

**Title:** NMR experimental data on magnon-BEC time crystals in superfluid  $^3\text{He}$  in the presence of a free surface

**Description:** This dataset supplements described experimental results in the article "Nonlinear two-level dynamics of quantum time crystals". The dataset contains experimental measurements of magnon-BEC time crystals in the B phase of superfluid  $^3\text{He}$  in a cylindrical sample container in the presence of a free surface. Two spatially separate time crystals are observed: one in the bulk of the liquid, and one touching the free surface located above the bulk crystal.

A nuclear demagnetization cryostat and NMR spectrometer were used in the experiments. Experimental measurements were carried out using the Low Temperature Laboratory, which is part of the OtaNano research infrastructure of Aalto University and an access site in the European Microkelvin Platform.

The  $^3\text{He}$  sample was confined within a 150-mm-long cylindrical container with  $\varnothing 6\text{mm}$  inner diameter, made from quartz glass. The experimental volume is connected to another volume of bulk B phase, used for thermometry and coupling to nuclear demagnetization stage. This volume contains a commercial quartz tuning fork with 32 kHz resonance frequency. Static magnetic field of 17–25 mT corresponding to NMR frequencies of 560–833kHz is created using a coil oriented parallel to the axis of the sample container. The NMR pick-up coil, oriented perpendicular to both main magnets, is a part of a tuned tank circuit with quality factor roughly  $Q \sim 100$ . We use a cold preamplifier, thermalized to a bath of liquid helium. The signal is then fed to a room temperature preamplifier, and from that to a lock-in amplifier which mixes down the signal w.r.t. the reference frequency. The lock-in output signal is then digitised at 48000Hz sampling rate. The sample can be cooled down to  $\sim 130\text{ mK}$  using ROTA nuclear demagnetization refrigerator. The earth's magnetic field is compensated using two saddle-shaped coils installed around the refrigerator.

The paper contains two experimental records. The data in Figs 3 and 4 were recorded using the following settings: The current in the pinch coil used to control the magnetic trap shape (4 turns, diameter  $\sim 1\text{cm}$ ) was 200mA. The excitation pulse used was at 833755Hz, pulse length was 2000 cycles, and the pulse amplitude was adjusted to achieve the desired ground state time crystal population. A cold preamplifier and a room temperature preamplifier was used. The maximum of the recorded signal corresponds to about 20 degree tilt angle of the magnetisation at the maximum of the time crystal wave function. The lock-in amplifier reference frequency was 834000Hz. The thermometer fork width was 500mHz, while the intrinsic width of the fork was 75.6mHz.

The data in Fig 5. was recorded in similar fashion, but the lock-in frequency was 624000Hz, the pulse frequency was 623282Hz, the pinch coil current was 250mA, and the thermometer fork width was 120mHz.

The numerical simulations were carried out using the ode113 partial differential equation solver from Matlab. The simulation codes are available from the corresponding author upon reasonable request.

#### **Content of the dataset parts:**

1. [Fig3\_panel\_a\_raw.csv] Raw experimental data file for Figures 3 and 4 in the article. The file is in csv format, as recorded from the pick-up coil after preamplification at 48kHz sampling rate. Note that the frequency axis is inverted as the lock-in reference frequency is higher than the signal frequency. The Larmor frequency is 1185Hz in the units of the raw data.

