d\mathbf{m}}{dt} = -\gamma_0 \mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha \left(\mathbf{m} \times \frac{d\mathbf{m}}{dt} \right) - \frac{\hbar \theta_{\text{SH}} J}{2meM_s t_{\text{FM}}} \mathbf{m} \times (\mathbf{m} \times \boldsymbol{\sigma}) \quad (3)$$

135 with normalized magnetization \mathbf{m} , the gyromagnetic ratio γ_0 , effective field vector \mathbf{H}_{eff} ,
 136 damping constant α , spin Hall angle θ_{SH} , current density J , saturation magnetization M_s ,
 137 thickness of the FM layer t_{FM} , and $\boldsymbol{\sigma} = -\vec{u}_y$ in our model. The intrinsic magnetic material
 138 parameters used in our simulation are adopted from experimental data of Co/Pt materials
 139 reported in Ref. [48] and are used in simulations reports [49-53]: Exchange stiffness $A_{\text{intra}} =$
 140 15 pJ m^{-1} , Gilbert damping coefficient $\alpha = 0.3$, saturation magnetization $M_s = 580 \text{ kA m}^{-1}$,
 141 perpendicular magnetic anisotropy (PMA) constant $K = 0.8 \text{ MJ m}^{-3}$, DMI strength $D = 3.5 \text{ MJ}$
 142 m^{-2} and spin Hall angle $\theta_{\text{SH}} = 0.2$. To balance the accuracy and efficiency, all models are
 143 discretized into tetragonal elements with the size of $2 \text{ nm} \times 2 \text{ nm} \times 2 \text{ nm}$.

