Transmission Based Characterisation of Superconducting Metamaterial

Samuel Kern, Pavol Neilinger, Daniel Manca, Evgeni Il'ichev, Matthias Schmelz, Jürgen Kunert, Gregor Oelsner, Ronny Stolz and Miroslav Grajcar

Superconducting Hybrids @ Extreme (June 28-July 2, 2021, Hotel Patria, Štrbské Pleso, Slovakia)

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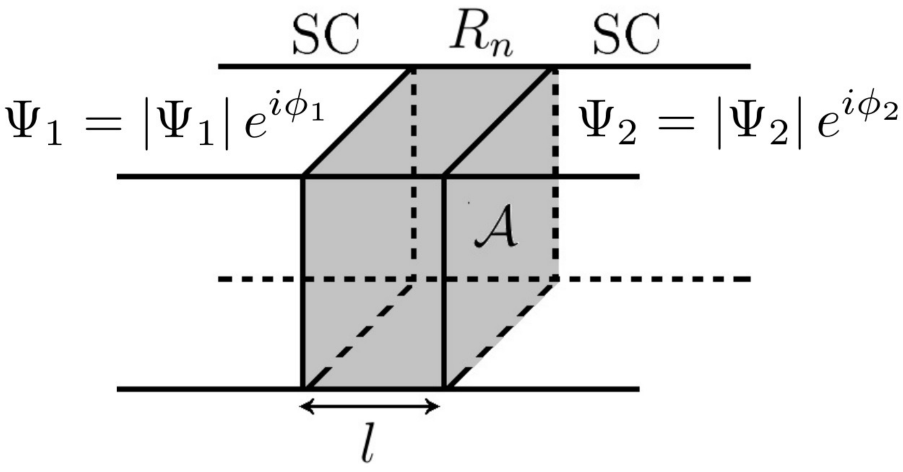
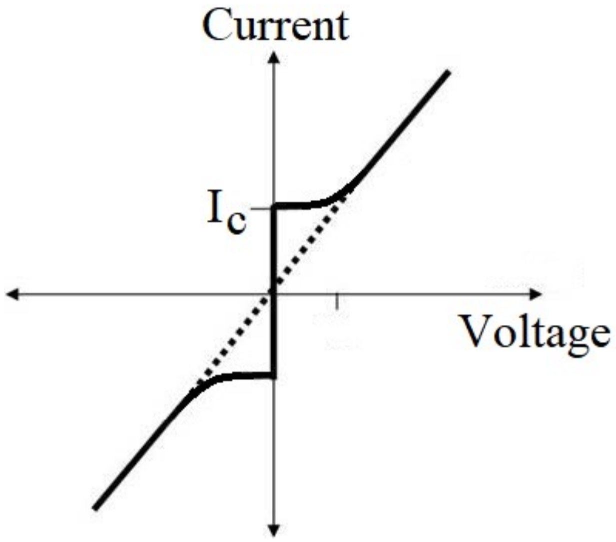
Jospehson junction





- Nonlinear inductance

$$I_s = I_c \sin(\phi), \text{ where } \phi = \phi_1 - \phi_2$$

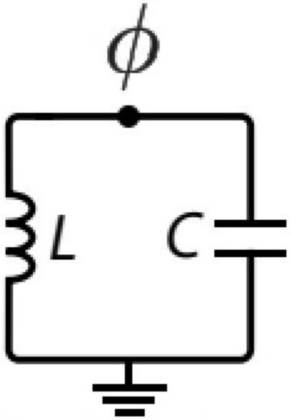
$$L = L_J\left(1 + \left(\frac{I}{I_c}\right)^2\right), \quad \text{where} \quad L_J = \frac{\Phi_0}{2\pi I_c}$$

