Practical parametric amplification with tunable working frequency

Abstract: Superconducting traveling wave parametric amplifiers (TWPA) based on kinetic or Josephson nonlinear inductance are known to be broadband and low noise. A high gain of TWPA can be achieved for long and ideally matched system. In practice, the unmatched nonlinear impedance of the transmission line with finite size results in resonant character of its response, and thus limits the bandwidth of the amplifier. Nevertheless, many applications do not require wide bandwidth, which increases the total noise. Instead, wide tunability of frequency range would be preferred. We present design of 14 mm long superconducting coplanar waveguide with 2080 Josephson junctions arranged as a metamaterial with proper dispersion. The highest amplification achieved was 15 dB within 35 MHz bandwidth around central frequency f0 = 6.2 GHz was achieved with the dynamic range of TWPA around 10dB. The f0 can be widely tuned in frequency ranges 3.2 – 4.3 GHz and 5.1 – 6.4 GHz by pump frequency and pump power. In the first frequency region the average amplification is around 4 dB with 8 dB peak and in the second region the gain oscillates between 6 dB and 15 dB. Similar behaviour was obtained theoretically within the coupled mode theory. The properties of the stepped impedance resonator and the role of the metamaterial are studied by numerical solution of nonlinear telegrapher's wave equations utilizing the finite element method.

S. Kern, P. Neilinger[1,2], Evgeni Il'ichev[3] , Matthias Schmelz[3], Jürgen Kunert[3], Ronny Stolz[3] and M. Grajcar[*1,2]

[1] *Department of Experimental Physics, Comenius University, Mlynská dolina F1, 84248, Bratislava, Slovakia*
[2] *Institute of Physics, Slovak Academy of Sciences, Dúbravská cesta 9, 84511, Bratislava, Slovakia*
[3] *Institute of Photonic Technology, D-07702 Jena, Germany*

