Cooperation increases robustness to ecological disturbance in microbial cross-feeding networks

Functions to calculate Entropy and Assortativity and Robustness

Entropy

In[ ]:=
```
Entrop[x_] :=
 N[Total[(-(x / Total[x])[[#]] Log[(x / Total[x])[[#]]] &) /@ Range[Length[x]]]]
```

In[ ]:=
```
RelatEntrop5[x_] := (
  elem25 = Cases[Flatten[x], Except[0]][[1]];
  y25 = Count[x[[#]], elem25] & /@ Range[Length[x]];
  Entrop[y25] / Entrop[Table[Total[y25] / Length[y25] // N, {Length[y25]}]])
```

Assortativity

Figures Main Text

Figure 1

In[ ]:=
```
jnl = 0  1  0  0  0
      0  0  1  1  1
      1  1  1  1  0;
      1  1  1  1  1
      0  0  1  0  1
```

In[•]:=
```
jnl
```

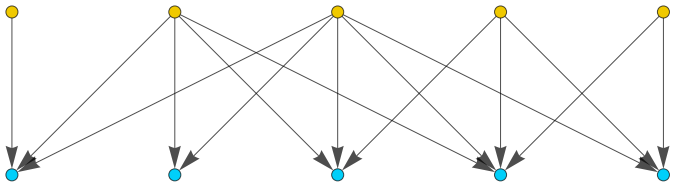
Out[•]=
```
{{0, 1, 0, 0, 0}, {0, 0, 1, 1, 1}, {1, 1, 1, 1, 0}, {1, 1, 1, 1, 1}, {0, 0, 1, 0, 1}}
```

In[•]:=
```
bacol = RGBColor[0.94, 0.79, 0.0]
metcol = RGBColor[0.0, 0.81, 0.98];
```

Out[•]=

In[•]:=
```
MakeNetworkH[x_] := (
  numBact = Length[x];
  numMetab = Length[x[[1]]];

  VectBact = Range[numBact];
  VectMetab = Range[numBact + 1, numBact + numMetab];

  Bact = StringInsert[ToString /@ Array[# &, {numBact}], "B", 1];
  Metab = StringInsert[ToString /@ Array[# &, {numMetab}], "M", 1];

  elem = Cases[Flatten[x], Except[0.]][[1]];

  color = Join[# → bacol & /@ VectBact, # → metcol & /@ VectMetab];

  conex = Flatten[Cases[Position[x, elem], {y_, z_} → {y → z + numBact}]];

  equiv = Join[#[[1]] → #[[2]] & /@ Partition[Riffle[VectBact, Bact], {2}],
    #[[1]] → #[[2]] & /@ Partition[Riffle[VectMetab, Metab], {2}]];

  Graph[conex, (*VertexLabels→equiv,*)VertexStyle → color, EdgeStyle → Black]
 )

MakeNetworkH[jnl /. {0 → 0., 1 → 0.1}]
```

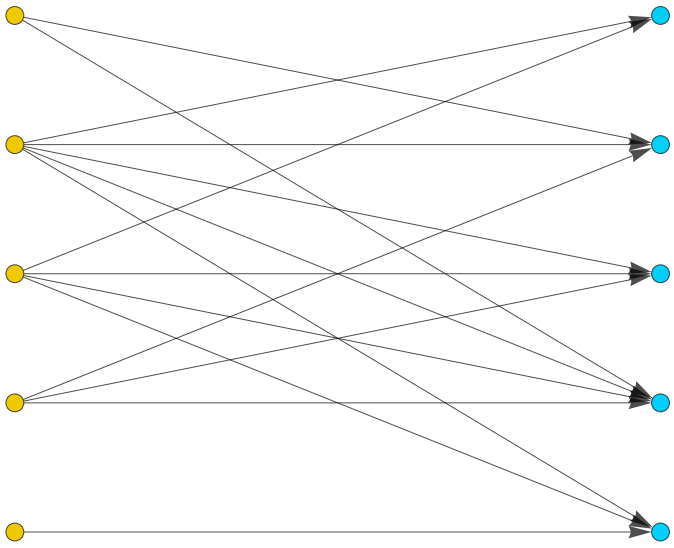
Out[•]=



In[ ]:=

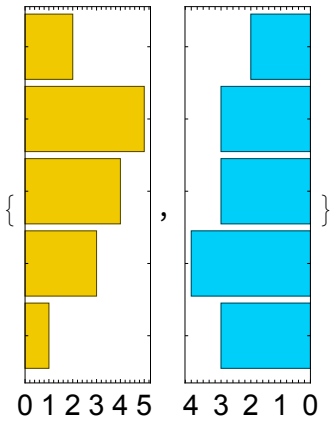
```
MakeNetworkV[x_] := (
  numBact = Length[x];
  numMetab = Length[x[[1]]];

  VectBact = Range[numBact];
  VectMetab = Range[numBact + 1, numBact + numMetab];

  Bact = StringInsert[ToString /@ Array[# &, {numBact}], "B", 1];
  Metab = StringInsert[ToString /@ Array[# &, {numMetab}], "M", 1];

  elem = Cases[Flatten[x], Except[0.]][[1]];

  color = Join[# → bacol & /@ VectBact, # → metcol & /@ VectMetab];

  conex = Flatten[Cases[Position[x, elem], {y_, z_} → {y → z + numBact}]];

  equiv = Join[#[[1]] → #[[2]] & /@ Partition[Riffle[VectBact, Bact], {2}],
    #[[1]] → #[[2]] & /@ Partition[Riffle[VectMetab, Metab], {2}]];

  Graph[conex(*,VertexLabels→equiv*), VertexStyle → color,
   EdgeStyle → Black, GraphLayout → "BipartiteEmbedding"]
 )

MakeNetworkV[jnl /. {0 → 0., 1 → 0.1}]
```

Out[ ]=



In[•]:=
```
{BarChart[Style[Reverse[{2, 5, 4, 3, 1}], bacol], AspectRatio → 3,
  BarOrigin → Left, Frame → True, FrameStyle → Directive[Black, FontSize → 15]],
 BarChart[Style[Reverse[{2, 3, 3, 4, 3}], metcol], AspectRatio → 3,
  BarOrigin → Right, Frame → True, FrameStyle → Directive[Black, FontSize → 15]]}
```

Out[•]=



In[ ]:=

```
Knum = 0.2;
ccrnum = 0.05;
qqrnum = 0.3;
ddrnum = 0.00015;
OMrnum = 1;
nurum = 1500;
den2rum = 2;

corrpar0 = 10^3;
corrpar1 = 10^4;
corrpar2 = 10^6;

KKr := RandomVariate[
    GammaDistribution[ corrpar0 Sqrt[Knum], (1 / corrpar0) Sqrt[Knum]], 1][[1]];
ccr := RandomVariate[GammaDistribution[ corrpar1 Sqrt[ccrnum],
     (1 / corrpar1) Sqrt[ccrnum]], 1][[1]];
qqr := RandomVariate[GammaDistribution[ corrpar0 Sqrt[qqrnum],
     (1 / corrpar0) Sqrt[qqrnum]], 1][[1]];
ddr := RandomVariate[GammaDistribution[ corrpar1 Sqrt[ddrnum],
     (1 / corrpar1) Sqrt[ddrnum]], 1][[1]];
OMr := RandomVariate[GammaDistribution[ corrpar0 Sqrt[OMrnum],
     (1 / corrpar0) Sqrt[OMrnum]], 1][[1]];
nur := (*nurum*) RandomVariate[GammaDistribution[
     corrpar2 Sqrt[nurum], (1 / corrpar2) Sqrt[nurum]], 1][[1]] ;
denr2 := (*den2rum*) RandomVariate[GammaDistribution[
     corrpar2 Sqrt[den2rum], (1 / corrpar2) Sqrt[den2rum]], 1][[1]] ;


parR = Join[Table[KKr, {5}], Table[ccr, {25}],
  Table[qqr, {5}], Table[ddr, {25}], Table[OMr, {25}], {nur}, {denr2}]
```

Out[•]= {0.201708, 0.185204, 0.208104, 0.205954, 0.218417, 0.0487218, 0.0501919,
0.0509234, 0.0494254, 0.051401, 0.0508852, 0.0505626, 0.0495632, 0.0500902,
0.0492636, 0.0493688, 0.0495445, 0.0504626, 0.050156, 0.0511484, 0.049766,
0.0503801, 0.0497853, 0.0479322, 0.0497703, 0.0508918, 0.0512489,
0.0492538, 0.0492022, 0.0506193, 0.289793, 0.300632, 0.295286, 0.302297,
0.304867, 0.000147542, 0.000137388, 0.000152496, 0.000138806, 0.000131574,
0.000139522, 0.000158137, 0.00016046, 0.000149519, 0.000147383, 0.000145509,
0.000139634, 0.000130288, 0.000140393, 0.000165934, 0.000150473,
0.000133193, 0.0001695, 0.000140701, 0.000157057, 0.000166298, 0.000161074,
0.000146195, 0.000147866, 0.000128872, 0.988162, 0.988359, 0.981767,
1.03073, 0.995803, 0.991643, 1.05086, 0.969885, 0.995799, 1.04009, 1.01,
0.988782, 0.965214, 1.03514, 1.01186, 1.01487, 0.997826, 1.01909, 0.976864,
0.969848, 0.978366, 1.02317, 0.999135, 1.00615, 1.0117, 1499.87, 2.00153}

In[•]:= **parR = %**

Out[•]= {0.201708, 0.185204, 0.208104, 0.205954, 0.218417, 0.0487218, 0.0501919,
0.0509234, 0.0494254, 0.051401, 0.0508852, 0.0505626, 0.0495632, 0.0500902,
0.0492636, 0.0493688, 0.0495445, 0.0504626, 0.050156, 0.0511484, 0.049766,
0.0503801, 0.0497853, 0.0479322, 0.0497703, 0.0508918, 0.0512489,
0.0492538, 0.0492022, 0.0506193, 0.289793, 0.300632, 0.295286, 0.302297,
0.304867, 0.000147542, 0.000137388, 0.000152496, 0.000138806, 0.000131574,
0.000139522, 0.000158137, 0.00016046, 0.000149519, 0.000147383, 0.000145509,
0.000139634, 0.000130288, 0.000140393, 0.000165934, 0.000150473,
0.000133193, 0.0001695, 0.000140701, 0.000157057, 0.000166298, 0.000161074,
0.000146195, 0.000147866, 0.000128872, 0.988162, 0.988359, 0.981767,
1.03073, 0.995803, 0.991643, 1.05086, 0.969885, 0.995799, 1.04009, 1.01,
0.988782, 0.965214, 1.03514, 1.01186, 1.01487, 0.997826, 1.01909, 0.976864,
0.969848, 0.978366, 1.02317, 0.999135, 1.00615, 1.0117, 1499.87, 2.00153}

In[•]:=
```
fNewSaitoRTmax[Net_, Dh_, parR_, tmax_] := (

  dB1 =
   B1[t] (-B1[t] κ1 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
        M5[t]/(denK + M5[t])) - (c1,1 + c1,2 + c1,3 + c1,4 + c1,5 + Dh) B1[t];
  dB2 = B2[t] (-B2[t] κ2 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
```

```
        M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c2,1 + c2,2 + c2,3 + c2,4 + c2,5 + Dh) B2[t];
dB3 = B3[t] (-B3[t] κ3 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
        M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c3,1 + c3,2 + c3,3 + c3,4 + c3,5 + Dh) B3[t];
dB4 = B4[t] (-B4[t] κ4 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
        M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c4,1 + c4,2 + c4,3 + c4,4 + c4,5 + Dh) B4[t];
dB5 = B5[t] (-B5[t] κ5 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
        M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c5,1 + c5,2 + c5,3 + c5,4 + c5,5 + Dh) B5[t];

dM1 = -M1[t] q1 +
   (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t]))
    (-B1[t] d1,1 - B2[t] d1,2 - B3[t] d1,3 - B4[t] d1,4 - B5[t] d1,5) +
  B1[t] Ω1,1 + B2[t] Ω1,2 + B3[t] Ω1,3 + B4[t] Ω1,4 + B5[t] Ω1,5;
dM2 = -M2[t] q2 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) (-B1[t] d2,1 - B2[t] d2,2 - B3[t] d2,3 - B4[t] d2,4 - B5[t] d2,5) +
  B1[t] Ω2,1 + B2[t] Ω2,2 + B3[t] Ω2,3 + B4[t] Ω2,4 + B5[t] Ω2,5;
dM3 = -M3[t] q3 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) (-B1[t] d3,1 - B2[t] d3,2 - B3[t] d3,3 - B4[t] d3,4 - B5[t] d3,5) +
  B1[t] Ω3,1 + B2[t] Ω3,2 + B3[t] Ω3,3 + B4[t] Ω3,4 + B5[t] Ω3,5;
dM4 = -M4[t] q4 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) (-B1[t] d4,1 - B2[t] d4,2 - B3[t] d4,3 - B4[t] d4,4 - B5[t] d4,5) +
  B1[t] Ω4,1 + B2[t] Ω4,2 + B3[t] Ω4,3 + B4[t] Ω4,4 + B5[t] Ω4,5;
dM5 = -M5[t] q5 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) (-B1[t] d5,1 - B2[t] d5,2 - B3[t] d5,3 - B4[t] d5,4 - B5[t] d5,5) +
  B1[t] Ω5,1 + B2[t] Ω5,2 + B3[t] Ω5,3 + B4[t] Ω5,4 + B5[t] Ω5,5;

par = {
```

```
κ1 → parR[[1]], κ2 → parR[[2]], κ3 → parR[[3]], κ4 → parR[[4]], κ5 → parR[[5]],

c1,1 → parR[[6]] × Net[[1]][[1]],
c1,2 → parR[[7]] × Net[[1]][[2]], c1,3 → parR[[8]] × Net[[1]][[3]],
c1,4 → parR[[9]] × Net[[1]][[4]], c1,5 → parR[[10]] × Net[[1]][[5]],
c2,1 → parR[[11]] × Net[[2]][[1]], c2,2 → parR[[12]] × Net[[2]][[2]],
c2,3 → parR[[13]] × Net[[2]][[3]], c2,4 → parR[[14]] × Net[[2]][[4]],
c2,5 → parR[[15]] × Net[[2]][[5]],
c3,1 → parR[[16]] × Net[[3]][[1]], c3,2 → parR[[17]] × Net[[3]][[2]],
c3,3 → parR[[18]] × Net[[3]][[3]], c3,4 → parR[[19]] × Net[[3]][[4]],
c3,5 → parR[[20]] × Net[[3]][[5]],
c4,1 → parR[[21]] × Net[[4]][[1]], c4,2 → parR[[22]] × Net[[4]][[2]],
c4,3 → parR[[23]] × Net[[4]][[3]], c4,4 → parR[[24]] × Net[[4]][[4]],
c4,5 → parR[[25]] × Net[[4]][[5]],
c5,1 → parR[[26]] × Net[[5]][[1]], c5,2 → parR[[27]] × Net[[5]][[2]],
c5,3 → parR[[28]] × Net[[5]][[3]], c5,4 → parR[[29]] × Net[[5]][[4]],
c5,5 → parR[[30]] × Net[[5]][[5]],

q1 → parR[[31]], q2 → parR[[32]],
q3 → parR[[33]], q4 → parR[[34]], q5 → parR[[35]],

d1,1 → parR[[36]], d1,2 → parR[[37]],
d1,3 → parR[[38]], d1,4 → parR[[39]], d1,5 → parR[[40]],
d2,1 → parR[[41]], d2,2 → parR[[42]], d2,3 → parR[[43]],
d2,4 → parR[[44]], d2,5 → parR[[45]],
d3,1 → parR[[46]], d3,2 → parR[[47]], d3,3 → parR[[48]],
d3,4 → parR[[49]], d3,5 → parR[[50]],
d4,1 → parR[[51]], d4,2 → parR[[52]], d4,3 → parR[[53]],
d4,4 → parR[[54]], d4,5 → parR[[55]],
d5,1 → parR[[56]], d5,2 → parR[[57]], d5,3 → parR[[58]],
d5,4 → parR[[59]], d5,5 → parR[[60]],

Ω1,1 → parR[[61]] × Net[[1]][[1]],
Ω1,2 → parR[[62]] × Net[[1]][[2]], Ω1,3 → parR[[63]] × Net[[1]][[3]],
Ω1,4 → parR[[64]] × Net[[1]][[4]], Ω1,5 → parR[[65]] × Net[[1]][[5]],
Ω2,1 → parR[[66]] × Net[[2]][[1]], Ω2,2 → parR[[67]] × Net[[2]][[2]],
Ω2,3 → parR[[68]] × Net[[2]][[3]], Ω2,4 → parR[[69]] × Net[[2]][[4]],
Ω2,5 → parR[[70]] × Net[[2]][[5]],
Ω3,1 → parR[[71]] × Net[[3]][[1]], Ω3,2 → parR[[72]] × Net[[3]][[2]],
Ω3,3 → parR[[73]] × Net[[3]][[3]], Ω3,4 → parR[[74]] × Net[[3]][[4]],
Ω3,5 → parR[[75]] × Net[[3]][[5]],
Ω4,1 → parR[[76]] × Net[[4]][[1]], Ω4,2 → parR[[77]] × Net[[4]][[2]],
Ω4,3 → parR[[78]] × Net[[4]][[3]], Ω4,4 → parR[[79]] × Net[[4]][[4]],
Ω4,5 → parR[[80]] × Net[[4]][[5]],
```

```
   Ω5,1 → parR[[81]] × Net[[5]][[1]], Ω5,2 → parR[[82]] × Net[[5]][[2]],
   Ω5,3 → parR[[83]] × Net[[5]][[3]], Ω5,4 → parR[[84]] × Net[[5]][[4]],
   Ω5,5 → parR[[85]] × Net[[5]][[5]],
   nuK → parR[[86]],
   denK → parR[[87]]




  };



B10 = 1500;
B20 = 1500;
B30 = 1500;
B40 = 1500;
B50 = 1500;
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;


sol =
 NDSolve[
  {
    B1'[t] == dB1,
    B2'[t] == dB2,
    B3'[t] == dB3,
    B4'[t] == dB4,
    B5'[t] == dB5,

    M1'[t] == dM1,
    M2'[t] == dM2,
    M3'[t] == dM3,
    M4'[t] == dM4,
    M5'[t] == dM5,


    B1[0] == B10,
    B2[0] == B20,
    B3[0] == B30,
```

```
      B4[0] == B40,
      B5[0] == B50,
      M1[0] == M10,
      M2[0] == M20,
      M3[0] == M30,
      M4[0] == M40,
      M5[0] == M50

     } /. par,
    {B1, B2, B3, B4, B5, M1, M2, M3, M4, M5},
    {t, 0, tmax}];

  {B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax],
     M1[tmax], M2[tmax], M3[tmax], M4[tmax], M5[tmax]} /. sol /. par

 )
```

In[•]:=
```
Clear[sol, par]
```

In[•]:=
```
fNewSaitoRTmax[jnl, 0, parR, 10]
```

Out[•]=
```
{{7411.26, 8071.17, 7182.77, 7257.51,
  6844.08, 651.412, 41 108.1, 70 568.4, 89 319., 18 195.1}}
```

```
(*D=500*)
```

In[•]:=
```
fNewSaitoRTmax[jnl, 500, parR, 10]
```

Out[•]=
```
{{4922.12, 5360.22, 4770.13, 4819.69,
  4545.36, 458.675, 27 311.4, 46 863.8, 59 310.9, 12 103.5}}
```

```
(*D=1000*)
```

In[•]:=
```
fNewSaitoRTmax[jnl, 1000, parR, 10]
```

Out[•]=
```
{{-9.78236 × 10^-11, -1.06406 × 10^-10, -1.0062 × 10^-10, -1.02805 × 10^-10,
  -9.4534 × 10^-11, 0.67394, 1.06016, 1.39155, 1.51629, 0.790323}}
```

```
In[•]:= t1Low = fNewSaitoRTmax[jnl, 0, parR, #] & /@ Range[30];

In[•]:= t2Low = fNewSaitoRTmax[jnl, 500, parR, #] & /@ Range[70];

In[•]:= t2High = fNewSaitoRTmax[jnl, 1000, parR, #] & /@ Range[70];
```

```
In[•]:= xsz1 = {#[[1]], #[[2]], #[[3]], #[[4]], #[[5]]} & /@ Flatten[t1Low, 1]
Out[•]= {{7375.3, 8032.02, 7147.92, 7222.29, 6810.88},
 {7394.65, 8053.08, 7166.67, 7241.24, 6828.74},
 {7401.87, 8060.95, 7173.67, 7248.31, 6835.41},
 {7405.55, 8064.96, 7177.24, 7251.92, 6838.81},
 {7407.7, 8067.3, 7179.32, 7254.02, 6840.8},
 {7409.06, 8068.78, 7180.64, 7255.35, 6842.05},
 {7409.95, 8069.75, 7181.5, 7256.22, 6842.87},
 {7410.55, 8070.4, 7182.08, 7256.81, 6843.43},
 {7410.97, 8070.86, 7182.49, 7257.22, 6843.81},
 {7411.26, 8071.17, 7182.77, 7257.51, 6844.08},
 {7411.46, 8071.4, 7182.97, 7257.71, 6844.27},
 {7411.61, 8071.56, 7183.11, 7257.85, 6844.41},
 {7411.72, 8071.67, 7183.21, 7257.95, 6844.5},
 {7411.79, 8071.75, 7183.29, 7258.03, 6844.57},
 {7411.84, 8071.81, 7183.34, 7258.08, 6844.62},
 {7411.88, 8071.85, 7183.38, 7258.12, 6844.66},
 {7411.91, 8071.89, 7183.4, 7258.14, 6844.68},
 {7411.93, 8071.91, 7183.42, 7258.16, 6844.7},
 {7411.95, 8071.92, 7183.44, 7258.18, 6844.72},
 {7411.96, 8071.93, 7183.45, 7258.19, 6844.73},
 {7411.96, 8071.94, 7183.45, 7258.2, 6844.73},
 {7411.97, 8071.95, 7183.46, 7258.2, 6844.74},
 {7411.97, 8071.95, 7183.46, 7258.21, 6844.74},
 {7411.98, 8071.96, 7183.47, 7258.21, 6844.74},
 {7411.98, 8071.96, 7183.47, 7258.21, 6844.75},
 {7411.98, 8071.96, 7183.47, 7258.21, 6844.75},
 {7411.98, 8071.96, 7183.47, 7258.21, 6844.75},
 {7411.98, 8071.96, 7183.47, 7258.21, 6844.75},
 {7411.98, 8071.96, 7183.47, 7258.21, 6844.75},
 {7411.98, 8071.96, 7183.47, 7258.21, 6844.75}}
```

```
In[•]:= xsz1[[1]]
Out[•]= {7375.3, 8032.02, 7147.92, 7222.29, 6810.88}
```

In[•]:=
```
Total[xsz1[[1]]]
```

Out[•]=
```
36 588.4
```

In[•]:=
```
xsz2 = xsz1 / Total[xsz1[[1]]]
```

Out[•]=
```
{{0.201575, 0.219524, 0.19536, 0.197393, 0.186148},
 {0.202104, 0.220099, 0.195873, 0.197911, 0.186637},
 {0.202301, 0.220314, 0.196064, 0.198104, 0.186819},
 {0.202402, 0.220424, 0.196162, 0.198203, 0.186912},
 {0.20246, 0.220488, 0.196219, 0.19826, 0.186966},
 {0.202497, 0.220528, 0.196254, 0.198296, 0.187},
 {0.202522, 0.220555, 0.196278, 0.19832, 0.187023},
 {0.202538, 0.220573, 0.196294, 0.198336, 0.187038},
 {0.20255, 0.220585, 0.196305, 0.198347, 0.187049},
 {0.202557, 0.220594, 0.196313, 0.198355, 0.187056},
 {0.202563, 0.2206, 0.196318, 0.198361, 0.187061},
 {0.202567, 0.220604, 0.196322, 0.198365, 0.187065},
 {0.20257, 0.220607, 0.196325, 0.198368, 0.187067},
 {0.202572, 0.22061, 0.196327, 0.19837, 0.187069},
 {0.202574, 0.220611, 0.196328, 0.198371, 0.187071},
 {0.202575, 0.220612, 0.196329, 0.198372, 0.187072},
 {0.202575, 0.220613, 0.19633, 0.198373, 0.187072},
 {0.202576, 0.220614, 0.196331, 0.198373, 0.187073},
 {0.202576, 0.220614, 0.196331, 0.198374, 0.187073},
 {0.202577, 0.220614, 0.196331, 0.198374, 0.187074},
 {0.202577, 0.220615, 0.196331, 0.198374, 0.187074},
 {0.202577, 0.220615, 0.196332, 0.198374, 0.187074},
 {0.202577, 0.220615, 0.196332, 0.198374, 0.187074},
 {0.202577, 0.220615, 0.196332, 0.198374, 0.187074},
 {0.202577, 0.220615, 0.196332, 0.198375, 0.187074},
 {0.202577, 0.220615, 0.196332, 0.198375, 0.187074},
 {0.202577, 0.220615, 0.196332, 0.198375, 0.187074},
 {0.202577, 0.220615, 0.196332, 0.198375, 0.187074},
 {0.202577, 0.220615, 0.196332, 0.198375, 0.187074},
 {0.202577, 0.220615, 0.196332, 0.198375, 0.187074}}
```

In[•]:=
```
zsz1 = {#[[1]], #[[2]], #[[3]], #[[4]], #[[5]]} & /@ Flatten[t2Low, 1];
```

In[•]:=
```
zsz2 = zsz1 / Total[xsz1[[1]]];
```

In[•]:=
```
xsz2[[1]][[1]]
```

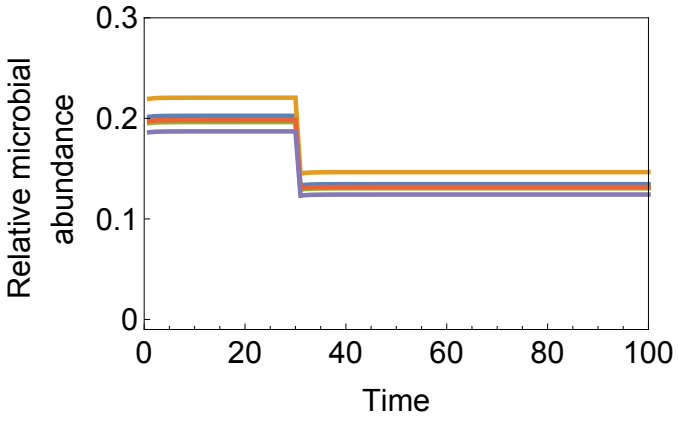
Out[•]=
```
0.201575
```

```
In[•]:= xsz2[[1]][[2]]
Out[•]= 0.219524

In[•]:= xsz2[[1]][[3]]
Out[•]= 0.19536

In[•]:= xsz2[[1]][[4]]
Out[•]= 0.197393

In[•]:= xsz2[[1]][[5]]
Out[•]= 0.186148
```

```
In[•]:= pop1 = Join[
          Partition[Riffle[Range[30], xsz2[[All, 1]]], {2}],
          Partition[Riffle[Range[31, 100], zsz2[[All, 1]]], {2}]
         ];

In[•]:= pop2 = Join[
          Partition[Riffle[Range[30], xsz2[[All, 2]]], {2}],
          Partition[Riffle[Range[31, 100], zsz2[[All, 2]]], {2}]
         ];

In[•]:= pop3 = Join[
          Partition[Riffle[Range[30], xsz2[[All, 3]]], {2}],
          Partition[Riffle[Range[31, 100], zsz2[[All, 3]]], {2}]
         ];

In[•]:= pop4 = Join[
          Partition[Riffle[Range[30], xsz2[[All, 4]]], {2}],
          Partition[Riffle[Range[31, 100], zsz2[[All, 4]]], {2}]
         ];

In[•]:= pop5 = Join[
          Partition[Riffle[Range[30], xsz2[[All, 5]]], {2}],
          Partition[Riffle[Range[31, 100], zsz2[[All, 5]]], {2}]
         ];
```

In[•]:=
```
ListPlot[{pop1, pop2, pop3, pop4, pop5}, Joined → True, Frame → True,
 ImageSize → 350, FrameLabel → {"Time", "Relative microbial \n abundance"},
 FrameStyle → Directive[Black, FontSize → 16],
 PlotStyle → {Thickness[0.009]}, PlotRange → {{0.0, 100}, {-0.01, 0.3}},
 FrameTicks → {{{0, 0.1, 0.2, 0.3}, None}, {Automatic, None}}]
```

Out[•]=



In[•]:=
```
ksz1 = {#[[1]], #[[2]], #[[3]], #[[4]], #[[5]]} & /@ Flatten[t2High, 1]
```

In[•]:=
```
ksz2 = zsz1 / Total[xsz1[[1]]]
```

In[•]:=
```
ksz2[[1]]
```

Out[•]=
```
{0.133247, 0.145107, 0.129133, 0.130474, 0.123048}
```

In[•]:=
```
If[# ≤ 1, 0, #] & /@ ksz2[[1]]
```

Out[•]=
```
{0, 0, 0, 0, 0}
```

In[•]:=
```
ConstantArray[0, {70}]
```

Out[•]=
```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

In[•]:=
```
pop1b = Join[
    Partition[Riffle[Range[30], xsz2[[All, 1]]], {2}],
    Partition[Riffle[Range[31, 100], ConstantArray[0, {70}]], {2}]
   ];
```

In[•]:=
```
pop2b = Join[
    Partition[Riffle[Range[30], xsz2[[All, 2]]], {2}],
    Partition[Riffle[Range[31, 100], ConstantArray[0, {70}]], {2}]
   ];
```

```
In[•]:= pop3b = Join[
          Partition[Riffle[Range[30], xsz2[[All, 3]]], {2}],
          Partition[Riffle[Range[31, 100], ConstantArray[0, {70}]], {2}]
         ];

In[•]:= pop4b = Join[
          Partition[Riffle[Range[30], xsz2[[All, 4]]], {2}],
          Partition[Riffle[Range[31, 100], ConstantArray[0, {70}]], {2}]
         ];

In[•]:= pop5b = Join[
          Partition[Riffle[Range[30], xsz2[[All, 5]]], {2}],
          Partition[Riffle[Range[31, 100], ConstantArray[0, {70}]], {2}]
         ];
```

```
In[•]:= ListPlot[{pop1b, pop2b, pop3b, pop4b, pop5b}, Joined → True, Frame → True,
         ImageSize → 350, FrameLabel → {"Time", "Relative microbial \n abundance"},
         FrameStyle → Directive[Black, FontSize → 16],
         PlotStyle → {Thickness[0.009]}, PlotRange → {{0.0, 100}, {-0.01, 0.3}},
         FrameTicks → {{{0, 0.1, 0.2, 0.3}, None}, {Automatic, None}}]
```

Out[•]=

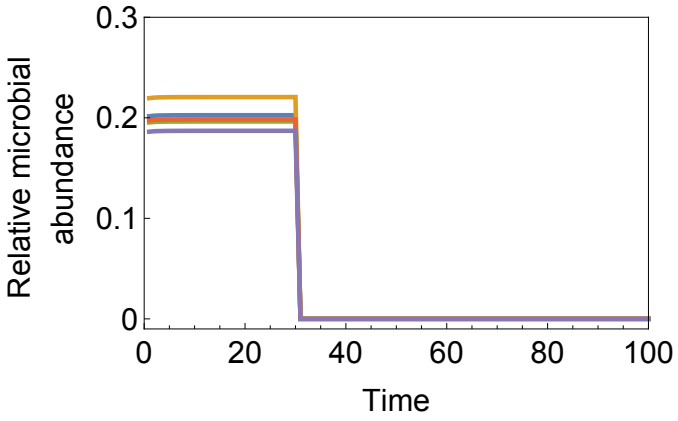


Figure 2

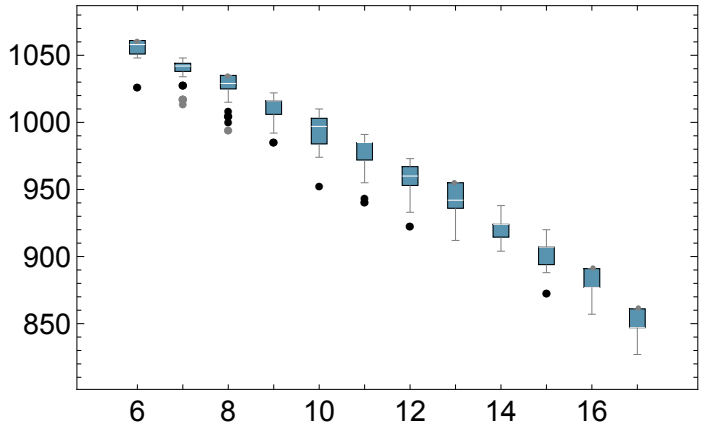
Here we use functions from the “1_ColimitationModel.nb” file.

In[•]:=
```
AuxoComm6 = robustnessNewSaito /@ hk6;
AuxoComm7 = robustnessNewSaito /@ hk7;
AuxoComm8 = robustnessNewSaito /@ hk8;
AuxoComm9 = robustnessNewSaito /@ hk9;
AuxoComm10 = robustnessNewSaito /@ hk10;
AuxoComm11 = robustnessNewSaito /@ hk11;
AuxoComm12 = robustnessNewSaito /@ hk12;
AuxoComm13 = robustnessNewSaito /@ hk13;
AuxoComm14 = robustnessNewSaito /@ hk14;
AuxoComm15 = robustnessNewSaito /@ hk15;
AuxoComm16 = robustnessNewSaito /@ hk16;
AuxoComm17 = robustnessNewSaito /@ hk17;
```

In[•]:=
```
Lik = {AuxoComm6, AuxoComm7, AuxoComm8, AuxoComm9, AuxoComm10, AuxoComm11,
   AuxoComm12, AuxoComm13, AuxoComm14, AuxoComm15, AuxoComm16, AuxoComm17};
```

In[•]:=
```
BoxWhiskerChart[Lik, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{coco}}, Frame → True,
 ChartLabels → {"6", "", "8", "", "10", "", "12", "", "14", "", "16", ""},
 BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15]]
```

Out[•]=



For a system of 6x6:

In[•]:=
```
fNewSaito6[Net_, Dh_] := (

  dB1 =
   B1[t] (-B1[t] κ1 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
```

```
        M5[t]/(denK + M5[t]) * M6[t]/(denK + M6[t])) - (c1,1 + c1,2 + c1,3 + c1,4 + c1,5 + c1,6 + Dh) B1[t];
dB2 = B2[t] (-B2[t] κ2 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) *
        M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t]) * M6[t]/(denK + M6[t])) -
   (c2,1 + c2,2 + c2,3 + c2,4 + c2,5 + c2,6 + Dh) B2[t];
dB3 = B3[t] (-B3[t] κ3 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) *
        M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t]) * M6[t]/(denK + M6[t])) -
   (c3,1 + c3,2 + c3,3 + c3,4 + c3,5 + c3,6 + Dh) B3[t];
dB4 = B4[t] (-B4[t] κ4 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) *
        M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t]) * M6[t]/(denK + M6[t])) -
   (c4,1 + c4,2 + c4,3 + c4,4 + c4,5 + c4,6 + Dh) B4[t];
dB5 = B5[t] (-B5[t] κ5 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) *
        M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t]) * M6[t]/(denK + M6[t])) -
   (c5,1 + c5,2 + c5,3 + c5,4 + c5,5 + c5,6 + Dh) B5[t];
dB6 = B6[t] (-B6[t] κ6 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) *
        M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t]) * M6[t]/(denK + M6[t])) -
   (c6,1 + c6,2 + c6,3 + c6,4 + c6,5 + c6,6 + Dh) B6[t];

dM1 = -M1[t] q1 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) *
        M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t]) * M6[t]/(denK + M6[t]))
    (-B1[t] d1,1 - B2[t] d1,2 - B3[t] d1,3 - B4[t] d1,4 - B5[t] d1,5 - B6[t] d1,6) +
   B1[t] Ω1,1 + B2[t] Ω1,2 + B3[t] Ω1,3 + B4[t] Ω1,4 + B5[t] Ω1,5 + B6[t] Ω1,6;
dM2 = -M2[t] q2 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) *
        M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t]) * M6[t]/(denK + M6[t]))
    (-B1[t] d2,1 - B2[t] d2,2 - B3[t] d2,3 - B4[t] d2,4 - B5[t] d2,5 - B6[t] d2,6) +
   B1[t] Ω2,1 + B2[t] Ω2,2 + B3[t] Ω2,3 + B4[t] Ω2,4 + B5[t] Ω2,5 + B6[t] Ω2,6;
dM3 = -M3[t] q3 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) *
        M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t]) * M6[t]/(denK + M6[t]))
```