2. [Fig3\_panel\_b\_raw.csv] Raw simulation data file for Figures 3 and 4 in the article. The file format is identical with the experiment described in item 1 in this list.
3. [Fig3\_panel\_a.csv] Windowed Fourier analysis of the raw data in csv format. First column is frequency separation from the Larmor frequency in Hz as indicated by the header. First row is time in seconds, also indicated by a header. The corresponding absolute values of the FFT amplitudes [abs(FFT)] for each (frequency,time) pair are given by the matrix in mV enveloped by the first column and the first row. The Blackman time window width used is  $\pm 0.5208$ s, and the time step between the overlapping windows is 0.1042s. The raw data frequency spectrum was corrected according to the measured lock-in amplification frequency profile (Stanford SR844, 6db/octave, time constant 100 $\mu$ s).
3. [Fig3\_panel\_b.csv] Windowed Fourier analysis of the simulation raw data in csv format. First column is frequency separation from the Larmor frequency in Hz as indicated by the header. First row is time in seconds, also indicated by a header. The corresponding absolute values of the FFT amplitudes [abs(FFT)] for each (frequency,time) pair are given by the matrix in mV enveloped by the first column and the first row. The Blackman time window width used is  $\pm 0.5208$ s, and the time step between the overlapping windows is 0.1042s.
4. The data in Figure 3 panel c can be obtained by subtracting the above Fourier files point-wise from another.
5. [Fig4\_panel\_a.csv] Ground state and excited state frequencies, as shown in Figure 3 panels a (experiment) and panel b (simulation). The experimental frequencies are extracted by tracing the maxima of the peaks in the Fourier spectrum in time, and simulation frequencies are the calculated eigenfrequencies. Column contents are as labeled by the header line in the CSV file.
6. [Fig4\_panel\_b.csv] Undressed bulk and surface time crystal frequencies, as extracted from the simulation in Fig3 panel b. Column contents are as labeled by the header line in the CSV file.
7. [Fig4\_panel\_c.csv] Squared ground state and excited state signal amplitudes (=populations), as shown in Figure 3 panel a (experiment), and simulated populations corresponding to the simulation in Fig3 panel b. Both are normalised so that the population is one at the level crossing. The experimental state amplitudes are extracted by tracing the maxima of the peaks in the Fourier spectrum in time. Column contents are as labeled by the header line in the CSV file.
8. [Fig4\_panel\_d.csv] Undressed bulk and surface time crystal populations and the bulk time crystal population evolution with population decay removed, as extracted from the simulation in Fig3 panel b, using the same normalisation as in item 7 in this list. Column contents are as labeled by the header line in the CSV file.
9. [Fig5\_panel\_a\_raw.csv] Raw experimental data file for Figures 5 in the article. The file is in csv format, as recorded from the pick-up coil after preamplification at 48kHz sampling rate. Note that the frequency axis is inverted as the lock-in reference frequency is higher than the signal frequency. The Larmor frequency is 1060Hz in the units of the raw data.
10. [Fig5\_panel\_a.csv] Windowed Fourier analysis of the raw data in csv format. First column is frequency separation from the Larmor frequency in Hz as indicated by the header. First row is time in seconds, also indicated by a header. The corresponding absolute values of the FFT amplitudes [abs(FFT)] for each (frequency,time) pair are given by the matrix in mV enveloped by the first column and the first row. The Blackman time window width used is  $\pm 0.5208$ s, and the time step between the overlapping windows is 0.1042s. The raw data frequency spectrum was corrected according to

the measured lock-in amplification frequency profile (Stanford SR844, 6db/octave, time constant 100 $\mu$ s).

11. [Fig5\_panel\_b.csv] Ground state and excited state frequencies, as shown in panel a. The frequencies are extracted by tracing the maxima of the peaks in the Fourier spectrum in time. Column contents are as labeled by the header line in the CSV file.

12. [Fig5\_panel\_c.csv] Ground state and excited state signal amplitudes ( $=\sqrt{\text{population}}$ ), as shown in Figure 3 panel a, normalised so that the total population is one at the level crossing. The amplitudes are extracted by tracing the maxima of the peaks in the Fourier spectrum in time. Column contents are as labeled by the header line in the CSV file.

12. [Fig5\_panel\_d.csv] Bulk sideband separation from the bulk trace, and excited state frequency separation from the bulk trace, as shown in panel a. Column contents are as labeled by the header line in the CSV file.

12. [Fig5\_panel\_e.csv] Two-level coupling as extracted from the amplitude of the bulk side band and the bulk main trace in panel a. Column contents are as labeled by the header line in the CSV file.