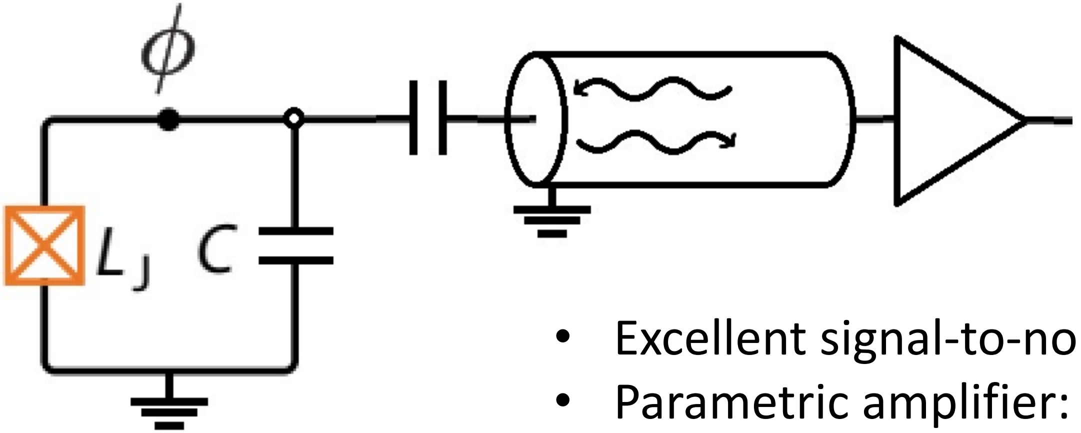
- Temperature dependance

$$I_c R_n = \frac{\pi \Delta(T)}{2e} \tanh\left(\frac{\Delta(T)}{2kT}\right)$$

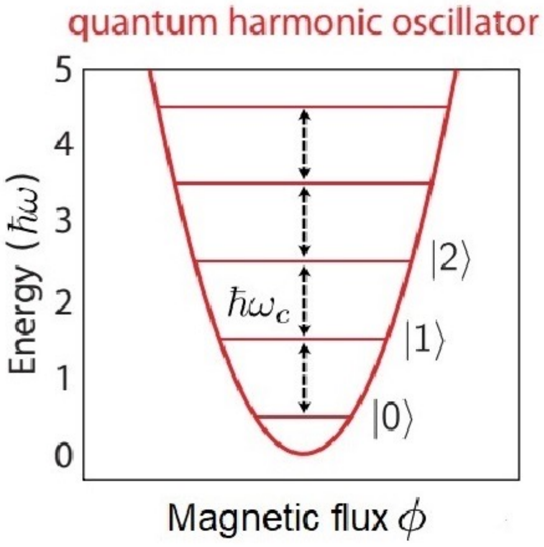
$$\frac{\Delta(T)}{\Delta(0)} = \tanh\left(1.74\sqrt{\frac{T_c}{T} - 1}\right)$$

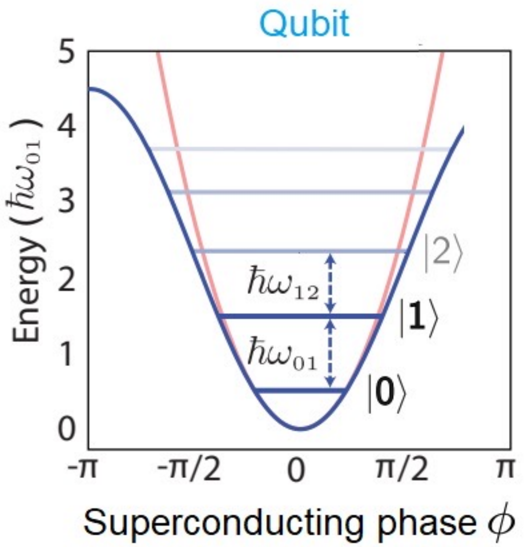
Superconducting qubit

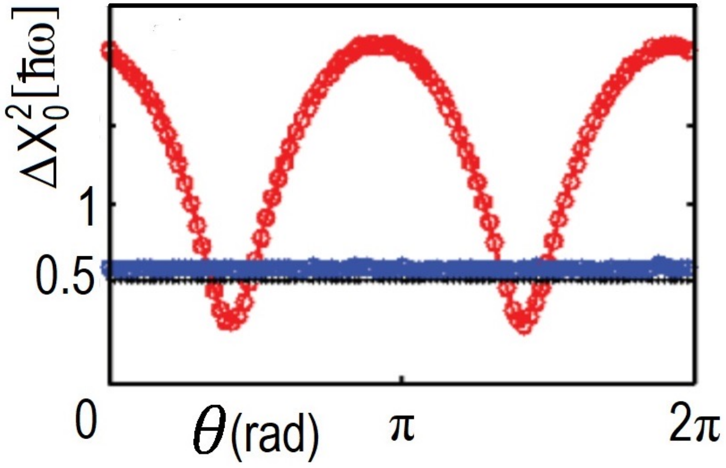




- Excellent signal-to-noise ratio required
- Parametric amplifier: phase-sensitive regime
  - Beyond the quantum limit

