Josephson Junctions traveling wave parametric amplifier: simulations, theory and experiment.

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Abstract

Superconducting traveling wave parametric amplifiers (TWPA), based on kinetic or Josephson nonlinear inductance, are known to be broadband and low noise. A high gain of TWPA can be achieved for long and ideally matched system. In practice, the unmatched nonlinear impedance of the transmission line with finite size results in resonant character of its response, and thus limits the bandwidth of the amplifier. Nevertheless, many applications do not require wide bandwidth, which would increase the total noise. Instead, wide tunability of frequency range would be preferred. We present the design of a 14 mm long superconducting coplanar waveguide with 2080 Josephson junctions arranged as a metamaterial with proper dispersion. The highest amplification of 15 dB within a 35 MHz bandwidth around the central frequency $f_0 = 6.2$ GHz was achieved, with the dynamic range of TWPA around 10 dB. The f_0 can be widely tuned in frequency ranges 3.2 – 4.3 GHz and 5.1 – 6.4 GHz by pump frequency and pump power. In the first frequency region the average amplification is around 4 dB with 8 dB peak and in the second region the gain oscillates between 6 dB and 15 dB. Similar behaviour was obtained theoretically within the coupled mode theory. The properties of the stepped impedance resonator and the role of the metamaterial are studied by means of numerical solution of nonlinear telegrapher's wave equations utilizing the finite element method.

Design of TWPA with Josephson junctions metamaterial

- Coplanar waveguide with Josephson junctions metamaterial - nonlinear medium based on inductance of Josephson junction (JJ):

$$L = L_J\left(1 + \left(\frac{I}{I_c}\right)^2\right), \quad \text{where} \quad L_J = \frac{\Phi_0}{2\pi I_c}$$

- Classical nonlinear electrodynamics model of JJ array: Transmission line (TL) with nonlinear inductance
- phase velocity $v_p = \frac{1}{LC} \approx 0.1c$
- phase velocity $Z_0 = \frac{L}{C} \approx 200\,\Omega$
- short step impedance resonators - stop bands and phase matching for fixed pump frequency
- Josephson junctions metamaterial with stopbands

