Neutrosophic Sets and Systems, Vol. 49, 2022
University of New Mexico

Attribute based Double Bounded Rough Neutrosophic Sets in Facial Expression Detection

Praba B [1], Pooja S [2] and Nethraa Sivakumar [3]

[1] Department of Mathematics; prabab@ssn.edu.in
[2] Department of Electronics and Communication; pooja18112@ece.ssn.edu.in
[3] Department of Electronics and Communication; nethraa18096@ece.ssn.edu.in
SSN College of Engineering, Kalavakkam, Tamil Nadu-603110

Abstract: In this paper, a hybrid intelligent structure called "Double Bounded Rough Neutrosophic Sets" is defined, which is a combination of Neutrosophic sets theory and Rough sets theory. Further, the Attribute based Double Bounded Rough Neutrosophic Sets was implemented using this hybrid intelligent structure for Facial Expression Detection on real time data. Facial expression detection is becoming increasingly important to understand one's emotion automatically and efficiently and is rich in applications. This paper implements some of these applications of facial expression such as: differentiating between Genuine and Fake smiles, prediction of Depression, determining the Degree of Closeness to a particular Attribute/Expression and detection of fake expression during an examination. With the onset of COVID – 19 pandemic, majority of people are choosing to wear masks. A suitable method to detect Facial Expression with and without mask is also implemented. Double Bounded Rough Neutrosophic Sets proposed in this paper is found to yield better results as compared to that of individual structures (Neutrosophic sets theory or Rough sets theory)

Keywords: Double Bounded Rough Neutrosophic Sets, Facial Expression Detection, Facial key points, Neutrosophic sets, Fuzzy set, Rough Set

1. Introduction

Non-verbal communication constitutes a key part of understanding one's emotion, thought process and mentality. Facial expressions, body language and movements/gestures primarily make up non-verbal communication. Hence, biometrics like facial recognition are essential for conversational user experience. Facial recognition is being employed as a standard safety feature in various applications. With latest developments, it is getting increasingly efficient to detect emotions and sentiment through the facial expression of a person. These expressions can further be used to differentiate between different emotions, such as sad, angry, happy, etc.

Counselling systems, lie detection, etc are some among the wide array of applications that automatic facial expression detection has. Facial expressions form a critical aspect of how we communicate, interact and develop impressions of people who we observe and are influenced by. Behavioural scientists like Darwin in 1872 [1,2,3] and Suwa *et al* in 1978 presented an early attempt to automatically analyse facial expressions by tracking the motion of 20 identified spots on an image sequence.

Following this, computer systems were developed which helped us understand and use this natural form of human communication. Research carried out by psychologists [4] indicates that only 7% of the actual information is transmitted orally, and 38% by auxiliary language, such as the rhythm and speed of speech, tone, etc. 55% of information is transmitted by the expression of face. Thus, most of the valuable information can be obtained by facial expression recognition and it provides the best way to judge a person's mental state.

Having said this, there have been numerous methodologies to determine facial expressions. Some of these methodologies involve Neutrosophic sets theory and Rough sets theory which have been implemented in “Facial Expression Recognition Based on Rough Set Theory and SVM” [5], “Face Recognition with Triangular Fuzzy Set-Based Local Cross Patterns in Wavelet Domain” [6], “Facial Expression Recognition based on Fuzzy Networks” [7], etc. These methods are certainly emerging as powerful tools for managing uncertainty, indeterminate, incomplete and imprecise information. This paper mainly focuses on a hybrid intelligent structure called “Rough Neutrosophic Sets” and also introduces “Double Bounded Rough Neutrosophic Sets” which are used for facial expression recognition. The significance of introducing these hybrid set structures is that the computational techniques based on any one of these individual structures will not always yield the best results, but a fusion of two or more of these often provide better results.

2. Materials and Methods

In this section, we give the definitions that are required to study the forth coming sections.

The source code for detection of Facial Expression is publicly available at:

https://github.com/Nethraasivakumar/Facial-Expression-Detection-Using-Double-Bounded-Rough-Neutrosophic-Sets-

https://github.com/poojasrini/Facial-Expression-Detection-using-Double-Bounded-Rough-Neutrosophic-Sets

2.1 Preliminaries:

Definition 2.1.1: Fuzzy set [8]

Fuzzy sets can be considered as an extension and gross oversimplification of classical sets. If X is a collection of objects denoted generically by x, then a fuzzy set A in X is a set of ordered pairs:

$$A = \{(x\ ,\mu_a(x))|x \in X\}$$

$\mu_a$is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in A that maps X to the membership space M (when M contains only the two points 0 and 1, A is nonfuzzy and $\mu_a(x)$ is identical to the characteristic function of a nonfuzzy set). The range of the membership function is a subset of the non-negative real numbers whose supremum is finite. Elements with a zero degree of membership are normally not listed.

Definition 2.1.2: Rough set [9]

Let $I = (U, A)$ be an information system, where U is a non-empty set of finite objects, called the universe and A is a non-empty finite set of fuzzy attributes defined by $\mu_a: U \rightarrow [0,1]$, $a \in A$, is a fuzzy set. Formally for any set $P \subseteq A$, there is an associated equivalence relation called Indiscernibility relation defined as follows:

$$IND(P) = \{(x, y) \in U^2 \mid \forall\, a \in P, \mu_a(x) = \mu_a(y)\}$$

The partition induced by $IND(P)$ consists of equivalence classes defined by:

$$[x]_p = \{y \in U \mid (x, y) \in IND(P)\}$$

For any $X \subseteq U$, define the lower approximation space $p_{-}(X)$ such that

$$p_{-}(X) = \{x \in U \mid [x]_P \subseteq X\}$$

Also, define the upper approximation space $p^{-}(X)$ such that

$$p^{-}(X) = \{x \in U \mid [x]_p \cap X \neq \emptyset\}$$

A rough set corresponding to X, where X is an arbitrary subset of U in the approximation space P, we mean the ordered pair $\{p_{-}(x), p^{-}(X)\}$ and it is denoted by $RS(X)$.