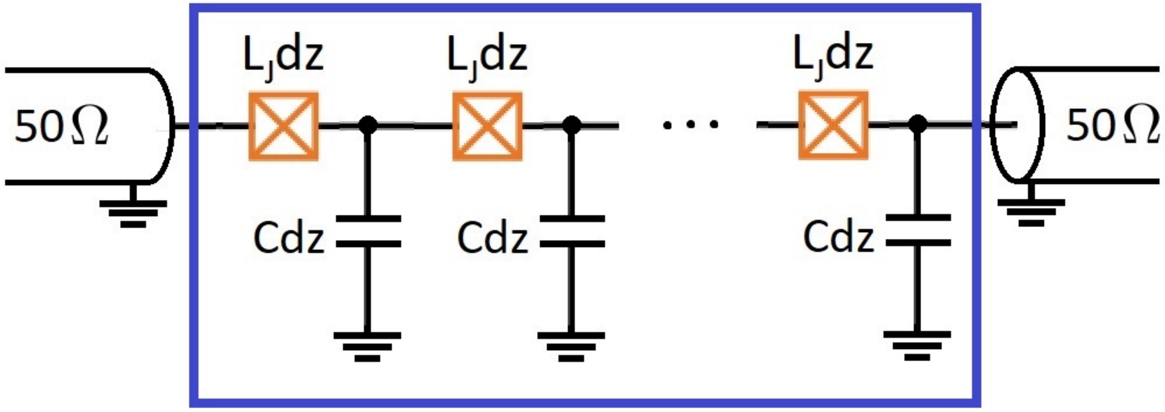






Mallet, F. et al. Quant. St. Tomogr. of an Itinerant. Sqeezed Microw. Field. *Phys. Rev. Lett*. (2011)

Parametric amplifier



$Z_0 \approx 200\ \Omega$

- Nonlinear telegrapher's equation

$$\frac{\partial^2 I}{\partial z^2} - L_0 C \frac{\partial^2 I}{\partial t^2} = \frac{L_0 C}{6 I_J^2} \frac{\partial^2 I^3}{\partial t^2}$$

- Coupled mode theory

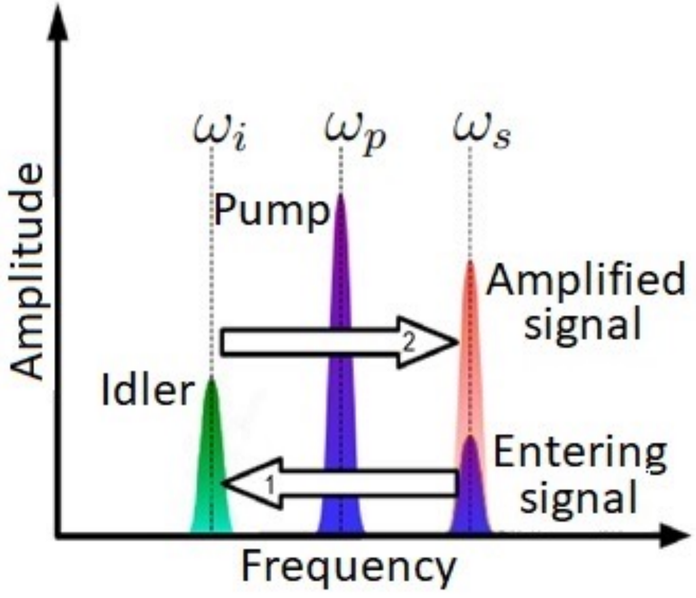
Phase matching β=0

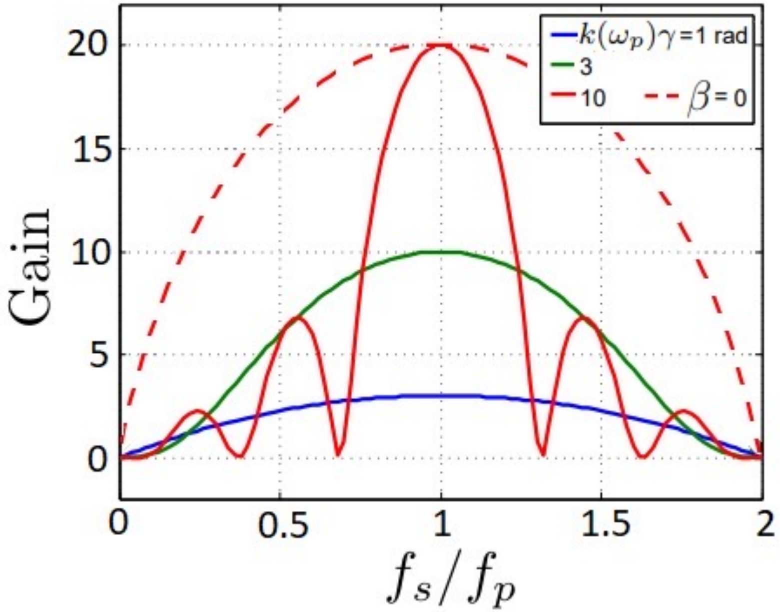
$$\gamma = \frac{|I_p|^2}{16 I_J^2}$$

$$\beta = \Delta k(1+2\gamma) - 2k(\omega_p)\gamma$$

$$K = \sqrt{k(\omega_s)k(\omega_i)\gamma^2 - \frac{\beta^2}{4}}$$

$$G \equiv \frac{P_s(z)}{P_s(0)} = \left(cosh^2(Kz) + \left(\frac{\beta}{2K}\right)^2 sinh^2(Kz)\right)$$