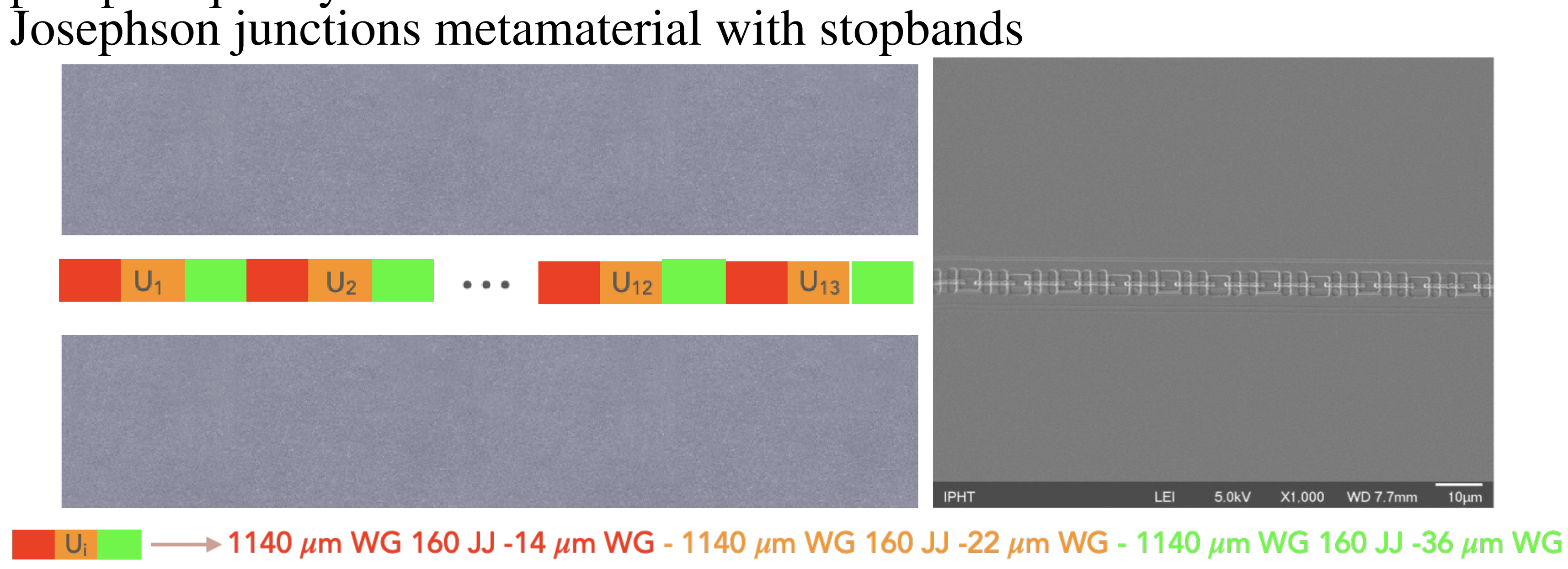

Figure 1: Linear array of 2080 (0.7 x 0.7 µm2) Josephson junctions with three sections inside a coplanar waveguide. Critical current $I_c \approx 8\mu$A ($j_c \approx 1.7$ kA/cm^2), junction capacitance $C_J \approx 30$ fF .

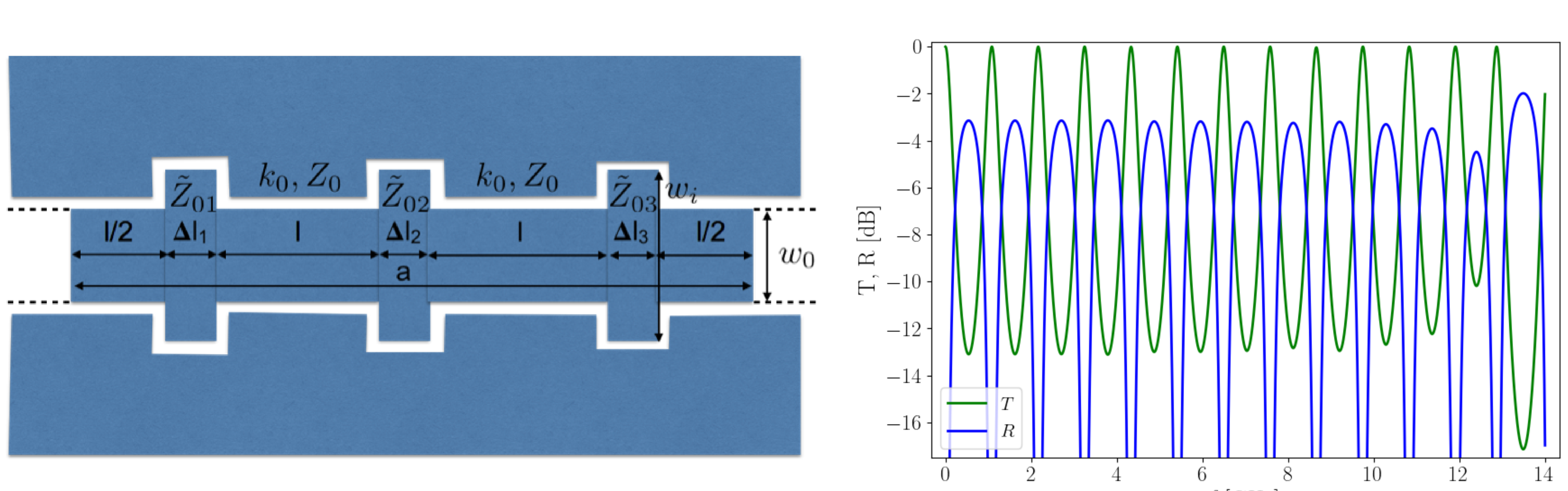

Figure 2: Left: illustrative coplanar waveguide analogue of unit cell of the metamaterial. Right: Transmission and reflection of coplanar waveguide with the JJ metamaterial.

