

Observation of Quantum Corrections to Conductivity of Disordered MoC and NbN Films up to Optical Frequencies

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Superconducting hybrids at Extreme, Štrbské pleso
28. 6. 2021

Strongly disordered superconductors

- Variety of applications
 - Parametric amplifiers
 - Superconducting quantum bits
 - Superconducting nanowire-based single-photon detectors

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 - Sheet resistance $R_{\square} = \rho/t$ (at room temperature)
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 - Electron concentration n
- Need for a robust characterization method of the electric properties of disordered films in the metallic state

- Drude model:

$$\sigma(\omega) = \sigma'(\omega) + i\sigma''(\omega) = \frac{\sigma_0}{1 - i\omega/\Gamma}$$

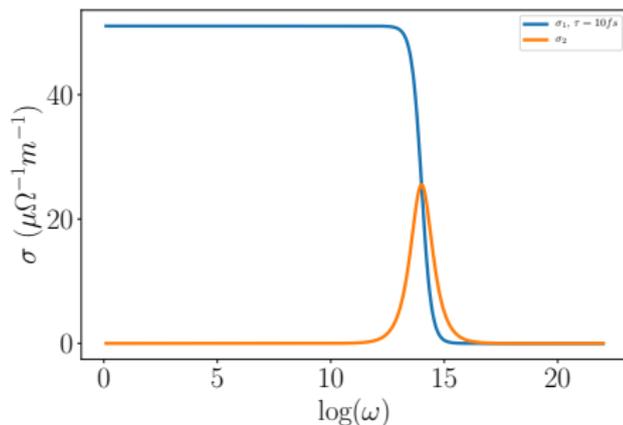
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Optical conductivity of metals

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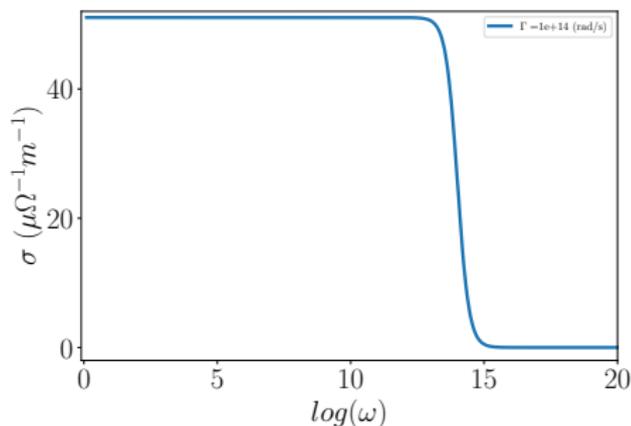


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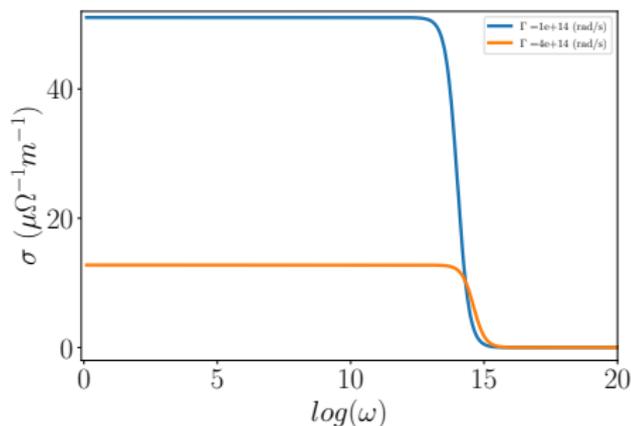


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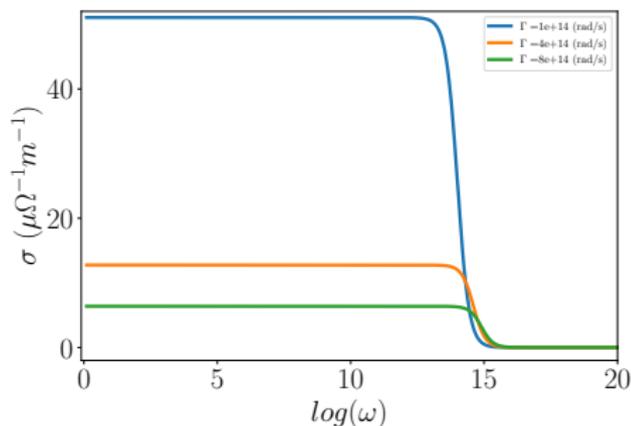


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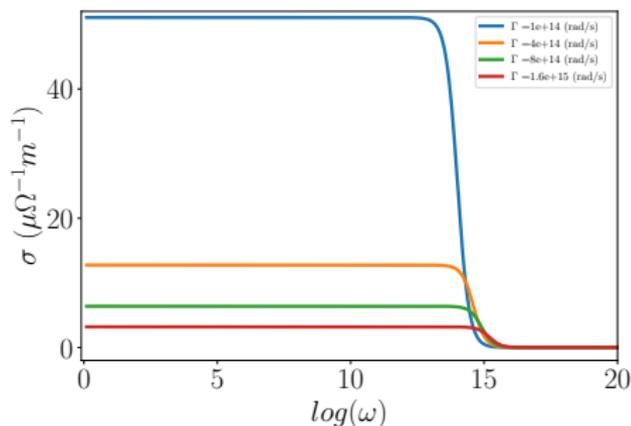


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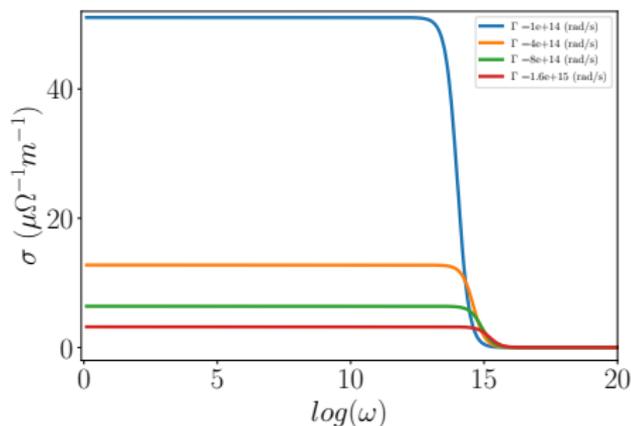


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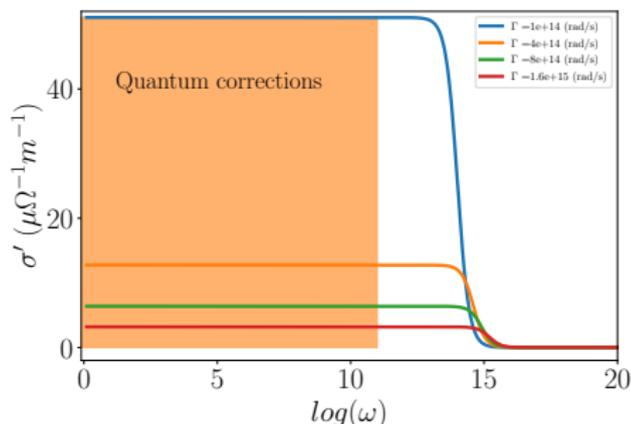
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$$\sigma(\omega) = \sigma_{\text{reg}}(\omega) + \delta\sigma(\omega)$$

