LINEAR DISPLACEMENT FROM A DIFFERENT PERSPECTIVE

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Abstract. This paper shows relation of displacement with acceleration, jerk and snap differently than it is currently established.

1. displacement during constant acceleration

If a point particle starts moving with constant velocity u, then it will cross u units of distance in first unit of time, u units of distance in second unit of time and so on. Over t units of time where $t \in \mathbb{Z}_+$, the particle will cross $u + u + \cdots + u = \sum_t u$ units of distance.

During the occurrence of constant acceleration a, in first unit of time, the particle will cross u and an additional a units of distance and their sum is $u + a$ units of distance; therefore we can say, $u + a$ was the 'crossing velocity' of the particle *during* the first unit of time and this becomes the initial velocity for the second unit of time.

In second unit of time, the particle will cross $u + a$ and an additional a units of distance and their sum is $u + a + a \Rightarrow u + 2a$ units of distance; we can say, the 'crossing velocity' of the particle *during* the second unit of time was $u + 2a$ and this becomes the initial velocity for third unit of time.

We can keep doing this process of calculation to get the crossed distance (by the particle) for each unit of time. We can notice from the process mentioned above, the t-th unit of time would give out the calculated crossed distance as $u + ta$.

We can make a connection of this process to the similarity of a simple arithmetic progression $u+a, u+2a, \ldots, u+ta$. This way, we get the displacement of the particle over the period of t as $r = \sum_t (u + ta)$. If acceleration a occurs for $(t + \tau)$ units of time where $0 < \tau < 1$ then, $r = \sum_t (u + ta) + \tau(u + (t + 1)a)$; since, over the next period of τ after t, the particle will cross $(u + ta)\tau$ and an additional $a\tau$ units of distance. If a occurs only for τ unit (fractional), then $r = (u + a)\tau$.

2. sketching out a general equation

(The symbols j and s represents kinematic terms jerk and snap respectively.)

In place of u, if we write $u + at$ to show the t-th term of the sequence $u + a, u + 2a, \ldots, u + ta$ then we get a recurrence relation $A_t = A_{t-1} + a$ with initial condition $A_0 = u$. We would get a few nested recurrence relations if we write $a + jt$ in place of a, $j + st$ in place of j and so on.

$$A_t = A_{t-1} + B_t,\ A_0 = u; \qquad B_t = B_{t-1} + C_t,\ B_0 = a; \qquad C_t = C_{t-1} + s,\ C_0 = j$$

By calculating for A_t, we get the t-th term of a new sequence as,

$A_t = A_0 + \sum_t (B_0 + \sum_t (C_0 + \sum_t s))$

From this, we get displacement as $r = \sum_t A_t$

For additional τ amount of time, where $0 < \tau < 1$, displacement is $r = \sum_t A_t + \tau A_{t+1}$

References

[1] Wikipedia. Fourth, fifth, and sixth derivatives of position.