Thermal Time Constant - TTC

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The term "*Time Constant*" is well defined in Academic Dictionaries and Encyclopedias [1] as "*the time it takes the system's step response to reach approximately 63.2% of its final (asymptotic) value*". A more sophisticated explanation regarding the "*63.2%*" value can be found on Wikipedia [2]. Wikipedia also describes the "*Thermal Time Constant*" term [3]. A detailed explanation regarding the "*63.2%*" value in the context of the building thermal mass may be found in the publication "*Thermal Response of Buildings*" by London South Bank University [4]. According to this publication the exact value of "63.2%" is as follows:

Formula 1 - Time Constant value

$$1 - 1/e = 0.632$$

e - Euler's Number

Formula 2 - Euler's Number

$$e = \sum_{i=0}^{\infty} \frac{1}{i!}$$

i – Integer: 1, 2, 3 …

Considering the approximate value of "e" as 2.71828182845905, the "*63.2%*" value can be calculated more precisely as 63.2120558828558%. Euler's Number can be generated in MS-Excel applying the following formula:

=EXP(1)

and the Time Constant Value:

=1-1/EXP(1)

In most cases, the term is related to electronic components rather than to building elements. The definition of this term can be found on the U.S.Sensor Corp Internet site [5] related to the electronic component: The Thermal Time Constant is a measurement of the time required for the thermistor to respond to a change in the ambient temperature. The technical definition of Thermal Time Constant is, "*The time required for a thermistor to change 63.2% of the total difference between its initial and final body temperature when subjected to a step function change in temperature, under zero power conditions*". By replacing the word "thermistor" by "building element", we can define the Thermal Time Constant of the building element accordingly: *The Thermal Time Constant is a measurement of the time required for the building element to respond to a change in the ambient temperature*. The technical definition of Thermal Time Constant is: "*The time required for a building element to change 63.2% of the total difference between its initial and final body temperature when subjected to a step function change in temperature, under zero power conditions*". The unit of TTC is hour (TTC is expressed in hours).

The formula for the determination of Thermal Time Constant in a building envelope is explained in the publication "*Thermal Time Constant – TTC*" [7] section 7.2.

An example of the building envelope element may be an external wall, which consists of a few layers. The example of the layers of the external wall may be the outside stone layer, thermal insulation layer, concrete layer and inside plaster.

Formula 3 - TTC of building envelope element [7]

$$TTC = \sum_{z=1}^{L} TTC_z$$

TTC	- Thermal Time Constant of the building element (like the external wall) [hr]
L	- number of layers in the building element
z	- serial number of the layer between 1 and L
z=1	- serial number of the first layer
TTC_z	- TTC of layer z [hr]

The formula for the TTC of layer z (TTC_z) involves the sum of thermal resistances of all previous layers starting from the outside surface of the building including the external laminar layer, half of the thermal resistance of the current layer (layer z) and thermal mass of the current layer (layer z).

Formula 4 - TTC_z of building envelope element [7]

$$TTC_z = (\sum_{i=0}^{z-1} r_i + 0.5 r_z) Cth_z$$

TTC_z	- TTC of layer z [hr]
i	- serial number of the layer between 0 and (z-1)
r_i	- thermal resistance of layer “i” starting from the outside of the building [m^2,°C/W]
r_0	- thermal resistance of the external laminar layer, layer 0 = 0.04 (m^2,°C)/W [7]
r_z	- thermal resistance of layer “z”, the current layer [(m^2,°C)/W]
Cth_z	- thermal mass of layer “z”, the current layer [Whr/(m^2,°C)]

Formula 5 - Thermal resistance of layer “i” [7]

$$r_i = \frac{d_i}{\lambda_i}$$

r_i	- thermal resistance of layer “i” [(m^2,°C)/W]
d_i	- thickness of layer “i" [m]
λ_i	- specific thermal conductivity of layer “i” [W/(m,°C)]

Formula 6 - Thermal mass of layer “z” [7]

$$Cth_z = m_z cp_z$$

Cth_z	- thermal mass of layer “z”, the current layer [Whr/(m^2,°C)]
m_z	- mass of layer “z” [kg/m^2]
cp_z	- isobaric specific heat of layer “z” [Whr/kg,°C]

The following example may clarify the meaning of TTC. In this example, the building TTC is 40 hours, as calculated with formula 3. The room temperature of the building was kept at 18°C for a long time. At the starting point, all internal and external energy sources were switched off, while the ambient temperature is 8°C. In this example, we shall consider that the ambient temperature remains constant day and night at 8°C and there is no solar radiation. The temperature of rooms in the building will start to go down. This is not a linear process. Theoretically, the room temperature will reach 8°C at an endless time (never) but will get close to it slower and slower. The difference between the 18°C room temperature of the building and the 8°C ambient temperature is 10°C. 63.2% of this difference is 6.3°C. TTC of 40 hours means that the room temperature of the building will drop by 6.32°C, from 18°C to 11.68°C in 40 hours, if the outside temperature remains constant at 8°C and there are no internal or external energy sources and no solar radiation.