■ Normal conductivity:

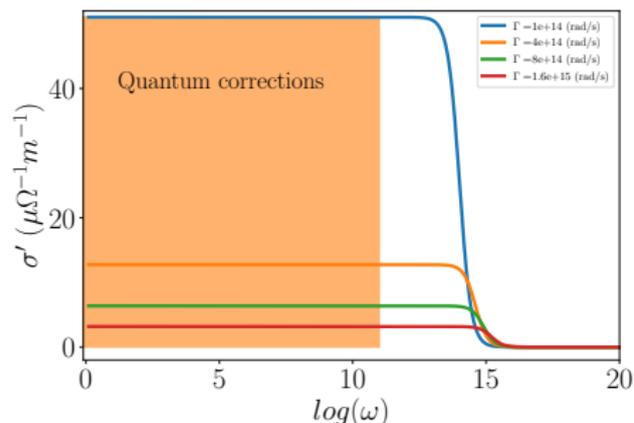
$$\sigma_{\text{reg}}(\omega) \approx \sigma_0$$

■ Quantum correction $\delta\sigma(\omega)$:

- Weak-localization
- Interaction effects

■ The same functional form

- 3D: $\delta\sigma(\omega) \sim \sqrt{\omega}$
- 2D: $\delta\sigma(\omega) \sim \ln(\omega)$



■ Temperature dependence

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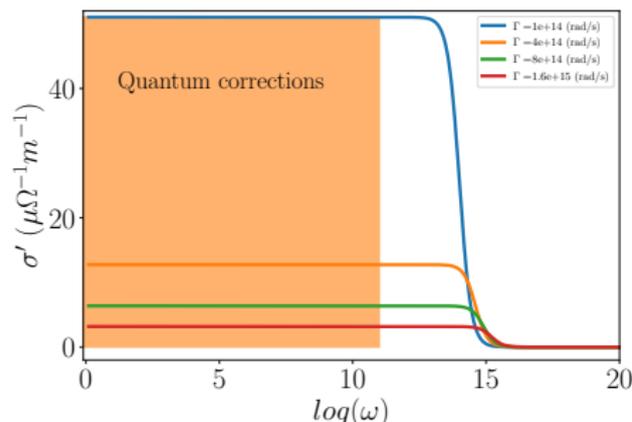
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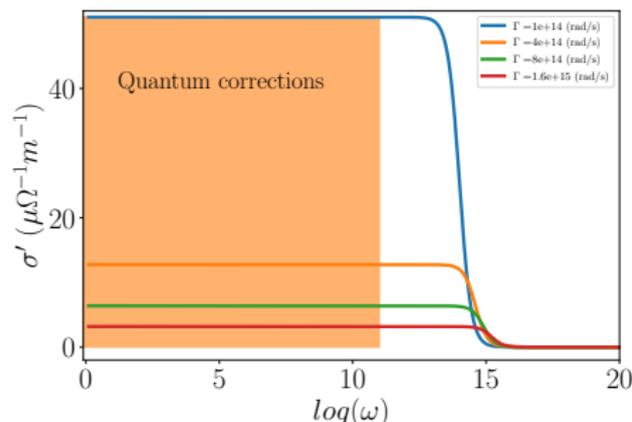
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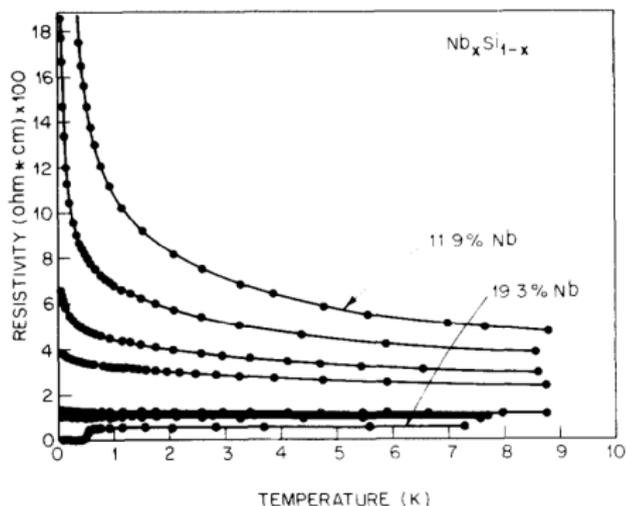


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- Generally considered low-temperature phenomena
- Theoretically should be present up to $\omega^* \sim \Gamma$

Transport measurements at low temperatures

- Amorphous $\text{Nb}_x\text{Si}_{1-x}$, stoichiometry x - tuning of disorder
- Resistivity measurements from 20mK to 9K:

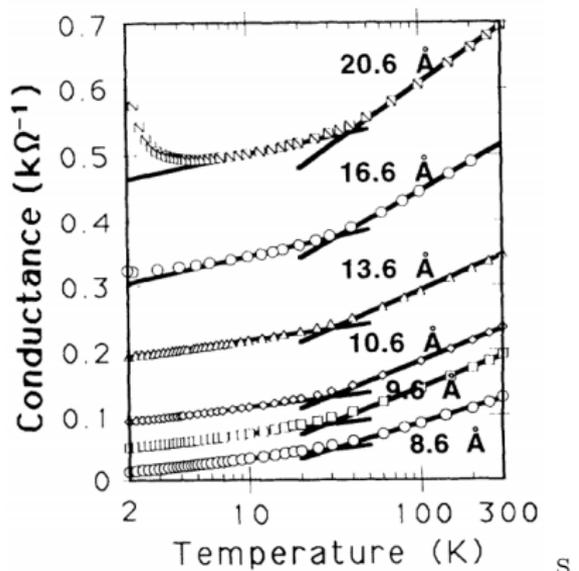
$$\sigma(T) = \sigma_0 + b\sqrt{T}$$



D.J. Bishop, E. G. Spencer, and R. C. Dynes, Solid State Electron. 38, 73 (1985)

Transport measurements of MoC

- MoC, thickness - tuned disorder
- 2D weak-localization and e - e interaction effects up to RT