Metamaterial

- Dispersion relation

$$\begin{pmatrix} V^+(x+l) \\ V^-(x+l) \end{pmatrix} = M_e \begin{pmatrix} V^+(x) \\ V^-(x) \end{pmatrix}$$

$$M_e = \begin{pmatrix} e^{i\frac{n_a \omega}{v_p} l_a} & 0 \\ 0 & e^{-i\frac{n_a \omega}{v_p} l_a} \end{pmatrix} M_{ab}^{-1} \begin{pmatrix} e^{i\frac{n_b \omega}{v_p} l_b} & 0 \\ 0 & e^{-i\frac{n_b \omega}{v_p} l_b} \end{pmatrix} M_{ab}$$

$$2lk = \arccos\left(\mathrm{Tr} M_e\right)$$

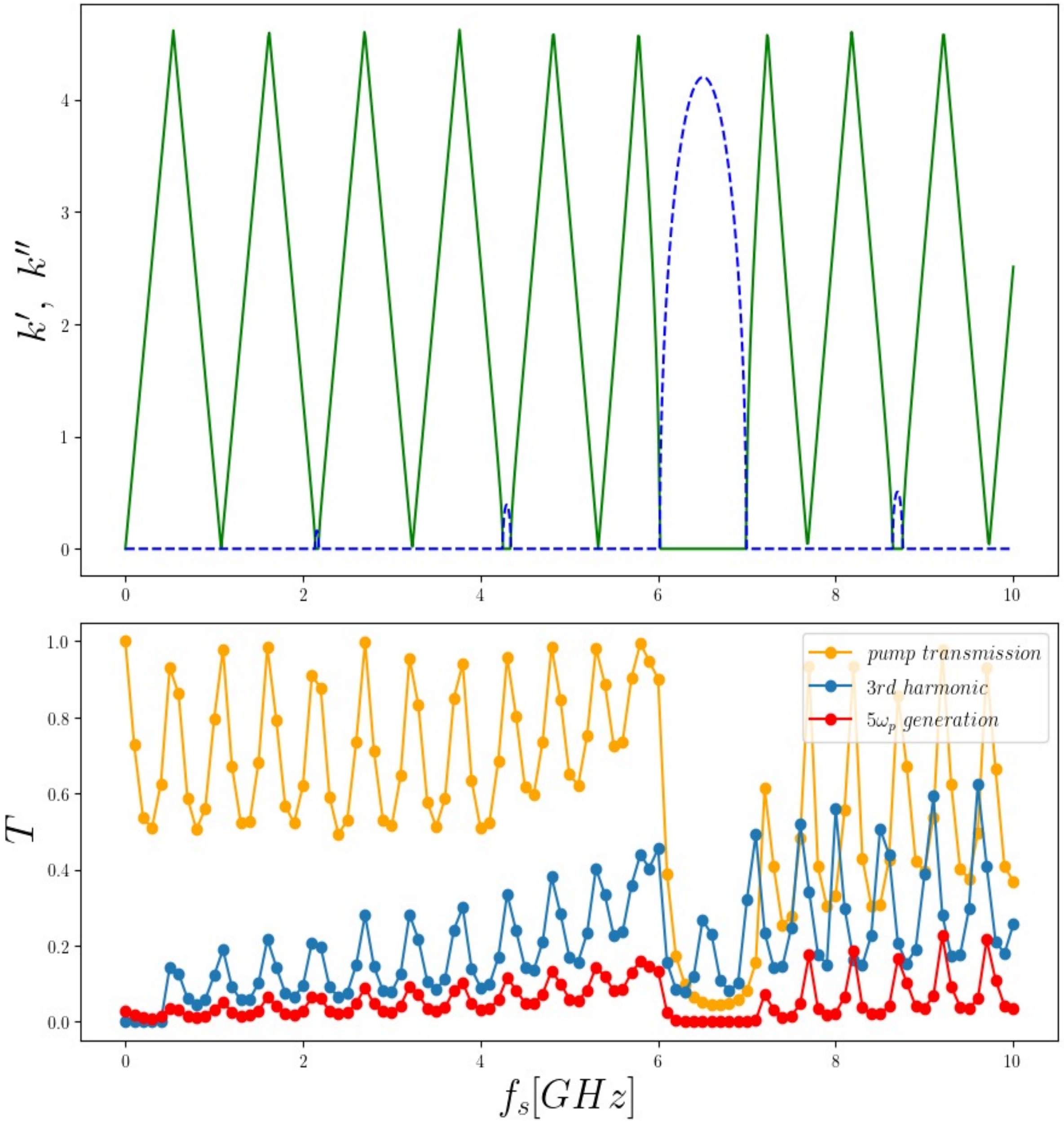
- Finite difference approach

$$\tilde{V}_i(\tau + \mathrm{d}\tau) = \tilde{V}_i(\tau) - \nu(\xi)\lambda(\xi)\frac{\mathrm{d}\tau}{\mathrm{d}\xi}(\tilde{I}_i(\tau) - \tilde{I}_{i-1}(\tau))$$

