Transmission Based Characterisation of Superconducting Metamaterial

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Abstract

Waveguides with superconducting Josephson junction-based metamaterial are widely used as parametric amplifiers in experiments on superconducting qubits and microwave circuit quantum electrodynamics. However, the precise estimation of power entering the device is crucial for the estimation of the amplifier's gain and noise temperature. This is nontrivial when the measurement tract is not symmetrical. We present a basic framework for the analysis of the properties of such nonlinear systems and the calibration of the input power. Utilizing measurements with varied temperature and input signal power, we estimate the additional attenuation of the input line, thus demonstrating the precise calibration procedure. Moreover, the properties of the metamaterial are estimated.

Introduction

- Inductance of Josephson junction (JJ) is nonlinear in current:

$$L = L_J\left(1 + \left(\frac{I}{I_c}\right)^2\right), \quad \text{where} \quad L_J = \frac{\Phi_0}{2\pi I_c}. \tag{1}$$

- The Josephson inductance is temperature dependent, as $I_c = I_c(T)$.
- BCS temperature dependence of $I_C(T)$ [1])

$$I_cR_n = \frac{\pi\Delta(T)}{2e}\tanh\left(\frac{\Delta(T)}{2kT}\right), \tag{2}$$

- Temperature-dependance of superconducting gap $\Delta(T)$ is

$$\frac{\Delta(T)}{\Delta(0)} = \tanh\left(1.74\sqrt{\frac{T_c}{T} - 1}\right). \tag{3}$$

- Classical nonlinear electrodynamics model of JJ array:

  Transmission line (TL) with nonlinear inductance
  - phase velocity $v_p = \frac{1}{LC} \approx 0.1c$
  - phase velocity $Z_0 = \frac{L}{C} \approx 200\ \Omega$

