

**AN EVALUATION OF VORTEX-OSCILLATIONS OF THE  
JOSEPHSON VORTEX LATTICE IN HIGH-T<sub>c</sub>-  
SUPERCONDUCTORS**

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**ABSTRACT**

We have studied vortex lattice oscillations of the Josephson vortex lattice at small field and frequency for high T<sub>c</sub>- superconductors. We have obtained the simple relation for the dynamic dielectric constant which the perturbation of the superconducting, phase induces by the oscillating electric field. We have computed the loss-function as a function of frequency for c-axis and in plane dissipation parameters. These parameters are inversely proportional to the anisotropy. This case of weak and strong dissipation are realized in Bi<sub>2</sub>Sr<sub>2</sub>CaCu<sub>2</sub>O<sub>x</sub> and underdoped YBa<sub>2</sub>Cu<sub>3</sub>O<sub>x</sub> respectively. We have also explored the evaluation of the loss- function with the increasing magnetic field. We observed in additional peak in the loss function below the JPR peak which was observed experimentally in underdoped YBCO. We established that this peak appears due to the frequency dependents of the in plane contribution to losses.

**KEY WORDS** : Vortex oscillations module, Josephson plasma resonance (JPR), Josephson vortices, vortex oscillation theory, vortex parameters, linear vortex mass, viscosity, coefficient, DC flux-below conductivity.

**INTRODUCTION** :

Collective oscillation in superconductors have been discussed in many plane in the literature phase oscillation with an acoustic spectrum has been founded by Bogolyubov<sup>1</sup> and Anderson<sup>2</sup> with neutral electrons.

In conventional superconductors coulomb interaction transfers. Such words into plasma oscillations with a frequency higher than the energy gap.

These oscillations differ slightly from plasma oscillations in normal metals weakly damped collective oscillations in real superconductor (the Carlson – Goldman modes) have been found experimentally<sup>3</sup> and explained theoretically<sup>4,5</sup>.

The modes exist only near the transition temperature  $T_c$ . It has a line spectrum and is associated with state, oscillators in the phase of the mode parameters in an electric field.

In cuprate superconductor, the superconductivity only the c-axis is maintained by the Josephson Current only between the CuO<sub>2</sub> layers the Josephson current flowing along the C-axis is coupled with the electromagnetic field, generating the Josephson plasma whose frequency appears in the range of 10 GHz and 1 THz. The frequency dispersion for the transverse and longitudinal plasma modes were calculated the Josephson plasma strongly interacts with vortices and the values of the plasma frequency depends on the vortex state.

The interaction is so strong that the plasma can be extended by the vortex flow generated by an external electric current.<sup>6-8</sup> In this paper using the theoretical formalism of A.E. Koshelev<sup>9</sup> and A.E. Kospelev and M.J.W. Dodgson,<sup>10</sup> we have theoretically evaluated vortex oscillations at small field and frequency. We have studied in role of in-plane dissipation and c-axis dissipation by evaluating loss-function as a function of frequency  $\omega$  for different values of parameters  $h$ ,  $N^2 \lambda_c$  and  $\nu_{ab}$

We have also compared the result of loss-function as a function of frequency with numerical solution and vortex oscillation mode for different values of  $\lambda$ ,  $\nu_{ab}$ ,  $h$  and  $N^2$  we observed that value of the vortex oscillation mode describes the high frequency response up to half of the plasma frequency. We have also evaluated frequency dependent loss function near JPR frequency of different field for  $\lambda_c = 0.01$  and  $\nu_{ab} = 0.1$  we have also evaluated the loss-function for the same  $h = 0.2$  and  $0.4$  for different  $N^2$  keeping  $\nu_c = 0.01$  and  $\nu_{ab} = 0.1$ .

In this evaluation, we observed that the loss function depends upon the vortex lattice superconductor above the JPR peak. We have also evaluated the frequency dependent loss-function of different field for  $\nu_c = 0.32$  and  $\nu_{ab} = 6.0$

In this study we have evaluated the loss- function with increasing in plane filed for under doped YBCO. Finally we evaluated the renal part of conductivity  $\sigma_1$  as a function of  $\omega$  keeping  $v_{ab}=6.0, v_c = 0.32, N_2 = 2$  and  $h = 1$ .