```
    (-B1[t] d3,1 - B2[t] d3,2 - B3[t] d3,3 - B4[t] d3,4 - B5[t] d3,5 - B6[t] d3,6) +
   B1[t] Ω3,1 + B2[t] Ω3,2 + B3[t] Ω3,3 + B4[t] Ω3,4 + B5[t] Ω3,5 + B6[t] Ω3,6;
dM4 = -M4[t] q4 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) *
      M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t]) * M6[t]/(denK + M6[t]))
    (-B1[t] d4,1 - B2[t] d4,2 - B3[t] d4,3 - B4[t] d4,4 - B5[t] d4,5 - B6[t] d4,6) +
   B1[t] Ω4,1 + B2[t] Ω4,2 + B3[t] Ω4,3 + B4[t] Ω4,4 + B5[t] Ω4,5 + B6[t] Ω4,6;
dM5 = -M5[t] q5 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) *
      M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t]) * M6[t]/(denK + M6[t]))
    (-B1[t] d5,1 - B2[t] d5,2 - B3[t] d5,3 - B4[t] d5,4 - B5[t] d5,5 - B6[t] d5,6) +
   B1[t] Ω5,1 + B2[t] Ω5,2 + B3[t] Ω5,3 + B4[t] Ω5,4 + B5[t] Ω5,5 + B6[t] Ω5,6;
dM6 = -M6[t] q6 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) *
      M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t]) * M6[t]/(denK + M6[t]))
    (-B1[t] d6,1 - B2[t] d6,2 - B3[t] d6,3 - B4[t] d6,4 - B5[t] d6,5 - B6[t] d6,6) +
   B1[t] Ω6,1 + B2[t] Ω6,2 + B3[t] Ω6,3 + B4[t] Ω6,4 + B5[t] Ω6,5 + B6[t] Ω6,6;

KK = 0.2;
cc = 0.05;
qq = 0.3;
dd = 0.00015;
OM = 1;
nu = 1500;
den = 2;

tmax = 1000;
par = {
  κ1 → KK, κ2 → KK, κ3 → KK, κ4 → KK, κ5 → KK, κ6 → KK,

  c1,1 → cc Net[[1]][[1]], c1,2 → cc Net[[1]][[2]], c1,3 → cc Net[[1]][[3]],
  c1,4 → cc Net[[1]][[4]], c1,5 → cc Net[[1]][[5]], c1,6 → cc Net[[1]][[6]],
  c2,1 → cc Net[[2]][[1]], c2,2 → cc Net[[2]][[2]], c2,3 → cc Net[[2]][[3]],
  c2,4 → cc Net[[2]][[4]], c2,5 → cc Net[[2]][[5]], c2,6 → cc Net[[2]][[6]],
  c3,1 → cc Net[[3]][[1]], c3,2 → cc Net[[3]][[2]], c3,3 → cc Net[[3]][[3]],
  c3,4 → cc Net[[3]][[4]], c3,5 → cc Net[[3]][[5]], c3,6 → cc Net[[3]][[6]],
  c4,1 → cc Net[[4]][[1]], c4,2 → cc Net[[4]][[2]], c4,3 → cc Net[[4]][[3]],
  c4,4 → cc Net[[4]][[4]], c4,5 → cc Net[[4]][[5]], c4,6 → cc Net[[4]][[6]],
  c5,1 → cc Net[[5]][[1]], c5,2 → cc Net[[5]][[2]], c5,3 → cc Net[[5]][[3]],
  c5,4 → cc Net[[5]][[4]], c5,5 → cc Net[[5]][[5]], c5,6 → cc Net[[5]][[6]],
```

```
  c6,1 → cc  Net[[6]][[1]], c6,2 → cc Net[[6]][[2]], c6,3 → cc Net[[6]][[3]],
  c6,4 → cc Net[[6]][[4]], c6,5 → cc Net[[6]][[5]], c6,6 → cc Net[[6]][[6]],

  q1 → qq, q2 → qq, q3 → qq, q4 → qq, q5 → qq, q6 → qq,

  d1,1 → dd, d1,2 → dd, d1,3 → dd, d1,4 → dd, d1,5 → dd, d1,6 → dd,
  d2,1 → dd, d2,2 → dd, d2,3 → dd, d2,4 → dd, d2,5 → dd, d2,6 → dd,
  d3,1 → dd, d3,2 → dd, d3,3 → dd, d3,4 → dd, d3,5 → dd, d3,6 → dd,
  d4,1 → dd, d4,2 → dd, d4,3 → dd, d4,4 → dd, d4,5 → dd, d4,6 → dd,
  d5,1 → dd, d5,2 → dd, d5,3 → dd, d5,4 → dd, d5,5 → dd, d5,6 → dd,
  d6,1 → dd, d6,2 → dd, d6,3 → dd, d6,4 → dd, d6,5 → dd, d6,6 → dd,

  Ω1,1 → OM Net[[1]][[1]], Ω1,2 → OM Net[[1]][[2]], Ω1,3 → OM Net[[1]][[3]],
  Ω1,4 → OM Net[[1]][[4]], Ω1,5 → OM Net[[1]][[5]], Ω1,6 → OM Net[[1]][[6]],
  Ω2,1 → OM Net[[2]][[1]], Ω2,2 → OM Net[[2]][[2]], Ω2,3 → OM Net[[2]][[3]],
  Ω2,4 → OM Net[[2]][[4]], Ω2,5 → OM Net[[2]][[5]], Ω2,6 → OM Net[[2]][[6]],
  Ω3,1 → OM Net[[3]][[1]], Ω3,2 → OM Net[[3]][[2]], Ω3,3 → OM Net[[3]][[3]],
  Ω3,4 → OM Net[[3]][[4]], Ω3,5 → OM Net[[3]][[5]], Ω3,6 → OM Net[[3]][[6]],
  Ω4,1 → OM Net[[4]][[1]], Ω4,2 → OM Net[[4]][[2]], Ω4,3 → OM Net[[4]][[3]],
  Ω4,4 → OM Net[[4]][[4]], Ω4,5 → OM Net[[4]][[5]], Ω4,6 → OM Net[[4]][[6]],
  Ω5,1 → OM Net[[5]][[1]], Ω5,2 → OM Net[[5]][[2]], Ω5,3 → OM Net[[5]][[3]],
  Ω5,4 → OM Net[[5]][[4]], Ω5,5 → OM Net[[5]][[5]], Ω5,6 → OM Net[[5]][[6]],
  Ω6,1 → OM Net[[6]][[1]], Ω6,2 → OM Net[[6]][[2]], Ω6,3 → OM Net[[6]][[3]],
  Ω6,4 → OM Net[[6]][[4]], Ω6,5 → OM Net[[6]][[5]], Ω6,6 → OM Net[[6]][[6]],
  nuK → nu,
  denK → den


 };


B10 = 1500;
B20 = 1500;
B30 = 1500;
B40 = 1500;
B50 = 1500;
B60 = 1500;
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;
M60 = 10;
```

```
sol =
 NDSolve[
  {
    B1'[t] == dB1,
    B2'[t] == dB2,
    B3'[t] == dB3,
    B4'[t] == dB4,
    B5'[t] == dB5,
    B6'[t] == dB6,

    M1'[t] == dM1,
    M2'[t] == dM2,
    M3'[t] == dM3,
    M4'[t] == dM4,
    M5'[t] == dM5,
    M6'[t] == dM6,


    B1[0] == B10,
    B2[0] == B20,
    B3[0] == B30,
    B4[0] == B40,
    B5[0] == B50,
    B6[0] == B60,
    M1[0] == M10,
    M2[0] == M20,
    M3[0] == M30,
    M4[0] == M40,
    M5[0] == M50,
    M6[0] == M60

   } /. par,
  {B1, B2, B3, B4, B5, B6, M1, M2, M3, M4, M5, M6},
  {t, 0, tmax}];

{B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax], B6[tmax], M1[tmax],
   M2[tmax], M3[tmax], M4[tmax], M5[tmax], M6[tmax]} /. sol /. par;

Min[{B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax], B6[tmax]} /. sol /. par]
```

In[•]:=
```
robustnessNewSaito6[NetTop_] := (
  n1 = 1;
  n2 = 5000;
  mid = (n1 + n2) / 2;

  While[(n1 ≠ mid && n2 ≠ mid),
   (If[fNewSaito6[NetTop, mid] < 1, n2 = mid, n1 = mid];
    mid = Floor[N[(n1 + n2) / 2]];); {n1, n2, mid}]; mid

 )
```

In[•]:=
```
N6 = {
   {1, 1, 1, 1, 1, 1},
   {0, 0, 0, 1, 0, 0},
   {0, 1, 1, 1, 1, 1},
   {1, 0, 0, 1, 0, 1},
   {1, 1, 1, 0, 0, 0},
   {1, 1, 1, 1, 0, 1}};
```

In[•]:=
```
fNewSaito6[N6, 0]
```
Out[•]= 5552.55

In[•]:=
```
robustnessNewSaito6[N6]
```
Out[•]= 938

In[5702]:=
```
hk6a14 = NestWhile[AppNe[#, 6, 14] &, {NetWorkGen[6, 14]}, Length[#] ≠ 100 &];
hk6a15 = NestWhile[AppNe[#, 6, 15] &, {NetWorkGen[6, 15]}, Length[#] ≠ 100 &];
hk6a16 = NestWhile[AppNe[#, 6, 16] &, {NetWorkGen[6, 16]}, Length[#] ≠ 100 &];
hk6a17 = NestWhile[AppNe[#, 6, 17] &, {NetWorkGen[6, 17]}, Length[#] ≠ 100 &];
hk6a18 = NestWhile[AppNe[#, 6, 18] &, {NetWorkGen[6, 18]}, Length[#] ≠ 100 &];
hk6a19 = NestWhile[AppNe[#, 6, 19] &, {NetWorkGen[6, 19]}, Length[#] ≠ 100 &];
hk6a20 = NestWhile[AppNe[#, 6, 20] &, {NetWorkGen[6, 20]}, Length[#] ≠ 100 &];
hk6a21 = NestWhile[AppNe[#, 6, 21] &, {NetWorkGen[6, 21]}, Length[#] ≠ 100 &];
hk6a22 = NestWhile[AppNe[#, 6, 22] &, {NetWorkGen[6, 22]}, Length[#] ≠ 100 &];
hk6a23 = NestWhile[AppNe[#, 6, 23] &, {NetWorkGen[6, 23]}, Length[#] ≠ 100 &];
hk6a24 = NestWhile[AppNe[#, 6, 24] &, {NetWorkGen[6, 24]}, Length[#] ≠ 100 &];
hk6a25 = NestWhile[AppNe[#, 6, 25] &, {NetWorkGen[6, 25]}, Length[#] ≠ 100 &];
hk6a26 = NestWhile[AppNe[#, 6, 26] &, {NetWorkGen[6, 26]}, Length[#] ≠ 100 &];
```

In[5715]:=
```
Rob6Aux14 = robustnessNewSaito6 /@ hk6a14;
Rob6Aux15 = robustnessNewSaito6 /@ hk6a15;
Rob6Aux16 = robustnessNewSaito6 /@ hk6a16;
Rob6Aux17 = robustnessNewSaito6 /@ hk6a17;
Rob6Aux18 = robustnessNewSaito6 /@ hk6a18;
Rob6Aux19 = robustnessNewSaito6 /@ hk6a19;
Rob6Aux20 = robustnessNewSaito6 /@ hk6a20;
Rob6Aux21 = robustnessNewSaito6 /@ hk6a21;
Rob6Aux22 = robustnessNewSaito6 /@ hk6a22;
Rob6Aux23 = robustnessNewSaito6 /@ hk6a23;
Rob6Aux24 = robustnessNewSaito6 /@ hk6a24;
Rob6Aux25 = robustnessNewSaito6 /@ hk6a25;
Rob6Aux26 = robustnessNewSaito6 /@ hk6a26;
```

In[5728]:=
```
Lik6 = {Rob6Aux14, Rob6Aux15, Rob6Aux16, Rob6Aux17, Rob6Aux18, Rob6Aux19, Rob6Aux20,
   Rob6Aux21, Rob6Aux22, Rob6Aux23, Rob6Aux24, Rob6Aux25, Rob6Aux26};
```

In[5729]:=
```
BoxWhiskerChart[Lik6, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{coco}}, Frame → True,
 ChartLabels → {"14", "", "16", "", "18", "", "20", "", "22", "", "24", "", "25", ""},
 BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15]]
```

Out[5729]=

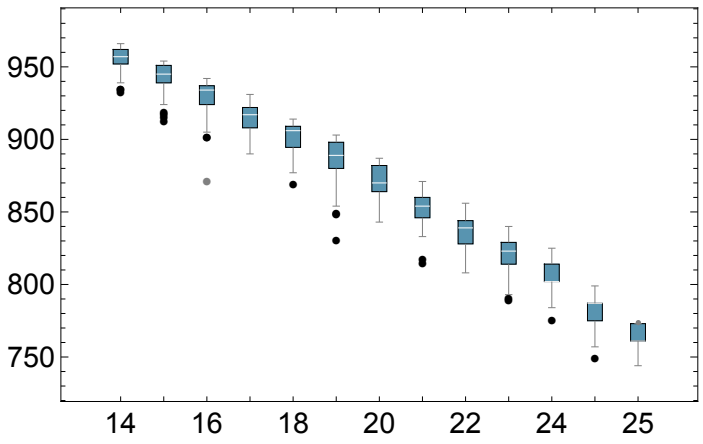


Figure 3

In[ ]:=

```
fNewSaitoPlotFig3[Net_, Dh_, tmax_] := (

 dB1 =
  B1[t] (-B1[t] κ1 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) - (c1,1 + c1,2 + c1,3 + c1,4 + c1,5 + Dh) B1[t];
 dB2 = B2[t] (-B2[t] κ2 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
      M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c2,1 + c2,2 + c2,3 + c2,4 + c2,5 + Dh) B2[t];
 dB3 = B3[t] (-B3[t] κ3 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
      M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c3,1 + c3,2 + c3,3 + c3,4 + c3,5 + Dh) B3[t];
 dB4 = B4[t] (-B4[t] κ4 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
      M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c4,1 + c4,2 + c4,3 + c4,4 + c4,5 + Dh) B4[t];
 dB5 = B5[t] (-B5[t] κ5 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
      M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c5,1 + c5,2 + c5,3 + c5,4 + c5,5 + Dh) B5[t];

 dM1 = -M1[t] q1 +
   (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t]))
     (-B1[t] d1,1 - B2[t] d1,2 - B3[t] d1,3 - B4[t] d1,4 - B5[t] d1,5) +
   B1[t] Ω1,1 + B2[t] Ω1,2 + B3[t] Ω1,3 + B4[t] Ω1,4 + B5[t] Ω1,5;
 dM2 = -M2[t] q2 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) (-B1[t] d2,1 - B2[t] d2,2 - B3[t] d2,3 - B4[t] d2,4 - B5[t] d2,5) +
   B1[t] Ω2,1 + B2[t] Ω2,2 + B3[t] Ω2,3 + B4[t] Ω2,4 + B5[t] Ω2,5;
 dM3 = -M3[t] q3 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
```

```
        M5[t]/(denK + M5[t])) (-B1[t] d3,1 - B2[t] d3,2 - B3[t] d3,3 - B4[t] d3,4 - B5[t] d3,5) +
   B1[t] Ω3,1 + B2[t] Ω3,2 + B3[t] Ω3,3 + B4[t] Ω3,4 + B5[t] Ω3,5;
dM4 = -M4[t] q4 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
        M5[t]/(denK + M5[t])) (-B1[t] d4,1 - B2[t] d4,2 - B3[t] d4,3 - B4[t] d4,4 - B5[t] d4,5) +
   B1[t] Ω4,1 + B2[t] Ω4,2 + B3[t] Ω4,3 + B4[t] Ω4,4 + B5[t] Ω4,5;
dM5 = -M5[t] q5 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
        M5[t]/(denK + M5[t])) (-B1[t] d5,1 - B2[t] d5,2 - B3[t] d5,3 - B4[t] d5,4 - B5[t] d5,5) +
   B1[t] Ω5,1 + B2[t] Ω5,2 + B3[t] Ω5,3 + B4[t] Ω5,4 + B5[t] Ω5,5;

KK = 0.2;
cc = 0.05;
qq = 0.3;
dd = 0.00015;
OM = 1;
nu = 1500;
den = 2;


par = {
  κ1 → KK, κ2 → KK, κ3 → KK, κ4 → KK, κ5 → KK,

  c1,1 → cc Net[[1]][[1]], c1,2 → cc Net[[1]][[2]],
  c1,3 → cc Net[[1]][[3]], c1,4 → cc Net[[1]][[4]], c1,5 → cc Net[[1]][[5]],
  c2,1 → cc Net[[2]][[1]], c2,2 → cc Net[[2]][[2]], c2,3 → cc Net[[2]][[3]],
  c2,4 → cc Net[[2]][[4]], c2,5 → cc Net[[2]][[5]],
  c3,1 → cc Net[[3]][[1]], c3,2 → cc Net[[3]][[2]], c3,3 → cc Net[[3]][[3]],
  c3,4 → cc Net[[3]][[4]], c3,5 → cc Net[[3]][[5]],
  c4,1 → cc Net[[4]][[1]], c4,2 → cc Net[[4]][[2]], c4,3 → cc Net[[4]][[3]],
  c4,4 → cc Net[[4]][[4]], c4,5 → cc Net[[4]][[5]],
  c5,1 → cc Net[[5]][[1]], c5,2 → cc Net[[5]][[2]], c5,3 → cc Net[[5]][[3]],
  c5,4 → cc Net[[5]][[4]], c5,5 → cc Net[[5]][[5]],

  q1 → qq, q2 → qq, q3 → qq, q4 → qq, q5 → qq,

  d1,1 → dd, d1,2 → dd, d1,3 → dd, d1,4 → dd, d1,5 → dd,
  d2,1 → dd, d2,2 → dd, d2,3 → dd, d2,4 → dd, d2,5 → dd,
  d3,1 → dd, d3,2 → dd, d3,3 → dd, d3,4 → dd, d3,5 → dd,
  d4,1 → dd, d4,2 → dd, d4,3 → dd, d4,4 → dd, d4,5 → dd,
  d5,1 → dd, d5,2 → dd, d5,3 → dd, d5,4 → dd, d5,5 → dd,
```

```
  Ω1,1 → OM Net[[1]][[1]], Ω1,2 → OM Net[[1]][[2]],
  Ω1,3 → OM Net[[1]][[3]], Ω1,4 → OM Net[[1]][[4]], Ω1,5 → OM Net[[1]][[5]],
  Ω2,1 → OM Net[[2]][[1]], Ω2,2 → OM Net[[2]][[2]], Ω2,3 → OM Net[[2]][[3]],
  Ω2,4 → OM Net[[2]][[4]], Ω2,5 → OM Net[[2]][[5]],
  Ω3,1 → OM Net[[3]][[1]], Ω3,2 → OM Net[[3]][[2]], Ω3,3 → OM Net[[3]][[3]],
  Ω3,4 → OM Net[[3]][[4]], Ω3,5 → OM Net[[3]][[5]],
  Ω4,1 → OM Net[[4]][[1]], Ω4,2 → OM Net[[4]][[2]], Ω4,3 → OM Net[[4]][[3]],
  Ω4,4 → OM Net[[4]][[4]], Ω4,5 → OM Net[[4]][[5]],
  Ω5,1 → OM Net[[5]][[1]], Ω5,2 → OM Net[[5]][[2]], Ω5,3 → OM Net[[5]][[3]],
  Ω5,4 → OM Net[[5]][[4]], Ω5,5 → OM Net[[5]][[5]],
  nuK → nu,
  denK → den


 };


B10 = 1500;
B20 = 1500;
B30 = 1500;
B40 = 1500;
B50 = 1500;
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;


sol =
 NDSolve[
  {
    B1'[t] == dB1,
    B2'[t] == dB2,
    B3'[t] == dB3,
    B4'[t] == dB4,
    B5'[t] == dB5,

    M1'[t] == dM1,
    M2'[t] == dM2,
    M3'[t] == dM3,
    M4'[t] == dM4,
    M5'[t] == dM5,
```

```
    B1[0] == B10,
    B2[0] == B20,
    B3[0] == B30,
    B4[0] == B40,
    B5[0] == B50,
    M1[0] == M10,
    M2[0] == M20,
    M3[0] == M30,
    M4[0] == M40,
    M5[0] == M50

   } /. par,
  {B1, B2, B3, B4, B5, M1, M2, M3, M4, M5},
  {t, 0, tmax}];

{B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax],
   M1[tmax], M2[tmax], M3[tmax], M4[tmax], M5[tmax]} /. sol /. par

(*Min[{B1[tmax],B2[tmax],B3[tmax],B4[tmax],B5[tmax]}/.sol/.par]*)

)
```

The function “fNewSaito” solves the ODE system and gives the lowest microbial population size (this is used to calculate the Robustness). The function “fNewSaito” receives a network and a disturbance value as arguments.

In[ ]:=

```
fNewSaito[Net_, Dh_] := (

 dB1 =
  B1[t] (-B1[t] κ1 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) - (c1,1 + c1,2 + c1,3 + c1,4 + c1,5 + Dh) B1[t];
 dB2 = B2[t] (-B2[t] κ2 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
      M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c2,1 + c2,2 + c2,3 + c2,4 + c2,5 + Dh) B2[t];
 dB3 = B3[t] (-B3[t] κ3 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
```

```
        M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c3,1 + c3,2 + c3,3 + c3,4 + c3,5 + Dh) B3[t];
dB4 = B4[t] (-B4[t] κ4 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
        M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c4,1 + c4,2 + c4,3 + c4,4 + c4,5 + Dh) B4[t];
dB5 = B5[t] (-B5[t] κ5 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
        M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c5,1 + c5,2 + c5,3 + c5,4 + c5,5 + Dh) B5[t];

dM1 = -M1[t] q1 +
   (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t]))
    (-B1[t] d1,1 - B2[t] d1,2 - B3[t] d1,3 - B4[t] d1,4 - B5[t] d1,5) +
   B1[t] Ω1,1 + B2[t] Ω1,2 + B3[t] Ω1,3 + B4[t] Ω1,4 + B5[t] Ω1,5;
dM2 = -M2[t] q2 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) (-B1[t] d2,1 - B2[t] d2,2 - B3[t] d2,3 - B4[t] d2,4 - B5[t] d2,5) +
   B1[t] Ω2,1 + B2[t] Ω2,2 + B3[t] Ω2,3 + B4[t] Ω2,4 + B5[t] Ω2,5;
dM3 = -M3[t] q3 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) (-B1[t] d3,1 - B2[t] d3,2 - B3[t] d3,3 - B4[t] d3,4 - B5[t] d3,5) +
   B1[t] Ω3,1 + B2[t] Ω3,2 + B3[t] Ω3,3 + B4[t] Ω3,4 + B5[t] Ω3,5;
dM4 = -M4[t] q4 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) (-B1[t] d4,1 - B2[t] d4,2 - B3[t] d4,3 - B4[t] d4,4 - B5[t] d4,5) +
   B1[t] Ω4,1 + B2[t] Ω4,2 + B3[t] Ω4,3 + B4[t] Ω4,4 + B5[t] Ω4,5;
dM5 = -M5[t] q5 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) (-B1[t] d5,1 - B2[t] d5,2 - B3[t] d5,3 - B4[t] d5,4 - B5[t] d5,5) +
   B1[t] Ω5,1 + B2[t] Ω5,2 + B3[t] Ω5,3 + B4[t] Ω5,4 + B5[t] Ω5,5;

KK = 0.2;
cc = 0.05;
qq = 0.3;
dd = 0.00015;
OM = 1;
nu = 1500;
den = 2;
```

```
tmax = 1000;
par = {
  κ1 → KK, κ2 → KK, κ3 → KK, κ4 → KK, κ5 → KK,

  c1,1 → cc Net[[1]][[1]], c1,2 → cc Net[[1]][[2]],
  c1,3 → cc Net[[1]][[3]], c1,4 → cc Net[[1]][[4]], c1,5 → cc Net[[1]][[5]],
  c2,1 → cc Net[[2]][[1]], c2,2 → cc Net[[2]][[2]], c2,3 → cc Net[[2]][[3]],
  c2,4 → cc Net[[2]][[4]], c2,5 → cc Net[[2]][[5]],
  c3,1 → cc Net[[3]][[1]], c3,2 → cc Net[[3]][[2]], c3,3 → cc Net[[3]][[3]],
  c3,4 → cc Net[[3]][[4]], c3,5 → cc Net[[3]][[5]],
  c4,1 → cc Net[[4]][[1]], c4,2 → cc  Net[[4]][[2]], c4,3 → cc  Net[[4]][[3]],
  c4,4 → cc  Net[[4]][[4]], c4,5 → cc  Net[[4]][[5]],
  c5,1 → cc  Net[[5]][[1]], c5,2 → cc Net[[5]][[2]], c5,3 → cc Net[[5]][[3]],
  c5,4 → cc Net[[5]][[4]], c5,5 → cc Net[[5]][[5]],

  q1 → qq, q2 → qq, q3 → qq, q4 → qq, q5 → qq,

  d1,1 → dd, d1,2 → dd, d1,3 → dd, d1,4 → dd, d1,5 → dd,
  d2,1 → dd, d2,2 → dd, d2,3 → dd, d2,4 → dd, d2,5 → dd,
  d3,1 → dd, d3,2 → dd, d3,3 → dd, d3,4 → dd, d3,5 → dd,
  d4,1 → dd, d4,2 → dd, d4,3 → dd, d4,4 → dd, d4,5 → dd,
  d5,1 → dd, d5,2 → dd, d5,3 → dd, d5,4 → dd, d5,5 → dd,

  Ω1,1 → OM Net[[1]][[1]], Ω1,2 → OM Net[[1]][[2]],
  Ω1,3 → OM Net[[1]][[3]], Ω1,4 → OM Net[[1]][[4]], Ω1,5 → OM Net[[1]][[5]],
  Ω2,1 → OM Net[[2]][[1]], Ω2,2 → OM Net[[2]][[2]], Ω2,3 → OM Net[[2]][[3]],
  Ω2,4 → OM Net[[2]][[4]], Ω2,5 → OM Net[[2]][[5]],
  Ω3,1 → OM Net[[3]][[1]], Ω3,2 → OM Net[[3]][[2]], Ω3,3 → OM Net[[3]][[3]],
  Ω3,4 → OM Net[[3]][[4]], Ω3,5 → OM Net[[3]][[5]],
  Ω4,1 → OM Net[[4]][[1]], Ω4,2 → OM Net[[4]][[2]], Ω4,3 → OM Net[[4]][[3]],
  Ω4,4 → OM Net[[4]][[4]], Ω4,5 → OM Net[[4]][[5]],
  Ω5,1 → OM Net[[5]][[1]], Ω5,2 → OM Net[[5]][[2]], Ω5,3 → OM Net[[5]][[3]],
  Ω5,4 → OM Net[[5]][[4]], Ω5,5 → OM Net[[5]][[5]],
  nuK → nu,
  denK → den


 };


B10 = 1500;
B20 = 1500;
B30 = 1500;
```

```
B40 = 1500;
B50 = 1500;
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;


sol =
 NDSolve[
  {
    B1'[t] == dB1,
    B2'[t] == dB2,
    B3'[t] == dB3,
    B4'[t] == dB4,
    B5'[t] == dB5,

    M1'[t] == dM1,
    M2'[t] == dM2,
    M3'[t] == dM3,
    M4'[t] == dM4,
    M5'[t] == dM5,


    B1[0] == B10,
    B2[0] == B20,
    B3[0] == B30,
    B4[0] == B40,
    B5[0] == B50,
    M1[0] == M10,
    M2[0] == M20,
    M3[0] == M30,
    M4[0] == M40,
    M5[0] == M50

   } /. par,
  {B1, B2, B3, B4, B5, M1, M2, M3, M4, M5},
  {t, 0, tmax}];

{B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax],
   M1[tmax], M2[tmax], M3[tmax], M4[tmax], M5[tmax]} /. sol /. par;

Min[{B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax]} /. sol /. par]
```

```
)
```

The function “robustnessNewSaito” uses the previous function “fNewSaito” and calculates the Robustness. The function “robustnessNewSaito” simply receives a network as an argument.

In[•]:=
```
robustnessNewSaito[NetTop_] := (
  n1 = 1;
  n2 = 5000;
  mid = (n1 + n2) / 2;

  While[(n1 ≠ mid && n2 ≠ mid),
   (If[fNewSaito[NetTop, mid] < 1, n2 = mid, n1 = mid];
    mid = Floor[N[(n1 + n2) / 2]];); {n1, n2, mid}]; mid

 )
```

In[•]:=
```
NetK = {
    {0, 1, 0, 1, 0},
    {1, 0, 1, 1, 0},
    {1, 0, 1, 0, 1},
    {0, 1, 0, 1, 0},
    {0, 0, 0, 0, 1}
   };
```

In[•]:=
```
fNewSaitoPlotFig3[NetK, 0, 1]
```

Out[•]=
```
{{6661.78, 6661.53, 6661.53, 6661.78,
  6662.03, 5755.31, 11495., 11495.2, 5755.31, 16.0556}}
```

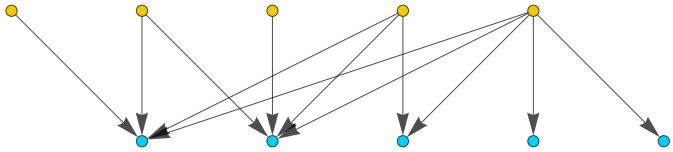
In[•]:=
```
Neta = {
    {1, 0, 0, 0, 0},
    {0, 1, 0, 0, 0},
    {1, 1, 0, 0, 0},
    {1, 1, 1, 0, 0},
    {1, 1, 1, 1, 1}
   };
```

In[•]:= **{RelatEntrop5[Neta], assortativity[Neta]}**

Out[•]= {0.884862, -0.562855}

In[•]:= **MakeNetworkH[Neta]**

Out[•]=



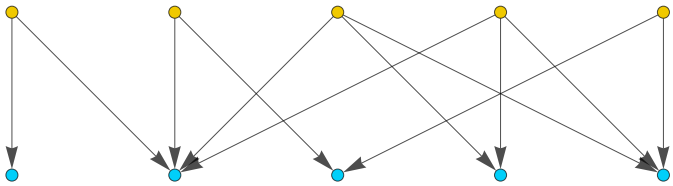
In[•]:= **Netb = {**
{1, 1, 0, 0, 0},
{0, 1, 1, 0, 0},
{0, 1, 0, 1, 1},
{0, 1, 0, 1, 1},
{0, 0, 1, 0, 1}
};

In[•]:= **{RelatEntrop5[Netb], assortativity[Netb]}**

Out[•]= {0.987318, 0.169031}

In[•]:= **MakeNetworkH[Netb]**

Out[•]=



In[•]:= **fNewSaitoPlotFig3[Neta, 0, 10]**

Out[•]= {{6656.97, 6656.97, 6656.72, 6656.47,
6655.97, 32.6134, 32.6134, 21 115.8, 42 198.2, 84 359.8}}

In[•]:= **fNewSaitoPlotFig3[Neta, 0, 1000]**

Out[•]= {{6656.97, 6656.97, 6656.72, 6656.47,
6655.97, 32.611, 32.611, 22 222.5, 44 411.6, 88 786.4}}

In[•]:= **fNewSaitoPlotFig3[Neta, 1000, 1000]**

Out[•]= $\{\{3.10255\times10^{-56}, 3.10255\times10^{-56}, 3.07545\times10^{-56}, 3.04847\times10^{-56}, 2.99491\times10^{-56},$
$-7.77599\times10^{-20}, -7.77599\times10^{-20}, -1.03358\times10^{-19}, -1.28951\times10^{-19}, -1.80121\times10^{-19}\}\}$

In[•]:= **robustnessNewSaito[Neta]**

Out[•]= 912

In[•]:= **robustnessNewSaito[Netb]**

Out[•]= 955

In[•]:= **fNewSaitoPlotFig3[Netb, 0, 10]**

Out[•]= {{7496.66, 7496.66, 7496.41, 7496.41,
7496.66, 20 787.8, 20 787., 44 526.8, 44 526.8, 20 787.}}

In[•]:= **fNewSaitoPlotFig3[Netb, 0, 1000]**

Out[•]= {{7496.8, 7496.8, 7496.55, 7496.55,
7496.8, 21 876.2, 21 875.3, 46 864.7, 46 864.7, 21 875.3}}

In[•]:= **fNewSaitoPlotFig3[Netb, 100, 1000]**

Out[•]= {{6996.61, 6996.61, 6996.36, 6996.36,
6996.61, 20 417.3, 20 416.4, 43 738.5, 43 738.5, 20 416.4}}

In[•]:= **(Flatten[fNewSaitoPlotFig3[Neta, 0, 1000]])[[1 ;; 5]]**

Out[•]= {6656.97, 6656.97, 6656.72, 6656.47, 6655.97}

In[•]:= **((Flatten[fNewSaitoPlotFig3[Neta, 0, 1000]])[[1 ;; 5]]) /**
Total[((Flatten[fNewSaitoPlotFig3[Neta, 0, 1000]])[[1 ;; 5]])]

Out[•]= {0.200011, 0.200011, 0.200003, 0.199995, 0.19998}

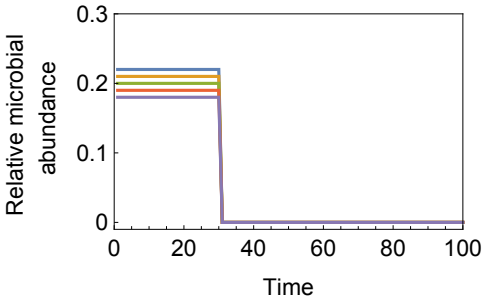
In[•]:= **((Flatten[fNewSaitoPlotFig3[Neta, 920, 1000]])[[1 ;; 5]]) /**
Total[((Flatten[fNewSaitoPlotFig3[Neta, 0, 1000]])[[1 ;; 5]])]

Out[•]= $\{2.30348\times10^{-62}, 2.30348\times10^{-62}, 2.31274\times10^{-62}, 2.32123\times10^{-62}, 2.33578\times10^{-62}\}$

```
In[•]:= (Flatten[fNewSaitoPlotFig3[Netb, 0, 1000]])[[1 ;; 5]]
Out[•]= {7496.8, 7496.8, 7496.55, 7496.55, 7496.8}

In[•]:= ((Flatten[fNewSaitoPlotFig3[Netb, 0, 1000]])[[1 ;; 5]]) /
         Total[((Flatten[fNewSaitoPlotFig3[Netb, 0, 1000]])[[1 ;; 5]])]
Out[•]= {0.200003, 0.200003, 0.199996, 0.199996, 0.200003}

In[•]:= ((Flatten[fNewSaitoPlotFig3[Netb, 920, 1000]])[[1 ;; 5]]) /
         Total[((Flatten[fNewSaitoPlotFig3[Netb, 0, 1000]])[[1 ;; 5]])]
Out[•]= {0.0771677, 0.0771677, 0.0771611, 0.0771611, 0.0771677}
```

In[•]:=
```
pop1b3 = Join[
    Partition[Riffle[Range[30], ConstantArray[0.22, {30}]], {2}],
    Partition[Riffle[Range[31, 100], ConstantArray[0, {70}]], {2}]
   ];

pop2b3 = Join[
    Partition[Riffle[Range[30], ConstantArray[0.21, {30}]], {2}],
    Partition[Riffle[Range[31, 100], ConstantArray[0, {70}]], {2}]
   ];

pop3b3 = Join[
    Partition[Riffle[Range[30], ConstantArray[0.2, {30}]], {2}],
    Partition[Riffle[Range[31, 100], ConstantArray[0, {70}]], {2}]
   ];

pop4b3 = Join[
    Partition[Riffle[Range[30], ConstantArray[0.19, {30}]], {2}],
    Partition[Riffle[Range[31, 100], ConstantArray[0, {70}]], {2}]
   ];

pop5b3 = Join[
    Partition[Riffle[Range[30], ConstantArray[0.18, {30}]], {2}],
    Partition[Riffle[Range[31, 100], ConstantArray[0, {70}]], {2}]
   ];