Table 1 - Room temperature after one TTC

	In	Out	ΔT	Time	Temp Drop	
	°C	°C	°C	hr	%	°C
Start	18.00	8.00	10.00	0		
TTC				40	63.2%	6.32
End	11.68			40		

According to the discussion at Physics Forum [6] "*after 4 thermal time constants, the temperature is considered to have stabilized*". 4 TTCs result in our case in 4x40hr=160hr, which means that the room temperature will fall almost to 8°C (will be stabilized) in 160 hours. In our example, the room temperature after 4 TTCs (4x40hr=160hr) will be 8.2°C, 0.2°C above the final temperature (8.0°C), far 1.8% to the final temperature.

Table 2 - Proximity to the target

TTCs	Time	Start	End	to Out	Temp Drop		Proximity to Target	
	hr	°C	°C	°C	%	°C	°C	%
0	0	18.0000		10.0000				
1	40	18.0000	11.6788	3.6788	63.2%	6.3212	3.6788	36.788%
2	80	11.6788	9.3534	1.3534	63.2%	2.3254	1.3534	13.534%
3	120	9.3534	8.4979	0.4979	63.2%	0.8555	0.4979	4.979%
4	160	8.4979	8.1832	0.1832	63.2%	0.3147	0.1832	1.832%
5	200	8.1832	8.0674	0.0674	63.2%	0.1158	0.0674	0.674%
6	240	8.0674	8.0248	0.0248	63.2%	0.0426	0.0248	0.248%
7	280	8.0248	8.0091	0.0091	63.2%	0.0157	0.0091	0.091%
8	320	8.0091	8.0034	0.0034	63.2%	0.0058	0.0034	0.034%
9	360	8.0034	8.0012	0.0012	63.2%	0.0021	0.0012	0.012%
10	400	8.0012	8.0005	0.0005	63.2%	0.0008	0.0005	0.005%

Based on the above calculation, a formula may be developed to calculate proximity to the ambient temperature as a function of the number of TTCs applied.

Formula 7 - Proximity to the target applying number of TTCs

$$dT = (1/e)^n$$

dT - proximity to the target, %
e - Euler's Number
n - number of TTCs applied

In MS-Excel the above formula for n=4 will have the following form:

=(1/EXP(1))^4

The observation "*after 4 thermal time constants, the temperature is considered to have stabilized*" [6] results in 1.8% proximity to the target. The proximity to the target of 0.1% will be achieved in 7 TTCs (7x40hr=280hr) and 0.01% in 9 TTCs (9x40hr=360hr).

The design of a building's exterior walls has an important influence on energy consumption, building cost and thermal comfort. It influences also heating and cooling devices' size.
Optimization of energy consumption of the building requires consideration of the building materials, their thickness and mass, type and thickness of insulation, and location of the insulation layer.
The correct way to optimize all the above parameters is the calculation of temperatures and energy consumption in a dynamic process, which means considering changes every few seconds. This may be done only with a computer program.
It is not easy to gain expertise in such programs. In addition, submission of input values to the program is time consuming and frustrating process, which many cases ends with errors.

The work “*Thermal Time Constant – TTC*” [7] analyses the application of the Thermal Time Constant method for the determination of thermal quality as an alternative to the dynamic simulation program. The Thermal Time Constant method is much easier, simpler and faster than a computer dynamic simulation program.
A few examples of exterior wall design were selected to calculate the energy consumption and room temperatures. Two locations were selected for the calculations, one with moderate summer and the second with moderate winter.
The dedicated dynamic simulation computer program, Building Energy Simulation and Optimization (BESO), was applied to gain data regarding the energy consumption and maximum and average room temperatures for the south-facing and north-facing rooms at each location.

Thermal Time Constant reflects the thermal inertia of the building.
Higher Thermal Time Constant (TTC) moderates the difference between day and night in the room temperature, flattening the room temperature extremes caused by ambient temperature and solar energy.
Higher TTC is not necessarily more expensive. To take advantage of high TTC is not so the question of additional investment, but the question of understanding the TTC concept.
In some cases consideration of TTC caused changes to the building's design and construction technique, bringing benefits to the contractor, the tenants and the national economy:

- Reduction of the building cost
- Climate comfort in the building's rooms, with or without air conditioning or space heating
- Mitigation of extreme temperatures in rooms, making it more uniform between day and night
- Saving energy for air conditioning and space heating
- Reduced size and price of air conditioning units and space heating radiators
- Saving of electricity infrastructure installation cost, as there is no more need for 3 phases for air conditioning
- Less noise from smaller air condition unit
- More rooms space
- Decrease in building weight
- Contribution to the national economy:
    - Energy conservation
    - Lower peak demand
    - Quality of construction
    - Green development
    - Energy CO_2 emissions reduction

Disregarding TTC leads to wrong decisions, wrong design, waste of money on building construction, increased energy consumption and not justified increase in cumulative CO_2 emissions.

Changes in this Version

- Change of formula for proximity to the target in percent
- Added formula for Euler’s Number
- Added formula for determination of TTC of building envelope
- Editorial corrections

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