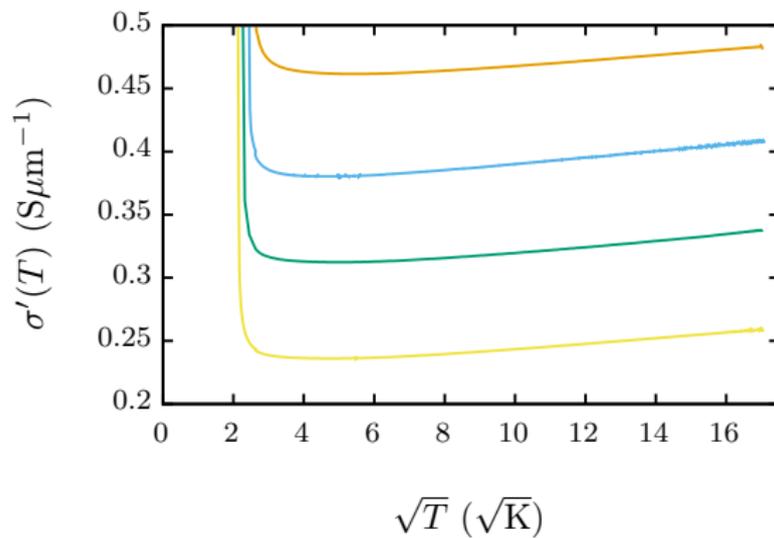
J. Lee et al., PRB **49**, 13882 (1994)

$$\sigma(T) = \sigma_0 + \{\alpha p + (1 - F)\} \ln(T)$$

- Magnetron sputtered in argon-acetylene atmosphere
- Mo - target and sapphire substrates
- $\text{Mo}_x\text{C}_{1-x}$; stoichiometry (x) or thickness (t) tuned disorder
- Two sets of thin film samples:
 - 1. set: R_{\square} from 420 to 720 Ω ; fixed $t=5$ nm, varied x
 - 2. set: R_{\square} from 100 to 220 Ω ; varied t , fixed x

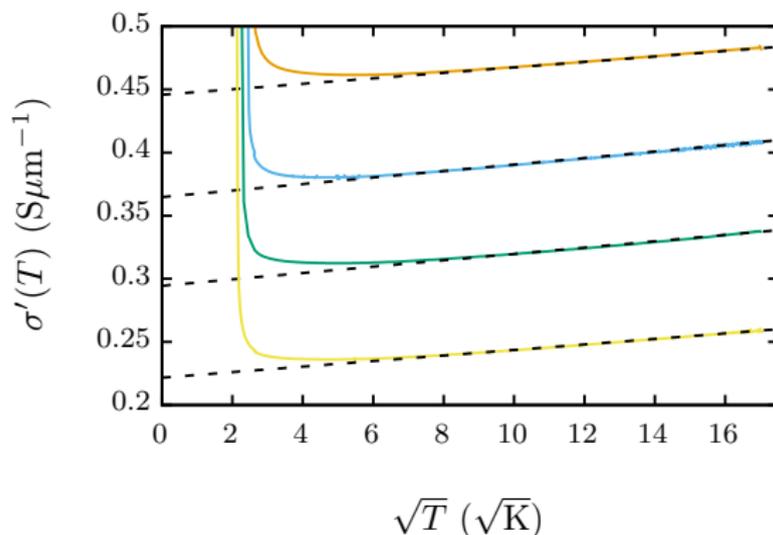
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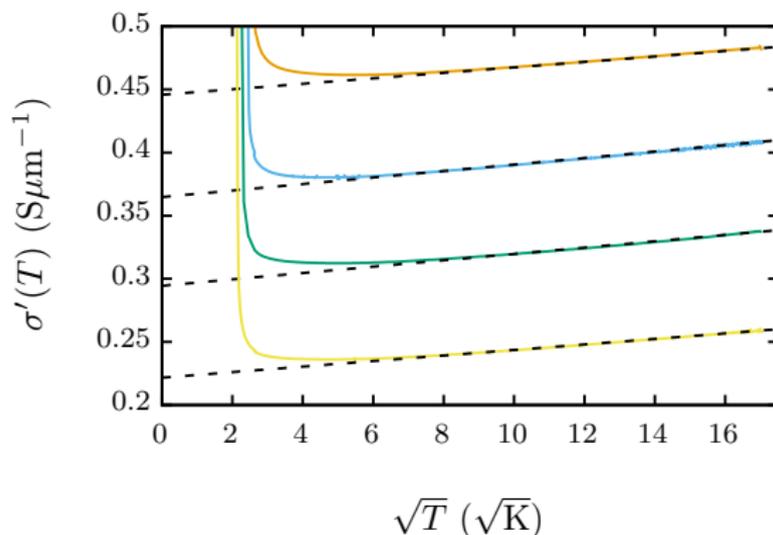
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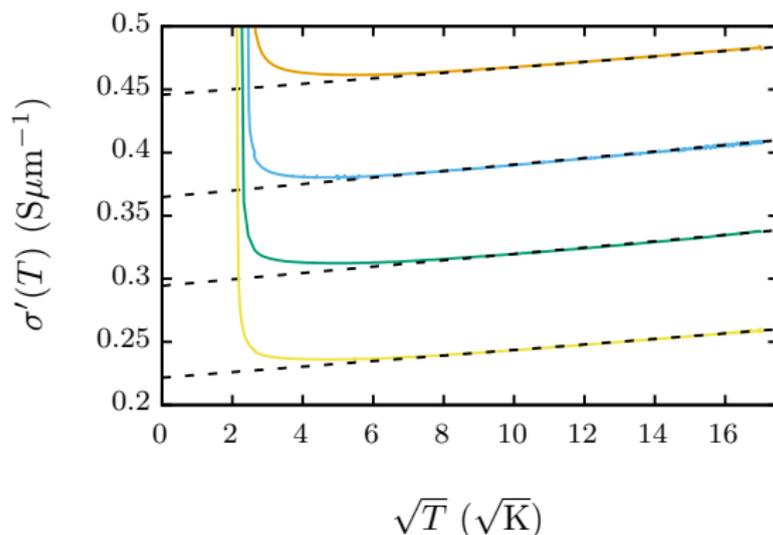
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- $\sigma = \sigma_0 + b\sqrt{T}$; 3D quantum corrections up to $T = 300K$
- How far do they extend? The temperature range is limited.

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 - for thin films: $t \ll c/\omega$, $t \ll c|g(\omega)|/\omega$, g' , $|g''| \lesssim 1$

$$\mathcal{T}(\omega) \approx \frac{\mathcal{T}_s(\omega)^2}{[1 + g'(\omega)/(n_s + 1)]^2 + [g''(\omega)/(n_s + 1)]^2}$$

