Quantum Mechanics, Information and Bound State Entanglement

Francesco R. Ruggeri Hanwell, N.B. March 16, 2022

In previous notes we discussed quantum mechanics as a statistical theory and also the idea of losing information i.e. exp(ik1 x) exp(ik2 x) = exp(i (k1+k2) x) for a product of free particle "probabilities" (1). In this note we wish to continue the investigation of the statistical behaviour of quantum bound states in terms of A= px - Et = classical action which treats x,t,p and E as independent variables. This leads to P(p) with no x information or P(x) with no p information and we argue is also responsible for features of entanglement in the quantum mechanic bound state which do not appear in classical statistical mechanics which retains pp/2m + V(x) = E for a single particle which is not colliding in an ideal gas.

Classical Statistical Mechanics

A feature of classical statistical mechanics is that it removes time, time being key in classical mechanics as everything may be expressed in terms of this variable i.e. x(t), v(t), acceleration(t) etc. For a single classical mechanical particle, one knows its momentum p, energy pp/2m, potential energy V(x) , position x, time t and acceleration at t at a point x(t). This same classical mechanical particle exists in an ideal gas, in fact many exist. What is the information known in the ideal gas? One may choose an x point, but there is no known t. V(x), p, pp/2m and acceleration at x are known. The loss of information regarding t seems to be replaced by a P(x) the probability for a particle to be at x which is proportional to exp(-V(x)/T) i.e. is the same for any p. There is also a probability P(p) and temperature T which provide information about the set of particles, but not really about the single particle if one knows its momentum is p and it does not collide for an amount of time delta t. During this delta t it is like a classical mechanical particle, but with t lost and P(x) gained.

The ideal gas makes use of two interaction scenarios, the first stochastic the other deterministic: two body elastic collisions and acceleration under V(x). Statistical mechanical features are linked to the elastic collisions because they are (A) unpredictable or stochastic and (B) are linked to the information ei+ej i.e. the energies pp/2m of the two colliding particles at a given x. In other words an elastic collision is only interested in the sum of the energies of the colliding particles. Given a sum E, any ei+ej=E are equivalent. On the other hand one has P(ei) and P(ej) and so even though these carry information linked to ei and ej particles a product P(ei)P(ej) must lose this information because the collision only depends on ei+ej=E. In other words:

P(ei)P(ej) relative = P(ei+ej) relative ((1))

This suggests P(ei) = exp(-ei/T)/ C(T) ((2)), in other words the Maxwell-Boltzmann distribution follows from knowledge of the stochastic interaction(s) involved and what information they require. One may note that there is a second interaction linked to V(x), but it is not stochastic with respect to changes in p. In other words as long as a collision does not occur, p accelerates as a classical particle. One has, however, lost the concept of time, but this may be removed in classical mechanics using a conservation of energy equation:

pp/2m + V(x) = E   ((2))

Probabilities are linked to stochastic interactions and we have argued that the form of relative probability exp(- pp/2mT) follows from an elastic collision only sensing ei+ej. Because of a second interaction due to V(x), pp depends on x if no collision occurs. Imagine V(x1)=0. Then one has P(p) relative = exp(-pp/2mT). At x2 assuming no collision occurs, pp/2m = p2p2/2m + V(x2).   Thus P(p,x) relative becomes exp(-1/T (pp/2m + V(x))) and one has a spatial probability P(x) as well as a momentum one P(p). In the classical case, one may still link p and x through ((2)) i.e. the particle accelerates classically when not colliding.

Thus the loss of time information is transferred into a conservation of energy equation ((2)) which results in P(x) as a new piece of information for any p particle replacing lost time information.