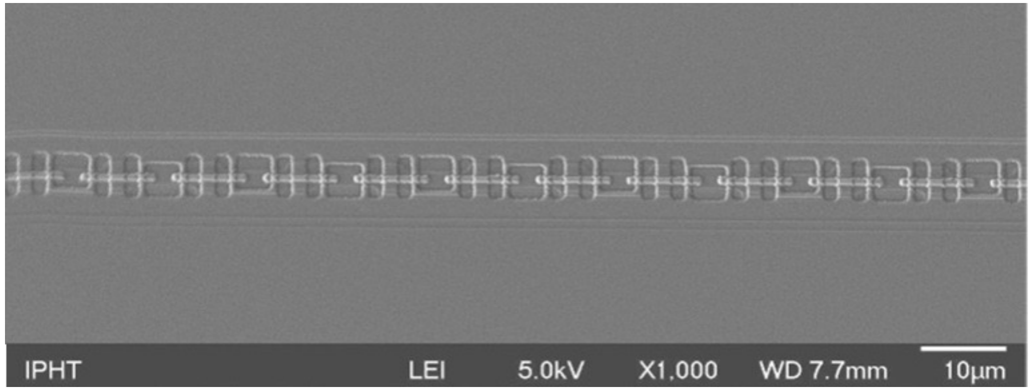
$$\tilde{I}_i(\tau + \mathrm{d}\tau) = \tilde{I}_i(\tau) - \frac{\nu(\xi)}{\lambda(\xi)(1 + \tilde{I}_i^2(\tau)/\tilde{I}_c)}\frac{\mathrm{d}\tau}{\mathrm{d}\xi}(\tilde{V}_i(\tau) - \tilde{V}_{i+1}(\tau))$$

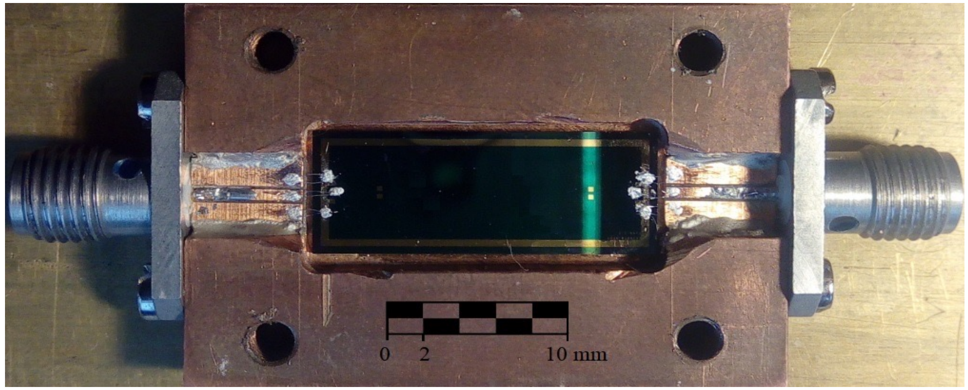
$$\tilde{I}_N(\tau + \mathrm{d}\tau) = \tilde{I}_0/\nu(0)$$