Definition 2.1.3: Neutrosophic set [10]

Neutrosophic sets are described by three functions: a membership function, indeterminacy function and a non-membership function that are independently related. The Rough Neutrosophic Set takes the form:

$$N = \{(x, \alpha N\,(x), \beta N\,(x), \gamma N\,(x)) | x \in X\}$$

which is characterized by a truth-membership function αN , an indeterminacy-membership function βN and falsity-membership function γN where the functions $\alpha N : X \rightarrow]0-, 1+[$, $\beta N : X \rightarrow]0-, 1+[$ and $\gamma N : X \rightarrow]0-, 1+[$ are real standard or non-standard subsets of $]0-, 1+[$. There is no restriction on the sum of $\alpha N\,(x), \beta N\,(x)$ and $\gamma N\,(x)$, therefore $0- \leq \alpha N\,(x) + \beta N\,(x) + \gamma N\,(x) \leq 3+$.

2.2 Attribute based Double Bounded Rough Neutrosophic Sets

In this section, we define Double Bounded Rough Neutrosophic Sets and some operations on these sets.

Let $I = (U, A)$ be an information system where U is a non-empty finite set of objects and A is a finite set of attributes possessed by the objects in view.

Let $F: A \rightarrow \rho(U)$ be a mapping such that for each $a \in A$, $F(a) \subseteq U$, containing those elements of U possessing the attribute a , we assume that $U\, F(a) = U, a \in A$.

Also let $N: U \rightarrow \rho(U)$ is a mapping that associates each $x \in U$ to a subset $N(x)$ consisting of the neighbours of x.

Note that the functions F and N are defined according to the systems under consideration and also using the expert knowledge. The function N can also be defined using the relation that prevails among the elements of U.

Now $I = (U, A, F, N)$ is called as a covering based N-information system. Throughout this section we consider this covering based N-information system.

Definition 2.2.1:

Let $I = (U, A, F, N)$ be a covering based N-information system. For any subset X of U define $N(X) = U\, N(x)\,, x \in X$

Definition 2.2.2:

Let $I = (U, A, F, N)$ be a covering based N-information system. For any subset X of U define:

$$DR_{-}(a \sim X) = N(F(a) \cap N(x))\,,$$

$$^{-}DR(a \sim X) = N(X) \cup (N(F(a)\,) \cap N(X)) \text{ and}$$

$$DR^{-}(a \sim X) = N(F(a)) \cup (N\big(F(a)\big) \cap N(X))$$

$DR_{-}(a \sim X)$ is called as the lower approximation of X with respect to the attribute a;

$^{-}DR(a \sim X)$ is called the left upper approximation of X with respect to the attribute a;

$DR^{-}(a \sim X)$ is called the right upper approximation of X with respect to the attribute a;

Definition 2.2.3:

For any subset $X(U)$ define $DRS(a \sim X) = (DR_{-}(a \sim X),\ ^{-}DR(a \sim X), DR^{-}(a \sim X))$ is called as the Double Bounded Rough Set of X with respect to the attribute a.

This rough set gives the definite, possible and unascertainable elements of X possessing the attribute a. Note that for each $a \in A, DRS(a \sim X)$ can be attained. This method of defining the Attribute based Double Bounded Rough Set will play a significant role in analysing the elements of X with respect to A.

Also, by evaluating the attribute based DBRS for various subsets of U with respect to a single attribute $a \in A$, the significance of $a \in A$ on the subsets can be easily compared.

This DBRS is called as the Attribute based Double Bounded Rough Set of X. Further if there is a set of parameters P defining the attributes and let for each

$p \in P, \mu_p : U \rightarrow [0,1]$ be a fuzzy set describing the degree of existence of the parameters on the elements of U . Then a Neutrosophic set can be defined for each $DRS(a \sim X)$ as follows,

Let,

$DR = \{DRS(a \sim X) | X \subseteq U, a \in A\}$

$DR_{-} = \{DR_{-}(a \sim X) \,| X \subseteq U, a \in A\}$

$^{-}DR = \{\,^{-}DR(a \sim X) \,| X \subseteq U, a \in A\}$

$DR^{-} = \{DR^{-}(a \sim X) \,| X \subseteq U, a \in A\}$

Definition 2.2.4:

Define a fuzzy set $\mu_{-}: DR_{-} \rightarrow [0,1]$ as follows,

$$\mu_{-}(DR_{-}(a\sim X)) = max\{min(\mu_p(x)\} \;, x \in DR_{-}(a\sim X)$$

similarly, $^{-}\mu$: $^{-}DR \rightarrow [0,1]$ by

$$^{-}\mu(\,^{-}DR(a\sim X)) = max\{min(\mu_p(x)\} \;, x \in \,^{-}DR(a\sim X) \; and$$

$\mu^{-} : DR^{-} \rightarrow [0,1]$ by

$$\mu^{-} \; (DR^{-}(a\sim X)) = max\{min(\mu_p(x)\} \;, x \in DR^{-}(a\sim X)$$

Hence fuzzy set,

$\bar{\mu}$: DR → [0,1] X [0,1] X [0,1] defined by

$$\bar{\mu}\;(DRS(a\sim X)) = \;(\mu_{-}(DR_{-}(a\sim X))\,,\,^{-}\mu(\,^{-}DR(a\sim X))\,,\mu^{-}\;(DR^{-}(a\sim X)))$$

constitutes a Neutrosophic fuzzy set on the set of all Attribute based Double Bounded N- rough sets.

Definition 2.2.5:

From the Neutrosophic fuzzy set, it is possible to predict the facial expression of the object/image.

