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Girth in Neutrosophic Graphs

Ideas | Approaches | Accessibility | Availability

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Report | Exposition | References | Research #22 2022

Abstract

In this book, some notions are introduced about "Girth in Neutrosophic Graphs." Three chapters are devised as "Neutrosophic Girth Based On Crisp Cycle in Neutrosophic Graphs", "Neutrosophic Girth Based On Neutrosophic Cycle in Neutrosophic Graphs" and "Neutrosophic Girth Polynomial Based On Neutrosophic Cycle and Crisp Cycle in Neutrosophic Graphs". Three manuscripts are cited as the references of these chapters which are my 62nd, 63rd, and 64th manuscripts. I've used my 62nd, 63rd, and 64th manuscripts to write this book.

In first chapter, there are some points as follow. New setting is introduced to study girth and neutrosophic girth arising from shortest cycles. Forming cycles from a sequence of consecutive vertices is key type of approach to have these notions namely girth and neutrosophic girth arising from shortest cycles. Two numbers are obtained but now both settings leads to approach is on demand which is finding minimum cardinality and minimum neutrosophic cardinality in the terms of vertices, which have edges which form a shortest cycle. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then Girth $\mathcal{G}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum crisp cardinality of vertices forming shortest cycle. If there isn't, then girth is ∞; neutrosophic girth $\mathcal{G}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of vertices forming shortest cycle. If there isn't, then girth is ∞. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs and wheel-neutrosophic graphs. The clarifications are also presented in both sections "Setting of Girth," and "Setting of Neutrosophic Girth," for introduced results and used classes. Neutrosophic number is reused in this way. It's applied to use the type of neutrosophic number in the way that, three values of a vertex are used and they've same share to construct this number to compare with other vertices. Summation of three values of vertex makes one number and applying it to a comparison. This approach facilitates identifying vertices which form girth and neutrosophic girth arising from shortest cycles. In both settings, some classes of well-known neutrosophic graphs are studied. Some clarifications for each result and each definition are provided. The cardinality of a set has eligibility to girth but the neutrosophic cardinality of a set has eligibility to call neutrosophic girth. Some results get more frameworks and perspective about these definitions. The way in that, a sequence of consecutive vertices forming a cycle, opens the way to do some approaches. These notions are applied into neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special neutrosophic graphs which are well-known, is an open way to pursue this study. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this chapter.

In second chapter, there are some points as follow. New setting is introduced to study girth and neutrosophic girth arising from shortest neutrosophic cycles. Forming neutrosophic cycles from a sequence of consecutive vertices is key type of approach to have these notions namely girth and neutrosophic girth arising from shortest neutrosophic cycles. Two numbers are obtained but now both settings leads to approach is on demand which is finding minimum cardinality and minimum neutrosophic cardinality in the terms of vertices, which have edges which form a shortest neutrosophic cycle. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then Girth $\mathcal{G}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum crisp cardinality of vertices forming shortest neutrosophic cycle. If there isn't, then girth is ∞; neutrosophic girth $\mathcal{G}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of vertices forming shortest neutrosophic cycle. If there isn't, then girth is ∞. As concluding results, there are some statements, remarks, examples and clarifications about some classes of strong neutrosophic graphs namely strong-path-neutrosophic graphs, strong-cycle-neutrosophic graphs, complete-neutrosophic graphs, strong-star-neutrosophic graphs, strong-complete-bipartite-neutrosophic graphs, strong-complete-t-partite-neutrosophic graphs and strong-wheel-neutrosophic graphs. The clarifications are also presented in both sections "Setting of Girth," and "Setting of Neutrosophic Girth," for introduced results and used classes. Neutrosophic number is reused in this way. It's applied to use the type of neutrosophic number in the way that, three values of a vertex are used and they've same share to construct this number to compare with other vertices. Summation of three values of vertex makes one number and applying it to a comparison. This approach facilitates identifying vertices which form girth and neutrosophic girth arising from shortest neutrosophic cycles. In both settings, some classes of well-known strong neutrosophic graphs are studied. Some clarifications for each result and each definition are provided. The cardinality of a set has eligibility to girth but the neutrosophic cardinality of a set has eligibility to call neutrosophic girth. Some results get more frameworks and perspective about these definitions. The way in that, a sequence of consecutive vertices forming a neutrosophic cycle, opens the way to do some approaches. These notions are applied into strong neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special strong neutrosophic graphs which are well-known, is an open way to pursue this study. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this chapter.

In third chapter, there are some points as follow. New setting is introduced to study girth polynomial and neutrosophic girth polynomial arising counting neutrosophic cycles and crisp cycles in strong neutrosophic graphs based on neutrosophic cycles and in neutrosophic graphs based on crisp cycles. Forming neutrosophic cycles from a sequence of consecutive vertices is key type of approach to have these notions namely girth polynomial and neutrosophic

girth polynomial arising from counting neutrosophic cycles and crisp cycles in strong neutrosophic graphs based on neutrosophic cycles and in neutrosophic graphs based on crisp cycles. Two numbers are obtained but now both settings leads to approach is on demand which is counting minimum cardinality and minimum neutrosophic cardinality in the terms of vertices, which have edges which form neutrosophic cycle and crisp cycles. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then girth polynomial $\mathcal{G}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is $n_1x^{m_1} + n_2x^{m_2} + \cdots + n_sx^3$ where n_i is the number of cycle with m_i as its crisp cardinality of the set of vertices of cycle; neutrosophic girth polynomial $\mathcal{G}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is $n_1x^{m_1} + n_2x^{m_2} + \cdots + n_sx^{m_s}$ where n_i is the number of cycle with m_i as its neutrosophic cardinality of the set of vertices of cycle. As concluding results, there are some statements, remarks, examples and clarifications about some classes of strong neutrosophic graphs namely (strong-)path-neutrosophic graphs, (strong-)cycle-neutrosophic graphs, complete-neutrosophic graphs, (strong-)star-neutrosophic graphs, (strong-)complete-bipartite-neutrosophic graphs, (strong-)complete-t-partite-neutrosophic graphs and (strong-)wheel-neutrosophic graphs. The clarifications are also presented in both sections "Setting of Girth Polynomial," and "Setting of Neutrosophic Girth Polynomial," for introduced results and used classes. Neutrosophic number is reused in this way. It's applied to use the type of neutrosophic number in the way that, three values of a vertex are used and they've same share to construct this number to compare with other vertices. Summation of three values of vertex makes one number and applying it to a comparison. This approach facilitates identifying vertices which form girth polynomial and neutrosophic girth polynomial arising from counting neutrosophic cycles and crisp cycles in strong neutrosophic graphs based on neutrosophic cycles and in neutrosophic graphs based on crisp cycles. In both settings, some classes of well-known (strong) neutrosophic graphs are studied. Some clarifications for each result and each definition are provided. The cardinality of a set has eligibility to define girth polynomial but the neutrosophic cardinality of a set has eligibility to define neutrosophic girth polynomial. Some results get more frameworks and perspective about these definitions. The way in that, a sequence of consecutive vertices forming a neutrosophic cycle and crisp cycles, opens the way to do some approaches. These notions are applied into strong neutrosophic graphs and neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special strong neutrosophic graphs and neutrosophic graphs which are well-known, is an open way to pursue this study. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this chapter.

The following references are cited by chapters.

[Ref1] Henry Garrett, "*Neutrosophic Girth Based On Crisp Cycle in Neutrosophic Graphs*", ResearchGate 2022 (doi: 10.13140/RG.2.2.14011.69923).

[Ref2] Henry Garrett, "*Finding Shortest Sequences of Consecutive Vertices in Neutrosophic Graphs*", ResearchGate 2022 (doi: 10.13140/RG.2.2.22924.59526).

[Ref3] Henry Garrett, "*Some Polynomials Related to Numbers in Classes of (Strong) Neutrosophic Graphs*", ResearchGate 2022 (doi: 10.13140/RG.2.2.36280.83204).

Abstract

Three chapters are devised as “Neutrosophic Girth Based On Crisp Cycle in Neutrosophic Graphs”, “Neutrosophic Girth Based On Neutrosophic Cycle in Neutrosophic Graphs” and “Neutrosophic Girth Polynomial Based On Neutrosophic Cycle and Crisp Cycle in Neutrosophic Graphs”.

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The words of mind and the minds of words, are too eligible to be in the stage of acknowledgements

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CHAPTER 1

Neutrosophic Girth Based On Crisp Cycle in Neutrosophic Graphs

The following sections are cited as [**Ref1**] which is my 62nd manuscript and I use prefix 62 as number before any labelling for items.

1.1 Neutrosophic Girth Based On Crisp Cycle in Neutrosophic Graphs

1.2 Abstract

New setting is introduced to study girth and neutrosophic girth arising from shortest cycles. Forming cycles from a sequence of consecutive vertices is key type of approach to have these notions namely girth and neutrosophic girth arising from shortest cycles. Two numbers are obtained but now both settings leads to approach is on demand which is finding minimum cardinality and minimum neutrosophic cardinality in the terms of vertices, which have edges which form a shortest cycle. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then Girth $\mathcal{G}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum crisp cardinality of vertices forming shortest cycle. If there isn't, then girth is ∞; neutrosophic girth $\mathcal{G}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of vertices forming shortest cycle. If there isn't, then girth is ∞. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs and wheel-neutrosophic graphs. The clarifications are also presented in both sections "Setting of Girth," and "Setting of Neutrosophic Girth," for introduced results and used classes. Neutrosophic number is reused in this way. It's applied to use the type of neutrosophic number in the way that, three values of a vertex are used and they've same share to construct this number to compare with other vertices. Summation of three values of vertex makes one number and applying it to a comparison. This approach facilitates identifying vertices which form girth and neutrosophic girth arising from shortest cycles. In both settings,

some classes of well-known neutrosophic graphs are studied. Some clarifications for each result and each definition are provided. The cardinality of a set has eligibility to girth but the neutrosophic cardinality of a set has eligibility to call neutrosophic girth. Some results get more frameworks and perspective about these definitions. The way in that, a sequence of consecutive vertices forming a cycle, opens the way to do some approaches. These notions are applied into neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special neutrosophic graphs which are well-known, is an open way to pursue this study. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.
Keywords: Girth, Neutrosophic Girth, Shortest Cycle

AMS Subject Classification: 05C17, 05C22, 05E45

1.3 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 1.3.1. *Is it possible to use mixed versions of ideas concerning "Neutrosophic Girth", "Girth" and "Neutrosophic Graph" to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Lack of connection amid two edges have key roles to assign girth and neutrosophic girth arising from shortest cycles. Thus they're used to define new ideas which conclude to the structure of girth and neutrosophic girth arising from shortest cycles. The concept of having common shortest cycle inspires us to study the behavior of vertices in the way that, some types of numbers, girth and neutrosophic girth arising from shortest cycles are the cases of study in the setting of individuals. In both settings, a corresponded number concludes the discussion. Also, there are some avenues to extend these notions.

The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In subsection "Preliminaries", new notions of girth and neutrosophic girth arising from shortest cycles, are highlighted, are introduced and are clarified as individuals. In section "Preliminaries", sequence of consecutive vertices forming cycles have the key role in this way. General results are obtained and also, the results about the basic notions of girth and neutrosophic girth arising from shortest cycles, are elicited. Some classes of neutrosophic graphs are studied in the terms of girth arising from shortest cycles, in section "Setting of Girth," as individuals. In section "Setting of Girth," girth is applied into individuals. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs and wheel-neutrosophic graphs. The clarifications are also presented in both sections "Setting of Girth," and "Setting of Neutrosophic Girth," for introduced results and used classes. In section “Applications in Time Table and Scheduling”, two applications are posed for quasi-complete and complete notions, namely complete-t-neutrosophic graphs and complete-neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are considered as individual. In section “Open Problems”, some problems and questions for further studies are proposed. In section “Conclusion and Closing Remarks”, gentle discussion about results and applications is featured. In section “Conclusion and Closing Remarks”, a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

1.4 Preliminaries

In this subsection, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.
Basic idea is about the model which is used. First definition introduces basic model.

Definition 1.4.1. (Graph).
$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

Definition 1.4.2. (Neutrosophic Graph And Its Special Case).
$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it’s graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_i v_j \in E$,

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic edge set**.

(iii) : $|V|$ is called **order** of NTG and it’s denoted by $\mathcal{O}(NTG)$.

(iv) : $\sum_{v \in V} \sigma(v)$ is called **neutrosophic order** of NTG and it’s denoted by $\mathcal{O}_n(NTG)$.

(v) : $|E|$ is called **size** of NTG and it’s denoted by $\mathcal{S}(NTG)$.

(vi) : $\sum_{e \in E} \sum_{i=1}^{3} \mu_i(e)$ is called **neutrosophic size** of NTG and it’s denoted by $\mathcal{S}_n(NTG)$.

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

Definition 1.4.3. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) : a sequence of vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}}$ is called **path** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$;

(ii) : **strength** of path $P : x_0, x_1, \cdots, x_{\mathcal{O}}$ is $\bigwedge_{i=0,\cdots,n-1} \mu(x_i x_{i+1})$;

(iii) : **connectedness** amid vertices x_0 and x_t is

$$\mu^{\infty}(x_0, x_t) = \bigvee_{P:x_0,x_1,\cdots,x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1});$$

(iv) : a sequence of vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}}$ is called **cycle** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$;

(v) : it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$;

(vi) : t-partite is **complete bipartite** if $t = 2$, and it's denoted by K_{σ_1,σ_2};

(vii) : complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1,σ_2};

$(viii)$: a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1,σ_2};

(ix) : it's **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$;

(x) : it's **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

Definition 1.4.4. (Girth and Neutrosophic Girth).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Girth** $\mathcal{G}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum crisp cardinality of vertices forming shortest cycle. If there isn't, then girth is ∞;

(ii) **neutrosophic girth** $\mathcal{G}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of vertices forming shortest cycle. If there isn't, then girth is ∞.

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.
In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 1.4.5. In Figure (1.1), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most

2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*ii*) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1, n_2, n_3$$

is corresponded to girth $\mathcal{G}(NTG)$ but neutrosophic length of this crisp cycle implies

$$n_1, n_2, n_3$$

isn't corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iii*) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's two crisp cycles with length two and three. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1, n_3, n_4, and with length four, n_1, n_2, n_3, n_4. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a sequence of consecutive vertices is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with two different length three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. The length of this path implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iv*) if n_1, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to girth $\mathcal{G}(NTG)$ and neutrosophic length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

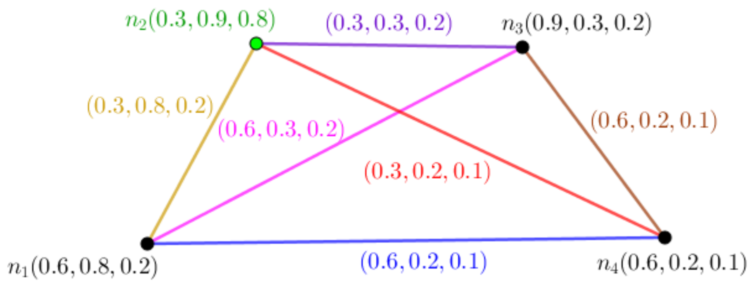

Figure 1.1: A Neutrosophic Graph in the Viewpoint of its Girth and its Neutrosophic Girth. 62NTG1

(v) 3 is girth and its corresponded sets are $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_2, n_3, n_4\}$;

(vi) 3.9 is neutrosophic girth and its corresponded set is $\{n_1, n_3, n_4\}$.

1.5 Setting of Girth

In this section, I provide some results in the setting of girth. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, star-neutrosophic graph, bipartite-neutrosophic graph, and t-partite-neutrosophic graph, are both of cases of study and classes which the results are about them.

Proposition 1.5.1. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{G}(NTG) = 3.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. The length of longest cycle is $\mathcal{O}(NTG)$. In other hand, there's a cycle if and only if $\mathcal{O}(NTG) \geq 3$. It's complete. So there's at least one neutrosophic cycle which its length is $\mathcal{O}(NTG) = 3$. By shortest cycle is on demand, the girth is three. Thus

$$\mathcal{G}(NTG) = 3.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.2. In Figure (1.2), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1, n_2, n_3$$

is corresponded to girth $\mathcal{G}(NTG)$ but neutrosophic length of this crisp cycle implies

$$n_1, n_2, n_3$$

isn't corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's two crisp cycles with length two and three. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1, n_3, n_4, and with length four, n_1, n_2, n_3, n_4. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a sequence of consecutive vertices is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with two different length three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. The length of this path implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

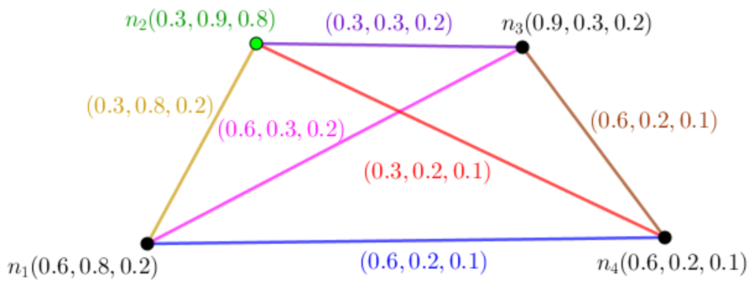

Figure 1.2: A Neutrosophic Graph in the Viewpoint of its Girth. 62NTG2

is corresponded to girth $\mathcal{G}(NTG)$ and neutrosophic length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 3 is girth and its corresponded sets are $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_2, n_3, n_4\}$;

(vi) 3.9 is neutrosophic girth and its corresponded set is $\{n_1, n_3, n_4\}$.

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

Proposition 1.5.3. *Let* $NTG : (V, E, \sigma, \mu)$ *be a path-neutrosophic graph. Then*

$$\mathcal{G}(NTG) = \infty.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. There's no crisp cycle. If $NTG : (V, E, \sigma, \mu)$ isn't a crisp cycle, then $NTG : (V, E, \sigma, \mu)$ isn't a neutrosophic cycle. There's no cycle from every version. Let $x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ be a path-neutrosophic graph. Since $x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is a sequence of consecutive vertices, there's no repetition of vertices in this sequence. So there's no cycle. Girth is corresponded to shortest cycle but there's no cycle. Thus it implies

$$\mathcal{G}(NTG) = \infty.$$

■

Example 1.5.4. There are two sections for clarifications.

(a) In Figure (1.3), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle.

So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. So adding points has to effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_3, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are four edges, n_1n_2, n_2n_3 and n_4n_5, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. So adding vertices has to effect to find a crisp cycle. There is only one path amid two given vertices. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4, n_5$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) ∞ is girth and there are no corresponded sets;

(*vi*) ∞ is neutrosophic girth and there are no corresponded sets.

(*b*) In Figure (1.4), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*ii*) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iii*) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. So adding points has to effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iv*) if n_1, n_2, n_3, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are four edges, n_1n_2, n_2n_3 and n_4n_5, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either

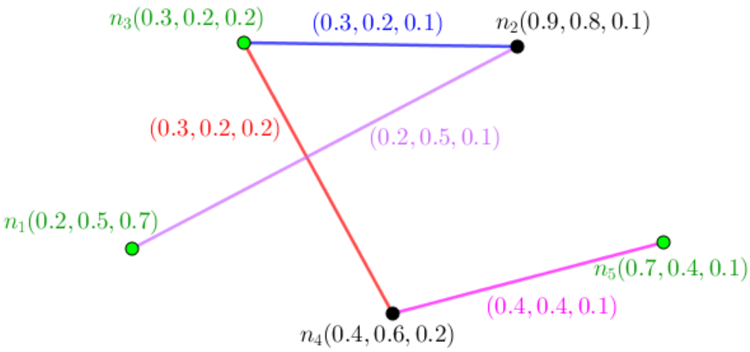

Figure 1.3: A Neutrosophic Graph in the Viewpoint of its Girth. 62NTG3

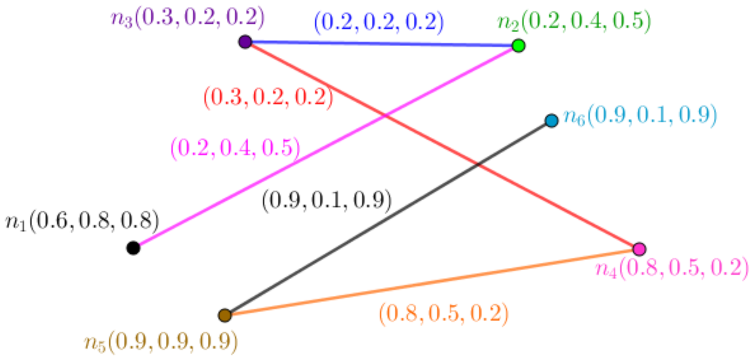

Figure 1.4: A Neutrosophic Graph in the Viewpoint of its Girth. 62NTG4

a neutrosophic cycle nor crisp cycle. So adding vertices has to effect to find a crisp cycle. There is only one path amid two given vertices. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4, n_5$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) ∞ is girth and there are no corresponded sets;

(vi) ∞ is neutrosophic girth and there are no corresponded sets.

Proposition 1.5.5. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(NTG) \geq 3$. Then*

$$\mathcal{G}(NTG) = \mathcal{O}(NTG).$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(NTG)}, x_1$ be a sequence of consecutive vertices of $NTG : (V, E, \sigma, \mu)$ such that

$$x_i x_{i+1} \in E, \ i = 1, 2, \cdots, \mathcal{O}(NTG) - 1, \ x_{\mathcal{O}(NTG)} x_1 \in E.$$

There are two paths amid two given vertices. The degree of every vertex is two. But there's one crisp cycle for every given vertex. So the efforts leads to one cycle for finding a shortest crisp cycle. For a given vertex x_i, the sequence of consecutive vertices

$$x_i, x_{i+1}, \cdots, x_{i-2}, x_{i-1}, x_i$$

is a corresponded crisp cycle for x_i. Every cycle has same length. The length is $\mathcal{O}(NTG)$. Thus the crisp cardinality of set of vertices forming shortest crisp cycle is $\mathcal{O}(NTG)$. It implies

$$\mathcal{G}(NTG) = \mathcal{O}(NTG).$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.6. There are two sections for clarifications.

(a) In Figure (1.5), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

  is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

  (ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

  is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. So adding points has to effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are six edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6$ and n_6n_1, according to corresponded neutrosophic path and it's neutrosophic cycle since it has two weakest edges, n_4n_5 and n_5n_6 with same values $(0.1, 0.1, 0.2)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is both of a neutrosophic cycle and crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is corresponded to both of girth $\mathcal{G}(NTG)$ and neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 6 is girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$;

(vi) $8.1 = \mathcal{O}(NTG)$ is neutrosophic girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$.

(b) In Figure (1.6), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. So adding points has to effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are five edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ and n_5n_1, according to corresponded neutrosophic path and it isn't neutrosophic cycle since it has only one weakest edge, n_1n_2, with value $(0.2, 0.5, 0.4)$ and not more. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is not a neutrosophic cycle but it is a crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_1$ is corresponded to both of girth $\mathcal{G}(NTG)$ and neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 5 is girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$;

(vi) $8.5 = \mathcal{O}(NTG)$ is neutrosophic girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$.

Proposition 1.5.7. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then*

$$\mathcal{G}(NTG) = \infty.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. Every vertex isn't a neighbor for every given vertex. Every vertex is a neighbor for center. Furthermore, center is only neighbor for any given vertex. So center is only neighbor for all vertices. It's possible to have some paths amid two given vertices but there's no crisp cycle. In other words, if $\mathcal{O}(NTG) > 2$, then there are at

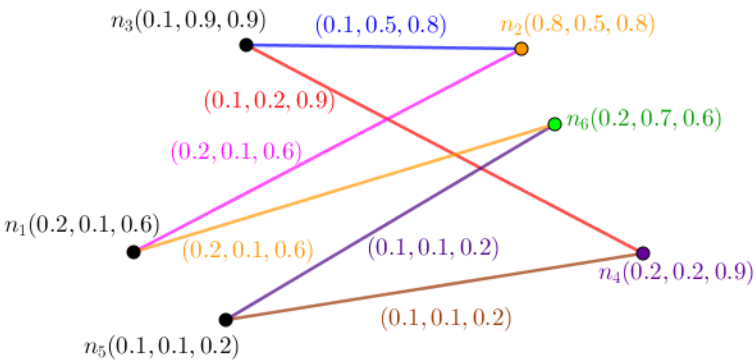

Figure 1.5: A Neutrosophic Graph in the Viewpoint of its Girth. 62NTG5

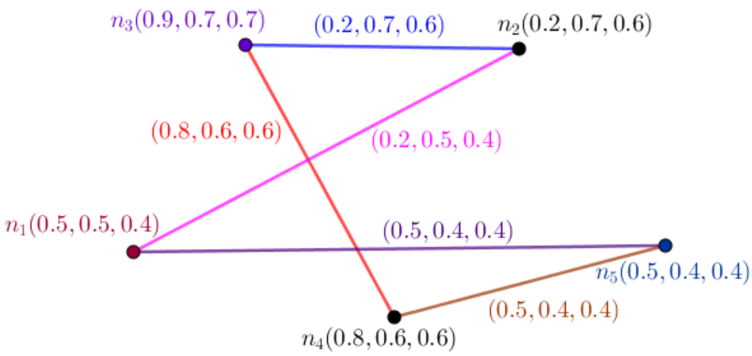

Figure 1.6: A Neutrosophic Graph in the Viewpoint of its Girth. 62NTG6

least three vertices x, y and z such that if x is a neighbor for y and z, then y and z aren't neighbors and x is center. To get more precise, if if x and y are neighbors then either x or y is center. Every edge have one common endpoint with other edges which is called center. Thus there is no triangle but there are some edges. One edge has two endpoints which one of them is center. There are no crisp cycle. Hence trying to find shortest cycle has no result. There is no crisp cycle. Then there is shortest crisp cycle. So

$$\mathcal{G}(NTG) = \infty.$$

■

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.8. There is one section for clarifications. In Figure (1.7), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a star and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this star implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. The length of this star implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a star and there are two edges, n_1n_2 and n_1n_3, according to corresponded neutrosophic star but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic star implies

$$n_1, n_2, n_3$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a star and there are three edges, n_1n_2, n_1n_3 and n_1n_4, according to corresponded neutrosophic star but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. So adding vertices has no effect to find a crisp cycle. The structure of this neutrosophic star implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_3, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a star and there are four edges, n_1n_2, n_1n_3, n_1n_4 and n_1n_5, according to corresponded neutrosophic star but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. So adding vertices has no effect to find a crisp cycle. There are some paths amid two given vertices. The structure of this neutrosophic star implies

$$n_1, n_2, n_3, n_4, n_5$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

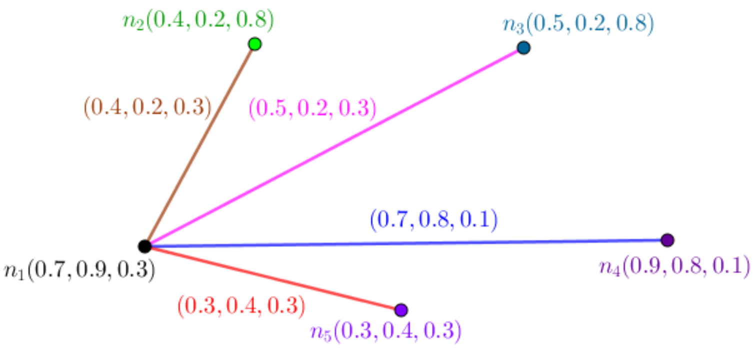

Figure 1.7: A Neutrosophic Graph in the Viewpoint of its Girth. 62NTG7

(v) ∞ is girth and there is no corresponded set;

(vi) ∞ is neutrosophic girth and there is no corresponded set.