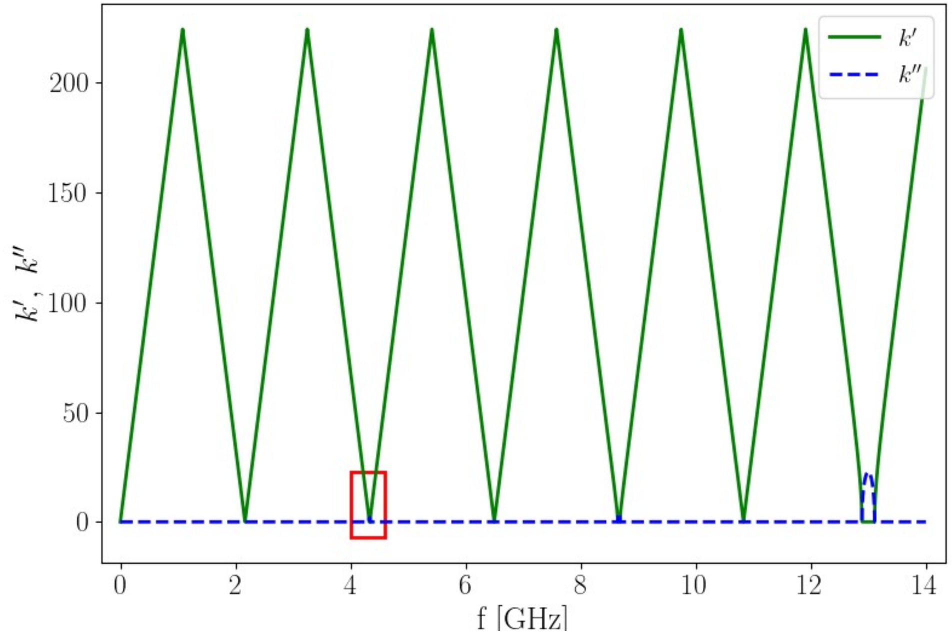
Metamaterial

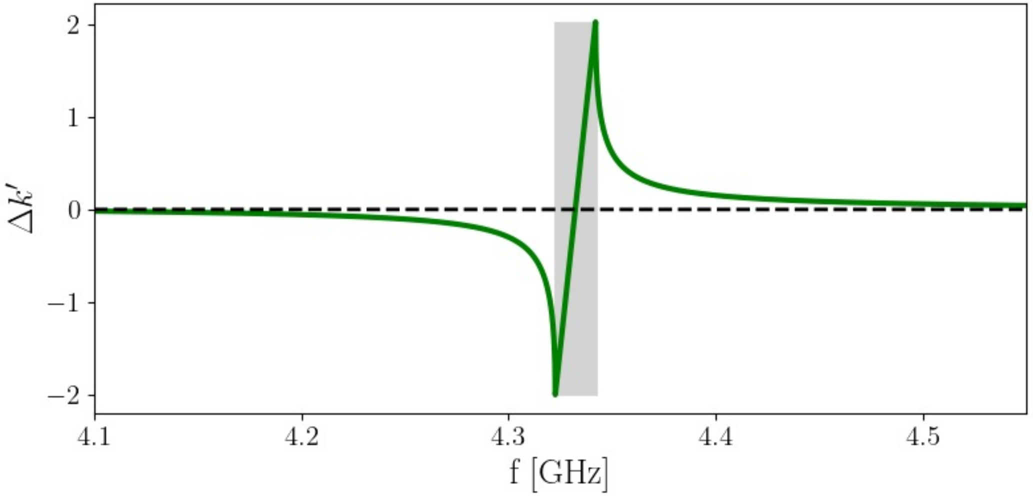
$$M = \prod_j \begin{pmatrix} e^{ik_j \Delta l_j} & 0 \\ 0 & e^{-ik_j \Delta l_j} \end{pmatrix} M_{Z_{0,j} Z_0} \begin{pmatrix} e^{ik_0 l} & 0 \\ 0 & e^{-ik_0 l} \end{pmatrix} M^{-1}_{Z_{0,j} Z_0}$$

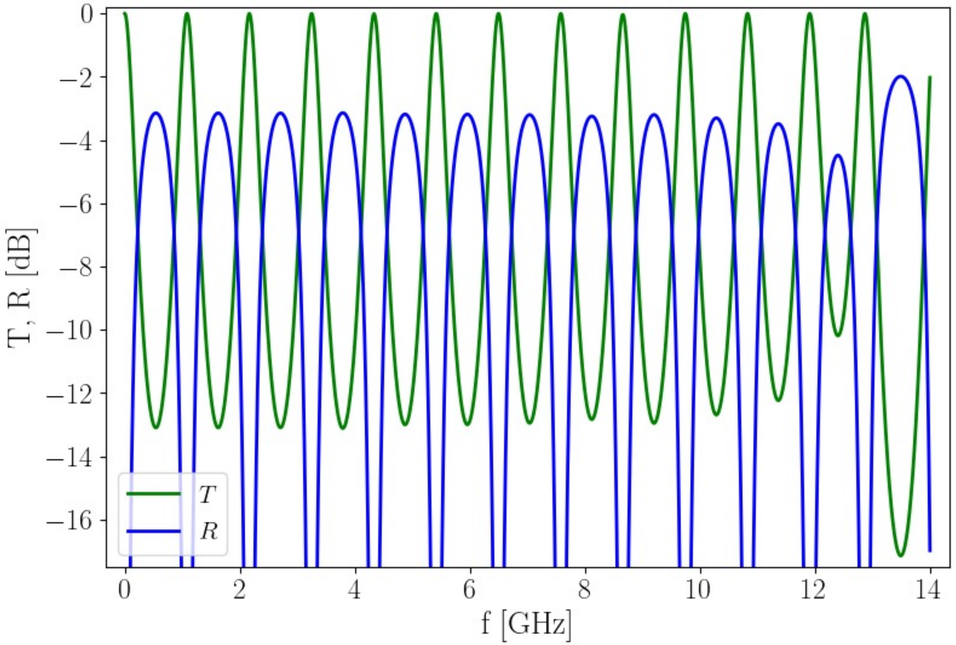
$$k = \frac{1}{2L} \arccos\Big(\mathrm{Tr} M\Big)$$