The attribute value can be calculated using the following expression:

Let: $\mu_{-}(DR_{-}(a\sim X))$ be denoted by T_a

$^{-}\mu(\,^{-}DR(a\sim X))$ be denoted by I_a

$\mu^{-}\;(DR^{-}(a\sim X))$ be denoted by F_a

General Formula to calculate Attribute "a" Value: [12]

$$V(A) = 2\left(\max\left(\left(\frac{T_A + I_A}{2}\right),\left(\frac{1 + I_A - F_A}{2}\right)\right) - \min\left(\left(\frac{T_A + I_A}{2}\right),\left(\frac{1 + I_A - F_A}{2}\right)\right)\right)$$

Example 2.1:

Let $U = \{x_1\,, x_2\,, x_3\,, x_4, x_5\}$, $A = \{\,a_1, a_2, a_3\}$

$F: A \rightarrow P(U)$ is defined by $F(a_1) = \;\{x_1, x_3\,\}, F(a_2) = \{\,x_2\,, x_4\}\,, F(a_3) = \{\,x_5\}$

$Let\ P = \{\,P1, P2\}$.The fuzzy set μ_{p_1} and μ_{p_2} are tabulated below

$\mu_p \backslash U$	x_1	x_2	x_3	x_4	x_5
μ_{p_1}	0.1	0.3	0.2	0.4	0.7
μ_{p_2}	0.4	0.3	0.8	0.6	0.9

Table 1: Fuzzy set values for objects $x_1\,, x_2\,, x_3\,, x_4, x_5$

$N(x_1) = \{x_1, x_2, x_3\}$

Lower approximation:

- $DR_{-}(a_1 \sim x_1) = F(a_1) \cap N(x_1) = \{x_1, x_3\} \cap \{x_1, x_2, x_3\} = \{x_1, x_3\}$

  $\mu_{-}(DR_{-}(a_1 \sim x_1)) = max\{min\{\mu_{p_1}(x_1), \mu_{p_1}(x_3)\}, min\{\mu_{p_2}(x_1), \mu_{p_2}(x_3)\}\}$

  $= max\{min\{0.1, 0.2\}, min\{0.4, 0.8\}\} = max\{0.1, 0.4\} = 0.4$

- $DR_{-}(a_2 \sim x_1) = F(a_2) \cap N(x_1) = \{x_2\}$

  $\mu_{-}(DR_{-}(a_2 \sim x_1)) = max\{min\{\mu_{p_1}(x_2)\}, min\{\mu_{p_2}(x_2)\}\} = 0.3$

- $DR_{-}(a_1 \sim x_1) = F(a_3) \cap N(x_1) = \{\ \}$

  $\mu_{-}(DR_{-}(a_3 \sim x_1)) = max\{min\{\ \}, min\{\ \}\} = 0$

Left upper approximation:

- $^{-}DR(a_1 \sim x_1) = N(x_1) \cup (F(a_1) \cap N(x_1))$
  $= \{x_1, x_2, x_3\} \cup (\{x_1, x_3\} \cap \{x_1, x_2, x_3\})$
  $= \{x_1, x_2, x_3\} \cup \{x_1, x_3\} = \{xx_1, x_2, x_3\}$

  $^{-}\mu(^{-}DR(a_1 \sim x_1)) = max\{min\{\mu_{p_1}(x_1), \mu_{p_1}(x_2), \mu_{p_1}(x_3)\}, min\{\mu_{p_2}(x_1), \mu_{p_2}(x_2), \mu_{p_2}(x_3)\}\}$

  $= max\{min\{0.1, 0.3, 0.2\}, min\{0.4, 0.3, 0.8\}\} = max\{0.1, 0.3\} = 0.3$

- $^{-}DR(a_2 \sim x_1) = N(x_1) \cup ((F(a_2)) \cap N(x_1)) = \{x_1, x_2, x_3\}$

  $^{-}\mu(^{-}DR(a_2 \sim x_1)) = max\{min\{\mu_{p_1}(x_1), \mu_{p_1}(x_2), \mu_{p_1}(x_3)\}, min\{\mu_{p_2}(x_1), \mu_{p_2}(x_2), \mu_{p_2}(x_3)\}\}$

  $= 0.3$

- $^{-}DR(a_3 \sim x_1) = N(x_1) \cup ((F(a_3)) \cap N(x_1)) = \{x_1, x_2, x_3\}$

  $^{-}\mu\left(^{-}DR(a_3 \sim x_1)\right) = max\{min\{\mu_{p_1}(x_1), \mu_{p_1}(x_2), \mu_{p_1}(x_3)\}, min\{\mu_{p_2}(x_1), \mu_{p_2}(x_2), \mu_{p_2}(x_3)\}\}$

  $= 0.3$

Right upper approximation:

- $DR^{-}(a_1 \sim x_1) = (F(a_1)) \cup ((F(a_1)) \cap N(x_1))$
  $= \{x_1, x_3\} \cup (\{x_1, x_3\} \cap \{x_1, x_2, x_3\})$
  $= \{x_1, x_3\} \cup \{x_1, x_3\} = \{x_1, x_3\}$
  $\mu^{-}(DR^{-}(a_1 \sim x_1)) = max\{min\{\mu_{p_1}(x_1), \mu_{p_1}(x_3)\}, min\{\mu_{p_2}(x_1), \mu_{p_2}(x_3)\}\}$

$$= max\{min\{0.1,0.2\}, min\{0.4,0.8\}\} = max\{0.1,0.4\} = 0.4$$

- $DR^{-}(a_2 \sim x_1) = (F(a_2)) \cup ((F(a_2)) \cap N(x_1)) = \{x_2, x_4\}$
  $\mu^{-}(DR^{-}(a_2 \sim x_1)) = max\{min\{\mu_{p_1}(x_2), \mu_{p_1}(x_4)\}, min\{\mu_{p_2}(x_2), \mu_{p_2}(x_4)\}\} = 0.3$

- $DR^{-}(a_3 \sim x_1) = (F(a_3)) \cup ((F(a_3)) \cap N(x_1)) = \{x_5\}$

  $\mu^{-}(DR^{-}(a_3 \sim x_1)) = max\{min\{\mu_{p_1}(x_5)\}, min\{\mu_{p_2}(x_5)\}\}$

  $= 0.7$

Result:

Table 2: Attributes versus Double Bounded Rough Neutrosophic Sets

a\approximation	$\mu_{-}(DR_{-}(a \sim x))$	$^{-}\mu(^{-}DR(a \sim x))$	$\mu^{-}(DR^{-}(a \sim x))$
a_1	0.4	0.3	0.4
a_2	0.3	0.3	0.3
a_3	0	0.3	0.7

2.3 Implementing Attribute based Double Bounded Rough Neutrosophic Sets to Detect Facial Expressions

The concepts of Double Bounded Rough Neutrosophic Sets were implemented in the decision-making process of detecting facial expressions of humans on real time data.

