PORTFOLIO OPTIMISATION FOR MALAYSIA'S TOP 30 AND MID 70 ASSETS USING MEAN-VARIANCE MODEL

Mohd Azdi Maasar, Aidah Najihah Abdul Aziz, Farah Najihah Ithnin, Nurin Adriana Ariff

Faculty of Computer and Mathematical Sciences
University of Technology Mara, Negeri Sembilan Branch, Seremban Campus, 73000 Kuala Pilah, Negeri Sembilan, Malaysia

**Corresponding author: nurinadrianariff@gmail.com*

Abstract

The goals of this research are to minimize the risk of losses for specified returns using the mean-variance model and to compare the risk and return valuations (in terms of in-sample and out-of-sample analysis) when the optimization is implemented on three different sets of assets. The assets consist of constituents of FBMKLCI, which represents the Top 30 Risky Asset and FBMM70, which represents the Mid 70 Risky Asset. The closing price data are drawn from Thomson Reuter Eikon. The mean-variance model is implemented using AMPL and the numerical results were analysed in Microsoft Excel. The general assumption on mean-variance is the higher the return, the higher the risk. Main findings show that the higher the expected return, the higher the risk at Top 30 Risky Asset. The number of assets that constructed the portfolios was more diversified as the risk decreased. While Mid 70 Risky Asset does not follow the general assumption of mean-variance. The combination of the two assets provides a more interesting outcome. The result improved in terms of the level of risk. The insertion of the really risky asset in a basket of assets somehow affects the behaviour of assets in terms of risk. We validate our in-sample portfolios by using out-of-sample analysis. The result shows that a combination of both Risky Assets gave better performance mainly for low and medium target returns.

Keywords: Mean-variance, Risk, Portfolio

Introduction

Uncertainty, Risk, and Decision Making

Uncertainty refers to situations that have more alternatives resulting from the outcome, but the probability is uncertain. This is due to a lack of information or knowledge about the condition. Hence, making it hard and impossible to predict the future outcome. Risk can be described as a situation in which loss of money during an investment provides disappointing returns (LeRoy & Singell Jr, 1987). In finance, risk can be defined as the standard deviation of the return values. The risk can be measured by theoretical model and as a prediction for future outcomes. Investors can minimise the risk by taking extra action or precautions. In general, an investment that carries a higher risk also has potential to get higher return. All investment carries some level of risk since the return is not guaranteed.

Every investor needs to make decisions since it is an important process in investment. Decision-making is an act of choosing the best alternative among two or more possible alternatives and putting it into practise. Most decision will take place under conditions of certainty, risk, and uncertainty. In certainty conditions, the decision maker knows the reasonable alternatives and the conditions that associated. In risk condition, it is about the availability of each alternatives, its potential payoffs, and cost associated with the estimation of probability (Steuer & Na, 2003). While in uncertainty conditions, the decision maker does not know the alternatives, the relation of risk, or the consequences of each alternatives. In finance, portfolio selection is a type of decision-making under risk because the decision maker must select a strategy before allocating his money in order to obtain high return (Roman & Mitra, 2009).

Portfolio Selection

Portfolio can be said as a basket of financial assets, which consist of stocks, bonds, and securities while, portfolio selection is a decision on how to distribute an amount of money among assets in order to obtain high return on investment (Roman & Mitra, 2009). The problem in this matter is that investors or practitioners are having a hard time in allocating the percentage of money to be distributed among assets. They need to deal with the decision-making under risk, which to minimise the risk of portfolio since the future return of assets are unknown. Portfolio selection problems focus on allocating an amount of capital onto a set of assets or securities such that the profit or the risk can be optimised in achieving investment goals.

One vital rule when choosing portfolio is that investors should maximise the value of future returns. However future is somehow unpredictable therefore, it is measure based on the performance return variables. Markowitz (1952) consider that expected return is desirable and variance of return is undesirable, meaning that risk is undesirable.

Following notation by Maasar et al. (2016), we consider the situation of which there is a set of n assets available for a trade. Since the future return is unknown, it can be denoted by R_j where $j \in \{1, \ldots, n\}$. Let x_j be the weight of the capital invested in asset j and $x = (x_1, \ldots, x_n)$ as the portfolio weight, which is the requirement of investment decisions. Finally, we denote the portfolio return, R_x as:

$$R_x = x_1 R_1 + \cdots + x_n R_n \quad (1)$$

where the weight $(x_1, \ldots, x_n)$ belongs to set of decision given as

$$X = (x_1, \ldots, x_n) | \sum_{j=1}^{n} x_j = 1, x_j \geq 0, \ \forall j = 1 \ldots n. \quad (2)$$

The above equation is the simplest way in setting the feasible set for future returns' requirement, where it must be positive and sum to one, meaning that buy to hold and any short selling[1] is not permitted.

