Theodor Kaluza's Theory of Everything: revisited

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Abstract

Using a metric based on solutions for the scalar of a 5-dimensional Kaluza model in the field equations for curved space-time allows to derive a convergent series of particle energies, to be quantized as a function of the fine-structure constant, α, with limits given by the energy values of the electron and the Higgs vacuum expectation value. The value of α can be given numerically by the gamma functions of the integrals involved, extending the formalism to N-dimensions yields a single expression for the electroweak coupling constants. The series expansion of the energy equation provides quantitative terms for particle energy, Coulomb and gravitational interaction. Additional terms in the field equations may give a value for the cosmological constant in the correct order of magnitude.

The model can be expressed ab initio without use of free parameters.

1 Introduction

Theory of everything is a somewhat ironic and pompous term and maybe an unachievable goal. Theodor Kaluza in 1919 developed a unified field theory of gravitation and electromagnetism that produced the formalism for the field equations of the general theory of relativity (GR) and Maxwell's equations thus unifying both major forces known at his time. His 5-dimensional model [1] is mainly known as Kaluza-Klein theory today, including the contributions of Klein [2] who introduced the idea of compactification and attempted to join the model with the emerging principles of quantum mechanics. This version became a progenitor of string theory. The classical Kaluza model was developed further as well [3], Wesson and coworkers elaborated a general non-compactified version to describe phenomena extending from particles to cosmological problems. The equations of 5D space-time may be separated in a 4D Einstein tensor and metric terms representing mass and the cosmological constant, Λ. Particles may be described as photon-like in 5D, traveling on time-like paths in 4D. This version is known as space-time-matter theory [4]. Both successor theories give general relationships rather than providing quantitative results for specific phenomena such as particle energy.

The model described in the following evolved from a heuristic approach and does not attempt to give a complete solution for a 5D theory but to demonstrate that Kaluza's ansatz provides very simple, parameter-free and in particular quantitative solutions for a wide range of phenomena. Basic equations from the existing literature may be used, with one significant simplification:

The terms in the metric tensor have to be dimensionless. Kaluza’s approach presupposes electromagnetic units. To reproduce the Einstein field equations (EFE) he choose a gravitational term to keep the electro-magnetic potential terms in the metric dimensionless, a rather unfitting combination [1]. This assumption will be dropped in this work, the equations will be interpreted in their entirety as related to electromagnetism. Gravitational phenomena will be recovered via a series expansion of the energy equation.

A necessary boundary condition in the equations used will be that all particles have to posses angular momentum, i.e. integer or half-integer spin or be composed of half-integer spin components (e.g. mesons). Spin might be considered to be implicit in Kaluza's ansatz as well, since electrodynamics allows solutions for circular polarized light.

For the model presented here it might be helpful to use the following visualization: a *photon* with its intrinsic angular momentum interpreted as having its E-vector rotating around a central axis of propagation will be transformed into an object that has the - still rotating - E-vector constantly oriented to a fixed point, the origin of a local coordinate system, resulting in an SO(3)-like object with *point charge* properties [2]. The vectors E, B and V of the propagation velocity [3] are supposed to be locally orthogonal and subject to the

1 In the closing remarks of [1] Kaluza considers to replace „die etwas fragwürdige Gravitationskonstante“ –„the somewhat questionable constant of gravitation“.

2 Neutral particles are supposed to exhibit nodes separating corresponding equal volume elements of reversed E-vector orientation and opposite polarity.

3 Referred to as “EBV-triple“ in the following; in the limit r -> r_n („particle radius“) => V -> c_0;

standard Maxwell equations, however, on the background of an appropriately curved space-time. Curvature of space-time based on an electromagnetic version of the field equations of GR will be strong enough to localize a photon in a self trapping kind of mechanism, yielding energy states in the range of the particle zoo. Such a qualitative model has some merits of its own, see [A1], quantitative results will be based on Kaluza's equations.

The basic proceeding will be as follows:

Kaluza's equations for flat 5D-space may be arranged to give [4, chapter 6.6]

1) Einstein-like equations for space-time curved by electromagnetic and scalar fields (equ. (5)),

2) Maxwell equations where the source depends on the scalar field,

3) a wave-like equation connecting the scalar with the electromagnetic tensor (equ. (6)).

Solutions for the scalar Φ of 3) in a flat 5D-metric will be used as general ansatz in a 4D-metric. This is considered to be a proof of concept only since it can not be expected that equations derived from classic GR provide an adequate ansatz for particles with spin. The results obtained seem to justify this to be a reasonable approximation. These are among others: electroweak coupling constants as geometric coefficients in 3 and 4 spatial dimensions; a convergent series of particle energies quantized as a function of the fine-structure constant, α, with limits given by electron and the Higgs vacuum expectation value (VEV) energy. The series expansion of the incomplete Γ-function in the energy expression for a point charge will include a term which at short range yields effects that may be associated with strong interaction, at long range gives a quantitative term for gravitational interaction. Additional scalar field terms of 1) may be considered to be a natural candidate for the cosmological constant, Λ, and using the basic coefficients of this model will give a result in the correct order of magnitude.

The equations presented in the following can essentially be expressed "ab initio" i.e. without free parameters.

Three well defined coefficients will enter the exponential of the pivotal function derived below:

- electromagnetic potential, A, in the static approximation of this work the electric potential, $A_{el} \approx e/(4\pi\varepsilon r)$, will be required in the solution for Φ (chpt. 2.2),
- a coefficient, σ, will represent the integration limit necessary to yield angular moment $J_z = \hbar/2$ (2.4, [A5]),
- a coefficient $\alpha_{Pl} = W_e/W_{Pl}$, the ratio of electron and Planck energies, is required by the expansion of the Γ-function in the energy expression (chpt. 2.11, 2.12).

The relation of the masses e, μ, π with α was noted first in 1952 by Nambu [5]. MacGregor calculated particle mass and constituent quark mass as *multiples* of α and related parameters [6].

To focus on the more fundamental relationships some minor aspects of the model are exiled to an appendix, related topics to be marked as [A]. Typical accuracy of the calculations is in the order of 0.001-0.0001 [4]. The difference between calculated and experimental results is consistent with a variation of input parameters related to elementary charge in an order of magnitude of QED corrections which are not included in this model.

2 Calculation

2.1 System of natural units

It is common to define natural electromagnetic units by referring them to the value of the speed of light. The same will be done here, thus subscript c will be used. Retaining SI units for length, time and energy the electromagnetic constants may be defined as:

$$c_0^2 = (\varepsilon_c \, \mu_c)^{-1} \tag{1}$$

with $\varepsilon_c = (2.998\text{E}+8\ [\text{m}^2/\text{Jm}]\ )^{-1} = (2.998\text{E}+8)^{-1}\ [\text{J/m}]$

$\mu_c = (2.998\text{E}+8\ [\text{Jm/s}^2]\ )^{-1} = (2.998\text{E}+8)^{-1}\ [\text{s}^2/\text{Jm}]$

From the Coulomb term $b_0 = e^2/(4\pi\varepsilon_0) = e_c^2/(4\pi\varepsilon_c) = 2.307\text{E-}28$ [Jm] follows for the square of the elementary charge: $e_c^2 = 9.671\text{E-}36$ [J²]. In the following $e_c = 3.110\text{E-}18$ [J] and $e_c/(4\pi\varepsilon_c) = 7.419\text{E-}11$ [m] will be used as natural unit of energy and length.

The constant G/c_0^4 [m/J] in the Einstein field equations (EFE) will be replaced by:

$$(8\pi)\,G/c_0^4 \quad => \quad \approx \quad \frac{1}{\varepsilon_c} \tag{2}$$

4 Including e.g. errors due to the numerical approximation of incomplete Γ-functions.

in an accordingly modified field equation ($T_{\alpha\beta}$ = stress-energy tensor):

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = \frac{1}{\varepsilon_c} T_{\alpha\beta} \tag{3}$$

2.2 Kaluza theory

Kaluza theory is an extension of general relativity to 5D-space-time with a metric given as [4, equ. 2.2]:

$$g_{AB} = \begin{bmatrix} (g_{\alpha\beta} - \kappa^2 \Phi^2 A_\alpha A_\beta) & -\kappa \Phi^2 A_\alpha \\ -\kappa \Phi^2 A_\beta & -\Phi^2 \end{bmatrix} \tag{4}$$

In (4) roman letters correspond to 5D [5], greek letters to 4D. κ^2 corresponds to the constant in the field equation (2), A is the electromagnetic potential. In the context of the electrostatic approximation of this model A will be assumed to be represented by the electric potential, $A_{el} = e_c/(4\pi\varepsilon_c r) = \rho_0/r$ [-] [6]. Assuming 5D-space-time to be flat, i.e. $R_{AB} = 0$, gives for the 4D-part of the field equations [4, equ. 2.3]:

$$G_{\alpha\beta} = \frac{\kappa^2 \Phi^2}{2} T_{\alpha\beta}^{EM} - \frac{1}{\Phi} \left(\nabla_\alpha (\partial_\alpha \Phi) - g_{\alpha\beta} \Box \Phi \right) \tag{5}$$

From $R_{44} = 0$ follows:

$$\Box \Phi = -\frac{\kappa^2 \Phi^3}{4} F_{\alpha\beta} F^{\alpha\beta} \tag{6}$$

In the following only derivations with respect to r of a spherical symmetric coordinate system will be considered. Equation (6) will be used to obtain an ansatz for a metric to get a solution of the 00-component in (3). A function Φ_N

$$\Phi_N \approx \left(\frac{\rho}{r}\right)^{N-1} e^{\nu/2} = \left(\frac{\rho}{r}\right)^{N-1} \exp\left(-\left(\frac{\rho}{2r}\right)^N\right) \quad ^{7} \tag{7}$$

yields solutions for an equation of general type of (6), where the term of highest order of exponential N, given by $\Phi'' \sim \rho^{3N-1}/r^{3N+1}$, may be interpreted to provide the terms for $A'(r) \sim \rho_0/r^2 \sim e_c/(4\pi\varepsilon_c r^2)$, see [A2]:

$$\Phi_N'' \sim \left(\frac{\rho^{3N-1}}{r^{3N+1}}\right) e^{\nu/2} \sim \boldsymbol{\Phi_N^3}\ e^{-\nu} (\boldsymbol{A_0}')^2 \approx \left[\left(\frac{\boldsymbol{\rho}}{\boldsymbol{r}}\right)^{\boldsymbol{N-1}} \boldsymbol{e^{\nu/2}}\right]^{\boldsymbol{3}} e^{-\nu} \left(\frac{\boldsymbol{\rho}}{\boldsymbol{r^2}}\right)^{\boldsymbol{2}} = \left(\frac{\rho}{r}\right)^{3N-3} e^{\nu/2} \left(\frac{\rho}{r^2}\right)^2 \tag{8}$$

The significance of (8) lies in providing the relation of exponential and pre-exponential terms and first of all in the requirement to contain $A \sim (\rho_0/r)$ in the exponent of Φ_N, to be used in the following.