Objective: To determine the facial expression of a person by classifying into 4 expressions: Sad, Angry, Happy and Surprised.

Data: A is a finite set of attributes possessed by the objects in view. The image of the person's face constitutes an object. Any object possesses one of the four attributes present in A: Sad, Angry, Happy and Surprised.

$$A = \{S, A, H, SU\}$$

Where: S represents Sad

A represents Angry

H represents Happy

SU represents Surprised

U is a non-empty finite set of objects/images. In this illustration, we have taken 200 objects as Universal set, U. The n[th] object is denoted x_n.

The images and the respective parameter values were obtained from the Kaggle Dataset provided by Dr Yoshua Bengio of the University of Montreal. [11]

$$U = \{x_1, x_2, x_3, \dots x_{200}\}$$

The real time data constituted the position of 15 feature points located at pivotal parts of the face/object. Each of these 15 feature points were divided into their respective x and y coordinates, hence resulting in a set of 30 parameters. These 30 parameters were represented by P.

Where:

$$P = \{ P_1, P_2, P_3, \dots P_{30} \}$$

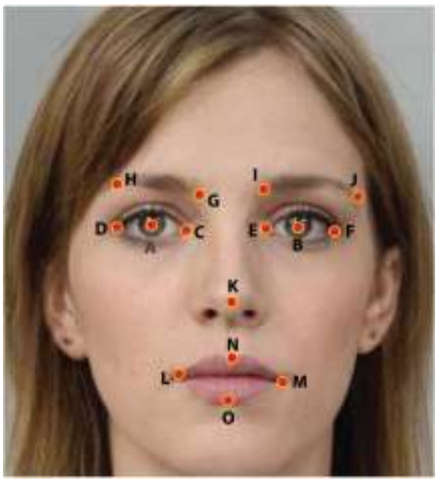

Figure 1: Location of facial feature points on the face

The 15 Facial Feature Points are:

$A\ (P_1, P_2)$
$B\ (P_3, P_4)$
$C\ (P_5, P_6)$
$D\ (P_7, P_8)$
$E\ (P_9, P_{10})$
$F\ (P_{11}, P_{12})$
$G\ (P_{13}, P_{14})$
$H\ (P_{15}, P_{16})$
$I\ (P_{17}, P_{18})$
$J\ (P_{19}, P_{20})$
$K\ (P_{21}, P_{22})$
$L\ (P_{23}, P_{24})$
$M\ (P_{25}, P_{26})$
$N\ (P_{27}, P_{28})$
$O\ (P_{29}, P_{30})$

Each of the 200 objects consists of these 30 parameters which are used to define their attribute. The tabulated form of the objects and their respective parameter values are given below. The values of the 30 attributes lie between [0,1].

Table 3: The parameter values for x_1 and x_2

Parameter	Name	$\mu_{P_i}(x_1)$	$\mu_{P_i}(x_2)$
P_1	left_eye_center_x	0.6701	0.6680
P_2	left_eye_center_y	0.3643	0.3572
P_3	right_eye_center_x	0.3120	0.3081
P_4	right_eye_center_y	0.3484	0.3452
P_5	left_eye_inner_corner_x	0.6131	0.6021
P_6	left_eye_inner_corner_y	0.3674	0.3662
P_7	left_eye_outer_corner_x	0.7367	0.7190
P_8	left_eye_outer_corner_y	0.3769	0.3572
P_9	right_eye_inner_corner_x	0.3754	0.3621
P_{10}	right_eye_inner_corner_y	0.3579	0.3512
P_{11}	right_eye_outer_corner_x	0.2549	0.2511
P_{12}	right_eye_outer_corner_y	0.3453	0.3452
P_{13}	left_eyebrow_inner_end_x	0.5624	0.6111
P_{14}	left_eyebrow_inner_end_y	0.2945	0.2822
P_{15}	left_eyebrow_outer_end_x	0.8191	0.7940
P_{16}	left_eyebrow_outer_end_y	0.3167	0.3032
P_{17}	right_eyebrow_inner_end_x	0.4451	0.4191

$\boldsymbol{P_{18}}$	right_eyebrow_inner_end_y	0.2724	0.2822
$\boldsymbol{P_{19}}$	right_eyebrow_outer_end_x	0.1757	0.2031
$\boldsymbol{P_{20}}$	right_eyebrow_outer_end_y	0.2819	0.2882
$\boldsymbol{P_{21}}$	nose_tip_x	0.5021	0.4881
$\boldsymbol{P_{22}}$	nose_tip_y	0.5798	0.5521
$\boldsymbol{P_{23}}$	mouth_left_corner_x	0.5877	0.5811
$\boldsymbol{P_{24}}$	mouth_left_corner_y	0.7953	0.7351
$\boldsymbol{P_{25}}$	mouth_right_corner_x	0.3659	0.3531
$\boldsymbol{P_{26}}$	mouth_right_corner_y	0.7922	0.7321
$\boldsymbol{P_{27}}$	mouth_center_top_lip_x	0.4863	0.4701
$\boldsymbol{P_{28}}$	mouth_center_top_lip_y	0.7319	0.6781
$\boldsymbol{P_{29}}$	mouth_center_bottom_lip_x	0.4736	0.4731
$\boldsymbol{P_{30}}$	mouth_center_bottom_lip_y	0.8904	0.8131

The parameter values for x_1 and x_2 are given above. All the values lie in [0,1].

Let $F: A \rightarrow \rho(U)$ be a mapping such that for each $a \in A$, $F(a) \subseteq U$. $F(a)$ constitutes those images which possess attribute 'a' such that $a \in A$. Therefore, the 200 images in U are categorised into the 4 attributes present in A. The four attributes are S (Sad), A(Angry), H (Happy) and SU (Surprised).