An efficient portfolio is the one that has the lowest risk at a specified target return. Portfolio optimisation is a technique for solving optimisation problem where we minimise the risk subject to a constraint on expected return. The efficient portfolios are obtained by solving optimisation problems that can be developed in various ways. The most common formulation is based on specified target on portfolio's targeted return while minimising its risk. Following notation by Roman & Mitra (2009), this paradigm is formulated as follows:

$$\min \varrho(R_x)$$
$$\text{Subject to: } E(R_x) \geq d \text{ and } x \in X \tag{3}$$

where $E(\cdot)$ represents the expected value operator and ϱ is denoted as the risk measure in the men-risk approach where d represents the desired level of expected return for the portfolio. This model will provide the efficient portfolio and create the efficient set.

Practitioners seek to optimally construct these portfolios by using the pioneer mean-risk model, which is Mean-Variance model. The problem always remain with "which set of asset will give efficient portfolio for which the risk is minimised for a specified level of return". It is common to see most of top or main risky asset in a country satisfy the mean-variance's assumption, thus making us curious to see the riskiness of less risky asset when mean-variance model is applied. We aim to address this

[1] Short selling is an act where the seller sells the stocks first and assuming that in the future, he is going to buy it all back for a cheaper price. The stocks involved are actually does not owned by him, but borrowed from others
i.e. broker.

problem by experimenting the performance of some efficient portfolios constructed under different set of assets, which are the Top 30 Risky Asset[2] and the Mid 70 Asset[3] . From the evaluation of the performance using in-sample and out-of-sample analyses, a practitioner can make a prompt comparison of which efficient portfolios may perform better than the other portfolios based on some specified constraints.

Methods

Variance

Risk measure used in this research is variance. Variance is widely used in statistic to measure spread around the expected value. The expected value of the square of deviation R_x from its own mean
$\sigma^2(R_x) = E((R_x) - E(R_x))^2$ (Roman & Mitra, 2009). The variance of linear combination of random variable is given as:

$$\sigma^2(sR_1 + tR_2) = s^2\sigma^2(R_1) + t^2\sigma^2(R_2) + 2st\,Cov(R_1, R_2) \tag{4}$$

where R_1, R_2 such a random variable, s, $t \in \mathbb{R}$ and $Cov(R_1, R_2) = E[(R_1 - E(R_1))(R_2 - E(R_2))]$ is the covariance of R_1 and R_2. The relation above is particularly useful to express the variance of the portfolio return $R_x = x_1R_1 + \cdots + x_nR_n$, as a result from choice $x = (x_1, \ldots, x_n)$ as:

$$\sigma^2(R_x) = \sum_{j=1}^{n} \sum_{k=1}^{n} x_j x_k \sigma_{jk} \tag{5}$$

where σ_{jk} denotes the co-variance between R_j and R_k. R_j and R_k is the returns of asset j and asset k. Thus, variance will expressed as a quadratic function of $(x_1, \ldots, x_n)$.

Mean-Variance Model

We used the mean-variance model to solve our portfolio solution problem formulated in equation (1) and (2) under the paradigm of (3). Consequently, this model will provide an efficient portfolio x having a return R_x. An efficient portfolio is considered efficient when risk is minimised at specified level either of target return or vice versa. For the general assumption of mean-variance efficient portfolio, it is high return, high risk (Roman & Mitra, 2009). The basic mean-variance optimisation model followed by (Markowitz, 1952):

$$\text{minimise} \sum_{j=1}^{n} \sum_{k=1}^{n} x_j x_k \sigma_{jk}$$

$$\text{Subject to: } \sum_{j=1}^{n} x_j \overline{r_j} \geq d$$

$$x_j \geq 0 \; for \; j = (1,2,3, \ldots, n)$$

$$\sum_{j=1}^{n} x_j = 1$$

[2] The Top 30 Risky Asset referring to the components of FTSE Bursa Malaysia KLCI (FBMKLCI) index and the main risky asset.
[3] The Mid 70 Risky Asset referring to the components of FTSE Bursa Malaysia Mid 70 (FBMKLCI) index and the less risky asset.

where,

n: Be the number of assets;
$\overline{r_j}$: Expected return of asset, j;

σ_{jk}: Co-variance between asset j and k;
x_j: The amount invested in asset $j(0 \leq x_i \leq 1)$ and
d: Level of target return for the portfolio.