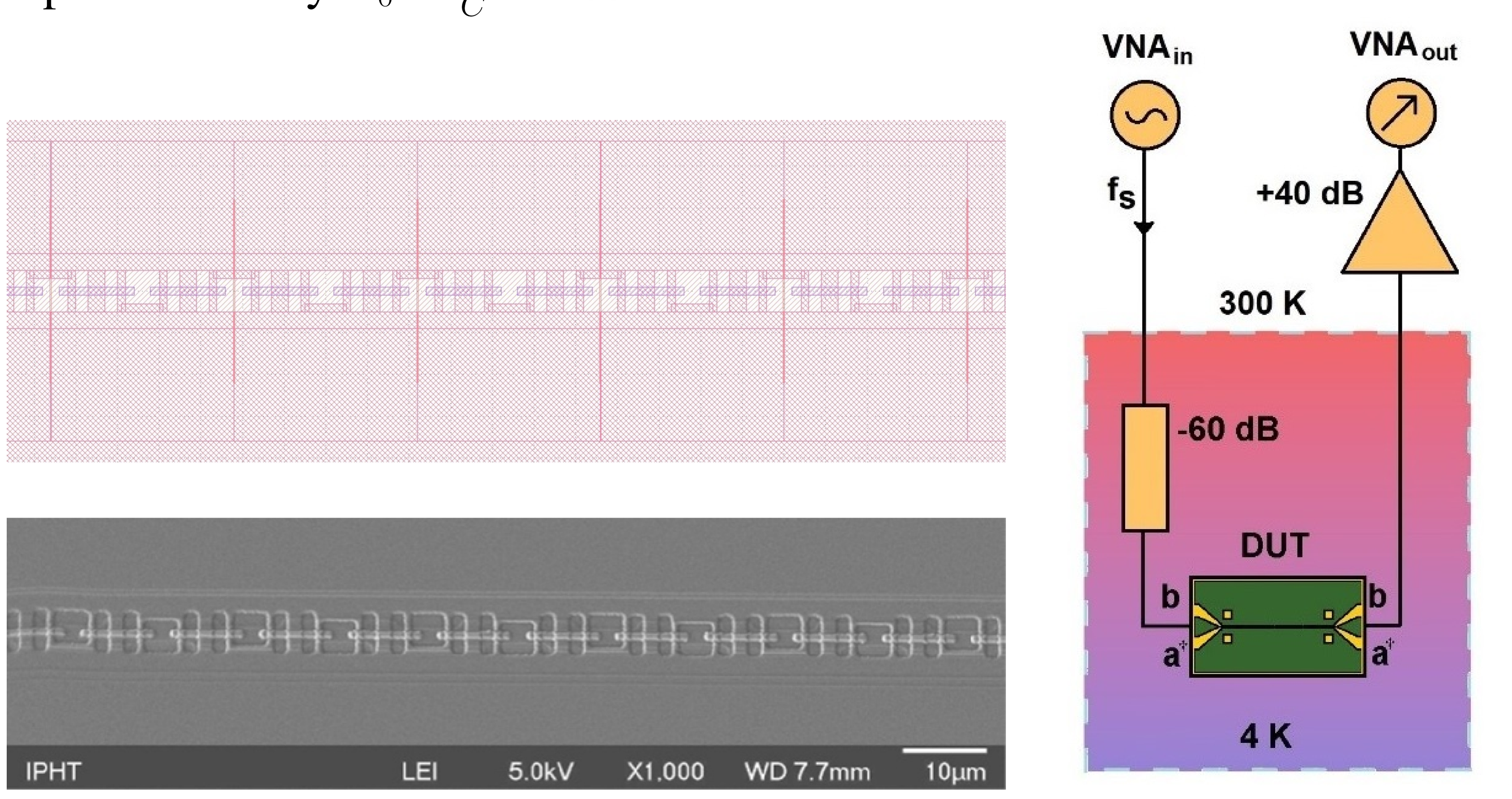

Figure 1: Left: Design of array of $0.7 \times 0.7\ \mu m^2$ niobium JJs (Top). SEM image of the middle wire of the waveguide with JJs (Bottom). Right: Scheme of the measurement set-up.

Experiment

- Coplanar waveguide - middle wire: array of niobium JJs with aluminium oxide barrier Fig 1 (left) [2].
- The sample characterised in $50\ \Omega$ RF measurement tract inside sorption He-3 refrigerator Fig. 1 (right).
- Tranmmission (Fig 2.) measured at temperatures $1\ \text{K} - 9\ \text{K} \approx T_c$ and signal power -30, -27, . . . , -12 dBm.
- Impedance mismatch ⇒ Fabry-Perrot resonances with reflection coefficients $\Gamma = (Z_0 - Z_L)/(Z_0 + Z_L)$:

$$T(f, v_p, \Gamma) = \frac{(1-\Gamma^2)^2}{(1-\Gamma^2\cos(2\frac{2\pi f}{v_p}l))^2 + (\Gamma^2\sin(2\frac{2\pi f}{v_p}l))^2}. \tag{4}$$

- The transmission analysed as two scatering elements in series

$$S_{21} = \frac{S^0_{21}S^1_{21}}{1 - S^0_{22}S^1_{11}e^{-2iq_0l}} \approx S^0_{21}S^1_{21} = T(f, v_p, \Gamma)T(f, v_p', \Gamma') \tag{5}$$