- Our experimental frequency range 80-1000 THz

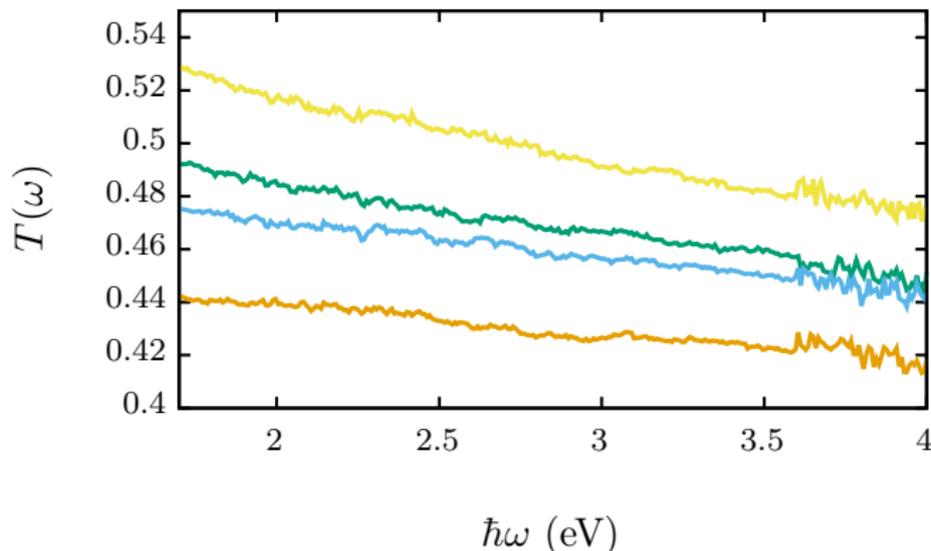
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- **Spectroscopic ellipsometry**
 - Reflection measurement, evaluation of σ_1 and σ_2
 - Our experimental frequency range 300-810 THz

Optical transmission spectra

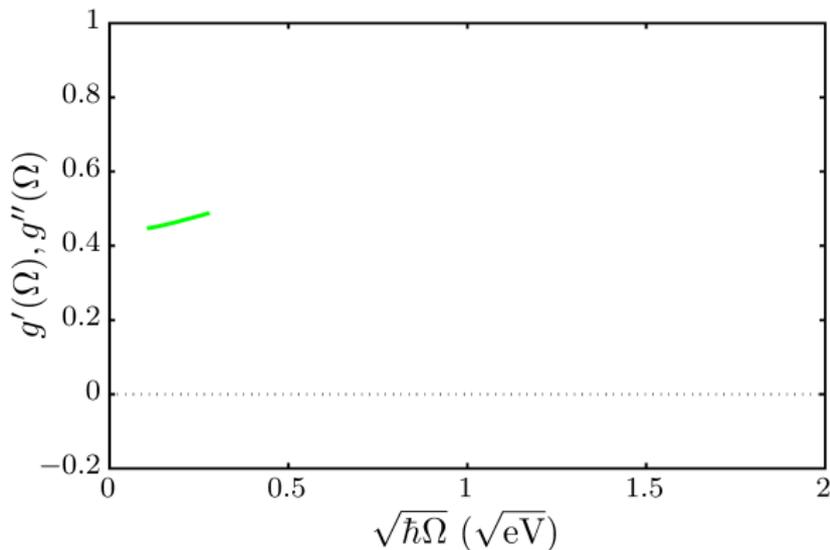
- 1. set of MoC: $R_{\square} = 420, 500, 590$ a 720Ω
- Smooth transmission spectra



- $T(\omega)$ decreases with frequency ω , the opposite is expected from the Drude model

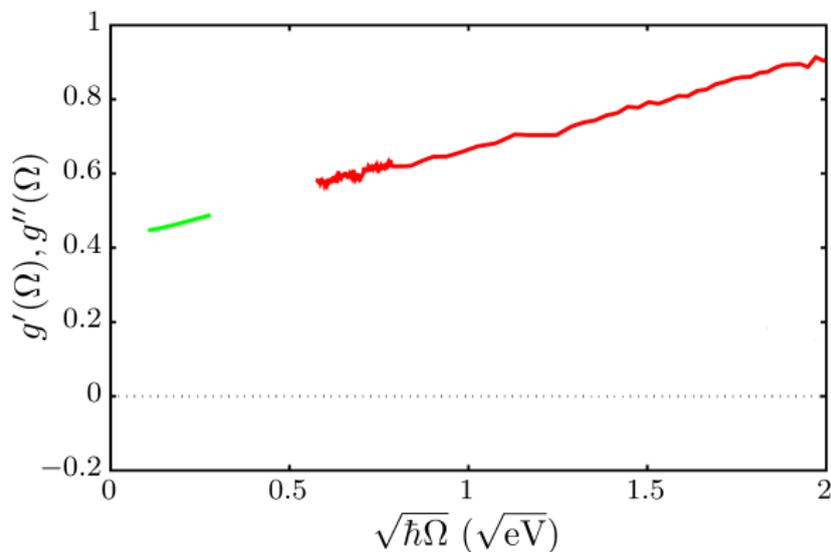
Optical conductivity of MoC with $R_{\square}=720\Omega$

- For interaction effects: $\Omega = \sqrt{\omega^2 + (\pi k_B T / \hbar)^2}$



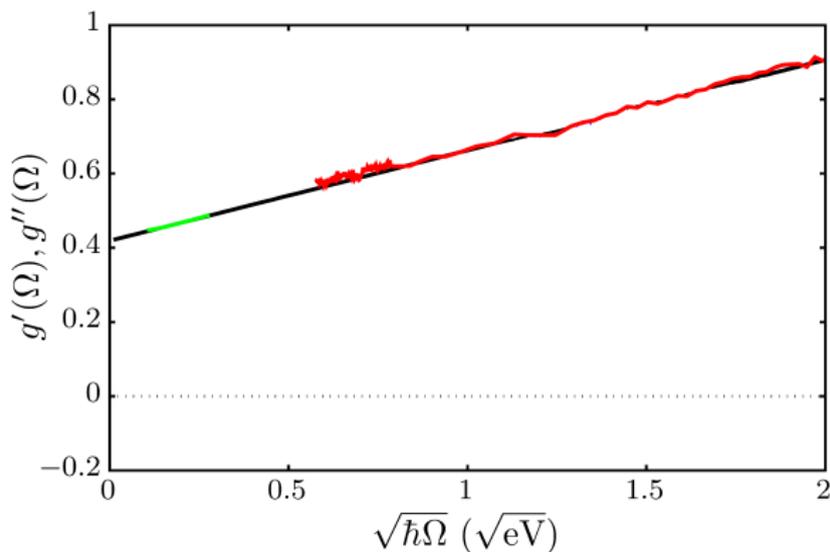
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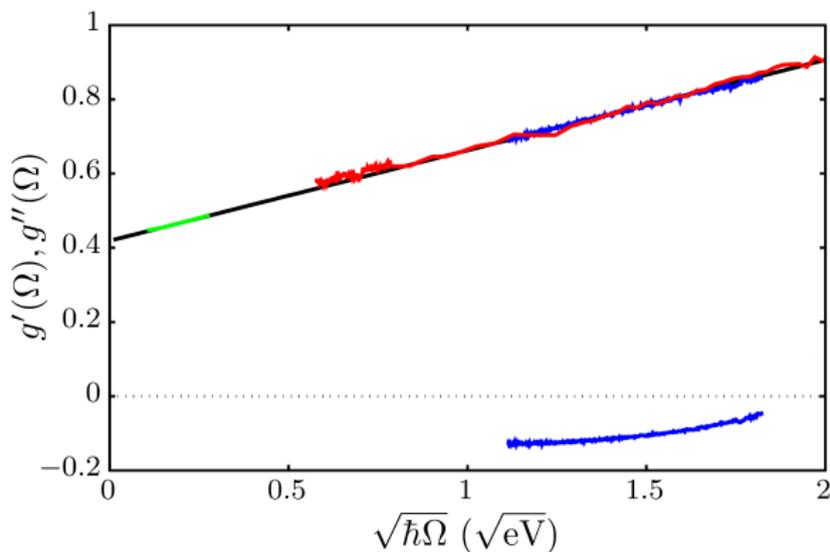
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- Agreement between 3 data sets, $\hbar\Omega \approx 14$ meV up to 4 eV