Proposition 1.5.9. *Let* $NTG : (V, E, \sigma, \mu)$ *be a complete-bipartite-neutrosophic graph. Then*

$$\mathcal{G}(NTG) = 4$$

where $\mathcal{O}(NTG) \geq 4$*. And*

$$\mathcal{G}(NTG) = \infty$$

where $\mathcal{O}(NTG) \leq 3$.

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-bipartite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. If $\mathcal{O}(NTG) \leq 3$, then it's neutrosophic path implying

$$\mathcal{G}(NTG) = \infty.$$

If $\mathcal{O}(NTG) \geq 4$, then it's possible to have two vertices in every part. In this case, four vertices form a crisp cycle which crisp cardinality of its vertices are four. It's impossible to have a crisp cycle which crisp cardinality of its vertices are three. Since the sequence of consecutive vertices are x_1, x_2, x_3 and there's no edge more. It implies there are two edges. It's neutrosophic path but neither crisp cycle nor neutrosophic cycle. So the first step of finding shortest crisp cycle is impossible but in second step, there's one crisp cycle corresponded to number four. Thus

$$\mathcal{G}(NTG) = 4$$

where $\mathcal{O}(NTG) \geq 4$. And

$$\mathcal{G}(NTG) = \infty$$

where $\mathcal{O}(NTG) \leq 3$. ∎

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.10. There is one section for clarifications. In Figure (1.8), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a complete-bipartite and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this complete-bipartite implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic complete-bipartite has neither a neutrosophic cycle nor crisp cycle. The length of this complete-bipartite implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-bipartite and there are two edges, n_1n_2 and n_1n_3, according to corresponded neutrosophic complete-bipartite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-bipartite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-bipartite implies

$$n_1, n_2, n_3$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-bipartite and there are two edges, n_1n_2 and n_2n_4, according to corresponded neutrosophic complete-bipartite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-bipartite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-bipartite implies

$$n_1, n_2, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a complete-bipartite and there are four edges, n_1n_2, n_1n_3, n_2n_4 and n_3n_4, according to corresponded neutrosophic complete-bipartite and it has neutrosophic cycle where n_2n_4 and n_3n_4 are two weakest edge with same amount $(0.3, 0.2, 0.3)$. First step is to

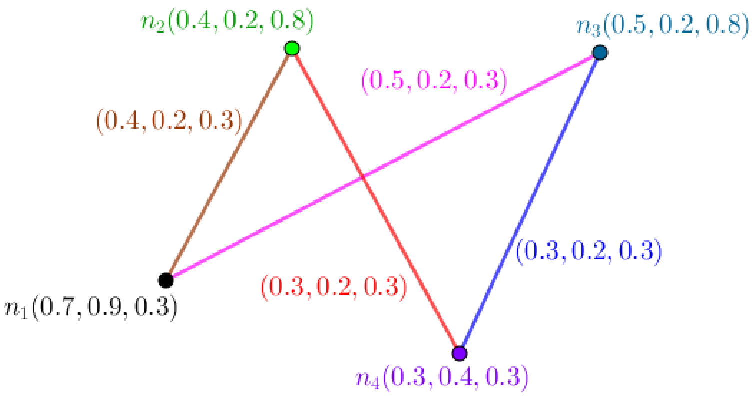

Figure 1.8: A Neutrosophic Graph in the Viewpoint of Girth. 62NTG8

have at least one crisp cycle for finding shortest cycle. Finding shortest cycle only has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-bipartite has both of a neutrosophic cycle and crisp cycle. So adding vertices has some effects to find a crisp cycle. The structure of this neutrosophic complete-bipartite implies

$$n_1, n_2, n_3, n_4$$

is corresponded to girth $\mathcal{G}(NTG)$ and uniqueness of this cycle implies the sequence

$$n_1, n_2, n_3, n_4$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 4 is girth and its corresponded sequence is n_1, n_2, n_3, n_4;

(vi) 5.8 is neutrosophic girth and its corresponded sequence is n_1, n_2, n_3, n_4.

Proposition 1.5.11. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-t-partite-neutrosophic graph. Then*

$$\mathcal{G}(NTG) = 3$$

where $t \geq 3$.

$$\mathcal{G}(NTG) = 4$$

where $t \leq 2$. And

$$\mathcal{G}(NTG) = \infty$$

where $\mathcal{O}(NTG) \leq 2$.

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-t-partite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. If $\mathcal{O}(NTG) \leq 2$, then it's neutrosophic path implying

$$\mathcal{G}(NTG) = \infty.$$

If $t \geq 3, \mathcal{O}(NTG) \geq 3$, then it has crisp cycle implying

$$\mathcal{G}(NTG) = 3.$$

If $t \geq 2, \mathcal{O}(NTG) \geq 4$, then it's possible to have two vertices in every part. In this case, four vertices form a crisp cycle which crisp cardinality of its vertices are four. It's impossible to have a crisp cycle which crisp cardinality of its vertices are three. Since the sequence of consecutive vertices are x_1, x_2, x_3 and there's no edge more. It implies there are two edges. It's neutrosophic path but neither crisp cycle nor neutrosophic cycle. So the first step of finding shortest crisp cycle is impossible but in second step, there's one crisp cycle corresponded to number four. Thus

$$\mathcal{G}(NTG) = 3$$

where $t \geq 3$.

$$\mathcal{G}(NTG) = 4$$

where $t \leq 2$. And

$$\mathcal{G}(NTG) = \infty$$

where $\mathcal{O}(NTG) \leq 2$. ■

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.12. There is one section for clarifications. In Figure (1.9), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a complete-t-partite and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this complete-t-partite implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The length of this complete-t-partite implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-t-partite and there are two edges, n_1n_2 and n_1n_3, according to corresponded neutrosophic complete-t-partite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_3$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-t-partite and there are two edges, n_1n_2 and n_2n_4, according to corresponded neutrosophic complete-t-partite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a complete-t-partite and there are four edges, n_1n_2, n_1n_5, n_2n_4 and n_5n_4, according to corresponded neutrosophic complete-t-partite and it has neutrosophic cycle where n_2n_4 and n_5n_4 are two weakest edge with same amount $(0.3, 0.2, 0.3)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle only has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has both of a neutrosophic cycle and crisp cycle. So adding vertices has some effects to find a crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_4, n_5$$

is corresponded to girth $\mathcal{G}(NTG)$ and minimum neutrosophic cardinality implies the sequence

$$n_1, n_2, n_4, n_5$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 4 is girth and its corresponded sequence is n_1, n_2, n_4, n_5;

(vi) 5.7 is neutrosophic girth and its corresponded sequence is n_1, n_2, n_4, n_5.

Proposition 1.5.13. *Let $NTG : (V, E, \sigma, \mu)$ be a wheel-neutrosophic graph. Then*

$$\mathcal{G}(NTG) = 3$$

where $t \geq 3$.

$$\mathcal{G}(NTG) = \infty$$

where $t \geq 2$.

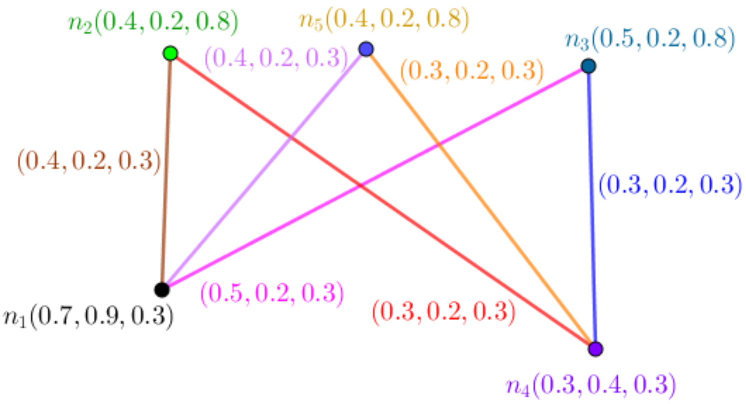

Figure 1.9: A Neutrosophic Graph in the Viewpoint of its Girth.

62NTG9

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a wheel-neutrosophic graph. The argument is elementary. Since all vertices of a path join to one vertex. Thus

$$\mathcal{G}(NTG) = 3$$

where $t \geq 3$.

$$\mathcal{G}(NTG) = \infty$$

where $t \geq 2$. ∎

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.5.14. There is one section for clarifications. In Figure (1.10), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If s_1, s_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a wheel and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this wheel implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic wheel has neither a neutrosophic cycle nor crisp cycle. The length of this wheel implies

$$s_1, s_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if s_4, s_2, s_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a wheel and there are two edges, s_3s_2 and s_4s_3, according to corresponded neutrosophic wheel but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic wheel has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic wheel implies

$$s_4, s_2, s_3$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if s_1, s_2, s_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a wheel and there are three edges, s_1s_2, s_2s_3 and s_1s_3 according to corresponded neutrosophic wheel but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one crisp cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic wheel has no neutrosophic cycle but it has crisp cycle. The structure of this neutrosophic wheel implies

$$s_1, s_2, s_3$$

is corresponded to girth $\mathcal{G}(NTG)$ but minimum neutrosophic cardinality implies

$$s_1, s_2, s_3$$

isn't corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if s_1, s_3, s_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a wheel and there are three edges, s_1s_4, s_4s_3 and s_1s_3 according to corresponded neutrosophic wheel and it has a neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one crisp cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic wheel has one neutrosophic cycle with two weakest edges s_1s_4 and s_3s_4 concerning same values $(0.1, 0.1, 0.5)$ and it has a crisp cycle. The structure of this neutrosophic wheel implies

$$s_1, s_3, s_4$$

is corresponded to girth $\mathcal{G}(NTG)$ and minimum neutrosophic cardinality implies

$$s_1, s_3, s_4$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 3 is girth and its corresponded sequences are s_1, s_3, s_4 and s_1, s_2, s_3 alongside s_1, s_4, s_5;

(vi) 3.8 is neutrosophic girth and its corresponded sequence is s_1, s_3, s_4.

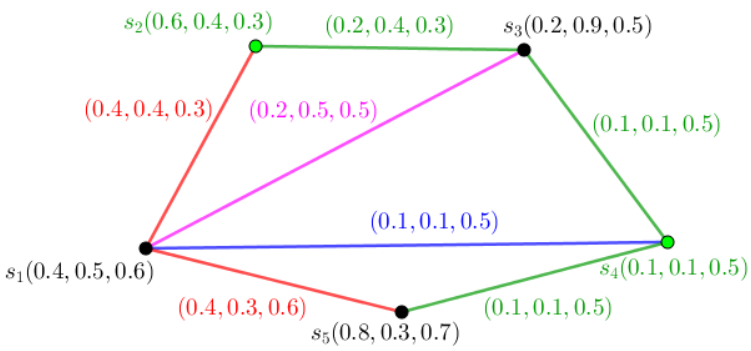

Figure 1.10: A Neutrosophic Graph in the Viewpoint of its Girth. 62NTG10

1.6 Setting of Neutrosophic Girth

In this section, I provide some results in the setting of neutrosophic girth. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, star-neutrosophic graph, bipartite-neutrosophic graph, and t-partite-neutrosophic graph, are both of cases of study and classes which the results are about them.

Proposition 1.6.1. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^3(\sigma_i(x) + \sigma_i(y) + \sigma_i(z))\}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. The length of longest cycle is $\mathcal{O}(NTG)$. In other hand, there's a cycle if and only if $\mathcal{O}(NTG) \geq 3$. It's complete. So there's at least one neutrosophic cycle which its length is $\mathcal{O}(NTG) = 3$. By shortest cycle is on demand, the girth is three. Thus

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^3(\sigma_i(x) + \sigma_i(y) + \sigma_i(z))\}.$$

∎

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.6.2. In Figure (1.11), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most

2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*ii*) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1, n_2, n_3$$

is corresponded to girth $\mathcal{G}(NTG)$ but neutrosophic length of this crisp cycle implies

$$n_1, n_2, n_3$$

isn't corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iii*) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's two crisp cycles with length two and three. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1, n_3, n_4, and with length four, n_1, n_2, n_3, n_4. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a sequence of consecutive vertices is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with two different length three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. The length of this path implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iv*) if n_1, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to girth $\mathcal{G}(NTG)$ and neutrosophic length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

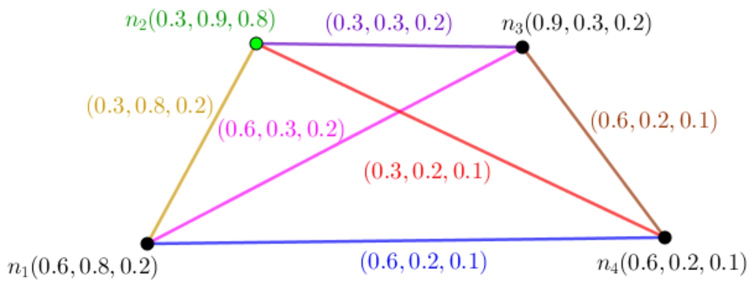

Figure 1.11: A Neutrosophic Graph in the Viewpoint of its Girth. 62NTG11

(v) 3 is girth and its corresponded sets are $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_2, n_3, n_4\}$;

(vi) 3.9 is neutrosophic girth and its corresponded set is $\{n_1, n_3, n_4\}$.

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

Proposition 1.6.3. *Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then*

$$\mathcal{G}_n(NTG) = \infty.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. There's no crisp cycle. If $NTG : (V, E, \sigma, \mu)$ isn't a crisp cycle, then $NTG : (V, E, \sigma, \mu)$ isn't a neutrosophic cycle. There's no cycle from every version. Let $x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ be a path-neutrosophic graph. Since $x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is a sequence of consecutive vertices, there's no repetition of vertices in this sequence. So there's no cycle. Girth is corresponded to shortest cycle but there's no cycle. Thus it implies

$$\mathcal{G}_n(NTG) = \infty.$$

■

Example 1.6.4. There are two sections for clarifications.

(a) In Figure (1.12), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. So adding points has to effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_3, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are four edges, n_1n_2, n_2n_3 and n_4n_5, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. So adding vertices has to effect to find a crisp cycle. There is only one path amid two given vertices. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4, n_5$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) ∞ is girth and there are no corresponded sets;

(vi) ∞ is neutrosophic girth and there are no corresponded sets.

(b) In Figure (1.13), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. So adding points has to effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_3, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are four edges, n_1n_2, n_2n_3 and n_4n_5, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. So adding vertices has to effect to find a crisp cycle. There is only one path amid two given vertices. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4, n_5$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

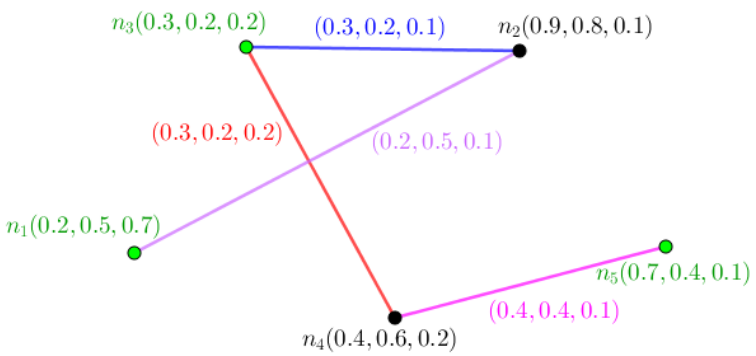

Figure 1.12: A Neutrosophic Graph in the Viewpoint of its Girth. 62NTG12

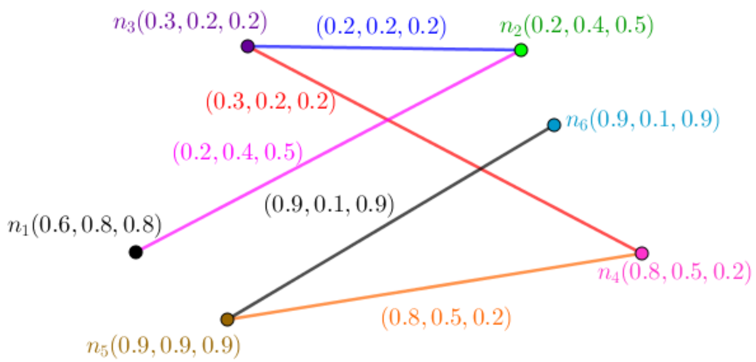

Figure 1.13: A Neutrosophic Graph in the Viewpoint of its Girth. 62NTG13

(v) ∞ is girth and there are no corresponded sets;

(vi) ∞ is neutrosophic girth and there are no corresponded sets.

Proposition 1.6.5. *Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $\mathcal{O}(NTG) \geq 3$. Then*

$$\mathcal{G}_n(NTG) = \mathcal{O}_n(NTG).$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(NTG)}, x_1$ be a sequence of consecutive vertices of $NTG : (V, E, \sigma, \mu)$ such that

$$x_i x_{i+1} \in E, \ i = 1, 2, \cdots, \mathcal{O}(NTG) - 1, \ x_{\mathcal{O}(NTG)} x_1 \in E.$$

There are two paths amid two given vertices. The degree of every vertex is two. But there's one crisp cycle for every given vertex. So the efforts leads to one cycle for finding a shortest crisp cycle. For a given vertex x_i, the sequence of consecutive vertices

$$x_i, x_{i+1}, \cdots, x_{i-2}, x_{i-1}, x_i$$

is a corresponded crisp cycle for x_i. Every cycle has same length. The length is $\mathcal{O}(NTG)$. Thus the crisp cardinality of set of vertices forming shortest crisp cycle is $\mathcal{O}(NTG)$. It implies

$$\mathcal{G}_n(NTG) = \mathcal{O}_n(NTG).$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.6.6. There are two sections for clarifications.

(a) In Figure (1.14), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. So adding points has to effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are six edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6$ and n_6n_1, according to corresponded neutrosophic path and it's neutrosophic cycle since it has two weakest edges, n_4n_5 and n_5n_6 with same values $(0.1, 0.1, 0.2)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is both of a neutrosophic cycle and crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is corresponded to both of girth $\mathcal{G}(NTG)$ and neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 6 is girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$;

(vi) $8.1 = \mathcal{O}(NTG)$ is neutrosophic girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$.

(b) In Figure (1.15), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

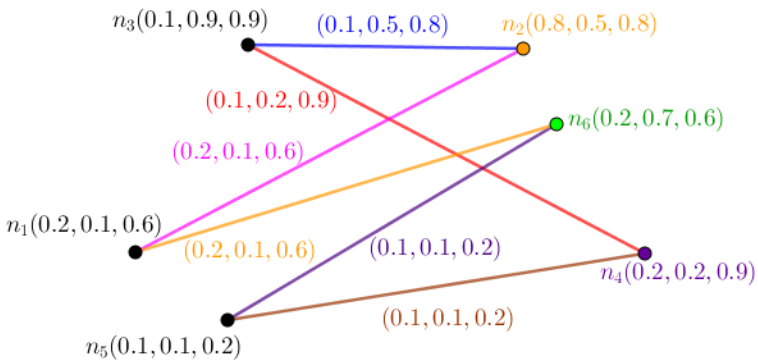

Figure 1.14: A Neutrosophic Graph in the Viewpoint of its Girth. 62NTG14

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. So adding points has to effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are five edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ and n_5n_1, according to corresponded neutrosophic path and it isn't neutrosophic cycle since it has only one weakest edge, n_1n_2, with value $(0.2, 0.5, 0.4)$ and not more. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is not a neutrosophic cycle but it is a crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_1$ is corresponded to both of girth $\mathcal{G}(NTG)$ and neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 5 is girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$;

(vi) $8.5 = \mathcal{O}(NTG)$ is neutrosophic girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$.

Proposition 1.6.7. *Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center c. Then*

$$\mathcal{G}_n(NTG) = \infty.$$

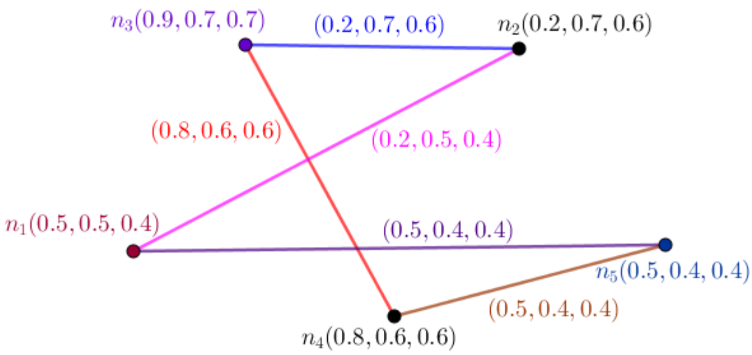

Figure 1.15: A Neutrosophic Graph in the Viewpoint of its Girth. 62NTG15

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. Every vertex isn't a neighbor for every given vertex. Every vertex is a neighbor for center. Furthermore, center is only neighbor for any given vertex. So center is only neighbor for all vertices. It's possible to have some paths amid two given vertices but there's no crisp cycle. In other words, if $\mathcal{O}(NTG) > 2$, then there are at least three vertices x, y and z such that if x is a neighbor for y and z, then y and z aren't neighbors and x is center. To get more precise, if if x and y are neighbors then either x or y is center. Every edge have one common endpoint with other edges which is called center. Thus there is no triangle but there are some edges. One edge has two endpoints which one of them is center. There are no crisp cycle. Hence trying to find shortest cycle has no result. There is no crisp cycle. Then there is shortest crisp cycle. So

$$\mathcal{G}_n(NTG) = \infty.$$

∎

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.6.8. There is one section for clarifications. In Figure (1.16), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a star and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this star implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. The length of this star implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a star and there are two edges, n_1n_2 and n_1n_3, according to corresponded neutrosophic star but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic star implies

$$n_1, n_2, n_3$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a star and there are three edges, n_1n_2, n_1n_3 and n_1n_4, according to corresponded neutrosophic star but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. So adding vertices has no effect to find a crisp cycle. The structure of this neutrosophic star implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_3, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a star and there are four edges, n_1n_2, n_1n_3, n_1n_4 and n_1n_5, according to corresponded neutrosophic star but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. So adding vertices has no effect to find a crisp cycle. There are some paths amid two given vertices. The structure of this neutrosophic star implies

$$n_1, n_2, n_3, n_4, n_5$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) ∞ is girth and there is no corresponded set;

(vi) ∞ is neutrosophic girth and there is no corresponded set.

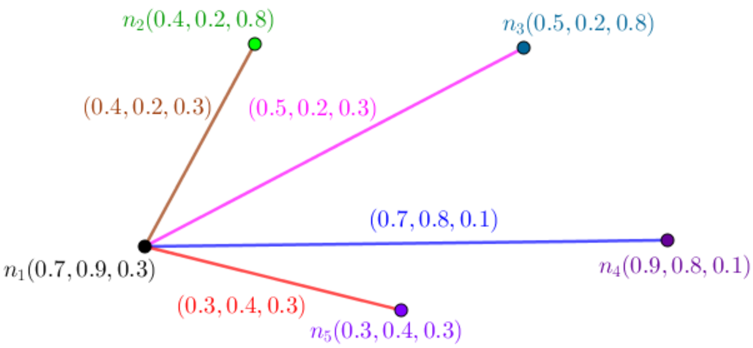

Figure 1.16: A Neutrosophic Graph in the Viewpoint of its Girth.

62NTG16

Proposition 1.6.9. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-bipartite-neutrosophic graph. Then*

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^{3}(\sigma_i(x) + \sigma_i(y) + \sigma_i(z) + \sigma_i(w))\}_{x,y \in V_1,\ z,w \in V_2}.$$

where $\mathcal{O}(NTG) \geq 4$ and $\min\{|V_1|, |V_2|\} \geq 2$. Also,

$$\mathcal{G}_n(NTG) = \infty$$

where $\mathcal{O}(NTG) \leq 3$.

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-bipartite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. If $\mathcal{O}(NTG) \leq 3$, then it's neutrosophic path implying

$$\mathcal{G}_n(NTG) = \infty.$$

If $\mathcal{O}(NTG) \geq 4$, then it's possible to have two vertices in every part. In this case, four vertices form a crisp cycle which crisp cardinality of its vertices are four. It's impossible to have a crisp cycle which crisp cardinality of its vertices are three. Since the sequence of consecutive vertices are x_1, x_2, x_3 and there's no edge more. It implies there are two edges. It's neutrosophic path but neither crisp cycle nor neutrosophic cycle. So the first step of finding shortest crisp cycle is impossible but in second step, there's one crisp cycle corresponded to number four. Thus

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^{3}(\sigma_i(x) + \sigma_i(y) + \sigma_i(z) + \sigma_i(w))\}_{x,y \in V_1,\ z,w \in V_2}.$$

where $\mathcal{O}(NTG) \geq 4$ and $\min\{|V_1|, |V_2|\} \geq 2$. Also,

$$\mathcal{G}_n(NTG) = \infty$$

where $\mathcal{O}(NTG) \leq 3$. ∎

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.6.10. There is one section for clarifications. In Figure (1.17), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a complete-bipartite and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this complete-bipartite implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic complete-bipartite has neither a neutrosophic cycle nor crisp cycle. The length of this complete-bipartite implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*ii*) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-bipartite and there are two edges, n_1n_2 and n_1n_3, according to corresponded neutrosophic complete-bipartite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-bipartite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-bipartite implies

$$n_1, n_2, n_3$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iii*) if n_1, n_2, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-bipartite and there are two edges, n_1n_2 and n_2n_4, according to corresponded neutrosophic complete-bipartite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-bipartite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-bipartite implies

$$n_1, n_2, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iv*) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a complete-bipartite and there are four edges, n_1n_2, n_1n_3, n_2n_4 and n_3n_4, according to corresponded neutrosophic complete-bipartite and it has neutrosophic cycle where n_2n_4 and n_3n_4 are two weakest edge with same amount $(0.3, 0.2, 0.3)$. First step is to

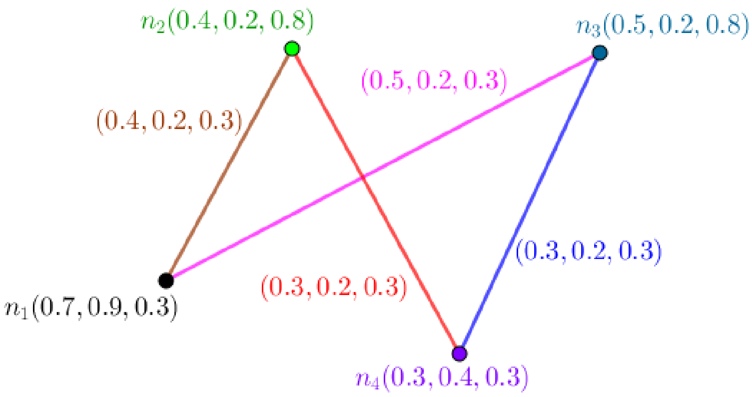

Figure 1.17: A Neutrosophic Graph in the Viewpoint of Girth. 62NTG17

have at least one crisp cycle for finding shortest cycle. Finding shortest cycle only has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-bipartite has both of a neutrosophic cycle and crisp cycle. So adding vertices has some effects to find a crisp cycle. The structure of this neutrosophic complete-bipartite implies

$$n_1, n_2, n_3, n_4$$

is corresponded to girth $\mathcal{G}(NTG)$ and uniqueness of this cycle implies the sequence

$$n_1, n_2, n_3, n_4$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 4 is girth and its corresponded sequence is n_1, n_2, n_3, n_4;

(vi) 5.8 is neutrosophic girth and its corresponded sequence is n_1, n_2, n_3, n_4.