144



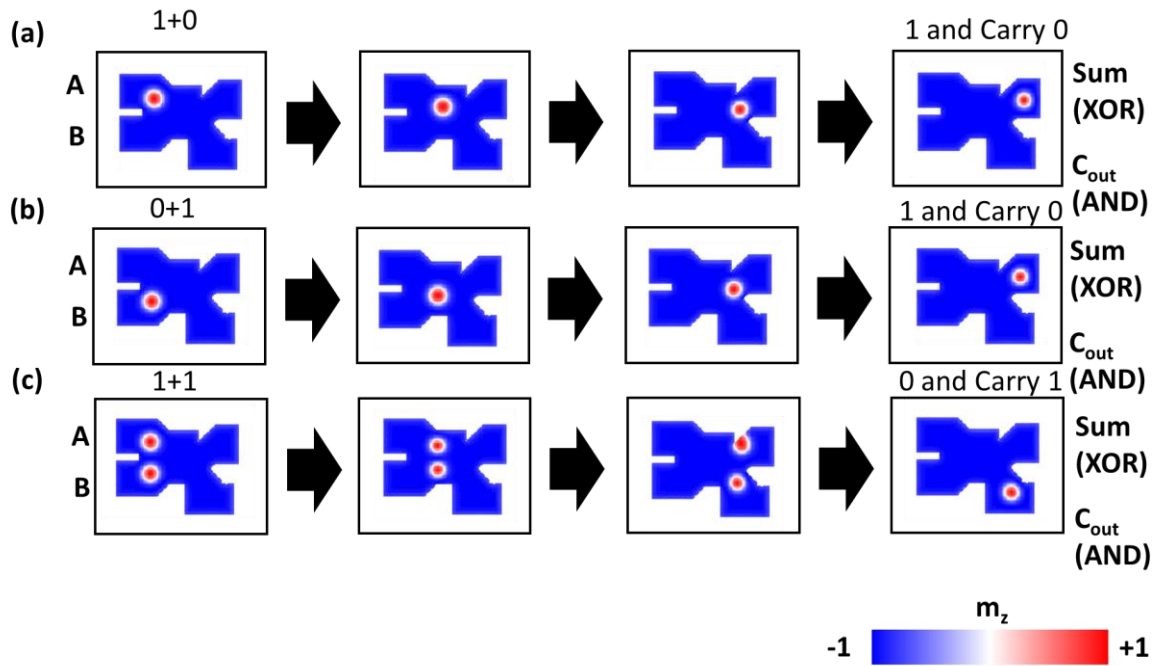
145

146 Fig. 1 Schematics diagram of the proposed SyAF bilayer single-bit half adder, single-bit full
 147 adder and logical XOR gate. (a) The XZ cut of the SyAF bilayer used. The thickness of each
 148 Ferromagnetic layer (FM) is 2 nm and two layers are separated by an Ru layer. The skyrmion
 149 motion is driven by the current perpendicular to the plane (CPP) approach. Charge current flows
 150 through heavy-metal layers along the x-direction, which gives rise to a spin current ($\mathbf{P} = +z$)
 151 perpendicularly injected to the bottom and top FM layer because of spin Hall effect. These
 152 Heavy metal layer's geometry is tetragonal in shape and are different from the FM layer. Hence,
 153 the skyrmion in the FM layers are driven by the spin current and move in the x direction. (b)
 154 The FM layer design of the single-bit half adder, the Input A and Input B are on the left. The
 155 Output is located on the top right of the channel and the Carry is located on the bottom right of
 156 the channel. Two 45° tilted edges are located beneath the center line of the channel to deflect
 157 skyrmions. (c) The FM layer design of the single-bit full adder, the Input A, Input B and Carry
 158 In are located on the left. The Output and Carry Out are located on the right. Two 45° tilted
 159 edges are located beneath the center line of the channel to deflect skyrmions. An edge is located
 160 on the top of the channel to annihilate unwanted skyrmions. (d) The FM layer design of the
 161 XOR logic gate. For the XOR logic gate, the Input A, Input B are located on the left. The
 162 Output is located on the centre right end of the channel.

163 **3 Results and discussion**

164 **3.1 Skyrmion-based single-bit half adder device**

165 A half adder is operated by compiling XOR and AND logical operation simultaneously
166 to generate SUM and CARRY outputs, respectively. Realization of half adder in conventional
167 CMOS circuit require 18 transistors with complex circuit connections [13]. Here, we propose
168 a different approach to realize single-bit half adder device using simple geometry and magnetic
169 skyrmions. The design of the half adder device includes two inputs and outputs with a crossover
170 region (Fig. 1b). Fig. 2 shows the operation of the proposed single-bit half adder. Skyrmions
171 present at the left input terminals travels to the right terminals for detection. For input (1, 0)
172 (Fig. 2a and supplementary video S1), the skyrmion nucleated at the upper input terminal is
173 diverged to the crossover region by a 45° tilted edge. Due to the skyrmion-edge repulsion, the
174 skyrmion moves in the centre of the crossover region and passes the triangular notch. Then, the
175 skyrmion is diverged by the second 45° tilted edge and is detected at the output terminals,
176 resulting in an output (1, 0). Similarly, for input (0, 1) (Fig. 2b and supplementary video S2),
177 the skyrmion nucleated at the lower input terminal is diverged to the crossover region by a 45°
178 tilted edge. Due to the skyrmion-edge repulsion, the skyrmion moves in the centre of the
179 crossover regions and passes the triangular notch. Then, the skyrmion is diverged by the second
180 45° tilted edge and is detected at the output terminals, resulting in an output (1, 0). Finally, for
181 input (1, 1) (Fig. 2c and supplementary video S3), two skyrmions encounter at the crossover
182 region and repel each other, due to skyrmion-skyrmion repulsion. Subsequently, the upper
183 skyrmion is annihilated at the triangular notch, and the lower skyrmion is diverged to the lower
184 output terminal by the second 45° tilted edge. Therefore, the input (1, 1) results in an output (0,
185 1). Input (0, 0) obviously generates an output (0, 0). The upper and lower output terminals are
186 corresponded to SUM and CARRY, respectively. As a result, the simple geometry shown in
187 Fig. 1b can perform the XOR and AND operation concurrently and can operate as a single-bit
188 half adder with delay time 1.1 ns.



190

191 Fig 2. Skyrmion-based single-bit full adder device operation for each input case. The device
 192 size is $150 \text{ nm} \times 300 \text{ nm}$, and the applied current density $j = 140 \text{ MA cm}^{-2}$. The operation
 193 time of the half adder is 1.1 ns.