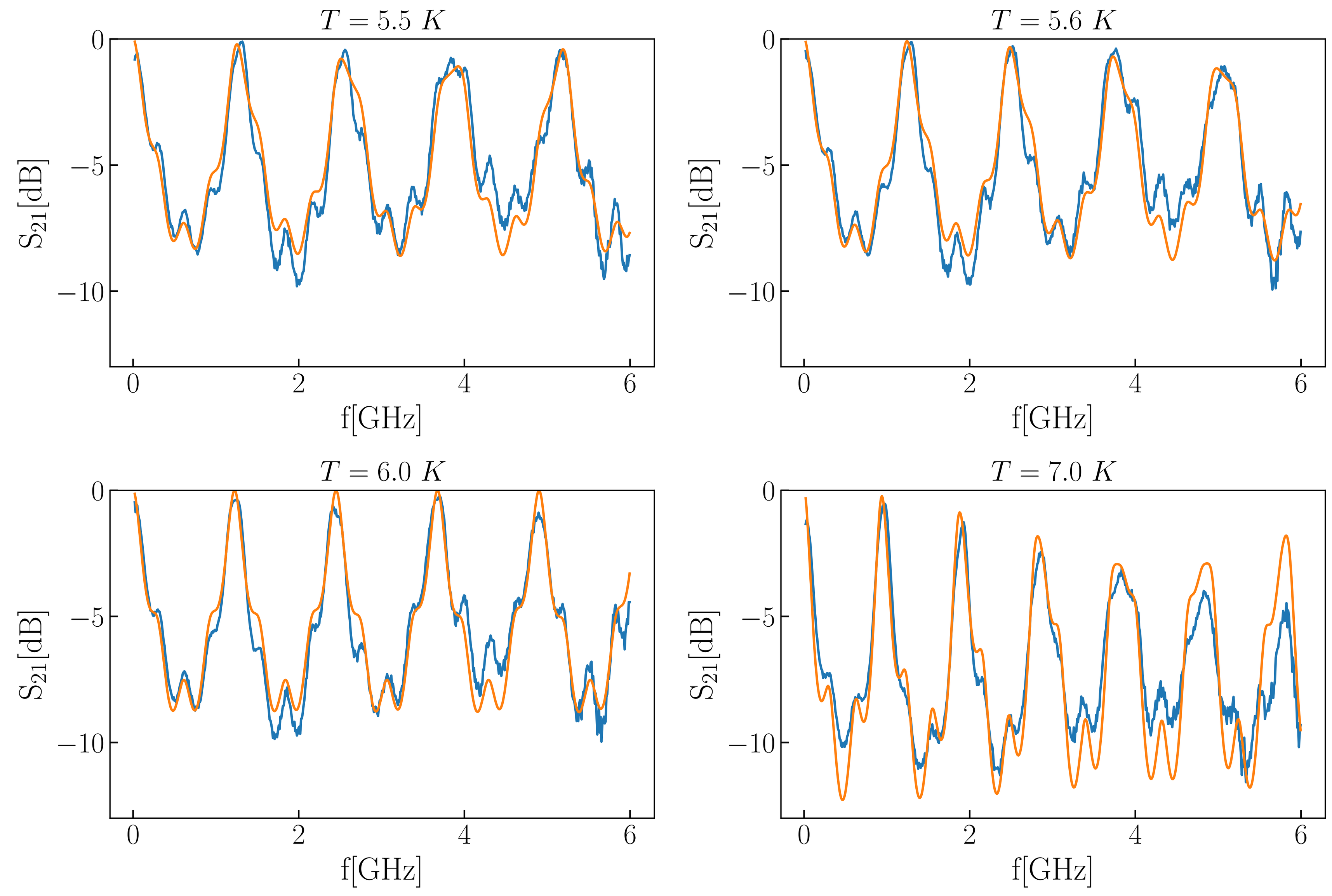

Figure 2: Transmission (S_{21} parameter) of the waveguide at different temperatures. Decrease of phase velocity is caused by temperature dependent Josephson inductance.