2.3 Example for metric, point charge energy

The following will be used as ansatz for a general metric.

$$g_{\mu\nu} = \left(\frac{\rho_0}{r}\right)^n \exp\left(-a\left(\frac{\rho}{r}\right)^3\right),\ -\left(\frac{\rho_0}{r}\right)^n \exp\left(-b\left(\frac{\rho}{r}\right)^3\right),\ -/+\ r^2,\ -/+\ r^2 \sin^2\vartheta \tag{9}$$

In a simple exponential ansatz for a metric such as given by [4, equ. 6.76f] coefficient n will be zero, n = 0. While this gives reasonable relative values for particle energy, see [A3], this corresponds to a photon interpretation of this model and may not yield much further information.

In the EBV-triple interpretation angular momentum has to be an indispensable property of particles and a description in the point charge picture including angular momentum as additional restraint provides much more information than the simpler photon picture. This path will be followed in the remainder of this work.

In [A4] it will be demonstrated that one may derive appropriate solutions from the EFE with n = 2, corresponding to Φ of (7) with N = 3 [8] and $e^{\nu/2}$ being squared. This is considered a proof of concept only [9].

In general it might be necessary to differentiate between ρ in the exponent and the pre-exponential term. In

5 (ct, r, ϑ, φ, 5th coord.) = (x0, x1, x2, x3, x4)

6 In an electrostatic approximation A_{el} in the metric is dimensionless and the replacement of the G-term of (2) by $1/\varepsilon_c$ does cancel in the term of the EM-stress-energy tensor, leaving E^2 in units of $[1/m^2]$ => $\kappa \approx 1$.

7 In the following any index for ν will be omitted, when necesseray $\varphi_N = e^{\nu/2}$ will be used;

8 Which is supposed to represent 3 spatial dimensions, see chpt. 2.6;

9 The considerations of 2.4 indicate that 1st order derivatives might play a role in an appropriate differential equation.

the following ρ_0 represents the Coulomb term while $\rho \sim \rho_0$ may include additional coefficients, to be discussed below (cf. equ. (22)).
The Einstein tensor component G_{00} will be:

$$G_{00} = \rho_0^2/r^4 e^v \qquad (10)$$

and using equ. (2)f will give:

$$\frac{\rho_0^{\;2}}{r^4} e^v \approx \frac{w}{\varepsilon_c} \quad => \quad \frac{\varepsilon_c \rho_0^{\;2}}{r^4} e^v \approx w \qquad (11)$$

The volume integral over (11) gives the particle energy according to:

$$W_n = \varepsilon_c \rho_0^{\;2} \int_0^{r_n} \frac{e^v}{r^4} d^3 r = 4\pi \varepsilon_c \rho_0^{\;2} \int_0^{r_n} \frac{e^v}{r^2} dr \qquad (12)$$

Solutions for integrals over e^v, with v according to (7), times some function of r can be given by:

$$\int_0^{r_n} \exp\left(-(\rho_n/r)^N\right) r^{-(m+1)} dr = \Gamma\left(m/N, (\rho_n/r_n)^3\right) \frac{\rho_n^{-m}}{N} = \int_{(\rho_n/r_n)^3}^{\infty} t^{\frac{m}{N}-1} e^{-t} dt \frac{\rho_n^{-m}}{N} \qquad (m = N-2) \qquad (13)$$

valid for N = {3; 4}, m = {-2; -1; 0; +1;+2}. The term $\Gamma(m/N, (\rho_n/r_n)^3)$ denotes the upper incomplete gamma function, given by the Euler integral of the second kind [10]. In the range of values relevant in this work, for m ≥ 1 the complete gamma function $\Gamma_{m/N}$ is a sufficient approximation, for m ≤ 0 the integrals have to be integrated numerically, requiring an integration limit, see 2.4.
Equation (12) will give as energy for a particle n:

$$W_{n,elstat} = 4\pi \varepsilon_c \rho_0^{\;2} \int_0^{r_n} \frac{e^v}{r^2} dr = b_0 \Gamma(1/3, (\rho_n/r_n)^3) \rho_n^{-1}/3 \approx b_0 \Gamma_{1/3} \rho_n^{-1}/3 \qquad (14)$$

including the integral for the energy of a point charge term modified by $e^v = \varphi^2$. Particles are supposed to be electromagnetic objects possessing photon-like properties, thus it will be assumed that particle energy has equal contributions of electric and magnetic energy, i.e.

$$W_n = W_{n,elstat} + W_{n,mag} = 2W_{n,elstat} \approx 2 b_0 \Gamma_{1/3} \rho_n^{-1}/3 \qquad (15)$$

2.4 Angular momentum, coefficient σ
The integral limits required for Euler integrals of (13) with m ≤ 0 are r_n („particle radius" of state n) in integrals over e^v and $(\rho_n/r_n)^3$ in the Euler integrals. The latter will be expressed via a constant defined as 8/σ (chosen to give coefficient σ in the exponent of e^v, see (22), [A5]):

$$(\rho_n/r_n)^3 = 8/\sigma \qquad (16)$$

whose value may be derived from the condition for angular momentum J_z = 1/2 [ħ]. A simple relation with angular momentum J_z for spherical symmetric states will be given by applying a semi-classical approach using

$$J_z = r_2 x\, p(r_1) = r_2 W_n(r_1)/c_0 \qquad (17)$$

with $W_{kin,n}$ = 1/2 W_n , using term $2b_0$ of equ. (15) as constant factor, integrating over a circular path of radius $|r_2| = |r_1|$. Equation (13) will give for m = 0:

$$J_z = \int_0^{r_n}\int_0^{2\pi} J_z(r,\varphi) d\varphi\, dr = 4\pi \frac{b_0}{c_0} \int_0^{r_n} e^v r^{-1} dr = 4\pi \alpha\hbar \int_0^{r_n} e^v r^{-1} dr = \frac{4\pi}{3} \alpha\hbar \int_{8/\sigma}^{\infty} t^{-1} e^{-t} dt \equiv 1/2\ [\hbar] \qquad (18)$$

To obtain J_z = 1/2 [ħ] the integral over $e^v r^{-1}$ of (18), *has to* yield $\alpha^{-1}/8\pi$.

$$\int_0^{r_n} e^v r^{-1} dr = 1/3 \int_{8/\sigma}^{\infty} t^{-1} e^{-t} dt \equiv \frac{\alpha^{-1}}{8\pi} \approx 5.45 \qquad (19)$$

Relation (19) may be used for a numerical calculation of the integration limit, 8/σ, giving a value of σ_0 for spherical symmetry as σ_0 = 1.810E+8 [-], to be used in the following.
The existence of such an integration limit implies a differential equation of a general type:

10 Euler integrals yield positive values, the sign convention of Γ-functions gives negative values for negative arguments. In the following the abbreviation $\Gamma_{-1/3}$ will be used for |Γ(-1/3)| etc.

$$-\frac{d^2 e^{v/2}}{dr^2} + \frac{\sigma\rho^3}{2r^4}\frac{de^{v/2}}{dr} - \frac{\rho^3}{2r^5}e^{v/2} = 0 \quad (20)$$ [11]

with a solution

$$e^{v/2} = \exp\left(-\left(\frac{\sigma\rho_n^3}{2r^3} + \left[\left(\frac{\sigma\rho_n^3}{2r^3}\right)^2 - 4\frac{\rho_n^3}{2r^3}\right]^{0.5}\right)/2\right) \quad (21)$$

where

$$\rho_n^3 \approx (\sigma_0\,\alpha(n)\,(e_c/(4\pi\varepsilon_C r))^3) \quad (22)$$

with $\alpha(n)$ being a particle specific coefficient, see 2.7. Within the parameter range of interest here $\varphi_n = e^{v/2} \approx \exp(-(\rho_n^3/(2r^3))$ with ρ_n^3 of equ. (22) may serve as an excellent approximation of (21) [12]. From (21) follows (16).

The model presented here does not consider QED-corrections and it is not obvious how and where to include those. Due to the nonlinearity of the Γ-functions a variation in the order of magnitude of QED-corrections i.e. 1.001 may result in variations of ≈ 1.007 in the values for particle energy and α.

2.5 Photon energy

In the following a term for length expressed via the Euler integral of (13) will be introduced for $\lambda_{C,n}$:

$$\lambda_{C,n} = \int_0^{\lambda_{C,n}} e^{2v}dr = \rho_n/3\int_{(\rho_n/\lambda_{C,n})^3}^{\infty} t^{-4/3}e^{-t}dt \approx \Gamma(-1/3,(\rho_n/\lambda_{C,n})^3)\ \rho_n/3 \quad (23)$$

In the limit $(\rho_x/r_x)^N \to 0$

$$\Gamma(-1/N, (\rho_x/r_x)^N) = \int_{(\rho_x/r_x)^N}^{\infty} t^{-(1/N+1)}e^{-t}dt \approx N\,(\rho_x/r_x)^{-1} \quad (24)$$

holds. Equation (24) inserted in the right side of (23) gives back $\lambda_{C,n}$, however, (23)f may be seen as expressing $\lambda_{C,n}$ in terms useful for this model, i.e. ρ_n, σ_0 and Γ-functions. Using equ. (24) for the incomplete Γ-function and multiplying r_x in the integration limit $(\rho_n/r_x)^3$ by √3, the ratio of total angular momentum and its z-component (see [A5, (69)]), gives in good approximation:

$$\lambda_{C,n} \approx 3^{1.5}\,\sigma_0^{1/3}/2\ \rho_n/3 \approx 36\,\pi^2\,\Gamma_{-1/3}\,\rho_n/3 \quad (25)$$

(last term for easier comparison with terms used below). With (25) energy of a photon can be expressed by:

$$W_{Phot,n} = hc_0/\lambda_{C,n} = hc_0 \Big/ \int^{\lambda_{C,n}} e^{2v}dr = \frac{2hc_0}{3^{0.5}\rho_n\sigma_0^{1/3}} \approx \frac{3hc_0}{36\pi^2\Gamma_{-1/3}\rho_n} \quad (26)$$

2.6 Fine-structure constant, α

The energy of a particle is assumed to be the same in both photon and point charge description. Equating (15) with (26)

$$W_{pc,n} = W_{Phot,n} = 2b_0\,\Gamma_{1/3}\,\rho_n/3 \approx \frac{3hc_0}{2\pi\,18\pi\,\Gamma_{-1/3}\rho_n} \quad (27)$$

and rearranging to emphasize the relationship of α with the gamma functions ($\Gamma_{1/3}$ = 2.679; $\Gamma_{-1/3}$ = 4.062) gives as first approximation (note: h => ħ):

$$\alpha^{-1} = \frac{hc_0}{2\pi b_0} = \frac{4\pi\Gamma_{1/3}\Gamma_{-1/3}}{0.998} \quad (28)$$ [13]

Equ. (28) is based on the integral over the 3-dimensional point charge term modified by the exponential term

11 This might give a hint at a relationship with quantum mechanics. Using the 3rd term with σ -> 1 in (45)f for elements of potential energy and an expansion by $(\hbar c_0)^2\alpha^2/b_0^2$ for the 2nd order term of (20), an approximate differential equation for this model may be given by $-(\alpha^2\hbar c_0)^2 r/b_0\ d^2\varphi(r)/dr^2 + rV(r)\ d\varphi(r)/dr - V(r)\varphi(r) = 0$ that resembles quantum mechanical terms. $V(r) \approx b_0\,\rho^3/(2\,r^4)/\sigma$; $c_0^2 r/b_0$ ~ „1/m"-term;

12 For higher angular terms, l -> ∞, σ will approach ≈ 1 (see 2.8) and (21) will approximate $e^{v/2}$ of equ. (7)ff as well.