Proposition 1.6.11. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-t-partite-neutrosophic graph. Then*

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^3 (\sigma_i(x) + \sigma_i(y) + \sigma_i(z))\}_{x \in V_1,\ y \in V_2,\ z \in V_3}.$$

where $t \geq 3$.

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^3 (\sigma_i(x) + \sigma_i(y) + \sigma_i(z) + \sigma_i(w))\}_{x,y \in V_1,\ z,w \in V_2}.$$

where $t \leq 2$. And

$$\mathcal{G}_n(NTG) = \infty$$

where $\mathcal{O}(NTG) \leq 2$.

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-t-partite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. If $\mathcal{O}(NTG) \leq 2$, then it's neutrosophic path implying

$$\mathcal{G}_n(NTG) = \infty.$$

If $t \geq 3, \mathcal{O}(NTG) \geq 3$, then it has crisp cycle implying

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^3(\sigma_i(x) + \sigma_i(y) + \sigma_i(z))\}_{x \in V_1,\ y \in V_2,\ z \in V_3}.$$

If $t \geq 2, \mathcal{O}(NTG) \geq 4$, then it's possible to have two vertices in every part. In this case, four vertices form a crisp cycle which crisp cardinality of its vertices are four. It's impossible to have a crisp cycle which crisp cardinality of its vertices are three. Since the sequence of consecutive vertices are x_1, x_2, x_3 and there's no edge more. It implies there are two edges. It's neutrosophic path but neither crisp cycle nor neutrosophic cycle. So the first step of finding shortest crisp cycle is impossible but in second step, there's one crisp cycle corresponded to number four. Thus

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^3(\sigma_i(x) + \sigma_i(y) + \sigma_i(z))\}_{x \in V_1,\ y \in V_2,\ z \in V_3}.$$

where $t \geq 3$.

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^3(\sigma_i(x) + \sigma_i(y) + \sigma_i(z) + \sigma_i(w))\}_{x,y \in V_1,\ z,w \in V_2}.$$

where $t \leq 2$. And

$$\mathcal{G}_n(NTG) = \infty$$

where $\mathcal{O}(NTG) \leq 2$. ■

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.6.12. There is one section for clarifications. In Figure (1.18), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a complete-t-partite and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this complete-t-partite implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The length of this complete-t-partite implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-t-partite and there are two edges, n_1n_2 and n_1n_3, according to corresponded neutrosophic complete-t-partite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_3$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-t-partite and there are two edges, n_1n_2 and n_2n_4, according to corresponded neutrosophic complete-t-partite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a complete-t-partite and there are four edges, n_1n_2, n_1n_5, n_2n_4 and n_5n_4, according to corresponded neutrosophic complete-t-partite and it has neutrosophic cycle where n_2n_4 and n_5n_4 are two weakest edge with same amount $(0.3, 0.2, 0.3)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle only has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has both of a neutrosophic cycle and crisp cycle. So adding vertices has some effects to find a crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_4, n_5$$

is corresponded to girth $\mathcal{G}(NTG)$ and minimum neutrosophic cardinality implies the sequence

$$n_1, n_2, n_4, n_5$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 4 is girth and its corresponded sequence is n_1, n_2, n_4, n_5;

(vi) 5.7 is neutrosophic girth and its corresponded sequence is n_1, n_2, n_4, n_5.

Proposition 1.6.13. *Let $NTG : (V, E, \sigma, \mu)$ be a wheel-neutrosophic graph. Then*

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^3(\sigma_i(x) + \sigma_i(y) + \sigma_i(z))\}_{xy,xz,zy\in E}.$$

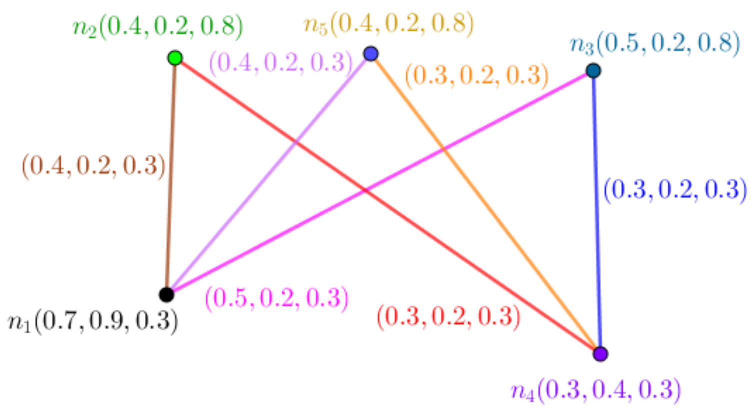

Figure 1.18: A Neutrosophic Graph in the Viewpoint of its Girth. 62NTG18

where $t \geq 3$.

$$\mathcal{G}_n(NTG) = \infty$$

where $t \geq 2$.

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a wheel-neutrosophic graph. The argument is elementary. Since all vertices of a path join to one vertex. Thus

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^{3}(\sigma_i(x) + \sigma_i(y) + \sigma_i(z))\}_{xy,xz,zy \in E}.$$

where $t \geq 3$.

$$\mathcal{G}_n(NTG) = \infty$$

where $t \geq 2$. ∎

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 1.6.14. There is one section for clarifications. In Figure (1.19), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If s_1, s_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a wheel and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this wheel implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic wheel has neither a neutrosophic cycle nor crisp cycle. The length of this wheel implies

$$s_1, s_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if s_4, s_2, s_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a wheel and there are two edges, s_3s_2 and s_4s_3, according to corresponded neutrosophic wheel but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic wheel has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic wheel implies

$$s_4, s_2, s_3$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if s_1, s_2, s_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a wheel and there are three edges, s_1s_2, s_2s_3 and s_1s_3 according to corresponded neutrosophic wheel but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one crisp cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic wheel has no neutrosophic cycle but it has crisp cycle. The structure of this neutrosophic wheel implies

$$s_1, s_2, s_3$$

is corresponded to girth $\mathcal{G}(NTG)$ but minimum neutrosophic cardinality implies

$$s_1, s_2, s_3$$

isn't corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if s_1, s_3, s_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a wheel and there are three edges, s_1s_4, s_4s_3 and s_1s_3 according to corresponded neutrosophic wheel and it has a neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one crisp cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic wheel has one neutrosophic cycle with two weakest edges s_1s_4 and s_3s_4 concerning same values $(0.1, 0.1, 0.5)$ and it has a crisp cycle. The structure of this neutrosophic wheel implies

$$s_1, s_3, s_4$$

is corresponded to girth $\mathcal{G}(NTG)$ and minimum neutrosophic cardinality implies

$$s_1, s_3, s_4$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 3 is girth and its corresponded sequences are s_1, s_3, s_4 and s_1, s_2, s_3 alongside s_1, s_4, s_5;

(vi) 3.8 is neutrosophic girth and its corresponded sequence is s_1, s_3, s_4.

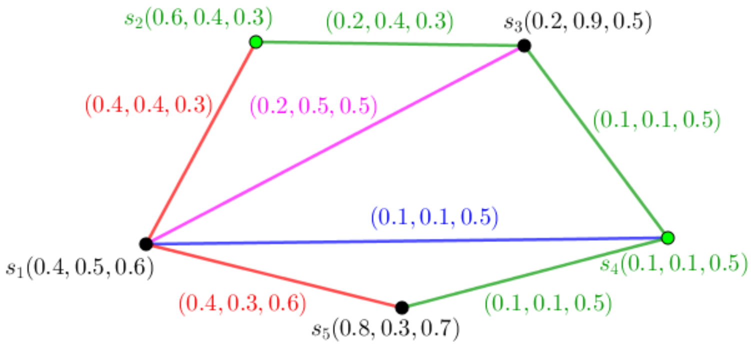

Figure 1.19: A Neutrosophic Graph in the Viewpoint of its Girth. 62NTG19

1.7 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are either complete models which mean complete connections are formed as individual and family of complete models with common neutrosophic vertex set or quasi-complete models which mean quasi-complete connections are formed as individual and family of quasi-complete models with common neutrosophic vertex set.
Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importance to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive sections. Beyond that, sometimes sections are not the same.

Step 3. (Model) The situation is designed as a model. The model uses data to assign every section and to assign to relation amid sections, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relations amid them. Table (1.1), clarifies about the assigned numbers to these situations.

Table 1.1: Scheduling concerns its Subjects and its Connections as a neutrosophic graph in a Model. 62tbl1

Sections of NTG	n_1	$n_2 \cdots$	n_5
Values	$(0.7, 0.9, 0.3)$	$(0.4, 0.2, 0.8) \cdots$	$(0.4, 0.2, 0.8)$
Connections of NTG	E_1	$E_2 \cdots$	E_6
Values	$(0.4, 0.2, 0.3)$	$(0.5, 0.2, 0.3) \cdots$	$(0.3, 0.2, 0.3)$

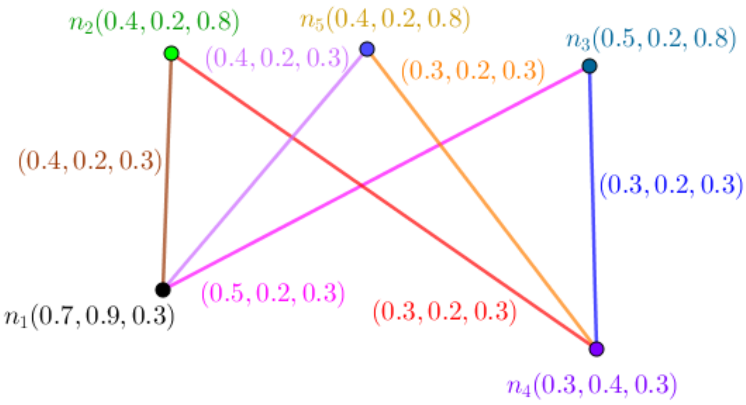

Figure 1.20: A Neutrosophic Graph in the Viewpoint of its Girth and its Neutrosophic Girth

62NTG20

1.8 Case 1: Complete-t-partite Model alongside its Girth and its Neutrosophic Girth

Step 4. (Solution) The neutrosophic graph alongside its girth and its neutrosophic girth as model, propose to use specific number. Every subject has connection with some subjects. Thus the connection is applied as possible and the model demonstrates quasi-full connections as quasi-possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is star, the number is different. Also, it holds for other types such that complete, wheel, path, and cycle. The collection of situations is another application of its girth and its neutrosophic girth when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are five subjects which are represented as Figure (1.20). This model is strong and even more it's quasi-complete. And the study proposes using specific number which is called its girth and its neutrosophic girth. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (1.20). In Figure (1.20), an complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a complete-t-partite and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this complete-t-partite implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The length of this complete-t-partite implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-t-partite and there are two edges, n_1n_2 and n_1n_3, according to corresponded neutrosophic complete-t-partite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_3$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-t-partite and there are two edges, n_1n_2 and n_2n_4, according to corresponded neutrosophic complete-t-partite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a complete-t-partite and there are four edges, n_1n_2, n_1n_5, n_2n_4 and n_5n_4, according to corresponded neutrosophic complete-t-partite and it has neutrosophic cycle where n_2n_4 and n_5n_4 are two weakest edge with same amount $(0.3, 0.2, 0.3)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle only has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has both of a neutrosophic cycle and crisp cycle. So adding vertices has some effects to find a crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_4, n_5$$

is corresponded to girth $\mathcal{G}(NTG)$ and minimum neutrosophic cardinality implies the sequence

$$n_1, n_2, n_4, n_5$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

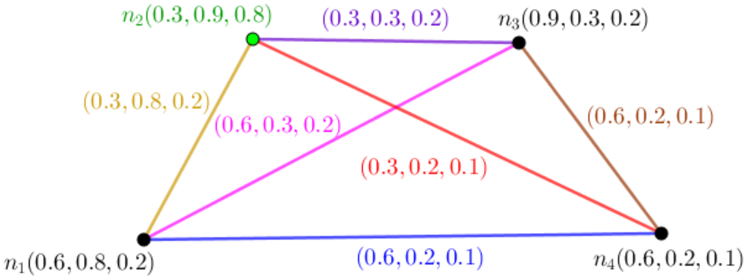

Figure 1.21: A Neutrosophic Graph in the Viewpoint of its Girth and its Neutrosophic Girth

62NTG21

(v) 4 is girth and its corresponded sequence is n_1, n_2, n_4, n_5;

(vi) 5.7 is neutrosophic girth and its corresponded sequence is n_1, n_2, n_4, n_5.

1.9 Case 2: Complete Model alongside its A Neutrosophic Graph in the Viewpoint of its Girth and its Neutrosophic Girth

Step 4. (Solution) The neutrosophic graph alongside its girth and its neutrosophic girth as model, propose to use specific number. Every subject has connection with every given subject in deemed way. Thus the connection applied as possible and the model demonstrates full connections as possible between parts but with different view where symmetry amid vertices and edges are the matters. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete multipartite, the number is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of its girth and its neutrosophic girth when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are four subjects which are represented in the formation of one model as Figure (1.21). This model is neutrosophic strong as individual and even more it's complete. And the study proposes using specific number which is called its girth and its neutrosophic girth for this model. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to these models as individual. A model as a collection of situations to compare them with another model as a collection of situations to get more precise. Consider Figure (1.21). There is one section for clarifications.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*ii*) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1, n_2, n_3$$

is corresponded to girth $\mathcal{G}(NTG)$ but neutrosophic length of this crisp cycle implies

$$n_1, n_2, n_3$$

isn't corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iii*) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's two crisp cycles with length two and three. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1, n_3, n_4, and with length four, n_1, n_2, n_3, n_4. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a sequence of consecutive vertices is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with two different length three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. The length of this path implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iv*) if n_1, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to girth $\mathcal{G}(NTG)$ and neutrosophic length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 3 is girth and its corresponded sets are $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_2, n_3, n_4\}$;

(vi) 3.9 is neutrosophic girth and its corresponded set is $\{n_1, n_3, n_4\}$.

1.10 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study. Notion concerning its girth and its neutrosophic girth are defined in neutrosophic graphs. Neutrosophic number is also reused. Thus,

Question 1.10.1. *Is it possible to use other types of its girth and its neutrosophic girth?*

Question 1.10.2. *Are existed some connections amid different types of its girth and its neutrosophic girth in neutrosophic graphs?*

Question 1.10.3. *Is it possible to construct some classes of neutrosophic graphs which have "nice" behavior?*

Question 1.10.4. *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

Problem 1.10.5. *Which parameters are related to this parameter?*

Problem 1.10.6. *Which approaches do work to construct applications to create independent study?*

Problem 1.10.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

1.11 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted.

This study uses two definitions concerning girth and neutrosophic girth arising from shortest cycles to study neutrosophic graphs. New neutrosophic number is reused which is too close to the notion of neutrosophic number but it's different since it uses all values as type-summation on them. Comparisons amid number, corresponded vertices and edges are done by using neutrosophic tool. The connections of vertices which aren't clarified by a cycle differ them from each other and put them in different categories to represent a number which is called girth and neutrosophic girth arising from shortest cycles. Further studies

Table 1.2: A Brief Overview about Advantages and Limitations of this Study

62tbl2

Advantages	Limitations
1. Neutrosophic Girth	1. Connections amid Classes
2. Girth	
3. Neutrosophic Number	2. Study on Families
4. Classes of Neutrosophic Graphs	
5. Shortest Cycles	3. Same Models in Family

could be about changes in the settings to compare these notions amid different settings of neutrosophic graphs theory. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (1.2), some limitations and advantages of this study are pointed out.

CHAPTER 2

Neutrosophic Girth Based On Neutrosophic Cycle in Neutrosophic Graphs

The following sections are cited as [**Ref2**] which is my 63rd manuscript and I use prefix 63 as number before any labelling for items.

2.1 Finding Shortest Sequences of Consecutive Vertices in Neutrosophic Graphs

2.2 Abstract

New setting is introduced to study girth and neutrosophic girth arising from shortest neutrosophic cycles. Forming neutrosophic cycles from a sequence of consecutive vertices is key type of approach to have these notions namely girth and neutrosophic girth arising from shortest neutrosophic cycles. Two numbers are obtained but now both settings leads to approach is on demand which is finding minimum cardinality and minimum neutrosophic cardinality in the terms of vertices, which have edges which form a shortest neutrosophic cycle. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then Girth $\mathcal{G}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum crisp cardinality of vertices forming shortest neutrosophic cycle. If there isn't, then girth is ∞; neutrosophic girth $\mathcal{G}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of vertices forming shortest neutrosophic cycle. If there isn't, then girth is ∞. As concluding results, there are some statements, remarks, examples and clarifications about some classes of strong neutrosophic graphs namely strong-path-neutrosophic graphs, strong-cycle-neutrosophic graphs, complete-neutrosophic graphs, strong-star-neutrosophic graphs, strong-complete-bipartite-neutrosophic graphs, strong-complete-t-partite-neutrosophic graphs and strong-wheel-neutrosophic graphs. The clarifications are also presented in both sections "Setting of Girth," and "Setting of Neutrosophic Girth," for introduced results and used classes. Neutrosophic number is reused in this way. It's applied to use the type of neutrosophic number in the way that, three values of a vertex are used and they've same share to construct this number to compare with other vertices. Summation of three values of vertex makes one number and applying it to a comparison. This approach

facilitates identifying vertices which form girth and neutrosophic girth arising from shortest neutrosophic cycles. In both settings, some classes of well-known strong neutrosophic graphs are studied. Some clarifications for each result and each definition are provided. The cardinality of a set has eligibility to girth but the neutrosophic cardinality of a set has eligibility to call neutrosophic girth. Some results get more frameworks and perspective about these definitions. The way in that, a sequence of consecutive vertices forming a neutrosophic cycle, opens the way to do some approaches. These notions are applied into strong neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special strong neutrosophic graphs which are well-known, is an open way to pursue this study. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.
Keywords: Girth, Neutrosophic Girth, Shortest Neutrosophic Cycle

AMS Subject Classification: 05C17, 05C22, 05E45

2.3 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 2.3.1. *Is it possible to use mixed versions of ideas concerning "Neutrosophic Girth", "Girth" and "Strong Neutrosophic Graph" to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of strong neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Lack of connection amid two edges have key roles to assign girth and neutrosophic girth arising from neutrosophic shortest cycles. Thus they're used to define new ideas which conclude to the structure of girth and neutrosophic girth arising from neutrosophic shortest cycles. The concept of having common shortest neutrosophic cycle inspires us to study the behavior of vertices in the way that, some types of numbers, girth and neutrosophic girth arising from neutrosophic shortest cycles are the cases of study in the setting of individuals. In both settings, a corresponded number concludes the discussion. Also, there are some avenues to extend these notions.
The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In subsection "Preliminaries", new notions of girth and neutrosophic girth arising from neutrosophic shortest cycles, are highlighted, are introduced and are clarified as individuals. In section "Preliminaries", sequence of consecutive vertices forming neutrosophic cycles have the key role in this way. General results are obtained and also, the results about the basic notions of girth and neutrosophic girth arising from shortest neutrosophic cycles, are elicited. Some classes of strong neutrosophic graphs are studied in the terms of girth arising from shortest neutrosophic cycles, in section "Setting of Girth," as individuals. In section "Setting of Girth," girth is applied into individuals. As concluding results, there are some statements, remarks, examples and clarifications about some classes of strong neutrosophic graphs namely strong-path-neutrosophic graphs, strong-cycle-neutrosophic

graphs, complete-neutrosophic graphs, strong-star-neutrosophic graphs, strong-complete-bipartite-neutrosophic graphs, strong-complete-t-partite-neutrosophic graphs and strong-wheel-neutrosophic graphs. The clarifications are also presented in both sections "Setting of Girth," and "Setting of Neutrosophic Girth," for introduced results and used classes. In section "Applications in Time Table and Scheduling", two applications are posed for quasi-complete and complete notions, namely complete-t-neutrosophic graphs and strong-complete-neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are considered as individual. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications is featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

2.4 Preliminaries

In this subsection, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.
Basic idea is about the model which is used. First definition introduces basic model.

Definition 2.4.1. (Graph).
$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

Definition 2.4.2. (Neutrosophic Graph And Its Special Case).
$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_i v_j \in E$,

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic edge set**.

(iii) : $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

(iv) : $\sum_{v \in V} \sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

(v) : $|E|$ is called **size** of NTG and it's denoted by $\mathcal{S}(NTG)$.

(vi) : $\sum_{e \in E} \sum_{i=1}^{3} \mu_i(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$.

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

Definition 2.4.3. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) : a sequence of vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}}$ is called **path** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$;

(ii) : **strength** of path $P : x_0, x_1, \cdots, x_{\mathcal{O}}$ is $\bigwedge_{i=0,\cdots,n-1} \mu(x_i x_{i+1})$;

(iii) : **connectedness** amid vertices x_0 and x_t is

$$\mu^{\infty}(x_0, x_t) = \bigvee_{P: x_0, x_1, \cdots, x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1});$$

(iv) : a sequence of vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}}$ is called **cycle** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$;

(v) : it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1, \sigma_2, \cdots, \sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$;

(vi) : t-partite is **complete bipartite** if $t = 2$, and it's denoted by K_{σ_1, σ_2};

(vii) : complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1, σ_2};

$(viii)$: a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1, σ_2};

(ix) : it's **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$;

(x) : it's **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

Definition 2.4.4. (Girth and Neutrosophic Girth).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **Girth** $\mathcal{G}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum crisp cardinality of vertices forming shortest neutrosophic cycle. If there isn't, then girth is ∞;

(ii) **neutrosophic girth** $\mathcal{G}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is minimum neutrosophic cardinality of vertices forming shortest neutrosophic cycle. If there isn't, then girth is ∞.

63thm

Theorem 2.4.5. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. If $NTG : (V, E, \sigma, \mu)$ is strong, then its crisp cycle is its neutrosophic cycle.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider u as a vertex of crisp cycle CYC, such that $\sigma(u) = \min \sigma(x)_{x \in V(CYC)}$. u has two neighbors y, z in CYC. Since NTG is strong, $\mu(uy) = \mu(uz) = \sigma(u)$. It implies there are two weakest edges in CYC. It means CYC is neutrosophic cycle. ■

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.
In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 2.4.6. In Figure (2.1), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1, n_2, n_3$$

is corresponded to girth $\mathcal{G}(NTG)$ but neutrosophic length of this crisp cycle implies

$$n_1, n_2, n_3$$

isn't corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's two crisp cycles with length two and three. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1, n_3, n_4, and with length four, n_1, n_2, n_3, n_4. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a sequence of consecutive vertices is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with two different length three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. The length of this path implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

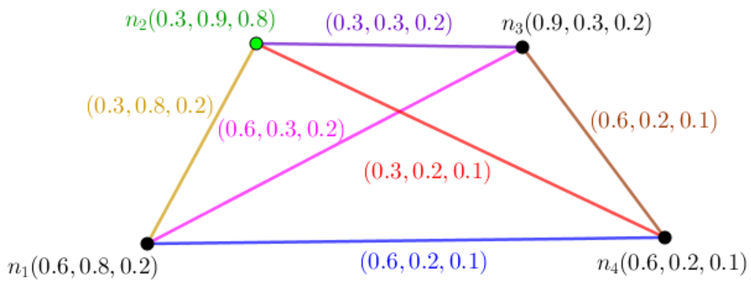

Figure 2.1: A Neutrosophic Graph in the Viewpoint of its Girth and its Neutrosophic Girth. 63NTG1

(iv) if n_1, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to girth $\mathcal{G}(NTG)$ and neutrosophic length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 3 is girth and its corresponded sets are $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_2, n_3, n_4\}$;

(vi) 3.9 is neutrosophic girth and its corresponded set is $\{n_1, n_3, n_4\}$.

2.5 Setting of Girth

In this section, I provide some results in the setting of girth. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, star-neutrosophic graph, bipartite-neutrosophic graph, and t-partite-neutrosophic graph, are both of cases of study and classes which the results are about them.

Proposition 2.5.1. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{G}(NTG) = 3.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. The length of longest cycle is $\mathcal{O}(NTG)$. In other hand, there's a cycle if and only if $\mathcal{O}(NTG) \geq 3$. It's complete. So there's at least one neutrosophic cycle which its length is $\mathcal{O}(NTG) = 3$. By shortest cycle is on demand, the girth is three. Thus

$$\mathcal{G}(NTG) = 3.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.2. In Figure (2.2), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1, n_2, n_3$$

is corresponded to girth $\mathcal{G}(NTG)$ but neutrosophic length of this crisp cycle implies

$$n_1, n_2, n_3$$

isn't corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's two crisp cycles with length two and three. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1, n_3, n_4, and with length four, n_1, n_2, n_3, n_4. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a sequence of consecutive vertices is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with

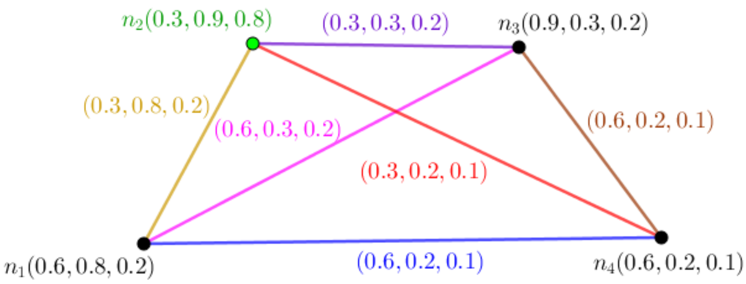

Figure 2.2: A Neutrosophic Graph in the Viewpoint of its Girth. 63NTG2

two different length three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. The length of this path implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to girth $\mathcal{G}(NTG)$ and neutrosophic length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 3 is girth and its corresponded sets are $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_2, n_3, n_4\}$;

(vi) 3.9 is neutrosophic girth and its corresponded set is $\{n_1, n_3, n_4\}$.