Classical Action of a Free Particle

The relativistic and nonrelativistic free particle classical actions are:

A= -mo t sqrt(1-vv) (c=1)   ((3a))   and A= t m/2 vv   ((3b))

We have argued in previous notes that   dA/dx partial = p ((4a)) and dA/dt = -E. ((4b)) In other words one may write v=x/t and treat x and t as independent. dA/dx partial = p follows from dL/dv = p (L=Lagrangian) for a free particle v=x/t. If x and t are independent, one no longer follows a classical mechanical picture as there is no x(t). Using this formalism also means that in the presence of V(x) one would need to use an ensemble of p’s in order to create a vrms which accelerates. Thus one has a statistical picture.

This statistical picture becomes even more unusual when one notes that ((4a)) and ((4b)) are equivalent to:

A= px - Et ((5))   with p, x, E and t all independent for a free particle

Consider the presence of a potential V(x). As argued above there is no acceleration because there is no sense of x(t) because x and t are independent. What is the interaction in the picture? Unlike a classical gas which has two types of interactions, there is only one interaction with V(x) in the quantum bound state. At first one might think that this is a deterministic interaction as in classical mechanics, but given that one has an ensemble of p’s, V(x) is stochastic delivering impulse hits which knock one p into another. By momentum conservation one expects:

P or particle + k hit from potential = p+k   ((6))

X does not appear in ((6)). From ((5)) one sees that A depends on p and E with E=pp/2m. In a bound state, however, one may have an overall fixed En for all p values. This En incorporates kinetic energy pp/2m and some kind of potential energy term for a particle with p. Consider a probability for a particle with p consistent with ((5)) which includes x and the operation d/dx to find p. This p may be made up of components such that the overall momentum is p. Then:

P(p1x)P(p2x) = P(px) where p1+p2=p because probabilities multiply in an AND situation

This suggests P(px) is proportional to exp(ipx) because one cannot have the function increase or decrease overall with x. As a result, one has a complex probability. Again like the classical case for which e1+e2 =E and only E was important for an elastic collision, impulses are linked to conservation of momentum and only total momentum is conserved which forces the form of the probability associated with a free particle i.e. exp(ipx).

Entanglement In Bound State Quantum Mechanics

In the above section it was found that a free quantum particle has a probability exp(ipx). In order to create an ensemble one has:

P(p/x) = a(p)exp(ipx) / W(x) where W(x) = Sum over p a(p) exp(ipx) ((7))

a(p) is a weight linked to a momentum probability P(p) although the specific details of are not yet clear. Furthermore there is a weight W(x) which involves entanglement of all of the ap's to produce a position dependent object. Given that this picture does not contain time one combines various exp(ipx) states as if they existed at x at one time. One may also consider P(x/p). If W(x) is a weight to be at x (which involves entanglement) then:

P(x/p) relative = W(x) exp(-ipx) and
P(x/p) = W(x) exp(-ipx) / Integral dx W(x)exp(-ipx) = W(x)exp(-ipx)/a(p) ((8))

Entanglement thus becomes a central feature of a quantum bound state.
Like in the classical statistical mechanical picture a loss of information about time t from a free particle p results in a spatial probability like weight W(x).

Unlike classical mechanics p and x are also independent, so one cannot use a mixed pp/2m + V(x) = E equation as was done with the ideal gas. As a result, one has a probability distribution P(p) with no information about x and a P(x) with no information about p reflecting this independence found in A=-Et+px. From ((7)) and ((8)) it may be shown that:

P(p) =a*(p)a(p) ((9a)) and P(x) = W*(x)W(x) ((9b)) with W(x)=W*(x) for a bound state

W(x) and a(p), however, are not independent, W(x) involves entanglement with all of the a(p)'s.