13 Alternatively $\alpha = 3^{0.5}2\pi/(\Gamma_{1/3}\sigma_0^{1/3})$ = 1.016 α^{-1}; Varying (18)ff by 1.00116^2 yields ≈ 1.003 α^{-1};

from (7), $\varphi_N^2 = e^v$, with N = 3, and a complementary integral to yield a dimensionless constant. This may be generalized to N dimensions (N ={3; 4}), to give a point charge term (S_N = geometric factor for N-dimensional surface, in case of 3D: 4π):

$$\int_0^r \varphi_N(r)^2 r^{-2(N-1)} \; d^N r \; = \; S_N \int_0^r \varphi_N(r)^2 r^{-(N-1)} dr \tag{29}$$

that has to be multiplied by a complementary integral

$$\int_0^r \varphi_N(r)^2 r^{(N-3)} dr \tag{30}$$

The exact result depends on the integration limit of the second integral, cf. [A6].
In terms of the Γ-functions both electroweak coupling constants can be given as

$$\alpha_N^{-1} \; = \; S_N \frac{\Gamma(+m/N)\Gamma(-m/N)}{m^2} \; = \; S_N \frac{\Gamma(+(N-2)/N)\Gamma(-(N-2)/N)}{(N-2)^2} \qquad \text{(m = N-2, cf. (13))} \tag{31}$$

Dimension - space	coupling constant	Value of *inverse* of coupling constant, α_N^{-1}	
4D	$\alpha_4 = \alpha_{weak}$	$2\pi^2 \Gamma_{+1/2} \Gamma_{-1/2} /4 \; = \; \pi^3 \; =$	31.0
3D	$\alpha_3 = \alpha$	$4\pi \, \Gamma_{+1/3} \Gamma_{-1/3} \; = 4\pi \, \Gamma_{+1/3} \Gamma_{-1/3} =$	136.8

Table 1: Values of electroweak coupling constants [14]

The ratio of α and α_{weak} represents the Weinberg angle, θ_W, and may be expressed as:

$$\sin^2\theta_W \; = \; \frac{\alpha}{\alpha_{weak}} \; = \; \frac{\pi^3}{4\,\Gamma_{1/3}\Gamma_{-1/3}} \; = \; 0.227 \tag{32}$$

(Experimental values: PDG [8]: $\sin^2\theta_W$ = 0.231, CODATA [9]: $\sin^2\theta_W$ = 0.222). The mass ratio of the W- and Z-bosons will be given by $\cos\theta_{W,calc} = (m_W/m_Z)_{calc}$ = 0.879 = 0.998 $(m_W/m_Z)_{exp}$ [10].

2.7 Quantization with powers of $1/3^n$ over α

Most relations given here are valid for any particle energy which should be expected as there is a continuous spectrum of energies according to special relativity. However, a particular set of energies may be identified by relaxing the condition of orthogonality of different states of quantum mechanics to requiring different states to a) be expressible in simple terms of a ground state coefficient, α_0, in the exponent of φ_n and b) to exhibit no dependence on intermediary states.

This may be illustrated best by considering the product of the point charge and the photon expression of energy, (15) and (26):

In a general case ρ_n may be given as product of $\rho_0 = e_c/(4\pi\varepsilon_c)$ [m], σ_0 and a partial product of particle specific dimensionless coefficients, α(n), of succeeding particles representing the ratio ρ_{n+1}^3 / ρ_n^3 in the exponential of φ_n as (α_0 = ground state coefficient):

$$\rho_n^3 \sim \alpha_0 \Pi_{k=1}^{n} \alpha(k) \qquad n = \{1;2;..\} \tag{33}$$

or ρ_n of the energy expression:

$$\rho_n \sim \alpha_0^{1/3} \Pi_{k=1}^{n} \alpha_k^{1/3} \qquad n = \{1;2;..\} \tag{34}$$

Inserting (34) in the product of the point charge and the photon expression of energy, (15) and (26), gives for the square of energy $W_n^2 = W_{pc,n} W_{phot,n}$:

$$W_n^2 \; = \; 2 b_0 h c_0 \frac{\int_{\lambda_{C,n}}^{r_n} e^v r^{-2} dr}{\int e^v \, dr} \; = \; \frac{4\pi b_0^2}{\alpha} \frac{\int_{\lambda_{C,n}}^{r_n} e^v r^{-2} dr}{\int e^v \, dr} \; = \; \frac{4\, b_0^2 \Gamma_{1/3}^2}{9[\alpha \boldsymbol{4\pi\,\Gamma_{1/3}\Gamma_{-1/3}}]\rho_n^2} \; = \; \left(\frac{2 b_0 \Gamma_{1/3}}{3\rho_n}\right)^2 \sim \frac{\alpha_0^{1/3}\alpha_1^{1/3}.....\alpha_n^{1/3}}{\alpha_0\alpha_1....\alpha_n} \tag{35}$$

The last expression of (35) is obtained by expanding the product of $\alpha_k^{2/3}$ included in ρ_n^2 of (35) with the product of $\alpha_k^{1/3}$.

All intermediate particle coefficients cancel out if a relation $\alpha_{n+1} = \alpha_n^{1/3}$ holds [15]:

14 All values and calculations of this work refer to a rest frame.

15 It is condition a) that requires the exponential of 1/3 in the equations. The reasoning of 2.7 implies a second solution

$$W_n^2 \sim \frac{\alpha_0^{1/3}\alpha_0^{1/9}....\alpha_0^{1/(3\wedge n)}\boldsymbol{\alpha_0^{1/(3\wedge(n+1))}}}{\boldsymbol{\alpha_0^1}\alpha_0^{1/3}\alpha_0^{1/9}....\alpha_0^{1/(3\wedge n)}} = \frac{\boldsymbol{\alpha_0^{1/(3\wedge(n+1))}}}{\boldsymbol{\alpha_0}} \tag{36}$$

Identifying α_0 as $\alpha_0 = \alpha^3$, i.e. the cube of the fine structure constant, and comparing with experimental particle data shows that an expression for particle energies can be given using the muon as reference state $W_\mu = W_0$:

$$W_n \sim \left(\frac{\alpha \wedge (1/3^n)}{\alpha^3}\right)^{0.5} = \frac{\alpha \wedge (0.5/3^n)}{\alpha^{1.5}} = \Pi_{k=0}^{n} \alpha \wedge (-1/3^k) \qquad n = \{0;1;2;..\} \tag{37}$$

The ratio W_n/W_μ will be given by the partial product of (37) with n = {1;2;..}.
In chpt. 2.11, 2.12 it will be demonstrated that a 3rd order relationship according to the terms of this work exists between Planck energy and ground state energy, implying the lowest charged particle, the electron, to correspond to a ground state term, however, requiring an ad hoc factor 1.513 ≈ 3/2 in addition to a modified $\alpha_0 = \alpha^9$. With W_e as ground state W_n would be given by (33)ff relative to the electron state as:

$$W_n/W_e \approx 1.513\,\Pi_{k=1}^{n} \alpha \wedge (-3/3^k) \qquad n = \{1;2;..\} \tag{38}$$

for spherical symmetric states, see table 2. Index n will indicate spherical symmetric solutions and serve in the following as equivalent of a radial quantum number. For the angular terms of $\Phi(r, \vartheta, \varphi)$, to be indicated by index l, only rudimentary results exist, their contribution will be assigned to parameter σ. The electron coefficient in the exponential of φ_e^2 and the energy term, equ. (15), would be given as:

$$\rho_e^3 \sim \alpha_e \approx 1.513^3\alpha^9 \approx (3/2)^3\alpha^9 \quad \text{and} \quad W_e \sim \alpha_e^{-1/3} \approx 2/3\ \alpha^{-3} \tag{39}$$

2.8 Upper limit of energy

Above relations refer to a point charge and thus to spherical symmetric states. Non-spherical particle states should exhibit lower values of σ [16]. The minimal possible value for σ is defined by the Γ-term in the integral expression for length, (23)f, and factor 8 in (16) to be:

$$\sigma_{min} = (2\,\Gamma_{-1/3}/3)^3 \tag{40}$$

leaving a term

$$\alpha_{l,\,max} \approx (1.513\,\alpha^{-1})^{-3} \approx 8/27\,\alpha^3 \tag{41}$$

as variable part in ρ_n^3 (see [A5]). The maximum angular contribution to W_{max} would be:

$$\Delta W_{max,\,angular} = 1.513\ \alpha^{-1} \approx 3/2\ \alpha^{-1} \tag{42}$$

According to (38) and (42), the maximum energy will be $W_{max} \approx W_e\ 9/4\ \alpha^{-2.5}$ = 4.05E-8 [J] (=1.03 Higgs vacuum expectation value, VEV = 246GeV = 3.941E-8 [J] [11]).
In the visualization of the EBV-triple sketched in the introduction the “rotating E-vector” might be interpreted to cover the whole angular range in the case of spherical symmetric states while an object with one angular node, as represented by the spherical harmonic y_1^0 or an atomic p-orbital, might be interpreted as forming a double cone. Increasing the number of angular nodes would close the angle of the cone leaving in the angular limit case, l -> ∞, a state of minimal angular extension representing the original vector, however, extending in both directions from the origin and featuring parity p= -1. Considering only „half“ such a state, extending in one direction only and having p = +1, would feature an energy of 1.01 W_{Higgs}, the energy value of the Higgs boson.