**MATERIALS AND METHODS:**

One develops a quantitative description the high frequency response a homogeneous layered superconductor valid for whole range of frequencies and fields. One relation the dynamic dielectric constants with the oscillating phases. The high frequency response is mainly determined by the c-axis and in plane dissipation parameters which are inversely proportional to the anisotropy.11

**DYNAMIC PHASE EQUATION, DIELECTRIC CONSTANT AND LOSS-FUNCTION:**

One writes Maxwell's equation for layered superconductor for fields and current in terms of gauge invariant phase difference between the layers.

$$\theta_n = \phi_{n+1} - \phi_n - (2\pi s / \phi_0) A_2$$

One writes these equations in the form of phase difference and magnetic field <sup>9-12</sup>

$$\frac{\sigma_c \phi_0}{2\pi s j_j} \frac{\partial \theta_n}{\partial t} + \sin \theta_n + \frac{1}{\omega_p^2} \frac{\partial^2 \theta_n}{\partial t^2} - \frac{C}{4\pi j_j} \frac{\partial B_n}{\partial x} \text{-----(1)}$$

$$\frac{1}{4\pi j_j} \frac{\partial D_z}{\partial t} \left( \frac{4\pi \sigma_{ab}}{c^2} \frac{\partial}{\partial t} + \frac{1}{\lambda_{ab}} \right) \left( \frac{\phi_0}{2\pi s} \frac{\partial \theta_n}{\partial x} - B_n \right) = \frac{\nabla_n^2 B_n}{s^2} \text{-----(2)}$$

Here the magnetic field along y- axis  $\sigma_{ab}$  and  $\sigma_c$  the component of the quasiparticles conductivity  $\lambda_{ab}$  and  $\lambda_c$  are the components of the London penetration depth,  $J_j$  is Josephson current density.

$$J_j = C \phi_0 / 8\pi^2 s \lambda_c^2, \omega_p \text{ is the plasma frequency.}$$

$$\omega_p = C / \sqrt{\xi_c} \lambda_c \cdot D_z \text{ is the external electric field and}$$

$$\nabla_n^2 B_n = B_{n+1} + B_{n-1} - 2B_n$$

Neglecting the charging effects. <sup>12</sup>The local electric field is connected with the phase difference by the Josephson relation.

$$E_z = \frac{\phi_0}{2\pi cs} \frac{\partial \theta_n}{\partial t} \text{-----(3)}$$

The average magnetic induction inside the superconductor By fixes the average phase gradient

$$(\partial \theta_n^{(0)} / \partial x) = 2\pi s B_y / \phi_0$$

Now, one uses a standard transformation to the reduced variables.

$$\begin{aligned} x / \lambda_j &\longrightarrow x \\ i \omega_p &\longrightarrow l \\ h_n &= 2\pi \gamma s^2 B_n / \phi_0 \end{aligned} \text{-----(4)}$$

Hence

$$\lambda_j = \gamma s$$

One introduces the dimension less parameters

$$\begin{aligned} l &= \frac{\lambda_{ab}}{s} \\ \nu_c &= \frac{4\pi\sigma_c}{\xi_c \omega_p} \\ \nu_{ab} &= \frac{4\pi\sigma_{ab} \lambda_{ab}^2 \omega_p}{c^2} \end{aligned} \text{-----(5)}$$

Both damping parameters  $\nu_c$  and  $\nu_{ab}$  inversely proportional to the anisotropy transfer  $\gamma$ .  $\gamma$  measures that the effecting damping is stronger in less anisotropic materials due to d-wave pairing in the high temperature superconductors both dissipation parameters  $\nu_c$  and  $\nu_{ab}$  do not vanish as T- O. Another important function of the high temperature superconductors is that the in-plane dissipation is much stronger than the C-axis dissipation<sup>12</sup>  $\nu_{ab} \gg \nu_c$ , this is the consequence of the rapid decreases of the in-plane scattering role with decreasing temperature. This manifest itself a large peak in the temperature dependents of the in-plane quasiparticle conductivity. <sup>13,14</sup>