$$\tilde{V}_0(\tau + \mathrm{d}\tau) = \nu(0)\tilde{I}_0 + S(\tau)$$



Experiment

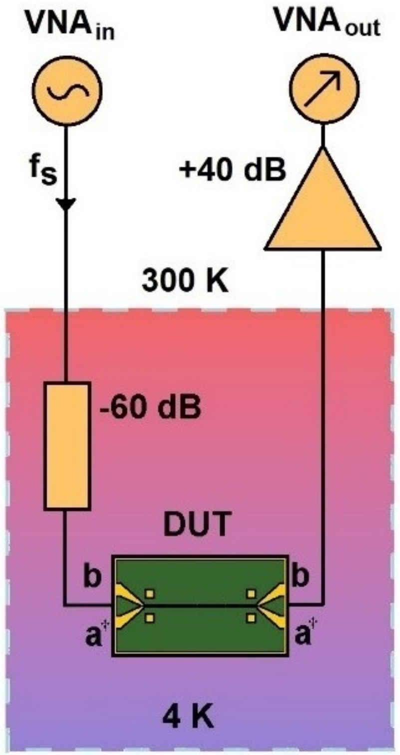




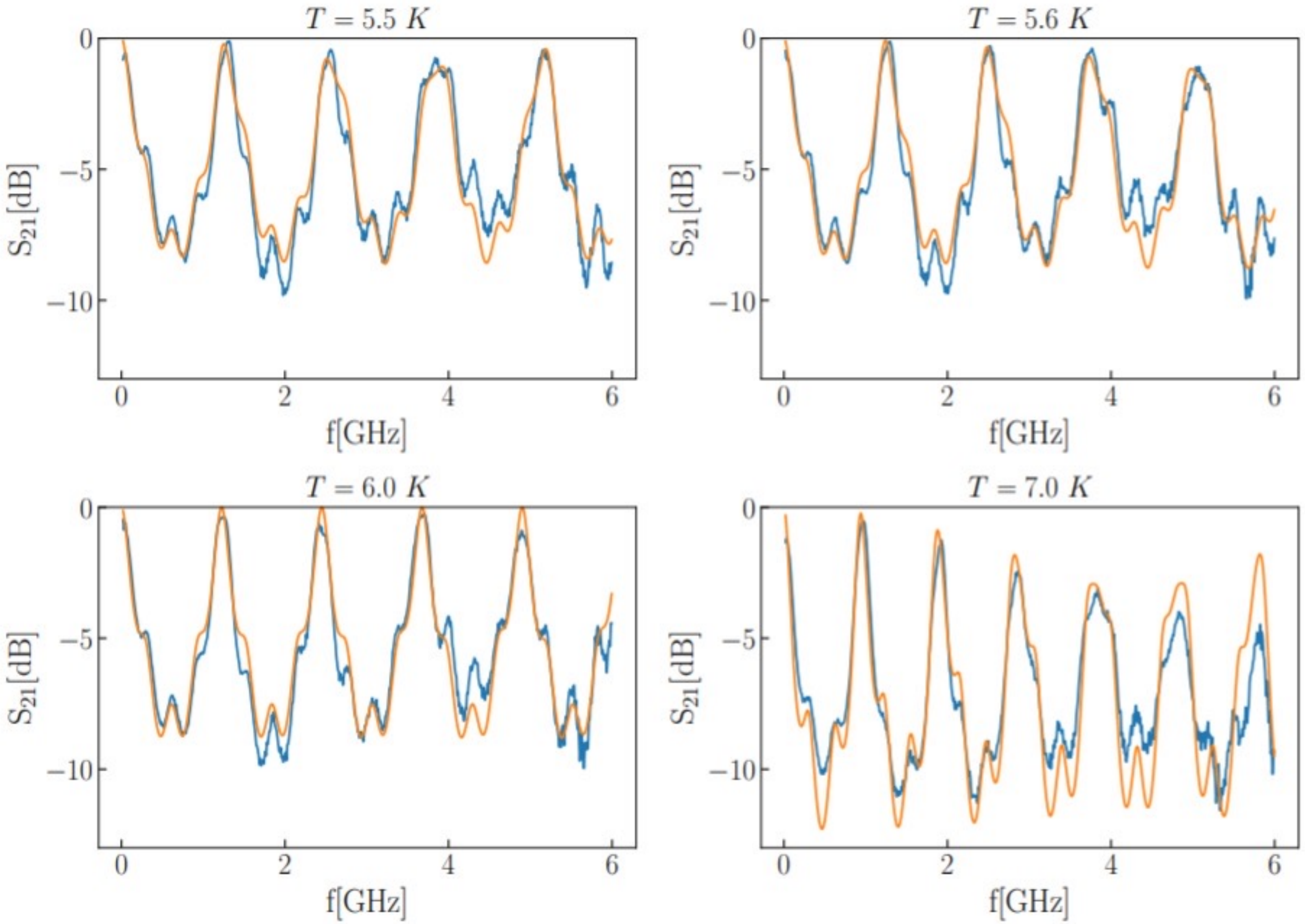
- Coplanar waveguide - middle wire: array of niobium JJs
- 50 Ω RF measurement tract inside sorption He-3 refrigerator
- Tranmsmission measured at temperatures 1 K – 9 K and signal power -30, -27, . . . , -12 dBm
- Impedance mismatch ⇒ Fabry-Perrot resonances

$$\Gamma = (Z_0 - Z_L)/(Z_0 + Z_L)$$

$$T(f, v_p, \Gamma) = \frac{(1-\Gamma^2)^2}{(1-\Gamma^2\cos(2\frac{2\pi f}{v_p}l))^2 + (\Gamma^2\sin(2\frac{2\pi f}{v_p}l))^2}$$