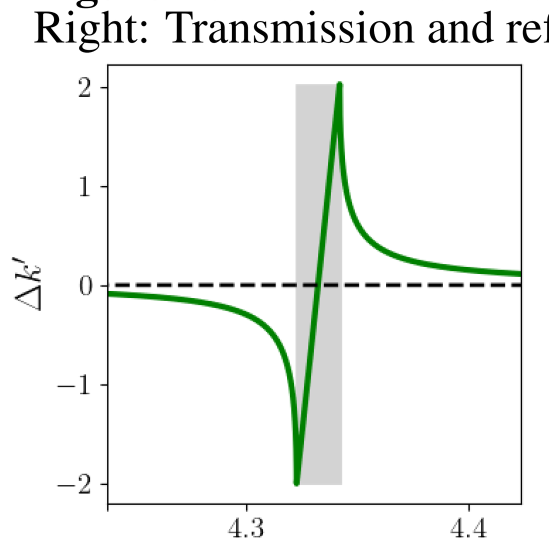

Figure 3: Deviation of k-number from linear dispersion relation at frequencies close to stopband.

- Transfer matrix of the metamaterial:

$$M = \prod_j \begin{pmatrix} e^{ik_j\Delta l_a} & 0 \\ 0 & e^{-ik_j\Delta l_a} \end{pmatrix} M_{Z_{0,j},Z_0} \begin{pmatrix} e^{ik_0 l} & 0 \\ 0 & e^{-ik_0\Delta l} \end{pmatrix} M^{-1}_{Z_{0,j},Z_0}. \quad (1)$$

- Dispersion relation:

$$k = \frac{1}{2L}\arccos\left(\mathrm{Tr}M\right). \quad (2)$$

- Phase matching condition:

$$\Delta k = \frac{2k_p\gamma}{1+2\gamma} \quad (3)$$

Theory vs. Experiment

- Impedance mismatch ⇒ Fabry-Perrot resonances with reflection coefficient $\Gamma(I_p) = (Z_0(I_p) - Z_L)/(Z_0(I_p) + Z_L)$, where $Z_L = 50\,\Omega$:

$$T(f) = \frac{(1-\Gamma^2(I_p))^2}{(1-\Gamma^2(I_p)\cos(2\frac{2\pi f}{v_p}l))^2 + (\Gamma^2(I_p)\sin(2\frac{2\pi f}{v_p}l))^2}$$

$$Q = \frac{2\pi f L}{v_p(I_p)(1-\Gamma^2(I_p))}$$

- Coupled mode theory in slowly varying envelope approx. for signal, pump and idler gives gain G:

$$\beta = \Delta k(1+2\gamma) - 2k(\omega_p)\gamma$$

$$K = \sqrt{k(\omega_s)k(\omega_i)\gamma^2 - \frac{\beta^2}{4}}$$

$$G \equiv \frac{P_s(z)}{P_s(0)} = \left(\cosh^2(Kz) + \left(\frac{\beta}{2K}\right)^2 \sinh^2(Kz)\right)$$