Proposed model

- Postulate for the σ :

$$\begin{aligned}\sigma'(\omega, T) &= \sigma_0 \left[1 - Q^2 + Q^2 \sqrt{\Omega/\Gamma} \right] && \text{if } \Omega < \omega^* \\ \sigma'(\omega, T) &= \frac{\sigma_0}{1 + (\Omega/\Gamma)^2} && \text{if } \Omega \geq \omega^*\end{aligned}$$

- 3 parameters:

1. Q - "Quantumness"
2. Γ - Scattering rate
3. $\sigma_0 = ne^2/m\Gamma$, whereas $\sigma(0, 0) = \sigma_0(1 - Q^2)$

- Fermi liquid theory, for interaction effects:

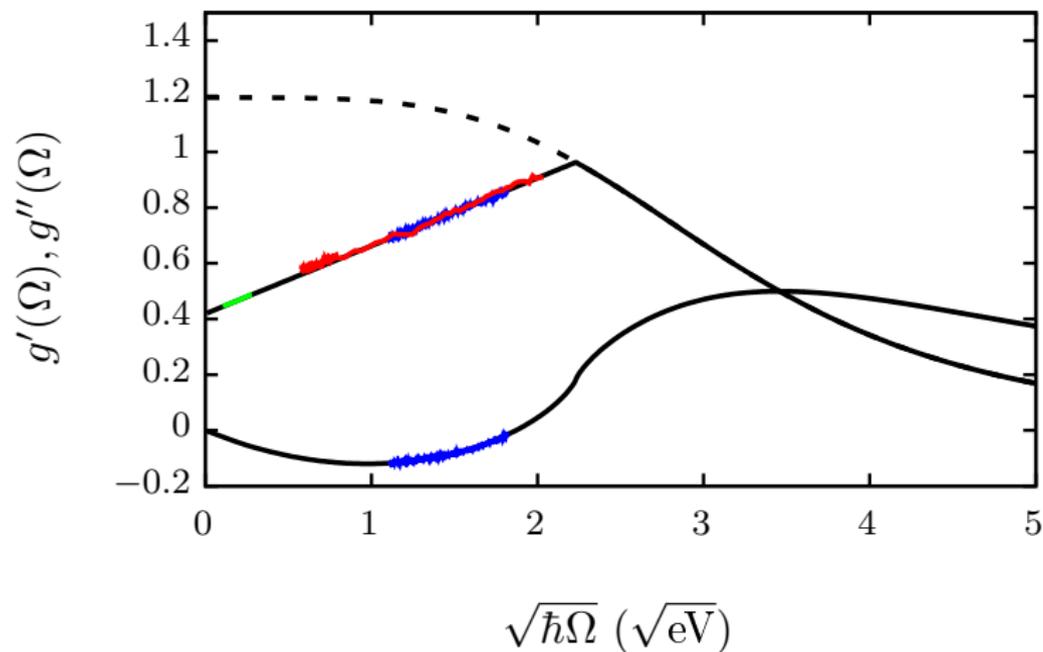
$$\Omega = \sqrt{\omega^2 + (\pi k_B T / \hbar)^2}$$

- Imaginary part σ'' given by Kramers-Kronig relations

- Contribution of bound electrons to permittivity

$$\sigma''_{\text{bound}}(\omega) = -\epsilon_0(\epsilon_\infty - 1)\omega, \quad \epsilon_\infty = 1.4$$

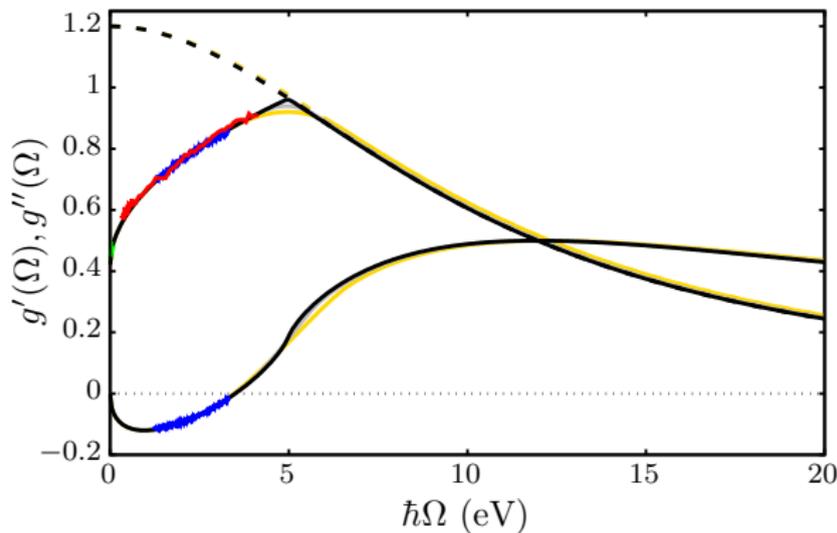
Conductivity $\sigma(\omega)$ fit



- Observation of quantum corrections to conductivity up to optical frequencies, Phys. Rev. B 100, 241106(R) (2019)

Smoothed Lorentzian model $\sigma(\omega)$

- Smoothing cups at $\omega = \omega^*$ - "tailoring" with 3rd-order polynomial $f(\omega)$ for $\omega_- < \omega < \omega_+$, for different ω_-, ω_+



Extracted parameters of MoC with $R_{\square}=720\Omega$

■ Lorentz vs. Gaussian

	Drude		Gaussian	
ϵ_{∞}	1.0	1.4	1.0	1.4
$g_0 = Z_0\sigma_0d$	1.25	1.20	1.32	1.26
Q^2	0.66	0.65	0.68	0.67
$\hbar\Gamma$ (eV)	11.5	10.1	13.6	12.0
n (10^{23} cm $^{-3}$)	4.1	3.5	5.1	4.3

■ Smoothened Lorentzian

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■ Scatter in σ_0 , Q , and $\Gamma \leq 15\%$