2.9 Other non-spherical symmetric states

Except for the limit case of 2.8 angular solutions for particle states are not known yet and to extend the model to such states assumptions have to be made.
Assuming the angular part to be related to spherical harmonics and exhibiting the corresponding nodes would give the analog of an atomic p-state for the 1st angular state, y_1^0. With the additional assumption that $W_{n,l} \sim 1/r_{n,l} \sim 1/V_{n,l}^{1/3}$ ($V_{n,l}$ = volume) is applicable for non-spherically symmetric states as well, this would give $W_1^0/W_0^0 = 3^{1/3} = 1.44$. A second partial product series of energies in addition to (38) corresponding to these values approximately fits the data, see tab. 2.
A change in angular momentum has to be expected for a transition from spherical symmetric states, y_0^0, to

given by $\alpha_{n+1} = \alpha_n^3$. Using the parameters of this work will result in values corresponding to α_0 (electron), $\alpha_0\ \alpha_0^3$ (≈Planck energy), $\alpha_0\ \alpha_0^3\ \alpha_0^9$ (≈ 1.4E+72[J]),

16 According to the geometric interpretation of [A5] as well as higher energy $W_{n,l}$ requiring lower $\rho_{n,l}$.

y_1^0 which is actually observed with ΔJ = ± 1 except for the pair μ/π with Δ J = 1/2.
The final version for the energy term according to (15), (22), (38)f and (49) would be:

$$W_n \approx 2b_0 \frac{\Gamma_{1/3}}{3} \rho_n^{-1} \approx 2b_0 \frac{\Gamma_{1/3}}{3} \left[1.513^{3\delta_{0,n}} \sigma_0 \alpha_{Pl} \alpha(n) \alpha(l) \left(\frac{e_c}{4\pi\varepsilon_c} \right)^3 \right]^{-1/3} \approx$$
$$2b_0 \frac{\Gamma_{1/3}}{3} \left[\mathbf{1.513^{3\delta_{0,n}}} \sigma_0 \frac{\alpha_{l,max}^{1/3}}{2} \; 1.513^3 \; \mathbf{\Pi_{k=0}^{n}} \boldsymbol{\alpha \wedge (9/3^k) \alpha(l)} \left(\frac{e_c}{4\pi\varepsilon_c} \right)^3 \right]^{-1/3} \qquad n = \{0;1;2;..\} \qquad (43)$$

(1.513^δ = extra coefficient for the electron only; α_{Pl} ratio of electron and Planck energy, comprising $\alpha_{l,max}$ and electron coefficient α_e, see 2.12; α(n) particle coefficient, excluding electron (n=0) (38)f; α(l) = angular coefficient, $\alpha(y_0) = 1$, $\alpha(y_1) = 1/3$, $\alpha(y_\infty) = \alpha_{l,max}$ (41);)
Expressing σ_0 by (67) W_e may be approximated as (1.513 ≈ 3/2):

$$W_e \approx 2b_0 \frac{\Gamma_{1/3}}{3} \left[\left(\frac{3}{2} \right)^3 \sigma_0 \alpha_{Pl} \left(\frac{e_c}{4\pi\varepsilon_c} \right)^3 \right]^{-1/3} \approx \frac{2e_c^2}{4\pi\varepsilon_c} \frac{\Gamma_{1/3}}{3} \left[\left(\frac{3}{2} \right)^5 8 \left(\frac{4\pi\Gamma_{-1/3}^3}{3} \right)^3 \frac{\alpha^{10}}{2} \left(\frac{e_c}{4\pi\varepsilon_c} \right)^3 \right]^{-1/3} \approx e_c \left[\frac{\Gamma_{1/3} \alpha^{-10/3}}{\pi \Gamma_{-1/3}^3 3^{5/3}} \right] \qquad (44)$$

2.10 Results of energy calculation

	n, l	$W_{n,Lit}$ [MeV]	α-coefficient in W_n $\alpha(n)^{-1/3}$ $[\alpha(l)^{-1/3}]$	W_{calc}/W_{Lit}	J	r_n [fm]
Planck	(-1,∞)	1.0 E+21*	$2\,(2/3\,\alpha^{-3})^3\,[3/2\,\alpha^{-1}]$ source term, relative to W_e !	0.9994 rel. to e !	-	-
e^{+-}	**0, 0**	**0.51**	**$2/3\,\alpha^{-3}$**	**1.005**	**1/2**	**1412**
μ^{+-}	**1, 0**	**105.66**	**$\alpha^{-3}\alpha^{-1}$**	**1.008**	**1/2**	**6.83**
π^{+-}	1, 1	139.57	$\alpha^{-3}\alpha^{-1}\,[3^{1/3}]$	1.100	0	4.74
K		495	see [A7]		0	
η^0	**2, 0**	**547.86**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}$**	**1.001**	**0**	**1.32**
ρ^0	2, 1	775.26	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3})\,[3^{1/3}]$	1.020	1	0.92
ω^0	2, 1	782.65	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3})\,[3^{1/3}]$	1.010	1	0.92
K*		894			1	
p^{+-}	**3, 0**	**938.27**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$**	**1.009**	**1/2**	**0.76**
n	**3, 0**	**939.57**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$**	**1.008**	**1/2**	**0.76**
η'		958	see [A7]		0	
Φ^0		1019	see [A7]		1	
Λ^0	**4, 0**	**1115.68**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}$**	**1.018**	**1/2**	**0.63**
Σ^0	**5, 0**	**1192.62**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81}$**	**1.012**	**1/2**	**0.61**
Δ	**∞, 0**	**1232.00**	**$\alpha^{-9/2}$**	**1.010**	**3/2**	**0.59**
Ξ		1318			1/2	
Σ^{*0}	3, 1	1383.70	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9})\,[3^{1/3}]$	0.987	3/2	0.53
Ω^-	4, 1	1672.45	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27})\,[3^{1/3}]$	0.980	3/2	0.45
N(1720)	5, 1	1720.00	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81})\,[3^{1/3}]$	1.012	3/2	0.43
tau^{+-}	∞, 1	1776.82	$(\alpha^{-9/2})\,[3^{1/3}]$	1.010	1/2	0.40
Higgs	∞,∞ **	1.25 E+5	$(\alpha^{-9/2})\,[3/2\,\alpha^{-1}]\,/2$	1.031	0	0.006
VEV	∞,∞ **	2.46 E+5	$(\alpha^{-9/2})\,[3/2\,\alpha^{-1}]$	1.04	0	0.003

Table 2: Particle energies; col.2: radial, angular quantum number, ** see 2.8; col.3: energy values of [11] except* (see (48)); col. 4: α-coefficient in W_n, (43); col.5: ratio of calculated energy, W_{calc} and literature value [11]; col.6: angular momentum J_z [ħ]; col.7: particle radius r_n [fm];

Table 2 presents the results of the energy calculation according to (43) for $\mathbf{y_0^0}$ **(bold)**, y_1^0. Only states given in [11] as 4-star, characterized as „*Existence is certain, and properties are at least fairly well explored*", are included, up to Σ'^0 all states given in [11] are listed. Coefficients given in col. 4 refer to the bold terms of

(43), including extra term of electron, 1.513 approximated by 3/2. Exponents of -9/2 for Δ and tauon are equal to the limit of the partial product of α(n), including the electron. "Particle radius", r_n, is calculated with equ. (16). For σ_0 the value calculated for J_z = 1/2 from chpt. 2.4 is used. Blanks in the table are discussed in [A7]. The values of physical constants are taken from [11].
To illustrate possible QED-Effects, a calculation of σ_0 with values of (18)f varying within +/-1.00116 gives a range of energy values of +/-1.006, varying within +/-1.00116^2 gives a range of energy values of +/-1.013 compared to the values given in table 2. Additional effects due to e.g. different charge in the nucleons may be expected.

The accuracy of ~1% of the values calculated for leptons, mesons and baryons is comparable to that for baryons in lattice-QED calculations, while for mesons the accuracy of the latter is >> 1% [12].

2.11 Expansion of the incomplete gamma function $\Gamma(1/3, (\rho_n/r)^3)$, „strong" force

The series expansion of $\Gamma(1/3,(\rho_n/r_n)^3)$ in the equation for calculating particle energy (14)f gives [13]:

$$\Gamma(1/3,\ (\rho_n/r)^3) \approx \Gamma_{1/3} - 3\left(\frac{\rho_n}{r}\right) + \frac{3}{4}\left(\frac{\rho_n}{r}\right)^4 + \frac{3}{7}\left(\frac{\rho_n}{r}\right)^7 + \ldots \qquad (45)$$

and for $W_n(r)$:

$$W_n(r) \approx W_n - 2b_0\frac{3\rho_n}{3\rho_n r} + 2b_0\frac{3}{4}\frac{\rho_n^4}{3\rho_n r^4} + 2b_0\frac{3}{7}\frac{\rho_n^7}{3\rho_n r^7} = W_n - \frac{2b_0}{r} + b_0\frac{\rho_n^3}{2r^4} + b_0\frac{\rho_n^6}{7r^7} + \ldots \qquad (46)$$

The 2nd term in (46) drops the particle specific factor ρ_n and gives twice [17] the electrostatic energy of two elementary charges at distance r. The 3rd term might be an appropriate choice for a potential energy term in the differential equation (cf. 2.4) as. As such it is supposed to be responsible for the localized character of a particle state and may play a similar role as the "strong force" of the standard model, an interaction observable e.g. in particle scattering.

In the standard model the leptons are distinguished from the hadrons by not being subject to the strong interaction and the tauon with its mass beyond the proton stands out as being grouped with the electron and the muon. According to this model it is suggestive to interpret strong interaction as evidenced in scattering events to be due to overlap of a wave function-like φ_n. Such an overlap should depend on: 1) comparable size and energy of wave functions, 2) sufficient *net* overlap. Condition 1) prevents neutrino or electron to exhibit effective interaction with hadrons, condition 2) prevents interaction of the tauon which is at the very end of the partial product series for y_1^0 and should exhibit a high, potentially infinite number of radial nodes, separating densely spaced volume elements of alternating wave function sign [18].

2.12 Gravitation

2.12.1 Planck scale

Gravitational effects may be recovered via the series expansion of chpt. 2.11, implying that the Coulomb term b_0 will be part of the expression for F_G, i.e. the ratio between gravitational and Coulomb force, e.g. for the electron, $F_{Gr,e}/F_{Co,e}$ = 2.41E-43, should be a completely separate, self-contained term. This is equivalent to assume that gravitational interaction is a higher order effect with respect to electromagnetic interaction and as such should be of less or equal strength compared to the latter. This suggests to use the expression

$$b_0 = G\, m_{Pl}^2 = G\, W_{Pl}^2/c_0^4 \qquad (47)$$

as definition for Planck terms, giving for the Planck energy, W_{Pl}:

$$W_{Pl} = c_0^2\,(b_0/G)^{0.5} = c_0^2\,(\alpha\hbar c_0/G)^{0.5} = 1.671\text{ E+8 [J]} \qquad (48)$$

The value of W_{Pl} according to definition (48) allows to give the ratio of W_e and W_{Pl} as (cf. (39), (41)f):

$$1.0003\frac{W_e}{W_{Pl}} = \frac{\alpha_{l,max}^{1/3}\alpha_e}{2} = 1.513^2\alpha^{10}/2 = 4.903\text{ E-22} = \alpha_{Pl} \qquad (49)$$

i.e. the relation between the electrostatic part of W_e, $W_{e,\,elst} = W_e/2$ and the electrostatically defined W_{Pl} [19] is

17 Due to adding up the electromagnetic contributions in (15): $W_n = 2W_{n,el} = 2W_{n,mag} = W_{n,el} + W_{n,mag}$
18 As for energy density ~ W_m/W_n^4: e/p ~ E-13, μ/p ~ 6E-4; μ/π ~ 1/3, with r of (16), i.e. in case of μ/π some distinctive effect could be expected; different symmetry may play an additional role.
19 Equivalent to doubling the value of W_{Pl} according to relation (15).

given by α_e, the electron coefficient in the exponent of φ_e, (39), corresponding to an extension of relation (38) for spherical symmetric states beyond the electron, times the angular limit factor according to (41)f. In the next chapter the relation of this term with the third term of the energy expansion (46) will be discussed.
The constant G may be given as

$$G = \frac{\alpha_{Pl}^2 c_0^4 b_0}{W_e^2} \tag{50}$$

i.e. since W_e may be expressed as function of π, $\Gamma_{1/3}$, $\Gamma_{-1/3}$ and e_c only ((28) and (44)) G may be expressed as a coefficient based on electromagnetic constants.