Table 4: $F(a)$ versus the attribute a

$\boldsymbol{a}$	$\boldsymbol{F(a)}$
$\boldsymbol{S}$	$\{x_{19}, x_{29}, x_{30}, x_{36}, x_{38}, x_{39}, x_{42}, x_{47}, x_{51}, x_{59}, x_{65}, x_{79}, x_{87}, x_{91}, x_{97}, x_{107}, x_{117}, x_{126}, x_{127}, x_{129}, x_{145}, x_{146}, x_{147}, x_{150}, x_{156}, x_{158}, x_{163}, x_{167}, x_{168}, x_{170}, x_{171}, x_{173}, x_{177}, x_{181}, x_{182}, x_{183}, x_{184}, x_{185}, x_{188}, x_{189}, x_{190}, x_{191}, x_{192}, x_{194}, x_{195}, x_{196}, x_{197}, x_{198}, x_{199}, x_{200}\}$
$\boldsymbol{A}$	$\{x_{24}, x_{28}, x_{31}, x_{33}, x_{34}, x_{35}, x_{40}, x_{44}, x_{50}, x_{52}, x_{54}, x_{56}, x_{58}, x_{70}, x_{80}, x_{83}, x_{92}, x_{93}, x_{94}, x_{95}, x_{96}, x_{99}, x_{100}, x_{105}, x_{108}, x_{111}, x_{112}, x_{114}, x_{115}, x_{120}, x_{121}, x_{128}, x_{132}, x_{133}, x_{134}, x_{135}, x_{136}, x_{137}, x_{138}, x_{139}, x_{142}, x_{143}, x_{152}, x_{153}, x_{160}, x_{174}, x_{180}, x_{186}, x_{187}, x_{193}\}$
$\boldsymbol{H}$	$\{x_3, x_4, x_5, x_9, x_{25}, x_{32}, x_{37}, x_{41}, x_{46}, x_{53}, x_{55}, x_{67}, x_{72}, x_{78}, x_{84}, x_{85}, x_{86}, x_{103}, x_{104}, x_{109}, x_{110}, x_{113}, x_{116}, x_{122}, x_{123}, x_{124}, x_{125}, x_{130}, x_{131}, x_{140}, x_{141}, x_{144}, x_{148}, x_{149}, x_{151}, x_{154}, x_{155}, x_{157}, x_{159}, x_{161}, x_{162}, x_{164}, x_{165}, x_{166}, x_{169}, x_{172}, x_{175}, x_{176}, x_{178}, x_{179}\}$
$\boldsymbol{SU}$	$\{x_1, x_2, x_6, x_7, x_8, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{20}, x_{21}, x_{22}, x_{23}, x_{26}, x_{27}, x_{43}, x_{45}, x_{48}, x_{49}, x_{57}, x_{60}, x_{61}, x_{62}, x_{63}, x_{64}, x_{66}, x_{68}, x_{69}, x_{71}, x_{73}, x_{74}, x_{75}, x_{76}, x_{77}, x_{81}, x_{82}, x_{88}, x_{89}, x_{90}, x_{98}, x_{101}, x_{102}, x_{106}, x_{118}, x_{119}\}$

For an image x_n from the Universal Set, the neighbours of x_n are denoted by $N(x_n)$.

$N(x_n)$ is a subset of the Universal Set and consists of images from the Universal set which lie in the neighbourhood of the given image x_n.

In order to compute $N(x_n)$, following steps were implemented.

Algorithm:

For each image $x_m \in$ U,

And for each parameter P_i $(i = 1,2,3 \dots,30)$,

1. Let 'q' be the absolute difference between the value of P_i for image x_n , and the mean value of P_i , where x_n is the given image under consideration.
2. Let 'r' be the absolute difference between the value of P_i for image x_m , and the mean value of P_i.
3. The absolute difference of 'q' and 'r' is computed and is denoted by 's'.
4. The value of 's' is compared with the threshold value for the parameter P_i .

The image x_m is said to fall in the neighbourhood of image x_n if at least 25 out of the 30 values of 's' fall within the threshold.

Threshold and number of parameters are subject to the system under study.

In this manner, the neighbourhood of a given image x_n is computed by carrying out the above steps for each image in the Universal Set. Hence, $N(x_n)$ is determined and is a subset of the Universal Set.

In the following table, we give examples for calculating the neighbourhood set.

Table 5: Neighbourhood sets $N(x_n)$ versus the object x_n

$\boldsymbol{x_n}$	$\boldsymbol{N(x_n)}$
$\boldsymbol{x_1}$	$\{x_1, x_{36}, x_{41}, x_{62}, x_{66}, x_{83}, x_{178}, x_{189}\}$
$\boldsymbol{x_2}$	$\{x_2, x_6, x_{16}, x_{111}\}$
$\boldsymbol{x_3}$	$\{x_3, x_{12}, x_{24}, x_{59}, x_{80}, x_{111}, x_{122}\}$
$\boldsymbol{x_4}$	$\{x_4, x_7, x_{13}, x_{15}, x_{17}, x_{20}, x_{21}, x_{22}, x_{25}, x_{27}, x_{36}, x_{38}, x_{49}, x_{50}, x_{71}, x_{76}, x_{81}, x_{88}, x_{89}, x_{90}, x_{93}, x_{94}, x_{106}, x_{108}, x_{109}, x_{114}, x_{118}, x_{124}, x_{138}, x_{147}, x_{152}, x_{153}, x_{163}, x_{168}, x_{171}, x_{178}, x_{186}, x_{189}\}$

Now $I = (U, A, F, N)$ is a covering based N-information system.

When $X = \{x_3\}$,

Figure 2: Image/object x_3

$$N(X) = \{x_3, x_{12}, x_{24}, x_{59}, x_{80}, x_{111}, x_{122}\}$$

Following table shows the Double Bounded Rough Sets with respect to X for each attribute.