Phase matching $\beta = 0 \;\Rightarrow$:

$$\Delta k = \frac{2 k_p \gamma}{(1 + 2\gamma)}$$







TWPA vs. Resonator

$$\beta = \Delta k(1+2\gamma) - 2k(\omega_p)\gamma$$

$$K = \sqrt{k(\omega_s)k(\omega_i)\gamma^2 - \frac{\beta^2}{4}}$$

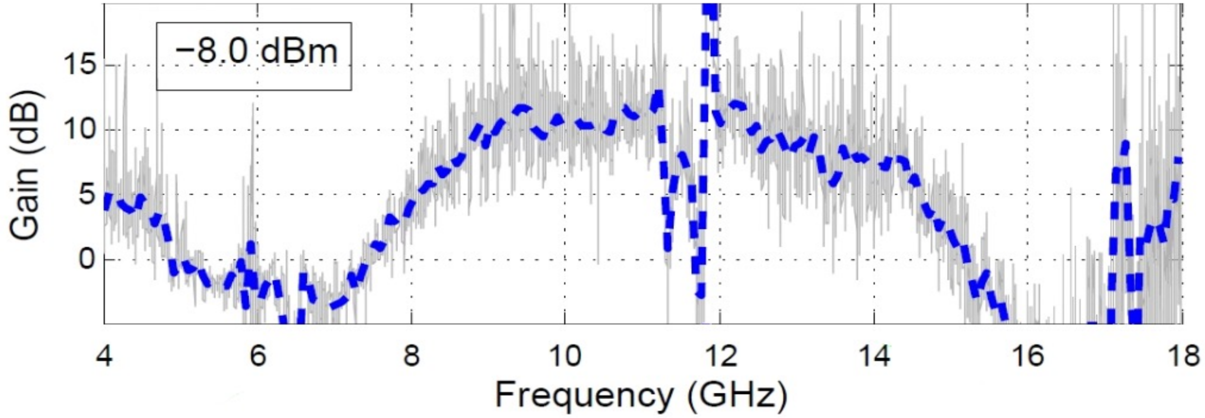
$$G \equiv \frac{P_s(z)}{P_s(0)} = \left(cosh^2(Kz) + \left(\frac{\beta}{2K}\right)^2 sinh^2(Kz)\right)$$

$$\gamma = \left(\frac{I_p}{4I_c}\right)^2$$

$$I_p^2 = 2QT(f)\frac{P}{Z_0}$$

$$T(f) = \frac{(1-\Gamma^2(I_p))^2}{(1-\Gamma^2(I_p)\cos(2\frac{2\pi f}{v_p}l))^2 + (\Gamma^2(I_p)\sin(2\frac{2\pi f}{v_p}l))^2}$$