2.12.2 Virtual states

Equ. (47)ff requires gravitational potential to be a factor of α_0 smaller than electrical potential for the electron. Angular momentum and therefore σ should have no significance for the states considered below, except for maybe the coefficient $\Gamma_{-1/3}/3$ that would have to appear in expressions for r as well. This will be neglected in the following [20]. The 3rd term in (46) will give the gravitational potential of the electron at $R \approx e_c/(4\pi\varepsilon_c)$, i.e. at the length scale considered to be a natural unit for length the correct relationship between electromagnetism and gravitation will be met.

$$W_3(e) \approx \frac{b_0 \rho_e^3}{2R^4} \approx \frac{b_0 \alpha_{Pl} (e_c/(4\pi\varepsilon_c))^3}{(e_c/(4\pi\varepsilon_c))^3 (e_c/(4\pi\varepsilon_c))} \approx \frac{b_0 \alpha_{Pl}}{e_c/(4\pi\varepsilon_c)} = e_c \alpha_{Pl} \tag{51}$$

What about the r^{-1} dependance of gravitation?
Within this model particles of energy W_n are not expected to actually exhibit something like a rigid radius. Even without relying on the uncertainty principle „virtual" particle states of energy $W_{VS} < W_n$ should be possible, extending in space up to a distance r_{VS}. Any such state adhering to the relations of this model would exhibit an according α(VS) in ρ_{VS}^3 and $r_{VS}^3 \approx \rho_{VS}^3 \approx \alpha(VS)$ i.e. these terms would cancel in (51) giving

$$W_3(e) \approx \frac{b_0 \alpha_{Pl} \alpha(VS) (e_c/(4\pi\varepsilon_c))^3}{\alpha(VS) (e_c/(4\pi\varepsilon_c))^3 ((\alpha(VS)^{1/3} e_c/(4\pi\varepsilon_c)))} \approx \frac{b_0 \alpha_{Pl}}{\alpha(VS)^{1/3} e_c/(4\pi\varepsilon_c)} = \frac{e_c \alpha_{Pl}}{R(VS)} \quad ^{21} \tag{52}$$

Equation (52) is a representation of the gravitational potential of the electron (while the 4th term of (46) would give the gravitational potential energy of two electrons). Terms for other particles may be obtained by inserting their energy values relative to the electron according to (38)f in (52) which might be interpreted as the intensity/frequency of the emergence of virtual states being proportional to the energy of the primary particle.
As a consequence of (52) the highest possible particle energy value will be α_{Pl}^{-1}, i.e. the value of the Planck energy relative to the electron, consistent with the energy relations of 2.7, 2.8 and the assumption used in the definition of equ. (47)f.

2.13 Cosmological constant Λ

The 2nd term on the right side of the full 5D equation (5), $\sim 1/\Phi\left(\nabla_\alpha(\partial_\alpha \Phi) - g_{\alpha\beta}\Box\Phi\right)$, might be considered to be a natural candidate for the cosmological constant term, $g_{\alpha\beta}\Lambda$. Its exact expression will depend on the complete 5D metric used. Nevertheless it will have to contain terms of type $g_{\alpha\beta}\Phi''/\Phi$ such as ρ_n^3/r^5 of G_{00} in [A3], [A4]. Referring the resulting expression to the natural unit of length used in this work, i.e. $R = e_c/(4\pi\varepsilon_c)$, will yield approximate values in the order of magnitude of critical, vacuum density, ρ_c, ρ_{vac} and of Λ. As in 2.12.2 the symmetry coefficient σ, related to angular momentum, will be dropped to give an expression of α_{Pl} and $R \approx e_c/(4\pi\varepsilon_c)$ only:

$$\frac{\Phi''}{\Phi} \approx \frac{\rho^3}{r^5} \approx \frac{\alpha_{Pl}}{(e_c/(4\pi\varepsilon_c))^5}\left(\frac{e_c}{4\pi\varepsilon_c}\right)^3 = \alpha_{Pl}\left(\frac{4\pi\varepsilon_c}{e_c}\right)^2 = 0.089\ [\mathrm{m}^{-2}] \tag{53}$$

Multiplied by ε_c this gives an energy density of 2.97E-10 [J/m^3].
Multiplied by the conversion factor for the electromagnetic and gravitational equations, equ. (2), $8\pi\varepsilon_c G/c_0^4$

20 i.e. approximating the angular limit of (40)ff; cf. the imaginary solution of (21) is independent of σ;
21 The term for gravitational attraction, $F_{m,n;R}$ between two particles, m and n at a distance $R_{m,n}$, would be obtained by using $1/b_0$ as proportionality constant: $F_{m,n;R} \approx W_{VS(m,r)} W_{VS(n,r)}/b_0 \approx b_0\ \alpha_0^2\ \alpha(n)\alpha(m) R_{m,n}^{-2}$

(53) gives as a rough estimate for Λ:

$$\alpha_{Pl} \frac{(4\pi)^2 \varepsilon_c^3}{e_c^2} \frac{8\pi G}{c_0^4} \approx \quad 6.17E\text{-}53\ [m^{-2}] \quad (54)$$ [22]

3 Discussion

The model presented above is far from being complete, however, it is quite minimalistic in its assumptions and parameters. It starts out from 5D-GR according to Kaluza plus the additional condition for particles to exhibit spin or be composed of spin-posessing components. Conventional GR does not accommodate for this, preventing a closed solution. In any case solutions for Φ have to be part of a 5D-metric and seem to be of general use.

This model works without free parameters, the natural constants needed are those of electrodynamics, specifically c_0, its constituent ε_c and elementary charge. Using only electromagnetic constants in the field equations is not only the simplest approach of a Kaluza-like model but justified by the possibility to recover the effects of gravitation and thus the original EFE via the series expansion of the energy term. While in the gravitational term of (51) the coefficient α_0 has a consistent base in all aspects of this model, the use of the Coulomb coefficient as radius of reference is admittedly a target-oriented choice though attractive in its simplicity and supported by giving a correct order of magnitude for the cosmological constant with the same approach.

It is a common thought that GR somehow has to be unified with quantum mechanics (QM). This model suggests that a combination of GR and EM as pioneered by Kaluza might be essentially sufficient to cover QM itself. Features of quantum mechanics that emerge from such an ansatz include quantization of energy, wave-character of particles and non-locality (cf. 2.12.2). Last not least the pivotal constant of quantum mechanics, Plancks constant, h, may be derived from the electromagnetic constants e_c, ε_c and geometry as expressed in α.

Formally in GR, EM and QM differential equations of 2^{nd} order are sufficient for the mathematical description and some congruence of GR with QM, such as the Klein-Gordon equation, is elaborated on in [4]. QM might be considered to be an effective theory for phenomena beyond the particle itself. Mass is a parameter replacing the integral over energy density of a particle, a QM wave function ψ represents effects of a wave based on EM and Φ in 5D-space-time.

As for the quantum field theory extensions of QM, QED corrections are a very likely amendment of this model.

Concerning electroweak interaction, weak interaction has no place in the series expansion of energy derived from the Γ-function. Yet there seems to be an intricate relationship of electroweak interaction with the EBV-triple interpretation of this model. A rotating orthogonal vector triple of E, B and V with the E-vector constantly oriented to a point of origin implies charge, a rest frame (mass), intrinsic chirality (left- and right-handeness of E, B, V), i.e. properties pertaining to electroweak theory and Higgs-mechanism. In particular rotation establishes a direct relationship with the SO(3) / SU(2) representation of weak interaction [23].

The prominent position of the energy levels corresponding to Higgs boson and vacuum expectation energy at the upper end of the energy series is startling. The upper limit of the particle states considered here would be reduced to a one-dimensional object as far as the E-field is concerned, while all other states might be figured as originating from an average over a trajectory of such a state, maybe giving it some significance beyond a mere upper limit [24].

This model obviously relies mainly on a function of radial coordinates of a spherical coordinate system. The lack of detailed solutions for angular components and thus of information about structure and symmetry is a crucial obstacle to further investigate a connection of weak isospin and hypercharge with states of this model. The possibility to derive both electroweak coupling constants in different dimension indicates that a simple 3D-rotational picture alone will not be sufficient to map this model on electroweak theory.

While by and large it seems that the concepts of electroweak theory may fit quite well especially to the EBV-triple aspects of this model, a congruence with quantum chromodynamics (QCD) and the properties of

22 Λ ~ 1.11E -52 $[m^{-2}]$ with Hubble constant H_0 = 67.66 [km/s/Mpc] [14]

23 Spin ½, 1etc. may be modeled by attributing differing phase to E, B and V in a quaternion representation, see [15];

24 It has been speculated about a relationship of the Higgs-field with the 5th coordinate and the scalar Φ in [4].

quarks is far from obvious. Supposed differences in orientation of E-vector, chirality, spin etc. of volume elements of particle states might be a source for emulating quark properties yet without more detailed structural information one is confined to speculation ([A8] gives some, concerning fractal charges 1/3, 2/3).

At this point this may leave the tauon as an irresolvable contradiction to the SM though not to experimental evidence. The similar scattering behaviour of leptons may be explained within this model, see 2.11. The distinctive interaction with neutrinos might in turn be based on the absence of the effects of „strong‟ interaction.

Neutrinos themselves are a blind spot of this model. As mentioned in the electroweak interaction section, a multi-dimensional approach seems to be needed to address this problem.

Conclusion

The model presented here provides a coherent, quantitative and parameter-free formalism based on Kaluza and including spin. It reduces the particle zoo to one elementary particle, the 5D-photon, covers effects of the „4 forces‟ gravitation, electromagnetism, „strong‟ and „weak‟ force and it connects phenomena ranging from particle to cosmological scale.

In particular it yields

- a single expression for the values of the electroweak coupling constants,
- a single energy expression comprising particle energy, electromagnetic and gravitational interaction,
- a convergent series of particle energies quantized as a function of the fine-structure constant, α, with electron and the Higgs VEV energy as lower and upper limit,
- a term for the cosmological constant, Λ, in the correct order of magnitude.

The model does not only work without free parameters but allows to reduce the conventional set of fundamental constants from:

e, c_0, h, G, α, α_{weak}, energy of elementary particles

to:

e, c_0.