Table 6: Double Bounded Rough Sets with respect to X versus attribute a

	$DR\_(a{\sim}X)$	$^{-}DR(a{\sim}X)$	$DR^{-}(a{\sim}X)$
S	$\{x_3, x_6, x_{12}, x_{14}, x_{27}, x_{36}, x_{59}, x_{87}, x_{91}, x_{109}, x_{117}, x_{122}, x_{127}, x_{142}, x_{163}, x_{180}, x_{188}, x_{189}\}$	$\{x_3, x_6, x_{12}, x_{14}, x_{24}, x_{27}, x_{36}, x_{59}, x_{80}, x_{87}, x_{91}, x_{109}, x_{111}, x_{117}, x_{122}, x_{127}, x_{142}, x_{163}, x_{180}, x_{188}, x_{189}\}$	$\{x_1, x_3, x_4, x_5, x_6, x_7, x_{12}, x_{13}, x_{14}, x_{15} \ldots, x_{191}, x_{192}, x_{193}, x_{194}, x_{195}, x_{196}, x_{197}, x_{198}, x_{199}, x_{200}\}$
A	$\{x_2, x_3, x_6, x_8, x_{12}, x_{16}, x_{24}, x_{30}, x_{35}, x_{46}, x_{52}, x_{58}, x_{80}, x_{99}, x_{111}, x_{116}, x_{128}, x_{130}, x_{131}, x_{142}\}$	$\{x_2, x_3, x_6, x_8, x_{12}, x_{16}, x_{24}, x_{30}, x_{35}, x_{46}, x_{52}, x_{58}, x_{59}, x_{80}, x_{99}, x_{111}, x_{116}, x_{122}, x_{128}, x_{130}, x_{131}, x_{142}\}$	$\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{11}, \ldots, x_{189}, x_{190}, x_{191}, x_{192}, x_{193}, x_{194}, x_{195}, x_{196}, x_{198}, x_{199}\}$
H	$\{x_3, x_6, x_{12}, x_{24}, x_{36}, x_{44}, x_{59}, x_{70}, x_{80}, x_{82}, x_{101}, x_{111}, x_{118}, x_{122}, x_{127}, x_{130}, x_{138}, x_{158}\}$	$\{x_3, x_6, x_{12}, x_{24}, x_{36}, x_{44}, x_{59}, x_{70}, x_{80}, x_{82}, x_{101}, x_{111}, x_{118}, x_{122}, x_{127}, x_{130}, x_{138}, x_{158}\}$	$\{x_1, x_3, x_4, x_5, x_6, x_7, x_9, x_{11}, x_{12}, x_{13} \ldots, x_{186}, x_{187}, x_{189}, x_{190}, x_{192}, x_{193}, x_{194}, x_{195}, x_{198}, x_{199}\}$
SU	$\{x_3, x_6, x_{12}, x_{13}, x_{17}, x_{20}, x_{21}, x_{22}, x_{24}, x_{27}, x_{36}, x_{59}, x_{67}, x_{69}, x_{70}, x_{77}, x_{107}, x_{116}, x_{118}, x_{122}, x_{137}, x_{161}, x_{186}, x_{192}, x_{197}\}$	$\{x_3, x_6, x_{12}, x_{13}, x_{17}, x_{20}, x_{21}, x_{22}, x_{24}, x_{27}, x_{36}, x_{59}, x_{67}, x_{69}, x_{70}, x_{77}, x_{80}, x_{107}, x_{111}, x_{116}, x_{118}, x_{122}, x_{137}, x_{161}, x_{186}, x_{192}, x_{197}\}$	$\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} \ldots, x_{190}, x_{191}, x_{192}, x_{193}, x_{194}, x_{195}, x_{196}, x_{197}, x_{198}, x_{199}\}$

From the Double Bounded Rough Sets, the elements of Neutrosophic set are obtained as follows:

Table 7: Neutrosophic sets versus attribute a

	μ_($DR(a{\sim}X)$)	¯μ(¯$DR(a{\sim}X)$)	μ¯ (DR^{-} $(a{\sim}X)$)
S	0.7816	0.7786	0.7597
A	0.7627	0.7627	0.7597
H	0.7786	0.7786	0.7676
SU	0.786	0.7786	0.7597

The Neutrsophic fuzzy set on the set of all attribute for the image x_3 is given by:

μ($DRS(S{\sim}X)$) = { 0.7816, 0.7786, 0.7597}
μ($DRS(A{\sim}X)$) = { 0.7627, 0.7627, 0.7597}
μ($DRS(H{\sim}X)$) = { 0.7786, 0.7786, 0.7676}
μ($DRS(SU{\sim}X)$) = { 0.7860, 0.7786, 0.7597}

From this Neutrosophic fuzzy set, it is possible to predict the facial expression of the object/image.

The attribute value can be calculated using the following expressions:

$$V(S) = 2\left(\max\left(\left(\frac{T_S + I_S}{2}\right), \left(\frac{1 + I_S - F_S}{2}\right)\right) - \min\left(\left(\frac{T_S + I_S}{2}\right), \left(\frac{1 + I_S - F_S}{2}\right)\right)\right)$$

$$V(A) = 2\left(\max\left(\left(\frac{T_A + I_A}{2}\right), \left(\frac{1 + I_A - F_A}{2}\right)\right) - \min\left(\left(\frac{T_A + I_A}{2}\right), \left(\frac{1 + I_A - F_A}{2}\right)\right)\right)$$

$$V(H) = 2\left(\max\left(\left(\frac{T_H + I_H}{2}\right), \left(\frac{1 + I_H - F_H}{2}\right)\right) - \min\left(\left(\frac{T_H + I_H}{2}\right), \left(\frac{1 + I_H - F_H}{2}\right)\right)\right)$$

$$V(SU) = 2\left(\max\left(\left(\frac{T_{SU} + I_{SU}}{2}\right),\left(\frac{1 + I_{SU} - F_{SU}}{2}\right)\right) - \min\left(\left(\frac{T_{SU} + I_{SU}}{2}\right),\left(\frac{1 + I_{SU} - F_{SU}}{2}\right)\right)\right)$$

Substituting Values from the fuzzy Neutrosophic Set, the following are obtained:

$V(S)$ = 0.5413

$V(A)$ = 0.5224

$V(H)$ = 0.5462

$V(SU)$ = 0.5457

The attribute having the highest value is most likely to be the attribute possessed by the image.

Conclusion**:** The Person is Happy.

3. Results

Implication of Attribute Based Double Bounded Rough Neutrosophic Sets to Detect Facial Expressions:

3.1 By implementing Attribute based Double Bounded Rough Neutrosophic Sets, it is possible to detect the expression of a person with real time data.