For an oscillating external field and using a complex parameter

$$D_z(t) = D_z \exp(-i \omega t)$$

One obtains for small oscillation

$$\left[-i\nu_c\varpi + C_n(x) - \varpi^2\right]\theta_n - l^2 \frac{\partial n}{\partial x} = \frac{i\omega}{4\pi j_l} D_z \quad \text{-----(6)}$$

$$\frac{\partial \theta_n}{\partial x} - h_n + \frac{l^2}{1 - i\nu_{ab}\varpi} \nabla_n^2 h_n = 0 \quad \text{-----(7)}$$

Where  $\varpi = \frac{\omega}{\omega_p}$

$$C_n(x) = \text{Cos}[\theta_n^{(0)}(x)]$$

The static phase  $\theta_n^{(0)}(x)$  are determined by the following reduced equation.

$$\frac{\partial^2 \theta_n^{(0)}}{\partial x^2} + \left(-\frac{1}{l^2} + \Delta_n\right) \text{Sin} \theta_n^{(0)} = 0 \quad \text{-----(8)}$$

Here  $(\theta_n^{(0)} / \partial x) = h = 2\pi\gamma^2 s^2 B y / \phi_0$

Now one introduces the reduces oscillating phase

$$\theta_n = \theta_n \frac{\omega_p D_z}{4\pi J_l}$$

From equation (6) and (7) are gets the following reduced equation.

$$\begin{aligned} &-\frac{\partial^2 \theta_n}{\partial x^2} + \left[\frac{1}{l^2} - \frac{1}{l - i\nu_{ab}\varpi} \nabla_n^2\right] \\ &[C_n(x) - \varpi^2 - i\nu_c\varpi]\theta_n = \frac{i\varpi}{l^2} \end{aligned} \quad \text{-----(9)}$$

From this Josephson relation (3) ones finds that

$$E_z = (-i\varpi / \xi_z) \bar{\theta} D_z \quad \text{-----(10)}$$

Here  $\bar{\theta}$  is the oscillating phase

$$\xi_c(\varpi) = -\xi_c l (-i\varpi \bar{\theta}) \quad \text{-----(11)}$$

For zero magnetic field the oscillating phase is given by

$$\theta_n = \bar{\theta} \left[ \frac{-i\varpi}{1 - \varpi^2 - i\nu_c\varpi} \right] \quad \text{-----(12)}$$

In these cases, equation (11) gives the well-known results for the dynamic dielectric constant.

$$\xi_c^0(\omega) = D_z / E_2$$

Then loss function  $\bar{l}_0(\omega) = I_m \left( -\frac{1}{\xi_c^0(\omega)} \right)$

In this of real units we get

$$\xi_c^0(\omega) = \xi_c - \frac{\xi_c \omega_p^2}{\omega^2} + \frac{4\pi\sigma_c}{\omega} \text{-----(13)}$$

$$L_0(\omega) = \frac{4\pi\omega^2 \sigma_c / \xi_c^2}{(\omega^2 - \omega_p^2)^2 + (4\pi\omega\sigma_c / \xi_c)^2} \text{-----(14)}$$

The zero field loss function has a peak at the Josephson plasma frequency width which determined only by the C-axis quasiparticle conductivity.

**HIGH-FIELD REGIME:**

The statics phase solution at high fields for transverse lattices is given by.

$$\phi_n^{(0)} = \frac{\pi n(n-1)}{2} + \frac{2}{h^2} \sin(h_x + \pi_n) \text{-----(15)}$$

At high field one can neglect rapidly oscillating  $C_n(x)$ .