194

195 3.2 Skyrmion-based single-bit full adder device

196 A single-bit full adder is operated by compiling two binary inputs and a carry-in digit to
 197 generate SUM and CARRY-OUT outputs. Hence, it comprises three inputs and two outputs.
 198 The previous skyrmion-based full adders are constructed by connecting two half adders and
 199 one OR gate properly [10, 13]. Here, we implement single-bit half adder device using simple
 200 geometry with design similar to the half adder, as shown in Fig. 1c. Fig. 3 and Supplementary
 201 Movies S4-S10 shows the operating process of the full adder. The skyrmions presented at the
 202 left input terminals travel to the right for detection at the output terminals. The final outputs
 203 represent SUM (upper branch) and CARRY-OUT (lower branch).

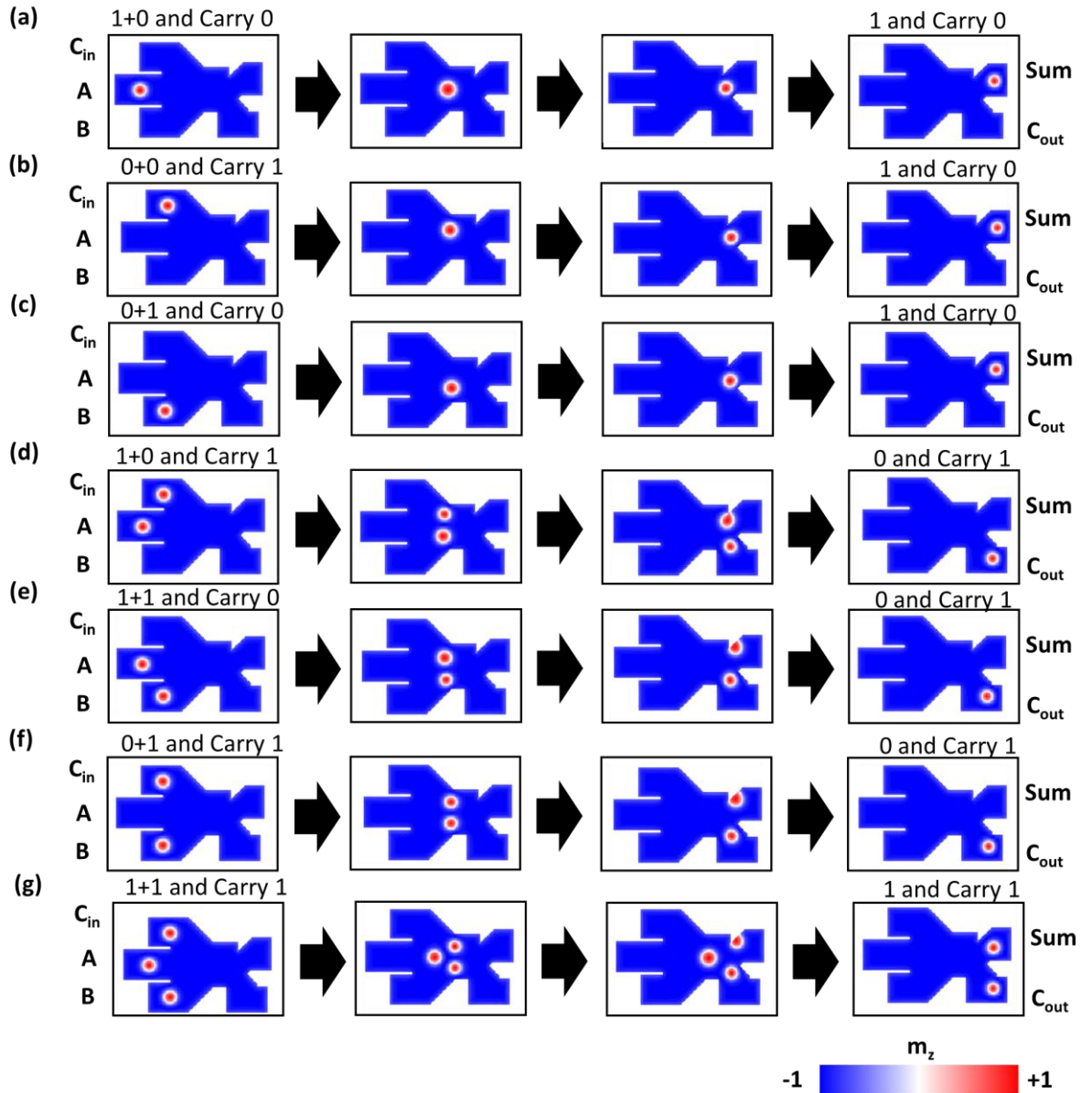
204 We first demonstrate the operation of full adder with input (0,1,0), (1,0,0) and (0,0,1). For
205 input (0, 1, 0), the skyrmion nucleates on the left terminal will pass through the triangular notch
206 and is diverged to the SUM region by the second 45° tilted edge, as show in Fig. 3a (see
207 Supplementary Movies S4). For Input (1, 0, 0) and (0, 0, 1), the skyrmion nucleates on the left
208 terminal will be diverged to the crossover region by the 45° tilted edge. Due to the skyrmion-
209 edge repulsion, the skyrmions will align to the centre of the crossover region and pass through
210 the triangular notch. The skyrmion will be diverged to the upper output terminal for detection
211 by the second 45° tilted edge, as shown in Fig 3b and Fig. 3c (see Supplementary Movies S5-
212 S6). Therefore, the input (0,1,0), (1,0,0) and (0,0,1) results in an output (1,0).