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

Proposition 2.5.3. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-path-neutrosophic graph. Then*

$$\mathcal{G}(NTG) = \infty.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-path-neutrosophic graph. There's no crisp cycle. If $NTG : (V, E, \sigma, \mu)$ isn't a crisp cycle, then $NTG : (V, E, \sigma, \mu)$ isn't a neutrosophic cycle. There's no cycle from every version. Let $x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ be a path-neutrosophic graph. Since $x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$

is a sequence of consecutive vertices, there's no repetition of vertices in this sequence. So there's no cycle. Girth is corresponded to shortest cycle but there's no cycle. Thus it implies

$$\mathcal{G}(NTG) = \infty.$$

■

Example 2.5.4. There are two sections for clarifications.

(a) In Figure (2.3), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_3, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are four edges, n_1n_2, n_2n_3 and n_4n_5, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding vertices has no effect to find a crisp cycle. There is only one path amid two given vertices. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4, n_5$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) ∞ is girth and there are no corresponded sets;

(vi) ∞ is neutrosophic girth and there are no corresponded sets.

(b) In Figure (2.4), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle

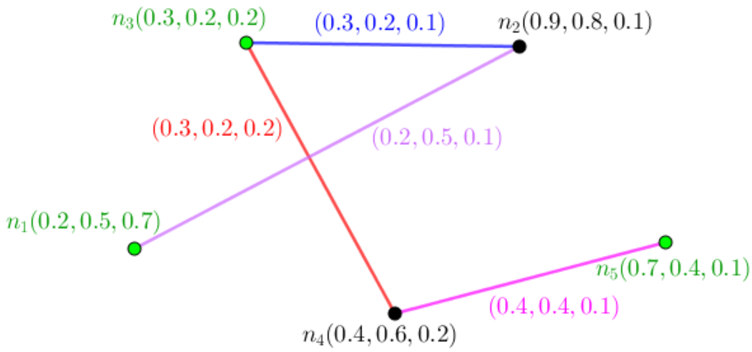

Figure 2.3: A Neutrosophic Graph in the Viewpoint of its Girth. 63NTG3

for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_3, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are four edges, n_1n_2, n_2n_3 and n_4n_5, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding vertices has no effect to find a crisp cycle. There is only one path amid two given vertices. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4, n_5$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) ∞ is girth and there are no corresponded sets;

(vi) ∞ is neutrosophic girth and there are no corresponded sets.

Proposition 2.5.5. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-cycle-neutrosophic graph where $\mathcal{O}(NTG) \geq 3$. Then*

$$\mathcal{G}(NTG) = \mathcal{O}(NTG).$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(NTG)}, x_1$ be a sequence of consecutive vertices of $NTG :$

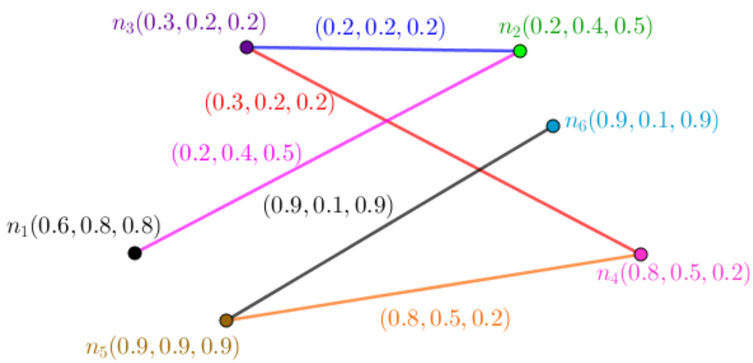

Figure 2.4: A Neutrosophic Graph in the Viewpoint of its Girth. 63NTG4

(V, E, σ, μ) such that

$$x_i x_{i+1} \in E, \ i = 1, 2, \cdots, \mathcal{O}(NTG) - 1, \ x_{\mathcal{O}(NTG)} x_1 \in E.$$

There are two paths amid two given vertices. The degree of every vertex is two. But there's one crisp cycle for every given vertex. So the efforts leads to one cycle for finding a shortest crisp cycle. For a given vertex x_i, the sequence of consecutive vertices

$$x_i, x_{i+1}, \cdots, x_{i-2}, x_{i-1}, x_i$$

is a corresponded crisp cycle for x_i. Every cycle has same length. The length is $\mathcal{O}(NTG)$. Thus the crisp cardinality of set of vertices forming shortest crisp cycle is $\mathcal{O}(NTG)$. By Theorem (3.4.5),

$$\mathcal{G}(NTG) = \mathcal{O}(NTG).$$

∎

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.6. There are two sections for clarifications.

(a) In Figure (2.5), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are six edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6$ and n_6n_1, according to corresponded neutrosophic path and it's neutrosophic cycle since it has two weakest edges, n_4n_5 and n_5n_6 with same values $(0.1, 0.1, 0.2)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is both of a neutrosophic cycle and crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is corresponded to both of girth $\mathcal{G}(NTG)$ and neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 6 is girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$;

(vi) $8.1 = \mathcal{O}(NTG)$ is neutrosophic girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$.

(b) In Figure (2.6), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are five edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ and n_5n_1, according to corresponded neutrosophic path and it isn't neutrosophic cycle since it has only one weakest edge, n_1n_2, with value $(0.2, 0.5, 0.4)$ and not more. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is not a neutrosophic cycle but it is a crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_1$ is corresponded to both of girth $\mathcal{G}(NTG)$ and neutrosophic girth $\mathcal{G}_n(NTG)$;

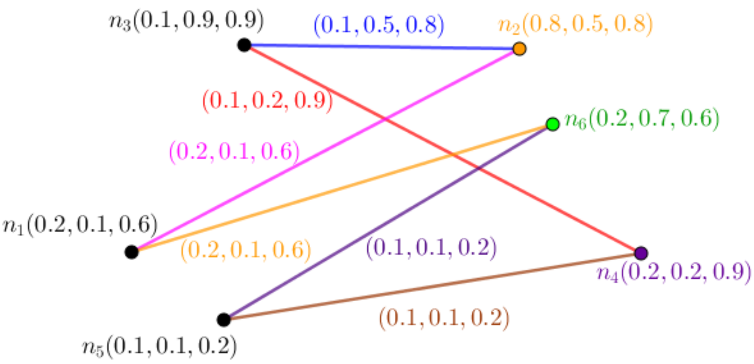

Figure 2.5: A Neutrosophic Graph in the Viewpoint of its Girth. 63NTG5

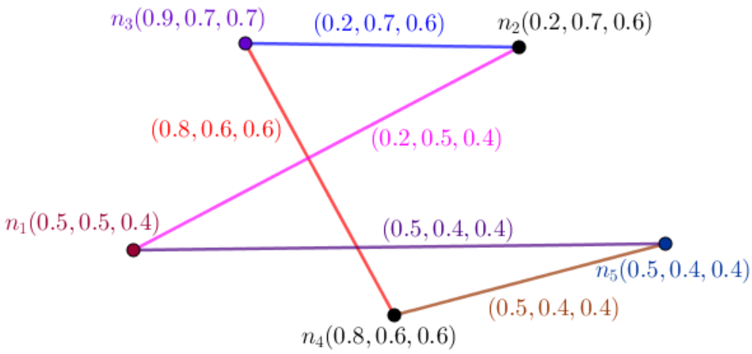

Figure 2.6: A Neutrosophic Graph in the Viewpoint of its Girth. 63NTG6

(v) 5 is girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$;

(vi) $8.5 = \mathcal{O}(NTG)$ is neutrosophic girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$.

Proposition 2.5.7. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-star-neutrosophic graph with center c. Then*

$$\mathcal{G}(NTG) = \infty.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-star-neutrosophic graph. Every vertex isn't a neighbor for every given vertex. Every vertex is a neighbor for center. Furthermore, center is only neighbor for any given vertex. So center is only neighbor for all vertices. It's possible to have some paths amid two given vertices but there's no crisp cycle. In other words, if $\mathcal{O}(NTG) > 2$, then there are at least three vertices x, y and z such that if x is a neighbor for y and z, then y and z aren't neighbors and x is center. To get more precise, if if x and y are neighbors then either x or y is center. Every edge have one common endpoint with other edges which is called center. Thus there is no triangle but there are some edges. One edge has two endpoints which one of them is center. There are no crisp cycle. Hence trying to find shortest cycle has no result. There is no crisp cycle. Then there is shortest crisp cycle. So

$$\mathcal{G}(NTG) = \infty.$$

■

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.8. There is one section for clarifications. In Figure (2.7), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a star and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this star implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. The length of this star implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a star and there are two edges, n_1n_2 and n_1n_3, according to corresponded neutrosophic star but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic star implies

$$n_1, n_2, n_3$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a star and there are three edges, n_1n_2, n_1n_3 and n_1n_4, according to corresponded neutrosophic star but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. So adding vertices has no effect to find a crisp cycle. The structure of this neutrosophic star implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

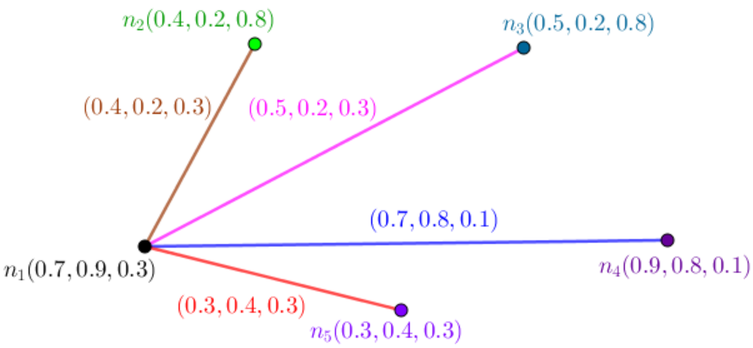

Figure 2.7: A Neutrosophic Graph in the Viewpoint of its Girth.

63NTG7

(iv) if n_1, n_2, n_3, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a star and there are four edges, n_1n_2, n_1n_3, n_1n_4 and n_1n_5, according to corresponded neutrosophic star but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. So adding vertices has no effect to find a crisp cycle. There are some paths amid two given vertices. The structure of this neutrosophic star implies

$$n_1, n_2, n_3, n_4, n_5$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) ∞ is girth and there is no corresponded set;

(vi) ∞ is neutrosophic girth and there is no corresponded set.

Proposition 2.5.9. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-complete-bipartite-neutrosophic graph. Then*

$$\mathcal{G}(NTG) = 4$$

where $\mathcal{O}(NTG) \geq 4$. And

$$\mathcal{G}(NTG) = \infty$$

where $\mathcal{O}(NTG) \leq 3$.

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-complete-bipartite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. If $\mathcal{O}(NTG) \leq 3$, then it's neutrosophic path implying

$$\mathcal{G}(NTG) = \infty.$$

If $\mathcal{O}(NTG) \geq 4$, then it's possible to have two vertices in every part. In this case, four vertices form a crisp cycle which crisp cardinality of its vertices are four. It's impossible to have a crisp cycle which crisp cardinality of its vertices are three. Since the sequence of consecutive vertices are x_1, x_2, x_3 and there's no edge more. It implies there are two edges. It's neutrosophic path but neither crisp cycle nor neutrosophic cycle. So the first step of finding shortest crisp cycle is impossible but in second step, there's one crisp cycle corresponded to number four. By Theorem (3.4.5),

$$\mathcal{G}(NTG) = 4$$

where $\mathcal{O}(NTG) \geq 4$. And

$$\mathcal{G}(NTG) = \infty$$

where $\mathcal{O}(NTG) \leq 3$. ∎

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.10. There is one section for clarifications. In Figure (2.8), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a complete-bipartite and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this complete-bipartite implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic complete-bipartite has neither a neutrosophic cycle nor crisp cycle. The length of this complete-bipartite implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-bipartite and there are two edges, n_1n_2 and n_1n_3, according to corresponded neutrosophic complete-bipartite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-bipartite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-bipartite implies

$$n_1, n_2, n_3$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-bipartite and there are two edges, n_1n_2 and n_2n_4, according to corresponded neutrosophic complete-bipartite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-bipartite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-bipartite implies

$$n_1, n_2, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a complete-bipartite and there are four edges, n_1n_2, n_1n_3, n_2n_4 and n_3n_4, according to corresponded neutrosophic complete-bipartite and it has neutrosophic cycle where n_2n_4 and n_3n_4 are two weakest edge with same amount $(0.3, 0.2, 0.3)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle only has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-bipartite has both of a neutrosophic cycle and crisp cycle. So adding vertices has some effects to find a crisp cycle. The structure of this neutrosophic complete-bipartite implies

$$n_1, n_2, n_3, n_4$$

is corresponded to girth $\mathcal{G}(NTG)$ and uniqueness of this cycle implies the sequence

$$n_1, n_2, n_3, n_4$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 4 is girth and its corresponded sequence is n_1, n_2, n_3, n_4;

(vi) 5.8 is neutrosophic girth and its corresponded sequence is n_1, n_2, n_3, n_4.

Proposition 2.5.11. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-complete-t-partite-neutrosophic graph. Then*

$$\mathcal{G}(NTG) = 3$$

where $t \geq 3$.

$$\mathcal{G}(NTG) = 4$$

where $t \leq 2$. And

$$\mathcal{G}(NTG) = \infty$$

where $\mathcal{O}(NTG) \leq 2$.

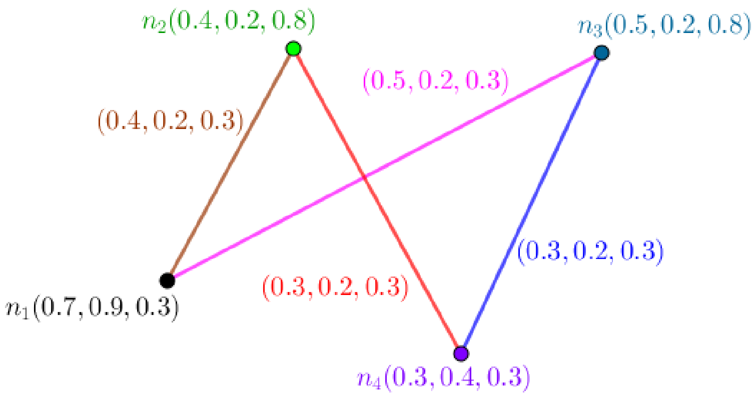

Figure 2.8: A Neutrosophic Graph in the Viewpoint of Girth. 63NTG8

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-complete-t-partite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. If $\mathcal{O}(NTG) \leq 2$, then it's neutrosophic path implying

$$\mathcal{G}(NTG) = \infty.$$

If $t \geq 3, \mathcal{O}(NTG) \geq 3$, then it has crisp cycle implying

$$\mathcal{G}(NTG) = 3.$$

If $t \geq 2, \mathcal{O}(NTG) \geq 4$, then it's possible to have two vertices in every part. In this case, four vertices form a crisp cycle which crisp cardinality of its vertices are four. It's impossible to have a crisp cycle which crisp cardinality of its vertices are three. Since the sequence of consecutive vertices are x_1, x_2, x_3 and there's no edge more. It implies there are two edges. It's neutrosophic path but neither crisp cycle nor neutrosophic cycle. So the first step of finding shortest crisp cycle is impossible but in second step, there's one crisp cycle corresponded to number four. By Theorem (3.4.5),

$$\mathcal{G}(NTG) = 3$$

where $t \geq 3$.

$$\mathcal{G}(NTG) = 4$$

where $t \leq 2$. And

$$\mathcal{G}(NTG) = \infty$$

where $\mathcal{O}(NTG) \leq 2$. ■

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.12. There is one section for clarifications. In Figure (2.9), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a complete-t-partite and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this complete-t-partite implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The length of this complete-t-partite implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*ii*) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-t-partite and there are two edges, n_1n_2 and n_1n_3, according to corresponded neutrosophic complete-t-partite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_3$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iii*) if n_1, n_2, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-t-partite and there are two edges, n_1n_2 and n_2n_4, according to corresponded neutrosophic complete-t-partite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iv*) if n_1, n_2, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a complete-t-partite and there are four edges, n_1n_2, n_1n_5, n_2n_4 and n_5n_4, according to corresponded neutrosophic complete-t-partite and it has neutrosophic cycle where n_2n_4 and n_5n_4 are two weakest edge with same amount $(0.3, 0.2, 0.3)$. First step is to

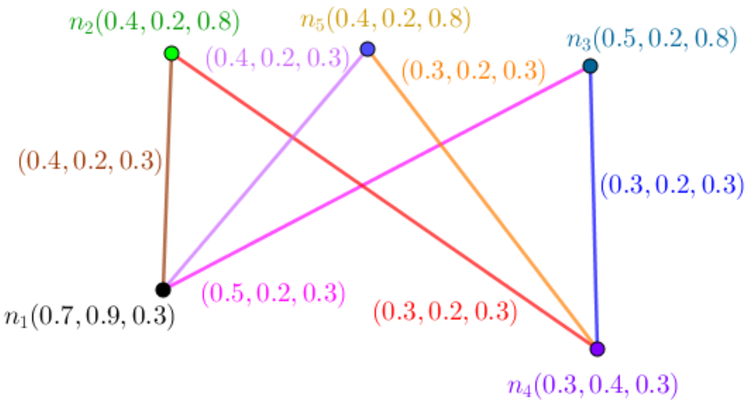

Figure 2.9: A Neutrosophic Graph in the Viewpoint of its Girth. 63NTG9

have at least one crisp cycle for finding shortest cycle. Finding shortest cycle only has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has both of a neutrosophic cycle and crisp cycle. So adding vertices has some effects to find a crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_4, n_5$$

is corresponded to girth $\mathcal{G}(NTG)$ and minimum neutrosophic cardinality implies the sequence

$$n_1, n_2, n_4, n_5$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 4 is girth and its corresponded sequence is n_1, n_2, n_4, n_5;

(vi) 5.7 is neutrosophic girth and its corresponded sequence is n_1, n_2, n_4, n_5.

Proposition 2.5.13. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-wheel-neutrosophic graph. Then*

$$\mathcal{G}(NTG) = 3$$

where $t \geq 3$.

$$\mathcal{G}(NTG) = \infty$$

where $t \geq 2$.

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-wheel-neutrosophic graph. The argument is elementary. Since all vertices of a path join to one vertex. By Theorem (3.4.5),

$$\mathcal{G}(NTG) = 3$$

where $t \geq 3$.

$$\mathcal{G}(NTG) = \infty$$

where $t \geq 2$. ∎

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.5.14. There is one section for clarifications. In Figure (2.10), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If s_1, s_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a wheel and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this wheel implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic wheel has neither a neutrosophic cycle nor crisp cycle. The length of this wheel implies

$$s_1, s_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if s_4, s_2, s_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a wheel and there are two edges, s_3s_2 and s_4s_3, according to corresponded neutrosophic wheel but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic wheel has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic wheel implies

$$s_4, s_2, s_3$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if s_1, s_2, s_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a wheel and there are three edges, s_1s_2, s_2s_3 and s_1s_3 according to corresponded neutrosophic wheel but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one crisp cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic wheel has no neutrosophic cycle but it has crisp cycle. The structure of this neutrosophic wheel implies

$$s_1, s_2, s_3$$

is corresponded to girth $\mathcal{G}(NTG)$ but minimum neutrosophic cardinality implies

$$s_1, s_2, s_3$$

isn't corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

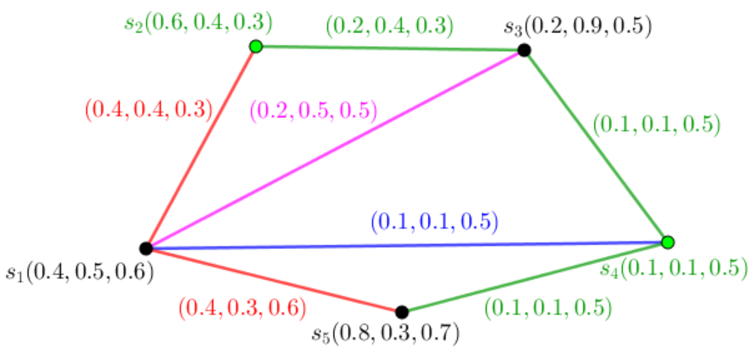

Figure 2.10: A Neutrosophic Graph in the Viewpoint of its Girth. 63NTG10

(iv) if s_1, s_3, s_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a wheel and there are three edges, s_1s_4, s_4s_3 and s_1s_3 according to corresponded neutrosophic wheel and it has a neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one crisp cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic wheel has one neutrosophic cycle with two weakest edges s_1s_4 and s_3s_4 concerning same values $(0.1, 0.1, 0.5)$ and it has a crisp cycle. The structure of this neutrosophic wheel implies

$$s_1, s_3, s_4$$

is corresponded to girth $\mathcal{G}(NTG)$ and minimum neutrosophic cardinality implies

$$s_1, s_3, s_4$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 3 is girth and its corresponded sequences are s_1, s_3, s_4 and s_1, s_2, s_3 alongside s_1, s_4, s_5;

(vi) 3.8 is neutrosophic girth and its corresponded sequence is s_1, s_3, s_4.

2.6 Setting of Neutrosophic Girth

In this section, I provide some results in the setting of neutrosophic girth. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, star-neutrosophic graph, bipartite-neutrosophic graph, and t-partite-neutrosophic graph, are both of cases of study and classes which the results are about them.

Proposition 2.6.1. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^{3}(\sigma_i(x) + \sigma_i(y) + \sigma_i(z))\}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. The length of longest cycle is $\mathcal{O}(NTG)$. In other hand, there's a cycle if and only if $\mathcal{O}(NTG) \geq 3$. It's complete. So there's at least one neutrosophic cycle which its length is $\mathcal{O}(NTG) = 3$. By shortest cycle is on demand, the girth is three. Thus

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^{3}(\sigma_i(x) + \sigma_i(y) + \sigma_i(z))\}.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.6.2. In Figure (2.11), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1, n_2, n_3$$

is corresponded to girth $\mathcal{G}(NTG)$ but neutrosophic length of this crisp cycle implies

$$n_1, n_2, n_3$$

isn't corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's two crisp cycles with length two and three. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1, n_3, n_4, and with length four, n_1, n_2, n_3, n_4. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a

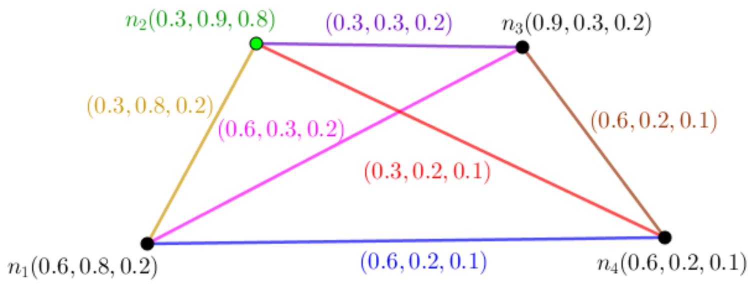

Figure 2.11: A Neutrosophic Graph in the Viewpoint of its Girth. 63NTG11

sequence of consecutive vertices is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with two different length three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. The length of this path implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to girth $\mathcal{G}(NTG)$ and neutrosophic length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 3 is girth and its corresponded sets are $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_2, n_3, n_4\}$;

(vi) 3.9 is neutrosophic girth and its corresponded set is $\{n_1, n_3, n_4\}$.

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

Proposition 2.6.3. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-path-neutrosophic graph. Then*

$$\mathcal{G}_n(NTG) = \infty.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-path-neutrosophic graph. There's no crisp cycle. If $NTG : (V, E, \sigma, \mu)$ isn't a crisp cycle, then $NTG : (V, E, \sigma, \mu)$

isn't a neutrosophic cycle. There's no cycle from every version. Let $x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ be a path-neutrosophic graph. Since $x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is a sequence of consecutive vertices, there's no repetition of vertices in this sequence. So there's no cycle. Girth is corresponded to shortest cycle but there's no cycle. Thus it implies

$$\mathcal{G}_n(NTG) = \infty.$$

■

Example 2.6.4. There are two sections for clarifications.

(*a*) In Figure (2.12), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*ii*) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iii*) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_3, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are four edges, n_1n_2, n_2n_3 and n_4n_5, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding vertices has no effect to find a crisp cycle. There is only one path amid two given vertices. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4, n_5$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) ∞ is girth and there are no corresponded sets;

(vi) ∞ is neutrosophic girth and there are no corresponded sets.

(b) In Figure (2.13), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle

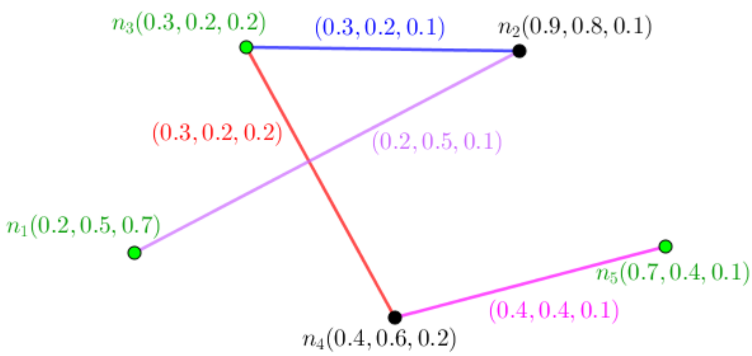

Figure 2.12: A Neutrosophic Graph in the Viewpoint of its Girth. 63NTG12

for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_3, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are four edges, n_1n_2, n_2n_3 and n_4n_5, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding vertices has no effect to find a crisp cycle. There is only one path amid two given vertices. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4, n_5$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) ∞ is girth and there are no corresponded sets;

(vi) ∞ is neutrosophic girth and there are no corresponded sets.