Transmission analysis



$$S_{21} = \frac{S_{21}^0 S_{21}^1}{1 - S_{22}^0 S_{11}^1 e^{-2iq_0 l}} \approx S_{21}^0 S_{21}^1 =$$

$$= T(f, v_p, \Gamma) T(f, v_p', \Gamma')$$

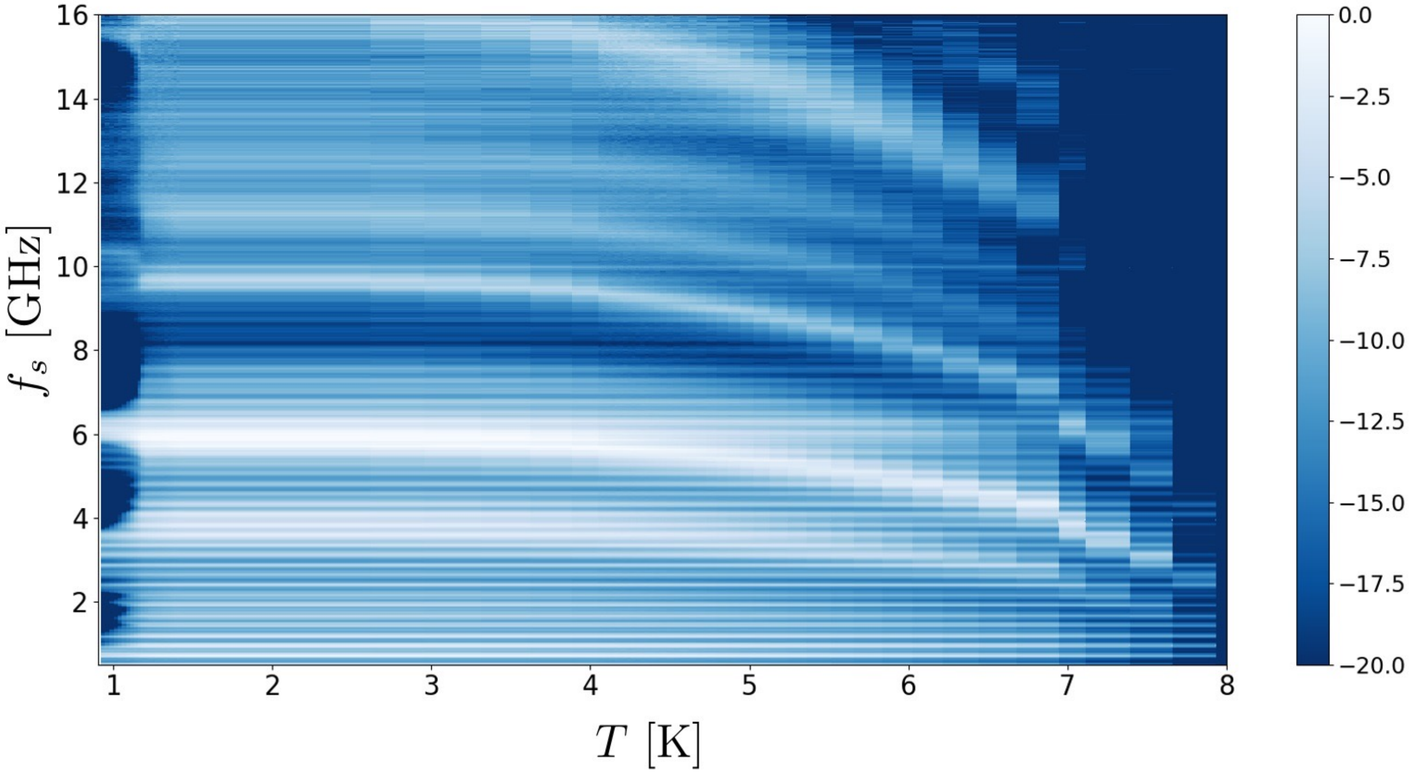
$$v_p = \frac{1}{LC} \qquad Z_0 = \frac{L}{C}$$

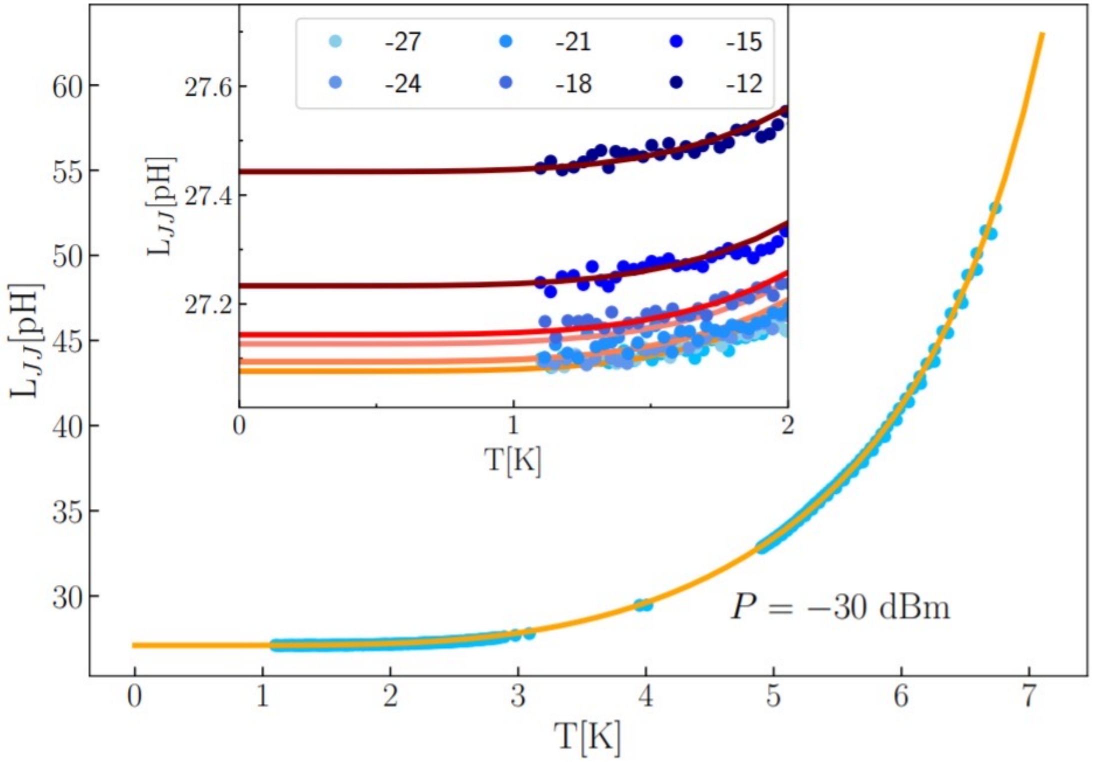
Z_0 [Ω]	v_p [c]	L [pH/μm]	C [fF/μm]
179	0,11	5,24	0,16