→ Prolongation procedure is robust

Characteristic parameters of MoC with $R_{\square}=720\Omega$

- $Q \approx 0.82 \pm 0.01$ strong quantum corrections
 - $\sigma'(0) = (1 - Q^2)\sigma_0 \approx \sigma_0/3$
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- Normalized conductivity $g_0 \approx 1.26 \pm 0.06$
 - ⇒ $n = (4.3 \pm 0.8) \times 10^{23} \text{ cm}^{-3}$
 - surprisingly large electron concentration

- Large $\Gamma \implies$ Electronic bands separated by $\leq \hbar\Gamma$ merge
 - Concentration of Mo $n_{\text{at}} = 5.1 \times 10^{22} \text{ cm}^{-3}$
 - Valence electron configuration within $\pm\hbar\Gamma$ from ε_F
Mo: $4d^5 5s^1$ a C: $2s^2 2p^2$
 - Corresponding electron density
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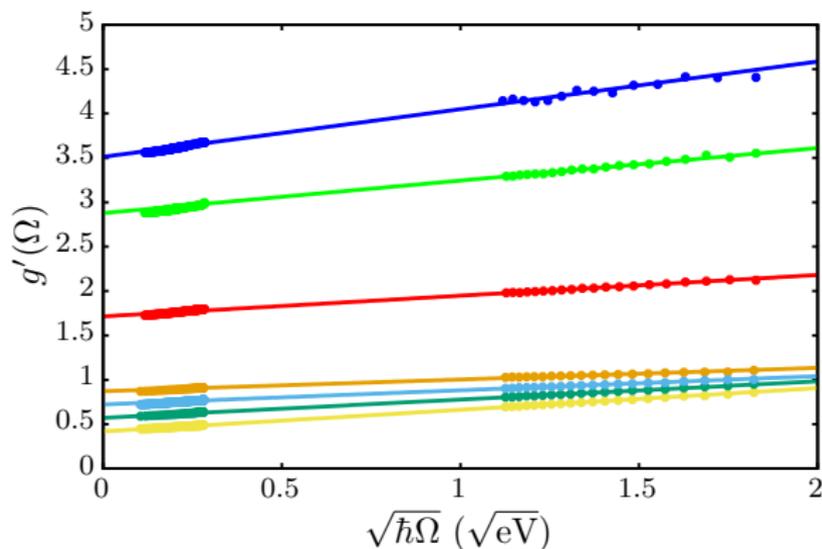
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- Absence of interband transitions \rightarrow Broad electronic band
- High Fermi energy $\epsilon_F = 20.65$, $k_F l \approx 3.5$
- Similarly, highly disordered NbN $n \approx 4.2 \times 10^{23} \text{ cm}^{-3}$
- Large n should be a generic property of dirty metals

Optical conductivity with the 2. set of MoC

- Broad range of $R_{\square} = 100, 120, 220, 420, 500, 590$ a 720Ω



- Details of microstructure are not important
- The control parameter is the degree of disorder

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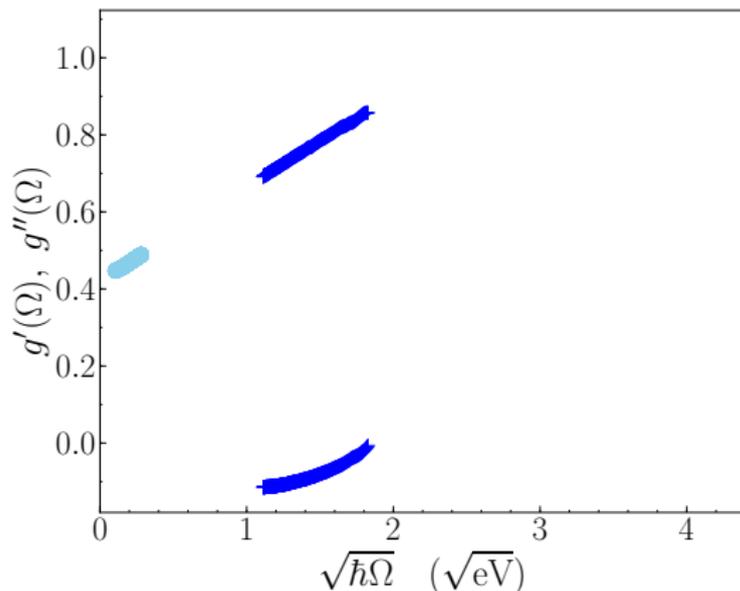
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- Numerical extrapolation method for complex conductivity of disordered metals, S. Kern et al., PRB **103**, 134205 (2021)

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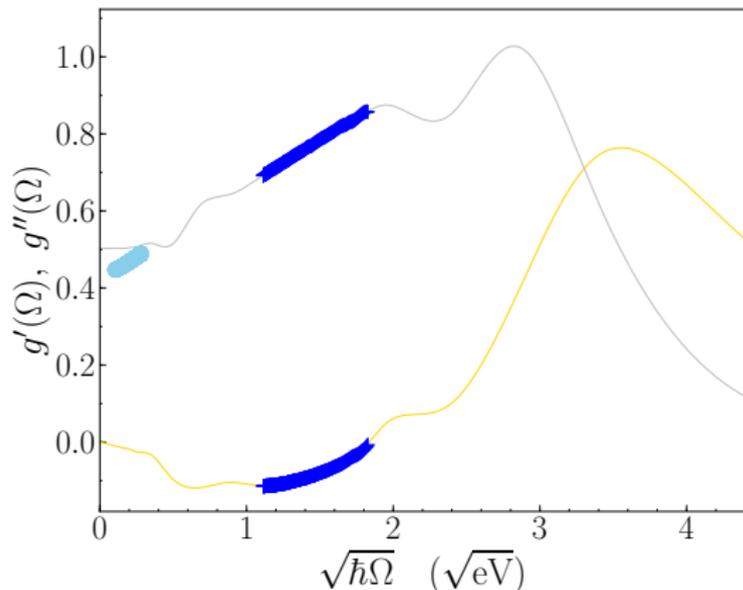
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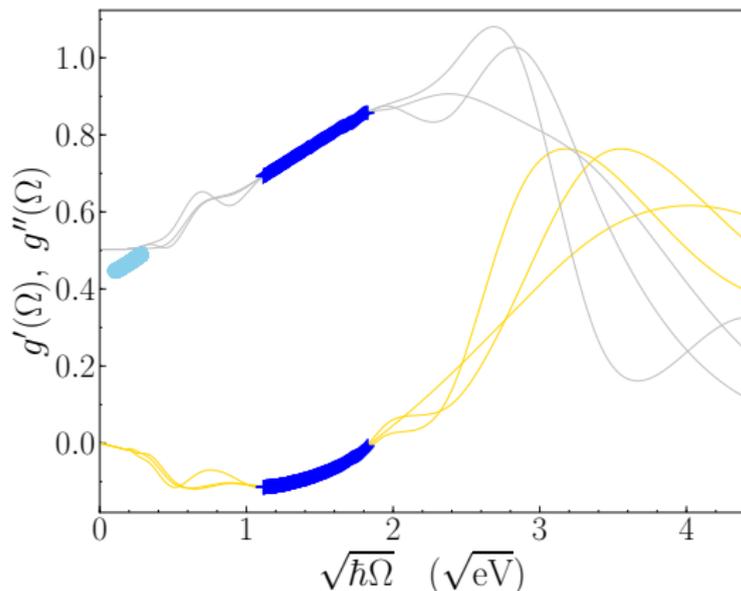
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Prolongation of optical conductivity

