2021 I-RIM Conference
October 8-10, Rome, Italy
ISBN: 9788894580525
DOI:10.5281/zenodo.6367957

# Whole-Body Balancing Control of Two-Wheeled Humanoids

Grazia Zambella[1], Gianluca Lentini[2], Manolo Garabini[1], Giorgio Grioli[2], Manuel G. Catalano[2]
Alessandro Palleschi[1], Lucia Pallottino[1], Antonio Bicchi[1,2], Alessandro Settimi[1], Danilo Caporale[1]

***Abstract*— Control of two-wheeled humanoid robots poses several challenges due to the unstable dynamics of their mobile base and the coupling between upper and lower body dynamics. In this work, we present the latest results of our research on control of two-wheeled unstable humanoid robots. In particular, we present a whole-body dynamic control approach to stabilize this kind of robots around the upward position. We consider the nominal constrained dynamics of the robot in the quasi-velocities and applied to it a computed torque control approach. To validate this control method, we report the experimental results obtained on AlterEgo.**

## I. INTRODUCTION

Two-wheeled humanoids, due to the unstable dynamics of their mobile base, offer more fast and efficient locomotion in a structured environment and this makes them very interesting systems for many applications. From the control point of view, one of the main challenges is the underactuation, which complicates the control design. Usually, the underactuation is at the pitch joint ($\phi$ in Fig. 1). Traditionally, it is tackled by decoupling the control laws used to regulate the upper body and the mobile base motion. In [1], an LQR balancing control is applied to a wheeled inverse pendulum robot. In this case, robot upper body movement is treated as a disturbance. In [2], the authors adopted a pole-placement controller to control the mobile base. In [3] a learning-based method has been applied for dexterous control of a mobile manipulator. In [4], inspired by [5], the authors proposed a whole-body control framework for *Golem Krang* [6]. They applied a hierarchical approach where a Quadratic Programming low-level controller is used to compute the joint torques ensuring the robot balancing and locomotion while performing other tasks with the upper body.

Our research line aims at developing whole-body control approaches for two-wheeled humanoid robots that make the robot upper body actively cooperate with the mobile base for balancing obtaining a human-like behaviour. In this work, we show our whole-body control law presented in [7] that guarantees the respect of kinematic constraints by designing the stabilizing controller directly on the constrained dynamics of the robot.

## II. ROBOT MODEL

At first, we focus on the kinematic and dynamic models of a two-wheeled humanoid. We suppose that it is moving on flat plane without rolling around its sagittal axis.

For the kinematics, with reference to Fig. 1, we describe the robot configuration defining the Lagrangian coordinates

[1]Centro di Ricerca E. Piaggio e Dipartimento di Ingegneria dell'Informazione, Università di Pisa, Pisa, Italia
[2]Istituto Italiano di Tecnologia, via Morego, 30, 16163 Genova, Italia

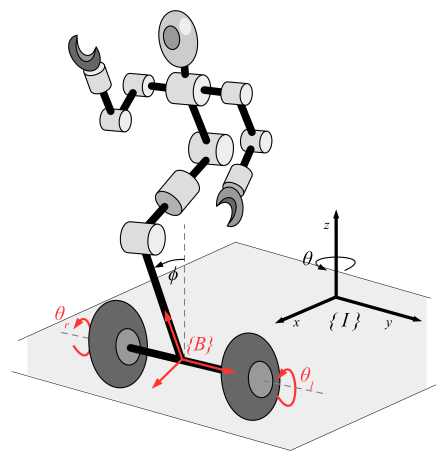


Fig. 1: Two-wheeled humanoid kinematic scheme

vector $q$ and the generalized velocities $\dot{q}$ as

$$q = \begin{bmatrix} q_{fb} \\ q_{mp} \\ q_{ub} \end{bmatrix}, \qquad \dot{q} = \begin{bmatrix} \dot{q}_{fb} \\ \dot{q}_{mp} \\ \dot{q}_{ub} \end{bmatrix}, \qquad q, \dot{q} \in \mathbb{R}^{n+n_{fb}}.$$

where $q_{fb} = \begin{bmatrix} x & y & \theta & \phi \end{bmatrix}^T \in \mathbb{R}^{n_{fb}}$ describes the position and orientation of the base frame $\{\mathcal{B}\}$ with respect to $\{\mathcal{I}\}$, $q_{mp} \in \mathbb{R}^{n_{mp}}$ is the vector which groups the DoFs of the mobile platform and $q_{ub} \in \mathbb{R}^{n_{ub}}$ the vector of the actuated upper body joint angles. $n = n_{mp} + n_{ub}$ represents the actuated degrees of freedom (DoFs).