Then one obtains the solution.

$$\phi_n = \frac{C_n(x/2)}{\omega^2 - (1 - i\nu_{ab}\omega)h^2/4 + i\nu_c\omega} \text{-----(16)}$$

Finally one obtained:

$$\frac{\xi_c(\omega)}{\xi_c} = 1 + \frac{i\nu_c}{\omega} - \frac{2}{h^2\omega^2} - \frac{[\frac{1}{i\omega^2}]}{\omega^2 + i\nu_c\omega - (1 - \nu_{ab}\omega^2)h^2/4} \text{-----(17)}$$

In this low frequency regime

$$\nu_c, \nu_{ab} \ll 1$$

This loss function has a peak at  $\omega = h/2$

Corresponding to homogeneous plasma mode.<sup>15</sup> such linear growth of plasma frequency with field has been observed in underdoped <sup>16</sup>

BSCCO (Bi<sub>2</sub>Sr<sub>2</sub>CaCu<sub>2</sub>O<sub>8</sub>)

**VORTEX OSCILLATION AT SMALL FIELDS AND FREQUENCIES:**

Now one calculation the phenomenological theory of vortex oscillations. It describes the response  $\omega \ll \omega_p$  of the vortex lattice at small frequency for the Abrikosov vortex lattice theory was developed collay and clean.<sup>17</sup>

The dynamic dielectric constant for the Josephson vortex lattice has been derived<sup>18</sup> consider a superconductor in the vortex state carrying ac superconductor.

$J_j \propto \exp(-i\omega t)$  along the c-axis

The ac electric field consist of London term and the contribution from the vortex oscillation<sup>15,16</sup>

$$E_z = \frac{4\pi\lambda_c^2}{c^2} i\omega j_s - \frac{By}{c} i\omega t \text{-----(18)}$$

The vortex oscillation u can be formed from the equation<sup>17</sup>

$$(-\rho_j\omega^2 - i\eta_j\omega + k) u = \frac{\phi_0}{c} j_s \text{-----(19)}$$

Here  $\rho_j$  is the linear mass of the Josephson vortex,<sup>19</sup>  $\eta_j$  is viscosity coefficient<sup>19,20</sup> and K is spring constant due to pairing. The viscosity of an isolated Josephson Vortex has been calculated considering the dissipation coset by both c-axis and in-plane quasiparticle transport.

$$\eta_j = \frac{\xi_c \omega_p \phi_0^2}{\pi(4\pi c s)^2 \gamma} (C_c \nu_c + C_{ab} \nu_{ab}) \text{-----(20)}$$

Where the numerical constant  $C_c$  and  $C_{ab}$  are derived by the phase distribution of an isolated Josephson vortex  $\phi_b^{(0)}$

$$C_c = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} du \left[ \frac{\partial(\phi_n^{(0)})}{\partial u} \right] = 9.0 \text{-----(21)}$$

$$C_{ab} = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} du \left( \frac{\partial^2(\phi_n^{(0)})}{\partial u^2} \right)^2 = 2.4 \text{-----(22)}$$

The linear mass of the Josephson vortex is distributed by the kinetic energy  $E_K$ . This for many vortex is expressed as.

$$E_k \int \frac{d^2 \gamma \xi_c E^2}{8\pi} = s \sum_u \int d^2 \gamma \frac{\xi_c}{8\pi} \left( \frac{\phi_0}{2\pi c s} \right)^2 \theta_n^2 \text{-----(23)}$$

By definition  $E_k = L_y \rho_j u^2 / 2$

The linear vortex mass is given by

$$\rho_j = \frac{C_c \xi \phi_0^2}{2\pi\gamma(4\pi c_s)^2} \text{-----(24)}$$

$$\text{Here } E_z = \left( -4\pi\lambda_c^2 + \frac{B_y \phi_0}{-\rho_j \omega^2 - i\eta_j \omega + k} \right) \frac{i\omega j}{c^2} \text{-----(25)}$$

The total conductivity  $\sigma_c(B_\gamma, \omega)$  is given by

$$\sigma_c(B_\gamma, \omega) = \sigma_c n / \omega - \frac{\xi_c \omega_p^2}{4\pi L \omega} \left[ 1 + \frac{1}{\rho_j \omega^2 - i\eta_j \omega + k} \frac{B_\gamma \phi_0}{4\pi\lambda_c^2} \right]^{-1} \text{-----(26)}$$