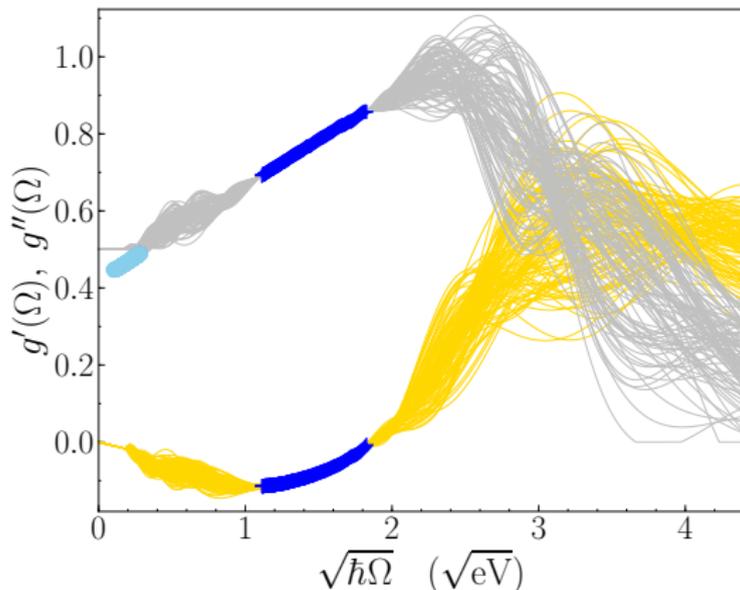
- Numerical extrapolation method for complex conductivity of disordered metals



- S. Kern et al., Phys. Rev. B **103**, 134205 (2021)

Prolongation of optical conductivity

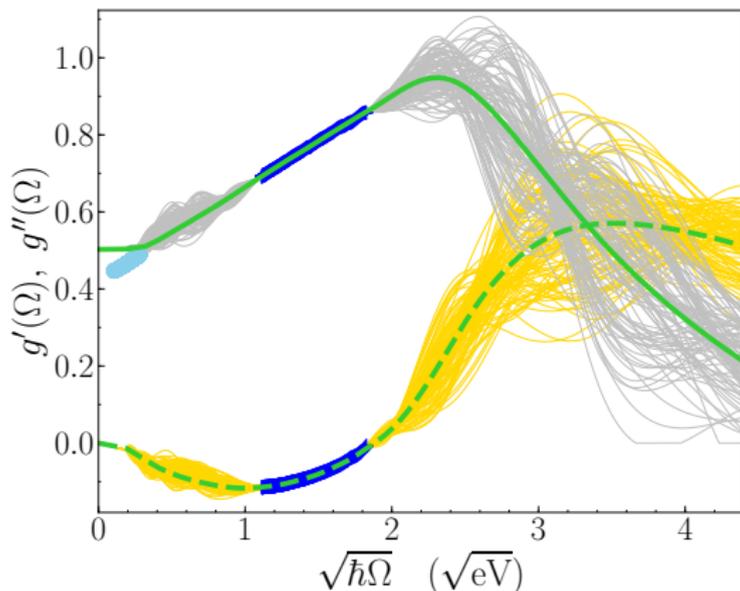
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- S. Kern et al., Phys. Rev. B **103**, 134205 (2021)

Prolongation of optical conductivity

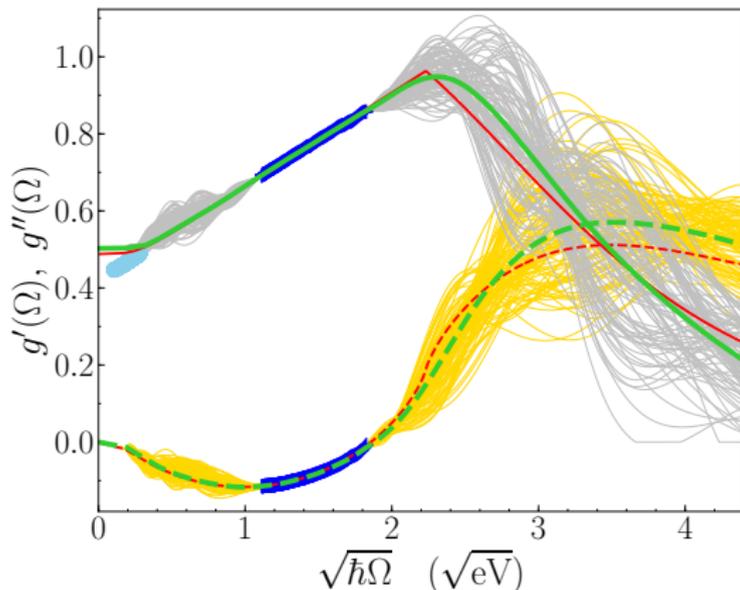
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- S. Kern et al., Phys. Rev. B **103**, 134205 (2021)