Proposition 2.6.5. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-cycle-neutrosophic graph where $\mathcal{O}(NTG) \geq 3$. Then*

$$\mathcal{G}_n(NTG) = \mathcal{O}_n(NTG).$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(NTG)}, x_1$ be a sequence of consecutive vertices of $NTG :$

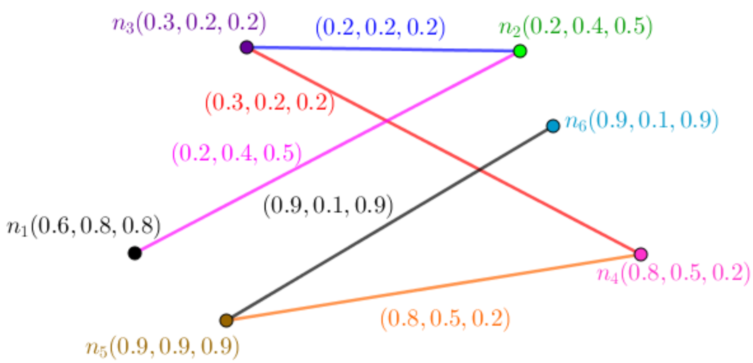

Figure 2.13: A Neutrosophic Graph in the Viewpoint of its Girth. 63NTG13

(V, E, σ, μ) such that

$$x_i x_{i+1} \in E, \ i = 1, 2, \cdots, \mathcal{O}(NTG) - 1, \ x_{\mathcal{O}(NTG)} x_1 \in E.$$

There are two paths amid two given vertices. The degree of every vertex is two. But there's one crisp cycle for every given vertex. So the efforts leads to one cycle for finding a shortest crisp cycle. For a given vertex x_i, the sequence of consecutive vertices

$$x_i, x_{i+1}, \cdots, x_{i-2}, x_{i-1}, x_i$$

is a corresponded crisp cycle for x_i. Every cycle has same length. The length is $\mathcal{O}(NTG)$. Thus the crisp cardinality of set of vertices forming shortest crisp cycle is $\mathcal{O}(NTG)$. By Theorem (3.4.5),

$$\mathcal{G}_n(NTG) = \mathcal{O}_n(NTG).$$

∎

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.6.6. There are two sections for clarifications.

(a) In Figure (2.14), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are six edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6$ and n_6n_1, according to corresponded neutrosophic path and it's neutrosophic cycle since it has two weakest edges, n_4n_5 and n_5n_6 with same values $(0.1, 0.1, 0.2)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is both of a neutrosophic cycle and crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is corresponded to both of girth $\mathcal{G}(NTG)$ and neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 6 is girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$;

(vi) $8.1 = \mathcal{O}(NTG)$ is neutrosophic girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$.

(b) In Figure (2.15), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are five edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ and n_5n_1, according to corresponded neutrosophic path and it isn't neutrosophic cycle since it has only one weakest edge, n_1n_2, with value $(0.2, 0.5, 0.4)$ and not more. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is not a neutrosophic cycle but it is a crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_1$ is corresponded to both of girth $\mathcal{G}(NTG)$ and neutrosophic girth $\mathcal{G}_n(NTG)$;

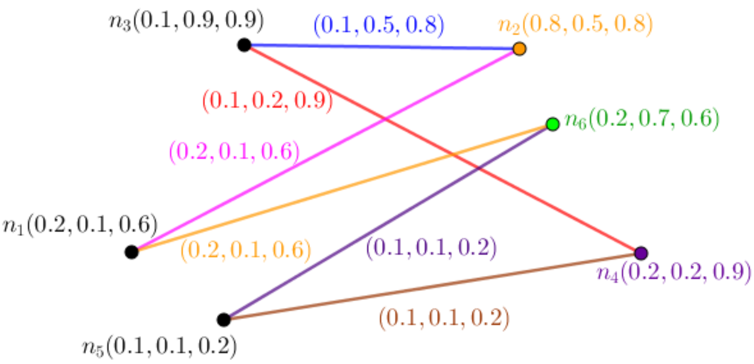

Figure 2.14: A Neutrosophic Graph in the Viewpoint of its Girth.

63NTG14

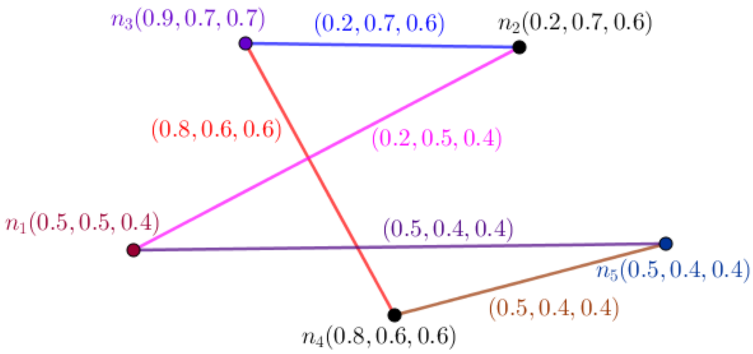

Figure 2.15: A Neutrosophic Graph in the Viewpoint of its Girth.

63NTG15

(v) 5 is girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$;

(vi) $8.5 = \mathcal{O}(NTG)$ is neutrosophic girth and its corresponded set is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$.

Proposition 2.6.7. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-star-neutrosophic graph with center c. Then*

$$\mathcal{G}_n(NTG) = \infty.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-star-neutrosophic graph. Every vertex isn't a neighbor for every given vertex. Every vertex is a neighbor for center. Furthermore, center is only neighbor for any given vertex. So center is only neighbor for all vertices. It's possible to have some paths amid two given vertices but there's no crisp cycle. In other words, if $\mathcal{O}(NTG) > 2$, then there are at least three vertices x, y and z such that if x is a neighbor for y and z, then y and z aren't neighbors and x is center. To get more precise, if if x and y are neighbors then either x or y is center. Every edge have one common endpoint with other edges which is called center. Thus there is no triangle but there are some edges. One edge has two endpoints which one of them is center. There are no crisp cycle. Hence trying to find shortest cycle has no result. There is no crisp cycle. Then there is shortest crisp cycle. So

$$\mathcal{G}_n(NTG) = \infty.$$

■

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.6.8. There is one section for clarifications. In Figure (2.16), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a star and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this star implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. The length of this star implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a star and there are two edges, n_1n_2 and n_1n_3, according to corresponded neutrosophic star but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic star implies

$$n_1, n_2, n_3$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a star and there are three edges, n_1n_2, n_1n_3 and n_1n_4, according to corresponded neutrosophic star but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. So adding vertices has no effect to find a crisp cycle. The structure of this neutrosophic star implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

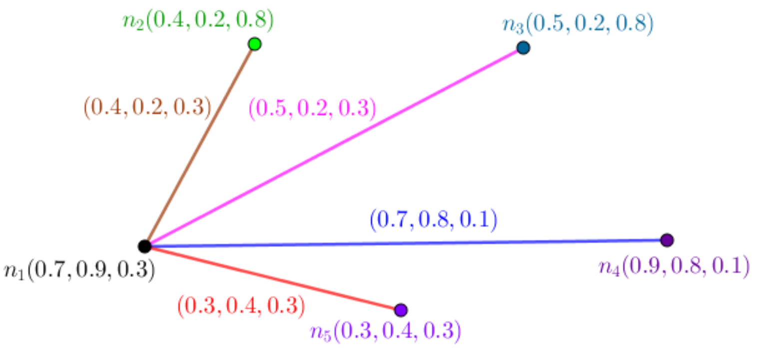

Figure 2.16: A Neutrosophic Graph in the Viewpoint of its Girth. 63NTG16

(*iv*) if n_1, n_2, n_3, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a star and there are four edges, n_1n_2, n_1n_3, n_1n_4 and n_1n_5, according to corresponded neutrosophic star but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. So adding vertices has no effect to find a crisp cycle. There are some paths amid two given vertices. The structure of this neutrosophic star implies

$$n_1, n_2, n_3, n_4, n_5$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*v*) ∞ is girth and there is no corresponded set;

(*vi*) ∞ is neutrosophic girth and there is no corresponded set.

Proposition 2.6.9. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-complete-bipartite-neutrosophic graph. Then*

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^{3}(\sigma_i(x) + \sigma_i(y) + \sigma_i(z) + \sigma_i(w))\}_{x,y\in V_1,\ z,w\in V_2}.$$

where $\mathcal{O}(NTG) \geq 4$ and $\min\{|V_1|, |V_2|\} \geq 2$. Also,

$$\mathcal{G}_n(NTG) = \infty$$

where $\mathcal{O}(NTG) \leq 3$.

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-complete-bipartite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. If $\mathcal{O}(NTG) \leq 3$, then it's neutrosophic path implying

$$\mathcal{G}_n(NTG) = \infty.$$

If $\mathcal{O}(NTG) \geq 4$, then it's possible to have two vertices in every part. In this case, four vertices form a crisp cycle which crisp cardinality of its vertices are four. It's impossible to have a crisp cycle which crisp cardinality of its vertices are three. Since the sequence of consecutive vertices are x_1, x_2, x_3 and there's no edge more. It implies there are two edges. It's neutrosophic path but neither crisp cycle nor neutrosophic cycle. So the first step of finding shortest crisp cycle is impossible but in second step, there's one crisp cycle corresponded to number four. By Theorem (3.4.5),

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^{3}(\sigma_i(x) + \sigma_i(y) + \sigma_i(z) + \sigma_i(w))\}_{x,y \in V_1,\ z,w \in V_2}.$$

where $\mathcal{O}(NTG) \geq 4$ and $\min\{|V_1|, |V_2|\} \geq 2$. Also,

$$\mathcal{G}_n(NTG) = \infty$$

where $\mathcal{O}(NTG) \leq 3$. ■

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.6.10. There is one section for clarifications. In Figure (2.17), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a complete-bipartite and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this complete-bipartite implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic complete-bipartite has neither a neutrosophic cycle nor crisp cycle. The length of this complete-bipartite implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-bipartite and there are two edges, n_1n_2 and n_1n_3, according to corresponded neutrosophic complete-bipartite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-bipartite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-bipartite implies

$$n_1, n_2, n_3$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-bipartite and there are two edges, n_1n_2 and n_2n_4, according to corresponded neutrosophic complete-bipartite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-bipartite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-bipartite implies

$$n_1, n_2, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a complete-bipartite and there are four edges, n_1n_2, n_1n_3, n_2n_4 and n_3n_4, according to corresponded neutrosophic complete-bipartite and it has neutrosophic cycle where n_2n_4 and n_3n_4 are two weakest edge with same amount $(0.3, 0.2, 0.3)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle only has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-bipartite has both of a neutrosophic cycle and crisp cycle. So adding vertices has some effects to find a crisp cycle. The structure of this neutrosophic complete-bipartite implies

$$n_1, n_2, n_3, n_4$$

is corresponded to girth $\mathcal{G}(NTG)$ and uniqueness of this cycle implies the sequence

$$n_1, n_2, n_3, n_4$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 4 is girth and its corresponded sequence is n_1, n_2, n_3, n_4;

(vi) 5.8 is neutrosophic girth and its corresponded sequence is n_1, n_2, n_3, n_4.

Proposition 2.6.11. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-complete-t-partite-neutrosophic graph. Then*

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^{3}(\sigma_i(x) + \sigma_i(y) + \sigma_i(z))\}_{x\in V_1,\ y\in V_2,\ z\in V_3}.$$

where $t \geq 3$.

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^{3}(\sigma_i(x) + \sigma_i(y) + \sigma_i(z) + \sigma_i(w))\}_{x,y\in V_1,\ z,w\in V_2}.$$

where $t \leq 2$. And

$$\mathcal{G}_n(NTG) = \infty$$

where $\mathcal{O}(NTG) \leq 2$.

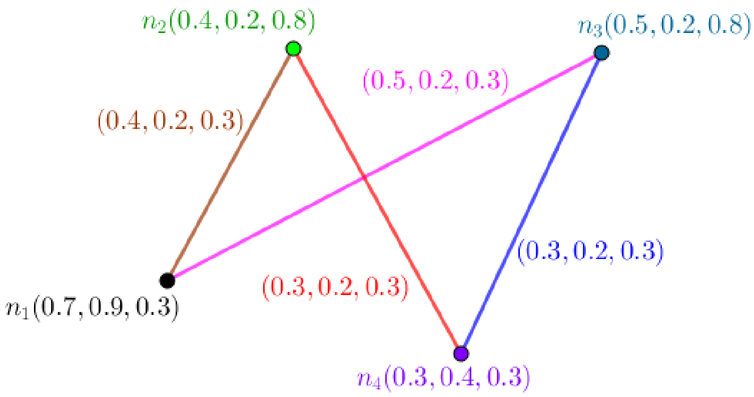

Figure 2.17: A Neutrosophic Graph in the Viewpoint of Girth. 63NTG17

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-complete-t-partite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. If $\mathcal{O}(NTG) \leq 2$, then it's neutrosophic path implying

$$\mathcal{G}_n(NTG) = \infty.$$

If $t \geq 3, \mathcal{O}(NTG) \geq 3$, then it has crisp cycle implying

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^{3}(\sigma_i(x) + \sigma_i(y) + \sigma_i(z))\}_{x \in V_1,\ y \in V_2,\ z \in V_3}.$$

If $t \geq 2, \mathcal{O}(NTG) \geq 4$, then it's possible to have two vertices in every part. In this case, four vertices form a crisp cycle which crisp cardinality of its vertices are four. It's impossible to have a crisp cycle which crisp cardinality of its vertices are three. Since the sequence of consecutive vertices are x_1, x_2, x_3 and there's no edge more. It implies there are two edges. It's neutrosophic path but neither crisp cycle nor neutrosophic cycle. So the first step of finding shortest crisp cycle is impossible but in second step, there's one crisp cycle corresponded to number four. By Theorem (3.4.5),

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^{3}(\sigma_i(x) + \sigma_i(y) + \sigma_i(z))\}_{x \in V_1,\ y \in V_2,\ z \in V_3}.$$

where $t \geq 3$.

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^{3}(\sigma_i(x) + \sigma_i(y) + \sigma_i(z) + \sigma_i(w))\}_{x,y \in V_1,\ z,w \in V_2}.$$

where $t \leq 2$. And

$$\mathcal{G}_n(NTG) = \infty$$

where $\mathcal{O}(NTG) \leq 2$. ∎

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.6.12. There is one section for clarifications. In Figure (2.18), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(*i*) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a complete-t-partite and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this complete-t-partite implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The length of this complete-t-partite implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*ii*) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-t-partite and there are two edges, n_1n_2 and n_1n_3, according to corresponded neutrosophic complete-t-partite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_3$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iii*) if n_1, n_2, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-t-partite and there are two edges, n_1n_2 and n_2n_4, according to corresponded neutrosophic complete-t-partite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(*iv*) if n_1, n_2, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a complete-t-partite and there are four edges, n_1n_2, n_1n_5, n_2n_4 and n_5n_4, according to corresponded neutrosophic complete-t-partite and it has neutrosophic cycle where n_2n_4 and n_5n_4 are two weakest edge with same amount $(0.3, 0.2, 0.3)$. First step is to

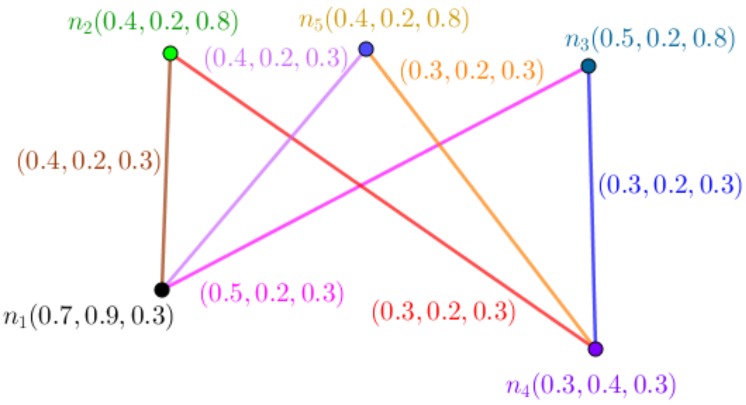

Figure 2.18: A Neutrosophic Graph in the Viewpoint of its Girth. 63NTG18

have at least one crisp cycle for finding shortest cycle. Finding shortest cycle only has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has both of a neutrosophic cycle and crisp cycle. So adding vertices has some effects to find a crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_4, n_5$$

is corresponded to girth $\mathcal{G}(NTG)$ and minimum neutrosophic cardinality implies the sequence

$$n_1, n_2, n_4, n_5$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 4 is girth and its corresponded sequence is n_1, n_2, n_4, n_5;

(vi) 5.7 is neutrosophic girth and its corresponded sequence is n_1, n_2, n_4, n_5.

Proposition 2.6.13. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-wheel-neutrosophic graph. Then*

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^{3}(\sigma_i(x) + \sigma_i(y) + \sigma_i(z))\}_{xy,xz,zy\in E}.$$

where $t \geq 3$.

$$\mathcal{G}_n(NTG) = \infty$$

where $t \geq 2$.

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-wheel-neutrosophic graph. The argument is elementary. Since all vertices of a path join to one vertex. By Theorem (3.4.5),

$$\mathcal{G}_n(NTG) = \min\{\Sigma_{i=1}^{3}(\sigma_i(x) + \sigma_i(y) + \sigma_i(z))\}_{xy,xz,zy\in E}.$$

where $t \geq 3$.

$$\mathcal{G}_n(NTG) = \infty$$

where $t \geq 2$. ∎

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 2.6.14. There is one section for clarifications. In Figure (2.19), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If s_1, s_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a wheel and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this wheel implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic wheel has neither a neutrosophic cycle nor crisp cycle. The length of this wheel implies

$$s_1, s_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if s_4, s_2, s_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a wheel and there are two edges, s_3s_2 and s_4s_3, according to corresponded neutrosophic wheel but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic wheel has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic wheel implies

$$s_4, s_2, s_3$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if s_1, s_2, s_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a wheel and there are three edges, s_1s_2, s_2s_3 and s_1s_3 according to corresponded neutrosophic wheel but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one crisp cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic wheel has no neutrosophic cycle but it has crisp cycle. The structure of this neutrosophic wheel implies

$$s_1, s_2, s_3$$

is corresponded to girth $\mathcal{G}(NTG)$ but minimum neutrosophic cardinality implies

$$s_1, s_2, s_3$$

isn't corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

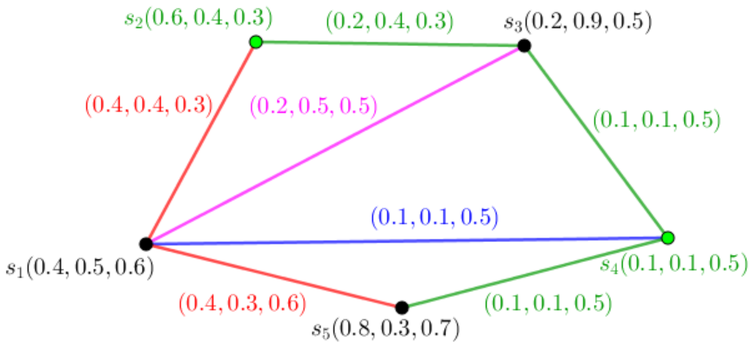

Figure 2.19: A Neutrosophic Graph in the Viewpoint of its Girth. 63NTG19

(iv) if s_1, s_3, s_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a wheel and there are three edges, s_1s_4, s_4s_3 and s_1s_3 according to corresponded neutrosophic wheel and it has a neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one crisp cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic wheel has one neutrosophic cycle with two weakest edges s_1s_4 and s_3s_4 concerning same values $(0.1, 0.1, 0.5)$ and it has a crisp cycle. The structure of this neutrosophic wheel implies

$$s_1, s_3, s_4$$

is corresponded to girth $\mathcal{G}(NTG)$ and minimum neutrosophic cardinality implies

$$s_1, s_3, s_4$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 3 is girth and its corresponded sequences are s_1, s_3, s_4 and s_1, s_2, s_3 alongside s_1, s_4, s_5;

(vi) 3.8 is neutrosophic girth and its corresponded sequence is s_1, s_3, s_4.

2.7 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are either complete models which mean complete connections are formed as individual and family of complete models with common neutrosophic vertex set or quasi-complete models which mean quasi-complete connections are formed as individual and family of quasi-complete models with common neutrosophic vertex set.

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importance to avoid mixing up.

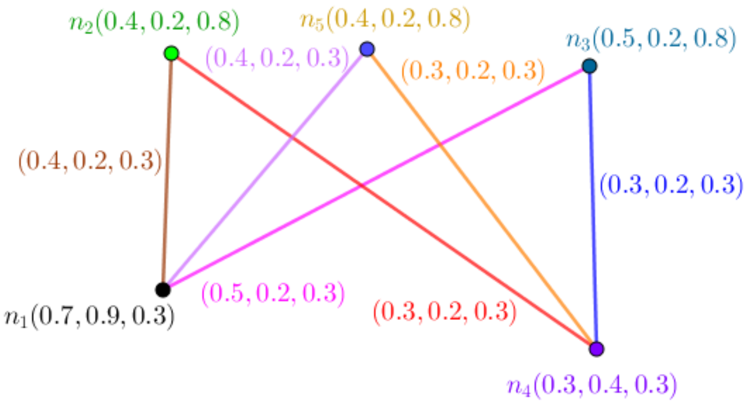

Figure 2.20: A Neutrosophic Graph in the Viewpoint of its Girth and its Neutrosophic Girth

63NTG20

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive sections. Beyond that, sometimes sections are not the same.

Step 3. (Model) The situation is designed as a model. The model uses data to assign every section and to assign to relation amid sections, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relations amid them. Table (2.1), clarifies about the assigned numbers to these situations.

Table 2.1: Scheduling concerns its Subjects and its Connections as a neutrosophic graph in a Model.

63tbl1

Sections of NTG	n_1	$n_2 \cdots$	n_5
Values	$(0.7, 0.9, 0.3)$	$(0.4, 0.2, 0.8) \cdots$	$(0.4, 0.2, 0.8)$
Connections of NTG	E_1	$E_2 \cdots$	E_6
Values	$(0.4, 0.2, 0.3)$	$(0.5, 0.2, 0.3) \cdots$	$(0.3, 0.2, 0.3)$

2.8 Case 1: Complete-t-partite Model alongside its Girth and its Neutrosophic Girth

Step 4. (Solution) The neutrosophic graph alongside its girth and its neutrosophic girth as model, propose to use specific number. Every subject has connection with some subjects. Thus the connection is applied as possible and the model demonstrates quasi-full connections as quasi-possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is star, the number is different. Also, it holds for other types such that complete, wheel, path, and cycle. The collection of situations is another application of its girth and its neutrosophic girth when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are five subjects which are represented as Figure (2.20). This model is strong and even more it's quasi-complete. And the study proposes using specific number which is called its girth and its neutrosophic girth. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (2.20). In Figure (2.20), an complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a complete-t-partite and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this complete-t-partite implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The length of this complete-t-partite implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-t-partite and there are two edges, n_1n_2 and n_1n_3, according to corresponded neutrosophic complete-t-partite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_3$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-t-partite and there are two edges, n_1n_2 and n_2n_4, according to corresponded neutrosophic complete-t-partite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor

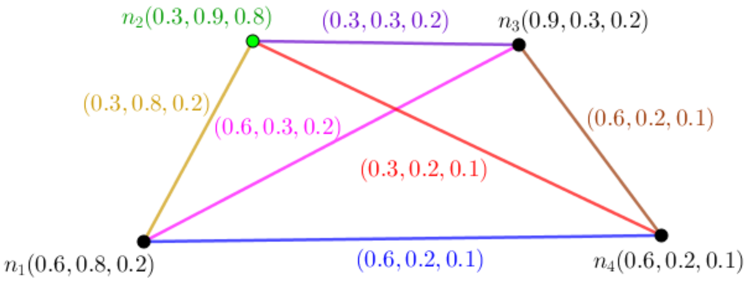

Figure 2.21: A Neutrosophic Graph in the Viewpoint of its Girth and its Neutrosophic Girth

63NTG21

crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a complete-t-partite and there are four edges, n_1n_2, n_1n_5, n_2n_4 and n_5n_4, according to corresponded neutrosophic complete-t-partite and it has neutrosophic cycle where n_2n_4 and n_5n_4 are two weakest edge with same amount $(0.3, 0.2, 0.3)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle only has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has both of a neutrosophic cycle and crisp cycle. So adding vertices has some effects to find a crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_4, n_5$$

is corresponded to girth $\mathcal{G}(NTG)$ and minimum neutrosophic cardinality implies the sequence

$$n_1, n_2, n_4, n_5$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 4 is girth and its corresponded sequence is n_1, n_2, n_4, n_5;

(vi) 5.7 is neutrosophic girth and its corresponded sequence is n_1, n_2, n_4, n_5.

2.9 Case 2: Complete Model alongside its A Neutrosophic Graph in the Viewpoint of its Girth and its Neutrosophic Girth

Step 4. (Solution) The neutrosophic graph alongside its girth and its neutrosophic girth as model, propose to use specific number. Every subject has connection with every given subject in deemed way. Thus the connection applied as possible and the model demonstrates full connections as possible between parts but with different view where symmetry amid vertices and edges are the matters. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete multipartite, the number is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of its girth and its neutrosophic girth when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are four subjects which are represented in the formation of one model as Figure (2.21). This model is neutrosophic strong as individual and even more it's complete. And the study proposes using specific number which is called its girth and its neutrosophic girth for this model. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to these models as individual. A model as a collection of situations to compare them with another model as a collection of situations to get more precise. Consider Figure (2.21). There is one section for clarifications.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1, n_2, n_3$$

is corresponded to girth $\mathcal{G}(NTG)$ but neutrosophic length of this crisp cycle implies

$$n_1, n_2, n_3$$

isn't corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's two crisp cycles with length two and three. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1, n_3, n_4, and with length four, n_1, n_2, n_3, n_4. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a sequence of consecutive vertices is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with two different length three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. The length of this path implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth $\mathcal{G}(NTG)$ nor neutrosophic girth $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to girth $\mathcal{G}(NTG)$ and neutrosophic length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to neutrosophic girth $\mathcal{G}_n(NTG)$;

(v) 3 is girth and its corresponded sets are $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_2, n_3, n_4\}$;

(vi) 3.9 is neutrosophic girth and its corresponded set is $\{n_1, n_3, n_4\}$.

2.10 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study. Notion concerning its girth and its neutrosophic girth are defined in neutrosophic graphs. Neutrosophic number is also reused. Thus,

Question 2.10.1. *Is it possible to use other types of its girth and its neutrosophic girth?*

Question 2.10.2. *Are existed some connections amid different types of its girth and its neutrosophic girth in neutrosophic graphs?*

Question 2.10.3. *Is it possible to construct some classes of neutrosophic graphs which have "nice" behavior?*

Question 2.10.4. *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

Problem 2.10.5. *Which parameters are related to this parameter?*

Problem 2.10.6. *Which approaches do work to construct applications to create independent study?*

Problem 2.10.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

2.11 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted.
This study uses two definitions concerning girth and neutrosophic girth arising from shortest cycles to study strong neutrosophic graphs. New neutrosophic number is reused which is too close to the notion of neutrosophic number but it's different since it uses all values as type-summation on them. Comparisons amid number, corresponded vertices and edges are done by using neutrosophic tool. The connections of vertices which aren't clarified by a neutrosophic cycle differ them from each other and put them in different categories to represent

Table 2.2: A Brief Overview about Advantages and Limitations of this Study 63tbl2

Advantages	Limitations
1. Neutrosophic Girth	1. Connections amid Classes
2. Girth	
3. Neutrosophic Number	2. Study on Families
4. Classes of Strong Neutrosophic Graphs	
5. Shortest Cycles	3. Same Models in Family

a number which is called girth and neutrosophic girth arising from shortest neutrosophic cycles. Further studies could be about changes in the settings to compare these notions amid different settings of strong neutrosophic graphs theory. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (2.2), some limitations and advantages of this study are pointed out.