The real part of the conductivity,

$$\sigma_{cs}(B_\gamma, \omega) = \frac{R_c [\sigma_{cs}(\omega) + \eta_j \xi_c \omega_p^2 B_\gamma \phi_0 / (4\pi\lambda_c)^2]}{(k + B_\gamma \phi_0 / (4\pi\lambda_c^2) - \rho^2 \omega^2)^2 + (\eta_j \omega)^2} \text{-----(27)}$$

The dynamic dielectric metal and conductivity is related with the equation.

$$\xi_c / \omega = \xi_c - 4\pi\sigma_c \omega / i\omega \text{-----(28)}$$

We gets

$$\frac{\xi_c(h, \omega)}{\xi_c} = 1 + \frac{i\nu_c}{\omega} - \frac{(1/\omega^2)}{1 - 2\pi h / [c_c / \omega^2 + i\nu_c \omega] + C_{ab}(\nu_{ab} \omega)} \text{-----(29)}$$

Some recent results<sup>21</sup> also reveal the same behavior.

**RESULT AND DISCUSSION :**

Using the theoretical formalism of A.E. Koshebev and A.E. Koshebev<sup>9</sup> and M.J.W. Dodgson.<sup>10</sup> We have theoretically evaluated vortex oscillation of Josephson vortex lattice for small fields and frequency. In Table T<sub>1</sub>, we have shown the evaluated result of loss function as a function of frequency  $\omega$  for in plane dissipation by keeping  $h = 0.5$ ,  $N_2 = 2$  and  $\nu_c = 0.1$ ,  $N_2$  is the number of vortices we have calculated the loss function as a function of frequency  $\omega$  for different value of  $\nu_{ab}$  starting to  $\nu_{ab} = 0.4, 0.6, 1.0$  and  $2.0$ . Our evaluated results. Show that there is a beak of the loss-function as a frequency of  $\omega$  for each value  $\nu_{ab}$ . In Table T<sub>2</sub> be repeated the results for c-axis dissipation. In this calculation, we have kept the value of  $h = 0.5$ ,  $N_2 =$



2 and  $\nu_{ab} = 0.4$ , we have evaluated the loss function as a function of frequency  $\omega$  for different values of  $\nu_c$  our evaluated results show that for the value of  $\nu_c = 0.1$  peak is large and gradually the high of the peak reduces as the value of  $\nu_c$  increases. In Table T<sub>3</sub>, T<sub>4</sub> and T<sub>5</sub> we have compared. Our theoretical results with exact Numerical solution<sup>21</sup> and vortex oscillation modle<sup>22</sup> keeping.

(a)  $\nu_c = 0.1$   $\nu_{ab} = 1.0$   $h = 0.5$  and  $N_2 = 2$  (b)  $\nu_c = 0.1$ ,  $\nu_{ab} = 4.0$ ,  $h = 0.5$  and  $N_2 = 2$  (c)  $\nu_c = 0.1$ ,  $\nu_{ab} = 0.1, 0.12$  and  $N_2 = 6$  our theoretically evaluated results for vortex oscillation modle is in good agreement with the exact numerical solution. In table T<sub>6</sub>, we have evaluated the frequency dependents loss-function near JPR frequency for different value of  $h$  and  $N_2$  keeping. He value of  $\nu_c = 0.01$  and  $\nu_{ab} = 0.1$ . In this case also we obtained that there is a peak for each set of values for  $h$  and  $N_2$ . In the table T<sub>7</sub> and T<sub>8</sub>, we repeated the calculation of loss- function as a function of frequency for some values of  $h = 0.2$  and  $0.4$  keeping  $\nu_c = 0.01$  and  $\nu_{ab} = 0.1$  for different value of  $N_2$ . We observed that there is a peak for  $N_2 = 2, 3$  and  $6$  as  $\omega = 1.05$  and  $1.0$ . In table T<sub>9</sub>. We have evaluated the frequency dependent loss-function at different fields for  $\nu_c = 0.32$  and  $\nu_{ab} = 6.0$  keeping  $h = 0.2, 0.5$  and  $1.0$  and  $N_2 = 2, 3$  and  $6$  we observed that in this care also there is a peak at  $\omega = 1.0$  for each set of values  $h$  and  $N_2$ . In table T<sub>10</sub> we have shown that he evaluated the loss-function as a function of frequency ( $c_m^{-1}$ ) for underdoped YBCO,  $T_c - 65$  k E||C and H||CuO<sub>2</sub>. Our evaluated results show that the loss function of frequency  $\omega$  has a peak for  $H = 0T, 2T$  and  $4T$  our evaluated results show that the loss-function increase with the increasing in-plane field. In the last Table T<sub>11</sub>, we have shown the frequency dependents of the real part of conductivity  $\sigma_1$  with  $\nu_c = 0.32$  and  $\nu_{ab} = 6.0$ ,  $N_2 = 2$  and  $h = 1$ . Our evaluated results show that  $\sigma_1$  decreases with  $\omega$ .