ListPlot[{pop1b3, pop2b3, pop3b3, pop4b3, pop5b3}, Joined → True, Frame → True,
 ImageSize → 250, FrameLabel → {"Time", "Relative microbial \n abundance"},
 FrameStyle → Directive[Black, FontSize → 12],
 PlotStyle → {Thickness[0.009]}, PlotRange → {{0.0, 100}, {-0.01, 0.3}},
 FrameTicks → {{{0, 0.1, 0.2, 0.3}, None}, {Automatic, None}}]
```

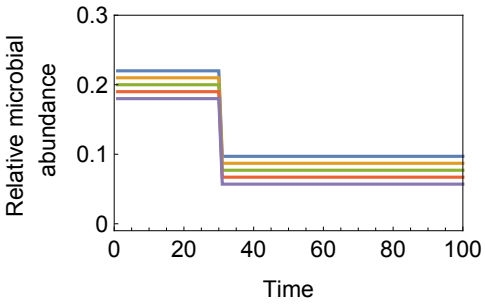
Out[•]=




```
0.077
```

In[•]:=
```
pop1b32 = Join[
    Partition[Riffle[Range[30], ConstantArray[0.22, {30}]], {2}],
    Partition[Riffle[Range[31, 100], ConstantArray[0.097, {70}]], {2}]
   ];

pop2b32 = Join[
    Partition[Riffle[Range[30], ConstantArray[0.21, {30}]], {2}],
    Partition[Riffle[Range[31, 100], ConstantArray[0.087, {70}]], {2}]
   ];

pop3b32 = Join[
    Partition[Riffle[Range[30], ConstantArray[0.2, {30}]], {2}],
    Partition[Riffle[Range[31, 100], ConstantArray[0.077, {70}]], {2}]
   ];

pop4b32 = Join[
    Partition[Riffle[Range[30], ConstantArray[0.19, {30}]], {2}],
    Partition[Riffle[Range[31, 100], ConstantArray[0.067, {70}]], {2}]
   ];

pop5b32 = Join[
    Partition[Riffle[Range[30], ConstantArray[0.18, {30}]], {2}],
    Partition[Riffle[Range[31, 100], ConstantArray[0.057, {70}]], {2}]
   ];

ListPlot[{pop1b32, pop2b32, pop3b32, pop4b32, pop5b32}, Joined → True, Frame → True,
 ImageSize → 250, FrameLabel → {"Time", "Relative microbial \n abundance"},
 FrameStyle → Directive[Black, FontSize → 12],
 PlotStyle → {Thickness[0.009]}, PlotRange → {{0.0, 100}, {-0.01, 0.3}},
 FrameTicks → {{{0, 0.1, 0.2, 0.3}, None}, {Automatic, None}}]
```

Out[•]=



```
AuxoComm13 = robustnessNewSaito /@ hk13;
```

One requirement for calculating Assortativity is that all values for rows and columns shouldn't be zero. Therefore, networks containing full auxotrophs cannot be included for calculating Assortativity. Still, we include them for all other analyses.

In[•]:=
```
hk32 = NestWhile[AppNe[#, 5, 13] &, {NetWorkGen[5, 13]}, Length[#] ≠  100 &];
```

In[•]:=
```
Length[hk32]
```

Out[•]=
```
100
```

In[•]:=
```
hk32T = Transpose /@ hk32;
```

Here we see that there are 14 Networks containing 14 full Auxotrophs:

In[•]:=
```
MemberQ[#, {0, 0, 0, 0, 0}] & /@ hk32T
```

Out[•]=
```
{False, False, False, False, False, False, False, False, False, False, False, False,
 True, False, True, False, False, False, False, False, False, False, False,
 False, False, False, True, True, False, False, False, False, False, False,
 False, False, False, False, False, True, False, False, False, False, False,
 False, False, False, False, True, True, False, False, False, False, False,
 False, False, False, True, False, False, False, True, False, False, False,
 False, True, False, False, False, False, False, False, True, False, False,
 False, False, False, False, False, True, False, False, False, False, False,
 False, True, False, False, False, False, True, False, False, False, False}
```

In[•]:=
```
Count[MemberQ[#, {0, 0, 0, 0, 0}] & /@ hk32T, True]
```

Out[•]=
```
14
```

In[•]:=
```
MemberQ[#, {0, 0, 0, 0, 0}] & /@ hk32
```

Out[•]=
```
{False, False, False, False, False, False, False, False, False, False, False, False,
 False, False, False, False, False, False, False, False, False, False, False,
 False, False, False, False, False, False, False, False, False, False, False,
 False, False, False, False, False, False, False, False, False, False, False,
 False, False, False, False, False, False, False, False, False, False, False,
 False, False, False, False, False, False, False, False, False, False, False,
 False, False, False, False, False, False, False, False, False, False, False,
 False, False, False, False, False, False, False, False, False, False, False,
 False, False, False, False, False, False, False, False, False, False, False}
```

In[•]:=
```
Count[MemberQ[#, {0, 0, 0, 0, 0}] & /@ hk32, True]
```

Out[•]=
```
0
```

Therefore , we calculate another set of 100 network without full Auxotrophs:

In[5991]:=
```
hk32New = NestWhile[AppNe[#, 5, 13] &, {NetWorkGen[5, 13]}, Length[#] ≠ 200 &];
```

In[5992]:=
```
Length[hk32New]
```

Out[5992]=
```
200
```

In[5993]:=
```
hk32TNew = Transpose /@ hk32New;
```

Here we see that there are 14 Networks containing 14 full Auxotrophs:

In[5994]:=
```
MemberQ[#, {0, 0, 0, 0, 0}] & /@ hk32TNew
```

Out[5994]=
```
{False, False, False, False, False, False, False, True, False, False, True, True,
 False, True, False, False, False, False, False, False, False, True, False,
 False, False, False, False, False, True, False, False, False, False, False,
 False, False, True, False, False, True, False, True, False, True, False, False,
 False, False, False, False, False, False, False, True, False, True, False,
 False, False, False, False, False, False, False, False, False, False, True,
 False, False, False, False, False, False, False, False, False, False, True,
 True, False, False, False, False, False, False, False, False, False, False,
 False, False, False, False, False, False, False, False, False, True, False,
 False, False, False, False, False, False, True, False, False, False, False,
 False, False, False, False, False, False, False, False, False, False, False,
 False, False, False, False, True, False, False, False, False, False, False,
 False, False, False, False, False, False, False, False, False, True, False,
 False, False, False, False, False, False, False, False, False, False, False,
 False, False, False, False, False, False, True, False, False, False, True,
 False, True, False, False, True, False, False, False, False, True, False,
 False, False, False, False, False, True, False, False, False, False, False,
 False, False, False, False, False, False, False, False, False, False, False}
```

In[5995]:=
```
Count[MemberQ[#, {0, 0, 0, 0, 0}] & /@ hk32TNew, True]
```

Out[5995]=
```
25
```

In[5996]:=
```
jop = Flatten[Position[MemberQ[#, {0, 0, 0, 0, 0}] & /@ hk32TNew, False]]
```

Out[5996]=
```
{1, 2, 3, 4, 5, 6, 7, 9, 10, 13, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27,
 28, 30, 31, 32, 33, 34, 35, 36, 38, 39, 41, 43, 45, 46, 47, 48, 49, 50, 51, 52,
 53, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75,
 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97,
 98, 99, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115,
 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 129, 130, 131, 132,
 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149,
 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165,
 166, 168, 170, 171, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 185,
 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200}
```

In[5997]:=
```
jop1 = RandomSample[jop, 98]
```

Out[5997]=
```
{77, 89, 2, 18, 27, 162, 32, 120, 75, 127, 104, 195, 197, 36, 39, 96, 179,
 84, 187, 185, 57, 140, 147, 98, 148, 154, 102, 166, 173, 38, 174, 116, 24,
 152, 183, 101, 130, 139, 87, 52, 111, 13, 142, 65, 41, 151, 196, 35, 69,
 194, 170, 198, 121, 112, 141, 186, 123, 138, 182, 131, 70, 50, 146, 23, 62,
 165, 95, 143, 132, 193, 181, 119, 135, 99, 178, 171, 9, 192, 58, 134, 55,
 64, 133, 161, 97, 31, 176, 33, 71, 115, 43, 49, 60, 159, 113, 10, 175, 153}
```

In[5998]:=
```
hk32Newfilt = hk32New[[#]] & /@ jop1;
```

In[5999]:=
```
hk32NewfiltCheck = Transpose /@ hk32Newfilt;
Count[MemberQ[#, {0, 0, 0, 0, 0}] & /@ hk32NewfiltCheck, True]
```

Out[6000]=
```
0
```

Also, we include the networks highlighted in our previous simulations (Here named “Neta”, and “Netb”):

In[ ]:=
```
Neta
```

Out[ ]=
```
{{1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {1, 1, 0, 0, 0}, {1, 1, 1, 0, 0}, {1, 1, 1, 1, 1}}
```

In[6001]:=
```
hk32NewfiltPlusNetAandB = Join[hk32Newfilt, {Neta, Netb}];
```

In[6002]:=
```
Entro12Prod = RelatEntrop5 /@ hk32NewfiltPlusNetAandB
```

Out[6002]=
```
{0.987318, 0.942631, 0.987318, 0.960225, 0.960225, 0.884862, 0.942631, 0.987318,
 0.942631, 0.942631, 0.960225, 0.987318, 0.942631, 0.911954, 0.942631, 0.987318,
 0.915539, 0.987318, 0.987318, 0.960225, 0.942631, 0.987318, 0.942631, 0.987318,
 0.987318, 0.942631, 0.987318, 0.960225, 0.942631, 0.960225, 0.987318, 0.960225,
 0.960225, 0.987318, 0.942631, 0.960225, 0.942631, 0.897944, 0.942631, 0.915539,
 0.960225, 0.942631, 0.942631, 0.942631, 0.987318, 0.987318, 0.969724, 0.942631,
 0.897944, 0.942631, 0.942631, 0.915539, 0.960225, 0.987318, 0.942631, 0.960225,
 0.987318, 0.942631, 0.987318, 0.987318, 0.960225, 0.987318, 0.942631, 0.942631,
 0.915539, 0.987318, 0.987318, 0.987318, 0.987318, 0.960225, 0.884862, 0.884862,
 0.884862, 0.969724, 0.942631, 0.960225, 0.969724, 0.897944, 0.942631,
 0.960225, 0.942631, 0.915539, 0.915539, 0.987318, 0.942631, 0.942631,
 0.942631, 0.942631, 0.987318, 0.987318, 0.969724, 0.987318, 0.960225,
 0.915539, 0.960225, 0.960225, 0.960225, 0.960225, 0.884862, 0.987318}
```

In[6003]:=
```
{Min[Entro12Prod], Max[Entro12Prod]}
```

Out[6003]=
```
{0.884862, 0.987318}
```

In[6004]:=
```
Assort12Prod = assortativity /@ hk32NewfiltPlusNetAandB
```

Out[6004]=
```
{-0.169031, 0.0571429, 0., 0.267261, 0., -0.485247, -0.455677, -0.169031,
 -0.2, 0.180702, 0.293689, -0.274721, -0.412096, -0.412096, -0.2, -0.507093,
 0.333333, -0.267261, 0., 0.451754, -0.169031, 0., -0.676123, -0.338062,
 0.333333, -0.358345, 0., -0.0714286, -0.239046, 0.142857, -0.107833, 0.267261,
 0.0734223, 0.169031, -0.455677, -0.230556, 0.0451754, -0.185745, -0.169031,
 -0.485247, -0.0714286, -0.496929, -0.628571, -0.225877, -0.169031, -0.169031,
 -0.242821, -0.412096, -0.562855, -0.285714, -0.412096, -0.25, -0.0903508, 0.,
 -0.371429, -0.361403, -0.107833, 0.0451754, -0.169031, 0.169031, -0.285714,
 0., -0.496929, -0.361403, -0.53033, 0., -0.169031, -0.534522, -0.412082,
 -0.368285, -0.403473, -0.346314, -0.230556, -0.239046, -0.169031, 0.267261,
 -0.188982, -0.464363, -0.285714, -0.225877, 0.169031, -0.0845154, -0.253546,
 -0.338062, 0.169031, 0.451754, -0.496929, -0.496929, 0., 0., -0.176777, 0.,
 -0.267261, 0., -0.496929, -0.403473, 0., -0.361403, -0.562855, 0.169031}
```

In[6005]:=
```
{Min[Assort12Prod], Max[Assort12Prod]}
```

Out[6005]=
```
{-0.676123, 0.451754}
```

Out[•]=
```
{-0.676123, 0.316228}
```

In[6006]:=
```
Robust12prod = robustnessNewSaito /@ hk32NewfiltPlusNetAandB
```

Out[6006]=
```
{955, 936, 955, 942, 942, 912, 936, 955, 936, 936, 942, 955, 936, 926, 936, 955, 922, 955,
 955, 942, 936, 955, 936, 955, 955, 936, 955, 942, 936, 942, 955, 942, 942, 955, 936,
 942, 936, 915, 936, 922, 942, 936, 936, 936, 955, 955, 949, 936, 915, 936, 936, 922,
 942, 955, 936, 942, 955, 936, 955, 955, 942, 955, 936, 936, 922, 955, 955, 955,
 955, 942, 912, 912, 912, 949, 936, 942, 949, 915, 936, 942, 936, 922, 922, 955,
 936, 936, 936, 936, 955, 955, 949, 955, 942, 922, 942, 942, 942, 942, 912, 955}
```

In[6007]:=
```
{Min[Robust12prod], Max[Robust12prod]}
```

Out[6007]=
```
{912, 955}
```

In[6008]:=
```
Position[Entro12Prod, Min[Entro12Prod]]
```

Out[6008]=
```
{{6}, {71}, {72}, {73}, {99}}
```

In[6009]:=
```
Position[Entro12Prod, Max[Entro12Prod]]
```

Out[6009]=
```
{{1}, {3}, {8}, {12}, {16}, {18}, {19}, {22}, {24}, {25}, {27}, {31}, {34}, {45}, {46},
 {54}, {57}, {59}, {60}, {62}, {66}, {67}, {68}, {69}, {84}, {89}, {90}, {92}, {100}}
```

In[6010]:=
```
Assort12Prod[[99]]
```

Out[6010]=
```
-0.562855
```

In[6011]:=
```
hk32NewfiltPlusNetAandB[[99]]
Count[Flatten[hk32NewfiltPlusNetAandB[[99]]], 0]
```

Out[6011]=
```
{{1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {1, 1, 0, 0, 0}, {1, 1, 1, 0, 0}, {1, 1, 1, 1, 1}}
```

Out[6012]=
```
13
```

In[6013]:=
```
hk32NewfiltPlusNetAandB[[99]]
```

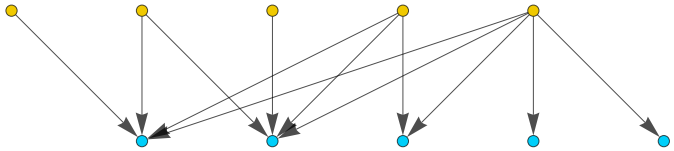
Out[6013]=
```
{{1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {1, 1, 0, 0, 0}, {1, 1, 1, 0, 0}, {1, 1, 1, 1, 1}}
```

In[6014]:=
```
Neta
```

Out[6014]=
```
{{1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {1, 1, 0, 0, 0}, {1, 1, 1, 0, 0}, {1, 1, 1, 1, 1}}
```

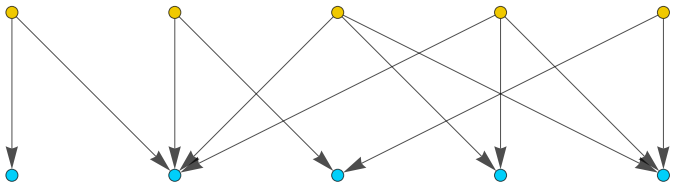
In[6015]:=
```
MakeNetworkH[hk32NewfiltPlusNetAandB[[99]]]
```

Out[6015]=



In[6016]:=
```
MakeNetworkH[hk32NewfiltPlusNetAandB[[100]]]
```

Out[6016]=



In[6017]:=
```
Assort12Prod[[#]] & /@ Flatten[Position[Entro12Prod, Max[Entro12Prod]]]
```

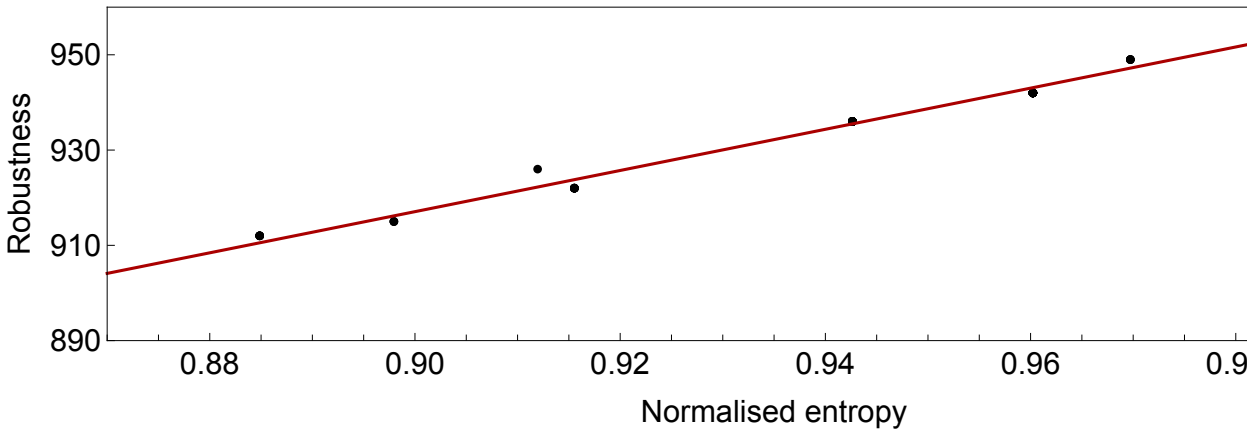
Out[6017]=
```
{-0.169031, 0., -0.169031, -0.274721, -0.507093, -0.267261, 0., 0., -0.338062,
 0.333333, 0., -0.107833, 0.169031, -0.169031, -0.169031, 0., -0.107833, -0.169031,
 0.169031, 0., 0., -0.169031, -0.534522, -0.412082, -0.338062, 0., 0., 0., 0.169031}
```

In[6018]:=
```
{Min[Robust12prod], Max[Robust12prod]}
```

Out[6018]=
```
{912, 955}
```

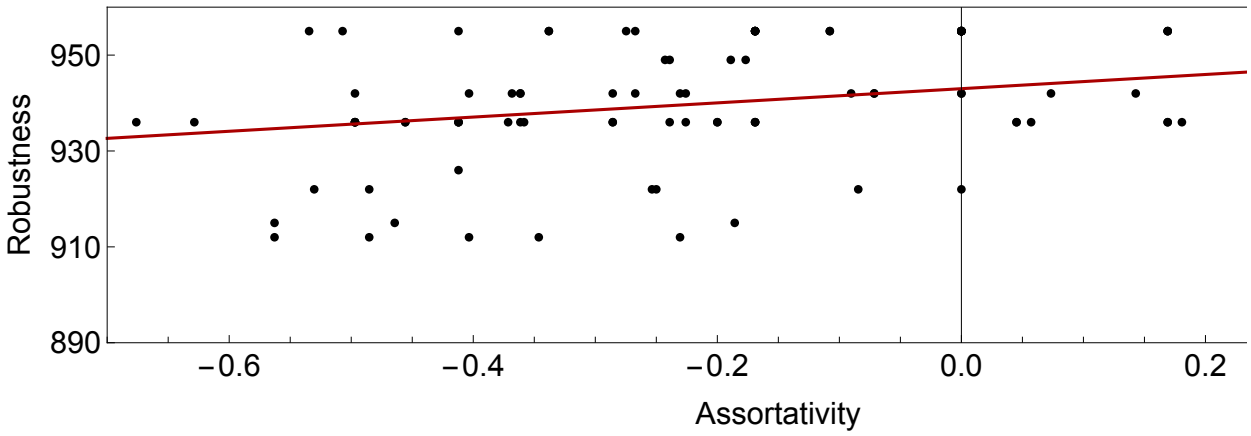
In[6019]:=
```
lineRNewSaitoProd12 =
  Fit[Partition[Riffle[Entro12Prod, Robust12prod], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Entro12Prod, Robust12prod], {2}],
  Frame → True, FrameLabel → {"Normalised entropy", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 16],
  PlotStyle → {Black, PointSize[Medium]},
  PlotRange → {{0.87, 1}, {890, 960}}, AspectRatio → 0.25,
  FrameTicks → {{{890, 910, 930, 950}, None}, {Automatic, None}}],
 Plot[lineRNewSaitoProd12, {x, 0.87, 1}, AspectRatio → 0.25,
  FrameTicks → {{{890, 910, 930, 950}, None}, {Automatic, None}},
  PlotStyle → Darker[Red]], ImageSize → 765]
```

Out[6020]=



In[6021]:=
```
lineAssoRobRProd12NewSaito =
  Fit[Partition[Riffle[Assort12Prod, Robust12prod], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Assort12Prod, Robust12prod], {2}],
  Frame → True, FrameLabel → {"Assortativity", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 16],
  PlotStyle → {Black, PointSize[Medium]},
  PlotRange → {{-0.7, 0.4}, {890, 960}}, AspectRatio → 0.25,
  FrameTicks → {{{890, 910, 930, 950}, None}, {Automatic, None}}],
 Plot[lineAssoRobRProd12NewSaito, {x, -0.7, 0.4},
  FrameTicks → {{{890, 910, 930, 950}, None}, {Automatic, None}},
  AspectRatio → 0.25, PlotStyle → Darker[Red]], ImageSize → 765]
```

Out[6022]=



In[6023]:=
```
SpearmanRankTest[Entro12Prod, Robust12prod, "TestDataTable"]
```

Out[6023]=

	Statistic	P-Value
Spearman Rank	0.999642	6.12073×10^{-156}

In[6024]:=
```
SpearmanRankTest[Assort12Prod, Robust12prod, "TestDataTable"]
```

Out[6024]=

	Statistic	P-Value
Spearman Rank	0.348218	0.000384958

Figure 4

```
fNewSaitoOVx[Net_, Dh_, coop_] := (

  dB1 =
```

$$dB_1 = B_1[t]\left(-B_1[t]\,\kappa_1 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - \left(c_{1,1} + c_{1,2} + c_{1,3} + c_{1,4} + c_{1,5} + Dh\right) B_1[t];$$

$$dB_2 = B_2[t]\left(-B_2[t]\,\kappa_2 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - \left(c_{2,1} + c_{2,2} + c_{2,3} + c_{2,4} + c_{2,5} + Dh\right) B_2[t];$$

$$dB_3 = B_3[t]\left(-B_3[t]\,\kappa_3 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - \left(c_{3,1} + c_{3,2} + c_{3,3} + c_{3,4} + c_{3,5} + Dh\right) B_3[t];$$

$$dB_4 = B_4[t]\left(-B_4[t]\,\kappa_4 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - \left(c_{4,1} + c_{4,2} + c_{4,3} + c_{4,4} + c_{4,5} + Dh\right) B_4[t];$$

$$dB_5 = B_5[t]\left(-B_5[t]\,\kappa_5 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - \left(c_{5,1} + c_{5,2} + c_{5,3} + c_{5,4} + c_{5,5} + Dh\right) B_5[t];$$

$$dM_1 = -M_1[t]\,q_1 + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right)\left(-B_1[t]\,d_{1,1} - B_2[t]\,d_{1,2} - B_3[t]\,d_{1,3} - B_4[t]\,d_{1,4} - B_5[t]\,d_{1,5}\right) + B_1[t]\,\Omega_{1,1} + B_2[t]\,\Omega_{1,2} + B_3[t]\,\Omega_{1,3} + B_4[t]\,\Omega_{1,4} + B_5[t]\,\Omega_{1,5};$$

$$dM_2 = -M_2[t]\,q_2 + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right)\left(-B_1[t]\,d_{2,1} - B_2[t]\,d_{2,2} - B_3[t]\,d_{2,3} - B_4[t]\,d_{2,4} - B_5[t]\,d_{2,5}\right) + B_1[t]\,\Omega_{2,1} + B_2[t]\,\Omega_{2,2} + B_3[t]\,\Omega_{2,3} + B_4[t]\,\Omega_{2,4} + B_5[t]\,\Omega_{2,5};$$

$$dM_3 = -M_3[t]\,q_3 + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right)\left(-B_1[t]\,d_{3,1} - B_2[t]\,d_{3,2} - B_3[t]\,d_{3,3} - B_4[t]\,d_{3,4} - B_5[t]\,d_{3,5}\right) + B_1[t]\,\Omega_{3,1} + B_2[t]\,\Omega_{3,2} + B_3[t]\,\Omega_{3,3} + B_4[t]\,\Omega_{3,4} + B_5[t]\,\Omega_{3,5};$$

$$dM_4 = -M_4[t]\,q_4 + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right)\left(-B_1[t]\,d_{4,1} - B_2[t]\,d_{4,2} - B_3[t]\,d_{4,3} - B_4[t]\,d_{4,4} - B_5[t]\,d_{4,5}\right) + B_1[t]\,\Omega_{4,1} + B_2[t]\,\Omega_{4,2} + B_3[t]\,\Omega_{4,3} + B_4[t]\,\Omega_{4,4} + B_5[t]\,\Omega_{4,5};$$

$$dM_5 = -M_5[t]\,q_5 + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} *$$

```
        M5[t]/(denK + M5[t]) ) (- B1[t] d5,1 - B2[t] d5,2 - B3[t] d5,3 - B4[t] d5,4 - B5[t] d5,5) +
   B1[t] Ω5,1 + B2[t] Ω5,2 + B3[t] Ω5,3 + B4[t] Ω5,4 + B5[t] Ω5,5;

KK = 0.2;
cc = 0.05;
qq = 0.3;
dd = 0.00015;
OM = 1;
nu = 1500;
den = 2;


op = coop; (*Number of links with overExpression*)
posNe = Position[Net, 1];
(*Positions in the matrix where there are links (=1)*)
RaN = RandomSample[posNe, op];
(*Random sample of op links that will be overproduced*)

costincr = 1.3; (*Term multiplying the cost link*)
overprodincr = 1.15;
(*Term multiplying the overproduction link*)


NewNetCost = Net cc;
Table[NewNetCost[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] =
  NewNetCost[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] * costincr, {i, Length[RaN]}];

NewNetOvProd = Net OM;
Table[NewNetOvProd[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] =
  NewNetOvProd[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] * overprodincr, {i,
  Length[RaN]}];


tmax = 1000;
par = {
  κ1 → KK, κ2 → KK, κ3 → KK, κ4 → KK, κ5 → KK,

  c1,1 → NewNetCost[[1]][[1]],
  c1,2 → NewNetCost[[1]][[2]], c1,3 → NewNetCost[[1]][[3]],
  c1,4 → NewNetCost[[1]][[4]], c1,5 → NewNetCost[[1]][[5]],
  c2,1 → NewNetCost[[2]][[1]], c2,2 → NewNetCost[[2]][[2]],
  c2,3 → NewNetCost[[2]][[3]], c2,4 → NewNetCost[[2]][[4]],
  c2,5 → NewNetCost[[2]][[5]],
```

```
  c3,1 → NewNetCost[[3]][[1]], c3,2 → NewNetCost[[3]][[2]],
  c3,3 → NewNetCost[[3]][[3]], c3,4 → NewNetCost[[3]][[4]],
  c3,5 → NewNetCost[[3]][[5]],
  c4,1 → NewNetCost[[4]][[1]], c4,2 → NewNetCost[[4]][[2]],
  c4,3 → NewNetCost[[4]][[3]], c4,4 → NewNetCost[[4]][[4]],
  c4,5 → NewNetCost[[4]][[5]],
  c5,1 → NewNetCost[[5]][[1]], c5,2 → NewNetCost[[5]][[2]],
  c5,3 → NewNetCost[[5]][[3]], c5,4 → NewNetCost[[5]][[4]],
  c5,5 → NewNetCost[[5]][[5]],

  q1 → qq, q2 → qq, q3 → qq, q4 → qq, q5 → qq,

  d1,1 → dd, d1,2 → dd, d1,3 → dd, d1,4 → dd, d1,5 → dd,
  d2,1 → dd, d2,2 → dd, d2,3 → dd, d2,4 → dd, d2,5 → dd,
  d3,1 → dd, d3,2 → dd, d3,3 → dd, d3,4 → dd, d3,5 → dd,
  d4,1 → dd, d4,2 → dd, d4,3 → dd, d4,4 → dd, d4,5 → dd,
  d5,1 → dd, d5,2 → dd, d5,3 → dd, d5,4 → dd, d5,5 → dd,

  Ω1,1 → NewNetOvProd[[1]][[1]],
  Ω1,2 → NewNetOvProd[[1]][[2]], Ω1,3 → NewNetOvProd[[1]][[3]],
  Ω1,4 → NewNetOvProd[[1]][[4]], Ω1,5 → NewNetOvProd[[1]][[5]],
  Ω2,1 → NewNetOvProd[[2]][[1]], Ω2,2 → NewNetOvProd[[2]][[2]],
  Ω2,3 → NewNetOvProd[[2]][[3]], Ω2,4 → NewNetOvProd[[2]][[4]],
  Ω2,5 → NewNetOvProd[[2]][[5]],
  Ω3,1 → NewNetOvProd[[3]][[1]], Ω3,2 → NewNetOvProd[[3]][[2]],
  Ω3,3 → NewNetOvProd[[3]][[3]], Ω3,4 → NewNetOvProd[[3]][[4]],
  Ω3,5 → NewNetOvProd[[3]][[5]],
  Ω4,1 → NewNetOvProd[[4]][[1]], Ω4,2 → NewNetOvProd[[4]][[2]],
  Ω4,3 → NewNetOvProd[[4]][[3]], Ω4,4 → NewNetOvProd[[4]][[4]],
  Ω4,5 → NewNetOvProd[[4]][[5]],
  Ω5,1 → NewNetOvProd[[5]][[1]], Ω5,2 → NewNetOvProd[[5]][[2]],
  Ω5,3 → NewNetOvProd[[5]][[3]], Ω5,4 → NewNetOvProd[[5]][[4]],
  Ω5,5 → NewNetOvProd[[5]][[5]],
  nuK → nu,
  denK → den


 };


B10 = 1500;
B20 = 1500;
B30 = 1500;
B40 = 1500;
```

```
B50 = 1500;
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;


sol =
 NDSolve[
  {
    B1'[t] == dB1,
    B2'[t] == dB2,
    B3'[t] == dB3,
    B4'[t] == dB4,
    B5'[t] == dB5,

    M1'[t] == dM1,
    M2'[t] == dM2,
    M3'[t] == dM3,
    M4'[t] == dM4,
    M5'[t] == dM5,


    B1[0] == B10,
    B2[0] == B20,
    B3[0] == B30,
    B4[0] == B40,
    B5[0] == B50,
    M1[0] == M10,
    M2[0] == M20,
    M3[0] == M30,
    M4[0] == M40,
    M5[0] == M50

   } /. par,
  {B1, B2, B3, B4, B5, M1, M2, M3, M4, M5},
  {t, 0, tmax}];

{B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax],
   M1[tmax], M2[tmax], M3[tmax], M4[tmax], M5[tmax]} /. sol /. par;

Min[{B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax]} /. sol /. par]
```

```
)
```

In[•]:=
```
robustnessNewSaitoOVx[NetTop_, coop_] := (
  n1 = 1;
  n2 = 5000;
  mid = (n1 + n2) / 2;

  While[(n1 ≠ mid && n2 ≠ mid),
   (If[fNewSaitoOVx[NetTop, mid, coop] < 1, n2 = mid, n1 = mid];
    mid = Floor[N[(n1 + n2) / 2]];); {n1, n2, mid}]; mid