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Appendix

[A1] Rotating orthogonal vector triple of E, B and V – EBV-triple

Defining particles as being based on an orthogonal vector triple of E, B and V of the propagation velocity [25] rotating in 3D with the E-vector constantly oriented to a fixed point in a local coordinate system (EBV-Triple) has the following consequences [26]:

1) The orientation of the E-vector allows 2 charges, switching orientation in equal volume or time elements allows neutral particles;

2) A center of rotation defines a rest frame; energy in a rest frame = mass;

3) Lateral extension of the E-field would imply infinitely growing energy density and curvature of space for r -> 0;

4) Rotating EBV-triples may be modeled by quaternions representing the E-vector - with 3 phases for E, B, V – establishing a relationship to SO(3), SU(2) - symmetry of electroweak theory;

5) Simple rotations of such an EBV-triple with different angular frequency of E and B, relative to V, $n\,\omega_E = n\,\omega_B = \omega_V$, yields in-phase solutions for n = 2, i.e. S =1/2, with an effective field strength of the E, B-vectors of 2/3, maybe related to this factor appearing in electron energy and magnetic moments (quaternion calculation, [15]);

6) Chirality, Helicity, Spin

Geometry allows for 2 different chiral orientations of the EBV-triple (right- left-handed), each with 2 different spin orientations. This has the following consequences:

- Chirality is an absolute attribute independent of mass or reference frame.
- 2 chiral orientations times 2 spin orientations gives 4 possible states. Within this model a meson consists of two EBV-triple components of opposite partial charge (assumed in the following toy model to be of 1/2). Thus there are 4 x 4, i.e. 16 distinctive basic meson states, half of them with spin $J_z = 0$, pseudoscalar mesons, half with spin $J_z = 1$, vector mesons. In a naive interpretation of electron-positron annihilation results with vector mesons as intermediates and partial charge of 1/2 the sum of charges in the scattering, Σz_i^2, would give 8 vector mesons with 2 contributions of squared charge $(1/2)^2$, i.e. $\Sigma z_i^2 = 4$, in very good agreement with the experimental results in the energy range beyond the ϒ-meson [16].
- If each particle (-component) has a well defined chirality, there would exist a forbidden transition for different chiral particles, being a possible explanation for the unobserved transition μ -> e + γ or the stability of the proton;
- Phenomena such as "handedness" in electromagnetism, the "chirality" of the weak interaction and matter-antimatter asymmetry might be based on a general preference for one chiral set of states.

[A2] Scalar potential Φ

The solutions for the scalar Φ depend on the complete metric used. The main problem to obtain $R_{44} = 0$ is to eliminate the terms of lowest order in ρ, which lack coefficients in their terms enabling an easy cancellation of them. The easiest method to get a solution of order N is to use spherical coordinates of dimension N+1. Using e.g. the line element (6.76) for a 4D metric of [4, equ. 6.76]

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - e^{\mu} r^2 (d\vartheta^2 + \sin^2\vartheta\, d\varphi^2) \quad (55)$$

and $A_\alpha = (A_{el}, 0, 0, 0)$ gives as solution for equation (8) (cf [4, equ. 6.77], prime corresponds to derivation with respect to r).

$$\Phi'' + \left(\frac{\nu' - \lambda' + 2\mu'}{2} + \frac{2}{r}\right)\Phi' - \frac{1}{2}\Phi^3 e^{-\nu} (A_{el}')^2 = 0 \quad (56)$$

and can be solved with function (7) for N = 2 , giving:

$$\Phi_2' = \left[-\left(\frac{\rho}{r^2}\right)e^{\nu} + \left(\frac{\rho^3}{r^4}\right)\right]e^{\nu} \quad (57)$$

and

$$\Phi_2'' = \left[2\left(\frac{\rho}{r^3}\right) - 4\left(\frac{\rho^3}{r^5}\right) + 2\left(\frac{\rho^5}{r^7}\right)\right]e^{\nu} \quad (58)$$

The ρ^1 terms cancel in (56), the ρ^3 terms can be eliminated by appropriate choice of v', λ' and μ', a remaining factor in the ρ^5 term has to be compensated by assuming a corresponding factor in A_{el}. For N = 3 hyperspherical coordinates with the line element

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - e^{\mu} r^2 (d\psi^2 + \sin^2\psi (d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)) \quad (59)$$

25 Orthogonal spatial „Dreibein"

26 This is a simplified picture neglecting the role of 4th and 5th dimension that might have an influence in attributing states of this model relative to states of the SM.

may be used. A more complex metric of the kind used in [A3], [A4] may be used as well to solve equation (8). [27]

[A3] Metric / simple

The following uses the metric of (9) with n= 0, i.e.

$$g_{\mu\nu} = \exp\left(-a\left(\frac{\rho}{r}\right)^3\right), \; -\exp\left(-b\left(\frac{\rho}{r}\right)^3\right), \; -r^2, \; -r^2\sin^2\vartheta \tag{60}$$

$\Gamma_{01}^{0} = \Gamma_{10}^{0}$	$= + 3/2\ a\ \rho^3/r^4$	$\Gamma_{00}^{1} = + 3/2\ a\ \rho^3/r^4 e^{(a-b)v}$
Γ_{11}^{1}	$= + 3/2\ b\ \rho^3/r^4$	
$\Gamma_{12}^{2} = \Gamma_{21}^{2} = \Gamma_{13}^{3} = \Gamma_{31}^{3}$	$= + 1/r^1$	$\Gamma_{22}^{1} = -r\ e^{-bv} = \Gamma_{33}^{1}/\sin^2\vartheta$
$\Gamma_{23}^{3} = \Gamma_{32}^{3} = \cot\vartheta$		$\Gamma_{33}^{2} = -\sin\vartheta\cos\vartheta$

$R_{00} = e^{(a-b)v}\,[(+ 6\ a\ \rho^3/r^5 - 9/2\ a\ (a-b)\ \rho^6/r^8) + 2(\Gamma_{01}^{0}\,\Gamma_{00}^{1}) - \Gamma_{00}^{1}\,(\Gamma_{10}^{0} + \Gamma_{11}^{1} + 2\Gamma_{12}^{2})]$
$= e^{(a-b)v}\,[+ 6a\ \rho^3/r^5 - 9/2a(a-b)\ \rho^6/r^8 - \Gamma_{00}^{1}\,(-\Gamma_{10}^{0} + \Gamma_{11}^{1} + 2\Gamma_{12}^{2})]$
$= e^{(a-b)v}\,[+ 6a\ \rho^3/r^5 - 9/2a(a-b)\ \rho^6/r^8 + (- 3/2\ a\ \rho^3/r^4)\,(+2/r^1)]$
$= e^{(a-b)v}\,[+ 6a\ \rho^3/r^5 - 9/2\ a(a-b)\ \rho^6/r^8 - 3a\rho^3/r^5]$
$R_{00} = e^{(a-b)v}\,[+ 3a\rho^3/r^5 - 9/2a(a-b)\rho^6/r^8]$
$R_{11} = [- 6a\ \rho^3/r^5 - 6b\ \rho^3/r^5 - 2/r^2 + 6b\rho^3/r^5 + \Gamma_{10}^{0}\,\Gamma_{01}^{0} + \Gamma_{11}^{1}\Gamma_{11}^{1} + 2\Gamma_{12}^{2}\,\Gamma_{21}^{2} - \Gamma_{11}^{1}\,(\Gamma_{10}^{0} + \Gamma_{11}^{1} + 2\Gamma_{12}^{2})]$
$= [-2/r^2 - 6\ a\ \rho^3/r^5 + \Gamma_{10}^{0}\,\Gamma_{01}^{0} + 2\Gamma_{12}^{2}\,\Gamma_{21}^{2} - \Gamma_{11}^{1}\,(\Gamma_{10}^{0} + 2\Gamma_{12}^{2})]$
$= [-2/r^2 - 6a\ \rho^3/r^5 + 9/4\ a^2\ \rho^6/r^8 + 2/r^2 + (- 3/2\ b\ \rho^3/r^4)\,(+2/r^1 + 3/2\ a\ \rho^3/r^4]$
$= [- 6a\ \rho^3/r^5 + 9/4\ a^2\ \rho^6/r^8 - 9/4\ ab\ \rho^6/r^8]$
$R_{11} = [- 6a\ \rho^3/r^5 + 9/4(+a^2 - ab)\rho^6/r^8]$
$R_{22} = - 1 + e^{-bv}\,[- 3b\ \rho^3/r^3 + 2(\Gamma_{21}^{2}\,\Gamma_{22}^{1}) - \Gamma_{22}^{1}\,(\Gamma_{10}^{0} + \Gamma_{11}^{1} + \Gamma_{12}^{2} + \Gamma_{13}^{3})]$
$= - 1 + e^{-bv}\,[- 3b\ \rho^3/r^3 - \Gamma_{22}^{1}\,(\Gamma_{10}^{0} + \Gamma_{11}^{1} - \Gamma_{12}^{2} + \Gamma_{13}^{3})]$
$= - 1 + e^{-bv}\,[- 3b\ \rho^3/r^3 + r^1\,(+ 3/2(a + b)\rho^3/r^3]$
$= - 1 + e^{-bv}\,[- 3b\ \rho^3/r^3 + 3/2(a + b)\rho^3/r^3$
$R_{22} = - 1 + e^{-bv}\,[+3/2(+a - b)\rho^3/r^3]$
$g^{00}R_{00} = e^{-bv}\,[+ 3a\rho^3/r^5 - 9/2(a^2 - ab)\rho^6/r^8]$
$g^{11}R_{11} = - e^{-bv}\,[a- 6a\rho^3/r^5 + 9/4(+a^2 - ab)\rho^6/r^8]$
$g^{22}R_{22} + g^{33}R_{33} = + 2/r^2 + e^{-bv}\,[(- 3(+a - b)\rho^3/r^5]$

The solutions for R will be:

$R = + 2/r^2 + e^{-bv}\,[(+ 6a + 3b)\ \rho^3/r^5 - 27/4(+a^2 - 3b)\ \rho^6/r^8]$

G_{00} will be:

$G_{00} = e^{(a-b)v}\,[+ 3a\rho^3/r^5 - 18/4a(a-b)\rho^6/r^8] - e^{av}/r^2 - e^{(a-b)v}\,[(+ 3a + 3/2b)\ \rho^3/r^5 - 27/8(+ a^2 - ab)\rho^6/r^8]$
$= - e^{av}/r^2 + e^{(a-b)v}\,[- 3/4b\ \rho^3/r^5 - 9/8(+ a^2 - ab)\rho^6/r^8]$

All terms ρ^n/r^{n+2} will give the same type of results in integrals $\int e^v\ \rho^n/r^{n+2}\ d^3r \approx \int e^v\ \rho^n/r^n\ dr \approx \rho$ [28]. For illustration purpose only the first term with n = 0, a = 1 will be chosen:

$G_{00} \approx - e^v/r^2$

According to chpt 2.3 this will result in:

$$-\frac{e^v}{r^2} \approx -\frac{w}{\varepsilon_c} \quad => \quad \frac{\varepsilon_c\, e^v}{r^2} \approx w \tag{61}$$

The volume integral over (61) gives the particle energy according to (using (13) with m = -1; σ neglected):

$$W_n = \varepsilon_c \int_0^{r_n} \frac{e^v}{r^2}\, d^3r = 4\pi\varepsilon_c \int_0^{r_n} e^v\, dr = 4\pi\varepsilon_c \Gamma_{-1/3}/3\ \rho_n = \Gamma_{-1/3}/3\ e_c\ \alpha_0^{*\,1/3}\alpha(n)^{*\,1/3} \tag{62}$$

To yield a unique solution the same 3rd power relation for particle coefficients as in 2.7 is required to refer to a ground state term α_0* as well (with $\alpha(n)^* \sim \alpha(n)^{-1}$ and $\alpha_0^* = 8/27\ \alpha^{-9} \approx \alpha_0^{-1}$ as first guess) to reproduce the series of relative energies. This results in a ground state / electron energy of ≈ 7.2E-12J, roughly in the order of magnitude for W_e .