**NUMERICAL RESULTS :**

**Table – T<sub>1</sub>**

Role of in-plane dissipation has been calculated by evaluating the loss function as a function of frequency for  $\omega$  for  $h = 0.5$ ,  $N_2 = 2$  and  $\nu_c = 0.1$ ,  $N_2$  is the No. of vortices. Loss-function

$\omega$	$\nu_{ab} = 0.4$	$\nu_{ab} = 0.6$	$\nu_{ab} = 1.0$	$\nu_{ab} = 2.0$
0	0.256	0.357	0.432	0.547
0.2	0.387	0.439	0.557	0.678
0.4	0.645	0.756	0.842	0.952
0.6	0.873	0.952	1.107	1.128
0.8	0.982	1.167	1.273	1.337
1.0	5.432	5.543	6.127	6.232
1.2	4.678	4.862	5.282	5.878
1.4	3.226	3.462	4.586	4.956
1.6	2.469	2.586	3.259	3.875
1.8	1.586	1.678	2.108	2.432
2.0	1.325	1.546	2.078	2.257

**Table – T<sub>2</sub>**

Role of c-axis dissipation has been calculated by evaluating the loss function as a function of frequency  $\omega$  keeping  $h = 0.5, N_2 = 2, \nu_{ab} = 0.4$  Loss-function

$\omega$	$\nu_c = 0.1$	$\nu_c = 0.2$	$\nu_c = 0.3$	$\nu_c = 0.4$
0	0.142	0.227	0.325	0.407
0.2	0.256	0.322	0.478	0.478
0.4	0.486	0.498	0.532	0.596
0.6	0.863	0.786	0.869	0.985
0.8	0.956	1.589	1.672	1.953
1.0	6.127	5.328	4.767	3.832
1.2	5.874	4.358	3.884	2.547
1.4	4.587	3.875	3.089	1.875
1.6	3.874	3.083	2.957	1.324
1.7	3.228	2.775	1.986	0.967
1.8	2.232	1.874	1.852	0.874
2.0	1.872	1.753	1.659	0.677

**Table – T<sub>3</sub>**

Comparison between Vortex oscillation mode with exact numerical calculation for the loss function as a function of frequency with different values of  $\nu_c, \nu_{ab}, h$  and  $N_2$  Loss-function

$$\nu_c = 0.1, \nu_{ab} = 1.0, h = 0.5 \text{ and } N_2 = 2$$

Frequency $\omega$	Exact Numerical Calculation	Vortex oscillation mode
0.00	0.00	0.00
0.2	0.232	0.145
0.4	0.478	0.356
0.6	0.765	0.678
0.7	0.867	0.753
0.8	0.976	0.896
0.9	1.123	1.108
0.95	2.674	2.107
1.0	3.435	2.935
1.15	4.536	3.035

**Table – T<sub>4</sub>**

Comparison between Vortex oscillation mode with exact numerical calculation for the loss function as a function of frequency with different values of  $\nu_c, \nu_{ab}, h$  and  $N_2$  Loss-function

$$\nu_c = 0.1, \nu_{ab} = 4.0, h = 0.5 \text{ and } N_2 = 2$$

$\omega$	Exact Numerical Calculation	Vortex oscillation mode
0.00	0.00	0.00
0.20	0.032	0.015
0.4	0.057	0.048
0.6	0.185	0.175
0.7	0.287	0.273
0.8	0.356	0.308
0.9	0.889	0.787
1.0	0.952	0.892
1.10	1.102	1.087
1.15	1.173	1.257
1.20	1.227	1.108