$\boldsymbol{V(S) = 0.5466}$
$\boldsymbol{V(A) = 0.2403}$
$\boldsymbol{V(H) = 0.2324}$
$\boldsymbol{V(SU) = 0.5343}$
Person is Sad.

$\boldsymbol{V(S) = 0.5272}$
$\boldsymbol{V(A) = 0.5852}$
$\boldsymbol{V(H) = 0.5366}$
$\boldsymbol{V(SU) = 0.5577}$
Person is Angry.

$\boldsymbol{V(S) = 0.5272}$
$\boldsymbol{V(A) = 0.5303}$
$\boldsymbol{V(H) = 0.5422}$
$\boldsymbol{V(SU) = 0.5272}$
Person is Happy**.**

$V(S) = 0.2403$
$V(A) = 0.2403$
$V(H) = 0.2324$
$V(SU) = 0.7164$
Person is Surprised.

Figure 3: Values of attributes and predicted facial expression for each image

3.2 Clinicians realize that making an accurate diagnosis relies on the provision of reliable information by patients and their family members and that timely, astute, and compassionate care depends on effective bidirectional communications (between the patient and the physician) [13]. Unfortunately, both patients and physicians are often challenged by complicated communications; each group withholds, distorts, obfuscates, fabricates, or lies about information that is crucial to the doctor-patient relationship and to effective treatment. Such untruths and manipulation of information can damage relationships and compromise clinical care.

Facial cues lead to detection of lies and hence can be incorporated in order to detect any sort of miscommunication by the patient.

It is possible to differentiate between genuine smiles and fake smiles using our proposed method. This is often not obvious when seen with naked eye. The advantage of this is that we get a deeper and more realistic insight about the patient's emotion. Below are two images of a patient taken at different instances. A lower (indeterminacy + non-membership) value indicates a realistic smile. As the (indeterminacy + non- membership) value increases, the smile becomes fake.

$V(S) = 0.2403$
$V(A) = 0.2403$
$V(H) = 0.5422$
$V(SU) = 0.2403$
The patient is smiling.

$D(S) = T_S + I_S = 1.5571$
$D(A) = T_A + I_A = 1.5571$
$D(H) = T_H + I_H = 1.5422$
$D(SU) = T_{SU} + I_{SU} = 1.5571$

$D(H)$ is NOT the Highest.
GENUINE SMILE!

$V(S) = 0.5272$
$V(A) = 0.5343$
$V(H) = 0.5352$
$V(SU) = 0.5343$
The patient is smiling.

$D(S) = T_S + I_S = 1.5272$
$D(A) = T_A + I_A = 1.5343$
$D(H) = T_H + I_H = 1.5352$
$D(SU) = T_{SU} + I_{SU} = 1.5343$

$D(H)$ is Highest.
FAKE SMILE!

Figure 4: Illustration showing the distinction between detection of genuine and fake smile

3.3 Sadness is most often the primary emotion that gets transformed into anger. As a result of suppressing their full expression, the energy "becomes" anger. Sadness turns into anger when we realize all our sadness won't resolve the problem. The combination of sadness and anger generally indicates depression. This kind of emotion can be detected when the $V(S) = V(A)$.

$V(S) = 0.6141$
$V(A) = 0.6141$
$V(H) = 0.2324$
$V(SU) = 0.2403$

Person is Sad and Angry
(Possibly Depressed)

Figure 5 : Detection based on combination of expressions

3.4 While detecting facial expressions, it is very important to know how closely the person's expression resembles the detected expression, i.e., the surety/precision of the output.
Using Double Bounded Neutrosophic Sets, we can predict how closely an image resembles any expression. This degree of closeness is denoted by Q(a).

$V(S) = 0.2403$
$V(A) = 0.5659$
$V(H) = 0.2324$
$V(SU) = 0.2403$
The Preson is Angry.

$Q(S) = 25.03\ \%$
$Q(A) = 58.94\ \%$
$Q(H) = 24.21\ \%$
$Q(SU) = 25.03\ \%$

The degree of closeness to Anger is 58.94 %

Figure 6: Calculation of degree of closeness to the detected attribute

3.5 With the onset of the corona virus pandemic, most people are choosing to wear masks on a regular basis. Thus, many of the feature points on the face will be hidden, which makes it difficult to detect the person's actual expression. However, by using Attribute based Double Bounded Rough Neutrosophic Sets the person's true expression can be detected just by using the feature points in and around the eyes. The image below shows that the prediction of the person's actual expression is possible with and without the mask.

			
$V(S) = 0.5652$	$V(S) = 0.5376$	$V(S) = 0.2403$	$V(S) = 0.2403$
$V(A) = 0.5413$	$V(A) = 0.2403$	$V(A) = 0.6452$	$V(A) = 0.5193$
$V(H) = 0.2324$	$V(H) = 0.2403$	$V(H) = 0.2324$	$V(H) = 0.2403$
$V(SU) = 0.2403$	$V(SU) = 0.2403$	$V(SU) = 0.5681$	$V(SU) = 0.2403$
Person is Sad	Person is Sad	Person is Angry	Person is Angry

$V(S) = 0.5272$	$V(S) = 0.2403$	$V(S) = 0.2403$	$V(S) = 0.2403$
$V(A) = 0.5303$	$V(A) = 0.2403$	$V(A) = 0.2403$	$V(A) = 0.2403$
$V(H) = 0.5422$	$V(H) = 0.5413$	$V(H) = 0.2324$	$V(H) = 0.2403$
$V(SU) = 0.5272$	$V(SU) = 0.2403$	$V(SU) = 0.5457$	$V(SU) = 0.5275$
Person is Happy	Person is Happy	Person is Surprised	Person is Surprised

Figure 7: Detection of facial expression with and without mask

3.6 All over the world various educational institutes are now slowly moving towards conducting exams online, even competitive exams like GRE, GMAT and English language tests like TOEFL. As more and more exams are conducted online, students tend to involve themselves in various malpractices. Proctors find it difficult to assess each and every student's movement and expression because some might be faking it. But, using this Attribute based Double Bounded Rough Neutrosophic Sets, it becomes easy for the invigilators to detect if the student is actually faking an expression or not. Thus, it ensures that the students don't cheat and helps the universities in getting quality results.