$$\gamma = \left(\frac{I_p}{4I_c}\right)^2, \quad I_p^2 = 2QT(f)\frac{P}{Z_0}$$

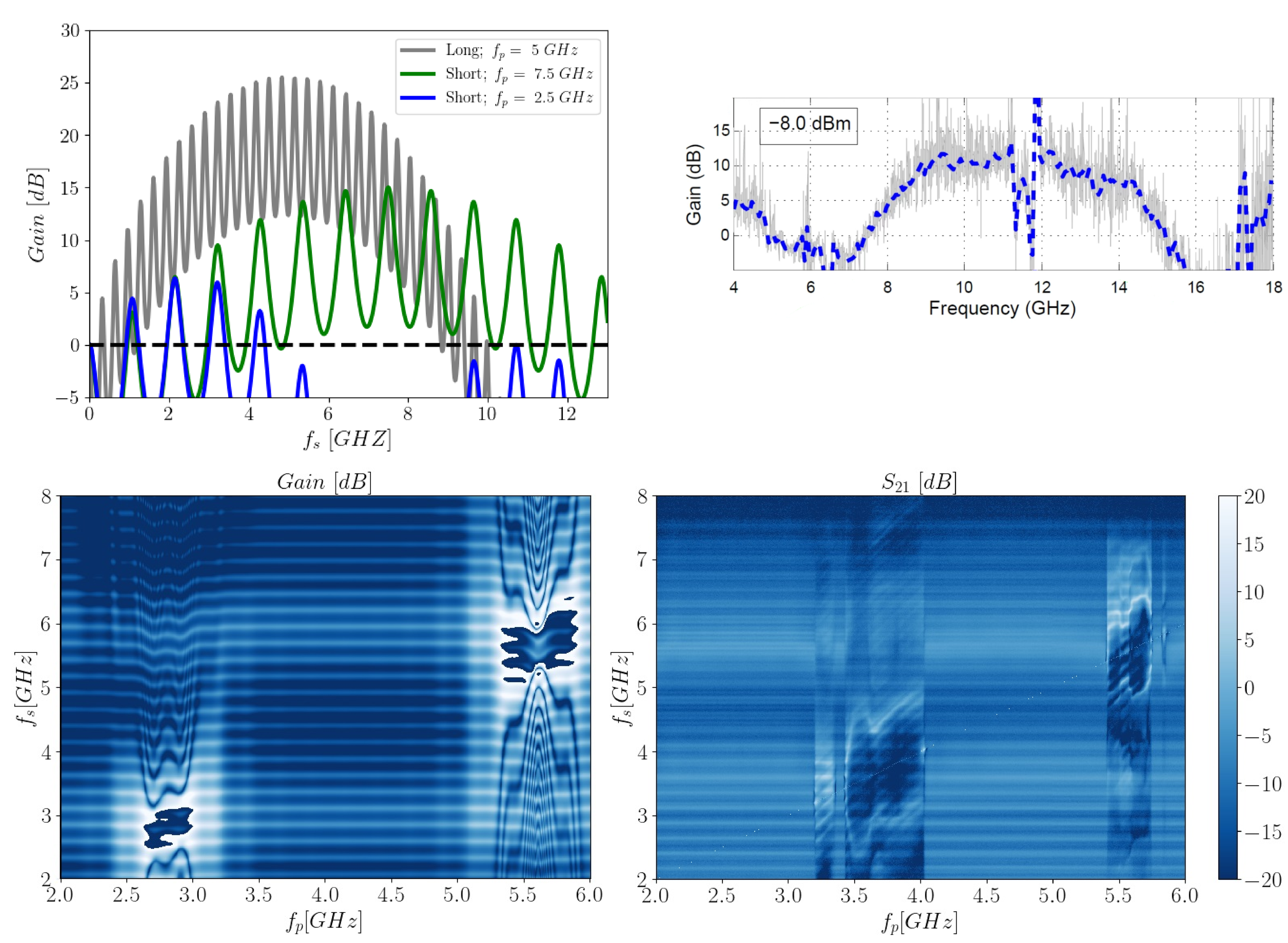

Figure 4: Comparison of calculated gain (left panels) with experimental data obtained on our sample (right bottom panel) and by Eom et al. (right upper panel)

Conclusions

- 14 mm long superconducting coplanar waveguide with 2080 Josephson junctions arranged as a metamaterial with proper dispersion exhibits parametric amplification.
- Parameters: Gain 15 dB, 35 MHz bandwidth, dynamic range 10 dB.
- Central frequency f_0 can be widely tuned in frequency ranges 3.2–4.3 GHz and 5.1–6.4 GHz by pump frequency and pump power.
- The properties of the stepped impedance resonator and the role of the metamaterial are studied by numerical solution of nonlinear telegrapher's wave equations utilizing the finite element method.

References

[1] S. Anders, M. Schmelz, L. Fritzsch, R. Stolz, V. Zakosarenko, T. Schönau and H.-G. Meyer, *Supercond. Sci.Technol.* 22, 064012 (2009).

[2] Byeong Ho Eom, Peter K. Day, Henry G. LeDuc, and Jonas Zmuidzinas *Nature Physics* 8, 623 (2012)

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