Best of Option Analytics

1. Overview

A best of option is an option whose payment is based on the gain of the best performance asset in a basket at maturity.

A worst of option is an option whose payment is based on the gain of the worst performance asset in a basket at maturity. This category also contains worst of option, option on the maximum, option on minimum, and rainbow.

The payoffs of those options are given by

- Best of assets or cash: $payoff_T = \max\left(S_1^T,..., S_n^T, K\right)$
- Worst of assets or cash: $payoff_t = \min\left(S_1^T,..., S_n^T, K\right)$
- Call on max: $payoff_T = \max\left(\max\left(S_1^T,..., S_n^T\right) - K, 0\right) = \max(S_1^T,..., S_n^T, K) - K$
- Call on min: $payoff_T = \max\left(\min\left(S_1^T,..., S_n^T\right) - K, 0\right)$
- Put on max: $payoff_T = \max\left(K - \max\left(S_1^T,..., S_n^T\right), 0\right)$
- Put on min: $payoff_T = \max\left(K - \min\left(S_1^T,..., S_n^T\right), 0\right)$

A best of option pays the option holder the best return at maturity among a given set of portfolios, where each portfolio may be defined by a set of weights on the same underlying basket or different baskets.

These are chooser options that return the best performing among several baskets of funds or indices that reflect growth, moderate and conservative investment styles. The returns could be based on average (Asian), single currency or quanto. The final payoff could be capped and floored.

Most commonly, portfolio weights are all positive and sum to 1. Note however, that weights are allowed to take on zero or negative values and need not necessarily sum to 1, but these values can considerably change the price and risk profile of the structure.

2. Valuation

There is a generic formula for 2 assets S_1 and S_2

i) Best of 2 assets plus cash A+B+C

ii) Maximum of 2 assets call A+B+C- Xe^{-rT}

iii) Better of 2 assets A+B+C (where X=0)

iv) Maximum of 2 assets put ii-iii+ Xe^{-rT}

v) Minimum of 2 assets call EuroCall(S_1)+EuroCall(S_2)-ii

vi) Worse of 2 assets EuroCall(S_1)+EuroCall(S_2)-iii

vii) Minimum of 2 asset put v-vi+ Xe^{-rT}

where

$$A = S_1 e^{-D_1 T}\left[N(d_3) - M(-d_1, d_3; \rho_1)\right]$$

$$B = S_2 e^{-D_2 T}\left[N(d_4) - M(-d_2, d_4; \rho_2)\right]$$

$$C = Xe^{-rT} M(-d_1 + \sigma_1\sqrt{T}, -d_2 + \sigma_2\sqrt{T}; p)$$

Where

N ~ cumulative normal

M ~ bivariate cumulative normal

$$d_1 = \frac{\ln(S_1 / X) + \left(r - D_1 + 0.5\sigma_1^2\right)T}{\sigma_1\sqrt{T}}$$

$$d_2 = \frac{\ln(S_2 / X) + \left(r - D_2 + 0.5\sigma_2^2\right)T}{\sigma_2\sqrt{T}}$$

$$d_3 = \frac{\ln(S_1 / S_2) + \left(D_2 - D_1 + 0.5\sigma_A^2\right)T}{\sigma_A\sqrt{T}}$$

$$d_4 = \frac{\ln(S_2 / S_1) + \left(D_1 - D_2 + 0.5\sigma_A^2\right)T}{\sigma_A\sqrt{T}}$$

$$\sigma_A = \sqrt{\sigma_1^w + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$$

$$\rho_1 = \frac{\rho\sigma_2 - \sigma_1}{\sigma_A}$$

$$\rho_2 = \frac{\rho\sigma_1 - \sigma_2}{\sigma_A}$$

Reference:

https://finpricing.com/lib/EqCorrelationSwap.html