$P = -30$ dBm

$T \to 0$ K

Temperature dependance





Josephson junction properties

R_n [Ω]	T_c [K]	$\Delta(0)$ [meV]	I_c [μA]	J_c [A/cm^2]
179	8,65	1,40	12,2	1500

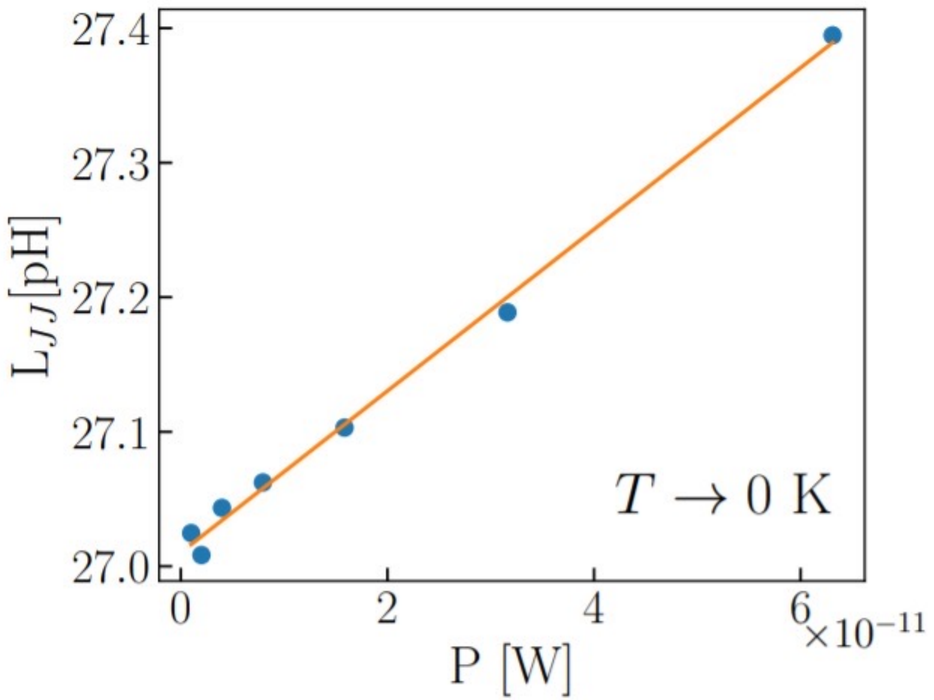
$$\Delta(0) \approx 1.76\, k_B T_c$$

Nonlinearity in power

$$I^2 = 2QT(f, v_p, \Gamma)\frac{P_{res}}{Z_0}, \quad \text{where} \quad P_{res} \equiv \alpha P_{in} \neq P_{in}$$

$$L = L_J\left(1 + \left(\frac{I}{I_c}\right)^2\right), \quad \text{where} \quad L_J = \frac{\Phi_0}{2\pi I_c}$$

$$L_{JJ}(T \to 0, P_{in}) = \frac{\Phi_0}{2\pi I_c} + \frac{\Phi_0}{2\pi I_c^3}\frac{\alpha P_{in}}{Z_0}$$



Power calibration

$$\alpha = -4.7 \text{ dB}$$

Additional attenuation of the measurement tract input line

Thank you

Acknowledgements: This work was supported by the Slovak Research and Development Agency under the contract APVV-16-0372, APVV-18-0358 and by the QuantERA grant SiUCs, by SAS-MTVS.