Furthermore a(p) involves entanglement with the potential V(x) and other a(pi)'s. To see this consider:

{-1/2m Sum over p a(p)pp/2m exp(ipx) } / W(x) + V(x) = En ((10))

Writing V(x) as Sum over k Vk exp(ikx), multiplying by W(x) = Sum over p a(p)exp(ipx) and collecting coefficients of exp(ipx) yields:

pp/2m a(p) + Sum over k Vk a(p-k) = En a(p)   ((11))

((11)) shows the entanglement of various a(p)'s clearly. Furthermore if one writes a Shannon's entropy for P(p) i.e.   -Sum over p P(p)ln(P(p)) then:

Sp = - Sum over p 2P(p) ln(a(p) = -Sum over p 2P(p) ln{ [Sum over k Vk a(p-k)] / (En-pp/2m) }

Thus Sp also reflects this entanglement. The entanglement occurs because the formalism used keeps p and x independent as well as p and x independent of t.

Conclusion

In conclusion, we argue that writing a free particle action A=-Et+px treats x,t, p and E as independent variables. Already in classical statistical mechanics (ideal gas) one removes time, but still keeps the classical link pp/2m + V(x) =E so that p and x are not independent. A stochastic interaction is needed to create a statistical system and this is provided by two body elastic collisions which conserve ei+ej. Thus even though P(ei) and P(ej) give separate information about particles with ei and ej, P(ei)P(ej) relative = P(ei+ej) i.e. information is lost and so P(ei) = exp(-ei/T) / C(T). Information considerations yield the form of P(ei). Here ei=pp/2m. The second interaction present involves V(x) acting on a non-colliding particle. In such a case, one may use pp/2m + V(x) and replace exp(-ei/T) with exp(-1/T (pp/2m + V(x)). Thus a loss of information about time yields a new piece of information namely P(x).

It is possible to write a free particle classical action A in terms of v=x/t and treat x and t as independent. This means that one does not use the classical mechanical approach which depends on x(t), but rather a statistical one because a particle may be at any point x. This approach may be taken further to treat p,x, E and t as independent as in: A= -Et+px (which holds relativistically and nonrelativisitcally). In such a case, p,x and t are independent. Thus one cannot have a picture with acceleration or have time appear with p or x. Furthermore one cannot even use pp/2m + V(x) which is applied to a single particle with momentum p in an ideal gas (when it is not colliding).

Again one looks for probabilities linked to dA/dx partial = p. Given that A depends on px , we consider P(px). Due to conservation of momentum a given p could consist of p1+p2 so: P(p1x)P(p2x)= P((p1+p2)x) which leads to exp(ipx) as a probability form.

To treat a problem with a potential, one must consider free particle probabilities exp(ipx) in an ensemble i.e. P(p/x) = a(p)exp(ipx) / W(x) where W(x) = Sum over p a(p)exp(ipx). There is not only an a(p) weight, but a W(x) which involves entanglement with all a(p) values. These are linked to P(p) and P(x), but given that p and x are independent in A= -Et+px, if one considers P(p) there is no information about x and for P(x) no information about p.

Further entanglement appears if one considers an overall average conservation of energy at each x: -1/2m Sum over p pp/2m a(p)exp(ipx) / W(x) +V(x) = En and writes V(x)=Sum over k Vkexp(ikx) and W(x)=Sum over p a(p)exp(ipx). Multiplying by W(x) and collecting exp(ipx) coefficients leads to: pp/2m a(p) + Sum over k Vk a(p-k) = En a(p). This shows again entanglement of an a(p) with Vk's and other a(p-k)'s. Thus entanglement is the price to pay for making x,p and t independent. (Note A=-Et+px, but E is replaced by En for all p.) If one writes Sp = momentum portion of entropy = - Sum over p P(p)2 ln(a(p)) then ln(a(p)) = ln{ [Sum over k Vk a(p-k)]/(En-pp/2m) } which shows that the information ln(a(p)) involves both entanglement with Vk's and other a(p-k)'s. Thus the consequences of keeping p,x and t independent are large, namely interference of exp(ipx)'s and entanglement.

References

1. Ruggeri, Francesco R. Product Probability in Quantum Mechanics With one Factor Linked to Maximum Entropy and Information (preprint,zenodo, 2021)