As regards the dynamics, we need to take into account the pure rolling constraints between the wheels and the ground. These constraints are non-holonomic and can be written in the Pfaffian form $J_c(q)\ \dot{q} = 0$ where, define $n_c$ the number of independent equations of the constraints, $J_c(q) \in \mathbb{R}^{n_c \times (n+n_{fb})}$ is the so-called Pfaffian matrix. In this way, we can write the dynamic equations in the standard form used for constrained systems that is given by

$$\begin{cases} M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + J_c^T(q)\lambda = U\tau + J_f(q)^T f \\ \dot{J}_c\dot{q} + J_c\ddot{q} = 0, \end{cases} \tag{1}$$

where

- $M(q)$ is the inertia matrix;
- $C(q,\dot{q})\dot{q}+G(q) = c(q,\dot{q})$ is the generalized force vector containing the Coriolis, centrifugal and gravity terms;
- $\lambda$ is a vector of Lagrange multipliers;
- $f$ is the vector of external forces and $J_f(q)$ is the kinematic jacobian corresponding to the point of application of the external forces;
- $\tau \in \mathbb{R}^n$ is the vector of actuated joints torques and $U = [0_{n\times n_{fb}}\ \mathbf{I}_n]^T$ is the matrix that maps these torques to the space of generalized forces.

Fig. 2: Experiment - *Dynamic disturbance*. When an external disturbance is applied, the robot arms motion contributes visibly to stabilize the pitch angle

## III. WHOLE-BODY DYNAMIC CONTROL

The presented control law computes the actuation torques $\tau$ so that the robot accelerations $\ddot{q}$ are equal to some desired, constraints-consistent, accelerations $\ddot{q}_d$.

To achieve this, let $\nu \in \mathbb{R}^{n+n_{fb}-n_c}$ be the quasi-velocity vector, i.e. a vector of velocities such that $\dot{q} = S(q)\nu$ where $S(q)$ is a linear operator that satisfies the relation $J_c(q)\ S(q) = 0$.

Therefore, we define the following quasi-velocity vector

$$\nu = \begin{bmatrix} v & \dot{\theta} & \dot{\phi} & \dot{q}_{ub}^T \end{bmatrix}^T,$$

with $v \in \mathbb{R}$ is the forward linear speed of the base link. We rewrite the constrained dynamics (1) in terms of quasi-velocities obtaining

$$\tilde{M}(q)\dot{\nu} + \tilde{c}(q, \dot{q}, \nu) = \tilde{U}\tau, \tag{2}$$

where:

$$\tilde{M}(q) = S^T(q)\ M(q)S(q);$$

$$\tilde{c}(q, \dot{q}, \nu) = S^T(q)\left(M(q)\dot{S}(q, \dot{q})\nu + C(q, \dot{q})S(q)\nu + G(q)\right);$$

$$\tilde{U} = S^T(q)\ U.$$

In this way, the control problem is reduced to choose the generalized torques $\tau$ that satisfy the following relation

$$\dot{\nu}^d - \dot{\nu} + K_d(\nu^d - \nu) + K_p \int_0^t (\nu^d - \nu) = 0, \tag{3}$$

with $\nu^d$ the desired quasi-velocity vector and $\dot{\nu}^d$ and $\int \nu^d$ respectively the derivative and the integral of $\nu^d$.