213 The operation of full adder with input (1,0,1), (0,1,1) and (1,1,0) is demonstrated in Fig.
214 3d-3f (see Supplementary Movies S7-S9). Similar to the operation with input (0,1,0), (1,0,0)
215 and (0,0,1), skyrmions nucleated in the input terminals enter the crossover region. Within the
216 crossover region, two skyrmions encounter and repel each other, due to skyrmion-skyrmion
217 repulsion. Subsequently, the upper skyrmion is annihilated at the triangular notch. The lower
218 skyrmion is diverged by the 45° tilted edge to the lower output terminal. Therefore, the input
219 (1,1,0), (0,1,1) and (1,1,0) results in an output (0,1).

220 The operation of full adder with input (1,1,1) is demonstrated in Fig. 3g (see
221 Supplementary Movies S10). Similar to the operation with input (1,1,0), (0,1,1) and (1,0,1),
222 skyrmions nucleated in the inputs enters the crossover region. Within the crossover region, three
223 skyrmions encounter and repel each other. Two skyrmions travel simultaneously with one
224 skyrmions is repel backwards. Thus, the upper skyrmion is annihilated at the triangular notch.
225 The lower skyrmion and the skyrmions at the back are diverged by the 45° tilted edge to the
226 lower and upper output terminal, respectively. Therefore, the input (1,1,1) results in an output
227 (1,1). In addition, Input (0,0) obviously generates an output (0,0). We have investigated the
228 reliability of the full adder and found the proper range of the operation current density to be
229 from 120-145 $MA\ cm^{-2}$.

230 It is interesting to note that the skyrmion arrive the upper and lower output terminal in
231 1.95 ns and 1.7ns, respectively. Therefore, the CARRY-OUT result can be read and pass to the
232 subsequent adder 0.25 ns before the OUTPUT is computed, which can further reduce the delay
233 time in constructing multi-bits adder. On the other hand, when we compare it with the full
234 adders that are constructed by combination of half adders, our design does not have the
235 synchronizing issue of arrival time between two half adders and one OR gate. Therefore, the
236 full adder is more robust and scalable. Additionally, ref. [13] suggests that cascading single-bit
237 skyrmion adders can realize energy efficient multi-bits adder without wire connection. Similar
238 cascading approach is also feasible to build energy efficient multi-bits adder with our proposed
239 adder, information can be found in supporting information 1.

240 In addition, we have performed simulations on our design when the notch is displaced
241 by ± 5 nm in the x direction (Supporting information Figure S2 and S3). The execution of
242 addition operation is not affected by the displace notch, showing there is tolerance for defects
243 to some degree in our design.



244

245 Fig 3. Skyrmion-based single-bit full adder device operation for each input case. The device
 246 size is $190 \text{ nm} \times 300 \text{ nm}$, and the applied current density $j = 140 \text{ MA cm}^{-2}$. The operation
 247 time of the full adder is 1.95 ns .