 )
```

In[•]:=
```
robustnessNewSaitoOVx[NetK, 5]
```

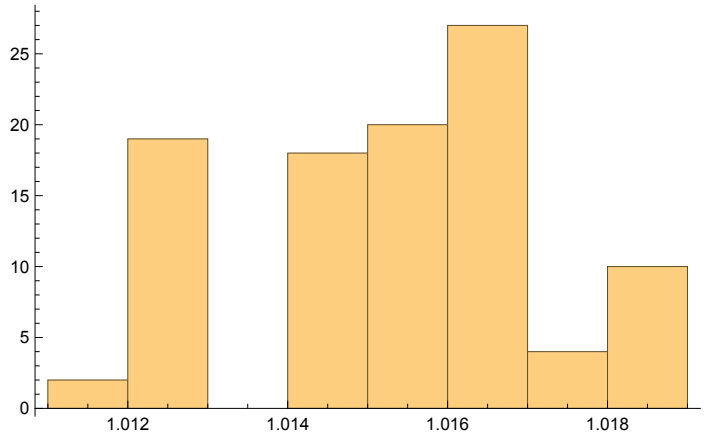
Out[•]=
```
941
```

In[•]:=
```
coopn = Table[robustnessNewSaitoOVx[NetK, 5], {100}]
```

Out[•]=
```
{936, 935, 940, 938, 936, 936, 939, 940, 941, 938, 936, 939, 936, 939, 938, 937, 941, 939,
 938, 938, 938, 937, 935, 939, 936, 939, 937, 939, 940, 939, 941, 936, 937, 938, 937,
 937, 937, 938, 939, 939, 937, 937, 941, 939, 939, 937, 937, 936, 936, 937, 939, 939,
 939, 937, 939, 936, 936, 936, 936, 941, 938, 938, 938, 936, 937, 939, 938, 941,
 941, 939, 936, 938, 941, 939, 941, 936, 936, 938, 937, 938, 936, 937, 937, 938,
 938, 939, 939, 937, 939, 939, 939, 940, 936, 938, 938, 939, 938, 939, 939, 941}
```

In[•]:=
```
Histogram[coopn / 924]
```

Out[•]=



In[•]:=
```
Select[N[coopn / 924], # ≥ 1 &] // Length
```

Out[•]=
```
100
```

In[•]:=
```
Mean[N[coopn / 924]]
```

Out[•]=
```
1.01518
```

In[•]:= {Min[N[coopn / 924]], Max[N[coopn / 924]]}

Out[•]= {1.0119, 1.0184}

In[•]:= SignedRankTest[coopn / 924, 1]

Out[•]= 2.67154×10^{-18}

In[•]:=
```
coop5to15 =
  {Table[robustnessNewSaitoOVx[#, 5], {20}], Table[robustnessNewSaitoOVx[#, 10],
     {20}], Table[robustnessNewSaitoOVx[#, 15], {20}]} &;
```

In[•]:= 25 - 17

Out[•]= 8

In[•]:= 25 - 6

Out[•]= 19

In[•]:= 25 - 8

Out[•]= 17

{6, 7, 8}

In[•]:= AuxoComm6[[1 ;; 3]]

Out[•]= {1058, 1061, 1058}

In[•]:= Timing[coop5to15[hk6[[1]]];]

Out[•]= {12.6454, Null}

In[•]:= 12 × 100 × 3

Out[•]= 3600

In[•]:=
```
3600. / 60
```
Out[•]=
```
60.
```

In[•]:=
```
wf6 = coop5to15 /@ hk6;
wf7 = coop5to15 /@ hk7;
wf8 = coop5to15 /@ hk8;
```

In[•]:=
```
robustnessNewSaito[hk6[[1]]]
```
Out[•]=
```
1058
```

In[•]:=
```
wf / 1058.
```
Out[•]=
```
{{1.00851, 1.00851, 1.00945, 1.00851, 1.00851, 1.00756, 1.00851, 1.00851, 1.00756,
  1.00851, 1.00756, 1.00851, 1.00851, 1.00756, 1.00756, 1.00851, 1.00756, 1.00756,
  1.00851, 1.00567, 1.00851, 1.00851, 1.00945, 1.00851, 1.00756, 1.00662,
  1.00851, 1.00851, 1.00662, 1.00851, 1.00567, 1.00756, 1.00756, 1.00851,
  1.00756, 1.00756, 1.00756, 1.00756, 1.00851, 1.00756, 1.00851, 1.00756,
  1.00756, 1.00945, 1.00756, 1.00662, 1.00851, 1.00851, 1.00945, 1.00851},
 {1.01512, 1.01607, 1.01512, 1.01607, 1.01607, 1.01607, 1.01512, 1.01607,
  1.01607, 1.01607, 1.01701, 1.01607, 1.01607, 1.01607, 1.01607, 1.01607, 1.01701,
  1.01512, 1.01607, 1.01512, 1.01607, 1.01607, 1.01512, 1.01607, 1.01607, 1.01323,
  1.01701, 1.01607, 1.01512, 1.01607, 1.01701, 1.01418, 1.01607, 1.01607,
  1.01512, 1.01607, 1.01701, 1.01323, 1.01607, 1.01701, 1.01607, 1.01607,
  1.01607, 1.01512, 1.01607, 1.01607, 1.01607, 1.01607, 1.01607, 1.01512},
 {1.02268, 1.02457, 1.02363, 1.02363, 1.02363, 1.02363, 1.02268, 1.02363,
  1.02268, 1.02268, 1.02363, 1.02268, 1.02268, 1.02363, 1.02457, 1.02457, 1.02363,
  1.02363, 1.02363, 1.02268, 1.02363, 1.02363, 1.02363, 1.02363, 1.02363, 1.02268,
  1.02268, 1.02363, 1.02268, 1.02363, 1.02363, 1.02363, 1.02268, 1.02363,
  1.02268, 1.02268, 1.02363, 1.02363, 1.02268, 1.02268, 1.02457, 1.02363,
  1.02363, 1.02268, 1.02268, 1.02363, 1.02363, 1.02363, 1.02363, 1.02268}}
```

In[•]:=
```
SignedRankTest[wf[[1]] / 1058, 1]
```
Out[•]=
```
3.85519 × 10^-10
```

In[•]:=
```
SignedRankTest[wf[[2]] / 1058, 1]
```
Out[•]=
```
2.25707 × 10^-10
```

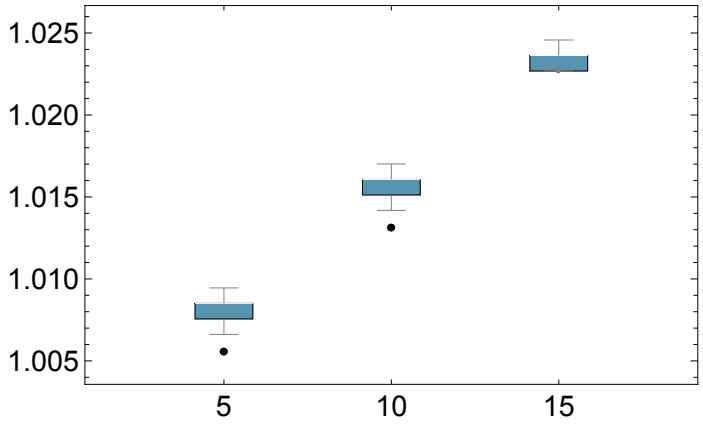
In[•]:=
```
SignedRankTest[wf[[3]] / 1058, 1]
```

Out[•]=
$$2.58042 \times 10^{-10}$$

In[•]:=
```
wf1 = Join[{ConstantArray[1, {50}]}, wf / robustnessNewSaito[hk6[[1]]]];
```

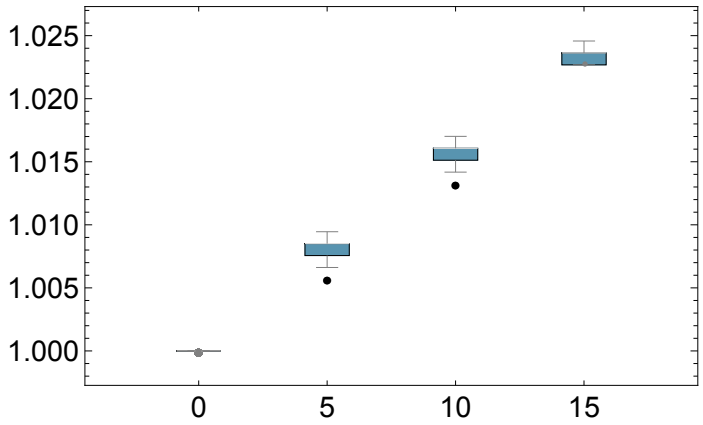
In[•]:=
```
BoxWhiskerChart[wf / 1058., "Outliers", ChartBaseStyle → EdgeForm[Dashing[0.99]],
 ChartStyle → {{coco}}, Frame → True, ChartLabels → {"5", "10", "15"},
 BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15]]
```

Out[•]=



In[•]:=
```
BoxWhiskerChart[wf1, "Outliers", ChartBaseStyle → EdgeForm[Dashing[0.99]],
 ChartStyle → {{coco}}, Frame → True, ChartLabels → {"0", "5", "10", "15"},
 BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15]]
```

Out[•]=



In[•]:=
```
wf6 // Length
```

Out[•]=
```
100
```

In[•]:=
```
N[wf6[[1]] / AuxoComm6[[1]]]
```

Out[•]=
```
{{1.00756, 1.00756, 1.00851, 1.00851, 1.00756, 1.00756,
  1.00662, 1.00567, 1.00756, 1.00756, 1.00851, 1.00756, 1.00851,
  1.00756, 1.00756, 1.00756, 1.00756, 1.00851, 1.00662, 1.00662},
 {1.01607, 1.01512, 1.01701, 1.01512, 1.01607, 1.01607, 1.01607,
  1.01512, 1.01607, 1.01701, 1.01512, 1.01607, 1.01512, 1.01512,
  1.01607, 1.01512, 1.01607, 1.01512, 1.01418, 1.01607},
 {1.02268, 1.02363, 1.02268, 1.02457, 1.02363, 1.02363, 1.02268,
  1.02363, 1.02363, 1.02363, 1.02457, 1.02363, 1.02363, 1.02268,
  1.02363, 1.02457, 1.02363, 1.02363, 1.02363, 1.02268}}
```

In[•]:=
```
wf6Normalized = N[wf6[[#]] / AuxoComm6[[#]]] & /@ Range[100]
```

In[•]:=
```
wf7Normalized = N[wf7[[#]] / AuxoComm7[[#]]] & /@ Range[100]
```

In[•]:=
```
wf8Normalized = N[wf8[[#]] / AuxoComm8[[#]]] & /@ Range[100]
```

In[•]:=
```
wf6Normalized // Length
```

Out[•]=
```
100
```

In[•]:=
```
wf6Normalized[[1]]
```

Out[•]=
```
{{1.00756, 1.00756, 1.00851, 1.00851, 1.00756, 1.00756,
  1.00662, 1.00567, 1.00756, 1.00756, 1.00851, 1.00756, 1.00851,
  1.00756, 1.00756, 1.00756, 1.00756, 1.00851, 1.00662, 1.00662},
 {1.01607, 1.01512, 1.01701, 1.01512, 1.01607, 1.01607, 1.01607,
  1.01512, 1.01607, 1.01701, 1.01512, 1.01607, 1.01512, 1.01512,
  1.01607, 1.01512, 1.01607, 1.01512, 1.01418, 1.01607},
 {1.02268, 1.02363, 1.02268, 1.02457, 1.02363, 1.02363, 1.02268,
  1.02363, 1.02363, 1.02363, 1.02457, 1.02363, 1.02363, 1.02268,
  1.02363, 1.02457, 1.02363, 1.02363, 1.02363, 1.02268}}
```

In[•]:=
```
wf6NormalizedWith5Coop = wf6Normalized[[#]][[1]] & /@ Range[100]
```

In[•]:=
```
wf6NormalizedWith10Coop = wf6Normalized[[#]][[2]] & /@ Range[100]
```

In[•]:=
```
wf6NormalizedWith15Coop = wf6Normalized[[#]][[3]] & /@ Range[100]
```

In[•]:=
```
wf7NormalizedWith5Coop = wf7Normalized[[#]][[1]] & /@ Range[100]
```

In[•]:=
```
wf7NormalizedWith10Coop = wf7Normalized[[#]][[2]] & /@ Range[100]
```

In[•]:=
```
wf7NormalizedWith15Coop = wf7Normalized[[#]][[3]] & /@ Range[100]
```

In[•]:=
```
wf8NormalizedWith5Coop = wf8Normalized[[#]][[1]] & /@ Range[100]
```

In[•]:=
```
wf8NormalizedWith10Coop = wf8Normalized[[#]][[2]] & /@ Range[100]
```

In[•]:=
```
wf8NormalizedWith15Coop = wf8Normalized[[#]][[3]] & /@ Range[100]
```

In[•]:=
```
allcoopWith6Auxo = {Flatten[wf6NormalizedWith5Coop],
  Flatten[wf6NormalizedWith10Coop], Flatten[wf6NormalizedWith15Coop]}
```

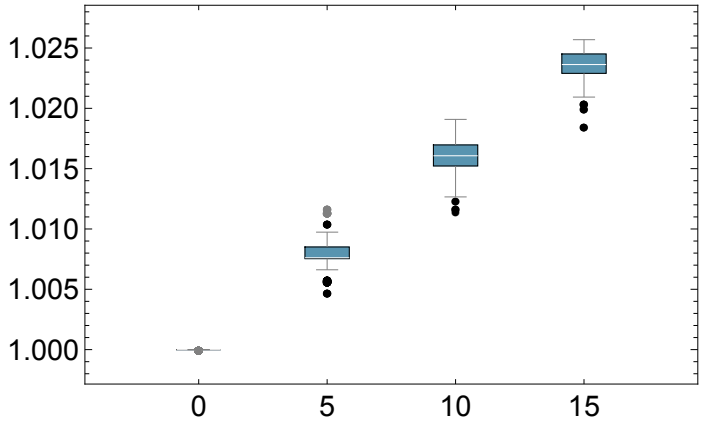
In[•]:=
```
Length[Flatten[wf6NormalizedWith5Coop]]
```

Out[•]=
```
2000
```

In[•]:=
```
allcoopWith6AuxoPlusAuxo = Join[{ConstantArray[1, {2000}]}, allcoopWith6Auxo]
```

In[ ]:=
```
BoxWhiskerChart[allcoopWith6AuxoPlusAuxo, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{coco}},
 Frame → True, ChartLabels → {"0", "5", "10", "15"},
 BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15]]
```

Out[ ]=



In[ ]:=
```
RGBColor[0.23921568627450981, 0.45098039215686275, 0.24705882352941178, 0.6]
```

Out[ ]=

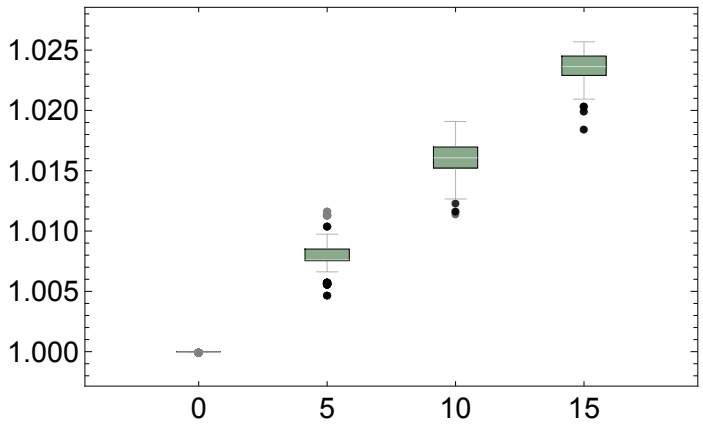
In[ ]:=
```
RGBColor[0.23921568627450981, 0.45098039215686275, 0.24705882352941178]
```

Out[ ]=

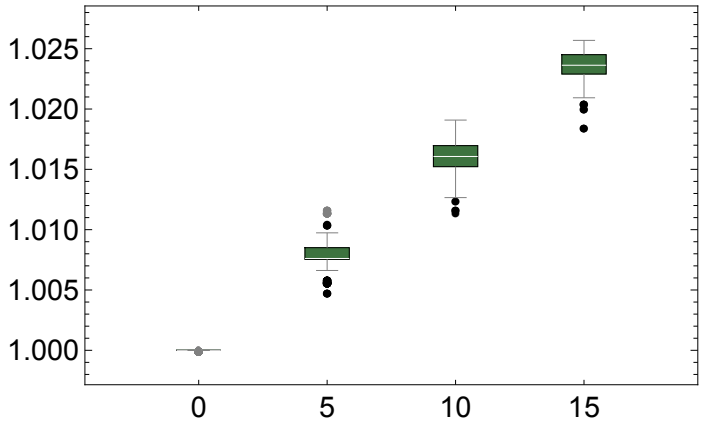
In[ ]:=
```
gree1 =
  RGBColor[0.23921568627450981, 0.45098039215686275, 0.24705882352941178, 0.6];
gree2 = RGBColor[0.23921568627450981, 0.45098039215686275, 0.24705882352941178];
```

In[ ]:=
```
BoxWhiskerChart[allcoopWith6AuxoPlusAuxo, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{gree1}},
 Frame → True, ChartLabels → {"0", "5", "10", "15"},
 BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15]]
```

Out[ ]=



In[•]:=
```
BoxWhiskerChart[allcoopWith6AuxoPlusAuxo, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{gree2}},
 Frame → True, ChartLabels → {"0", "5", "10", "15"},
 BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15]]
```

Out[•]=


In[•]:=
```
SignedRankTest[allcoopWith6AuxoPlusAuxo[[2]], 1]
SignedRankTest[allcoopWith6AuxoPlusAuxo[[3]], 1]
SignedRankTest[allcoopWith6AuxoPlusAuxo[[4]], 1]
```

Out[•]= 0.

Out[•]= 0.

Out[•]= 0.

In[•]:=
```
allcoopWith6AuxoPlusAuxo // Length
```

Out[•]= 4

In[•]:=
```
allcoopWith6AuxoPlusAuxo[[2]]
```

In[•]:=
```
wf1
```

In[•]:=
```
wf1 // Length
```

Out[•]= 4

In[•]:=
```
wf1[[1]] // Length
```

Out[•]= 50

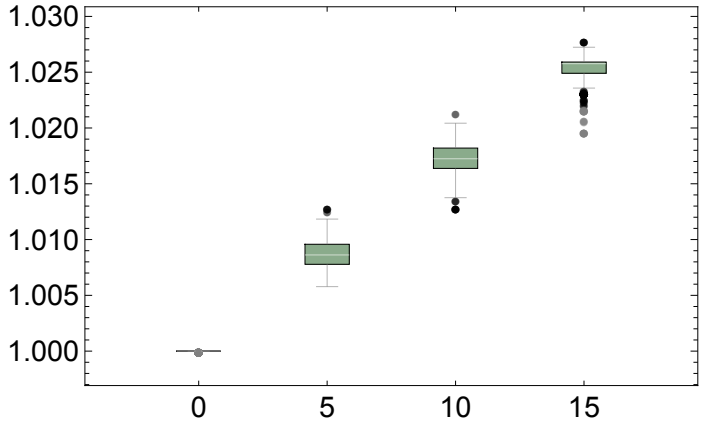
```
(*For 7 auxotrophies networks*)
```

In[ ]:=
```
allcoopWith7Auxo = {Flatten[wf7NormalizedWith5Coop],
  Flatten[wf7NormalizedWith10Coop], Flatten[wf7NormalizedWith15Coop]}
```

In[ ]:=
```
allcoopWith7AuxoPlusAuxo = Join[{ConstantArray[1, {2000}]}, allcoopWith7Auxo]
```

In[ ]:=
```
BoxWhiskerChart[allcoopWith7AuxoPlusAuxo, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{gree1}},
 Frame → True, ChartLabels → {"0", "5", "10", "15"},
 BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15]]
```

Out[ ]=



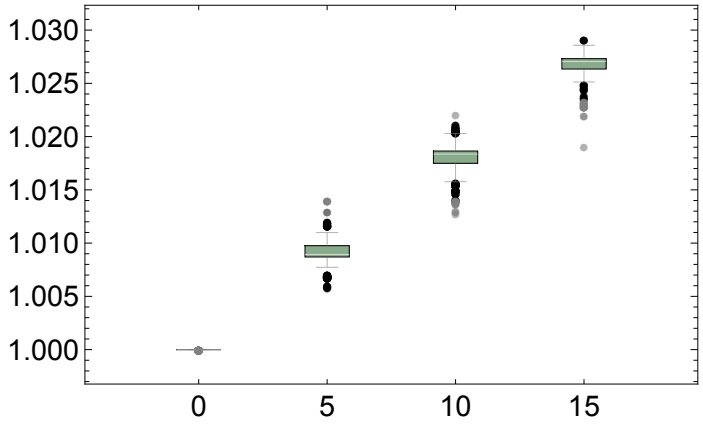

```
(*For 8 auxotrophies networks*)
```

In[ ]:=
```
allcoopWith8Auxo = {Flatten[wf8NormalizedWith5Coop],
  Flatten[wf8NormalizedWith10Coop], Flatten[wf8NormalizedWith15Coop]}
```

In[ ]:=
```
allcoopWith8AuxoPlusAuxo = Join[{ConstantArray[1, {2000}]}, allcoopWith8Auxo]
```

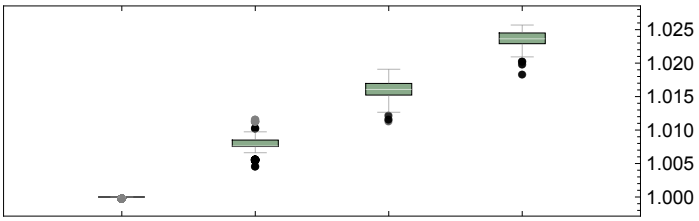
In[ ]:=
```
BoxWhiskerChart[allcoopWith8AuxoPlusAuxo, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{gree1}},
 Frame → True, ChartLabels → {"0", "5", "10", "15"},
 BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15]]
```

Out[ ]=



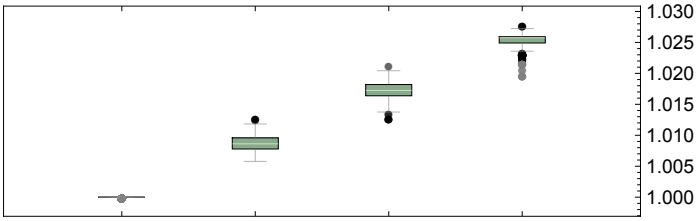
In[ ]:=
```
BoxWhiskerChart[allcoopWith6AuxoPlusAuxo, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{gree1}},
 Frame → True, FrameTicks → {{None, All}, {None, All}}, BarSpacing → 1.9,
 FrameStyle → Directive[Black, FontSize → 10], AspectRatio → 0.33]
```

Out[ ]=



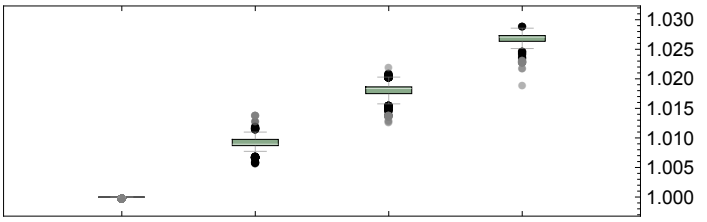
In[ ]:=
```
BoxWhiskerChart[allcoopWith7AuxoPlusAuxo, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{gree1}},
 Frame → True, FrameTicks → {{None, All}, {None, All}}, BarSpacing → 1.9,
 FrameStyle → Directive[Black, FontSize → 10], AspectRatio → 0.33]
```

Out[ ]=



In[•]:=
```
BoxWhiskerChart[allcoopWith8AuxoPlusAuxo, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{gree1}},
 Frame → True, FrameTicks → {{None, All}, {None, All}}, BarSpacing → 1.9,
 FrameStyle → Directive[Black, FontSize → 10], AspectRatio → 0.33]
```

Out[•]=



In[•]:=
```
NetK = {
    {0, 1, 0, 1, 0},
    {1, 0, 1, 1, 0},
    {1, 0, 1, 0, 1},
    {0, 1, 0, 1, 0},
    {0, 0, 0, 0, 1}
   };
```

In[•]:=
```
Position[NetK, 1]
```

Out[•]=
```
{{1, 2}, {1, 4}, {2, 1}, {2, 3}, {2, 4}, {3, 1}, {3, 3}, {3, 5}, {4, 2}, {4, 4}, {5, 5}}
```

In[•]:=
```
Neta
```

Out[•]=
```
{{1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {1, 1, 0, 0, 0}, {1, 1, 1, 0, 0}, {1, 1, 1, 1, 1}}
```

In[•]:=
```
Netb
```

Out[•]=
```
{{1, 1, 0, 0, 0}, {0, 1, 1, 0, 0}, {0, 1, 0, 1, 1}, {0, 1, 0, 1, 1}, {0, 0, 1, 0, 1}}
```

In[•]:=
```
robustnessNewSaito[Neta]
```

Out[•]=
```
912
```

In[•]:=
```
robustnessNewSaito[Netb]
```

Out[•]=
```
955
```

```
(* For Neta *)
```

```
{{3, 1}, {3, 2}, {4, 1}, {4, 2}, {4, 3}}
{{1, 1}, {2, 2}, {3, 2}, {5, 4}, {5, 5}}

(* For Netb *)
{{2, 2}, {3, 2}, {3, 5}, {4, 2}, {4, 5}}
{{1, 1}, {2, 3}, {3, 4}, {4, 4}, {5, 3}}
```

In[ ]:=

```
fNewSaitoOVSpecificOV[Net_, Dh_] := (

 dB1 =
  B1[t] (-B1[t] κ1 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) - (c1,1 + c1,2 + c1,3 + c1,4 + c1,5 + Dh) B1[t];
 dB2 = B2[t] (-B2[t] κ2 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
      M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c2,1 + c2,2 + c2,3 + c2,4 + c2,5 + Dh) B2[t];
 dB3 = B3[t] (-B3[t] κ3 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
      M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c3,1 + c3,2 + c3,3 + c3,4 + c3,5 + Dh) B3[t];
 dB4 = B4[t] (-B4[t] κ4 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
      M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c4,1 + c4,2 + c4,3 + c4,4 + c4,5 + Dh) B4[t];
 dB5 = B5[t] (-B5[t] κ5 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
      M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c5,1 + c5,2 + c5,3 + c5,4 + c5,5 + Dh) B5[t];

 dM1 = -M1[t] q1 +
   (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t]))
    (-B1[t] d1,1 - B2[t] d1,2 - B3[t] d1,3 - B4[t] d1,4 - B5[t] d1,5) +
   B1[t] Ω1,1 + B2[t] Ω1,2 + B3[t] Ω1,3 + B4[t] Ω1,4 + B5[t] Ω1,5;
```

```
dM2 = -M2[t] q2 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) (-B1[t] d2,1 - B2[t] d2,2 - B3[t] d2,3 - B4[t] d2,4 - B5[t] d2,5) +
  B1[t] Ω2,1 + B2[t] Ω2,2 + B3[t] Ω2,3 + B4[t] Ω2,4 + B5[t] Ω2,5;
dM3 = -M3[t] q3 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) (-B1[t] d3,1 - B2[t] d3,2 - B3[t] d3,3 - B4[t] d3,4 - B5[t] d3,5) +
  B1[t] Ω3,1 + B2[t] Ω3,2 + B3[t] Ω3,3 + B4[t] Ω3,4 + B5[t] Ω3,5;
dM4 = -M4[t] q4 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) (-B1[t] d4,1 - B2[t] d4,2 - B3[t] d4,3 - B4[t] d4,4 - B5[t] d4,5) +
  B1[t] Ω4,1 + B2[t] Ω4,2 + B3[t] Ω4,3 + B4[t] Ω4,4 + B5[t] Ω4,5;
dM5 = -M5[t] q5 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) (-B1[t] d5,1 - B2[t] d5,2 - B3[t] d5,3 - B4[t] d5,4 - B5[t] d5,5) +
  B1[t] Ω5,1 + B2[t] Ω5,2 + B3[t] Ω5,3 + B4[t] Ω5,4 + B5[t] Ω5,5;

KK = 0.2;
cc = 0.05;
qq = 0.3;
dd = 0.00015;
OM = 1;
nu = 1500;
den = 2;


(*################################################
 ##########################################################################*)
(*RaN={{3,1},{3,2},{4,1},{4,2},{4,3}};*)
(*specific provided links that will be overproduced*)
(*RaN={{1,1},{2,2},{3,2},{5,4},{5,5}};*)

(*RaN={{2,2},{3,2},{3,5},{4,2},{4,5}};*)
RaN = {{1, 1}, {2, 3}, {3, 4}, {4, 4}, {5, 3}};
(*################################################
 ##########################################################################*)

costincr = 1.3; (*Term multiplying the cost link*)
overprodincr = 1.15;
(*Term multiplying the overproduction link*)
```

```
NewNetCost = Net cc;
Table[NewNetCost[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] =
  NewNetCost[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] * costincr, {i, Length[RaN]}];

NewNetOvProd = Net OM;
Table[NewNetOvProd[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] =
  NewNetOvProd[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] * overprodincr, {i,
  Length[RaN]}];


tmax = 1000;
par = {
  κ1 → KK, κ2 → KK, κ3 → KK, κ4 → KK, κ5 → KK,

  c1,1 → NewNetCost[[1]][[1]],
  c1,2 → NewNetCost[[1]][[2]], c1,3 → NewNetCost[[1]][[3]],
  c1,4 → NewNetCost[[1]][[4]], c1,5 → NewNetCost[[1]][[5]],
  c2,1 → NewNetCost[[2]][[1]], c2,2 → NewNetCost[[2]][[2]],
  c2,3 → NewNetCost[[2]][[3]], c2,4 → NewNetCost[[2]][[4]],
  c2,5 → NewNetCost[[2]][[5]],
  c3,1 → NewNetCost[[3]][[1]], c3,2 → NewNetCost[[3]][[2]],
  c3,3 → NewNetCost[[3]][[3]], c3,4 → NewNetCost[[3]][[4]],
  c3,5 → NewNetCost[[3]][[5]],
  c4,1 → NewNetCost[[4]][[1]], c4,2 → NewNetCost[[4]][[2]],
  c4,3 → NewNetCost[[4]][[3]], c4,4 → NewNetCost[[4]][[4]],
  c4,5 → NewNetCost[[4]][[5]],
  c5,1 → NewNetCost[[5]][[1]], c5,2 → NewNetCost[[5]][[2]],
  c5,3 → NewNetCost[[5]][[3]], c5,4 → NewNetCost[[5]][[4]],
  c5,5 → NewNetCost[[5]][[5]],

  q1 → qq, q2 → qq, q3 → qq, q4 → qq, q5 → qq,

  d1,1 → dd, d1,2 → dd, d1,3 → dd, d1,4 → dd, d1,5 → dd,
  d2,1 → dd, d2,2 → dd, d2,3 → dd, d2,4 → dd, d2,5 → dd,
  d3,1 → dd, d3,2 → dd, d3,3 → dd, d3,4 → dd, d3,5 → dd,
  d4,1 → dd, d4,2 → dd, d4,3 → dd, d4,4 → dd, d4,5 → dd,
  d5,1 → dd, d5,2 → dd, d5,3 → dd, d5,4 → dd, d5,5 → dd,

  Ω1,1 → NewNetOvProd[[1]][[1]],
  Ω1,2 → NewNetOvProd[[1]][[2]], Ω1,3 → NewNetOvProd[[1]][[3]],
  Ω1,4 → NewNetOvProd[[1]][[4]], Ω1,5 → NewNetOvProd[[1]][[5]],
  Ω2,1 → NewNetOvProd[[2]][[1]], Ω2,2 → NewNetOvProd[[2]][[2]],
```

```
  Ω2,3 → NewNetOvProd[[2]][[3]], Ω2,4 → NewNetOvProd[[2]][[4]],
  Ω2,5 → NewNetOvProd[[2]][[5]],
  Ω3,1 → NewNetOvProd[[3]][[1]], Ω3,2 → NewNetOvProd[[3]][[2]],
  Ω3,3 → NewNetOvProd[[3]][[3]], Ω3,4 → NewNetOvProd[[3]][[4]],
  Ω3,5 → NewNetOvProd[[3]][[5]],
  Ω4,1 → NewNetOvProd[[4]][[1]], Ω4,2 → NewNetOvProd[[4]][[2]],
  Ω4,3 → NewNetOvProd[[4]][[3]], Ω4,4 → NewNetOvProd[[4]][[4]],
  Ω4,5 → NewNetOvProd[[4]][[5]],
  Ω5,1 → NewNetOvProd[[5]][[1]], Ω5,2 → NewNetOvProd[[5]][[2]],
  Ω5,3 → NewNetOvProd[[5]][[3]], Ω5,4 → NewNetOvProd[[5]][[4]],
  Ω5,5 → NewNetOvProd[[5]][[5]],
  nuK → nu,
  denK → den


 };


B10 = 1500;
B20 = 1500;
B30 = 1500;
B40 = 1500;
B50 = 1500;
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;


sol =
 NDSolve[
  {
    B1'[t] == dB1,
    B2'[t] == dB2,
    B3'[t] == dB3,
    B4'[t] == dB4,
    B5'[t] == dB5,

    M1'[t] == dM1,
    M2'[t] == dM2,
    M3'[t] == dM3,
    M4'[t] == dM4,
    M5'[t] == dM5,
```

```
      B1[0] == B10,
      B2[0] == B20,
      B3[0] == B30,
      B4[0] == B40,
      B5[0] == B50,
      M1[0] == M10,
      M2[0] == M20,
      M3[0] == M30,
      M4[0] == M40,
      M5[0] == M50

     } /. par,
    {B1, B2, B3, B4, B5, M1, M2, M3, M4, M5},
    {t, 0, tmax}];

  {B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax],
     M1[tmax], M2[tmax], M3[tmax], M4[tmax], M5[tmax]} /. sol /. par;

  Min[{B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax]} /. sol /. par]

 )
```

In[ ]:=

```
robustnessNewSaitoOVSpecificOV[NetTop_] := (
  n1 = 1;
  n2 = 5000;
  mid = (n1 + n2) / 2;

  While[(n1 ≠ mid && n2 ≠ mid),
   (If[fNewSaitoOVSpecificOV[NetTop, mid] < 1, n2 = mid, n1 = mid];
    mid = Floor[N[(n1 + n2) / 2]];); {n1, n2, mid}]; mid

 )
```

```
(*************************)
(* For: {{3,1},{3,2},{4,1},{4,2},{4,3}} : *)
In[•]:= robustnessNewSaitoOVSpecificOV[Neta]
Out[•]= 924
(*************************)
```

```
(*************************)
(* For: {{1,1},{2,2},{3,2},{5,4},{5,5}} : *)
In[•]:= robustnessNewSaitoOVSpecificOV[Neta]
Out[•]= 930
(*************************)
```

```
(*************************)
(* For: {{2,2},{3,2},{3,5},{4,2},{4,5}} : *)
In[•]:= robustnessNewSaitoOVSpecificOV[Netb]
Out[•]= 966
(*************************)
```

```
(*************************)
(* For: {{1,1},{2,3},{3,4},{4,4},{5,3}} : *)
In[•]:= robustnessNewSaitoOVSpecificOV[Netb]
Out[•]= 969
(*************************)
```

Figure S1: In file “2_ColimitationModelOnlyMetabolitesD.nb”

Figure S2: In file “3_ColimitationModelBothMicroAndMetD.nb”

Figure S3: In file “4_LiebigModel.nb”

Figure S4: In file “5_AdditiveModel.nb”

Figure S5: In file “6_OpenSystemColimitationModel.nb”

Figure S6: Correlation Smallest Population size in the community and Robustness

1) Saito Colimitation Model

```
In[•]:= NetK = {
          {0, 1, 0, 1, 0},
          {1, 0, 1, 1, 0},
          {1, 0, 1, 0, 1},
          {0, 1, 0, 1, 0},
          {0, 0, 0, 0, 1}
         };

In[•]:= fNewSaito[NetK, 0]

Out[•]= 6661.43
```

In[•]:=
```
PopMin0R = Parallelize[fNewSaito[#, 0] & /@ hk8]
```

Out[•]=
```
{7497.06, 7496.81, 7497.06, 7497.62, 7496.75, 7496.81, 6660.49, 7497.62, 7497.62,
 7497.06, 7497.37, 7497.37, 7496.75, 7497.37, 7497.32, 7497.06, 7497.06, 7496.81,
 7497.37, 6660.74, 7497.06, 7497.06, 7497.37, 7497.06, 7497.06, 7497.62, 7497.37,
 6660.49, 7497.62, 7497.37, 7497.37, 6660.49, 7497.37, 7497.62, 7497.62, 7497.62,
 7497.06, 7497.32, 7497.62, 7497.62, 7497.06, 7497.62, 7497.32, 7497.62,
 7497.62, 7497.37, 7497.06, 7497.06, 7497.62, 7497.06, 7497.06, 7497.37,
 7497.06, 6660.49, 7497.62, 7497.37, 6660.24, 7497.62, 7497.62, 7497.37,
 7497.06, 7497.06, 7497.62, 7497.62, 7497.37, 7497.06, 7497.06, 7497.62,
 7497.62, 7497.32, 7497.37, 7497.06, 7497.62, 7497.62, 7497.62, 7497.62,
 7497.37, 7496.81, 7497.06, 7497.62, 7496.81, 7497.62, 7497.37, 7497.06,
 7497.06, 7497.62, 7497.32, 7497.06, 7497.06, 7497.06, 6660.24, 7497.62,
 7497.62, 7497.32, 7497.62, 7497.37, 7497.06, 7497.06, 7497.37, 7497.62}
```

In[•]:=
```
RobustNewSaitoR8 = Parallelize[robustnessNewSaito /@ hk8]
```

Out[•]=
```
{1025, 1019, 1025, 1035, 1015, 1019, 1005, 1035, 1035, 1025, 1029, 1029,
 1015, 1029, 1031, 1025, 1025, 1019, 1029, 1009, 1025, 1025, 1029, 1025,
 1025, 1035, 1029, 1005, 1035, 1029, 1029, 1005, 1029, 1035, 1035, 1035,
 1025, 1031, 1035, 1035, 1025, 1035, 1031, 1035, 1035, 1029, 1025, 1025,
 1035, 1025, 1025, 1029, 1025, 995, 1035, 1029, 1001, 1035, 1035, 1029, 1025,
 1025, 1035, 1035, 1029, 1025, 1025, 1035, 1035, 1031, 1029, 1025, 1035, 1035,
 1035, 1035, 1029, 1019, 1025, 1035, 1019, 1035, 1029, 1025, 1025, 1035, 1031,
 1025, 1025, 1025, 995, 1035, 1035, 1031, 1035, 1029, 1025, 1025, 1029, 1035}
```

In[•]:=
```
Length[%]
```

Out[•]=
```
100
```

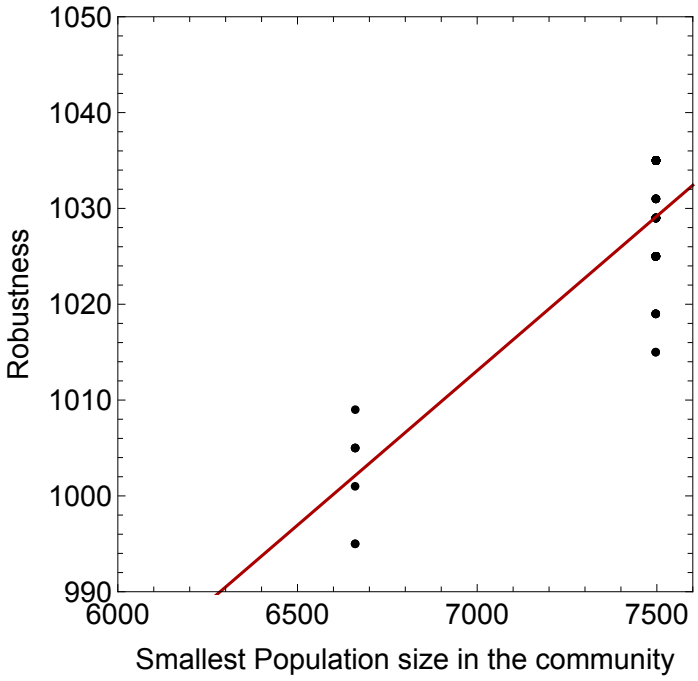
In[•]:=
```
{Min[PopMin0R], Max[PopMin0R]}
{Min[RobustNewSaitoR8], Max[RobustNewSaitoR8]}
```

Out[•]=
```
{6660.24, 7497.62}
```

Out[•]=
```
{995, 1035}
```

In[•]:=
```
linePopMinRobSaitoRColim =
  Fit[Partition[Riffle[PopMin0R, RobustNewSaitoR8], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[PopMin0R, RobustNewSaitoR8], {2}], Frame → True,
  FrameLabel → {"Smallest Population size in the community", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]}, PlotRange → {{6000, 7600}, {990, 1050}},
  AspectRatio → 1.0], Plot[linePopMinRobSaitoRColim,
  {x, 6000, 7600}, AspectRatio → 0.5, PlotStyle → Darker[Red]]]