$$Q = \frac{2\pi f L}{v_p(I_p)(1-\Gamma^2(I_p))}$$



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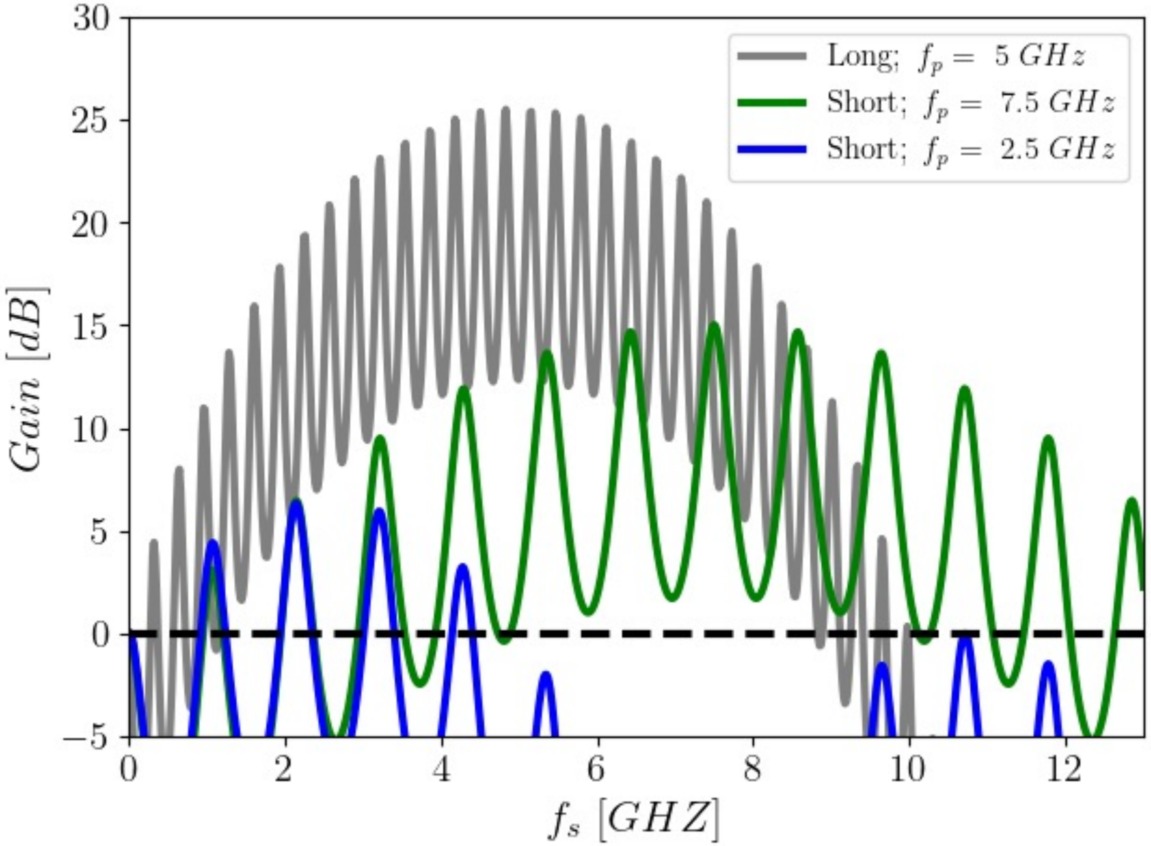
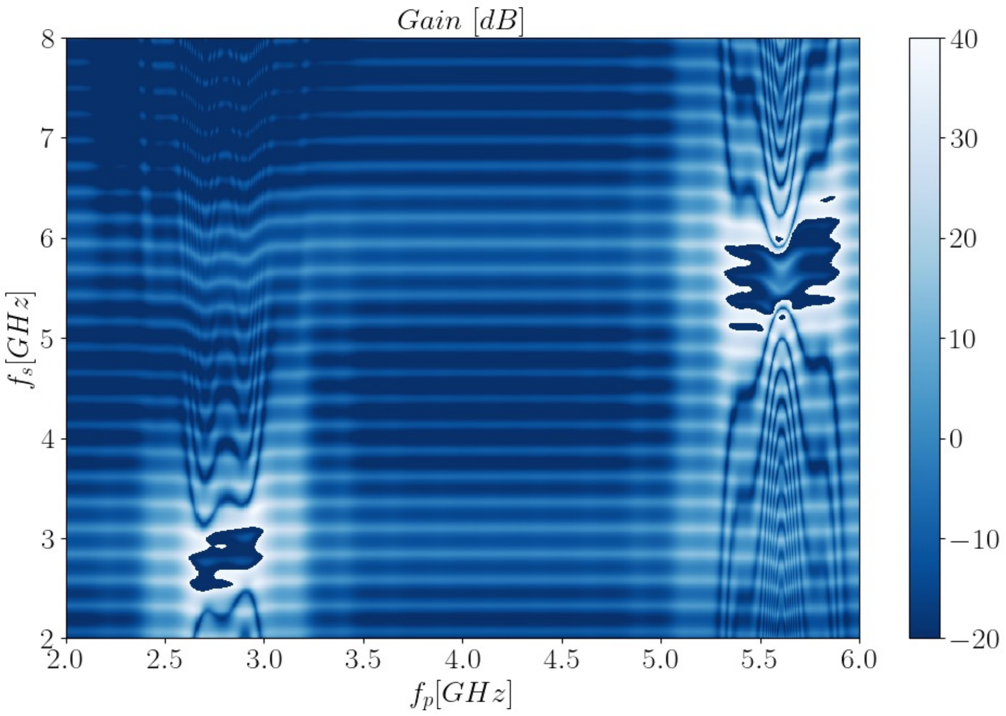
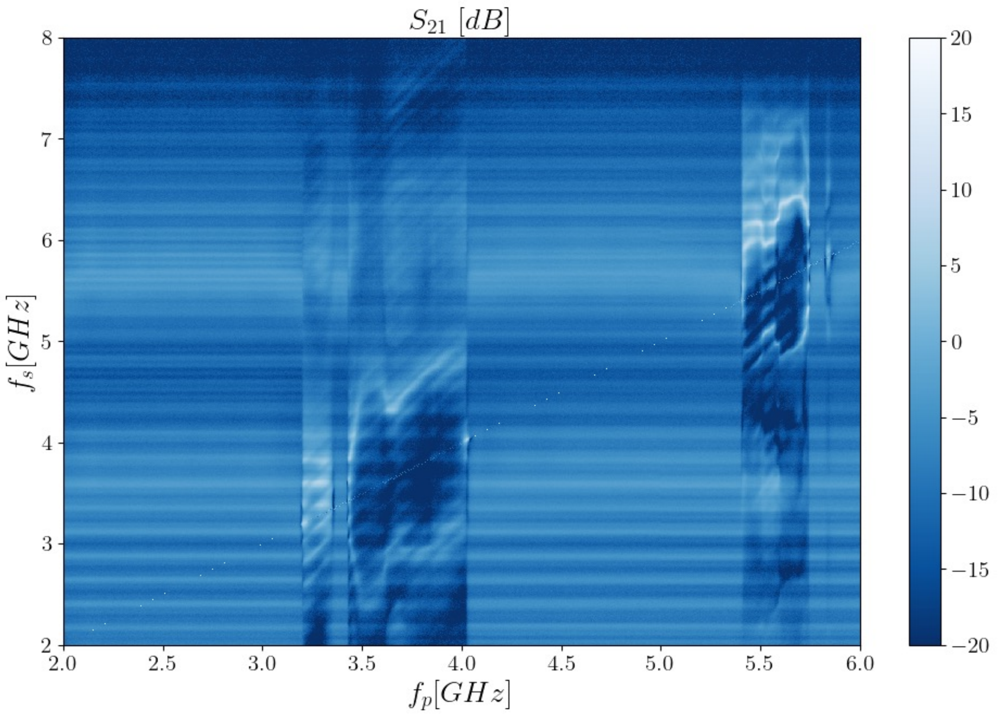


Experiment





$$\Gamma(I_p) = \frac{Z_0(I_p) - Z_l}{Z_0(I_p) + Z_l}; \quad Z_0(I_p) = \sqrt{\frac{L(1 + I_p^2/I_c^2)}{C}};$$

$$v_p(I_p) = \frac{1}{CL(1 + I_p^2/I_c^2)}$$

Nonlinear response solved iteratively