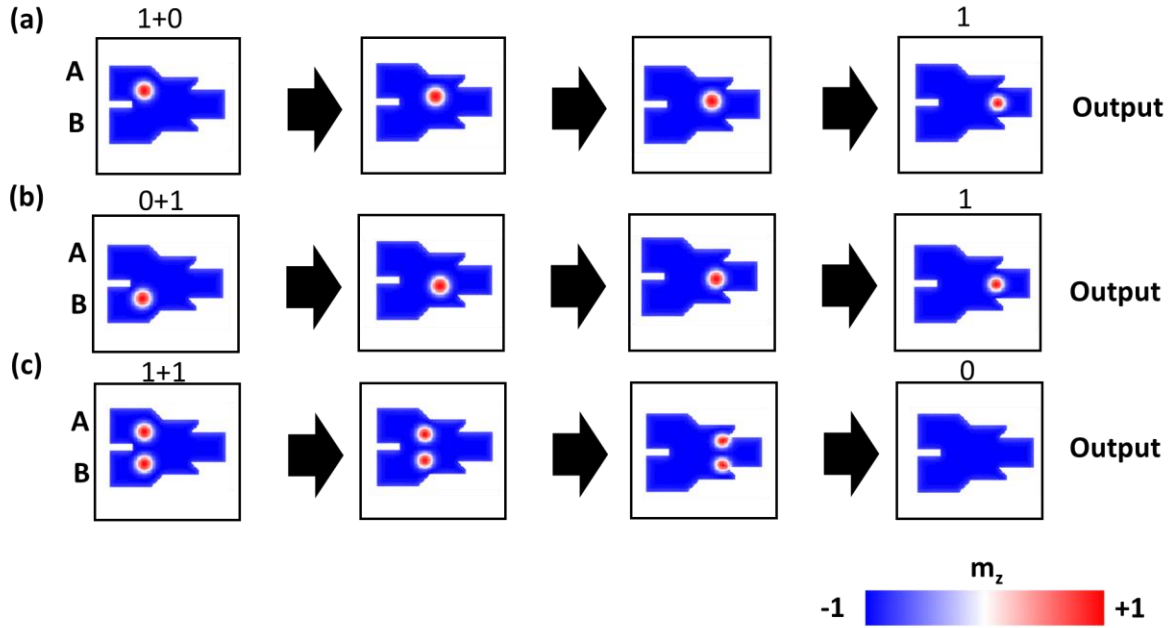
248 3.3 Skyrmion-based XOR logic gate

249 A half adder is a combination of XOR and AND logic gate, as demonstrated in section
 250 A. However, for high-speed computation, we designed an alternative version of XOR logic gate
 251 that operate faster than the half adder. The proposed Skyrmion-based XOR logic gate is shown
 252 in Fig. 4. Input A and Input B are located on the left and the output are located on the right.
 253 Different from the half adder, the output region is at the centre of the channel. We first

254 demonstrate the XOR logic gate operation in Fig.4 (see Supplementary Movies S11-S13). The
255 logical XOR gate is operated such that $0 + 0 = 0$, $1 + 0 = 1$, $0 + 1 = 1$ and $1 + 1 = 0$. The process
256 of $0 + 0 = 0$ in XOR gate is trivial, where no skyrmions are presents in input and output. We
257 interpret the process $1 + 0 = 1$ and $0 + 1 = 1$ as follow: When a skyrmion is placed on either
258 Input A or Input B, the skyrmion is diverged to the crossover region. Due to skyrmion edge
259 repulsion, the skyrmion travels at the centre and reaches the output terminal for detection. On
260 the other hand, the important process $1 + 1 = 0$ is represented as follow: Skyrmions are placed
261 on both Input A and Input B, both skyrmions enter the crossover region and repel each other.
262 Then, two skyrmions travel simultaneously and are annihilated by the edges of the output
263 terminal. Thus, no skyrmions reaches the output region, representing logical 0 as the output of
264 XOR gate. The operation time for the XOR logic gate is 0.7 ns, which is much lower than 1.1
265 ns in half adder.

266 XOR logic gate is important in constructing adder-subtractor. A multi-bit ripple-carry
267 adder-subtractor is demonstrated in Fig. 5a, where the circuit is composed of multiple full
268 adders and XOR logic gate, in which the carry-out of each full adder is the carry-in of the
269 subsequent full adder. XOR logic gate provide Control input to the adder-subtractor logic
270 circuit. When the Control Input is logical 0, the XOR logic gate is a connection line. Thus, the
271 logical state in Input B is passed to the full adder without alternation and addition operation is
272 executed. On the other hand, when Control Input is logical 1, XOR logic gate is turn into a NOT
273 gate. Thus, the logical state in Input B will be flipped and passes to the full adder. In such
274 condition, the logical computation $A - B = A + \bar{B} + 1$ is realized in circuit and subtraction
275 operation is executed. Hence, by cascading or wire-connected the full adder and XOR logic
276 gate proposed in this study, a quick response skyrmion-based multi-bit ripple-carry adder-
277 subtractor can be realized.