SpearmanRankTest[PopMin0R, RobustNewSaitoR8, "TestDataTable"]
```

Out[•]=



Out[•]=

	Statistic	P-Value
Spearman Rank	0.949827	2.88359×10^{-51}

2) Saito Core Model with both Bacteria and Metabolites affected by Disturbance

In[]:=
```
NetK = {
    {0, 1, 0, 1, 0},
    {1, 0, 1, 1, 0},
    {1, 0, 1, 0, 1},
    {0, 1, 0, 1, 0},
    {0, 0, 0, 0, 1}
   };
```

In[]:=
```
fNewSaitoBth[NetK, 0]
```

Out[]=
```
6661.43
```

In[]:=
```
PopMin0Rmb = Parallelize[fNewSaitoBth[#, 0] & /@ hk8]
```

In[]:=
```
RobustNewSaitoMMR7 = Parallelize[robustnessNewSaitoBth /@ hk8]
```

Out[]=
```
{349, 338, 349, 365, 334, 338, 286, 365, 365, 349, 353, 353, 334, 353, 360, 349, 349, 338,
 353, 289, 349, 349, 353, 349, 349, 365, 353, 286, 365, 353, 353, 286, 353, 365, 365,
 365, 349, 360, 365, 365, 349, 365, 360, 365, 365, 353, 349, 349, 365, 349, 349, 353,
 349, 275, 365, 353, 283, 365, 365, 353, 349, 349, 365, 365, 353, 349, 349, 365,
 365, 360, 353, 349, 365, 365, 365, 365, 353, 338, 349, 365, 338, 365, 353, 349,
 349, 365, 360, 349, 349, 349, 275, 365, 365, 360, 365, 353, 349, 349, 353, 365}
```

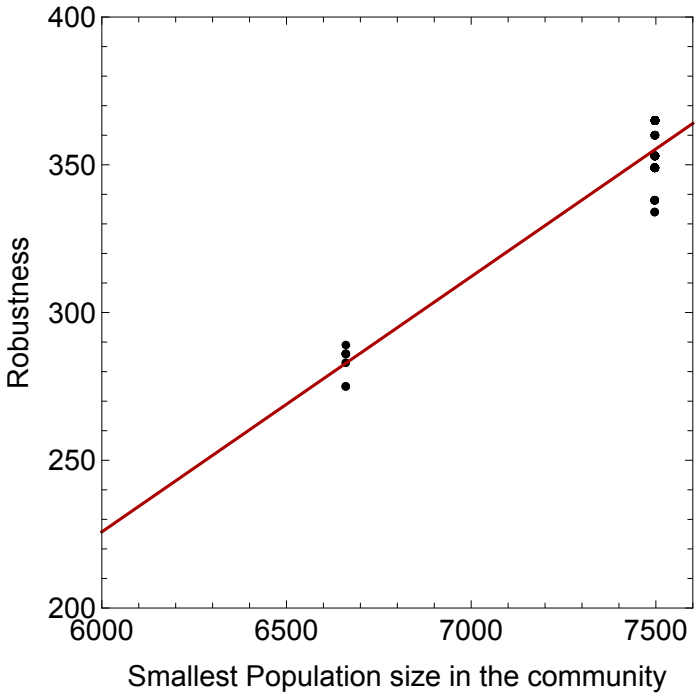
In[]:=
```
{Min[PopMin0Rmb], Max[PopMin0Rmb]}
{Min[RobustNewSaitoMMR7], Max[RobustNewSaitoMMR7]}
```

Out[]=
```
{6660.24, 7497.62}
```

Out[]=
```
{275, 365}
```

In[•]:=
```
linePopMinRobSaitoRmb =
  Fit[Partition[Riffle[PopMin0Rmb, RobustNewSaitoMMR7], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[PopMin0Rmb, RobustNewSaitoMMR7], {2}], Frame → True,
  FrameLabel → {"Smallest Population size in the community", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]}, PlotRange → {{6000, 7600}, {200, 400}},
  AspectRatio → 1.0], Plot[linePopMinRobSaitoRmb,
  {x, 6000, 7600}, AspectRatio → 0.5, PlotStyle → Darker[Red]]]

SpearmanRankTest[PopMin0Rmb, RobustNewSaitoMMR7, "TestDataTable"]
```

Out[•]=



Out[•]=

	Statistic	P-Value
Spearman Rank	0.949827	2.88359×10^{-51}

3) Saito Core Model with only Metabolites affected by Disturbance

In[•]:=
```
NetK = {
    {0, 1, 0, 1, 0},
    {1, 0, 1, 1, 0},
    {1, 0, 1, 0, 1},
    {0, 1, 0, 1, 0},
    {0, 0, 0, 0, 1}
   };
```

```
fNewSaitoOM[NetK, 0]
```

Out[•]= 6661.43

In[•]:= `PopMin0RmbOM = Parallelize[fNewSaitoOM[#, 0] & /@ hk8]`

In[•]:= `RobustNewSaitoMMR7OM = Parallelize[robustnessNewSaitoOM /@ hk8]`

Out[•]= {846, 800, 846, 915, 783, 800, 655, 915, 915, 846, 865, 865, 783, 865, 894, 846, 846, 800, 865, 668, 846, 846, 865, 846, 846, 915, 865, 655, 915, 865, 865, 655, 865, 915, 915, 915, 846, 894, 915, 915, 846, 915, 894, 915, 915, 865, 846, 846, 915, 846, 846, 865, 846, 613, 915, 865, 642, 915, 915, 865, 846, 846, 915, 915, 865, 846, 846, 915, 915, 894, 865, 846, 915, 915, 915, 915, 865, 800, 846, 915, 800, 915, 865, 846, 846, 915, 894, 846, 846, 846, 613, 915, 915, 894, 915, 865, 846, 846, 865, 915}

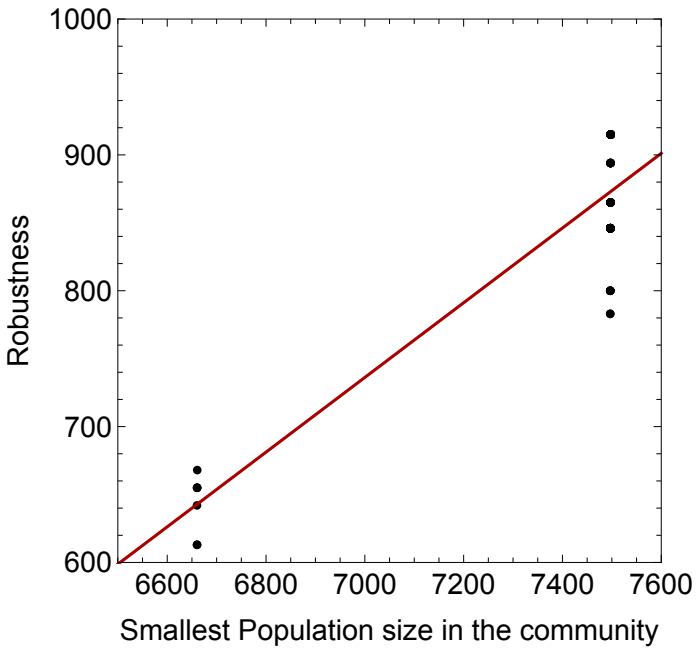
In[•]:=
```
{Min[PopMin0RmbOM], Max[PopMin0RmbOM]}
{Min[RobustNewSaitoMMR7OM], Max[RobustNewSaitoMMR7OM]}
```

Out[•]= {6660.24, 7497.62}

Out[•]= {613, 915}

In[•]:=
```
linePopMinRobSaitoRmbOM =
  Fit[Partition[Riffle[PopMin0RmbOM, RobustNewSaitoMMR7OM], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[PopMin0RmbOM, RobustNewSaitoMMR7OM], {2}], Frame →
   True, FrameLabel → {"Smallest Population size in the community", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]}, PlotRange → {{6500, 7600}, {600, 1000}},
  AspectRatio → 1.0], Plot[linePopMinRobSaitoRmbOM,
  {x, 6500, 7600}, AspectRatio → 0.5, PlotStyle → Darker[Red]]]

SpearmanRankTest[PopMin0RmbOM, RobustNewSaitoMMR7OM, "TestDataTable"]
```

Out[•]=



Out[•]=

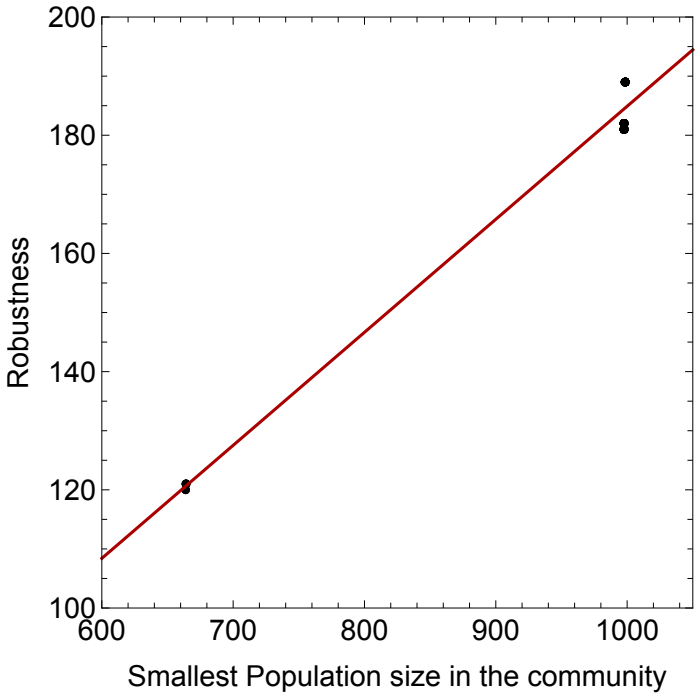
	Statistic	P-Value
Spearman Rank	0.949827	2.88359×10^{-51}

4) Liebig's Law Model

```
In[•]:= NetK = {
           {0, 1, 0, 1, 0},
           {1, 0, 1, 1, 0},
           {1, 0, 1, 0, 1},
           {0, 1, 0, 1, 0},
           {0, 0, 0, 0, 1}
          };

In[•]:= fNewLiebig[NetK, 0]
Out[•]= 665.02

In[•]:= PopMin0RmbLi = Parallelize[fNewLiebig[#, 0] & /@ hk8]

In[•]:= RobustNewSaitoMMR7Li = Parallelize[robustnessNewLiebig /@ hk8]
Out[•]= {181, 181, 182, 189, 181, 181, 121, 189, 189, 182, 181, 181, 182, 181, 189, 182, 181, 181,
         181, 121, 181, 181, 182, 182, 181, 189, 182, 121, 189, 182, 181, 121, 182, 189, 189,
         189, 182, 189, 189, 189, 181, 189, 189, 189, 189, 181, 181, 181, 189, 182, 181, 182,
         182, 121, 189, 181, 120, 189, 189, 182, 182, 182, 189, 189, 182, 181, 181, 189,
         189, 189, 181, 182, 189, 189, 189, 189, 181, 181, 181, 189, 181, 189, 182, 182,
         181, 189, 189, 182, 181, 181, 120, 189, 189, 189, 189, 182, 181, 182, 181, 189}

In[•]:= {Min[PopMin0RmbLi], Max[PopMin0RmbLi]}
        {Min[RobustNewSaitoMMR7Li], Max[RobustNewSaitoMMR7Li]}
Out[•]= {663.819, 998.6}

Out[•]= {120, 189}
```

In[•]:=
```
linePopMinRobSaitoRmbLi =
  Fit[Partition[Riffle[PopMin0RmbLi, RobustNewSaitoMMR7Li], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[PopMin0RmbLi, RobustNewSaitoMMR7Li], {2}], Frame →
   True, FrameLabel → {"Smallest Population size in the community", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]}, PlotRange → {{600, 1050}, {100, 200}},
  AspectRatio → 1.0], Plot[linePopMinRobSaitoRmbLi,
  {x, 600, 1050}, AspectRatio → 0.5, PlotStyle → Darker[Red]]]

SpearmanRankTest[PopMin0RmbLi, RobustNewSaitoMMR7Li, "TestDataTable"]
```

Out[•]=



Out[•]=

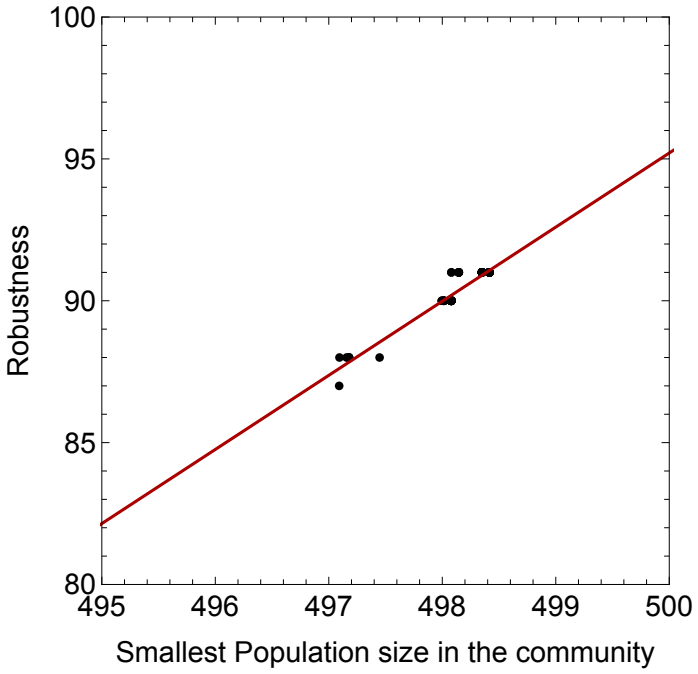
	Statistic	P-Value
Spearman Rank	0.90482	4.06642×10^{-38}

5) Additive Model

```
In[•]:= NetK = {
           {0, 1, 0, 1, 0},
           {1, 0, 1, 1, 0},
           {1, 0, 1, 0, 1},
           {0, 1, 0, 1, 0},
           {0, 0, 0, 0, 1}
          };

In[•]:= fNewMono[NetK, 0]
Out[•]= 497.327

In[•]:= PopMin0RmbAdd = Parallelize[fNewMono[#, 0] & /@ hk8]
Out[•]= {498.08, 498.014, 498.08, 498.415, 497.995, 498.014, 497.178, 498.415, 498.415,
         498.08, 498.349, 498.349, 497.996, 498.349, 498.145, 498.08, 498.08, 498.014,
         498.349, 497.448, 498.08, 498.08, 498.349, 498.08, 498.08, 498.415, 498.349,
         497.178, 498.415, 498.349, 498.349, 497.178, 498.349, 498.415, 498.415, 498.415,
         498.08, 498.145, 498.415, 498.415, 498.08, 498.415, 498.145, 498.415, 498.415,
         498.349, 498.08, 498.08, 498.415, 498.08, 498.08, 498.349, 498.08, 497.094, 498.415,
         498.349, 497.157, 498.415, 498.415, 498.349, 498.08, 498.08, 498.415, 498.415,
         498.349, 498.08, 498.08, 498.415, 498.415, 498.145, 498.349, 498.08, 498.415,
         498.415, 498.415, 498.415, 498.349, 498.014, 498.08, 498.415, 498.014, 498.415,
         498.349, 498.08, 498.08, 498.415, 498.145, 498.08, 498.08, 498.08, 497.091,
         498.415, 498.415, 498.145, 498.415, 498.349, 498.08, 498.08, 498.349, 498.415}

In[•]:= RobustNewSaitoMMR7Add = Parallelize[robustnessNewMono /@ hk8]
Out[•]= {90, 90, 91, 91, 90, 90, 88, 91, 91, 90, 91, 91, 90, 91, 91, 90, 90, 90, 91, 88,
         90, 90, 91, 90, 90, 91, 91, 88, 91, 91, 91, 88, 91, 91, 91, 91, 90, 91, 91, 91,
         90, 91, 91, 91, 91, 91, 90, 90, 91, 90, 90, 91, 90, 88, 91, 91, 88, 91, 91, 91,
         90, 90, 91, 91, 91, 90, 90, 91, 91, 91, 91, 90, 91, 91, 91, 91, 91, 90, 90, 91,
         90, 91, 91, 90, 90, 91, 91, 91, 90, 90, 87, 91, 91, 91, 91, 91, 90, 90, 91, 91}

In[•]:= {Min[PopMin0RmbAdd], Max[PopMin0RmbAdd]}
        {Min[RobustNewSaitoMMR7Add], Max[RobustNewSaitoMMR7Add]}
Out[•]= {497.091, 498.415}

Out[•]= {87, 91}
```

In[•]:=
```
linePopMinRobSaitoRmbAdd =
  Fit[Partition[Riffle[PopMin0RmbAdd, RobustNewSaitoMMR7Add], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[PopMin0RmbAdd, RobustNewSaitoMMR7Add], {2}], Frame →
   True, FrameLabel → {"Smallest Population size in the community", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]}, PlotRange → {{495, 500}, {80, 100}},
  AspectRatio → 1.0], Plot[linePopMinRobSaitoRmbAdd,
  {x, 450, 550}, AspectRatio → 0.5, PlotStyle → Darker[Red]]]

SpearmanRankTest[PopMin0RmbAdd, RobustNewSaitoMMR7Add, "TestDataTable"]
```

Out[•]=



Out[•]=

	Statistic	P-Value
Spearman Rank	0.868543	1.22749×10^{-31}

Figure S7: Local Stability Analysis Eigenvalues

In[5162]:=
```
fNewSaitoK[Net_, Dh_] := (

  dB1 =
   B1[t] (-B1[t] κ1 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
       M5[t]/(denK + M5[t])) - (c1,1 + c1,2 + c1,3 + c1,4 + c1,5 + Dh) B1[t];
  dB2 = B2[t] (-B2[t] κ2 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
```

```
        M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c2,1 + c2,2 + c2,3 + c2,4 + c2,5 + Dh) B2[t];
dB3 = B3[t] (-B3[t] κ3 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
        M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c3,1 + c3,2 + c3,3 + c3,4 + c3,5 + Dh) B3[t];
dB4 = B4[t] (-B4[t] κ4 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
        M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c4,1 + c4,2 + c4,3 + c4,4 + c4,5 + Dh) B4[t];
dB5 = B5[t] (-B5[t] κ5 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
        M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c5,1 + c5,2 + c5,3 + c5,4 + c5,5 + Dh) B5[t];

dM1 = -M1[t] q1 +
   (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t]))
    (-B1[t] d1,1 - B2[t] d1,2 - B3[t] d1,3 - B4[t] d1,4 - B5[t] d1,5) +
  B1[t] Ω1,1 + B2[t] Ω1,2 + B3[t] Ω1,3 + B4[t] Ω1,4 + B5[t] Ω1,5;
dM2 = -M2[t] q2 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) (-B1[t] d2,1 - B2[t] d2,2 - B3[t] d2,3 - B4[t] d2,4 - B5[t] d2,5) +
  B1[t] Ω2,1 + B2[t] Ω2,2 + B3[t] Ω2,3 + B4[t] Ω2,4 + B5[t] Ω2,5;
dM3 = -M3[t] q3 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) (-B1[t] d3,1 - B2[t] d3,2 - B3[t] d3,3 - B4[t] d3,4 - B5[t] d3,5) +
  B1[t] Ω3,1 + B2[t] Ω3,2 + B3[t] Ω3,3 + B4[t] Ω3,4 + B5[t] Ω3,5;
dM4 = -M4[t] q4 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) (-B1[t] d4,1 - B2[t] d4,2 - B3[t] d4,3 - B4[t] d4,4 - B5[t] d4,5) +
  B1[t] Ω4,1 + B2[t] Ω4,2 + B3[t] Ω4,3 + B4[t] Ω4,4 + B5[t] Ω4,5;
dM5 = -M5[t] q5 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) (-B1[t] d5,1 - B2[t] d5,2 - B3[t] d5,3 - B4[t] d5,4 - B5[t] d5,5) +
  B1[t] Ω5,1 + B2[t] Ω5,2 + B3[t] Ω5,3 + B4[t] Ω5,4 + B5[t] Ω5,5;

KK = 0.2;
cc = 0.05;
qq = 0.3;
```

```
dd = 0.00015;
OM = 1;
nu = 1500;
den = 2;

tmax = 1000;
par = {
  κ1 → KK, κ2 → KK, κ3 → KK, κ4 → KK, κ5 → KK,

  c1,1 → cc Net[[1]][[1]], c1,2 → cc Net[[1]][[2]],
  c1,3 → cc Net[[1]][[3]], c1,4 → cc Net[[1]][[4]], c1,5 → cc Net[[1]][[5]],
  c2,1 → cc Net[[2]][[1]], c2,2 → cc Net[[2]][[2]], c2,3 → cc Net[[2]][[3]],
  c2,4 → cc Net[[2]][[4]], c2,5 → cc Net[[2]][[5]],
  c3,1 → cc Net[[3]][[1]], c3,2 → cc Net[[3]][[2]], c3,3 → cc Net[[3]][[3]],
  c3,4 → cc Net[[3]][[4]], c3,5 → cc Net[[3]][[5]],
  c4,1 → cc Net[[4]][[1]], c4,2 → cc  Net[[4]][[2]], c4,3 → cc  Net[[4]][[3]],
  c4,4 → cc  Net[[4]][[4]], c4,5 → cc  Net[[4]][[5]],
  c5,1 → cc  Net[[5]][[1]], c5,2 → cc Net[[5]][[2]], c5,3 → cc Net[[5]][[3]],
  c5,4 → cc Net[[5]][[4]], c5,5 → cc Net[[5]][[5]],

  q1 → qq, q2 → qq, q3 → qq, q4 → qq, q5 → qq,

  d1,1 → dd, d1,2 → dd, d1,3 → dd, d1,4 → dd, d1,5 → dd,
  d2,1 → dd, d2,2 → dd, d2,3 → dd, d2,4 → dd, d2,5 → dd,
  d3,1 → dd, d3,2 → dd, d3,3 → dd, d3,4 → dd, d3,5 → dd,
  d4,1 → dd, d4,2 → dd, d4,3 → dd, d4,4 → dd, d4,5 → dd,
  d5,1 → dd, d5,2 → dd, d5,3 → dd, d5,4 → dd, d5,5 → dd,

  Ω1,1 → OM Net[[1]][[1]], Ω1,2 → OM Net[[1]][[2]],
  Ω1,3 → OM Net[[1]][[3]], Ω1,4 → OM Net[[1]][[4]], Ω1,5 → OM Net[[1]][[5]],
  Ω2,1 → OM Net[[2]][[1]], Ω2,2 → OM Net[[2]][[2]], Ω2,3 → OM Net[[2]][[3]],
  Ω2,4 → OM Net[[2]][[4]], Ω2,5 → OM Net[[2]][[5]],
  Ω3,1 → OM Net[[3]][[1]], Ω3,2 → OM Net[[3]][[2]], Ω3,3 → OM Net[[3]][[3]],
  Ω3,4 → OM Net[[3]][[4]], Ω3,5 → OM Net[[3]][[5]],
  Ω4,1 → OM Net[[4]][[1]], Ω4,2 → OM Net[[4]][[2]], Ω4,3 → OM Net[[4]][[3]],
  Ω4,4 → OM Net[[4]][[4]], Ω4,5 → OM Net[[4]][[5]],
  Ω5,1 → OM Net[[5]][[1]], Ω5,2 → OM Net[[5]][[2]], Ω5,3 → OM Net[[5]][[3]],
  Ω5,4 → OM Net[[5]][[4]], Ω5,5 → OM Net[[5]][[5]],
  nuK → nu,
  denK → den


 };
```

```
B10 = 1500;
B20 = 1500;
B30 = 1500;
B40 = 1500;
B50 = 1500;
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;


sol =
 NDSolve[
  {
    B1'[t] == dB1,
    B2'[t] == dB2,
    B3'[t] == dB3,
    B4'[t] == dB4,
    B5'[t] == dB5,

    M1'[t] == dM1,
    M2'[t] == dM2,
    M3'[t] == dM3,
    M4'[t] == dM4,
    M5'[t] == dM5,


    B1[0] == B10,
    B2[0] == B20,
    B3[0] == B30,
    B4[0] == B40,
    B5[0] == B50,
    M1[0] == M10,
    M2[0] == M20,
    M3[0] == M30,
    M4[0] == M40,
    M5[0] == M50

   } /. par,
  {B1, B2, B3, B4, B5, M1, M2, M3, M4, M5},
  {t, 0, tmax}];
```

```
  {B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax],
     M1[tmax], M2[tmax], M3[tmax], M4[tmax], M5[tmax]} /. sol /. par

  (*Min[{B1[tmax],B2[tmax],B3[tmax],B4[tmax],B5[tmax]}/.sol/.par]*)

 )
```

As an example let's take the following Network

In[5177]:=
```
NetK = {
    {0, 1, 0, 1, 0},
    {1, 0, 1, 1, 0},
    {1, 0, 1, 0, 1},
    {0, 1, 0, 1, 0},
    {0, 0, 0, 0, 1}
   };
```

In[5164]:=
```
fNewSaitoK[NetK, 0]
fNewSaitoK[NetK, 1]
```

Out[5164]=
```
{{6661.68, 6661.43, 6661.43, 6661.68,
  6661.93, 22 219.9, 44 425.5, 44 426.3, 22 219.9, 15.9422}}
```

Out[5165]=
```
{{6656.68, 6656.43, 6656.43, 6656.68,
  6656.93, 22 203.2, 44 392.1, 44 393., 22 203.2, 15.9421}}
```

Using the function f we can calculate the smallest value of a bacterial population in the community for a given disturbance vale. For example, let's take Disturbance value 1 and 70:

In[5166]:=
```
po = Flatten[fNewSaitoK[NetK, 0]]
```

Out[5166]=
```
{6661.68, 6661.43, 6661.43, 6661.68,
 6661.93, 22 219.9, 44 425.5, 44 426.3, 22 219.9, 15.9422}
```

In[5194]:=
$$dB_1 = B_1 \left(-B_1 \kappa_1 + nuK * \frac{M_1}{denK + M_1} * \frac{M_2}{denK + M_2} * \frac{M_3}{denK + M_3} * \frac{M_4}{denK + M_4} * \frac{M_5}{denK + M_5}\right) - (c_{1,1} + c_{1,2} + c_{1,3} + c_{1,4} + c_{1,5})\, B_1;$$

$$dB_2 = B_2 \left(-B_2 \kappa_2 + nuK * \frac{M_1}{denK + M_1} * \frac{M_2}{denK + M_2} * \frac{M_3}{denK + M_3} * \frac{M_4}{denK + M_4} * \frac{M_5}{denK + M_5}\right) - (c_{2,1} + c_{2,2} + c_{2,3} + c_{2,4} + c_{2,5})\, B_2;$$

```
dB3 = B3 (-B3 κ3 + nuK * M1/(denK + M1) * M2/(denK + M2) * M3/(denK + M3) * M4/(denK + M4) * M5/(denK + M5)) -
   (c3,1 + c3,2 + c3,3 + c3,4 + c3,5) B3;
dB4 = B4 (-B4 κ4 + nuK * M1/(denK + M1) * M2/(denK + M2) * M3/(denK + M3) * M4/(denK + M4) * M5/(denK + M5)) -
   (c4,1 + c4,2 + c4,3 + c4,4 + c4,5) B4;
dB5 = B5 (-B5 κ5 + nuK * M1/(denK + M1) * M2/(denK + M2) * M3/(denK + M3) * M4/(denK + M4) * M5/(denK + M5)) -
   (c5,1 + c5,2 + c5,3 + c5,4 + c5,5) B5;

dM1 = -M1 q1 + (nuK * M1/(denK + M1) * M2/(denK + M2) * M3/(denK + M3) * M4/(denK + M4) * M5/(denK + M5))
    (-B1 d1,1 - B2 d1,2 - B3 d1,3 - B4 d1,4 - B5 d1,5) + B1 Ω1,1 + B2 Ω1,2 + B3 Ω1,3 + B4 Ω1,4 + B5 Ω1,5;
dM2 = -M2 q2 + (nuK * M1/(denK + M1) * M2/(denK + M2) * M3/(denK + M3) * M4/(denK + M4) * M5/(denK + M5))
    (-B1 d2,1 - B2 d2,2 - B3 d2,3 - B4 d2,4 - B5 d2,5) + B1 Ω2,1 + B2 Ω2,2 + B3 Ω2,3 + B4 Ω2,4 + B5 Ω2,5;
dM3 = -M3 q3 + (nuK * M1/(denK + M1) * M2/(denK + M2) * M3/(denK + M3) * M4/(denK + M4) * M5/(denK + M5))
    (-B1 d3,1 - B2 d3,2 - B3 d3,3 - B4 d3,4 - B5 d3,5) + B1 Ω3,1 + B2 Ω3,2 + B3 Ω3,3 + B4 Ω3,4 + B5 Ω3,5;
dM4 = -M4 q4 + (nuK * M1/(denK + M1) * M2/(denK + M2) * M3/(denK + M3) * M4/(denK + M4) * M5/(denK + M5))
    (-B1 d4,1 - B2 d4,2 - B3 d4,3 - B4 d4,4 - B5 d4,5) + B1 Ω4,1 + B2 Ω4,2 + B3 Ω4,3 + B4 Ω4,4 + B5 Ω4,5;
dM5 = -M5 q5 + (nuK * M1/(denK + M1) * M2/(denK + M2) * M3/(denK + M3) * M4/(denK + M4) * M5/(denK + M5))
    (-B1 d5,1 - B2 d5,2 - B3 d5,3 - B4 d5,4 - B5 d5,5) + B1 Ω5,1 + B2 Ω5,2 + B3 Ω5,3 + B4 Ω5,4 + B5 Ω5,5;

KK = 0.2;
cc = 0.05;
qq = 0.3;
dd = 0.00015;
OM = 1;
nu = 1500;
den = 2;


par = {
    κ1 → KK, κ2 → KK, κ3 → KK, κ4 → KK, κ5 → KK,

    c1,1 → cc NetK[[1]][[1]], c1,2 → cc NetK[[1]][[2]],
    c1,3 → cc NetK[[1]][[3]], c1,4 → cc NetK[[1]][[4]], c1,5 → cc NetK[[1]][[5]],
    c2,1 → cc NetK[[2]][[1]], c2,2 → cc NetK[[2]][[2]], c2,3 → cc NetK[[2]][[3]],
    c2,4 → cc NetK[[2]][[4]], c2,5 → cc NetK[[2]][[5]],
    c3,1 → cc NetK[[3]][[1]], c3,2 → cc NetK[[3]][[2]], c3,3 → cc NetK[[3]][[3]],
    c3,4 → cc NetK[[3]][[4]], c3,5 → cc NetK[[3]][[5]],
```

```
    c4,1 → cc NetK[[4]][[1]], c4,2 → cc NetK[[4]][[2]], c4,3 → cc NetK[[4]][[3]],
    c4,4 → cc NetK[[4]][[4]], c4,5 → cc NetK[[4]][[5]],
    c5,1 → cc NetK[[5]][[1]], c5,2 → cc NetK[[5]][[2]], c5,3 → cc NetK[[5]][[3]],
    c5,4 → cc NetK[[5]][[4]], c5,5 → cc NetK[[5]][[5]],

    q1 → qq, q2 → qq, q3 → qq, q4 → qq, q5 → qq,

    d1,1 → dd, d1,2 → dd, d1,3 → dd, d1,4 → dd, d1,5 → dd,
    d2,1 → dd, d2,2 → dd, d2,3 → dd, d2,4 → dd, d2,5 → dd,
    d3,1 → dd, d3,2 → dd, d3,3 → dd, d3,4 → dd, d3,5 → dd,
    d4,1 → dd, d4,2 → dd, d4,3 → dd, d4,4 → dd, d4,5 → dd,
    d5,1 → dd, d5,2 → dd, d5,3 → dd, d5,4 → dd, d5,5 → dd,

    Ω1,1 → OM NetK[[1]][[1]], Ω1,2 → OM NetK[[1]][[2]],
    Ω1,3 → OM NetK[[1]][[3]], Ω1,4 → OM NetK[[1]][[4]], Ω1,5 → OM NetK[[1]][[5]],
    Ω2,1 → OM NetK[[2]][[1]], Ω2,2 → OM NetK[[2]][[2]], Ω2,3 → OM NetK[[2]][[3]],
    Ω2,4 → OM NetK[[2]][[4]], Ω2,5 → OM NetK[[2]][[5]],
    Ω3,1 → OM NetK[[3]][[1]], Ω3,2 → OM NetK[[3]][[2]], Ω3,3 → OM NetK[[3]][[3]],
    Ω3,4 → OM NetK[[3]][[4]], Ω3,5 → OM NetK[[3]][[5]],
    Ω4,1 → OM NetK[[4]][[1]], Ω4,2 → OM NetK[[4]][[2]], Ω4,3 → OM NetK[[4]][[3]],
    Ω4,4 → OM NetK[[4]][[4]], Ω4,5 → OM NetK[[4]][[5]],
    Ω5,1 → OM NetK[[5]][[1]], Ω5,2 → OM NetK[[5]][[2]], Ω5,3 → OM NetK[[5]][[3]],
    Ω5,4 → OM NetK[[5]][[4]], Ω5,5 → OM NetK[[5]][[5]],
    nuK → nu,
    denK → den