Therefore, applying the computed torque control approach to (2), we find

$$\tilde{\tau} = \tilde{U}\tau = \tilde{M}\left(\dot{\nu}^d + K_d(\nu^d - \nu) + K_p \int_0^t (\nu^d - \nu)\right) + \tilde{c} \tag{4}$$

where $K_p$ and $K_d$ are positive definite matrix.

Notice that, to find the actuated joints torques $\tau$, since $\tilde{U} \in \mathbb{R}^{(n_{fb}+n-n_c)\times n}$ is full column rank matrix, we can not directly invert (4). However, since the rows of $\tilde{U}$ related to the forward displacement and to the pitch angle are linearly dependent, the row related to the forward displacement is deleted. Indicating with $\tilde{U}_s$ the resulting submatrix, we compute

$$\tau = \tilde{U}_s^{-1}\tilde{\tau}_s,$$

where $\tilde{\tau}_s \in \mathbb{R}^n$ is the vector obtained neglecting the first element of $\tilde{\tau}$, corresponding to the generalized forces acting on the forward velocity.

## IV. EXPERIMENTAL RESULTS

To validate the proposed control approach, we tested it on AlterEgo [8], which is a two-wheeled robot with two 5DoFs arms, each equipped with variable stiffness actuators [9]. For these experiments, the actuators are controlled with a high level of stiffness and can be considered rigid without compromising the validity of the results.

We tested the control system capability to reject unknown external (static and dynamic) disturbances. To generate a static disturbance effect, we loaded the robot with a series of increasing weights. Instead, to reproduce dynamic disturbances, we pushed the robot in different parts of its body (for example on the shoulder, Fig. 2). All experiments have been performed on a flat, non-slippery, surface.

In all these preliminary experiments, despite the unmodeled dynamics in the robot arms, the results were satisfactory. As we can see in Fig. 2, the arms actively cooperate with the mobile base to stabilize the pitch angle. Moreover, once the pitch angle is stabilized, the robot slowly goes back to its initial sagittal displacement. For more details, see [7].

## V. CONCLUSIONS

In this work, we presented our method for whole-body nonlinear control of wheeled humanoids discussed in [7]. We show how to derive a control law for the constrained system as a computed torque in the quasi-velocities. We illustrate the idea behind the control design and prove its effectiveness in several experiments.

## REFERENCES


[1] S.Jeong *et al.* Wheeled inverted pendulum type assistant robot: design concept and mobile control. *Intelligent Service Robotics*, 1(4):313–320, 2008.

[2] T. Feng *et al.* Modeling and implementation of two-wheel self-balancing robot equipped with supporting arms. In *2011 6th IEEE Conference on Industrial Electronics and Applications*, pages 713–718. IEEE, 2011 .

[3] S. R. Kuindersma, E. Hannigan, and D. Ruiken R. A. Grupen. Dexterous mobility with the ubot-5 mobile manipulator. In *2009 International Conference on Advanced Robotics*, pages 1–7. IEEE, 2009.

[4] M. Zafar *et al.* Hierarchical optimization for whole-body control of wheeled inverted pendulum humanoids. *arXiv preprint arXiv:1810.03074*, 2018.

[5] M. Zafar *et al.* Whole body control of a wheeled inverted pendulum humanoid. In *2016 IEEE-RAS 16th International Conference on Humanoid Robots (Humanoids)*, pages 89–94. IEEE, 2016.

[6] M. Stilman *et al.* Golem krang: Dynamically stable humanoid robot for mobile manipulation. In *2010 IEEE International Conference on Robotics and Automation*, pages 3304–3309. IEEE, 2010.

[7] G. Zambella *et al.* Dynamic whole-body control of unstable wheeled humanoid robots. *IEEE Robotics and Automation Letters*, 4(4):3489–3496, 2019.

[8] G. Lentini *et al.* Alter-ego: A mobile robot with a functionally anthropomorphic upper body designed for physical interaction. *IEEE Robotics Automation Magazine*, 26(4):94–107, 2019.

[9] C. Della Santina *et al.* The quest for natural machine motion: An open platform to fast-prototyping articulated soft robots. *IEEE Robotics & Automation Magazine*, 24(1):48–56, 2017.