Results

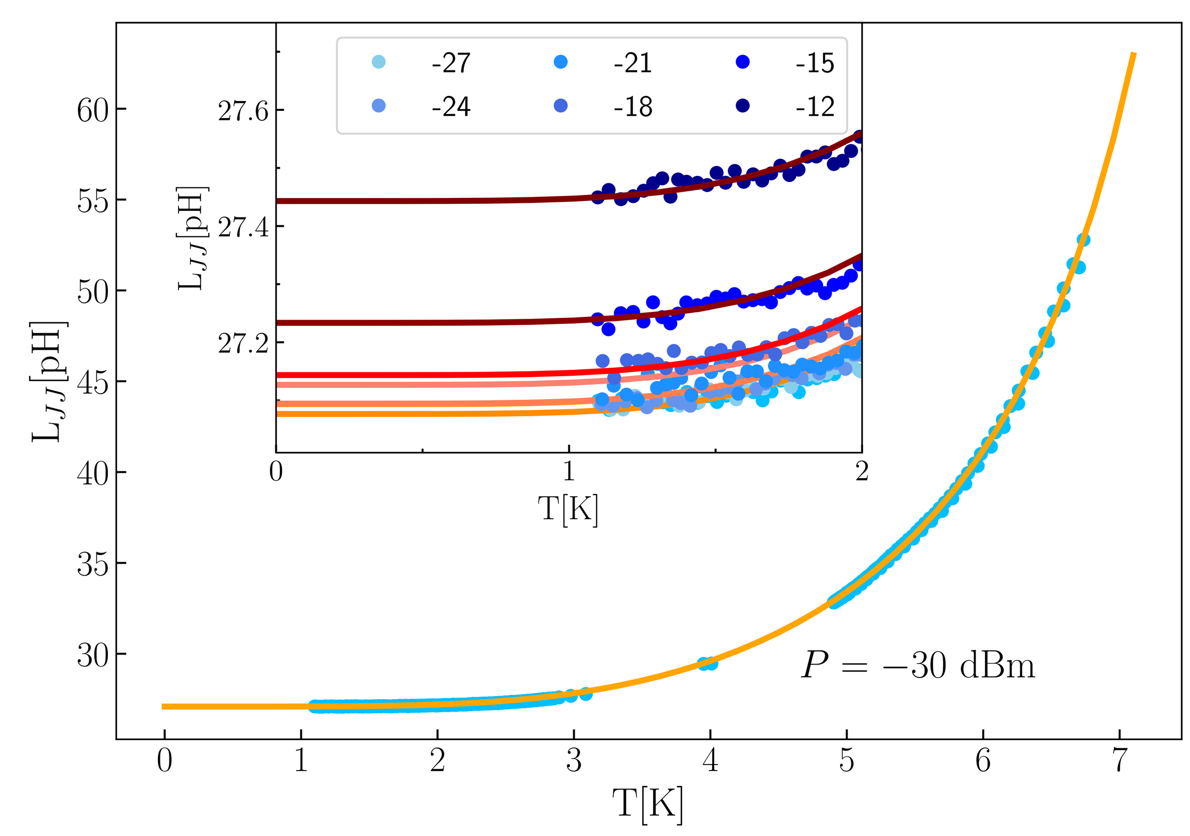

Figure 3: Inductance of single JJ determined from from transmission. Temperature dependence is fitted to equations (2) and (3) and superconducting parameters of JJ is obtained. Inset shows the fit for different signal powers.

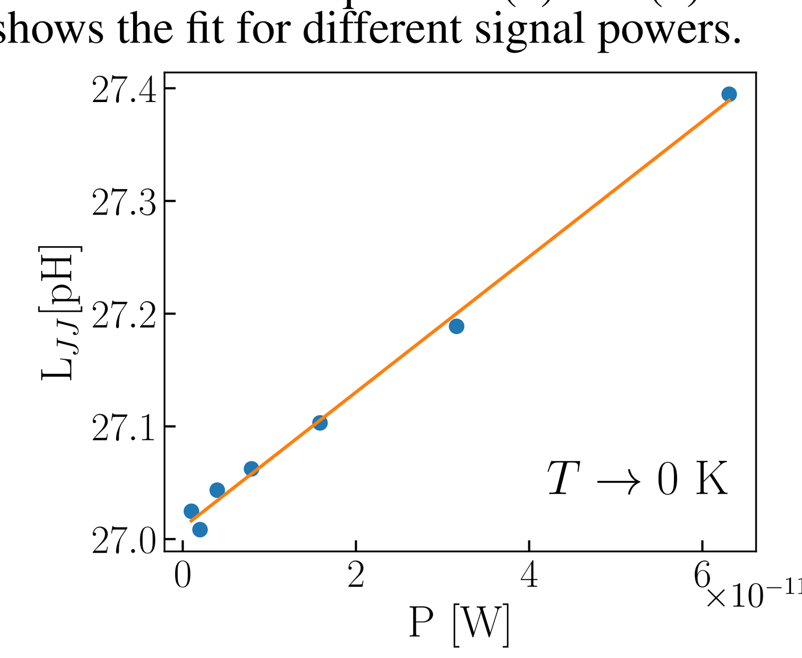

Figure 4: Blue dots: zero temperature limit of JJ inductance obtained from fits in Fig. 3. Orange curve is fit of the data to eq. (7).

- Current inside the waveguide [3]:

$$I^2 = 2QT(f, v_p, \Gamma)\frac{P_{in}}{Z_0} \tag{6}$$

- Power entering the waveguide $P_{in} = \alpha P \neq P$.

$$\Downarrow$$

$$L_{JJ}(T \to 0, P_{in}) = \frac{\Phi_0}{2\pi I_c} + \frac{\Phi_0}{2\pi I_c^3}\frac{\alpha P}{Z_0} \tag{7}$$

- Aditional attenuation $\alpha = -4.7$ dB.

Conclusions

A simple calibration method based on the temperature- and power-dependent transmission analysis was introduced. Basic properties of the superconducting waveguide with Josephson junctions were determined and the input power was estimated. The procedure was demonstrated experimentally on waveguide with metamaterial of Josephson junctions, which enabled us to calibrate the attenuation of the input line of the cryogenic microwave measurement system. This calibration can be useful in applications, where the precise magnitude of input power into a tested device is required, e.g., nonlinearity strength estimation or noise temperature measurement.

References

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