  };
```

In[5204]:=
```
Partition[Riffle[{B1, B2, B3, B4, B5, M1, M2, M3, M4, M5}, po], {2}]
```

Out[5204]=
$$\{\{B_1, 6661.68\}, \{B_2, 6661.43\}, \{B_3, 6661.43\}, \{B_4, 6661.68\}, \{B_5, 6661.93\}, \{M_1, 22\,219.9\}, \{M_2, 44\,425.5\}, \{M_3, 44\,426.3\}, \{M_4, 22\,219.9\}, \{M_5, 15.9422\}\}$$

In[5205]:=
```
parVar = (#[[1]] → #[[2]] &) /@
  Partition[Riffle[{B1, B2, B3, B4, B5, M1, M2, M3, M4, M5}, po], {2}]
```

Out[5205]=
$$\{B_1 \to 6661.68, B_2 \to 6661.43, B_3 \to 6661.43, B_4 \to 6661.68, B_5 \to 6661.93, M_1 \to 22\,219.9, M_2 \to 44\,425.5, M_3 \to 44\,426.3, M_4 \to 22\,219.9, M_5 \to 15.9422\}$$

In[5206]:=
```
dB1
```

Out[5206]=
$$B_1 \left(\frac{\text{nuK}\, M_1\, M_2\, M_3\, M_4\, M_5}{(\text{denK} + M_1)\,(\text{denK} + M_2)\,(\text{denK} + M_3)\,(\text{denK} + M_4)\,(\text{denK} + M_5)} - B_1\, \kappa_1 \right) - B_1\, (c_{1,1} + c_{1,2} + c_{1,3} + c_{1,4} + c_{1,5})$$

In[5207]:=
$$m1 = \begin{pmatrix} D[dB_1, B_1] & D[dB_1, B_2] & D[dB_1, B_3] & D[dB_1, B_4] & D[dB_1, B_5] & D[dB_1, M_1] & D[dB_1, M_2] & D[dB_1 \\ D[dB_2, B_1] & D[dB_2, B_2] & D[dB_2, B_3] & D[dB_2, B_4] & D[dB_2, B_5] & D[dB_2, M_1] & D[dB_2, M_2] & D[dB_2 \\ D[dB_3, B_1] & D[dB_3, B_2] & D[dB_3, B_3] & D[dB_3, B_4] & D[dB_3, B_5] & D[dB_3, M_1] & D[dB_3, M_2] & D[dB_3 \\ D[dB_4, B_1] & D[dB_4, B_2] & D[dB_4, B_3] & D[dB_4, B_4] & D[dB_4, B_5] & D[dB_4, M_1] & D[dB_4, M_2] & D[dB_4 \\ D[dB_5, B_1] & D[dB_5, B_2] & D[dB_5, B_3] & D[dB_5, B_4] & D[dB_5, B_5] & D[dB_5, M_1] & D[dB_5, M_2] & D[dB_5 \\ D[dM_1, B_1] & D[dM_1, B_2] & D[dM_1, B_3] & D[dM_1, B_4] & D[dM_1, B_5] & D[dM_1, M_1] & D[dM_1, M_2] & D[dM_1 \\ D[dM_2, B_1] & D[dM_2, B_2] & D[dM_2, B_3] & D[dM_2, B_4] & D[dM_2, B_5] & D[dM_2, M_1] & D[dM_2, M_2] & D[dM_2 \\ D[dM_3, B_1] & D[dM_3, B_2] & D[dM_3, B_3] & D[dM_3, B_4] & D[dM_3, B_5] & D[dM_3, M_1] & D[dM_3, M_2] & D[dM_3 \\ D[dM_4, B_1] & D[dM_4, B_2] & D[dM_4, B_3] & D[dM_4, B_4] & D[dM_4, B_5] & D[dM_4, M_1] & D[dM_4, M_2] & D[dM_4 \\ D[dM_5, B_1] & D[dM_5, B_2] & D[dM_5, B_3] & D[dM_5, B_4] & D[dM_5, B_5] & D[dM_5, M_1] & D[dM_5, M_2] & D[dM_5 \end{pmatrix}$$

In[5208]:=
$$D[dB_1, B_1]$$

Out[5208]=
$$\frac{\text{nuK}\, M_1\, M_2\, M_3\, M_4\, M_5}{(\text{denK} + M_1)\,(\text{denK} + M_2)\,(\text{denK} + M_3)\,(\text{denK} + M_4)\,(\text{denK} + M_5)} - 2\, B_1\, \kappa_1 - c_{1,1} - c_{1,2} - c_{1,3} - c_{1,4} - c_{1,5}$$

In[5209]:=
```
m1 /. par /. parVar
```

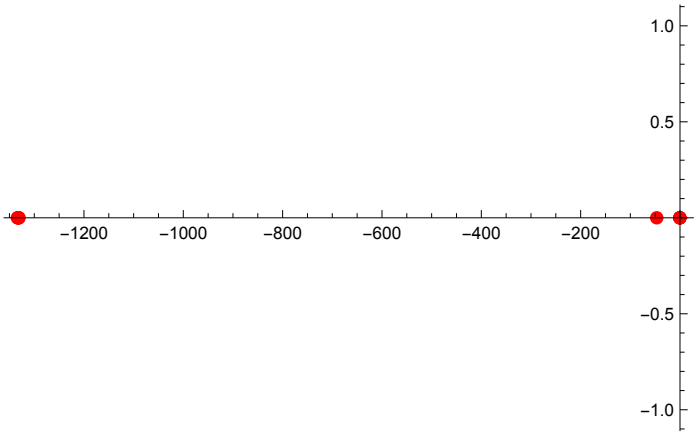
Out[5209]=
$$\{\{-1332.34, 0, 0, 0, 0, 0.0359532, 0.00899449, 0.00899415, 0.0359532, 62\,063.8\},$$
$$\{0, -1332.29, 0, 0, 0, 0.0359519, 0.00899415, 0.00899381, 0.0359519, 62\,061.5\},$$
$$\{0, 0, -1332.29, 0, 0, 0.0359519, 0.00899415, 0.00899381, 0.0359519, 62\,061.5\},$$
$$\{0, 0, 0, -1332.34, 0, 0.0359532, 0.00899449, 0.00899415, 0.0359532, 62\,063.8\},$$
$$\{0, 0, 0, 0, -1332.39, 0.0359546, 0.00899482, 0.00899449, 0.0359546, 62\,066.2\},$$
$$\{-0.199865, 0.800135, -0.199865, 0.800135, -0.199865, -0.300027,$$
$$-6.74581 \times 10^{-6}, -6.74556 \times 10^{-6}, -0.0000269647, -46.5475\},$$
$$\{0.800135, -0.199865, 0.800135, 0.800135, -0.199865, -0.0000269647,$$
$$-0.300007, -6.74556 \times 10^{-6}, -0.0000269647, -46.5475\},$$
$$\{0.800135, -0.199865, 0.800135, -0.199865, 0.800135, -0.0000269647,$$
$$-6.74581 \times 10^{-6}, -0.300007, -0.0000269647, -46.5475\},$$
$$\{-0.199865, 0.800135, -0.199865, 0.800135, -0.199865, -0.0000269647,$$
$$-6.74581 \times 10^{-6}, -6.74556 \times 10^{-6}, -0.300027, -46.5475\},$$
$$\{-0.199865, -0.199865, -0.199865, -0.199865, 0.800135, -0.0000269647,$$
$$-6.74581 \times 10^{-6}, -6.74556 \times 10^{-6}, -0.0000269647, -46.8475\}\}$$

In[5210]:=
```
eig = Eigenvalues[m1 /. par /. parVar]
```

Out[5210]=
$$\{-1334.06, -1332.34, -1332.32, -1332.29, -1330.66, -46.8151, -0.3, -0.3, -0.3, -0.3\}$$

In[5211]:=
```
ListPlot[({Re[#], Im[#]} &) /@ eig, PlotStyle → {PointSize[Large], Red}]
```

Out[5211]=



In[5215]:=
```
fNewSaitoEig[Net_] := (

 dB1 =
  B1[t] (-B1[t] κ1 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) - (c1,1 + c1,2 + c1,3 + c1,4 + c1,5) B1[t];
 dB2 = B2[t] (-B2[t] κ2 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
      M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c2,1 + c2,2 + c2,3 + c2,4 + c2,5) B2[t];
 dB3 = B3[t] (-B3[t] κ3 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
      M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c3,1 + c3,2 + c3,3 + c3,4 + c3,5) B3[t];
 dB4 = B4[t] (-B4[t] κ4 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
      M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c4,1 + c4,2 + c4,3 + c4,4 + c4,5) B4[t];
 dB5 = B5[t] (-B5[t] κ5 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
      M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c5,1 + c5,2 + c5,3 + c5,4 + c5,5) B5[t];
```

```
dM1 = -M1[t] q1 +
   (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) *
    (-B1[t] d1,1 - B2[t] d1,2 - B3[t] d1,3 - B4[t] d1,4 - B5[t] d1,5) +
   B1[t] Ω1,1 + B2[t] Ω1,2 + B3[t] Ω1,3 + B4[t] Ω1,4 + B5[t] Ω1,5;
dM2 = -M2[t] q2 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
       M5[t]/(denK + M5[t])) (-B1[t] d2,1 - B2[t] d2,2 - B3[t] d2,3 - B4[t] d2,4 - B5[t] d2,5) +
   B1[t] Ω2,1 + B2[t] Ω2,2 + B3[t] Ω2,3 + B4[t] Ω2,4 + B5[t] Ω2,5;
dM3 = -M3[t] q3 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
       M5[t]/(denK + M5[t])) (-B1[t] d3,1 - B2[t] d3,2 - B3[t] d3,3 - B4[t] d3,4 - B5[t] d3,5) +
   B1[t] Ω3,1 + B2[t] Ω3,2 + B3[t] Ω3,3 + B4[t] Ω3,4 + B5[t] Ω3,5;
dM4 = -M4[t] q4 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
       M5[t]/(denK + M5[t])) (-B1[t] d4,1 - B2[t] d4,2 - B3[t] d4,3 - B4[t] d4,4 - B5[t] d4,5) +
   B1[t] Ω4,1 + B2[t] Ω4,2 + B3[t] Ω4,3 + B4[t] Ω4,4 + B5[t] Ω4,5;
dM5 = -M5[t] q5 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
       M5[t]/(denK + M5[t])) (-B1[t] d5,1 - B2[t] d5,2 - B3[t] d5,3 - B4[t] d5,4 - B5[t] d5,5) +
   B1[t] Ω5,1 + B2[t] Ω5,2 + B3[t] Ω5,3 + B4[t] Ω5,4 + B5[t] Ω5,5;

KK = 0.2;
cc = 0.05;
qq = 0.3;
dd = 0.00015;
OM = 1;
nu = 1500;
den = 2;

tmax = 1000;
par = {
  κ1 → KK, κ2 → KK, κ3 → KK, κ4 → KK, κ5 → KK,

  c1,1 → cc Net[[1]][[1]], c1,2 → cc Net[[1]][[2]],
  c1,3 → cc Net[[1]][[3]], c1,4 → cc Net[[1]][[4]], c1,5 → cc Net[[1]][[5]],
  c2,1 → cc Net[[2]][[1]], c2,2 → cc Net[[2]][[2]], c2,3 → cc Net[[2]][[3]],
  c2,4 → cc Net[[2]][[4]], c2,5 → cc Net[[2]][[5]],
  c3,1 → cc Net[[3]][[1]], c3,2 → cc Net[[3]][[2]], c3,3 → cc Net[[3]][[3]],
  c3,4 → cc Net[[3]][[4]], c3,5 → cc Net[[3]][[5]],
```

```
  c4,1 → cc Net[[4]][[1]], c4,2 → cc Net[[4]][[2]], c4,3 → cc Net[[4]][[3]],
  c4,4 → cc Net[[4]][[4]], c4,5 → cc Net[[4]][[5]],
  c5,1 → cc Net[[5]][[1]], c5,2 → cc Net[[5]][[2]], c5,3 → cc Net[[5]][[3]],
  c5,4 → cc Net[[5]][[4]], c5,5 → cc Net[[5]][[5]],

  q1 → qq, q2 → qq, q3 → qq, q4 → qq, q5 → qq,

  d1,1 → dd, d1,2 → dd, d1,3 → dd, d1,4 → dd, d1,5 → dd,
  d2,1 → dd, d2,2 → dd, d2,3 → dd, d2,4 → dd, d2,5 → dd,
  d3,1 → dd, d3,2 → dd, d3,3 → dd, d3,4 → dd, d3,5 → dd,
  d4,1 → dd, d4,2 → dd, d4,3 → dd, d4,4 → dd, d4,5 → dd,
  d5,1 → dd, d5,2 → dd, d5,3 → dd, d5,4 → dd, d5,5 → dd,

  Ω1,1 → OM Net[[1]][[1]], Ω1,2 → OM Net[[1]][[2]],
  Ω1,3 → OM Net[[1]][[3]], Ω1,4 → OM Net[[1]][[4]], Ω1,5 → OM Net[[1]][[5]],
  Ω2,1 → OM Net[[2]][[1]], Ω2,2 → OM Net[[2]][[2]], Ω2,3 → OM Net[[2]][[3]],
  Ω2,4 → OM Net[[2]][[4]], Ω2,5 → OM Net[[2]][[5]],
  Ω3,1 → OM Net[[3]][[1]], Ω3,2 → OM Net[[3]][[2]], Ω3,3 → OM Net[[3]][[3]],
  Ω3,4 → OM Net[[3]][[4]], Ω3,5 → OM Net[[3]][[5]],
  Ω4,1 → OM Net[[4]][[1]], Ω4,2 → OM Net[[4]][[2]], Ω4,3 → OM Net[[4]][[3]],
  Ω4,4 → OM Net[[4]][[4]], Ω4,5 → OM Net[[4]][[5]],
  Ω5,1 → OM Net[[5]][[1]], Ω5,2 → OM Net[[5]][[2]], Ω5,3 → OM Net[[5]][[3]],
  Ω5,4 → OM Net[[5]][[4]], Ω5,5 → OM Net[[5]][[5]],
  nuK → nu,
  denK → den


 };


B10 = 1500;
B20 = 1500;
B30 = 1500;
B40 = 1500;
B50 = 1500;
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;


sol =
 NDSolve[
```

```
  {
    B1'[t] == dB1,
    B2'[t] == dB2,
    B3'[t] == dB3,
    B4'[t] == dB4,
    B5'[t] == dB5,

    M1'[t] == dM1,
    M2'[t] == dM2,
    M3'[t] == dM3,
    M4'[t] == dM4,
    M5'[t] == dM5,


    B1[0] == B10,
    B2[0] == B20,
    B3[0] == B30,
    B4[0] == B40,
    B5[0] == B50,
    M1[0] == M10,
    M2[0] == M20,
    M3[0] == M30,
    M4[0] == M40,
    M5[0] == M50

   } /. par,
  {B1, B2, B3, B4, B5, M1, M2, M3, M4, M5},
  {t, 0, tmax}];

parvar0 =
 Flatten[({B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax], M1[tmax], M2[tmax],
      M3[tmax], M4[tmax], M5[tmax]} /. sol /. par)];

parvar1 = Partition[Riffle[{B1, B2, B3, B4, B5, M1, M2, M3, M4, M5}, parvar0], {2}];
parvar = (#[[1]] → #[[2]] &) /@ parvar1;



dB1 = B1 (-B1 κ1 + nuK * M1/(denK + M1) * M2/(denK + M2) * M3/(denK + M3) * M4/(denK + M4) * M5/(denK + M5)) -
  (c1,1 + c1,2 + c1,3 + c1,4 + c1,5) B1;
dB2 = B2 (-B2 κ2 + nuK * M1/(denK + M1) * M2/(denK + M2) * M3/(denK + M3) * M4/(denK + M4) * M5/(denK + M5)) -
```

```
    (c2,1 + c2,2 + c2,3 + c2,4 + c2,5) B2;
dB3 = B3 (-B3 κ3 + nuK * M1/(denK + M1) * M2/(denK + M2) * M3/(denK + M3) * M4/(denK + M4) * M5/(denK + M5)) -
    (c3,1 + c3,2 + c3,3 + c3,4 + c3,5) B3;
dB4 = B4 (-B4 κ4 + nuK * M1/(denK + M1) * M2/(denK + M2) * M3/(denK + M3) * M4/(denK + M4) * M5/(denK + M5)) -
    (c4,1 + c4,2 + c4,3 + c4,4 + c4,5) B4;
dB5 = B5 (-B5 κ5 + nuK * M1/(denK + M1) * M2/(denK + M2) * M3/(denK + M3) * M4/(denK + M4) * M5/(denK + M5)) -
    (c5,1 + c5,2 + c5,3 + c5,4 + c5,5) B5;

dM1 = -M1 q1 + (nuK * M1/(denK + M1) * M2/(denK + M2) * M3/(denK + M3) * M4/(denK + M4) * M5/(denK + M5))
    (-B1 d1,1 - B2 d1,2 - B3 d1,3 - B4 d1,4 - B5 d1,5) + B1 Ω1,1 + B2 Ω1,2 + B3 Ω1,3 + B4 Ω1,4 + B5 Ω1,5;
dM2 = -M2 q2 + (nuK * M1/(denK + M1) * M2/(denK + M2) * M3/(denK + M3) * M4/(denK + M4) * M5/(denK + M5))
    (-B1 d2,1 - B2 d2,2 - B3 d2,3 - B4 d2,4 - B5 d2,5) + B1 Ω2,1 + B2 Ω2,2 + B3 Ω2,3 + B4 Ω2,4 + B5 Ω2,5;
dM3 = -M3 q3 + (nuK * M1/(denK + M1) * M2/(denK + M2) * M3/(denK + M3) * M4/(denK + M4) * M5/(denK + M5))
    (-B1 d3,1 - B2 d3,2 - B3 d3,3 - B4 d3,4 - B5 d3,5) + B1 Ω3,1 + B2 Ω3,2 + B3 Ω3,3 + B4 Ω3,4 + B5 Ω3,5;
dM4 = -M4 q4 + (nuK * M1/(denK + M1) * M2/(denK + M2) * M3/(denK + M3) * M4/(denK + M4) * M5/(denK + M5))
    (-B1 d4,1 - B2 d4,2 - B3 d4,3 - B4 d4,4 - B5 d4,5) + B1 Ω4,1 + B2 Ω4,2 + B3 Ω4,3 + B4 Ω4,4 + B5 Ω4,5;
dM5 = -M5 q5 + (nuK * M1/(denK + M1) * M2/(denK + M2) * M3/(denK + M3) * M4/(denK + M4) * M5/(denK + M5)) (-B1
      d5,1 - B2 d5,2 - B3 d5,3 - B4 d5,4 - B5 d5,5) + B1 Ω5,1 + B2 Ω5,2 + B3 Ω5,3 + B4 Ω5,4 + B5 Ω5,5;


m1 = ( D[dB1, B1]  D[dB1, B2]  D[dB1, B3]  D[dB1, B4]  D[dB1, B5]  D[dB1, M1]  D[dB1, M2]  D
       D[dB2, B1]  D[dB2, B2]  D[dB2, B3]  D[dB2, B4]  D[dB2, B5]  D[dB2, M1]  D[dB2, M2]  D
       D[dB3, B1]  D[dB3, B2]  D[dB3, B3]  D[dB3, B4]  D[dB3, B5]  D[dB3, M1]  D[dB3, M2]  D
       D[dB4, B1]  D[dB4, B2]  D[dB4, B3]  D[dB4, B4]  D[dB4, B5]  D[dB4, M1]  D[dB4, M2]  D
       D[dB5, B1]  D[dB5, B2]  D[dB5, B3]  D[dB5, B4]  D[dB5, B5]  D[dB5, M1]  D[dB5, M2]  D
       D[dM1, B1]  D[dM1, B2]  D[dM1, B3]  D[dM1, B4]  D[dM1, B5]  D[dM1, M1]  D[dM1, M2]  D
       D[dM2, B1]  D[dM2, B2]  D[dM2, B3]  D[dM2, B4]  D[dM2, B5]  D[dM2, M1]  D[dM2, M2]  D
       D[dM3, B1]  D[dM3, B2]  D[dM3, B3]  D[dM3, B4]  D[dM3, B5]  D[dM3, M1]  D[dM3, M2]  D
       D[dM4, B1]  D[dM4, B2]  D[dM4, B3]  D[dM4, B4]  D[dM4, B5]  D[dM4, M1]  D[dM4, M2]  D
       D[dM5, B1]  D[dM5, B2]  D[dM5, B3]  D[dM5, B4]  D[dM5, B5]  D[dM5, M1]  D[dM5, M2]  D


eig = Eigenvalues[m1 /. par /. parvar];
```

```
  ({Re[#], Im[#]} &) /@ eig

 )
```

In[5216]:=
```
NetK = {
    {0, 1, 0, 1, 0},
    {1, 0, 1, 1, 0},
    {1, 0, 1, 0, 1},
    {0, 1, 0, 1, 0},
    {0, 0, 0, 0, 1}
   };
```

In[5217]:=
```
fNewSaitoEig[NetK]
```

Out[5217]=
```
{{-1334.06, 0}, {-1332.34, 0}, {-1332.32, 0}, {-1332.29, 0},
 {-1330.66, 0}, {-46.8151, 0}, {-0.3, 0}, {-0.3, 0}, {-0.3, 0}, {-0.3, 0}}
```

Out[•]=
```
{{-401.278, 0.}, {-401.034, 0.}, {-401.034, 0.},
 {-401.034, 0.}, {-401.034, 0.}, {-106.872, 0.}, {-106.864, 0.},
 {-106.822, 0.}, {-106.755, 1.87371}, {-106.755, -1.87371}}
```

In[5218]:=
```
hk7[[1]]
```

Out[5218]=
```
{{1, 0, 1, 1, 1}, {1, 1, 1, 1, 0}, {1, 1, 1, 1, 1}, {1, 0, 0, 1, 0}, {1, 0, 1, 1, 0}}
```

In[5219]:=
```
Eig7 = fNewSaitoEig /@ hk7
```

In[5220]:=
```
Eig17 = fNewSaitoEig /@ hk17
```

(***)

In[5313]:=
```
Eig7 = fNewSaitoEig /@ hk7;
Eig8 = fNewSaitoEig /@ hk8;
Eig9 = fNewSaitoEig /@ hk9;
Eig10 = fNewSaitoEig /@ hk10;
Eig11 = fNewSaitoEig /@ hk11;
Eig12 = fNewSaitoEig /@ hk12;
Eig13 = fNewSaitoEig /@ hk13;
Eig14 = fNewSaitoEig /@ hk14;
Eig15 = fNewSaitoEig /@ hk15;
Eig16 = fNewSaitoEig /@ hk16;
Eig17 = fNewSaitoEig /@ hk17;
```

In[5222]:=
```
Eig7[[1]]
```

Out[5222]=
$$\{\{-1499.58, 0.\}, \{-1499.53, 0.\}, \{-1499.48, 0.\}, \{-1499.48, 0.\}, \{-1499.43, 0.\}, \{-0.3, 0.\}, \{-0.3, 3.22882 \times 10^{-14}\}, \{-0.3, -3.22882 \times 10^{-14}\}, \{-0.3, 0.\}, \{-0.299988, 0.\}\}$$

In[5225]:=
```
ReEig7 = (First /@ Eig7[[#]] &) /@ Range[100]
```

In[5226]:=
```
ReEig17 = (First /@ Eig17[[#]] &) /@ Range[100]
```

(***)

In[5324]:=
```
ReEig7 = (First /@ Eig7[[#]] &) /@ Range[100];
ReEig8 = (First /@ Eig8[[#]] &) /@ Range[100];
ReEig9 = (First /@ Eig9[[#]] &) /@ Range[100];
ReEig10 = (First /@ Eig10[[#]] &) /@ Range[100];
ReEig11 = (First /@ Eig11[[#]] &) /@ Range[100];
ReEig12 = (First /@ Eig12[[#]] &) /@ Range[100];
ReEig13 = (First /@ Eig13[[#]] &) /@ Range[100];
ReEig14 = (First /@ Eig14[[#]] &) /@ Range[100];
ReEig15 = (First /@ Eig15[[#]] &) /@ Range[100];
ReEig16 = (First /@ Eig16[[#]] &) /@ Range[100];
ReEig17 = (First /@ Eig17[[#]] &) /@ Range[100];
```

(*Imagin Parenth*)

In[5309]:=
```
(#[[2]] & /@ Eig7[[#]] &) /@ Range[100]
```

In[5311]:=
```
Select[Flatten[(#[[2]] & /@ Eig7[[#]] &) /@ Range[100]], # > 0.01 &]
```

Out[5311]=
```
{1.83969, 0.992709, 0.992709}
```

In[5312]:=
```
Select[Flatten[(#[[2]] & /@ Eig17[[#]] &) /@ Range[100]], # > 0.01 &]
```

Out[5312]=
```
{0.580954, 0.692303, 0.283037, 0.580951, 0.692304, 0.295027, 0.771589,
 0.295027, 0.580952, 0.295029, 0.692303, 0.283037, 0.283035, 0.580954,
 0.692302, 0.692304, 0.28304, 0.692302, 0.28304, 0.692304, 0.782002, 0.692304,
 0.782002, 1.01341, 0.283035, 0.692304, 0.929063, 0.283036, 0.295028,
 0.692306, 0.771586, 0.572608, 0.580954, 0.692302, 0.295027, 1.06118,
 0.692302, 0.692304, 0.692303, 0.580952, 0.692302, 0.580951, 0.692303,
 0.283038, 0.692303, 0.692305, 0.283041, 0.283035, 0.283037, 0.692303}
```

```
(*End Imagin Parenth*)
```

In[5227]:=
```
ReEig7[[1]]
ReEig17[[1]]
```

Out[5227]=
```
{-1499.58, -1499.53, -1499.48, -1499.48,
 -1499.43, -0.3, -0.3, -0.3, -0.3, -0.299988}
```

Out[5228]=
```
{-1331.04, -1330.36, -1330.36, -1330.3, -1329.64, -16.1537, -0.3, -0.3, -0.3, -0.3}
```

In[5232]:=
```
Partition[Riffle[ConstantArray[7, {10}], ReEig7[[1]]], {2}]
```

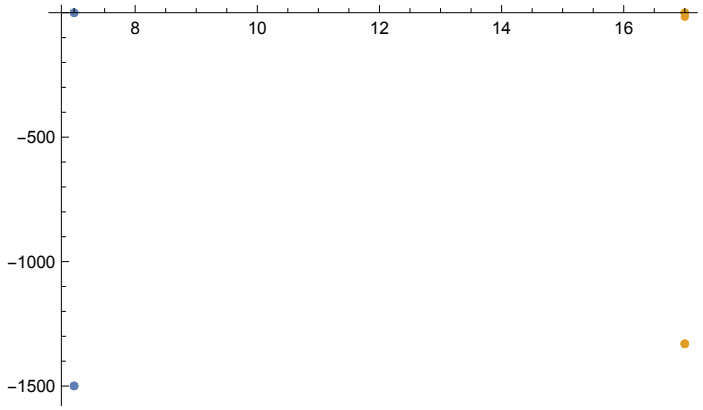
Out[5232]=
```
{{7, -1499.58}, {7, -1499.53}, {7, -1499.48}, {7, -1499.48},
 {7, -1499.43}, {7, -0.3}, {7, -0.3}, {7, -0.3}, {7, -0.3}, {7, -0.299988}}
```

In[5233]:=
```
Partition[Riffle[ConstantArray[17, {10}], ReEig17[[1]]], {2}]
```

Out[5233]=
```
{{17, -1331.04}, {17, -1330.36}, {17, -1330.36}, {17, -1330.3},
 {17, -1329.64}, {17, -16.1537}, {17, -0.3}, {17, -0.3}, {17, -0.3}, {17, -0.3}}
```

In[5234]:=
```
ListPlot[{Partition[Riffle[ConstantArray[7, {10}], ReEig7[[1]]], {2}],
  Partition[Riffle[ConstantArray[17, {10}], ReEig17[[1]]], {2}]}]
```

Out[5234]=



In[5236]:=
```
dw7 = Partition[Riffle[ConstantArray[7, {10}], ReEig7[[#]]], {2}] & /@ Range[100];
```

In[5237]:=
```
Length[dw7]
```

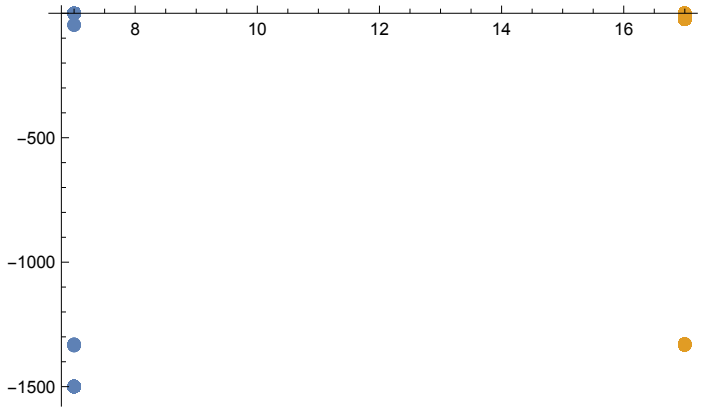
Out[5237]= 100

In[5239]:=
```
Adw7 = Partition[Flatten[dw7], {2}];
```

In[5243]:=
```
dw17 = Partition[Riffle[ConstantArray[17, {10}], ReEig17[[#]]], {2}] & /@ Range[100];
Adw17 = Partition[Flatten[dw17], {2}];
```

In[5247]:=
```
ListPlot[{Adw7, Adw17}, PlotStyle → PointSize[Large]]
```

Out[5247]=



In[5254]:=
```
ReEig7[[30]]
ReEig17[[30]]
```

Out[5254]=
```
{-1499.52, -1499.52, -1499.42, -1499.37,
 -1499.37, -0.300002, -0.3, -0.3, -0.3, -0.3}
```

Out[5255]=
```
{-1332.19, -1331.35, -1331.3, -1331.3, -1330.5, -23.7892, -0.3, -0.3, -0.3, -0.3}
```

In[5259]:=
```
E7 = Abs /@ Flatten[ReEig7];
```

In[5260]:=
```
E17 = Abs /@ Flatten[ReEig17];
```

In[5268]:=
```
BoxWhiskerChart[{E7, E17}, "Notched", ChartStyle → {Gray, Black},
 Frame → True, Frame → True, FrameTicks → {None, {5, 10, 20}, None, None},
 FrameTicksStyle → Directive[Black, 16]]
```

In[5263]:=
```
Hk = MannWhitneyTest[{E7, E17}, 0, "HypothesisTestData"];
Hk["PValueTable"]
Hk["TestStatisticTable"]
```

Out[5264]=

	P-Value
Mann-Whitney	4.32914×10^{-11}

Out[5265]=

	Statistic
Mann-Whitney	583292.

In[5266]:=
```
{Mean[E7], Mean[E17]}
```

Out[5266]=
```
{745.177, 667.427}
```

In[5299]:=
```
{Min[E7], Min[E17]}
```

Out[5299]=
```
{0.299974, 0.3}
```

In[5300]:=
```
{Max[E7], Max[E17]}
```

Out[5300]=
```
{1499.6, 1332.19}
```

```
(******)
```

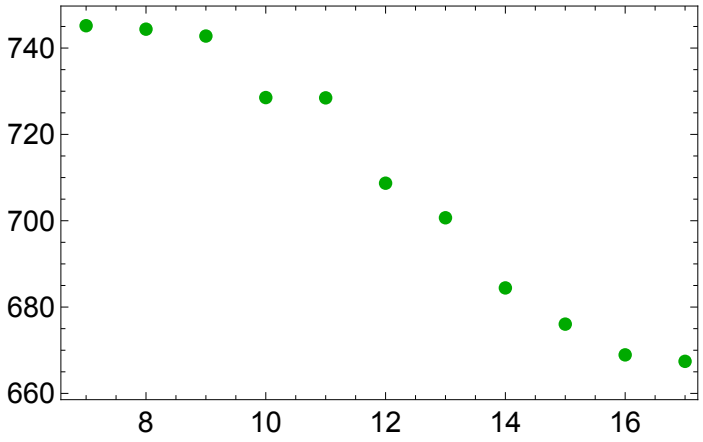
In[5335]:=
```
E7 = Abs /@ Flatten[ReEig7];
E8 = Abs /@ Flatten[ReEig8];
E9 = Abs /@ Flatten[ReEig9];
E10 = Abs /@ Flatten[ReEig10];
E11 = Abs /@ Flatten[ReEig11];
E12 = Abs /@ Flatten[ReEig12];
E13 = Abs /@ Flatten[ReEig13];
E14 = Abs /@ Flatten[ReEig14];
E15 = Abs /@ Flatten[ReEig15];
E16 = Abs /@ Flatten[ReEig16];
E17 = Abs /@ Flatten[ReEig17];
```

In[5347]:=
```
MeansEig = {Mean[E7], Mean[E8], Mean[E9], Mean[E10], Mean[E11],
  Mean[E12], Mean[E13], Mean[E14], Mean[E15], Mean[E16], Mean[E17]}
```

Out[5347]=
```
{745.177, 744.377, 742.784, 728.53, 728.465,
 708.695, 700.685, 684.435, 676.046, 668.924, 667.427}
```

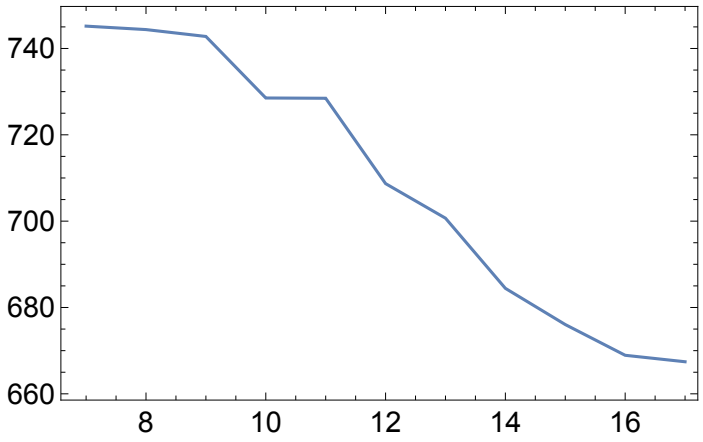
In[7544]:=
```
ax1 = ListPlot[Partition[Riffle[Range[7, 17], MeansEig], {2}],
  Frame → True, FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Darker[Green], PointSize[Large]}]
```

Out[7544]=



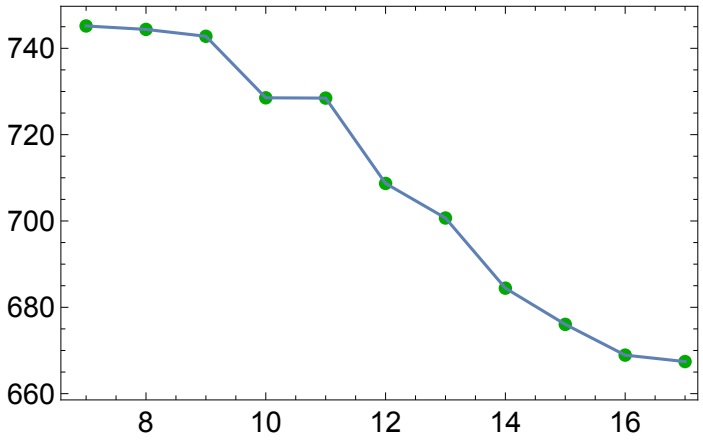
In[5604]:=
```
ax2 = ListPlot[Partition[Riffle[Range[7, 17], MeansEig], {2}],
  Joined → True, Frame → True, FrameStyle → Directive[Black, FontSize → 15]]
```

Out[5604]=



In[5605]:=
```
Show[ax1, ax2]
```

Out[5605]=



In[5395]:=
```
Stab8 = Abs /@ (Mean /@ ReEig8)
```

Out[5395]=
```
{749.896, 749.871, 749.896, 749.927, 749.865, 749.871, 670.909, 749.927, 749.927,
 749.896, 749.902, 749.902, 749.865, 749.902, 749.922, 749.896, 749.896, 749.871,
 749.902, 670.91, 749.896, 749.896, 749.902, 749.896, 749.896, 749.927, 749.902,
 670.909, 749.927, 749.902, 749.902, 670.909, 749.902, 749.927, 749.927, 749.927,
 749.896, 749.922, 749.927, 749.927, 749.896, 749.927, 749.922, 749.927,
 749.927, 749.902, 749.896, 749.896, 749.927, 749.896, 749.896, 749.902,
 749.896, 670.906, 749.927, 749.902, 670.886, 749.927, 749.927, 749.902,
 749.896, 749.896, 749.927, 749.927, 749.902, 749.896, 749.896, 749.927,
 749.927, 749.922, 749.902, 749.896, 749.927, 749.927, 749.927, 749.927,
 749.902, 749.871, 749.896, 749.927, 749.871, 749.927, 749.902, 749.896,
 749.896, 749.927, 749.922, 749.896, 749.896, 749.896, 670.884, 749.927,
 749.927, 749.922, 749.927, 749.902, 749.896, 749.896, 749.902, 749.927}
```

In[5396]:=
```
Entropy8 = RelatEntrop5 /@ hk8;
```

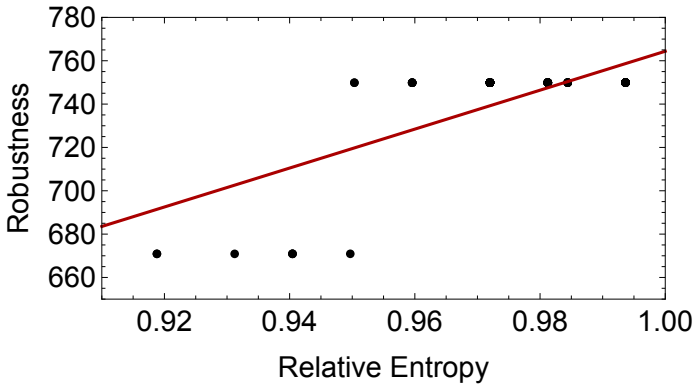
In[5397]:=
```
Assort8 = assortativity /@ hk8;
```

In[5398]:=
```
{Min[Stab8], Max[Stab8]}
```

Out[5398]=
```
{670.884, 749.927}
```

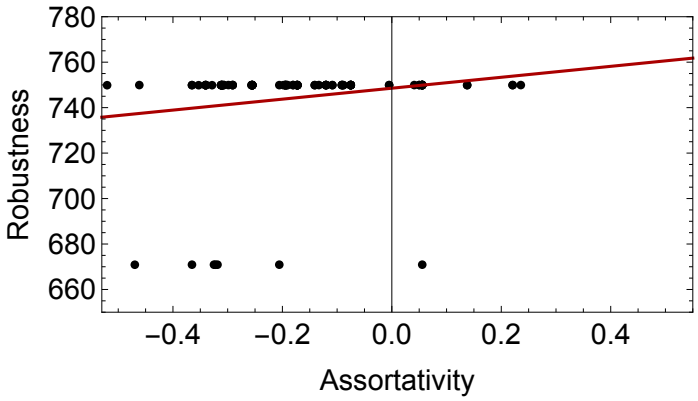
In[5691]:=
```
linerobustnessNewSaito25Stabil =
  Fit[Partition[Riffle[Entropy8, Stab8], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Entropy8, Stab8], {2}],
  Frame → True, FrameLabel → {"Relative Entropy", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]}, PlotRange → {{0.91, 1}, {650, 780}},
  AspectRatio → 0.5], Plot[linerobustnessNewSaito25Stabil,
  {x, 0.91, 1}, AspectRatio → 0.5, PlotStyle → Darker[Red]]]
```

Out[5692]=



In[5693]:=
```
lineAssoRobrobustnessNewSaito25Stabil =
  Fit[Partition[Riffle[Assort8, Stab8], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Assort8, Stab8], {2}],
  Frame → True, FrameLabel → {"Assortativity", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]}, PlotRange → {{-0.53, 0.55}, {650, 780}},
  AspectRatio → 0.5], Plot[lineAssoRobrobustnessNewSaito25Stabil,
  {x, -0.53, 0.55}, AspectRatio → 0.5, PlotStyle → Darker[Red]]]
```

Out[5694]=



In[5403]:=
```
SpearmanRankTest[Entropy8, Stab8, "TestDataTable"]
```

Out[5403]=

	Statistic	P-Value
Spearman Rank	0.967511	2.49184×10^{-60}

In[5404]:=
```
SpearmanRankTest[Assort8, Stab8, "TestDataTable"]
```

Out[5404]=

	Statistic	P-Value
Spearman Rank	0.549483	3.21176×10^{-9}

