Swaption Volatility Surfaces Research

Swaption Volatility

- Implied volatility is the volatility implied by the market price of an option based on the Black-Scholes option pricing model.
- An swaption volatility surface is a four-dimensional cube: implied volatility as a function of moneyness and expiry and tenor.
- Moneyness is defined as:

  Moneyness = Strike – Forward
- For swaption volatility surface, forward is the forward swap rate

$$Forward\quad Rate(ot,ut)=\begin{cases}\dfrac{P(0,ot)-P(0,ot+ut)}{ut*P(0,ot+ut)}, & \text{if } ut<1Y\\[2ex] \dfrac{P(0,ot)-P(0,ot+ut)}{0.5*\sum_{i=1}^{2*ut}P(0,ot+0.5*i)}, & \text{if } ut\geq 1Y\end{cases}$$

Swaption Volatility

- where *P(0,t)* is the value of zero coupon bond paying one dollar at the maturity date *t*; *ot* is the option term is and *ut* is the underlying term is *ut.*
- Vertical arbitrage free and horizontal arbitrage free conditions for swaption volatility surfaces depend on different strikes.
- There is no calendar arbitrage in swaption volatility surfaces as swaptions with different expiries and tenors have different underlying swaps.

Swaption Volatility

- The absence of triangular arbitrage condition is sufficient to exclude static arbitrages in swaption surfaces.

- The triangular arbitrage free conditions are

$$Sw(t_1, T_s, T_e, K) \leq Sw(t_2, T_s, T_e, K) \qquad \text{where } t_1 \leq t_2$$

$$Sw(T_1, T_1, T_3, K) \leq Sw(T_1, T_1, T_2, K) + Sw(T_2, T_2, T_3, K)$$
$$\text{where } T_1 \leq T_2 \leq T_3$$

Swaption Volatility

- For each term (expiry) and tenor of the swaption, conduct the following calibration procedure based on SABR model.
- The β parameter is estimated first and typically chosen a priri according to how the market prices are to be observed.
- Alternatively β can be estimated by a linear regression on a time series of ATM volatilities and of forward rates.
- After β is set, we can obtain α by using σ_{ATM} to solve the following equation

$$\sigma_{ATM} = \frac{\alpha}{f^{1-\beta}}\left\{1 + \left[\frac{\alpha^2(1-\beta)^2}{24f^{2(1-\beta)}} + \frac{\rho\beta v\alpha}{4f^{1-\beta}} + \frac{(2-3\rho^2)v^2}{24}\right]T\right\}$$

The Viete method is used to solve this equation

Swaption Volatility

- Given the α and β solved above, we can find the optimized value of (ρ, v) by minimizing the distance between the SABR model output volatilities and market volatilities across all strikes for each term and tenor.

$$min \sum_{i=1}^{n} \left[\sigma_i^{SABR}(\alpha, \beta, \rho, v) - \sigma_i^{Market}\right]^2$$

  The Levenberg-Marquardt least-squares optimization routine is used for optimization.

- After α, β, ρ, v calibrated, one can generate SABR volatility (swaption volatility) for any moneyness.
- Repeat the above process for each term and tenor.

Thank You

You can find more details at

https://finpricing.com/lib/EqSpread.html