Equ. (62) might be considered to be a representation for photon energy.

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27 Condition $R_{44} = 0$ may not have to be obeyed strictly. Using additional terms of Φ_N for canceling of similar terms of other $R_{\alpha\beta}$ components may increase the resources to obtain a specific solution.

28 Ignoring the solution a = b with diverging integrals.

[A4] Metric / point charge

The following uses the metric of (9) with n = 2, i.e.

$$g_{\mu\nu} = \left(\frac{\rho_0}{r}\right)^2 \exp\left(-a\left(\frac{\rho}{r}\right)^3\right),\ \ -\left(\frac{\rho_0}{r}\right)^2 \exp\left(-b\left(\frac{\rho}{r}\right)^3\right),\ \ -/+\ r^2,\ \ -/+\ r^2\sin^2\vartheta \tag{63}$$

$\Gamma_{01}^0 = \Gamma_{10}^0 \quad = -1/r^1 + 3/2\ a\ \rho^3/r^4 \qquad \Gamma_{00}^1 = -1/r^1\ e^{(a-b)v} + 3/2\ a\ \rho^3/r^4 e^{(a-b)v}$

$\Gamma_{11}^1 \quad = -1/r^1 + 3/2\ b\ \rho^3/r^4$

$\Gamma_{12}^2 = \Gamma_{21}^2 = \Gamma_{13}^3 = \Gamma_{31}^3 \quad = +1/r^1 \qquad \Gamma_{22}^1 = -/+\ r^3/\rho_0^2\ e^{-bv} \ = \Gamma_{33}^1/\sin^2\vartheta$

$\Gamma_{23}^3 = \Gamma_{32}^3 = \cot\vartheta \qquad \Gamma_{33}^2 = -\sin\vartheta\cos\vartheta$

$R_{00} = e^{(a-b)v}\ [(-1/r^2 + 3\ (a-b)\ \rho^3/r^5 + 6\ a\ \rho^3/r^5 - 9/2\ a\ (a-b)\ \rho^6/r^8) + 2(\Gamma_{01}^0\ \Gamma_{00}^1) - \Gamma_{00}^1\ (\Gamma_{10}^0 + \Gamma_{11}^1 + 2\Gamma_{12}^2)]$

$= e^{(a-b)v}\ [(-1/r^2 + (9a - 3b)\ \rho^3/r^5 - 9/2a(a-b)\ \rho^6/r^8 - \Gamma_{00}^1\ (-\Gamma_{10}^0 + \Gamma_{11}^1 + 2\Gamma_{12}^2)]$

$= e^{(a-b)v}\ [(-1/r^2 + (9a - 3b)\ \rho^3/r^5 - 9/2a(a-b)\ \rho^6/r^8 + (+1/r^1 - 3/2\ a\ \rho^3/r^4)\ (+2/r^1)]$

$= e^{(a-b)v}\ [(-1/r^2 + (9a - 3b)\ \rho^3/r^5 - 9/2\ a(a-b)\ \rho^6/r^8 + 2/r^2 - 3a\rho^3/r^5\ ]$

$R_{00} = e^{(a-b)v}\ [+1/r^2 + (6a - 3b)\rho^3/r^5 - 9/2a(a-b)\rho^6/r^8]$

$R_{11} = [+\ 1/r^2 - 6a\ \rho^3/r^5 + 1/r^2 - 6b\ \rho^3/r^5 - 2/r^2 - 1/r^2 + 6b\rho^3/r^5 + \Gamma_{10}^0\ \Gamma_{01}^0 + \Gamma_{11}^1\Gamma_{11}^1 + 2\Gamma_{12}^2\ \Gamma_{21}^2 - \Gamma_{11}^1\ (\Gamma_{10}^0 + \Gamma_{11}^1 + 2\Gamma_{12}^2\ )]$

$= [-1/r^2 - 6\ a\ \rho^3/r^5 + \Gamma_{10}^0\ \Gamma_{01}^0 + 2\Gamma_{12}^2\ \Gamma_{21}^2 - \Gamma_{11}^1\ (\Gamma_{10}^0 + 2\Gamma_{12}^2)]$

$= [-1/r^2 - 6a\ \rho^3/r^5 + 1/r^2 + 9/4\ a^2\ \rho^6/r^8 - 3\ a\ \rho^3/r^5 + 2/r^2 + (+\ 1/r^1 - 3/2\ b\ \rho^3/r^4)\ (+1/r^1 + 3/2\ a\ \rho^3/r^4\ ]$

$= [+2/r^2 - 9a\ \rho^3/r^5 + 9/4\ a^2\ \rho^6/r^8 + 1/r^2 + 3/2a\rho^3/r^5 - 3/2b\rho^3/r^5 - 9/4\ ab\ \rho^6/r^8]$

$R_{11} = [+3/r^2 - (15/2a + 3/2b)\rho^3/r^5 + 9/4(+a^2 - ab)\rho^6/r^8]$

$R_{22} = -1 + e^{(c-b)v}\ [(+3\ r^2/\rho_0^2 - 3b\rho^3/(r\rho_0^2) + 2(\Gamma_{21}^2\ \Gamma_{22}^1) - \Gamma_{22}^1\ (\Gamma_{10}^0 + \Gamma_{11}^1 + \Gamma_{12}^2 + \Gamma_{13}^3)]$

$= -1 + e^{(c-b)v}\ [(+/-3\ r^2/\rho_0^2 - 3b\rho^3/(r\rho_0^2) - \Gamma_{22}^1\ (\Gamma_{10}^0 + \Gamma_{11}^1 - \Gamma_{12}^2 + \Gamma_{13}^3)]$

$= -1 + e^{(c-b)v}\ [(+/-3\ r^2/\rho_0^2 - 3b\rho^3/(r\rho_0^2) + r^3/\rho^{*2}\ (-2/r^1 + 3/2(a + b)\rho^3/r^4\ )]$

$= -1 + e^{(c-b)v}\ [(+/-3\ r^2/\rho_0^2 - 3b\rho^3/(r\rho_0^2) - 2r^2/\rho_0^2 + 3/2(a + b)\rho^3/(r\rho_0^2)]$

$R_{22} = -1 + e^{(c-b)v}\ [(+/-\ 1r^2/\rho_0^2\ +/-\ 3/2(+a - b)\rho^3/(r\rho_0^2)]$

$g^{00}R_{00} = e^{-bv}\ [+1/\rho_0^2 + (6a - 3b)\rho^3/(r^3\rho_0^2) - 9/2a(a-b)\rho^6/(r^6\rho_0^2)]$

$g^{11}R_{11} = -\ e^{-bv}\ [+3/\rho_0^2 - (15/2a + 3/2b)\rho^3/(r^3\rho_0^2) + 9/4(+a^2 - ab)\rho^6/(r^6\rho_0^2)]$

$g^{22}R_{22} + g^{33}R_{33} = +/-\ 2/r^2\ -/+\ e^{-bv}\ [(+/-\ 2/\rho_0^2\ +/-\ 3(+a - b)\rho^3/(r^3\rho_0^2)]$

The two solutions for R with different sign of $R_{22,33}$ will be:

$R = +/-\ 2/r^2 + e^{-bv}\ [(-\ 4/\rho_0^2 + (+21/2a + 3/2b)\ \rho^3/(r^3\rho_0^2) - 9/4(+3a^2 - 3ab)\rho^6/(r^6\rho_0^2)]$

G_{00} will be:

$G_{00} = e^{(a-b)v}\ [+1/r^2 + (6a - 3b)\rho^3/r^5 - 9/8(4a^2 - 4ab)\rho^6/r^8]\ -/+\ \rho_0^2/r^4\ e^{av} + e^{(a-b)v}\ [(+\ 2/r^2 + (-21/4a - 3/4b)\ \rho^3/r^5 - 9/8(-3a^2 + 3ab)\rho^6/r^8] = -/+\ \rho_0^2/r^4\ e^{av} + e^{(a-b)v}\ [(+\ 3/r^2 + (+3/4a - 15/4b)\ \rho^3/r^5 - 9/8(+a^2 - ab)\rho^6/r^8]$

As in [A3] volume integrals over the ρ^n/r^{n+2} terms will yield results $\int e^{av}\ \rho^n/r^{n+2}\ d^3r \approx \rho \approx$ 1E-14 [m^{-1}] compared to the term $\int e^{av}\ \rho_0^2/r^4\ d^3r \approx \rho_0^2\ \rho^{-1} \approx$ 1E-6 [m^{-1}] (both with electron parameter), giving negligible contributions to particle energy within the parameter range discussed here. This leaves

$G_{00} = -/+\ e^{av}\ \rho_0^2/r^4$

For chpt. 2.3 a = 1 and positive sign is chosen, giving e^v as exponential term corresponding to Φ being squared in the metric (4).