CHAPTER 3

Neutrosophic Girth Polynomial Based On Neutrosophic Cycle and Crisp Cycle in Neutrosophic Graphs

The following sections are cited as [**Ref3**] which is my 64th manuscript and I use prefix 64 as number before any labelling for items.

3.1 Some Polynomials Related to Numbers in Classes of (Strong) Neutrosophic Graphs

3.2 Abstract

New setting is introduced to study girth polynomial and neutrosophic girth polynomial arising counting neutrosophic cycles and crisp cycles in strong neutrosophic graphs based on neutrosophic cycles and in neutrosophic graphs based on crisp cycles. Forming neutrosophic cycles from a sequence of consecutive vertices is key type of approach to have these notions namely girth polynomial and neutrosophic girth polynomial arising from counting neutrosophic cycles and crisp cycles in strong neutrosophic graphs based on neutrosophic cycles and in neutrosophic graphs based on crisp cycles. Two numbers are obtained but now both settings leads to approach is on demand which is counting minimum cardinality and minimum neutrosophic cardinality in the terms of vertices, which have edges which form neutrosophic cycle and crisp cycles. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then girth polynomial $\mathcal{G}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is $n_1x^{m_1} + n_2x^{m_2} + \cdots + n_sx^3$ where n_i is the number of cycle with m_i as its crisp cardinality of the set of vertices of cycle; neutrosophic girth polynomial $\mathcal{G}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is $n_1x^{m_1} + n_2x^{m_2} + \cdots + n_sx^{m_s}$ where n_i is the number of cycle with m_i as its neutrosophic cardinality of the set of vertices of cycle. As concluding results, there are some statements, remarks, examples and clarifications about some classes of strong neutrosophic graphs namely (strong-)path-neutrosophic graphs, (strong-)cycle-neutrosophic graphs, complete-neutrosophic graphs, (strong-)star-neutrosophic graphs, (strong-)complete-bipartite-neutrosophic graphs, (strong-)complete-t-partite-neutrosophic graphs and (strong-)wheel-neutrosophic graphs. The clarifications are also presented in both sections "Setting of Girth Polynomial," and "Setting of Neutrosophic Girth Polynomial," for introduced results and used classes. Neutrosophic number is reused in this way. It's applied to use the type of neutrosophic number in the way that, three values of a vertex are used and they've same share to construct this number to compare with other vertices. Summation of three values of vertex makes one number and applying it to a comparison. This approach facilitates identifying vertices which form girth polynomial and neutrosophic girth polynomial arising from counting neutrosophic cycles and crisp cycles in strong neutrosophic graphs based on neutrosophic cycles and in neutrosophic graphs based on crisp cycles. In both settings, some classes of well-known (strong) neutrosophic graphs are studied. Some clarifications for each result and each definition are provided. The cardinality of a set has eligibility to define girth polynomial but the neutrosophic cardinality of a set has eligibility to define neutrosophic girth polynomial. Some results get more frameworks and perspective about these definitions. The way in that, a sequence of consecutive vertices forming a neutrosophic cycle and crisp cycles, opens the way to do some approaches. These notions are applied into strong neutrosophic graphs and neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special strong neutrosophic graphs and neutrosophic graphs which are well-known, is an open way to pursue this study. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Keywords: Girth Polynomial, Neutrosophic Girth Polynomial, Counting

Neutrosophic Cycle and Crisp Cycle
AMS Subject Classification: 05C17, 05C22, 05E45

3.3 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 3.3.1. *Is it possible to use mixed versions of ideas concerning "Neutrosophic Girth Polynomial", "Girth Polynomial" and "(Strong) Neutrosophic Graph" to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of (strong) neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Lack of connection amid two edges have key roles to assign girth polynomial and neutrosophic girth polynomial arising from counting neutrosophic cycles and crisp cycles in strong neutrosophic graphs based on neutrosophic cycles and in neutrosophic graphs based on crisp cycles. Thus they're used to define new ideas which conclude to the structure of girth polynomial and neutrosophic girth polynomial arising from counting neutrosophic cycles and crisp cycles in strong neutrosophic graphs based on neutrosophic cycles and in neutrosophic graphs based on crisp cycles. The concept of having common number of neutrosophic cycle inspires us to study the behavior of vertices in the way that, some types of numbers, girth polynomial and neutrosophic girth polynomial arising from counting neutrosophic cycles

and crisp cycles in strong neutrosophic graphs based on neutrosophic cycles and in neutrosophic graphs based on crisp cycles are the cases of study in the setting of individuals. In both settings, a corresponded number concludes the discussion. Also, there are some avenues to extend these notions.
The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In subsection "Preliminaries", new notions of girth polynomial and neutrosophic girth polynomial arising from counting neutrosophic cycles and crisp cycles in strong neutrosophic graphs based on neutrosophic cycles and in neutrosophic graphs based on crisp cycles, are highlighted, are introduced and are clarified as individuals. In section "Preliminaries", sequence of consecutive vertices forming neutrosophic cycles and crisp cycles have the key role in this way. General results are obtained and also, the results about the basic notions of girth polynomial and neutrosophic girth polynomial arising from counting neutrosophic cycles and crisp cycles in strong neutrosophic graphs based on neutrosophic cycles and in neutrosophic graphs based on crisp cycles, are elicited. Some classes of (strong) neutrosophic graphs are studied in the terms of girth polynomial arising from counting neutrosophic cycles and crisp cycles in strong neutrosophic graphs based on neutrosophic cycles and in neutrosophic graphs based on crisp cycles, in section "Setting of Girth Polynomial," as individuals. In section "Setting of Girth Polynomial," girth polynomial is applied into individuals. As concluding results, there are some statements, remarks, examples and clarifications about some classes of (strong) neutrosophic graphs namely strong-path-neutrosophic graphs, (strong-)cycle-neutrosophic graphs, complete-neutrosophic graphs, (strong-)star-neutrosophic graphs, (strong-)complete-bipartite-neutrosophic graphs, (strong-)complete-t-partite-neutrosophic graphs and (strong-)wheel-neutrosophic graphs. The clarifications are also presented in both sections "Setting of Girth Polynomial," and "Setting of Neutrosophic Girth Polynomial," for introduced results and used classes. In section "Applications in Time Table and Scheduling", two applications are posed for quasi-complete and complete notions, namely complete-neutrosophic graphs and (strong-)complete-t-neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are considered as individual. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications is featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

3.4 Preliminaries

In this subsection, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.
Basic idea is about the model which is used. First definition introduces basic model.

Definition 3.4.1. (Graph).
$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

Definition 3.4.2. (Neutrosophic Graph And Its Special Case).
$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, and $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_i v_j \in E$,

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic edge set**.

(iii) : $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

(iv) : $\sum_{v \in V} \sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

(v) : $|E|$ is called **size** of NTG and it's denoted by $\mathcal{S}(NTG)$.

(vi) : $\sum_{e \in E} \sum_{i=1}^{3} \mu_i(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$.

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

Definition 3.4.3. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) : a sequence of vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}}$ is called **path** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$;

(ii) : **strength** of path $P : x_0, x_1, \cdots, x_{\mathcal{O}}$ is $\bigwedge_{i=0,\cdots,n-1} \mu(x_i x_{i+1})$;

(iii) : **connectedness** amid vertices x_0 and x_t is

$$\mu^{\infty}(x_0, x_t) = \bigvee_{P : x_0, x_1, \cdots, x_t} \bigwedge_{i=0,\cdots,t-1} \mu(x_i x_{i+1});$$

(iv) : a sequence of vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}}$ is called **cycle** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$;

(v) : it's **t-partite** where V is partitioned to t parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge xy implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1, \sigma_2, \cdots, \sigma_t}$ where σ_i is σ on $V_i^{s_i}$ instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_j^{s_i}| = s_i$;

(vi) : t-partite is **complete bipartite** if $t = 2$, and it's denoted by K_{σ_1, σ_2};

(vii) : complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1, σ_2};

$(viii)$: a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1, σ_2};

$(ix):$ it's **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$;

$(x):$ it's **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

Definition 3.4.4. (Girth Polynomial and Neutrosophic Girth Polynomial). Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) **girth polynomial** $\mathcal{G}(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is $n_1x^{m_1} + n_2x^{m_2} + \cdots + n_sx^3$ where n_i is the number of cycle with m_i as its crisp cardinality of the set of vertices of cycle;

(ii) **neutrosophic girth polynomial** $\mathcal{G}_n(NTG)$ for a neutrosophic graph $NTG : (V, E, \sigma, \mu)$ is $n_1x^{m_1} + n_2x^{m_2} + \cdots + n_sx^{m_s}$ where n_i is the number of cycle with m_i as its neutrosophic cardinality of the set of vertices of cycle.

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Theorem 3.4.5. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. If $NTG : (V, E, \sigma, \mu)$ is strong, then its crisp cycle is its neutrosophic cycle.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider u as a vertex of crisp cycle CYC, such that $\sigma(u) = \min \sigma(x)_{x \in V(CYC)}$. u has two neighbors y, z in CYC. Since NTG is strong, $\mu(uy) = \mu(uz) = \sigma(u)$. It implies there are two weakest edges in CYC. It means CYC is neutrosophic cycle. ■

For convenient usages, the word neutrosophic which is used in previous definition, won't be used, usually.
In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 3.4.6. In Figure (3.1), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1, n_2, n_3$$

is corresponded to girth polynomial $\mathcal{G}(NTG)$ but neutrosophic length of this crisp cycle implies

$$n_1, n_2, n_3$$

isn't corresponded to neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's two crisp cycles with length two and three. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1, n_3, n_4, and with length four, n_1, n_2, n_3, n_4. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a sequence of consecutive vertices is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with two different length three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. The length of this path implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to girth polynomial $\mathcal{G}(NTG)$ and neutrosophic length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) $x^4 + 3x^3$ is girth polynomial and its corresponded sets, for coefficient of smallest term, are $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_2, n_3, n_4\}$;

(vi) $x^{5.9} + x^5 + x^{4.5} + x^{4.3} + x^{3.9}$ is neutrosophic girth polynomial and its corresponded set, for coefficient of smallest term, is $\{n_1, n_3, n_4\}$.

3.5 Setting of Girth Polynomial

In this section, I provide some results in the setting of girth polynomial. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, star-neutrosophic graph, bipartite-neutrosophic graph, and t-partite-neutrosophic graph, are both of cases of study and classes which the results are about them.

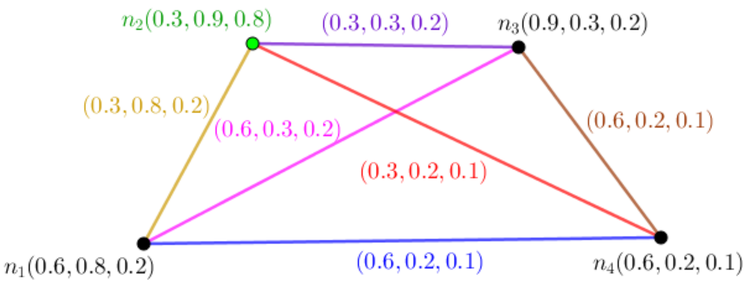

Figure 3.1: A Neutrosophic Graph in the Viewpoint of its girth polynomial and its Neutrosophic girth polynomial.

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Proposition 3.5.1. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{G}(NTG) = x^{\mathcal{O}(NTG)} + \mathcal{O}(NTG)x^{\mathcal{O}(NTG)-1} + \cdots + \binom{\mathcal{O}(NTG)}{3}x^3.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. The length of longest cycle is $\mathcal{O}(NTG)$. In other hand, there's a cycle if and only if $\mathcal{O}(NTG) \geq 3$. It's complete. So there's at least one neutrosophic cycle which its length is $\mathcal{O}(NTG) = 3$. By shortest cycle is on demand, the girth polynomial is three. Thus

$$\mathcal{G}(NTG) = x^{\mathcal{O}(NTG)} + \mathcal{O}(NTG)x^{\mathcal{O}(NTG)-1} + \cdots + \binom{\mathcal{O}(NTG)}{3}x^3.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.5.2. In Figure (3.2), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1, n_2, n_3$$

is corresponded to girth polynomial $\mathcal{G}(NTG)$ but neutrosophic length of this crisp cycle implies

$$n_1, n_2, n_3$$

isn't corresponded to neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's two crisp cycles with length two and three. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1, n_3, n_4, and with length four, n_1, n_2, n_3, n_4. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a sequence of consecutive vertices is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with two different length three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. The length of this path implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to girth polynomial $\mathcal{G}(NTG)$ and neutrosophic length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) $x^4 + 3x^3$ is girth polynomial and its corresponded sets, for coefficient of smallest term, are $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_2, n_3, n_4\}$;

(vi) $x^{5.9} + x^5 + x^{4.5} + x^{4.3} + x^{3.9}$ is neutrosophic girth polynomial and its corresponded set, for coefficient of smallest term, is $\{n_1, n_3, n_4\}$.

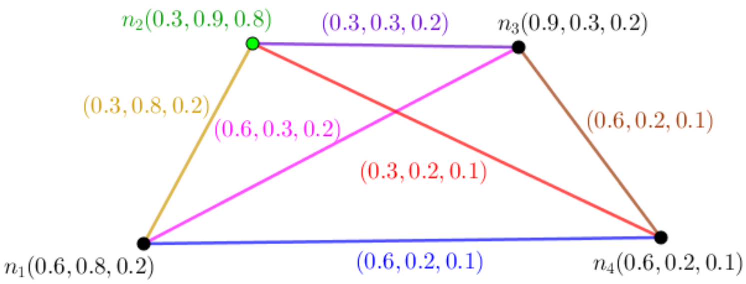

Figure 3.2: A Neutrosophic Graph in the Viewpoint of its girth polynomial. 64NTG2

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

Proposition 3.5.3. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-path-neutrosophic graph. Then*

$$\mathcal{G}(NTG) = 0.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-path-neutrosophic graph. There's no crisp cycle. If $NTG : (V, E, \sigma, \mu)$ isn't a crisp cycle, then $NTG : (V, E, \sigma, \mu)$ isn't a neutrosophic cycle. There's no cycle from every version. Let $x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ be a path-neutrosophic graph. Since $x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is a sequence of consecutive vertices, there's no repetition of vertices in this sequence. So there's no cycle. girth polynomial is corresponded to shortest cycle but there's no cycle. Thus it implies

$$\mathcal{G}(NTG) = 0.$$

∎

Example 3.5.4. There are two sections for clarifications.

(a) In Figure (3.3), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_3, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are four edges, n_1n_2, n_2n_3 and n_4n_5, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding vertices has no effect to find a crisp cycle. There is only one path amid two given vertices. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4, n_5$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) ∞ is girth polynomial and there are no corresponded sets;

(vi) ∞ is neutrosophic girth polynomial and there are no corresponded sets.

(b) In Figure (3.4), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(*ii*) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(*iii*) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(*iv*) if n_1, n_2, n_3, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are four edges, n_1n_2, n_2n_3 and n_4n_5, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding vertices has no effect to find a crisp cycle. There is only one path amid two given vertices. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4, n_5$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

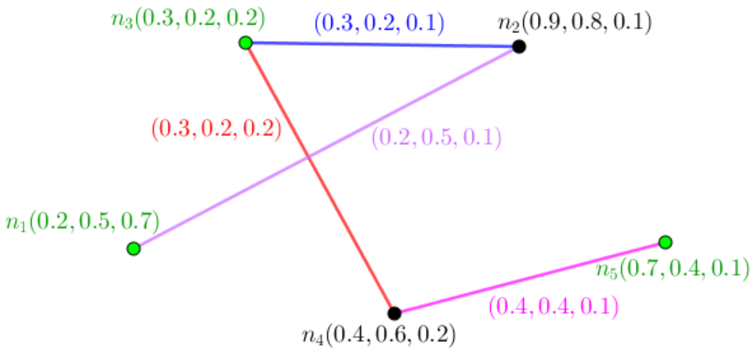

Figure 3.3: A Neutrosophic Graph in the Viewpoint of its girth polynomial. 64NTG3

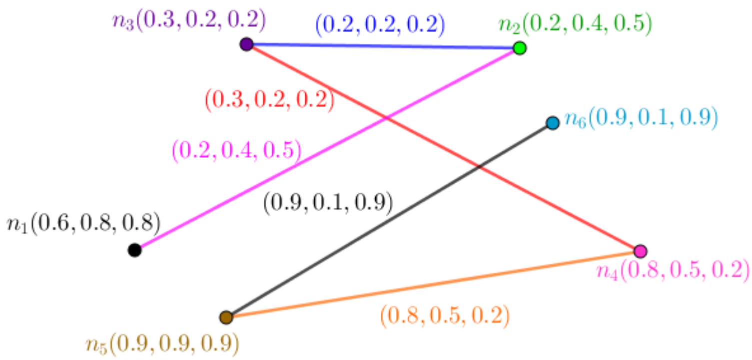

Figure 3.4: A Neutrosophic Graph in the Viewpoint of its girth polynomial. 64NTG4

(v) 0 is girth polynomial and there are no corresponded sets;

(vi) 0 is neutrosophic girth polynomial and there are no corresponded sets.

Proposition 3.5.5. *Let* $NTG : (V, E, \sigma, \mu)$ *be a strong-cycle-neutrosophic graph where* $\mathcal{O}(NTG) \geq 3$. *Then*

$$\mathcal{G}(NTG) = x^{\mathcal{O}(NTG)}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(NTG)}, x_1$ be a sequence of consecutive vertices of $NTG : (V, E, \sigma, \mu)$ such that

$$x_i x_{i+1} \in E, \ i = 1, 2, \cdots, \mathcal{O}(NTG) - 1, \ x_{\mathcal{O}(NTG)} x_1 \in E.$$

There are two paths amid two given vertices. The degree of every vertex is two. But there's one crisp cycle for every given vertex. So the efforts leads to one cycle for finding a shortest crisp cycle. For a given vertex x_i, the sequence of consecutive vertices

$$x_i, x_{i+1}, \cdots, x_{i-2}, x_{i-1}, x_i$$

is a corresponded crisp cycle for x_i. Every cycle has same length. The length is $\mathcal{O}(NTG)$. Thus the crisp cardinality of set of vertices forming shortest crisp cycle is $\mathcal{O}(NTG)$. By Theorem (3.4.5),

$$\mathcal{G}(NTG) = x^{\mathcal{O}(NTG)}.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.5.6. There are two sections for clarifications.

(a) In Figure (3.5), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are six edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6$ and n_6n_1, according to corresponded neutrosophic path and it's neutrosophic cycle since it has two weakest edges, n_4n_5 and n_5n_6 with same values $(0.1, 0.1, 0.2)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is both of a neutrosophic cycle and crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is corresponded to both of girth polynomial $\mathcal{G}(NTG)$ and neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) $x^{6=\mathcal{O}(NTG)}$ is girth polynomial and its corresponded set, for coefficient of smallest term, is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$;

(vi) $x^{8.1=\mathcal{O}_n(NTG)}$ is neutrosophic girth polynomial and its corresponded set, for coefficient of smallest term, is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$.

(b) In Figure (3.6), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are five edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ and n_5n_1, according to corresponded neutrosophic path and it isn't neutrosophic cycle since it has only one weakest edge, n_1n_2, with value $(0.2, 0.5, 0.4)$ and not more. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is not a neutrosophic cycle but it is a crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_1$ is corresponded to both of girth polynomial $\mathcal{G}(NTG)$ and neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) $x^{5=\mathcal{O}(NTG)}$ is girth polynomial and its corresponded set, for coefficient of smallest term, is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$;

(vi) $x^{8.5=\mathcal{O}_n(NTG)}$ is neutrosophic girth polynomial and its corresponded set, for coefficient of smallest term, is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$.

Proposition 3.5.7. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-star-neutrosophic graph with center c. Then*

$$\mathcal{G}(NTG) = 0.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-star-neutrosophic graph. Every vertex isn't a neighbor for every given vertex. Every vertex is a neighbor for center. Furthermore, center is only neighbor for any given vertex. So center is only neighbor for all vertices. It's possible to have some paths amid two given vertices but there's no crisp cycle. In other words, if $\mathcal{O}(NTG) > 2$, then there are at least three vertices x, y and z such that if x is a neighbor for y and z, then y and z aren't neighbors and x is center. To get more precise, if if x and y are neighbors then either x or y is center. Every edge have one common endpoint with other edges which is called center. Thus there is no triangle but there are some edges. One edge has two endpoints which one of them is

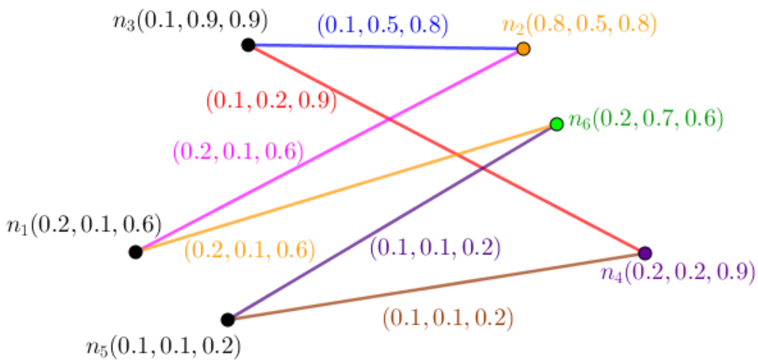

Figure 3.5: A Neutrosophic Graph in the Viewpoint of its girth polynomial. 64NTG5

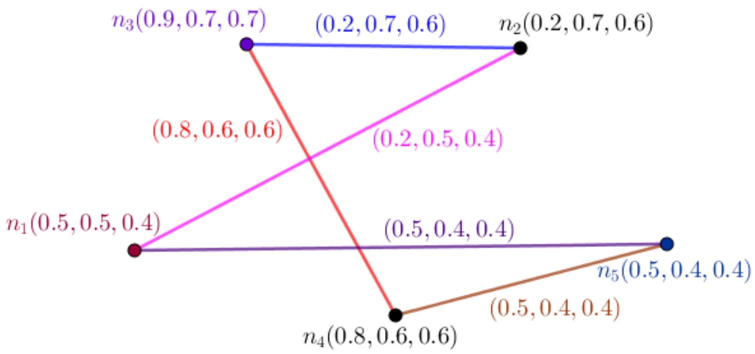

Figure 3.6: A Neutrosophic Graph in the Viewpoint of its girth polynomial. 64NTG6

center. There are no crisp cycle. Hence trying to find shortest cycle has no result. There is no crisp cycle. Then there is shortest crisp cycle. So

$$\mathcal{G}(NTG) = 0.$$

■

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.5.8. There is one section for clarifications. In Figure (3.7), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a star and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this star implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. The length of this star implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a star and there are two edges, n_1n_2 and n_1n_3, according to corresponded neutrosophic star but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic star implies

$$n_1, n_2, n_3$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a star and there are three edges, n_1n_2, n_1n_3 and n_1n_4, according to corresponded neutrosophic star but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. So adding vertices has no effect to find a crisp cycle. The structure of this neutrosophic star implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_3, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a star and there are four edges, n_1n_2, n_1n_3, n_1n_4 and n_1n_5, according to corresponded neutrosophic star but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. So adding vertices has no effect to find a crisp cycle. There are some paths amid two given vertices. The structure of this neutrosophic star implies

$$n_1, n_2, n_3, n_4, n_5$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) 0 is girth polynomial and there is no corresponded set;

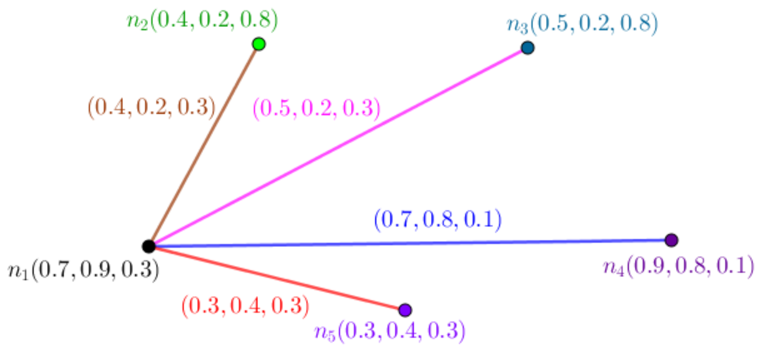

Figure 3.7: A Neutrosophic Graph in the Viewpoint of its girth polynomial. 64NTG7

(vi) 0 is neutrosophic girth polynomial and there is no corresponded set.

Proposition 3.5.9. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-complete-bipartite-neutrosophic graph. Then*

$$\mathcal{G}(NTG) = c_1 x^{2\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor} + c_2 x^{2\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor - 2} + \cdots + c_s x^4.$$

where $\mathcal{O}(NTG) \geq 4$. And

$$\mathcal{G}(NTG) = 0$$

where $\mathcal{O}(NTG) \leq 3$.