Here the objective function is to minimise the total variance, which is the value of risk related with the portfolio. The constraint used to ensure that the portfolio will achieve the target return, d. The total amount of stocks investment must be equal to one. Therefore, all cash is invested in the asset.

Numerical Setup and Implementations

The data of closing prices of constituents of FBMKLCI and FBMM70 are drawn from Thomson Reuter Eikon. We evaluate the 120 monthly return from September 2009 to September 2018 and implement the mean-variance model in AMPL using the in-sample data. Generally, in-sample analysis refers to past data that you have and wish to see the initial prediction and do the model selection. A specific criterion on efficient portfolio is determine by investor's preferences of risk (Roman & Mitra, 2009). We wish to see the movement of risk as we change the target return. Therefore, we setup low return – low risk portfolio, medium return – medium risk portfolio, and high return – high risk portfolio. An efficient portfolio may vary depending on the chosen target return, d. For both data set of FBMKLCI and FBMM70, we use $d_1 = 1\%$ to represent our low return low risk portfolio, $d_2 = 1.5\%$ to represent our medium return medium risk portfolio, and $d_3 = 2.55\%$ to represent our high return high risk portfolio. For the first in-sample portfolio, we consider 100 scenarios from October 2008 to November 2016 to be the return parameter, and one month rolling window approach is used for the following in- sample portfolios[4]. Then, we run the mean-variance model to obtain 9 optimised in-sample portfolios and compare the risk and return values.

A better way to test the assumptions of a model is to perform out-of-sample analysis. Out-of-sample analysis means to withhold some of the sample data from the model identification and estimation process, then use the model to make the predictions for the holdout data in order to its accuracy. We validate the optimised in-sample portfolios using the out-of-sample analysis. Back-testing is an important in validation process because it provides confirmation concerning a system's efficiency before implement the system for future use. At this stage, we use the remaining of 20 scenarios to construct the out-of-sample portfolios. Then, we calculate the realised return and standard deviation using the optimal portfolio weight of in-sample portfolios.

Results and Discussions

In-sample portfolio analysis

In this section, we compare the risk and return values for both FBMKLCI and FBMM70, each having the expected values of d_1, d_2, and d_3 (1%, 1.5%, *and* 2.55%). Note that it is desirable for investors to have smaller standard deviation for portfolio distribution as it indicates the level of risk.

For Top 30 Asset, as the level of targeted return increase, the standard deviation also increased. This indicates that the higher the return, the higher the level of risk and it is aligned with the mean-variance's assumption as stated earlier. As optimisation ran for 9 times, the low target return has decreasing value of standard deviation, followed by increasing number of components assets. The number of assets are more diversified in order to lower the level of risk. Similar to the medium and high target return, as the level of risk decreasing, the number of assets also diversified. For each portfolios, it can be seen that the consistency is obvious as target return increase, the standard deviation also increase. This strengthen our research our research that when dealing with unsystematic risk which is diversified risk, as the

[4] Rolling window approach is where the adding of next monthly return with the current monthly return and removing the oldest monthly return is made.

higher the target return, the higher the standard deviation. For Mid 70 Asset, the standard deviation also increase as the level of target return increase. This indicates that the higher the return, the higher the level of risk and it is aligned with mean-variance's assumption. However, for each portfolio, as the expected return increase, the number of assets that constructed the portfolio shows unfavourable movement. The number of assets initially increase and then rapidly decline. This situation consequently making the portfolio unaligned with the mean-variance's assumption. This happened due to the constituents frequently going in and out of the Mid 70 Asset during the 10-year period. Contradict with the result obtained in Top 30 Asset, Mid 70 Asset does not consists of the "really risky" assets as in the Top 30 Asset, thus mean-variance model may not do the works as presumed. Mean-variance model cannot capture the "riskiness" of the asset. Alternatively, we combine the both Top 30 and Mid 70 Risky Asset into one basket of asset.

Risk Movement for Combination of Top 30 and Mid 70 Asset

3.5
3
2.5
2
1.5
1
0.5
0

1 2 3 4 5 6 7 8 9

Efficient Portfolios

1% 1.50% 2.55%

Figure 1. Risk values for efficient in-sample portfolios for Combination of Top 30 and Mid 70 Risky Assets.