[A5] Coefficient σ, differential equation

For a differential equation and its solution such as (20)f and an Euler expression such as (19) to exhibit the same integration limits, r_n and $8/\sigma \sim \rho'^3/r_n^3$ of a general term ρ', requires the coefficient σ to be part of the exponent ν in φ:

Considering a general solution of a damped oscillation type equation with two coefficients, ρ' and ß ρ', where ß is an unknown coefficient,

$$\varphi = e^{\nu/2} = \exp\left(-\left(\frac{ß\rho'^3}{2r^3} + \left[\left(\frac{ß\rho'^3}{2r^3}\right)^2 - 4\frac{\rho'^3}{2r^3}\right]^{0.5}\right)/2\right) \tag{64}$$

gives at the limit r_n of the real solution (64):

$$\left(ß\rho'^3/r_n^3\right)^2 = 8\,\rho'^3/r^3 \tag{65}$$

Since the limit of the Euler integral is defined to be $8/\sigma = ß\ \rho'^3/r_n^3$:

$$\left(\frac{ß\rho'^3}{r_n^3}\right)^2 = \left(\frac{8}{\sigma}\right)^2 = 8\frac{\rho'^3}{r^3} \quad => \quad \left(\frac{8}{\sigma}\right) = \frac{ß\rho'^3}{r_n^3} = 8\frac{\sigma\rho'^3}{8r_n^3} \quad => \quad ß = \sigma \tag{66}$$

Analyzing the components of σ_0, in addition to the mandatory term for length, $\Gamma_{-1/3}/3$, of the integral (13) for m = -1, r_n

and σ_0 contain a factor ≈ 1.524 α^{-1}, close to the coefficient W_{Pl}/W_e ~ 1.513 α^{-1} (1.513 is in turn close to $\Gamma_{-1/3}/\Gamma_{1/3}$ =1.516). The latter suggests a geometrical interpretation:

$$1.513\ \alpha^{-1}\ \Gamma_{-1/3}/3 \approx \Gamma_{-1/3}/\Gamma_{1/3}\ \ 4\pi\ \Gamma_{-1/3}\ \Gamma_{1/3}\ \ \Gamma_{-1/3}/3 \approx \left(\frac{4\pi\,\Gamma_{-1/3}{}^3}{3}\right) = (\sigma_0/8)^{1/3} \qquad (67)$$

As a consequence a dimensionless volume-like term appears in the denominator of the energy expression (15), together with the coupling constants one of several hints that aspects of this model might be reducible to geometry.
The various expressions for spherical symmetry σ_0 used within this work may be summarized as:

$$\sigma_0 \approx 8\ (r_n/\rho_n)^3 \approx (1.513\ \alpha^{-1}\ 2/3\ \Gamma_{-1/3})^3 \approx (\alpha^{-1}\ \Gamma_{-1/3})^3 \approx 8\left(\frac{4\pi\,\Gamma_{-1/3}{}^3}{3}\right)^3 \qquad (68)$$

The integration limits for calculating angular momentum in z-direction, r_n of J_z, (17)ff, and (Compton-)wavelength, λ_C, supposed to represent the rotating E-vector and in turn total angular momentum J should be related by the factor √3 of the ratio J/J_z

$$\lambda_C / r_n = (1/2(1/2+1))^{0.5} / (1/2) = \sqrt{3} \qquad (69)$$ [29]

[A6] Coupling constant in N dimensions
3D case:
The 3D case of the coupling constant is easy to interprete, for the 4D-case some assumptions have to be made concerning the integration limit. The following gives an alternative, more detailed interpretation than 2.6.
The exact value of the product of the integrals (29)f, depends on the integration limit relevant for the second integral, i.e. the lower integration limit of the Euler integrals, which can be expressed as 3D volume with $\Gamma_{-1/3}$ as radius (67):

$$\rho_n^3/\lambda_{C,n}^3 = 8/(3^{1.5}\sigma_0) = \left(3^{0.5}\frac{4\pi}{3}\ \ \Gamma_{-1/3}{}^3\right)^{-3} \qquad (70)$$

The additional factor $3^{0.5}$ gives the ratio between r_n of equ. (16) and $\lambda_{C,n}$ as required in the expression for photon *energy*. This gives $\Gamma(-1/3, 8/\sigma_0) \approx 36\pi^2\Gamma_{-1/3}$ and

$$2\int_0^r \varphi_3(r)^2 r^{-2}\,dr \int_0^r \varphi_3(r)^2 dr = 2\left[\frac{\Gamma_{1/3}}{3}\right]\left[2\pi\,2\pi\,9\,\frac{\Gamma_{-1/3}}{3}\right] = 4\pi\,\Gamma_{1/3}\Gamma_{-1/3}\ \ \mathbf{2\pi} = \mathbf{2\pi}\ \ \alpha^{-1} \qquad (71)$$ [30]

The result of (71) yields a dimensionless constant α' = h c_0 4π ε/e^2 and it is a matter of choice to include 2π in the dimensionless coupling constant. Factor 9 cancels the corresponding factors from the Euler integrals. The remaining factor of 4π is needed to yield the correct value of α.
A general N-dimensional version of (70) may be given as:

$$8/\sigma_N = \left(3^{0.5\delta}\,V_N\ \ (\Gamma(-1/N))^N\right)^{-N/(N-2)} \qquad (72)$$

V_N is the coefficient for volume in N-D, coefficient $3^{0.5}$ will be omitted in 4D where coordinate r is considered to be directly related to energy via r_n ~ $1/W_n$ and r_n might be directly identified with $\lambda_{C,n}$; subscript in σ_N corresponds to dimension in the following.

4D case:
Using φ_4 according to the definition (7) and (72) for 4D:

$$\rho_n^4/r_n^4 = 8/\sigma_4 = \left(\frac{\pi^2}{2}\ \ (\Gamma_{-1/4})^4\right)^{-2} = 1.232\text{E-}7 \qquad (73)$$

as integration limit, with (13) the non-point charge integral in 4D will be given by:

$$\int_0^r \varphi_4(r)^2 r\,dr \sim \Gamma(-1/2, 8/\sigma_4) = \int_{8/\sigma_4}^{\infty} t^{-1.5}e^{-t}dt = 5687 \approx 16\pi^4\Gamma_{-1/2} \qquad (74)$$

The 4D equivalent of (71) will be:

$$2\int_0^r \varphi_4(r)^2 r^{-3}dr \int_0^r \varphi_4(r)^2 r\,dr \approx 2\left[\frac{\Gamma_{1/2}}{4}\right]\left[16\pi^4\frac{\Gamma_{-1/2}}{4}\right] = \frac{\pi^2}{2}\Gamma_{1/2}\Gamma_{-1/2}\ \ \mathbf{4\pi^2} = \pi^3\,\mathbf{4\pi^2} = \alpha_{weak}^{-1}\,\mathbf{4\pi^2} \qquad (75)$$

The interpretation is the same as in the 3D-case:
A $4\pi^2$ term originating from the second integral of equation (71) is required for turning h^2 into $\hbar^2$ since the integrals in (75) refer to ρ_n^2 and thus to the square of energy and h, ħ. Factor 16 cancels the corresponding factors from the Euler integrals. The remaining factor of $\pi^2/2$ is needed to yield the correct value of α_{weak}.
While the integral $\int\varphi_3(r)^2dr$ in 3D yields the wavelength of *one* photon, $\int\varphi_4(r)^2$ r dr may be considered as an integration

29 Alternatively: $\lambda_{C,n} = 3\rho h c_0/(2b_0\Gamma_{1/3}) = 3\pi\alpha^{-1}\rho/\Gamma_{1/3}$, $r_n = 3/2\ \alpha^{-1}\rho\ \Gamma_{-1/3}/3$ => $\lambda_{C,n}/r_n = 6\pi/(\Gamma_{1/3}\Gamma_{-1/3}) = 6\pi/(2\pi\sqrt{3}) = 3^{0.5}$
30 Factor 2 from adding electric and magnetic contributions to energy

over 1/W of *all* photons within the integration limits, giving a term $\int\varphi_4(\lambda)^2\ \lambda\ d\lambda \sim 1/W^2$.

2D case:
the 2D case is not as straightforward as the 4D case. The integral over the 1D point charge

$$\int_0^r \varphi_2(r)^2 r^{-1}\, dr = \Gamma\left(0, \rho_n^2/r_2^2\right)\ /2 \tag{76}$$

features Γ(0, x), with Γ(0, x) -> ∞ for x -> 0 and m = 0 in the equations above. Setting nevertheless m=1 in the 2D equivalent of the integration limit

$$\rho_n^2/\lambda_{C,n}^2 = 8/(3\sigma_2) = \left(3^{0.5}\pi\ \ \Gamma_{-1/2}{}^2\right)^{-2} \approx 1/4676 \tag{77}$$

and calculating $\Gamma(0, \rho_2^2/r_2^2)$ numerically gives $\int\varphi_2(r)^2\, r^{-1}\, dr \approx \Gamma(0, \rho_2^2/r_2^2)/2 = 7.872/2$. In the 2D case the complementary integral would be identical to the point charge integral, giving $2(\int\varphi_2(r)^2\, r^{-1}\, dr)^2 \approx 4\pi^3/4 = \pi^3$, i.e. the same value as 4D, maybe giving an alternate candidate for α_{weak}.

[A7] Additional particle states

Assignment of more particle states will not be obvious. The following gives some possible approaches.

[A7.1] Partial products

Additional partial product series will have to start with higher exponents n in α^(-1/3^n) giving smaller differences in energy while density of experimentally detected states is high. There might be a tendency of particles to exhibit a lower mean lifetime (MLT), making experimental detection of particles difficult [31]. To determine the factor y_l^m of higher angular states requires an appropriate ansatz for the differential equation yet to be found.

One more partial product might be inferred from considering d-like-orbital equivalents with a factor of $5^{1/3}$ as energy ratio relative to η giving the start of an additional partial product series at $5^{1/3}$ W(η) = 937MeV i.e. close to energy values of the first particles available as starting point, η', Φ^0. However, in general it is not expected that partial products can explain all values of particle energies.

[A7.2] Linear combinations

The first particle family that does not fit to the partial product series scheme are the kaons at ~ 495MeV. They might be considered to be linear combination states of π-states. The π-states of the y_1^0 series are assumed to exhibit one angular node, giving a charge distribution of +|+, -|- and +|-. A linear combination of two π-states would yield the basic symmetry properties of the 4 kaons as:

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          +               -            -            +
K+    +      +     K-  -     -   KSo +    +   KLo +     -
          +               -            -            -
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(+/- = charge)

providing two neutral kaons of different structure and parity, implying a decay with different parity and MLT values. For the charged Kaons, K^+, K^-, a configuration for wave function sign / chirality equal to the configuration for charge of K_S^o and K_L^o might be possible, giving two versions of P+ and P- parity of otherwise identical particles and corresponding decay modes not violating parity conservation.

[A7] Fractional charge

As far as e.g. fractional charge is concerned, a simple progression {1, 2, 3 …} is expectable in any model with states of increasing complexity. The symmetry of particles in the standard model (SM) starts with 1, 2, 3 as well, i.e. leptons, mesons and baryons, though the first ones are not part of QCD. Fractional charges of 1/3 or 2/3 might thus be expectable for baryons in this work. However, there are hints that such fractional charges may play a more fundamental role for particles of this model even starting at the electron state, such as e.g. factor ≈ 2/3 in the term for W_e (38), giving $\varepsilon_0 \int E(r)\varphi(r)^2\, dA = 2/3\ e_c$ at the maximum of the W(r) curve (numerical calculation), or a correspondence of $J_z = 1/2$ and average fields of 2/3 from quaternion calculations (see [15], 5) in [A1]). On the other hand charge 1/2 for mesons may be a more natural choice and may yield reasonable explanations for key experiments of particle physics as well (see 6) in [A1]).

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31 Which might explain missing particles of higher n in the y_0^0 and y_1^0 series as well.