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-complete-bipartite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. If $\mathcal{O}(NTG) \leq 3$, then it's neutrosophic path implying

$$\mathcal{G}(NTG) = 0.$$

If $\mathcal{O}(NTG) \geq 4$, then it's possible to have two vertices in every part. In this case, four vertices form a crisp cycle which crisp cardinality of its vertices are four. It's impossible to have a crisp cycle which crisp cardinality of its vertices are three. Since the sequence of consecutive vertices are x_1, x_2, x_3 and there's no edge more. It implies there are two edges. It's neutrosophic path but neither crisp cycle nor neutrosophic cycle. So the first step of finding shortest crisp cycle is impossible but in second step, there's one crisp cycle corresponded to number four. By Theorem (3.4.5),

$$\mathcal{G}(NTG) = c_1 x^{2\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor} + c_2 x^{2\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor - 2} + \cdots + c_s x^4.$$

where $\mathcal{O}(NTG) \geq 4$. And

$$\mathcal{G}(NTG) = 0$$

where $\mathcal{O}(NTG) \leq 3$. ∎

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.5.10. There is one section for clarifications. In Figure (3.8), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a complete-bipartite and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this complete-bipartite implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic complete-bipartite has neither a neutrosophic cycle nor crisp cycle. The length of this complete-bipartite implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-bipartite and there are two edges, n_1n_2 and n_1n_3, according to corresponded neutrosophic complete-bipartite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-bipartite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-bipartite implies

$$n_1, n_2, n_3$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-bipartite and there are two edges, n_1n_2 and n_2n_4, according to corresponded neutrosophic complete-bipartite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-bipartite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-bipartite implies

$$n_1, n_2, n_4$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a complete-bipartite and there are four

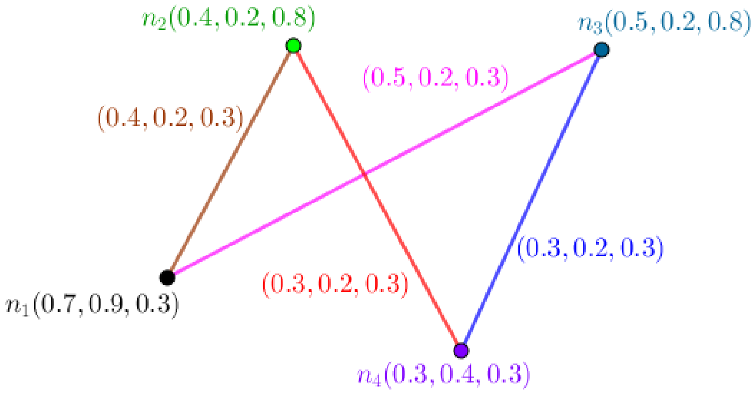

Figure 3.8: A Neutrosophic Graph in the Viewpoint of girth polynomial. 64NTG8

edges, n_1n_2, n_1n_3, n_2n_4 and n_3n_4, according to corresponded neutrosophic complete-bipartite and it has neutrosophic cycle where n_2n_4 and n_3n_4 are two weakest edge with same amount $(0.3, 0.2, 0.3)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle only has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-bipartite has both of a neutrosophic cycle and crisp cycle. So adding vertices has some effects to find a crisp cycle. The structure of this neutrosophic complete-bipartite implies

$$n_1, n_2, n_3, n_4$$

is corresponded to girth polynomial $\mathcal{G}(NTG)$ and uniqueness of this cycle implies the sequence

$$n_1, n_2, n_3, n_4$$

is corresponded to neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) $x^{4=\mathcal{O}(NTG)}$ is girth polynomial and its corresponded sequence is n_1, n_2, n_3, n_4;

(vi) $x^{5.8=\mathcal{O}_n(NTG)}$ is neutrosophic girth polynomial and its corresponded sequence is n_1, n_2, n_3, n_4.

Proposition 3.5.11. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-complete-t-partite-neutrosophic graph. Then*

$$\mathcal{G}(NTG) = c_1 x^{\mathcal{O}(NTG)} + c_2 x^{\mathcal{O}(NTG)-1} + \cdots + c_s x^3$$

where $t \geq 3$.

$$\mathcal{G}(NTG) = c_1 x^{2\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor} + c_2 x^{2\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor - 2} + \cdots + c_s x^4$$

where $t \leq 2$. And

$$\mathcal{G}(NTG) = 0$$

where $\mathcal{O}(NTG) \leq 2$.

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-complete-t-partite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. If $\mathcal{O}(NTG) \leq 2$, then it's neutrosophic path implying

$$\mathcal{G}(NTG) = 0.$$

If $t \geq 3, \mathcal{O}(NTG) \geq 3$, then it has crisp cycle implying

$$\mathcal{G}(NTG) = c_1 x^{\mathcal{O}(NTG)} + c_2 x^{\mathcal{O}(NTG)-1} + \cdots + c_s x^3.$$

If $t \geq 2, \mathcal{O}(NTG) \geq 4$, then it's possible to have two vertices in every part. In this case, four vertices form a crisp cycle which crisp cardinality of its vertices are four. It's impossible to have a crisp cycle which crisp cardinality of its vertices are three. Since the sequence of consecutive vertices are x_1, x_2, x_3 and there's no edge more. It implies there are two edges. It's neutrosophic path but neither crisp cycle nor neutrosophic cycle. So the first step of finding shortest crisp cycle is impossible but in second step, there's one crisp cycle corresponded to number four. By Theorem (3.4.5),

$$\mathcal{G}(NTG) = c_1 x^{\mathcal{O}(NTG)} + c_2 x^{\mathcal{O}(NTG)-1} + \cdots + c_s x^3$$

where $t \geq 3$.

$$\mathcal{G}(NTG) = c_1 x^{2\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor} + c_2 x^{2\lfloor \frac{\mathcal{O}(NTG)}{2} \rfloor - 2} + \cdots + c_s x^4$$

where $t \leq 2$. And

$$\mathcal{G}(NTG) = 0$$

where $\mathcal{O}(NTG) \leq 2$. ∎

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.5.12. There is one section for clarifications. In Figure (3.9), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a complete-t-partite and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this complete-t-partite implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The length of this complete-t-partite implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-t-partite and there are two edges, n_1n_2 and n_1n_3, according to corresponded neutrosophic complete-t-partite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_3$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-t-partite and there are two edges, n_1n_2 and n_2n_4, according to corresponded neutrosophic complete-t-partite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_4$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a complete-t-partite and there are four edges, n_1n_2, n_1n_5, n_2n_4 and n_5n_4, according to corresponded neutrosophic complete-t-partite and it has neutrosophic cycle where n_2n_4 and n_5n_4 are two weakest edge with same amount $(0.3, 0.2, 0.3)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle only has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has both of a neutrosophic cycle and crisp cycle. So adding vertices has some effects to find a crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_4, n_5$$

is corresponded to girth polynomial $\mathcal{G}(NTG)$ and minimum neutrosophic cardinality implies the sequence

$$n_1, n_2, n_4, n_5$$

is corresponded to neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) $36x^4$ is girth polynomial and some its corresponded sequences, for coefficient of smallest term, are

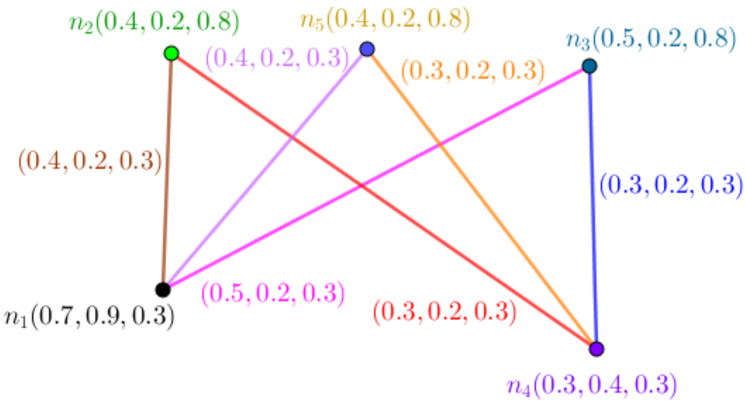

Figure 3.9: A Neutrosophic Graph in the Viewpoint of its girth polynomial. 64NTG9

$$n_1, n_2, n_4, n_3, n_1$$
$$n_1, n_2, n_4, n_5, n_1$$
$$n_1, n_5, n_4, n_3, n_1$$
$$n_1, n_5, n_4, n_2, n_1$$
$$n_1, n_3, n_4, n_5, n_1$$
$$n_1, n_3, n_4, n_2, n_1;$$

(vi) $x^{5.8} + 2x^{5.7}$ is neutrosophic girth polynomial and its corresponded sequences, for coefficient of smallest term, are

$$n_1, n_2, n_4, n_5, n_1$$
$$n_1, n_5, n_4, n_2, n_1$$

Proposition 3.5.13. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-wheel-neutrosophic graph. Then*

$$\mathcal{G}(NTG) = c_1 x^{\mathcal{O}(NTG)} + c_2 x^{\mathcal{O}(NTG)-1} + \cdots + c_s x^3$$

where $t \geq 3$.

$$\mathcal{G}(NTG) = 0$$

where $t \geq 2$.

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-wheel-neutrosophic graph. The argument is elementary. Since all vertices of a path join to one vertex. By Theorem (3.4.5),

$$\mathcal{G}(NTG) = c_1 x^{\mathcal{O}(NTG)} + c_2 x^{\mathcal{O}(NTG)-1} + \cdots + c_s x^3$$

where $t \geq 3$.

$$\mathcal{G}(NTG) = 0$$

where $t \geq 2$. ∎

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.5.14. There is one section for clarifications. In Figure (3.10), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If s_1, s_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a wheel and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this wheel implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic wheel has neither a neutrosophic cycle nor crisp cycle. The length of this wheel implies

$$s_1, s_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(ii) if s_4, s_2, s_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a wheel and there are two edges, s_3s_2 and s_4s_3, according to corresponded neutrosophic wheel but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic wheel has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic wheel implies

$$s_4, s_2, s_3$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iii) if s_1, s_2, s_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a wheel and there are three edges, s_1s_2, s_2s_3 and s_1s_3 according to corresponded neutrosophic wheel but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one crisp cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic wheel has no neutrosophic cycle but it has crisp cycle. The structure of this neutrosophic wheel implies

$$s_1, s_2, s_3$$

is corresponded to girth polynomial $\mathcal{G}(NTG)$ but minimum neutrosophic cardinality implies

$$s_1, s_2, s_3$$

isn't corresponded to neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

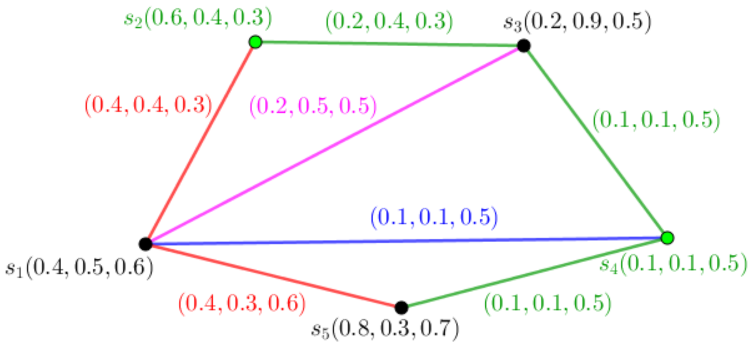

Figure 3.10: A Neutrosophic Graph in the Viewpoint of its girth polynomial. 64NTG10

(iv) if s_1, s_3, s_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a wheel and there are three edges, s_1s_4, s_4s_3 and s_1s_3 according to corresponded neutrosophic wheel and it has a neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one crisp cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic wheel has one neutrosophic cycle with two weakest edges s_1s_4 and s_3s_4 concerning same values $(0.1, 0.1, 0.5)$ and it has a crisp cycle. The structure of this neutrosophic wheel implies

$$s_1, s_3, s_4$$

is corresponded to girth polynomial $\mathcal{G}(NTG)$ and minimum neutrosophic cardinality implies

$$s_1, s_3, s_4$$

is corresponded to neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) $x^5 + 2x^4 + 3x^3$ is girth polynomial and its corresponded sequences, for coefficient of smallest term, are

$$s_1, s_3, s_4$$
$$s_1, s_2, s_3$$
$$s_1, s_4, s_5;$$

(vi) $x^{6.9} + x^{5.1} + x^{5.6} + x^{4.4} + x^4 + x^{3.8}$ is neutrosophic girth polynomial and its corresponded sequence is s_1, s_3, s_4.

3.6 Setting of Neutrosophic Girth Polynomial

In this section, I provide some results in the setting of neutrosophic girth polynomial. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, star-neutrosophic graph, bipartite-neutrosophic graph, and t-partite-neutrosophic graph, are both of cases of study and classes which the results are about them.

Proposition 3.6.1. *Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then*

$$\mathcal{G}_n(NTG) = x^{\mathcal{O}_n(NTG)} + \mathcal{O}(NTG)x^{\mathcal{O}(NTG)-\sum_{i=1}^{3}\sigma_i(x)} + \cdots + \binom{\mathcal{O}(NTG)}{3} x^{\min\{\Sigma_{i=1}^{3}(\sigma_i(x)+\sigma_i(y)+\sigma_i(z))\}}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. The length of longest cycle is $\mathcal{O}(NTG)$. In other hand, there's a cycle if and only if $\mathcal{O}(NTG) \geq 3$. It's complete. So there's at least one neutrosophic cycle which its length is $\mathcal{O}(NTG) = 3$. By shortest cycle is on demand, the girth polynomial is three. Thus

$$\mathcal{G}_n(NTG) = x^{\mathcal{O}_n(NTG)} + \mathcal{O}(NTG)x^{\mathcal{O}(NTG)-\sum_{i=1}^{3}\sigma_i(x)} + \cdots + \binom{\mathcal{O}(NTG)}{3} x^{\min\{\Sigma_{i=1}^{3}(\sigma_i(x)+\sigma_i(y)+\sigma_i(z))\}}.$$

■

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.6.2. In Figure (3.11), a complete neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1, n_2, n_3$$

is corresponded to girth polynomial $\mathcal{G}(NTG)$ but neutrosophic length of this crisp cycle implies

$$n_1, n_2, n_3$$

isn't corresponded to neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

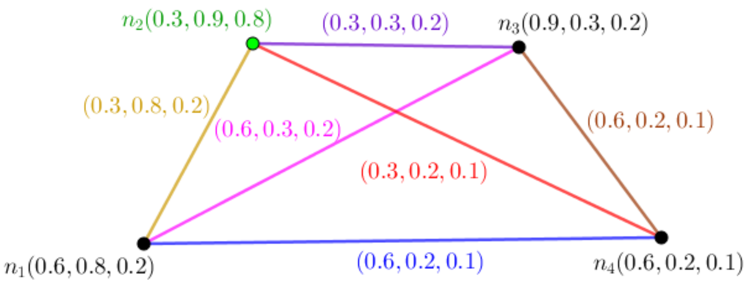

Figure 3.11: A Neutrosophic Graph in the Viewpoint of its girth polynomial. 64NTG11

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's two crisp cycles with length two and three. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1, n_3, n_4, and with length four, n_1, n_2, n_3, n_4. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a sequence of consecutive vertices is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with two different length three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. The length of this path implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to girth polynomial $\mathcal{G}(NTG)$ and neutrosophic length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) $x^4 + 3x^3$ is girth polynomial and its corresponded sets, for coefficient of smallest term, are $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_2, n_3, n_4\}$;

(vi) $x^{5.9} + x^5 + x^{4.5} + x^{4.3} + x^{3.9}$ is neutrosophic girth polynomial and its corresponded set, for coefficient of smallest term, is $\{n_1, n_3, n_4\}$.

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

Proposition 3.6.3. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-path-neutrosophic graph. Then*

$$\mathcal{G}_n(NTG) = 0.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-path-neutrosophic graph. There's no crisp cycle. If $NTG : (V, E, \sigma, \mu)$ isn't a crisp cycle, then $NTG : (V, E, \sigma, \mu)$ isn't a neutrosophic cycle. There's no cycle from every version. Let $x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ be a path-neutrosophic graph. Since $x_0, x_1, \cdots, x_{\mathcal{O}(NTG)}$ is a sequence of consecutive vertices, there's no repetition of vertices in this sequence. So there's no cycle. girth polynomial is corresponded to shortest cycle but there's no cycle. Thus it implies

$$\mathcal{G}_n(NTG) = 0.$$

■

Example 3.6.4. There are two sections for clarifications.

(a) In Figure (3.12), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

  (i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

  is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

  (ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

  is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

  (iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_3, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are four edges, n_1n_2, n_2n_3 and n_4n_5, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding vertices has no effect to find a crisp cycle. There is only one path amid two given vertices. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4, n_5$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) 0 is girth polynomial and there are no corresponded sets;

(vi) 0 is neutrosophic girth polynomial and there are no corresponded sets.

(b) In Figure (3.13), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_3, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are four edges, n_1n_2, n_2n_3 and n_4n_5, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding vertices has no effect to find a crisp cycle. There is only one path amid two given vertices. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4, n_5$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) 0 is girth polynomial and there are no corresponded sets;

(vi) 0 is neutrosophic girth polynomial and there are no corresponded sets.

Proposition 3.6.5. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-cycle-neutrosophic graph where $\mathcal{O}(NTG) \geq 3$. Then*

$$\mathcal{G}_n(NTG) = x^{\mathcal{O}_n(NTG)}.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-cycle-neutrosophic graph. Let $x_1, x_2, \cdots, x_{\mathcal{O}(NTG)}, x_1$ be a sequence of consecutive vertices of $NTG : (V, E, \sigma, \mu)$ such that

$$x_i x_{i+1} \in E,\ i = 1, 2, \cdots, \mathcal{O}(NTG) - 1,\ x_{\mathcal{O}(NTG)} x_1 \in E.$$

There are two paths amid two given vertices. The degree of every vertex is two. But there's one crisp cycle for every given vertex. So the efforts leads to one cycle for finding a shortest crisp cycle. For a given vertex x_i, the sequence of consecutive vertices

$$x_i, x_{i+1}, \cdots, x_{i-2}, x_{i-1}, x_i$$

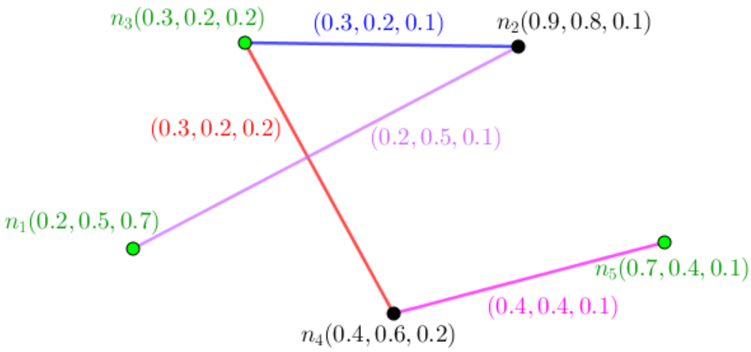

Figure 3.12: A Neutrosophic Graph in the Viewpoint of its girth polynomial. 64NTG12

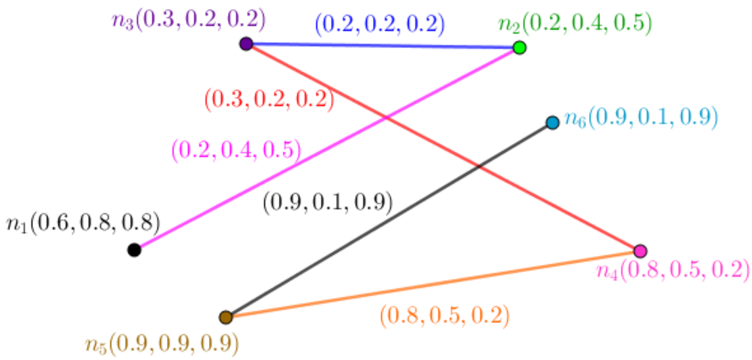

Figure 3.13: A Neutrosophic Graph in the Viewpoint of its girth polynomial. 64NTG13

is a corresponded crisp cycle for x_i. Every cycle has same length. The length is $\mathcal{O}(NTG)$. Thus the crisp cardinality of set of vertices forming shortest crisp cycle is $\mathcal{O}(NTG)$. By Theorem (3.4.5),

$$\mathcal{G}_n(NTG) = x^{\mathcal{O}_n(NTG)}.$$

■

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.6.6. There are two sections for clarifications.

(a) In Figure (3.14), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are six edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5, n_5n_6$ and n_6n_1, according to corresponded neutrosophic path and it's neutrosophic cycle since it has two weakest edges, n_4n_5 and n_5n_6 with same values $(0.1, 0.1, 0.2)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is both of a neutrosophic cycle and crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_6, n_1$ is corresponded to both of girth polynomial $\mathcal{G}(NTG)$ and neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) $x^{6=\mathcal{O}(NTG)}$ is girth polynomial and its corresponded set, for coefficient of smallest term, is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$;

(vi) $x^{8.1=\mathcal{O}_n(NTG)}$ is neutrosophic girth polynomial and its corresponded set, for coefficient of smallest term, is only $\{n_1, n_2, n_3, n_4, n_5, n_6, n_1\}$.

(b) In Figure (3.15), an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are two edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is either a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a path and there are three edges, n_1n_2 and n_2n_3, according to corresponded neutrosophic path but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. So adding points has no effect to find a crisp cycle. The structure of this neutrosophic path implies

$$n_1, n_2, n_3, n_4$$

is corresponded neither to girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iv) if $n_1, n_2, n_3, n_4, n_5, n_1$ is a sequence of consecutive vertices, then it's obvious that there's one cycle. It's also a path and there are five edges, $n_1n_2, n_2n_3, n_3n_4, n_4n_5$ and n_5n_1, according to corresponded

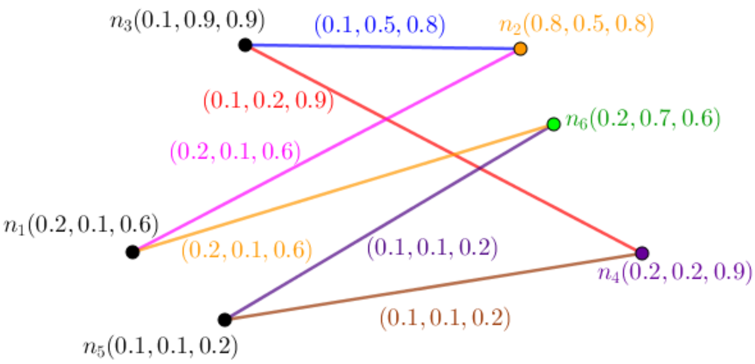

Figure 3.14: A Neutrosophic Graph in the Viewpoint of its girth polynomial. 64NTG14

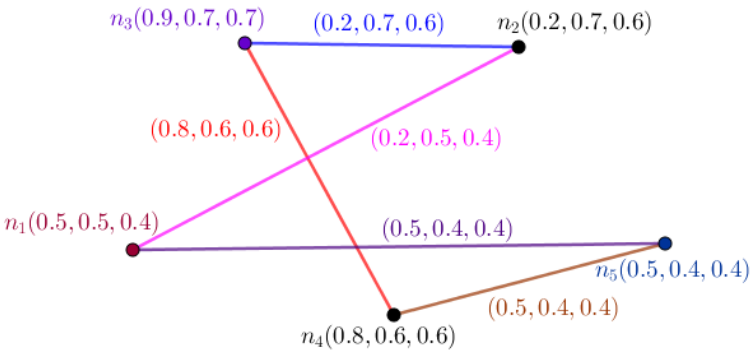

Figure 3.15: A Neutrosophic Graph in the Viewpoint of its girth polynomial. 64NTG15

neutrosophic path and it isn't neutrosophic cycle since it has only one weakest edge, n_1n_2, with value $(0.2, 0.5, 0.4)$ and not more. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic path is not a neutrosophic cycle but it is a crisp cycle. So adding vertices has effect on finding a crisp cycle. There are only two paths amid two given vertices. The structure of this neutrosophic path implies $n_1, n_2, n_3, n_4, n_5, n_1$ is corresponded to both of girth polynomial $\mathcal{G}(NTG)$ and neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) $x^{5=\mathcal{O}(NTG)}$ is girth polynomial and its corresponded set, for coefficient of smallest term, is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$;

(vi) $x^{8.5=\mathcal{O}_n(NTG)}$ is neutrosophic girth polynomial and its corresponded set, for coefficient of smallest term, is only $\{n_1, n_2, n_3, n_4, n_5, n_1\}$.

Proposition 3.6.7. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-star-neutrosophic graph with center c. Then*

$$\mathcal{G}_n(NTG) = 0.$$

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-star-neutrosophic graph. Every vertex isn't a neighbor for every given vertex. Every vertex is a neighbor for center. Furthermore, center is only neighbor for any given vertex. So center is only neighbor for all vertices. It's possible to have some paths amid two given vertices but there's no crisp cycle. In other words, if $\mathcal{O}(NTG) > 2$, then there are at least three vertices x, y and z such that if x is a neighbor for y and z, then y and z aren't neighbors and x is center. To get more precise, if if x and y are neighbors then either x or y is center. Every edge have one common endpoint with other edges which is called center. Thus there is no triangle but there are some edges. One edge has two endpoints which one of them is center. There are no crisp cycle. Hence trying to find shortest cycle has no result. There is no crisp cycle. Then there is shortest crisp cycle. So

$$\mathcal{G}_n(NTG) = 0.$$

■

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.6.8. There is one section for clarifications. In Figure (3.16), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a star and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this star implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. The length of this star implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a star and there are two edges, n_1n_2 and n_1n_3, according to corresponded neutrosophic star but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic star implies

$$n_1, n_2, n_3$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

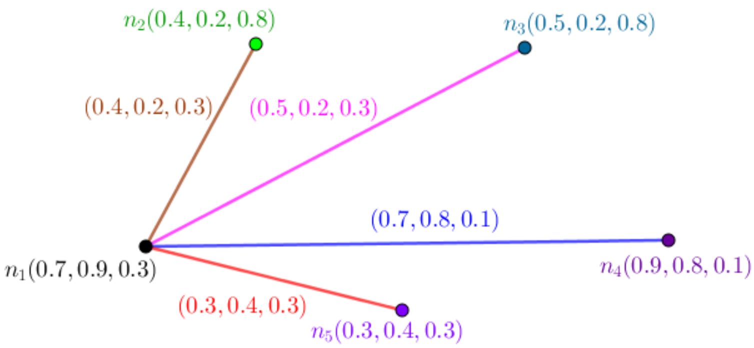

Figure 3.16: A Neutrosophic Graph in the Viewpoint of its girth polynomial. 64NTG16

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a star and there are three edges, n_1n_2, n_1n_3 and n_1n_4, according to corresponded neutrosophic star but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. So adding vertices has no effect to find a crisp cycle. The structure of this neutrosophic star implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_3, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a star and there are four edges, n_1n_2, n_1n_3, n_1n_4 and n_1n_5, according to corresponded neutrosophic star but it isn't neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic star has neither a neutrosophic cycle nor crisp cycle. So adding vertices has no effect to find a crisp cycle. There are some paths amid two given vertices. The structure of this neutrosophic star implies

$$n_1, n_2, n_3, n_4, n_5$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) 0 is girth polynomial and there is no corresponded set;

(vi) 0 is neutrosophic girth polynomial and there is no corresponded set.

Proposition 3.6.9. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-complete-bipartite-neutrosophic graph. Then*

$$\mathcal{G}_n(NTG) = c_1 x^{\mathcal{O}_n(NTG)} + c_2 x^{\mathcal{O}_n(NTG) - (\Sigma_{i=1}^{3}(\sigma_i(x)+\sigma_i(y)))} + \cdots + c_s x^{\min\{\Sigma_{i=1}^{3}(\sigma_i(x)+\sigma_i(y)+\sigma_i(z)+\sigma_i(w))\}_{x,y \in V_1,\ z,w \in V_2}}$$

where $\mathcal{O}(NTG) \geq 4$ *and* $\min\{|V_1|, |V_2|\} \geq 2$. *Also,*

$$\mathcal{G}_n(NTG) = 0$$

where $\mathcal{O}(NTG) \leq 3$.