The basket of asset is now contain the really risky asset and the "not so risky" asset. It is noticeable that as the target return goes higher, the standard deviation also goes higher, which indicates that level of risk goes higher as well. When both of Top 30 and Mid 70 Asset are combined, the level of risks are reduced compared to the previous findings on Top 30 and Mid 70 Asset alone. While for each portfolio obtained, each of them having the higher the expected return, the higher the level of risk. The number of component assets that constructed the portfolio also decreasing as the expected return goes high. Interestingly, this outcome for combination of assets are somehow affected due to the involvement of the less risky asset, where the result earlier on Top 30 Asset shows that the asset followed the mean- variance's characteristics and the other way around for Mid 70 Asset. This situation making a basket of assets becomes more desirable when being merged with the undesirable asset. Hence, we can conclude that the inclusion of Mid 70 Asset into the basket has caught the nature of mean-variance model as a risk minimisation tool.

Out-of-sample portfolio analysis

We analysed the realised return of the efficient portfolios for three different sets of data, each having the expected value of d_1, d_2, and d_3.

Table 1. The realised returns and standard deviation using second in-sample portfolio

	Top 30			**Mid 70**			**Top 30 + Mid 70**		
	d_1	d_2	d_3	d_1	d_2	d_3	d_1	d_2	d_3
Expected Value	0.003819	0.008325	0.018589	(0.001640)	0.002172	0.010676	**0.019837**	**0.022441**	**0.024869**
Standard Deviation	0.027818	0.028206	0.032717	0.030043	0.032854	0.041963	**0.021156**	**0.023932**	0.042150

Our recent numerical works show that the results of the analysis is inconsistent between portfolios. We analysed the favourable results according to the level of target return. The performance in the combination of both assets give better statistics. The lower and medium of target return in combination of both assets shows a desirable return by looking on the standard deviation. The value of standard deviation is lower compare to other assets. For high target return, the lower standard deviation is obtained in Top 30 Asset. Hence, the out-of-sample results differ as the targeted return differ. Based on the table, the combination of Top 30 and Mid 70 Asset gives desirable outcomes especially at the low and medium target return. The expected values (realised return) obtained also are relatively higher. While the standard deviation at the high target return is higher than the Top 30 Asset alone. Therefore, the combination of these two risky assets followed the result in in-sample portfolios.

Conclusion

Above all, we consider variance as the risk measure and mean-variance model as the mean-risk model for this research. Variance is an unsystematic risk which dealt with diversified risk. We observed that when dealing with main or top risky asset, the in-sample portfolios behave accordingly to the mean- variance's efficient portfolio's assumption that, as the higher the expected return, the higher the level of risk. The number of constituents that constructed the portfolios also diversified to reduce risk. However, throughout the research, we found that when mean-variance was applied at the FBMM70 asset, the outcome did not came out as expected. Mid 70 does not possessed the nature of mean-variance and this indicates that Mid 70 Asset is a less risky asset. This situation resulting us in combining both FBMKLCI and FBMM70 into one basket of assets. The interesting part begins here where the in-sample results show that this combination follow the mean-variance's assumption. The combination of the risky and the "not so risky" assets hold the mean-variance's assumption and at the same time provide a smaller level of risk compared to constructing portfolios based on risky asset only.

We validate the in-sample portfolios obtained earlier using out-of-sample analysis. The remaining 20 scenarios and the portfolio weight from in-sample portfolios were used to construct the out-of-sample analysis. The result shows that the combination of Top 30 and Mid 70 Assets gave better performance mainly at low and medium target return. Overall, we can conclude that the inclusion of less risky asset in a basket of assets will give a better result in terms of lowering the risk and future performance in terms of realised returns. As for future improvement, we are planning to observe on how many constituents in the less risky asset will be selected in the optimal portfolios of combination of both Top 30 and Mid 70 Assets.

References

LeRoy, S. F., & Singell Jr, L. D. (1987). Knight on risk and uncertainty. Journal of political economy, 95(2), 394- 406.

Maasar, M. A., & Roman, D. (2016). Portfolio optimisation using risky assets with options as derivative insurance. In 5th Student Conference on Operational Research (SCOR 2016). Schloss Dagstuhl-Leibniz-Zentrum für Informatik.

Markowitz, H. (1952). Portfolio selection. The journal of finance, 7(1), 77-91.

Roman, D., & Mitra, G. (2009). Portfolio selection models: A review and new directions. Wilmott Journal: The International Journal of Innovative Quantitative Finance Research, 1(2), 69-85.

Steuer, R. E., & Na, P. (2003). Multiple criteria decision making combined with finance: A categorized bibliographic study. European Journal of operational research, 150(3), 496-515.