278



279

280 Fig. 4. Skyrmion-based XOR logic gate operation for each input case. The device size is
 281 $130 \text{ nm} \times 200 \text{ nm}$, and the applied current density $j = 140 \text{ MA cm}^{-2}$. The operation time of
 282 the XOR logic gate is 0.7 ns .

283

284 3.4 Performance analysis of the proposed devices

285 The operation of half adder, full adder and XOR logic gate that operate without skyrmion Hall
 286 effect have been demonstrated on SyAF structure. We first estimate the total delay time of our
 287 full adder and half adder using the following equation:

$$288 \quad t_{operation} = t_{nucleation} + t_{propagation} + t_{detection} \quad (4)$$

289 The $t_{nucleation}$ of one skyrmion nucleation by MTJ is about 20 ps and the $t_{detection}$ by MTJ is
 290 about 5 ps [35]. Therefore, the operation time of our full adder and half adder is 1.98 ns and
 291 1.13 ns , which is only about 12% and 19% of the previous skyrmion-based adder [13]. The
 292 difference is contributed by the SyAF structure, which allows skyrmion to be diverged by the
 293 conduct's edge in high current density without annihilation, and can make great impact on
 294 computational devices. For example, ripple-carry adder is constructed by series connection of
 295 single-bit adder, as shown in Fig. 5b. In such configuration, the delay time of the n-bit ripple-

296 carry adder is the sum of all single-bit full adder delay times. Fig. 5c shows the total delay time
 297 of the n-bit ripple-carry adder for previous proposed devices [10, 13]. The blue and red symbols
 298 correspond to the delay time consumption based on [10] and [13], respectively. The Black
 299 symbols correspond to the delay time consumption of our design. Obviously, our adder design
 300 response quicker than the previous proposal [10] and [13]. For a 64-bit system ($n = 64$), the
 301 time delay in our design is about 126.4 ns, which is about 899 ns lower than previous skyrmion-
 302 based adder design [13]. The delay time can be further reduced to 110.65 ns by taking
 303 advantages of the 0.25 ns delay time different between the OUTPUT and CARRY-OUT of the
 304 single-bit adder. Since our proposed device are built on SyAF structure, it is possible to further
 305 increase the operation speed by minimizing the device for sub-10 nm skyrmion [33, 34].

306 We then evaluate the energy efficiency of our adder devices using the Energy-delay
 307 product $EDP_{operation} = E_{operation} \times t_{operation}$, where $E_{operation}$ is the energy during
 308 nucleation (Input), propagation and detection (Output), which are given by the following
 309 equation:

$$310 \quad E_{operation} = E_{nucleation} + E_{propagation} + E_{detection} \quad (5)$$

311 The $E_{Nucleation}$ of one skyrmion nucleation by MTJ is about 3.1 fJ, while the $E_{Detection}$
 312 by MTJ is about 0.8 fJ [17]. $E_{propagation}$ is calculated from the energy dissipation caused by the
 313 electric resistance of the HM layer:

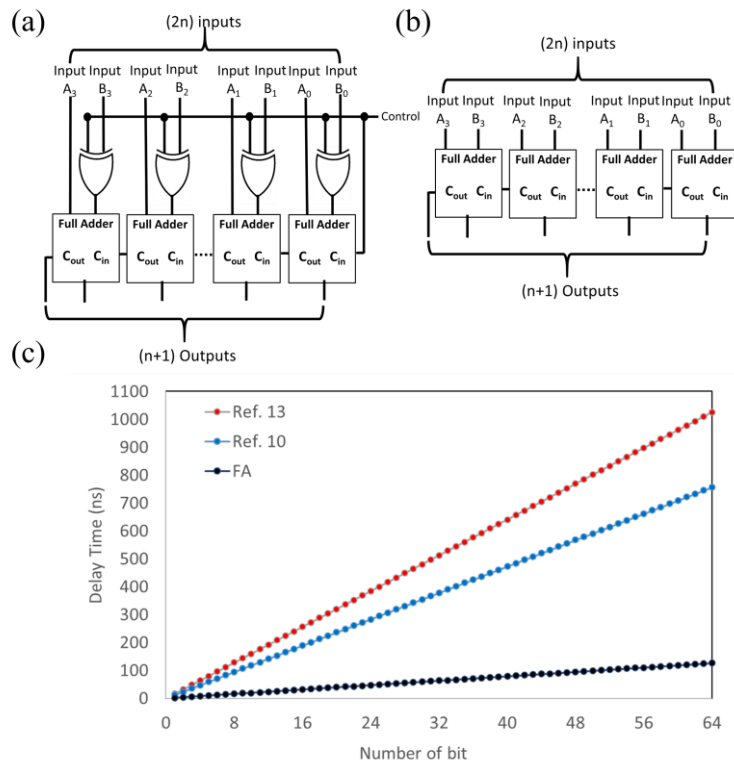
$$314 \quad E_{propagation} = I^2 R_{HM} \Delta t = \rho_{HM} V_{HM} j^2 t_{propagation} \quad (6)$$

315 where ρ_{HM} is the resistivity of the HM (Resistivity of Pt = $10.6 \times 10^{-8} \Omega \cdot m$), j is the current
 316 density, V_{HM} is the volume of the HM layer ($190 \times 300 \times 2.5 \text{ nm}^3$ for Full adder) and the
 317 $t_{propagation}$ of full adder is 1.95 ns. Therefore, the EDP of full adder, including I/O, are
 318 estimated to be 2.5×10^{-22} Js, which are only about 79% of the previous proposed skyrmion-
 319 based adders [13]. The difference in energy efficiency is contributed by the smaller propagation
 320 area and shorter propagation time in our design, which allows the same functionality with a

321 smaller number of logic gates than the previous proposal. On the other hand, the size of our full
322 adder is $190 \text{ nm} \times 300 \text{ nm}$, which is only 48% of the previous proposed skyrmion based full
323 adder and 0.06% of the transistor based full adder [54]), which is significant for making
324 compact computer chips.

325 On the other hand, our design of the skyrmion-based adder is implemented by the
326 skyrmion trajectory that is driven by SOT, which is one of the most commonly used methods
327 applied in skyrmion-based devices to manipulate the magnetization. Recently, other
328 approaches, such as the piezospintronics and topological anomalous Hall effect, have been
329 predicted and experimentally demonstrated to switch magnetization with lower energy and less
330 Joule heating effect [55-57]. These approaches on skyrmion-based device are worth studying
331 due to their potential to further improve the energy efficiency of our proposed full-adder.

332



334

335 Fig. 5. (a) Diagram of wire-connected n-bit ripple-carry adder-subtractor (b) Diagram of wire-
 336 connected n-bit ripple-carry adder (c) Time delay of various skyrmion-based n-bit- ripple-carry
 337 adder. Red squares represent estimated Time delay for [13], blue squares correspond to the
 338 estimated time delay for [10] and black squares correspond to our proposed full adder design.

339 **4 Conclusion**

340 In summary, novel designs of the skyrmion-based arithmetic circuits and logic gates on SyAF
 341 structure have been demonstrated. By introducing geometric notches and tilted edges on
 342 patterned nanotracks, we have successfully constructed the half adder, the full adder and the
 343 XOR logic gate based on skyrmions. Due to the simple geometries, the proposed logic gates
 344 have the advantage of low delay time and high energy efficient when compared with the
 345 conventional CMOS-based logic gates or skyrmion-based logic devices that have been
 346 proposed. Since the skyrmion Hall effect is suppressed on the SyAF structure, our proposed
 347 logic gate is less sensitive to electric current fluctuation. We have also demonstrated the
 348 strength of our proposed single-bit full adder by calculating the delay time and energy-delay

349 product of a multi-bit ripple carry adder circuit. Our proposed arithmetic circuit is compossible
350 with semiconductor fabrication technology in the industry. Therefore, the skyrmion-based
351 arithmetic circuit on the SyAF structure is a promising candidate for spintronics devices.

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