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-complete-bipartite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. If $\mathcal{O}(NTG) \leq 3$, then it's neutrosophic path implying

$$\mathcal{G}_n(NTG) = 0.$$

If $\mathcal{O}(NTG) \geq 4$, then it's possible to have two vertices in every part. In this case, four vertices form a crisp cycle which crisp cardinality of its vertices are four. It's impossible to have a crisp cycle which crisp cardinality of its vertices are three. Since the sequence of consecutive vertices are x_1, x_2, x_3 and there's no edge more. It implies there are two edges. It's neutrosophic path but neither crisp cycle nor neutrosophic cycle. So the first step of finding shortest crisp cycle is impossible but in second step, there's one crisp cycle corresponded to number four. By Theorem (3.4.5),

$$\mathcal{G}_n(NTG) = c_1 x^{\mathcal{O}_n(NTG)} + c_2 x^{\mathcal{O}_n(NTG) - (\Sigma_{i=1}^{3}(\sigma_i(x)+\sigma_i(y)))} + \cdots + c_s x^{\min\{\Sigma_{i=1}^{3}(\sigma_i(x)+\sigma_i(y)+\sigma_i(z)+\sigma_i(w))\}_{x,y \in V_1,\ z,w \in V_2}}$$

where $\mathcal{O}(NTG) \geq 4$ and $\min\{|V_1|, |V_2|\} \geq 2$. Also,

$$\mathcal{G}_n(NTG) = 0$$

where $\mathcal{O}(NTG) \leq 3$. ∎

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.6.10. There is one section for clarifications. In Figure (3.17), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a complete-bipartite and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this complete-bipartite implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic complete-bipartite has neither a neutrosophic cycle nor crisp cycle. The length of this complete-bipartite implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-bipartite and there are two edges, n_1n_2 and n_1n_3, according to corresponded neutrosophic complete-bipartite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-bipartite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-bipartite implies

$$n_1, n_2, n_3$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-bipartite and there are two edges, n_1n_2 and n_2n_4, according to corresponded neutrosophic complete-bipartite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-bipartite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-bipartite implies

$$n_1, n_2, n_4$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a complete-bipartite and there are four edges, n_1n_2, n_1n_3, n_2n_4 and n_3n_4, according to corresponded neutrosophic complete-bipartite and it has neutrosophic cycle where n_2n_4 and n_3n_4 are two weakest edge with same amount $(0.3, 0.2, 0.3)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle only has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-bipartite has both of a neutrosophic cycle and crisp cycle. So adding vertices has some effects to find a crisp cycle. The structure of this neutrosophic complete-bipartite implies

$$n_1, n_2, n_3, n_4$$

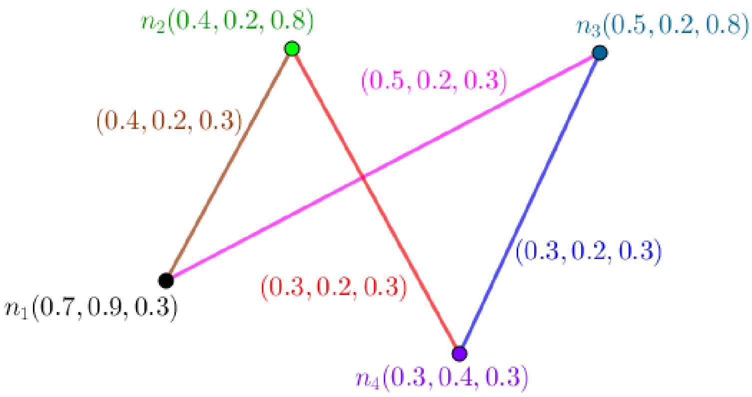

Figure 3.17: A Neutrosophic Graph in the Viewpoint of girth polynomial. 64NTG17

is corresponded to girth polynomial $\mathcal{G}(NTG)$ and uniqueness of this cycle implies the sequence

$$n_1, n_2, n_3, n_4$$

is corresponded to neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) $x^{4=\mathcal{O}(NTG)}$ is girth polynomial and its corresponded sequence is n_1, n_2, n_3, n_4;

(vi) $x^{5.8=\mathcal{O}_n(NTG)}$ is neutrosophic girth polynomial and its corresponded sequence is n_1, n_2, n_3, n_4.

Proposition 3.6.11. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-complete-t-partite-neutrosophic graph. Then*

$$\mathcal{G}_n(NTG) = c_1 x^{\mathcal{O}_n(NTG)} + c_2 x^{\mathcal{O}_n(NTG) - (\Sigma_{i=1}^3 (\sigma_i(x_1) + \sigma_i(x_2) + \cdots + + \sigma_i(x_t)))} + \cdots + c_s x^{\min\{\Sigma_{i=1}^3 (\sigma_i(x) + \sigma_i(y) + \sigma_i(z)\}_{x \in V_1,\ y \in V_2,\ z \in V_3}}$$

where $t \geq 3$.

$$\mathcal{G}_n(NTG) = c_1 x^{\mathcal{O}_n(NTG)} + c_2 x^{\mathcal{O}_n(NTG) - (\Sigma_{i=1}^3 (\sigma_i(x) + \sigma_i(y)))} + \cdots + c_s x^{\min\{\Sigma_{i=1}^3 (\sigma_i(x) + \sigma_i(y) + \sigma_i(z) + \sigma_i(w))\}_{x, y \in V_1,\ z, w \in V_2}}$$

where $t \leq 2$. And

$$\mathcal{G}_n(NTG) = 0$$

where $\mathcal{O}(NTG) \leq 2$.

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-complete-t-partite-neutrosophic graph. Every vertex is a neighbor for all vertices in another part. If $\mathcal{O}(NTG) \leq 2$, then it's neutrosophic path implying

$$\mathcal{G}_n(NTG) = 0.$$

If $t \geq 3, \mathcal{O}(NTG) \geq 3$, then it has crisp cycle implying

$$\mathcal{G}_n(NTG) = c_1 x^{\mathcal{O}_n(NTG)} + c_2 x^{\mathcal{O}_n(NTG) - (\Sigma_{i=1}^{3}(\sigma_i(x_1)+\sigma_i(x_2)+\cdots++\sigma_i(x_t)))} + \cdots + c_s x^{\min\{\Sigma_{i=1}^{3}(\sigma_i(x)+\sigma_i(y)+\sigma_i(z)\}_{x\in V_1,\ y\in V_2,\ z\in V_3}}$$

If $t \geq 2, \mathcal{O}(NTG) \geq 4$, then it's possible to have two vertices in every part. In this case, four vertices form a crisp cycle which crisp cardinality of its vertices are four. It's impossible to have a crisp cycle which crisp cardinality of its vertices are three. Since the sequence of consecutive vertices are x_1, x_2, x_3 and there's no edge more. It implies there are two edges. It's neutrosophic path but neither crisp cycle nor neutrosophic cycle. So the first step of finding shortest crisp cycle is impossible but in second step, there's one crisp cycle corresponded to number four. By Theorem (3.4.5),

$$\mathcal{G}_n(NTG) = c_1 x^{\mathcal{O}_n(NTG)} + c_2 x^{\mathcal{O}_n(NTG) - (\Sigma_{i=1}^{3}(\sigma_i(x_1)+\sigma_i(x_2)+\cdots++\sigma_i(x_t)))} + \cdots + c_s x^{\min\{\Sigma_{i=1}^{3}(\sigma_i(x)+\sigma_i(y)+\sigma_i(z)\}_{x\in V_1,\ y\in V_2,\ z\in V_3}}$$

where $t \geq 3$.

$$\mathcal{G}_n(NTG) = c_1 x^{\mathcal{O}_n(NTG)} + c_2 x^{\mathcal{O}_n(NTG) - (\Sigma_{i=1}^{3}(\sigma_i(x)+\sigma_i(y)))} + \cdots + c_s x^{\min\{\Sigma_{i=1}^{3}(\sigma_i(x)+\sigma_i(y)+\sigma_i(z)+\sigma_i(w))\}_{x,y\in V_1,\ z,w\in V_2}}$$

where $t \leq 2$. And

$$\mathcal{G}_n(NTG) = 0$$

where $\mathcal{O}(NTG) \leq 2$. ■

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.6.12. There is one section for clarifications. In Figure (3.18), a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a complete-t-partite and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this complete-t-partite implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The length of this complete-t-partite implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-t-partite and there are two edges, n_1n_2 and n_1n_3, according to corresponded neutrosophic complete-t-partite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_3$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_4 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a complete-t-partite and there are two edges, n_1n_2 and n_2n_4, according to corresponded neutrosophic complete-t-partite but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_4$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_2, n_4, n_5 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a complete-t-partite and there are four edges, n_1n_2, n_1n_5, n_2n_4 and n_5n_4, according to corresponded neutrosophic complete-t-partite and it has neutrosophic cycle where n_2n_4 and n_5n_4 are two weakest edge with same amount $(0.3, 0.2, 0.3)$. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle only has one result. Since there's one cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic complete-t-partite has both of a neutrosophic cycle and crisp cycle. So adding vertices has some effects to find a crisp cycle. The structure of this neutrosophic complete-t-partite implies

$$n_1, n_2, n_4, n_5$$

is corresponded to girth polynomial $\mathcal{G}(NTG)$ and minimum neutrosophic cardinality implies the sequence

$$n_1, n_2, n_4, n_5$$

is corresponded to neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) $36x^4$ is girth polynomial and some its corresponded sequences, for coefficient of smallest term, are

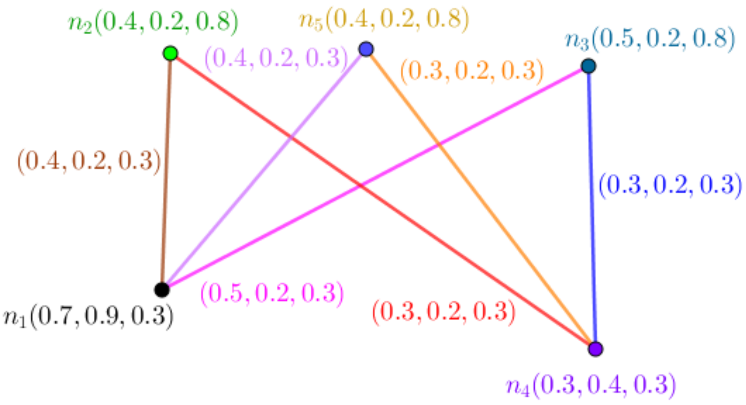

Figure 3.18: A Neutrosophic Graph in the Viewpoint of its girth polynomial. 64NTG18

$$n_1, n_2, n_4, n_3, n_1$$
$$n_1, n_2, n_4, n_5, n_1$$
$$n_1, n_5, n_4, n_3, n_1$$
$$n_1, n_5, n_4, n_2, n_1$$
$$n_1, n_3, n_4, n_5, n_1$$
$$n_1, n_3, n_4, n_2, n_1;$$

(vi) $x^{5.8} + 2x^{5.7}$ is neutrosophic girth polynomial and its corresponded sequences, for coefficient of smallest term, are

$$n_1, n_2, n_4, n_5, n_1$$
$$n_1, n_5, n_4, n_2, n_1$$

Proposition 3.6.13. *Let $NTG : (V, E, \sigma, \mu)$ be a strong-wheel-neutrosophic graph. Then*

$$\mathcal{G}_n(NTG) = x^{\mathcal{O}_n(NTG)} + c_2 x^{\mathcal{O}_n(NTG) - \sum_{i=1}^{3} \sigma_i(x)} + \cdots + c_{\mathcal{O}(NTG)-4} x^{\min\{\Sigma_{i=1}^{3}(\sigma_i(x)+\sigma_i(y)+\sigma_i(z))\}}.$$

where $t \geq 3$.

$$\mathcal{G}_n(NTG) = 0$$

where $t \geq 2$.

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a strong-wheel-neutrosophic graph. The argument is elementary. Since all vertices of a path join to one vertex. By Theorem (3.4.5),

$$\mathcal{G}_n(NTG) = x^{\mathcal{O}_n(NTG)} + c_2 x^{\mathcal{O}_n(NTG) - \sum_{i=1}^{3} \sigma_i(x)} +$$

$$\cdots + c_{\mathcal{O}(NTG)-4} x^{\min\{\Sigma_{i=1}^{3}(\sigma_i(x)+\sigma_i(y)+\sigma_i(z))\}}.$$

where $t \geq 3$.

$$\mathcal{G}_n(NTG) = 0$$

where $t \geq 2$. ■

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it's studied to apply the definitions on it, too.

Example 3.6.14. There is one section for clarifications. In Figure (3.19), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If s_1, s_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a wheel and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this wheel implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic wheel has neither a neutrosophic cycle nor crisp cycle. The length of this wheel implies

$$s_1, s_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(ii) if s_4, s_2, s_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a wheel and there are two edges, s_3s_2 and s_4s_3, according to corresponded neutrosophic wheel but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic wheel has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic wheel implies

$$s_4, s_2, s_3$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iii) if s_1, s_2, s_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a wheel and there are three edges, s_1s_2, s_2s_3 and s_1s_3 according to corresponded neutrosophic wheel but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one crisp cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic wheel has no neutrosophic cycle but it has crisp cycle. The structure of this neutrosophic wheel implies

$$s_1, s_2, s_3$$

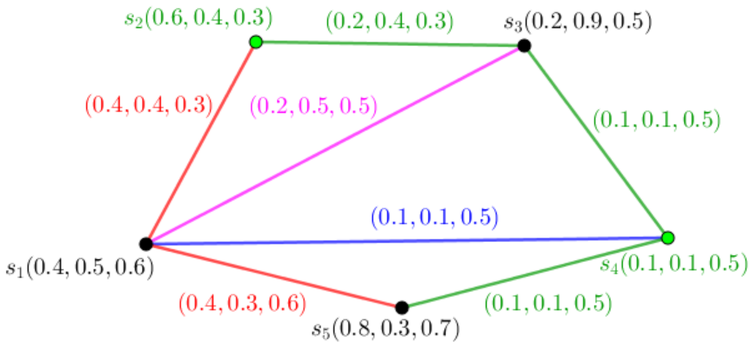

Figure 3.19: A Neutrosophic Graph in the Viewpoint of its girth polynomial. 64NTG19

is corresponded to girth polynomial $\mathcal{G}(NTG)$ but minimum neutrosophic cardinality implies

$$s_1, s_2, s_3$$

isn't corresponded to neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iv) if s_1, s_3, s_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a wheel and there are three edges, s_1s_4, s_4s_3 and s_1s_3 according to corresponded neutrosophic wheel and it has a neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one crisp cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic wheel has one neutrosophic cycle with two weakest edges s_1s_4 and s_3s_4 concerning same values $(0.1, 0.1, 0.5)$ and it has a crisp cycle. The structure of this neutrosophic wheel implies

$$s_1, s_3, s_4$$

is corresponded to girth polynomial $\mathcal{G}(NTG)$ and minimum neutrosophic cardinality implies

$$s_1, s_3, s_4$$

is corresponded to neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) $x^5 + 2x^4 + 3x^3$ is girth polynomial and its corresponded sequences, for coefficient of smallest term, are

$$s_1, s_3, s_4$$
$$s_1, s_2, s_3$$
$$s_1, s_4, s_5;$$

(vi) $x^{6.9} + x^{5.1} + x^{5.6} + x^{4.4} + x^4 + x^{3.8}$ is neutrosophic girth polynomial and its corresponded sequence is s_1, s_3, s_4.

3.7 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are either complete models which mean complete connections are formed as individual and family of complete models with common neutrosophic vertex set or quasi-complete models which mean quasi-complete connections are formed as individual and family of quasi-complete models with common neutrosophic vertex set.
Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importance to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive sections. Beyond that, sometimes sections are not the same.

Step 3. (Model) The situation is designed as a model. The model uses data to assign every section and to assign to relation amid sections, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relations amid them. Table (3.1), clarifies about the assigned numbers to these situations.

Table 3.1: Scheduling concerns its Subjects and its Connections as a neutrosophic graph in a Model. 64tbl1

Sections of NTG	n_1	$n_2 \cdots$	n_5
Values	$(0.7, 0.9, 0.3)$	$(0.4, 0.2, 0.8) \cdots$	$(0.4, 0.2, 0.8)$
Connections of NTG	E_1	$E_2 \cdots$	E_6
Values	$(0.4, 0.2, 0.3)$	$(0.5, 0.2, 0.3) \cdots$	$(0.3, 0.2, 0.3)$

3.8 Case 1: Complete-t-partite Model alongside its girth polynomial and its Neutrosophic girth polynomial

Step 4. (Solution) The neutrosophic graph alongside its girth polynomial and its neutrosophic girth polynomial as model, propose to use specific number. Every subject has connection with some subjects. Thus the connection is applied as possible and the model demonstrates quasi-full connections as quasi-possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is star, the number is different. Also, it holds for other types such that complete, wheel, path, and cycle. The collection of situations is another application of its girth polynomial and its neutrosophic girth polynomial when the notion of family is applied in

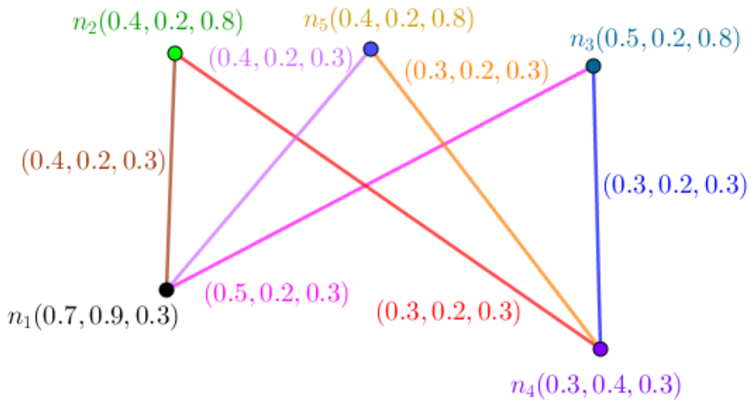

Figure 3.20: A Neutrosophic Graph in the Viewpoint of its girth polynomial and its Neutrosophic girth polynomial 64NTG20

the way that all members of family are from same classes of neutrosophic graphs. As follows, There are five subjects which are represented as Figure (3.20). This model is strong and even more it's quasi-complete. And the study proposes using specific number which is called its girth polynomial and its neutrosophic girth polynomial. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (3.20). In Figure (3.20), an complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) If s_1, s_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a wheel and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this wheel implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic wheel has neither a neutrosophic cycle nor crisp cycle. The length of this wheel implies

$$s_1, s_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(ii) if s_4, s_2, s_3 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's also a wheel and there are two edges, s_3s_2 and s_4s_3, according to corresponded neutrosophic wheel but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has no result. Since there's no cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic wheel has neither a neutrosophic cycle nor crisp cycle. The structure of this neutrosophic wheel implies

$$s_4, s_2, s_3$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iii) if s_1, s_2, s_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a wheel and there are three edges, s_1s_2, s_2s_3 and s_1s_3 according to corresponded neutrosophic wheel but it doesn't have neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one crisp cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic wheel has no neutrosophic cycle but it has crisp cycle. The structure of this neutrosophic wheel implies

$$s_1, s_2, s_3$$

is corresponded to girth polynomial $\mathcal{G}(NTG)$ but minimum neutrosophic cardinality implies

$$s_1, s_2, s_3$$

isn't corresponded to neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iv) if s_1, s_3, s_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a wheel and there are three edges, s_1s_4, s_4s_3 and s_1s_3 according to corresponded neutrosophic wheel and it has a neutrosophic cycle. First step is to have at least one crisp cycle for finding shortest cycle. Finding shortest cycle has one result. Since there's one crisp cycle. Neutrosophic cycle is a crisp cycle with at least two weakest edges. So this neutrosophic wheel has one neutrosophic cycle with two weakest edges s_1s_4 and s_3s_4 concerning same values $(0.1, 0.1, 0.5)$ and it has a crisp cycle. The structure of this neutrosophic wheel implies

$$s_1, s_3, s_4$$

is corresponded to girth polynomial $\mathcal{G}(NTG)$ and minimum neutrosophic cardinality implies

$$s_1, s_3, s_4$$

is corresponded to neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) $x^5 + 2x^4 + 3x^3$ is girth polynomial and its corresponded sequences, for coefficient of smallest term, are

$$s_1, s_3, s_4$$
$$s_1, s_2, s_3$$
$$s_1, s_4, s_5;$$

(vi) $x^{6.9} + x^{5.1} + x^{5.6} + x^{4.4} + x^4 + x^{3.8}$ is neutrosophic girth polynomial and its corresponded sequence is s_1, s_3, s_4.

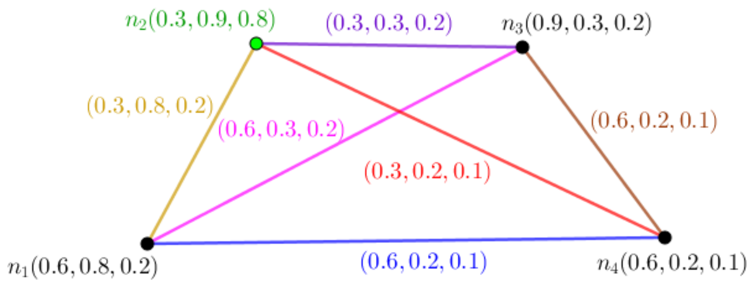

Figure 3.21: A Neutrosophic Graph in the Viewpoint of its girth polynomial and its Neutrosophic girth polynomial 64NTG21

3.9 Case 2: Complete Model alongside its A Neutrosophic Graph in the Viewpoint of its girth polynomial and its Neutrosophic girth polynomial

Step 4. (Solution) The neutrosophic graph alongside its girth polynomial and its neutrosophic girth polynomial as model, propose to use specific number. Every subject has connection with every given subject in deemed way. Thus the connection applied as possible and the model demonstrates full connections as possible between parts but with different view where symmetry amid vertices and edges are the matters. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete multipartite, the number is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of its girth polynomial and its neutrosophic girth polynomial when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are four subjects which are represented in the formation of one model as Figure (3.21). This model is neutrosophic strong as individual and even more it's complete. And the study proposes using specific number which is called its girth polynomial and its neutrosophic girth polynomial for this model. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to these models as individual. A model as a collection of situations to compare them with another model as a collection of situations to get more precise. Consider Figure (3.21). There is one section for clarifications.

(i) If n_1, n_2 is a sequence of consecutive vertices, then it's obvious that there's no crisp cycle. It's only a path and it's only one edge but it is neither crisp cycle nor neutrosophic cycle. The length of this path implies there's no cycle since if the length of a sequence of consecutive vertices is at most 2, then it's impossible to have cycle. So this neutrosophic path is neither a neutrosophic cycle nor crisp cycle. The length of this path implies

$$n_1, n_2$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(ii) if n_1, n_2, n_3 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it isn't neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there aren't two weakest edges which imply there is no neutrosophic cycle. So this crisp cycle isn't a neutrosophic cycle but it's crisp cycle. The crisp length of this crisp cycle implies

$$n_1, n_2, n_3$$

is corresponded to girth polynomial $\mathcal{G}(NTG)$ but neutrosophic length of this crisp cycle implies

$$n_1, n_2, n_3$$

isn't corresponded to neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iii) if n_1, n_2, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's two crisp cycles with length two and three. It's also a path and there are three edges but there are some crisp cycles but there are only two neutrosophic cycles with length three, n_1, n_3, n_4, and with length four, n_1, n_2, n_3, n_4. The length of this sequence implies there are some crisp cycles and there are two neutrosophic cycles since if the length of a sequence of consecutive vertices is at most 4 and it's crisp complete, then it's possible to have some crisp cycles and two neutrosophic cycles with two different length three and four. So this neutrosophic path forms some neutrosophic cycles and some crisp cycles. The length of this path implies

$$n_1, n_2, n_3, n_4$$

is corresponded to neither girth polynomial $\mathcal{G}(NTG)$ nor neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(iv) if n_1, n_3, n_4 is a sequence of consecutive vertices, then it's obvious that there's one crisp cycle. It's also a path and there are three edges but it is also neutrosophic cycle. The length of crisp cycle implies there's one cycle since if the length of a sequence of consecutive vertices is at most 3, then it's possible to have cycle but there are two weakest edges, n_3n_4 and n_1n_4, which imply there is one neutrosophic cycle. So this crisp cycle is a neutrosophic cycle and it's crisp cycle. The crisp length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to girth polynomial $\mathcal{G}(NTG)$ and neutrosophic length of this neutrosophic cycle implies

$$n_1, n_3, n_4$$

is corresponded to neutrosophic girth polynomial $\mathcal{G}_n(NTG)$;

(v) $x^4 + 3x^3$ is girth polynomial and its corresponded sets, for coefficient of smallest term, are $\{n_1, n_2, n_3\}$, $\{n_1, n_2, n_4\}$, and $\{n_2, n_3, n_4\}$;

(vi) $x^{5.9} + x^5 + x^{4.5} + x^{4.3} + x^{3.9}$ is neutrosophic girth polynomial and its corresponded set, for coefficient of smallest term, is $\{n_1, n_3, n_4\}$.

3.10 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study. Notion concerning its girth polynomial and its neutrosophic girth polynomial are defined in neutrosophic graphs. Neutrosophic number is also reused. Thus,

Question 3.10.1. *Is it possible to use other types of its girth polynomial and its neutrosophic girth polynomial?*

Question 3.10.2. *Are existed some connections amid different types of its girth polynomial and its neutrosophic girth polynomial in neutrosophic graphs?*

Question 3.10.3. *Is it possible to construct some classes of neutrosophic graphs which have "nice" behavior?*

Question 3.10.4. *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

Problem 3.10.5. *Which parameters are related to this parameter?*

Problem 3.10.6. *Which approaches do work to construct applications to create independent study?*

Problem 3.10.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

3.11 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted.
This study uses two definitions concerning girth polynomial and neutrosophic girth polynomial arising from counting cycles to study strong neutrosophic graphs based on neutrosophic cycles and neutrosophic graphs based on crisp cycles. New neutrosophic number is reused which is too close to the notion of neutrosophic number but it's different since it uses all values as type-summation on them. Comparisons amid number, corresponded vertices and edges are done by using neutrosophic tool. The connections of vertices which aren't clarified by a neutrosophic cycle differ them from each other and put them in different categories to represent a number which is called girth polynomial and neutrosophic girth polynomial arising from counting neutrosophic cycles and crisp cycles in strong neutrosophic graphs based on neutrosophic cycles and in neutrosophic graphs based on crisp cycles. Further studies could be about changes in the settings to compare these notions amid different settings

Table 3.2: A Brief Overview about Advantages and Limitations of this Study 64tbl2

Advantages	Limitations
1. Neutrosophic girth polynomial	1. Connections amid Classes
2. girth polynomial	
3. Neutrosophic Number	2. Study on Families
4. Classes of (Strong) Neutrosophic Graphs	
5. Counting Crisp Cycles and Neutrosophic Cycles	3. Same Models in Family

of strong neutrosophic graphs theory. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (3.2), some limitations and advantages of this study are pointed out.

Bibliography

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Ref3 [3] Henry Garrett, "*Some Polynomials Related to Numbers in Classes of (Strong) Neutrosophic Graphs*", ResearchGate 2022 (doi: 10.13140/RG.2.2.36280.83204).