$$\mu(DRS(S\sim X)) = \{ 0.7869, 0.7869, 0.7597 \}$$
$$\mu(DRS(A\sim X)) = \{ 0.8256, 0.8256, 0.7597 \}$$
$$\mu(DRS(H\sim X)) = \{ 0.7911, 0.7911, 0.7676 \}$$
$$\mu(DRS(SU\sim X)) = \{ 0.8109, 0.8109, 0.7597\}$$

$$V(S) = 0.5465$$
$$V(A) = 0.5852$$
$$V(H) = 0.5587$$
$$V(SU) = 0.5705$$

From above values, we can say that person is Angry.

$$D(S) = T_S + I_S = 1.5465$$
$$D(A) = T_A + I_A = 1.5852$$
$$D(H) = T_H + I_H = 1.5587$$
$$D(SU) = T_{SU} + I_{SU} = 1.5706$$

However, Sum of Falsity and Non-Membership Value is maximum for Anger. Hence, it can be concluded that person is faking the expression.

Figure 8: Cheating detection

4. Applications

Human beings have continually been seeking personal possessions (like nourishment, garments, vehicles, houses, fundamental information and data), ever since the birth of first mankind. It is turning out to be progressively significant that such important resources be preserved and protected by methods for security control. The types of technologies used in the access control systems are countless, throughout history. Traditional methodologies include security guard checks, elementary keypads, locks, passwords and entry codes. However, organisations now seek more progressed technologies with greater security and suitability. They seek an economical way for property protection, particularly in today's multifaceted society.

Fingerprint recognition, iris recognition, voice recognition, and facial recognition systems are some of the popular biometric systems in use today. These systems are being used in various organizations like banks, airports, social services offices, blood banks and other highly sensitive organizations. Biometrics play a very crucial role in today's society as they offer the most accurate authentication solution, and hence as a result of fast increasing technology, facial expression recognition becomes very important. The expressions that we emote are signals that carry high biological value. The key job that these facial articulations perform is that they transmit flags about the expresser's feeling, aims and conditions which are effective in social connection. It has always been a topic of discussion that the evolution of facial expression signalling systems have assisted adaptation. Hence the creditable transmission and decoding of such signals by human operators are of much significance.

Nonverbal communication cues such as facial expressions and other gestures play an important role in interpersonal relations. These cues assist speech by helping the listener to interpret the intended meaning of spoken words. Data from the images or any other visual feed are used in a variety of fields especially for Human Computer Interaction like computer vision, biometric security, social interaction, emotional and social intelligence.

5. Conclusions

A hybrid intelligent structure called "Double Bounded Rough Neutrosophic Sets" was defined. The Attribute based Double Bounded Rough Neutrosophic Sets was implemented for Facial Expression Detection and the following implications were discussed:

1. Detecting the facial expression of a person using real time data
2. Differentiating between Genuine and Fake smiles
3. Predicting if person might be Depressed
4. Determining the Degree of Closeness to a particular Attribute/Expression
5. With the onset of the corona virus pandemic, most people are choosing to wear masks on a regular basis. By using Attribute based Double Bounded Rough Neutrosophic Sets the person's true expression can be detected just by using the feature points in and around the eyes.
6. To check if a person is faking an expression or trying to cheat during an examination.

The results from our work helped us to understand the importance of Attribute based Double Bounded Rough Neutrosophic Sets and we were able to apply it for Facial Expression Detection and its various implications. The future work in this direction is to explore various other applications of double bounded rough Neutrosophic sets and detection of facial expressions using various other concepts.

Funding
This research received no external funding.

Acknowledgments
We thank the Management and Principal, Sri Sivasubramaniya Nadar College of Engineering for their support and the encouragement for the successful completion of the work.

Conflicts of Interest

The Authors declare no conflict of interest.

References

[1] Darwin, C.: The Expression of Emotions in Man and Animals. Murray, London (1872), reprinted by University of Chicago Press, (1965)

[2] Ekman, P.: The Argument and Evidence about Universals in Facial Expressions of Emotion,vol. 58,pp.143–164. Wiley, New York (1989)

[3] Scherer, K., Ekman, P.: Handbook of Methods in Nonverbal Behaviour Research, Cambridge University Press, Cambridge (1982)

[4] Tuark MA, Peintland AP. feeling recognition mistreatment eigenfaces. In: laptop Vision and Pattern Recognition. Proceedings, IEEE laptop Society Conference on. IEEE; 1991, p. 586-91.

[5] G. Wang et al. (Eds.) :Facial Expression Recognition Based on Rough Set Theory and SVM, RSKT 2006, LNAI 4062, pp. 772–777 (2006)

[6] Turker Tuncer, Sengul Dogan, Moloud Abdar , Mohammad Ehsan Basiri and Paweł Pławiak: Face Recognition with Triangular Fuzzy Set-Based Local Cross Patterns in Wavelet Domain - Symmetry, 11, 787, doi:10.3390/sym11060787 (2019)

[7] Facial Expression Recognition based on Fuzzy Networks-2016 International Conference on Computational Science and Computational Intelligence-978-1-5090-5510-4/16 ,DOI 10.1109/CSCI.2016.160

[8] Zimmermann, H.-J. (Hans-Jiirgen): Fuzzy set theory-and its applications / H.-J. Zimmermann:4th ed.(2001)

[9] B. Praba and R. Mohan , Rough Lattice,International Journal of Fuzzy Mathematics and Systems, Vol. 3, pp. 135-151,Number 2 (2013)

[10] S. Broumi, F. Smarandache, M. Dhar : Italian journal of pure and applied mathematics-N.32 , pp.493-502, (2014)

[11] https://www.kaggle.com/drgilermo/face-images-with-marked-landmark-points

[12] A Neutrosophic Multi-Criteria Decision Making Method, New Mathematics and Natural Computation Vol. 10, 143–162, DOI: 10.1142/S1793005714500070, No. 2 (2014)

[13] https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2736034/

Received: Dec. 10, 2021. Accepted: April 1, 2022.