```
(**********)
```

In[5405]:=
```
fNewSaitoEigOV[Net_, coop_] := (

  dB1 =
    B1[t] (-B1[t] κ1 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
        M5[t]/(denK + M5[t])) - (c1,1 + c1,2 + c1,3 + c1,4 + c1,5) B1[t];
  dB2 = B2[t] (-B2[t] κ2 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
        M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c2,1 + c2,2 + c2,3 + c2,4 + c2,5) B2[t];
```

```
dB3 = B3[t] (-B3[t] κ3 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
        M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c3,1 + c3,2 + c3,3 + c3,4 + c3,5) B3[t];
dB4 = B4[t] (-B4[t] κ4 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
        M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c4,1 + c4,2 + c4,3 + c4,4 + c4,5) B4[t];
dB5 = B5[t] (-B5[t] κ5 + nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
        M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) - (c5,1 + c5,2 + c5,3 + c5,4 + c5,5) B5[t];

dM1 = -M1[t] q1 +
   (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t]))
    (-B1[t] d1,1 - B2[t] d1,2 - B3[t] d1,3 - B4[t] d1,4 - B5[t] d1,5) +
   B1[t] Ω1,1 + B2[t] Ω1,2 + B3[t] Ω1,3 + B4[t] Ω1,4 + B5[t] Ω1,5;
dM2 = -M2[t] q2 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
        M5[t]/(denK + M5[t])) (-B1[t] d2,1 - B2[t] d2,2 - B3[t] d2,3 - B4[t] d2,4 - B5[t] d2,5) +
   B1[t] Ω2,1 + B2[t] Ω2,2 + B3[t] Ω2,3 + B4[t] Ω2,4 + B5[t] Ω2,5;
dM3 = -M3[t] q3 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
        M5[t]/(denK + M5[t])) (-B1[t] d3,1 - B2[t] d3,2 - B3[t] d3,3 - B4[t] d3,4 - B5[t] d3,5) +
   B1[t] Ω3,1 + B2[t] Ω3,2 + B3[t] Ω3,3 + B4[t] Ω3,4 + B5[t] Ω3,5;
dM4 = -M4[t] q4 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
        M5[t]/(denK + M5[t])) (-B1[t] d4,1 - B2[t] d4,2 - B3[t] d4,3 - B4[t] d4,4 - B5[t] d4,5) +
   B1[t] Ω4,1 + B2[t] Ω4,2 + B3[t] Ω4,3 + B4[t] Ω4,4 + B5[t] Ω4,5;
dM5 = -M5[t] q5 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
        M5[t]/(denK + M5[t])) (-B1[t] d5,1 - B2[t] d5,2 - B3[t] d5,3 - B4[t] d5,4 - B5[t] d5,5) +
   B1[t] Ω5,1 + B2[t] Ω5,2 + B3[t] Ω5,3 + B4[t] Ω5,4 + B5[t] Ω5,5;

KK = 0.2;
cc = 0.05;
qq = 0.3;
dd = 0.00015;
OM = 1;
```

```
nu = 1500;
den = 2;




op = coop; (*Number of links with overExpression*)
posNe = Position[Net, 1];
(*Positions in the matrix where there are links (=1)*)
RaN = RandomSample[posNe, op];
(*Random sample of op links that will be overproduced*)

costincr = 1.3; (*Term multiplying the cost link*)
overprodincr = 1.15;
(*Term multiplying the overproduction link*)


NewNetCost = Net cc;
Table[NewNetCost[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] =
  NewNetCost[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] * costincr, {i, Length[RaN]}];

NewNetOvProd = Net OM;
Table[NewNetOvProd[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] =
  NewNetOvProd[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] * overprodincr, {i,
  Length[RaN]}];


tmax = 1000;
par = {
  κ1 → KK, κ2 → KK, κ3 → KK, κ4 → KK, κ5 → KK,

  c1,1 → NewNetCost[[1]][[1]],
  c1,2 → NewNetCost[[1]][[2]], c1,3 → NewNetCost[[1]][[3]],
  c1,4 → NewNetCost[[1]][[4]], c1,5 → NewNetCost[[1]][[5]],
  c2,1 → NewNetCost[[2]][[1]], c2,2 → NewNetCost[[2]][[2]],
  c2,3 → NewNetCost[[2]][[3]], c2,4 → NewNetCost[[2]][[4]],
  c2,5 → NewNetCost[[2]][[5]],
  c3,1 → NewNetCost[[3]][[1]], c3,2 → NewNetCost[[3]][[2]],
  c3,3 → NewNetCost[[3]][[3]], c3,4 → NewNetCost[[3]][[4]],
  c3,5 → NewNetCost[[3]][[5]],
  c4,1 → NewNetCost[[4]][[1]], c4,2 → NewNetCost[[4]][[2]],
  c4,3 → NewNetCost[[4]][[3]], c4,4 → NewNetCost[[4]][[4]],
  c4,5 → NewNetCost[[4]][[5]],
  c5,1 → NewNetCost[[5]][[1]], c5,2 → NewNetCost[[5]][[2]],
```

```
  c5,3 → NewNetCost[[5]][[3]], c5,4 → NewNetCost[[5]][[4]],
  c5,5 → NewNetCost[[5]][[5]],

  r1,1 → RR, r1,2 → RR, r1,3 → RR, r1,4 → RR, r1,5 → RR,
  r2,1 → RR, r2,2 → RR, r2,3 → RR, r2,4 → RR, r2,5 → RR,
  r3,1 → RR, r3,2 → RR, r3,3 → RR, r3,4 → RR, r3,5 → RR,
  r4,1 → RR, r4,2 → RR, r4,3 → RR, r4,4 → RR, r4,5 → RR,
  r5,1 → RR, r5,2 → RR, r5,3 → RR, r5,4 → RR, r5,5 → RR,

  q1 → qq, q2 → qq, q3 → qq, q4 → qq, q5 → qq,

  d1,1 → dd, d1,2 → dd, d1,3 → dd, d1,4 → dd, d1,5 → dd,
  d2,1 → dd, d2,2 → dd, d2,3 → dd, d2,4 → dd, d2,5 → dd,
  d3,1 → dd, d3,2 → dd, d3,3 → dd, d3,4 → dd, d3,5 → dd,
  d4,1 → dd, d4,2 → dd, d4,3 → dd, d4,4 → dd, d4,5 → dd,
  d5,1 → dd, d5,2 → dd, d5,3 → dd, d5,4 → dd, d5,5 → dd,

  Ω1,1 → NewNetOvProd[[1]][[1]],
  Ω1,2 → NewNetOvProd[[1]][[2]], Ω1,3 → NewNetOvProd[[1]][[3]],
  Ω1,4 → NewNetOvProd[[1]][[4]], Ω1,5 → NewNetOvProd[[1]][[5]],
  Ω2,1 → NewNetOvProd[[2]][[1]], Ω2,2 → NewNetOvProd[[2]][[2]],
  Ω2,3 → NewNetOvProd[[2]][[3]], Ω2,4 → NewNetOvProd[[2]][[4]],
  Ω2,5 → NewNetOvProd[[2]][[5]],
  Ω3,1 → NewNetOvProd[[3]][[1]], Ω3,2 → NewNetOvProd[[3]][[2]],
  Ω3,3 → NewNetOvProd[[3]][[3]], Ω3,4 → NewNetOvProd[[3]][[4]],
  Ω3,5 → NewNetOvProd[[3]][[5]],
  Ω4,1 → NewNetOvProd[[4]][[1]], Ω4,2 → NewNetOvProd[[4]][[2]],
  Ω4,3 → NewNetOvProd[[4]][[3]], Ω4,4 → NewNetOvProd[[4]][[4]],
  Ω4,5 → NewNetOvProd[[4]][[5]],
  Ω5,1 → NewNetOvProd[[5]][[1]], Ω5,2 → NewNetOvProd[[5]][[2]],
  Ω5,3 → NewNetOvProd[[5]][[3]], Ω5,4 → NewNetOvProd[[5]][[4]],
  Ω5,5 → NewNetOvProd[[5]][[5]],
  nuK → nu,
  denK → den


 };


B10 = 1500;
B20 = 1500;
B30 = 1500;
B40 = 1500;
B50 = 1500;
```

```
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;


sol =
 NDSolve[
  {
    B1'[t] == dB1,
    B2'[t] == dB2,
    B3'[t] == dB3,
    B4'[t] == dB4,
    B5'[t] == dB5,

    M1'[t] == dM1,
    M2'[t] == dM2,
    M3'[t] == dM3,
    M4'[t] == dM4,
    M5'[t] == dM5,


    B1[0] == B10,
    B2[0] == B20,
    B3[0] == B30,
    B4[0] == B40,
    B5[0] == B50,
    M1[0] == M10,
    M2[0] == M20,
    M3[0] == M30,
    M4[0] == M40,
    M5[0] == M50

   } /. par,
  {B1, B2, B3, B4, B5, M1, M2, M3, M4, M5},
  {t, 0, tmax}];


parvar0 =
 Flatten[({B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax], M1[tmax], M2[tmax],
      M3[tmax], M4[tmax], M5[tmax]} /. sol /. par)];
parvar1 = Partition[Riffle[{B1, B2, B3, B4, B5, M1, M2, M3, M4, M5}, parvar0], {2}];
```

```
parvar = (#[[1]] → #[[2]] &) /@ parvar1;
```

$$dB_1 = B_1 \left(-B_1 \kappa_1 + nuK * \frac{M_1}{denK + M_1} * \frac{M_2}{denK + M_2} * \frac{M_3}{denK + M_3} * \frac{M_4}{denK + M_4} * \frac{M_5}{denK + M_5}\right) - (c_{1,1} + c_{1,2} + c_{1,3} + c_{1,4} + c_{1,5})\, B_1;$$

$$dB_2 = B_2 \left(-B_2 \kappa_2 + nuK * \frac{M_1}{denK + M_1} * \frac{M_2}{denK + M_2} * \frac{M_3}{denK + M_3} * \frac{M_4}{denK + M_4} * \frac{M_5}{denK + M_5}\right) - (c_{2,1} + c_{2,2} + c_{2,3} + c_{2,4} + c_{2,5})\, B_2;$$

$$dB_3 = B_3 \left(-B_3 \kappa_3 + nuK * \frac{M_1}{denK + M_1} * \frac{M_2}{denK + M_2} * \frac{M_3}{denK + M_3} * \frac{M_4}{denK + M_4} * \frac{M_5}{denK + M_5}\right) - (c_{3,1} + c_{3,2} + c_{3,3} + c_{3,4} + c_{3,5})\, B_3;$$

$$dB_4 = B_4 \left(-B_4 \kappa_4 + nuK * \frac{M_1}{denK + M_1} * \frac{M_2}{denK + M_2} * \frac{M_3}{denK + M_3} * \frac{M_4}{denK + M_4} * \frac{M_5}{denK + M_5}\right) - (c_{4,1} + c_{4,2} + c_{4,3} + c_{4,4} + c_{4,5})\, B_4;$$

$$dB_5 = B_5 \left(-B_5 \kappa_5 + nuK * \frac{M_1}{denK + M_1} * \frac{M_2}{denK + M_2} * \frac{M_3}{denK + M_3} * \frac{M_4}{denK + M_4} * \frac{M_5}{denK + M_5}\right) - (c_{5,1} + c_{5,2} + c_{5,3} + c_{5,4} + c_{5,5})\, B_5;$$

$$dM_1 = -M_1 q_1 + \left(nuK * \frac{M_1}{denK + M_1} * \frac{M_2}{denK + M_2} * \frac{M_3}{denK + M_3} * \frac{M_4}{denK + M_4} * \frac{M_5}{denK + M_5}\right) \left(-B_1 d_{1,1} - B_2 d_{1,2} - B_3 d_{1,3} - B_4 d_{1,4} - B_5 d_{1,5}\right) + B_1 \Omega_{1,1} + B_2 \Omega_{1,2} + B_3 \Omega_{1,3} + B_4 \Omega_{1,4} + B_5 \Omega_{1,5};$$

$$dM_2 = -M_2 q_2 + \left(nuK * \frac{M_1}{denK + M_1} * \frac{M_2}{denK + M_2} * \frac{M_3}{denK + M_3} * \frac{M_4}{denK + M_4} * \frac{M_5}{denK + M_5}\right) \left(-B_1 d_{2,1} - B_2 d_{2,2} - B_3 d_{2,3} - B_4 d_{2,4} - B_5 d_{2,5}\right) + B_1 \Omega_{2,1} + B_2 \Omega_{2,2} + B_3 \Omega_{2,3} + B_4 \Omega_{2,4} + B_5 \Omega_{2,5};$$

$$dM_3 = -M_3 q_3 + \left(nuK * \frac{M_1}{denK + M_1} * \frac{M_2}{denK + M_2} * \frac{M_3}{denK + M_3} * \frac{M_4}{denK + M_4} * \frac{M_5}{denK + M_5}\right) \left(-B_1 d_{3,1} - B_2 d_{3,2} - B_3 d_{3,3} - B_4 d_{3,4} - B_5 d_{3,5}\right) + B_1 \Omega_{3,1} + B_2 \Omega_{3,2} + B_3 \Omega_{3,3} + B_4 \Omega_{3,4} + B_5 \Omega_{3,5};$$

$$dM_4 = -M_4 q_4 + \left(nuK * \frac{M_1}{denK + M_1} * \frac{M_2}{denK + M_2} * \frac{M_3}{denK + M_3} * \frac{M_4}{denK + M_4} * \frac{M_5}{denK + M_5}\right) \left(-B_1 d_{4,1} - B_2 d_{4,2} - B_3 d_{4,3} - B_4 d_{4,4} - B_5 d_{4,5}\right) + B_1 \Omega_{4,1} + B_2 \Omega_{4,2} + B_3 \Omega_{4,3} + B_4 \Omega_{4,4} + B_5 \Omega_{4,5};$$

$$dM_5 = -M_5 q_5 + \left(nuK * \frac{M_1}{denK + M_1} * \frac{M_2}{denK + M_2} * \frac{M_3}{denK + M_3} * \frac{M_4}{denK + M_4} * \frac{M_5}{denK + M_5}\right) \left(-B_1 d_{5,1} - B_2 d_{5,2} - B_3 d_{5,3} - B_4 d_{5,4} - B_5 d_{5,5}\right) + B_1 \Omega_{5,1} + B_2 \Omega_{5,2} + B_3 \Omega_{5,3} + B_4 \Omega_{5,4} + B_5 \Omega_{5,5};$$

$$m1 = \begin{pmatrix} D[dB_1, B_1] & D[dB_1, B_2] & D[dB_1, B_3] & D[dB_1, B_4] & D[dB_1, B_5] & D[dB_1, M_1] & D[dB_1, M_2] & D[\\ D[dB_2, B_1] & D[dB_2, B_2] & D[dB_2, B_3] & D[dB_2, B_4] & D[dB_2, B_5] & D[dB_2, M_1] & D[dB_2, M_2] & D[\\ D[dB_3, B_1] & D[dB_3, B_2] & D[dB_3, B_3] & D[dB_3, B_4] & D[dB_3, B_5] & D[dB_3, M_1] & D[dB_3, M_2] & D[\\ D[dB_4, B_1] & D[dB_4, B_2] & D[dB_4, B_3] & D[dB_4, B_4] & D[dB_4, B_5] & D[dB_4, M_1] & D[dB_4, M_2] & D[\\ D[dB_5, B_1] & D[dB_5, B_2] & D[dB_5, B_3] & D[dB_5, B_4] & D[dB_5, B_5] & D[dB_5, M_1] & D[dB_5, M_2] & D[\\ D[dM_1, B_1] & D[dM_1, B_2] & D[dM_1, B_3] & D[dM_1, B_4] & D[dM_1, B_5] & D[dM_1, M_1] & D[dM_1, M_2] & D[\\ D[dM_2, B_1] & D[dM_2, B_2] & D[dM_2, B_3] & D[dM_2, B_4] & D[dM_2, B_5] & D[dM_2, M_1] & D[dM_2, M_2] & D[\\ D[dM_3, B_1] & D[dM_3, B_2] & D[dM_3, B_3] & D[dM_3, B_4] & D[dM_3, B_5] & D[dM_3, M_1] & D[dM_3, M_2] & D[\\ D[dM_4, B_1] & D[dM_4, B_2] & D[dM_4, B_3] & D[dM_4, B_4] & D[dM_4, B_5] & D[dM_4, M_1] & D[dM_4, M_2] & D[\\ D[dM_5, B_1] & D[dM_5, B_2] & D[dM_5, B_3] & D[dM_5, B_4] & D[dM_5, B_5] & D[dM_5, M_1] & D[dM_5, M_2] & D[\end{pmatrix}$$

```
eig = Eigenvalues[m1 /. par /. parvar];

({Re[#], Im[#]} &) /@ eig

)
```

In[5406]:=
```
NetK = {
   {0, 1, 0, 1, 0},
   {1, 0, 1, 1, 0},
   {1, 0, 1, 0, 1},
   {0, 1, 0, 1, 0},
   {0, 0, 0, 0, 1}
  };
```

In[5407]:=
```
fNewSaitoEig[NetK]
```

Out[5407]=
```
{{-1334.06, 0}, {-1332.34, 0}, {-1332.32, 0}, {-1332.29, 0},
 {-1330.66, 0}, {-46.8151, 0}, {-0.3, 0}, {-0.3, 0}, {-0.3, 0}, {-0.3, 0}}
```

In[5411]:=
```
fNewSaitoEigOV[NetK, 10]
```

Out[5411]=
```
{{-1495.43, 0}, {-1495.34, 0}, {-1495.33, 0}, {-1495.29, 0},
 {-1495.27, 0}, {-0.332402, 0}, {-0.3, 0}, {-0.3, 0}, {-0.3, 0}, {-0.3, 0}}
```

In[5609]:=
```
Timing[fNewSaitoEigOV[#, 5] & /@ hk8;]
```

Out[5609]=
```
{6.01476, Null}
```

In[5610]:=
```
6 × 100. / 60
```

Out[5610]=
```
10.
```

In[5490]:=
```
EigOV8coop5 = fNewSaitoEigOV[#, 5] & /@ hk8
EigOV8coop6 = fNewSaitoEigOV[#, 6] & /@ hk8
EigOV8coop7 = fNewSaitoEigOV[#, 7] & /@ hk8
EigOV8coop8 = fNewSaitoEigOV[#, 8] & /@ hk8
EigOV8coop9 = fNewSaitoEigOV[#, 9] & /@ hk8
EigOV8coop10 = fNewSaitoEigOV[#, 10] & /@ hk8
EigOV8coop11 = fNewSaitoEigOV[#, 11] & /@ hk8
EigOV8coop12 = fNewSaitoEigOV[#, 12] & /@ hk8
EigOV8coop13 = fNewSaitoEigOV[#, 13] & /@ hk8
EigOV8coop14 = fNewSaitoEigOV[#, 14] & /@ hk8
EigOV8coop15 = fNewSaitoEigOV[#, 15] & /@ hk8
```

In[5501]:=
```
ReEig8coop5 = (First /@ EigOV8coop5[[#]] &) /@ Range[100];
ReEig8coop6 = (First /@ EigOV8coop6[[#]] &) /@ Range[100];
ReEig8coop7 = (First /@ EigOV8coop7[[#]] &) /@ Range[100];
ReEig8coop8 = (First /@ EigOV8coop8[[#]] &) /@ Range[100];
ReEig8coop9 = (First /@ EigOV8coop9[[#]] &) /@ Range[100];
ReEig8coop10 = (First /@ EigOV8coop10[[#]] &) /@ Range[100];
ReEig8coop11 = (First /@ EigOV8coop11[[#]] &) /@ Range[100];
ReEig8coop12 = (First /@ EigOV8coop12[[#]] &) /@ Range[100];
ReEig8coop13 = (First /@ EigOV8coop13[[#]] &) /@ Range[100];
ReEig8coop14 = (First /@ EigOV8coop14[[#]] &) /@ Range[100];
ReEig8coop15 = (First /@ EigOV8coop15[[#]] &) /@ Range[100];
```

In[5512]:=
```
E8coop5 = Abs /@ Flatten[ReEig8coop5];
E8coop6 = Abs /@ Flatten[ReEig8coop6];
E8coop7 = Abs /@ Flatten[ReEig8coop7];
E8coop8 = Abs /@ Flatten[ReEig8coop8];
E8coop9 = Abs /@ Flatten[ReEig8coop9];
E8coop10 = Abs /@ Flatten[ReEig8coop10];
E8coop11 = Abs /@ Flatten[ReEig8coop11];
E8coop12 = Abs /@ Flatten[ReEig8coop12];
E8coop13 = Abs /@ Flatten[ReEig8coop13];
E8coop14 = Abs /@ Flatten[ReEig8coop14];
E8coop15 = Abs /@ Flatten[ReEig8coop15];
```

(* 100 Replicates for each n (= 5-15) cooperative links *)

In[ ]:=
```
Eigcoop5 = Table[fNewSaitoEigOV[#, 5] & /@ hk8, {100}];
Eigcoop10 = Table[fNewSaitoEigOV[#, 10] & /@ hk8, {100}];
Eigcoop15 = Table[fNewSaitoEigOV[#, 15] & /@ hk8, {100}];
```

In[ ]:=
```
Eigcoop6 = Table[fNewSaitoEigOV[#, 6] & /@ hk8, {100}];
Eigcoop7 = Table[fNewSaitoEigOV[#, 7] & /@ hk8, {100}];
Eigcoop8 = Table[fNewSaitoEigOV[#, 8] & /@ hk8, {100}];
Eigcoop9 = Table[fNewSaitoEigOV[#, 9] & /@ hk8, {100}];

Eigcoop11 = Table[fNewSaitoEigOV[#, 11] & /@ hk8, {100}];
Eigcoop12 = Table[fNewSaitoEigOV[#, 12] & /@ hk8, {100}];
Eigcoop13 = Table[fNewSaitoEigOV[#, 13] & /@ hk8, {100}];
Eigcoop14 = Table[fNewSaitoEigOV[#, 14] & /@ hk8, {100}];
```

In[ ]:=
```
Timing[Table[i, {i, 1, 10 000 000}];]
```
Out[ ]= {0.163432, Null}

In[ ]:=
```
Timing[Parallelize[Table[i, {i, 1, 10 000 000}]];]
```
Out[ ]= {0.371539, Null}

In[•]:=
```
Eigcoop5[[1]]
```

In[•]:=
```
ReEig8coop5i = Table[(First /@ Eigcoop5[[i]][[#]] &) /@ Range[100], {i, 1, 100}]
```

In[•]:=
```
E8coop5i = Abs /@ Flatten[ReEig8coop5i]
```

In[•]:=
```
ReEig8coop6i = Table[(First /@ Eigcoop6[[i]][[#]] &) /@ Range[100], {i, 1, 100}];
E8coop6i = Abs /@ Flatten[ReEig8coop6i]
```

In[•]:=
```
ReEig8coop7i = Table[(First /@ Eigcoop7[[i]][[#]] &) /@ Range[100], {i, 1, 100}];
E8coop7i = Abs /@ Flatten[ReEig8coop7i]
```

In[•]:=
```
ReEig8coop8i = Table[(First /@ Eigcoop8[[i]][[#]] &) /@ Range[100], {i, 1, 100}];
E8coop8i = Abs /@ Flatten[ReEig8coop8i]
```

In[•]:=
```
ReEig8coop9i = Table[(First /@ Eigcoop9[[i]][[#]] &) /@ Range[100], {i, 1, 100}];
E8coop9i = Abs /@ Flatten[ReEig8coop9i]
```

In[•]:=
```
ReEig8coop10i = Table[(First /@ Eigcoop10[[i]][[#]] &) /@ Range[100], {i, 1, 100}]
```

In[•]:=
```
E8coop10i = Abs /@ Flatten[ReEig8coop10i]
```

In[•]:=
```
ReEig8coop11i = Table[(First /@ Eigcoop11[[i]][[#]] &) /@ Range[100], {i, 1, 100}];
E8coop11i = Abs /@ Flatten[ReEig8coop11i]
```

In[•]:=
```
ReEig8coop12i = Table[(First /@ Eigcoop12[[i]][[#]] &) /@ Range[100], {i, 1, 100}];
E8coop12i = Abs /@ Flatten[ReEig8coop12i]
```

In[•]:=
```
ReEig8coop13i = Table[(First /@ Eigcoop13[[i]][[#]] &) /@ Range[100], {i, 1, 100}];
E8coop13i = Abs /@ Flatten[ReEig8coop13i]
```

In[•]:=
```
ReEig8coop14i = Table[(First /@ Eigcoop14[[i]][[#]] &) /@ Range[100], {i, 1, 100}];
E8coop14i = Abs /@ Flatten[ReEig8coop14i]
```

In[•]:=
```
ReEig8coop15i = Table[(First /@ Eigcoop15[[i]][[#]] &) /@ Range[100], {i, 1, 100}]
```

In[•]:=
```
E8coop15i = Abs /@ Flatten[ReEig8coop15i]
```

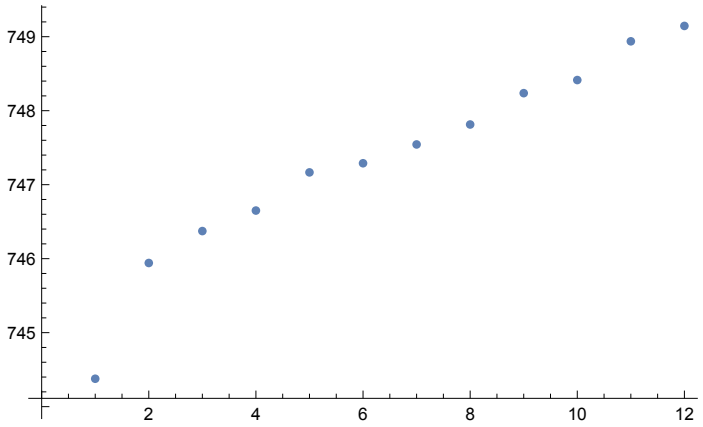
In[•]:=
```
{Mean[E8coop5i], Mean[E8coop10i], Mean[E8coop15i]}
```

Out[•]=
```
{745.942, 747.544, 749.146}
```

In[5664]:=
```
MeCoop = {Mean[E8], Mean[E8coop5i], Mean[E8coop6i], Mean[E8coop7i],
  Mean[E8coop8i], Mean[E8coop9i], Mean[E8coop10i], Mean[E8coop11i],
  Mean[E8coop12i], Mean[E8coop13i], Mean[E8coop14i], Mean[E8coop15i]}
ListPlot[MeCoop]
```

Out[5664]=
```
{744.377, 745.942, 746.373, 746.65, 747.166,
 747.29, 747.544, 747.813, 748.237, 748.414, 748.938, 749.146}
```

Out[5665]=



In[5666]:=
```
dataplo8Coop =
 Sort[Partition[Riffle[{0, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}, MeCoop], {2}]]
```

Out[5666]=
```
{{0, 744.377}, {5, 745.942}, {6, 746.373}, {7, 746.65},
 {8, 747.166}, {9, 747.29}, {10, 747.544}, {11, 747.813},
 {12, 748.237}, {13, 748.414}, {14, 748.938}, {15, 749.146}}
```

In[5667]:=
```
nlm8coop = NonlinearModelFit[dataplo8Coop, b x + a , {a, b}, x]
```

Out[5667]=
```
FittedModel[ 744.428 + 0.31589 x ]
```

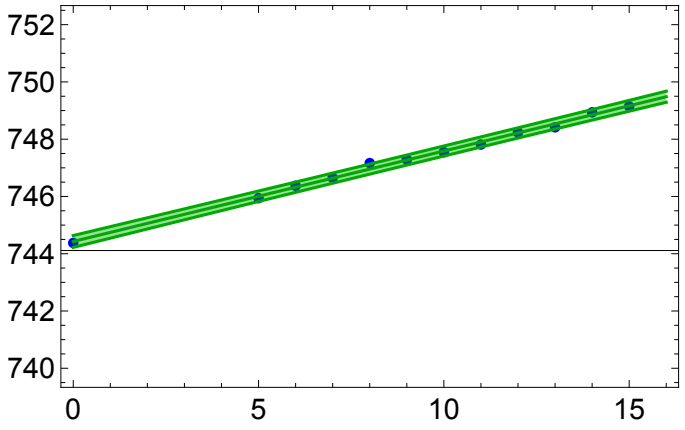
In[5668]:=
```
nlm8coop["ParameterTable"]
```

Out[5668]=

	Estimate	Standard Error	t-Statistic	P-Value
a	744.428	0.0659853	11 281.7	7.36786×10^{-37}
b	0.31589	0.0065712	48.0719	3.66097×10^{-13}

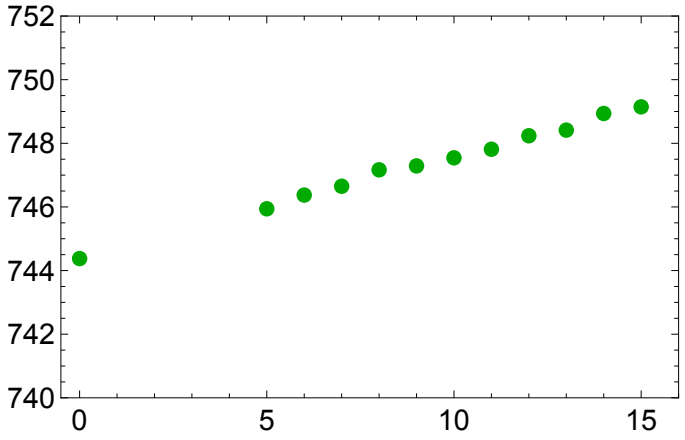
In[5671]:=
```
Show[
 ListPlot[dataplo8Coop, PlotStyle → {Blue, PointSize[0.017]}],

 Plot[{nlm8coop[x],
   nlm8coop["SinglePredictionBands", ConfidenceLevel → 0.9]}, {x, 0, 16},
  FillingStyle -> Directive[Opacity[0.4], Darker[Green]],
  Filling → {2 → {1}}, PlotStyle → Darker[Green]],
 PlotRange → {{0, 16}, {740, 752}},
 Frame → True,
 ImageSize → 350,
 FrameStyle → Directive[Black, FontSize → 15]

]
```

Out[5671]=



In[7546]:=
```
ListPlot[dataplo8Coop, PlotStyle → {Darker[Green], PointSize[0.025]},
 PlotRange → {{-0.5, 16}, {740, 752}},
 Frame → True,
 ImageSize → 350,
 FrameStyle → Directive[Black, FontSize → 15]]
```

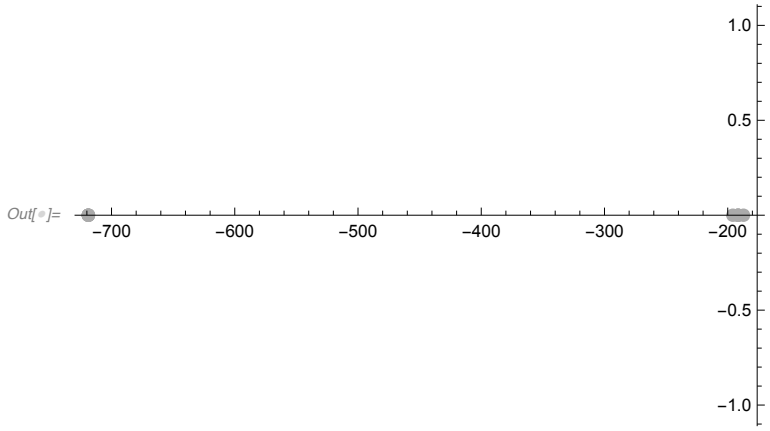
Out[7546]=



```
In[•]:= Eig7 // Length
Out[•]= 569

In[•]:= Eig7[[4]]
Out[•]= {{-718.912, 0}, {-718.76, 0}, {-718.76, 0}, {-718.76, 0}, {-718.76, 0},
         {-195.875, 0}, {-191.691, 0}, {-191.519, 0}, {-191.519, 0}, {-187.19, 0}}