**Table – T<sub>5</sub>**

Comparison between Vortex oscillation mode with exact numerical calculation for the loss function as a function of frequency  $\omega$  with different values of  $\nu_c, \nu_{ab}, h$  and  $N_2$  Loss-function

$$\nu_c = 0.01, \nu_{ab} = 0.1, h = 0.12, N_2 = 6$$

$\omega$	Exact Numerical Calculation	Vortex oscillation mode
0.00	0.00	0.000
0.20	0.047	0.038
0.40	0.058	0.047
0.50	0.045	0.035
0.60	0.032	0.0029
0.70	0.022	0.0232
0.80	0.038	0.034
0.90	0.049	0.052
1.00	0.057	0.062
1.10	0.075	0.070
1.15	0.088	0.082
1.20	0.092	0.090

**Table – T<sub>6</sub>**

We have evaluated the frequency dependents loss function near Josephson plasma resonance (JPR) frequency for different values of  $h$  and  $N_2$  keeping value of  $\nu_c = 0.01$  and  $\nu_{ab} = 0.1$  Loss-function

$$\nu_c = 0.01, \text{ and } \nu_{ab} = 0.1$$

$\omega$	$h=0.4$ $N_2=2$	$h=0.5$ $N_2=2$	$h=1.0$ $N_2=2$
0.8	0.000	2.329	4.566
0.9	7.605	27.329	32.466
1.0	9.626	55.058	48.058
1.10	42.058	62.328	72.055
1.20	10.542	53.439	48.059
1.25	12.337	40.055	33.745
1.30	9.864	27.562	29.456
1.35	7.523	19.156	20.158
1.40	6.329	10.544	11.569
1.50	4.567	6.539	8.322

**Table – T<sub>7</sub>**

In this table, we have evaluated loss function as a function of frequency  $\omega$  with the same  $h$  and different  $N_2$  for  $\nu_c = 0.01$  and  $\nu_{ab} = 0.1$  Loss-function for  $h = 0.2$ ,  $\nu_c = 0.01$ , and  $\nu_{ab} = 0.1$

$\omega$	$N_2 = 3$	$N_2 = 6$
0.90	0.005	0.018
0.95	1.102	2.247
1.00	3.307	7.439
1.05	55.428	62.528
1.10	43.546	48.032
1.15	32.249	40.085
1.20	29.558	31.586
1.25	19.087	27.088
1.30	12.123	18.532
1.35	9.469	12.058
1.40	7.322	9.059

**Table – T<sub>8</sub>**

In this table, we have evaluated loss function as a function of frequency  $\omega$  for the some  $h=0.4$  and two different values of  $N_2$  keeping  $\nu_c = 0.01$ , and  $\nu_{ab} = 0.1$  Loss-function

$\omega$	$N_2 = 2$	$N_2 = 3$
0.00	0.053	2.349
0.85	2.657	9.548
0.90	8.548	32.158
0.95	38.262	43.058
1.00	60.058	62.158
1.05	54.329	55.057
1.10	47.058	49.155
1.15	33.168	37.059
1.20	28.549	31.257
1.25	20.457	22.058
1.30	16.057	18.237
1.40	10.549	12.087

**Table – T<sub>9</sub>**

Evaluation of the frequency dependent loss-function at different fields for  $\nu_c = 0.32$  and  $\nu_{ab} = 6.0$  Loss-function

Frequency $\omega$	$h=0.2$ $N_2=6$	$h=0.5$ $N_2=3$	$h=1.0$ $N_2=2$
0	0.042	0.157	0.232
0.2	0.167	0.532	0.635
0.4	1.538	1.438	1.558
0.6	2.272	1.696	1.796
0.8	3.056	2.935	2.656
1.0	4.187	3.866	3.557
1.10	3.158	2.884	2.679
1.20	2.972	1.987	2.055
1.30	1.986	1.305	1.987
1.40	6.954	6.987	1.050
1.50	0.753	6.659	9.982