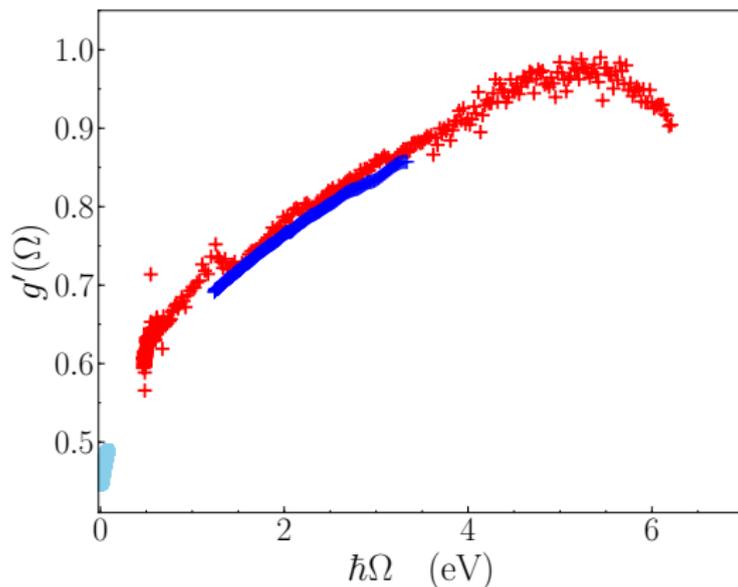
Prolongation of optical conductivity

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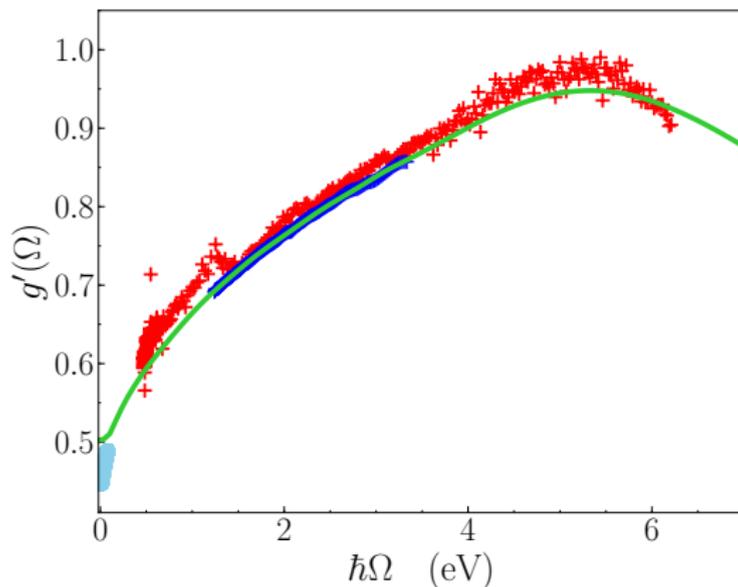
- S. Kern et al., Phys. Rev. B **103**, 134205 (2021)

- 5 nm MoC with $R_{\square}=720\Omega$



- Transmission spectra from 0.5 to 6 eV

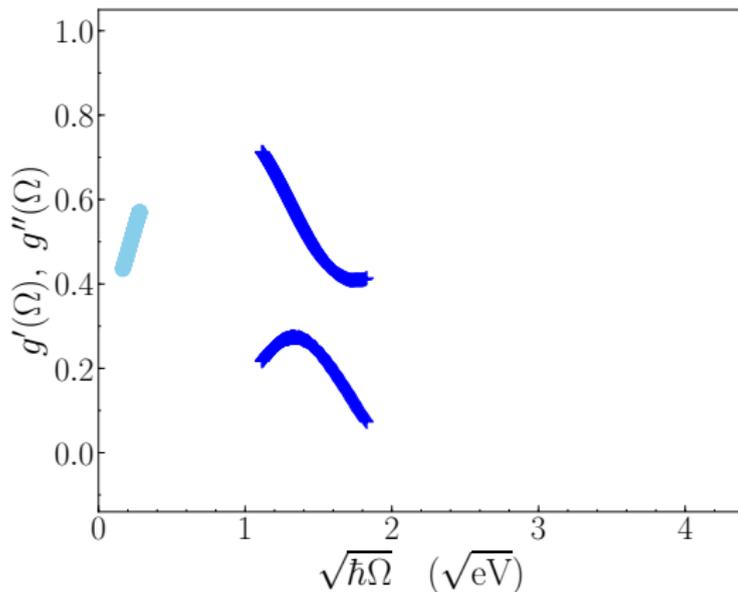
- 5 nm MoC with $R_{\square}=720\Omega$



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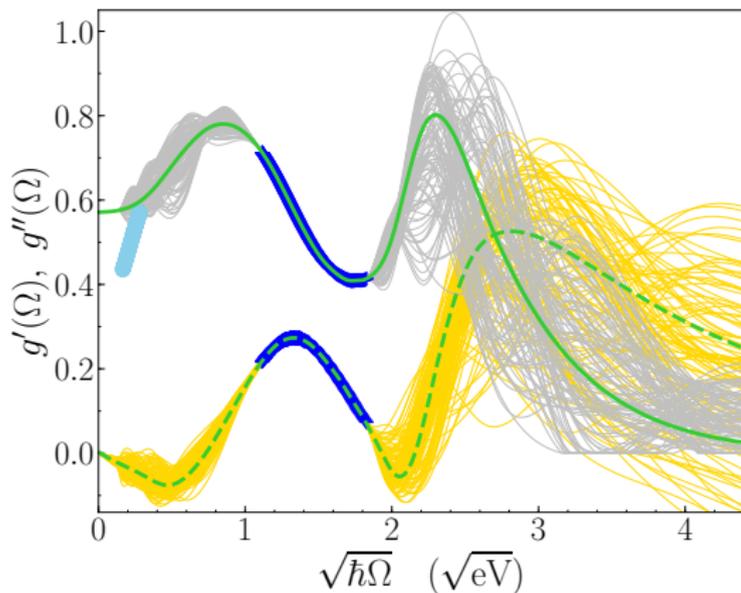
Optical conductivity of NbN

- Pulsed laser ablation; Nb-target, N₂ atmosphere
- Film thickness $t=3.5$ nm and $R_{\square}=655\Omega$



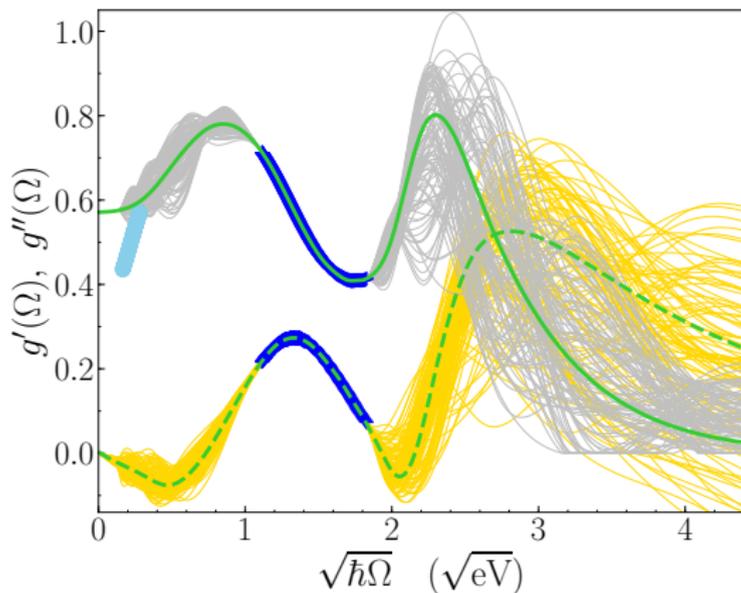
Numerical extrapolation of optical conductivity

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Numerical extrapolation of optical conductivity

- Pulsed laser ablation; Nb-target, N₂ atmosphere
- Film thickness $t=3.5$ nm and $R_{\square}=655\Omega$



- Strong deviation from Drude-Lorentz model

- Observation of quantum corrections to conductivity up to optical frequencies in highly disordered films
- Method to extract σ_0 , Q and Γ from the combined knowledge $\sigma'(\omega)$ and $\sigma''(\omega)$
- Estimation of the magnitude of the quantum correction
$$\delta\sigma' = -Q^2\sigma_0$$
- Numerical extrapolation method for complex conductivity of disordered metals

Thank you for your attention!

Neilinger et al., Phys. Rev. B **100**, 241106 (R) (2019)
S. Kern et al., Phys. Rev. B **103**, 134205 (2021)