In[•]:= EigOV7[[4]]
Out[•]= {{-760.605, 0}, {-760.455, 0}, {-760.455, 0}, {-760.455, 0}, {-760.455, 0},
         {-207.003, 0}, {-202.813, 0}, {-202.653, 0}, {-202.589, 0}, {-198.332, 0}}
```

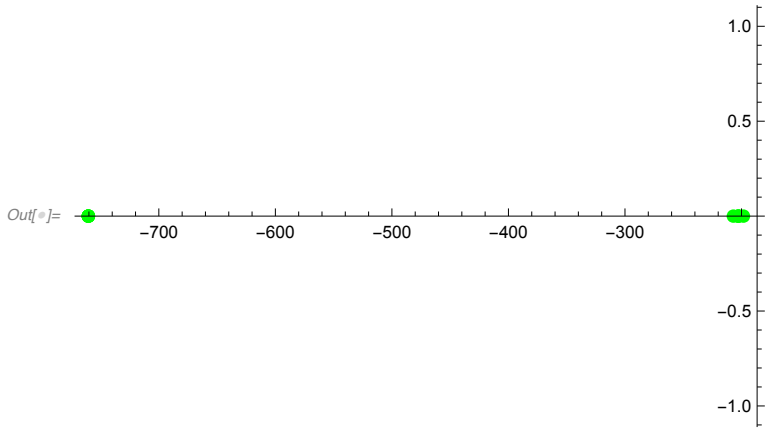
In[•]:=
```
ListPlot[Eig7[[4]], PlotStyle → {PointSize[Large], Lighter[Gray]}]
```

Out[•]=



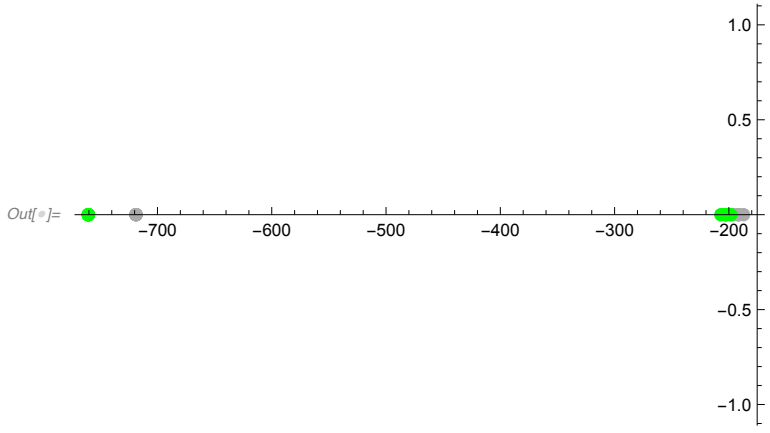
In[•]:=
```
ListPlot[EigOV7[[4]], PlotStyle → {PointSize[Large], Green}]
```

Out[•]=



In[•]:=
```
rg = 4;
ListPlot[{Eig7[[rg]], EigOV7[[rg]]},
 PlotStyle → {{PointSize[Large], Lighter[Gray]}, {PointSize[Large], Green}}]
AuxoComm7[[rg]]
```

Out[•]=



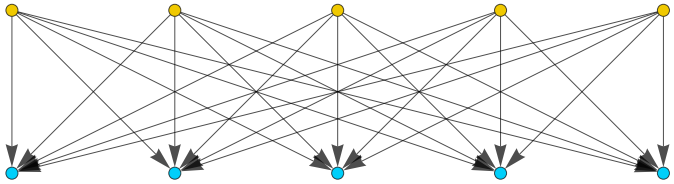
Out[•]=
```
{{0, 0, 0, 0, 1}, {0, 0, 0, 1, 1}, {1, 1, 1, 1, 1}, {1, 1, 1, 1, 1}, {1, 1, 1, 1, 1}}
```

In[•]:=
```
Length[EigOV7]
```

Out[•]=
```
569
```

In[•]:=
```
Net[AuxoComm7[[490]]]
```

Out[•]=



In[•]:=
```
Table[#[[2]] & /@ EigOV7[[i]], {i, 4, Length[EigOV7]}]
```

In[•]:=
```
nk = Complement[Table[{i}, {i, 1, Length[Eig7]}], Position[
    Table[#[[2]] & /@ Eig7[[i]], {i, 4, Length[Eig7]}], {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}]]
```

Out[•]=
```
{{13}, {66}, {67}, {71}, {77}, {85}, {86}, {88}, {95}, {113}, {122}, {158},
 {159}, {160}, {177}, {183}, {192}, {203}, {207}, {208}, {238}, {245},
 {246}, {248}, {253}, {254}, {255}, {256}, {265}, {269}, {270}, {284},
 {285}, {286}, {291}, {292}, {293}, {294}, {296}, {297}, {298}, {299}, {300},
 {301}, {302}, {303}, {305}, {306}, {307}, {308}, {312}, {313}, {320}, {325},
 {335}, {339}, {341}, {343}, {346}, {350}, {351}, {362}, {363}, {364}, {365},
 {367}, {368}, {380}, {384}, {385}, {389}, {395}, {396}, {398}, {402}, {409},
 {417}, {434}, {437}, {444}, {447}, {448}, {449}, {450}, {460}, {462}, {463},
 {466}, {467}, {468}, {470}, {471}, {474}, {475}, {495}, {508}, {510}, {527},
 {529}, {533}, {535}, {536}, {537}, {555}, {556}, {559}, {567}, {568}, {569}}
```

```
In[•]:= nkOV = Complement[Table[{i}, {i, 1, Length[EigOV7]}],
          Position[Table[#[[2]] & /@ EigOV7[[i]], {i, 4, Length[EigOV7]}],
           {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}]]
```

```
Out[•]= {{29}, {31}, {38}, {41}, {45}, {51}, {58}, {66}, {67}, {71}, {75}, {77}, {80}, {85},
         {87}, {88}, {95}, {104}, {111}, {112}, {113}, {117}, {122}, {127}, {151}, {158},
         {159}, {160}, {166}, {172}, {177}, {178}, {183}, {192}, {207}, {226}, {227}, {228},
         {231}, {234}, {235}, {238}, {242}, {245}, {246}, {247}, {248}, {253}, {256},
         {257}, {259}, {260}, {261}, {264}, {265}, {266}, {268}, {269}, {270}, {273},
         {280}, {282}, {284}, {285}, {286}, {288}, {290}, {293}, {294}, {296}, {297},
         {298}, {299}, {300}, {301}, {302}, {303}, {304}, {305}, {306}, {307}, {308},
         {313}, {318}, {320}, {335}, {341}, {343}, {346}, {349}, {350}, {353}, {355},
         {357}, {362}, {363}, {364}, {365}, {370}, {372}, {373}, {374}, {377}, {380},
         {384}, {386}, {389}, {390}, {395}, {396}, {404}, {406}, {409}, {413}, {414},
         {416}, {418}, {421}, {423}, {426}, {434}, {435}, {437}, {440}, {444}, {446},
         {448}, {449}, {450}, {451}, {452}, {454}, {455}, {460}, {461}, {462}, {467},
         {468}, {470}, {473}, {474}, {475}, {479}, {481}, {490}, {491}, {493}, {508},
         {510}, {518}, {523}, {524}, {527}, {529}, {530}, {533}, {535}, {536}, {537},
         {538}, {539}, {542}, {543}, {553}, {555}, {556}, {558}, {567}, {568}, {569}}
```

```
In[•]:= Intersection[nk, nkOV]
```

```
Out[•]= {{66}, {67}, {71}, {77}, {85}, {88}, {95}, {113}, {122}, {158}, {159},
         {160}, {177}, {183}, {192}, {207}, {238}, {245}, {246}, {248}, {253},
         {256}, {265}, {269}, {270}, {284}, {285}, {286}, {293}, {294}, {296},
         {297}, {298}, {299}, {300}, {301}, {302}, {303}, {305}, {306}, {307},
         {308}, {313}, {320}, {335}, {341}, {343}, {346}, {350}, {362}, {363}, {364},
         {365}, {380}, {384}, {389}, {395}, {396}, {409}, {434}, {437}, {444}, {448},
         {449}, {450}, {460}, {462}, {467}, {468}, {470}, {474}, {475}, {508}, {510},
         {527}, {529}, {533}, {535}, {536}, {537}, {555}, {556}, {567}, {568}, {569}}
```

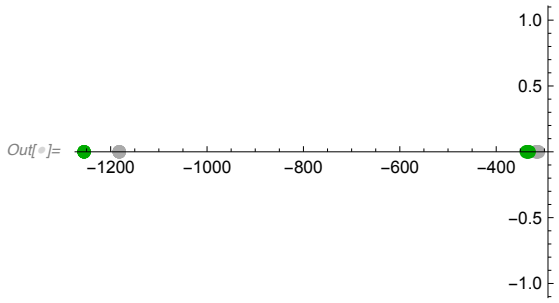
```
In[•]:= EigOV7[[#]] & /@ Flatten[nkOV]
```

```
In[•]:= #[[2]] & /@ (EigOV7[[#]] & /@ Flatten[nkOV])
```

```
In[•]:= eigGr[x_] := (rg = x;
         ListPlot[{Eig7[[rg]], EigOV7[[rg]]},
          PlotStyle → {{PointSize[Large], Lighter[Gray]},
            {PointSize[Large], Darker[Green]}}, ImageSize → 250]
         (*{ListPlot[{Eig7[[rg]],EigOV7[[rg]]},PlotStyle→
            {{PointSize[Large],Red},{PointSize[Large],Darker[Green]}},ImageSize→250]
          ,
          Net[AuxoComm7[[rg]]]}*))
```

```
In[•]:= eigGr[145]
```

Out[•]=




```
In[•]:= nkOV
```

```
Out[•]= {{29}, {31}, {38}, {41}, {45}, {51}, {58}, {66}, {67}, {71}, {75}, {77}, {80}, {85},
  {87}, {88}, {95}, {104}, {111}, {112}, {113}, {117}, {122}, {127}, {151}, {158},
  {159}, {160}, {166}, {172}, {177}, {178}, {183}, {192}, {207}, {226}, {227}, {228},
  {231}, {234}, {235}, {238}, {242}, {245}, {246}, {247}, {248}, {253}, {256},
  {257}, {259}, {260}, {261}, {264}, {265}, {266}, {268}, {269}, {270}, {273},
  {280}, {282}, {284}, {285}, {286}, {288}, {290}, {293}, {294}, {296}, {297},
  {298}, {299}, {300}, {301}, {302}, {303}, {304}, {305}, {306}, {307}, {308},
  {313}, {318}, {320}, {335}, {341}, {343}, {346}, {349}, {350}, {353}, {355},
  {357}, {362}, {363}, {364}, {365}, {370}, {372}, {373}, {374}, {377}, {380},
  {384}, {386}, {389}, {390}, {395}, {396}, {404}, {406}, {409}, {413}, {414},
  {416}, {418}, {421}, {423}, {426}, {434}, {435}, {437}, {440}, {444}, {446},
  {448}, {449}, {450}, {451}, {452}, {454}, {455}, {460}, {461}, {462}, {467},
  {468}, {470}, {473}, {474}, {475}, {479}, {481}, {490}, {491}, {493}, {508},
  {510}, {518}, {523}, {524}, {527}, {529}, {530}, {533}, {535}, {536}, {537},
  {538}, {539}, {542}, {543}, {553}, {555}, {556}, {558}, {567}, {568}, {569}}
```

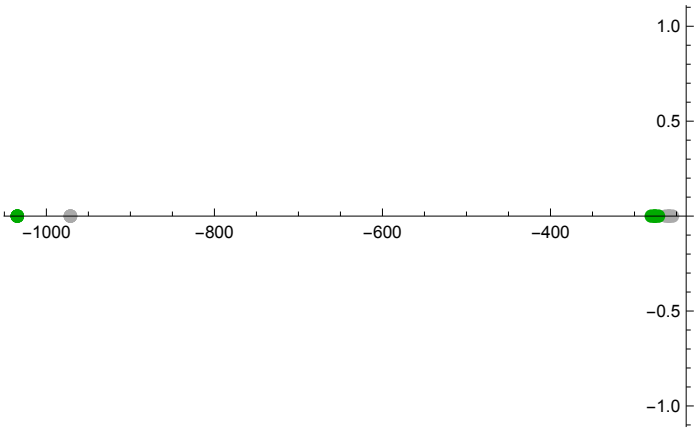
```
In[•]:= eigGr /@ Flatten[nkOV]
```

```
In[•]:= Partition[Riffle[Flatten[nkOV], eigGr /@ Flatten[nkOV]], {2}]
```

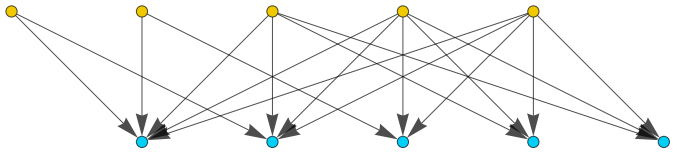
```
{29, 41, 80, 88, 256, 299}
```

In[•]:=
```
rg = 29;
ListPlot[{Eig7[[rg]], EigOV7[[rg]]}, PlotStyle →
  {{PointSize[Large], Lighter[Gray]}, {PointSize[Large], Darker[Green]}}]
AuxoComm7[[rg]];
Net[AuxoComm7[[rg]] /. {0 → 0., 1 → 0.1}]
```

Out[•]=

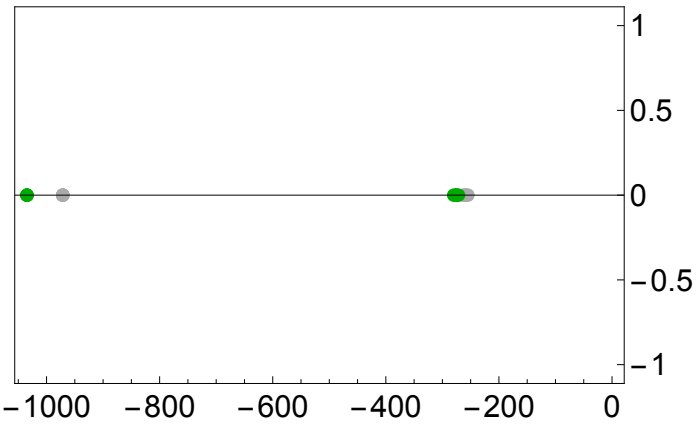


Out[•]=

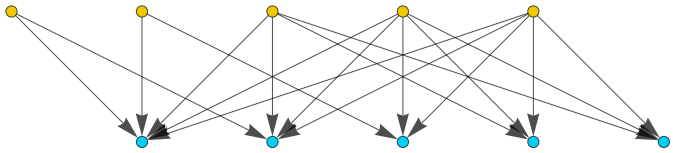


In[•]:=
```
rg = 29;
ListPlot[{Eig7[[rg]], EigOV7[[rg]]},
 PlotStyle → {{PointSize[Large], Lighter[Gray]}, {PointSize[Large], Darker[Green]}},
 AxesOrigin → {0, 0}, Frame → True,
 FrameTicks → {{None, {-1, -0.5, 0, 0.5, 1}}, {Automatic, None}},
 FrameTicksStyle → Directive[Black, 16]]
AuxoComm7[[rg]];
Net[AuxoComm7[[rg]] /. {0 → 0., 1 → 0.1}]
```

Out[•]=



Out[•]=



In[•]:=
```
AuxoComm7[[29]]
AuxoComm7[[41]]
```

Out[•]= {{0, 0, 0, 1, 1}, {0, 0, 1, 0, 1}, {1, 1, 0, 1, 1}, {1, 1, 1, 1, 1}, {1, 1, 1, 1, 1}}

Out[•]= {{0, 0, 0, 1, 1}, {0, 0, 1, 1, 1}, {1, 1, 0, 1, 1}, {1, 1, 1, 0, 1}, {1, 1, 1, 1, 1}}

In[•]:=
```
AuxoComm7[[29]] // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

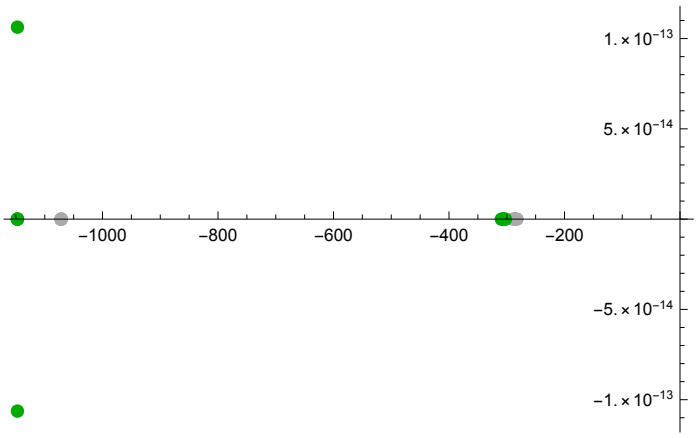
In[•]:=
```
AuxoComm7[[41]] // MatrixForm
```

Out[•]//MatrixForm=

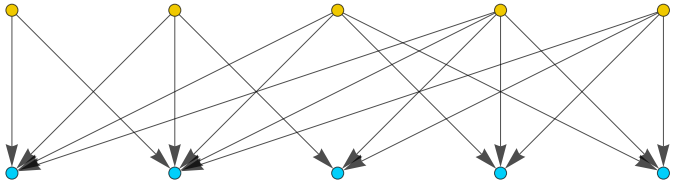
$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

In[•]:=
```
rg = 41;
ListPlot[{Eig7[[rg]], EigOV7[[rg]]},
 PlotStyle → {{PointSize[Large], Lighter[Gray]}, {PointSize[Large], Darker[Green]}},
 AxesOrigin → {0, 0}]
AuxoComm7[[rg]];
Net[AuxoComm7[[rg]] /. {0 → 0., 1 → 0.1}]
```

Out[•]=

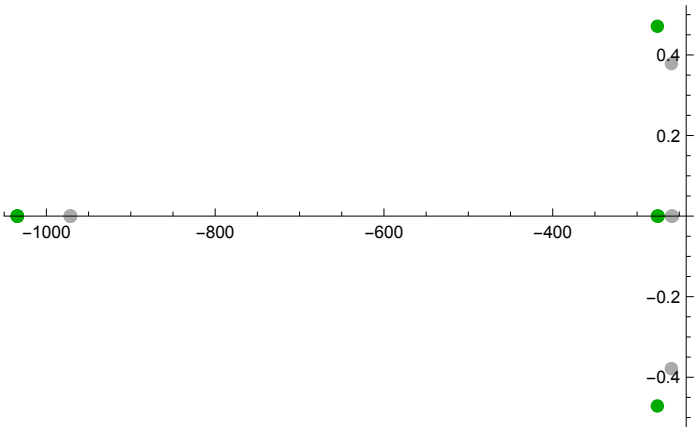


Out[•]=

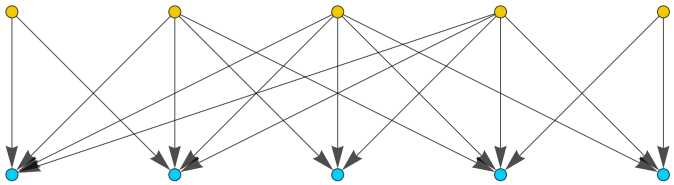


In[•]:=
```
rg = 80;
ListPlot[{Eig7[[rg]], EigOV7[[rg]]}, PlotStyle →
  {{PointSize[Large], Lighter[Gray]}, {PointSize[Large], Darker[Green]}}]
AuxoComm7[[rg]];
Net[AuxoComm7[[rg]] /. {0 → 0., 1 → 0.1}]
```

Out[•]=

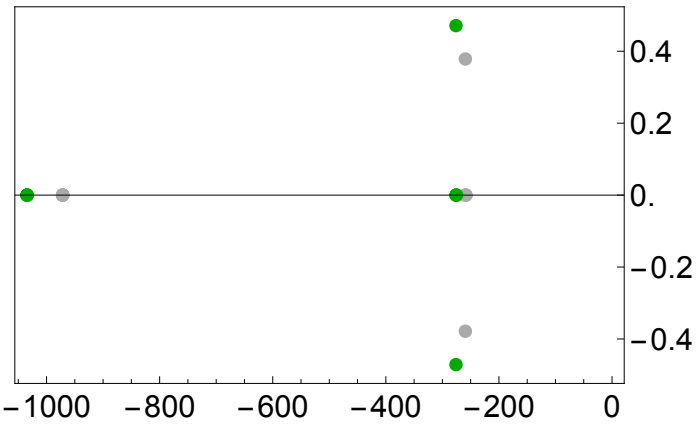


Out[•]=

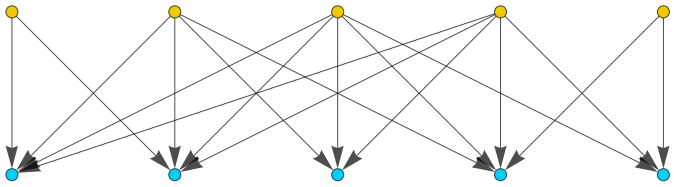


In[•]:=
```
rg = 80;
ListPlot[{Eig7[[rg]], EigOV7[[rg]]},
 PlotStyle → {{PointSize[Large], Lighter[Gray]}, {PointSize[Large], Darker[Green]}},
 AxesOrigin → {0, 0}, Frame → True,
 FrameTicks → {{None, {-0.4, -0.2, 0.0, 0.2, 0.4}}, {Automatic, None}},
 FrameTicksStyle → Directive[Black, 16]]
AuxoComm7[[rg]];
Net[AuxoComm7[[rg]] /. {0 → 0., 1 → 0.1}]
```

Out[•]=



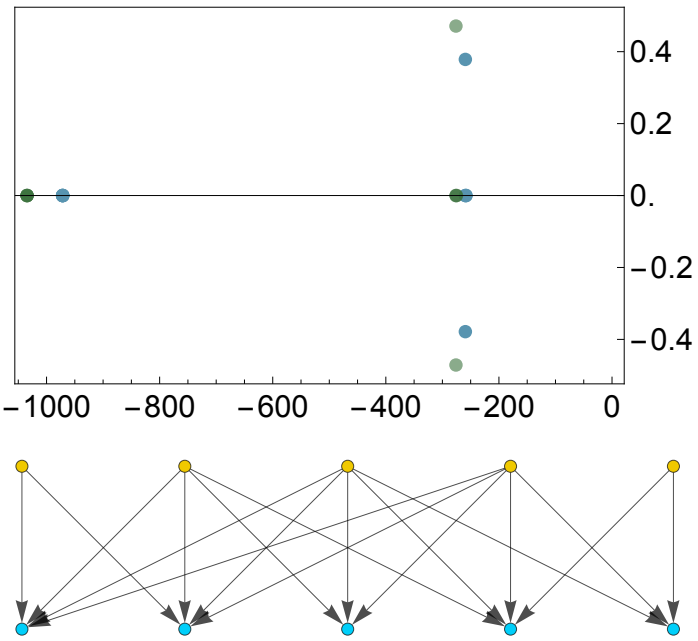
Out[•]=



```
RGBColor[0.34509803921568627, 0.5803921568627451, 0.6901960784313725],
RGBColor[0.23921568627450981, 0.45098039215686275, 0.24705882352941178, 0.6]
```

In[•]:=
```
rg = 80;
ListPlot[{Eig7[[rg]], EigOV7[[rg]]},
 PlotStyle → {{PointSize[Large], RGBColor[0.34509803921568627,
     0.5803921568627451, 0.6901960784313725]}, {PointSize[Large],
    RGBColor[0.23921568627450981, 0.4509803921568627, 0.24705882352941178, 0.6]}},
 AxesOrigin → {0, 0}, Frame → True, FrameTicks →
  {{None, {-0.4, -0.2, 0.0, 0.2, 0.4}}, {Automatic, None}},
 FrameTicksStyle → Directive[Black, 16]]
AuxoComm7[[rg]];
Net[AuxoComm7[[rg]] /. {0 → 0., 1 → 0.1}]
```

Out[•]=



Out[•]=

In[•]:=
```
AuxoComm7[[80]]
```

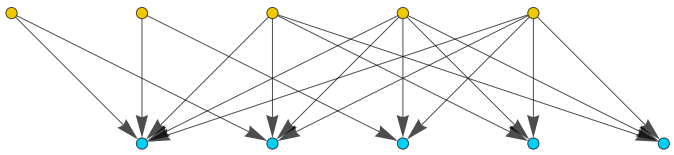
Out[•]=
```
{{0, 0, 0, 1, 1}, {0, 1, 1, 1, 1}, {1, 0, 1, 0, 0}, {1, 1, 1, 1, 1}, {1, 1, 1, 1, 1}}
```

In[•]:=
```
AuxoComm7[[29]]
```

Out[•]=
```
{{0, 0, 0, 1, 1}, {0, 0, 1, 0, 1}, {1, 1, 0, 1, 1}, {1, 1, 1, 1, 1}, {1, 1, 1, 1, 1}}
```

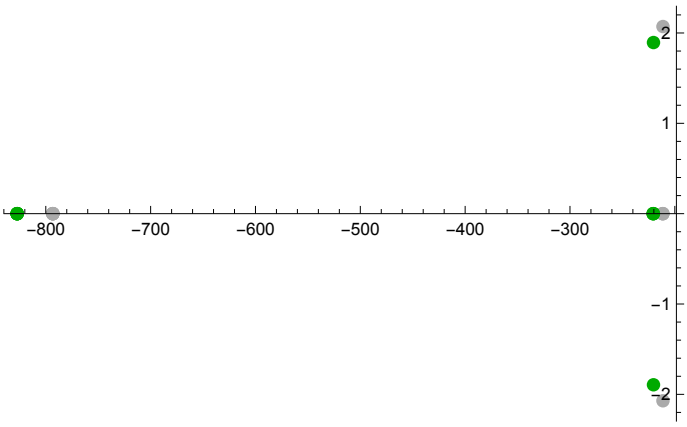
In[•]:=
```
Net[AuxoComm7[[29]] /. {0 → 0., 1 → 0.1}]
```

Out[•]=

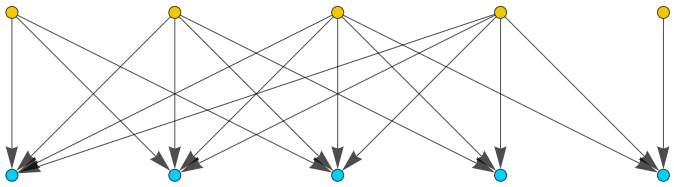


In[•]:=
```
rg = 256;
ListPlot[{Eig7[[rg]], EigOV7[[rg]]}, PlotStyle →
  {{PointSize[Large], Lighter[Gray]}, {PointSize[Large], Darker[Green]}}]
AuxoComm7[[rg]];
Net[AuxoComm7[[rg]] /. {0 → 0., 1 → 0.1}]
```

Out[•]=

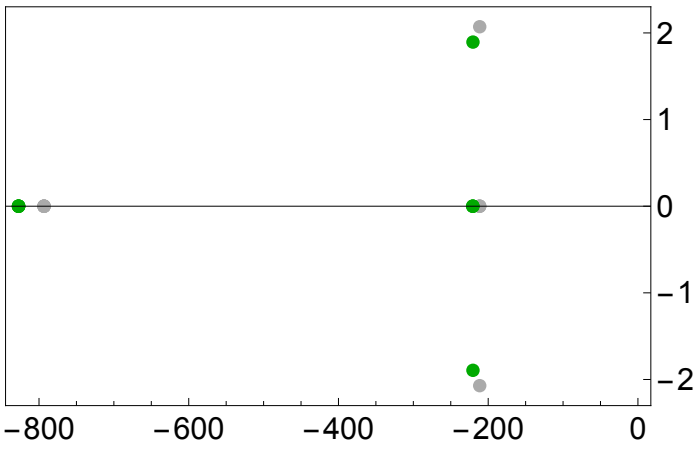


Out[•]=

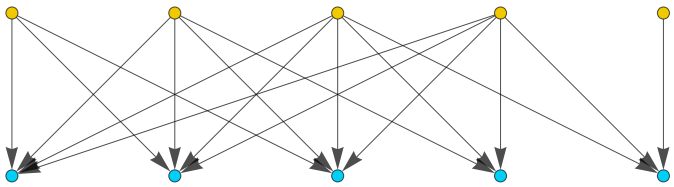


In[•]:=
```
rg = 256;
ListPlot[{Eig7[[rg]], EigOV7[[rg]]},
 PlotStyle → {{PointSize[Large], Lighter[Gray]}, {PointSize[Large], Darker[Green]}},
 AxesOrigin → {0, 0}, Frame → True,
 FrameTicks → {{None, {-2, -1, 0, 1, 2}}, {Automatic, None}},
 FrameTicksStyle → Directive[Black, 16]]
AuxoComm7[[rg]];
Net[AuxoComm7[[rg]] /. {0 → 0., 1 → 0.1}]
```

Out[•]=

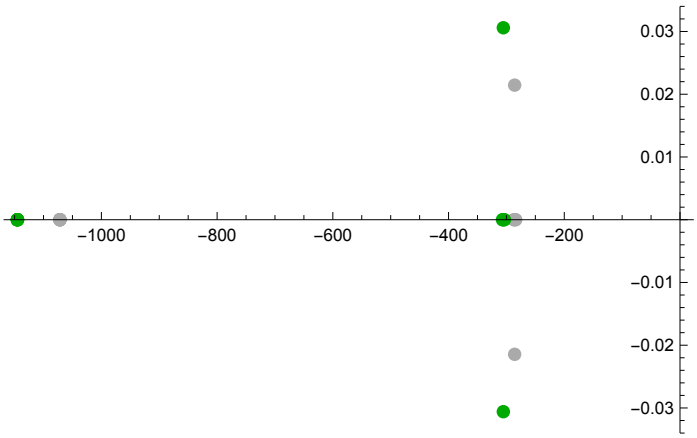


Out[•]=

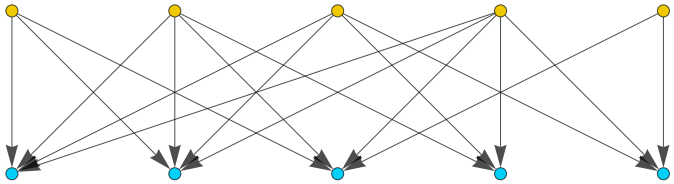


In[•]:=
```
rg = 259;
ListPlot[{Eig7[[rg]], EigOV7[[rg]]},
 PlotStyle → {{PointSize[Large], Lighter[Gray]}, {PointSize[Large], Darker[Green]}},
 AxesOrigin → {0, 0}]
AuxoComm7[[rg]];
Net[AuxoComm7[[rg]] /. {0 → 0., 1 → 0.1}]
```

Out[•]=

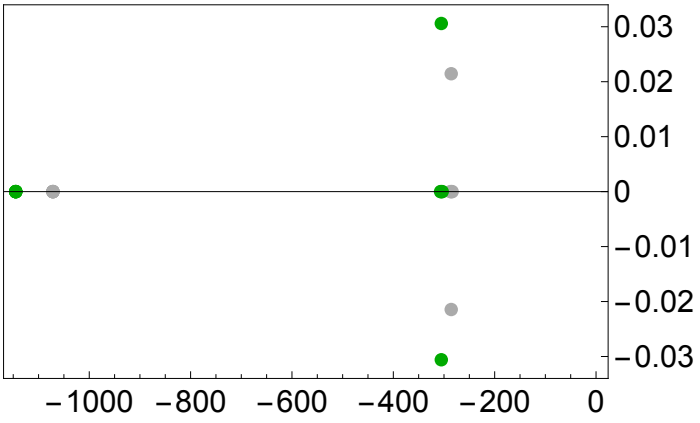


Out[•]=

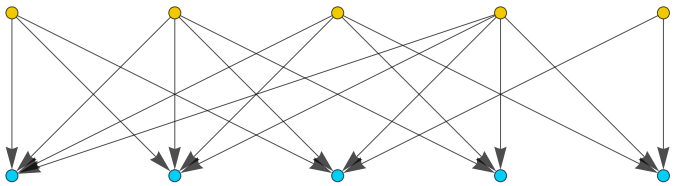


In[•]:=
```
rg = 259;
ListPlot[{Eig7[[rg]], EigOV7[[rg]]},
 PlotStyle → {{PointSize[Large], Lighter[Gray]}, {PointSize[Large], Darker[Green]}},
 AxesOrigin → {0, 0}, Frame → True,
 FrameTicks → {{None, {-0.03, -0.02, -0.01, 0, 0.01, 0.02, 0.03}}, {Automatic, None}},
 FrameTicksStyle → Directive[Black, 16]]
AuxoComm7[[rg]];
Net[AuxoComm7[[rg]] /. {0 → 0., 1 → 0.1}]
```

Out[•]=



Out[•]=

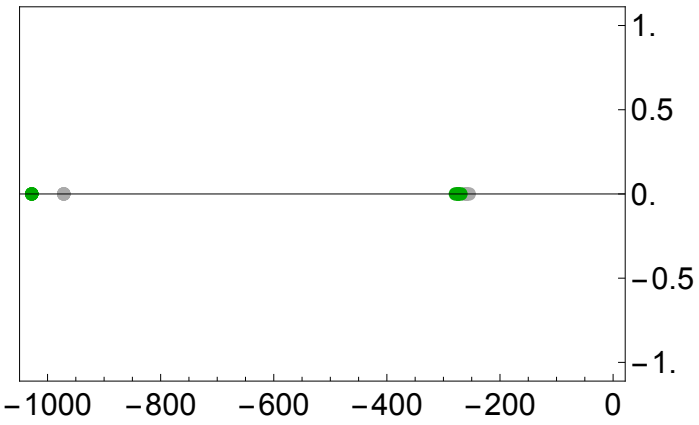


In[•]:=
```
EigOV7[[67]]
```

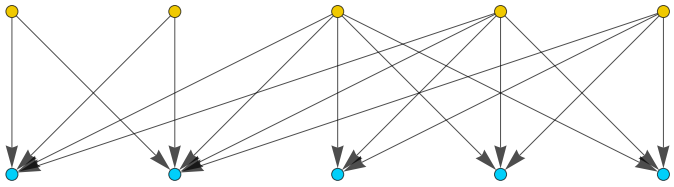
Out[•]=
```
{{-1160.15, 0}, {-1160.01, 0}, {-1160.01, 0}, {-1160.01, 0}, {-1160.01, 0},
 {-312.89, 0}, {-309.255, 0}, {-309.239, 0}, {-309.22, 0}, {-305.526, 0}}
```

In[•]:=
```
rg = 26;
ListPlot[{Eig7[[rg]], EigOV7[[rg]]},
 PlotStyle → {{PointSize[Large], Lighter[Gray]}, {PointSize[Large], Darker[Green]}},
 AxesOrigin → {0, 0}, Frame → True,
 FrameTicks → {{None, {-1.0, -0.5, 0.0, 0.5, 1.0}}, {Automatic, None}},
 FrameTicksStyle → Directive[Black, 16]]
AuxoComm7[[rg]];
Net[AuxoComm7[[rg]] /. {0 → 0., 1 → 0.1}]
```

Out[•]=



Out[•]=



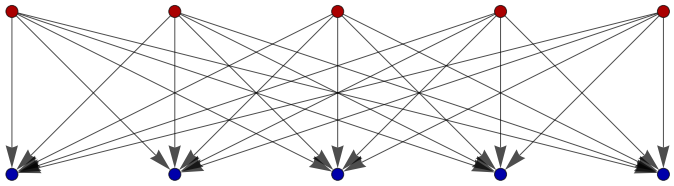
In[•]:=
```
MakeNetworkH[x_] := (
  numBact = Length[x];
  numMetab = Length[x[[1]]];

  VectBact = Range[numBact];
  VectMetab = Range[numBact + 1, numBact + numMetab];

  Bact = StringInsert[ToString /@ Array[# &, {numBact}], "B", 1];
  Metab = StringInsert[ToString /@ Array[# &, {numMetab}], "M", 1];

  elem = Cases[Flatten[x], Except[0.]][[1]];

  color = Join[# → bacol & /@ VectBact, # → metcol & /@ VectMetab];

  conex = Flatten[Cases[Position[x, elem], {y_, z_} → {y → z + numBact}]];

  equiv = Join[#[[1]] → #[[2]] & /@ Partition[Riffle[VectBact, Bact], {2}],
    #[[1]] → #[[2]] & /@ Partition[Riffle[VectMetab, Metab], {2}]];

  Graph[conex, (*VertexLabels→equiv,*)VertexStyle → color, EdgeStyle → Black]
 )

MakeNetworkH[jnl /. {0 → 0., 1 → 0.1}]
```

In[•]:=
```
AuxoComm7[[23]]
```

Out[•]=
```
{{0, 0, 0, 0, 1}, {1, 1, 1, 1, 0}, {1, 1, 1, 1, 0}, {0, 1, 1, 1, 1}, {1, 1, 1, 1, 1}}
```

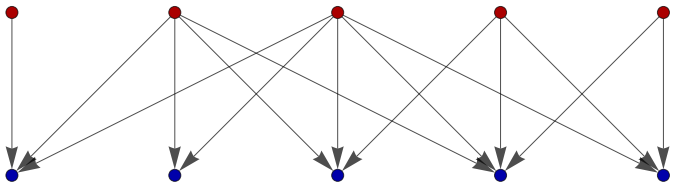
In[•]:=
```
MakeNetworkH[AuxoComm7[[23]] /. {0 → 0.1, 1 → 0.1}]
```

Out[•]=



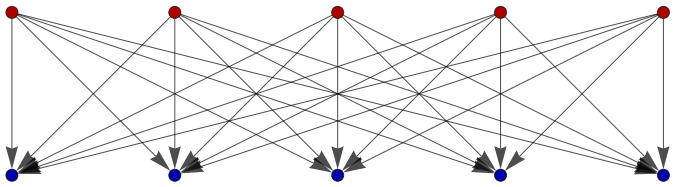
In[•]:=
```
MakeNetworkH[{{0.`, 0.1`, 0.`, 0.`, 0.`},
  {0.`, 0.`, 0.1`, 0.1`, 0.1`}, {0.1`, 0.1`, 0.1`, 0.1`, 0.`},
  {0.1`, 0.1`, 0.1`, 0.1`, 0.1`}, {0.`, 0.`, 0.1`, 0.`, 0.1`}}]
```

Out[•]=



In[•]:= `Net[AuxoComm7[[23]]]`

Out[•]=



In[•]:= `Net[AuxoComm7[[490]]]`

Out[•]=