**Table – T<sub>10</sub>**

In this table we have shown the evaluated values of the loss-function as a function of frequency ( $c_m^{-1}$ ) with increasing in plane field for underdoped YBCO. Loss-function YBa<sub>2</sub> CuO<sub>6.76</sub>

Frequency ( $c_m^{-1}$ )	<b>T<sub>C</sub> = 65K, T = 8K</b> <b>E  C, H  CuO<sub>2</sub></b>		
	<b>H=0T</b>	<b>H=2T</b>	<b>H=4 T</b>
0	0.00	0.032	0.042
20	0.027	0.043	0.058
40	0.035	0.056	0.067
60	0.087	0.097	0.092
80	0.095	1.032	1.107
100	1.032	2.057	3.055
110	1.097	1.006	1.010
120	8.072	0.982	0.092
130	0.055	0.073	0.060
140	0.047	0.058	0.065

**Table – T<sub>11</sub>**

Evaluated values of the frequency dependent of the loss-function, the real part of conductivity with  $\nu_c = 0.32$   $\nu_{ab} = 6.0$ ,  $N_2 = 2$ ,  $h = 1$

$\omega$	$\sigma_1$ (real part of conductivity)
0	3.237
0.2	2.586
0.4	1.749
0.6	1.328
0.8	0.986
0.9	0.774
1.0	0.765
1.10	0.932
1.20	0.657
1.30	0.468
1.40	0.396
1.50	0.286

**CONCLUSION :**

In this paper, we have obtained the frequency dependence of the loss-function of different magnetic fields including both of dilute and dense Josephson vortex lattice. We observed that in case of very strong in plane dissipation additional peak in the loss-function appears below the plasma frequency.

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**REFERENCES –**

1. N.N. Bogolyubov Steklov ' A New method in the theory of superconductivity' .(Izd and nuat SSSR, Moscow) Nov, 1968.
2. P. W. Anderson Random-Phase Approximation in the Theory of Superconductivity . Phys. Rev. 112, 1900 - Published 15 December (1958).
3. R. V. Carlson and A. M. Goldman Phys. Rev. Lett. (PRL) 34, 11 (1975).
4. Albert Schmid and Gerd Schön Phys. Rev. Lett. 34, 941 - ( 1975)
5. S.N. Artemenko and A.F volkov, sov. Phys. JETP 42, 896 (1975)
6. S. N. Artemenko and A. F. Volkov, Sov.Phys USP 22,295(1979)
7. V. Kresin and H. Morawitz, Phys. Rev. B 37, 7854 (1988).
8. V. Kresin and H. Morawitz, Phys. C 153-155, 1327 (1988).
9. A. E. Koshelev Phys. Rev. B 76, 054525 - ( 2007)
10. Josephson vortex ... Matthew J. W. Dodgson arXiv:1304.6735 v1 [cond-mat.suprcon] 24 Apr 2013.
11. Xiao Hu, Mengbo Luo, and Yuqiang Ma Phys. Rev. B 72, 174503 - ( 2005).
12. T. Koyama and M. Tachiki Phys. Rev. B 54, 16183 -(1996)
13. D. A. Bonn, P. Dosanjh, R. Liang, and W. N. Hardy Phys. Rev. Lett. 68, 2390 - (1992)
14. Kitano, H., Hanaguri, T., Tsuchiya, Y. et al. Journal of Low Temperature Physics 117, 1241-1245 (1999).
15. L. N. Bulaevskii, D. Domínguez, M. P. Maley, and A. R. Bishop Phys. Rev. B 55, 8482 (1997)
16. Takeya, T. Wada, R. Nakamura, and K. Kadowaki Phys. Rev. B 72, 014540 - (2005)
17. Mark W. Coffey and John R. Clem Phys. Rev. Lett. 67, 386 -(1991).
18. M. Tachiki, T. Koyama, and S. Takahashi Phys. Rev. B 50, 7065 - (1994)
19. Mark W. Coffey and John R. Clem Phys. Rev. B 44, 6903 - (1991).
20. A. E. Koshelev Phys. Rev. B 62, R3616(R) -(2000)
21. Vischenlev Ostrou kn cashimiri PHd series Delft Leidin (2018)