Cooperation increases robustness to ecological disturbance in microbial cross-feeding networks

Generating Random Networks

Functions to calculate Entropy and Assortativity

Entropy

Assortativity

1. Colimitation model

Solving the system of ODE

The function “fNewSaitoK” solves the ODE system and gives the population at steady state of the system. The function “fNewSaitoK” receives a network and a disturbance value as arguments.

In[7238]:=
```
v = 0.5;
```

In[7239]:=
```
fNewSaitoOPsyK[Net_, Dh_] := (
```

$$dB_1 = B_1[t] \left(-B_1[t]\, \kappa_1 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - \left(c_{1,1} + c_{1,2} + c_{1,3} + c_{1,4} + c_{1,5} + Dh\right) B_1[t];$$

$$dB_2 = B_2[t] \left(-B_2[t]\, \kappa_2 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} *\right.$$

$$\frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\Big) - \left(c_{2,1} + c_{2,2} + c_{2,3} + c_{2,4} + c_{2,5} + Dh\right) B_2[t];$$

$$dB_3 = B_3[t] \Big(-B_3[t]\, \varkappa_3 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\Big) - \left(c_{3,1} + c_{3,2} + c_{3,3} + c_{3,4} + c_{3,5} + Dh\right) B_3[t];$$

$$dB_4 = B_4[t] \Big(-B_4[t]\, \varkappa_4 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\Big) - \left(c_{4,1} + c_{4,2} + c_{4,3} + c_{4,4} + c_{4,5} + Dh\right) B_4[t];$$

$$dB_5 = B_5[t] \Big(-B_5[t]\, \varkappa_5 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\Big) - \left(c_{5,1} + c_{5,2} + c_{5,3} + c_{5,4} + c_{5,5} + Dh\right) B_5[t];$$

$$dM_1 = v - M_1[t]\, q_1 + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) \left(-B_1[t]\, d_{1,1} - B_2[t]\, d_{1,2} - B_3[t]\, d_{1,3} - B_4[t]\, d_{1,4} - B_5[t]\, d_{1,5}\right) + B_1[t]\, \Omega_{1,1} + B_2[t]\, \Omega_{1,2} + B_3[t]\, \Omega_{1,3} + B_4[t]\, \Omega_{1,4} + B_5[t]\, \Omega_{1,5};$$

$$dM_2 = v - M_2[t]\, q_2 + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) \left(-B_1[t]\, d_{2,1} - B_2[t]\, d_{2,2} - B_3[t]\, d_{2,3} - B_4[t]\, d_{2,4} - B_5[t]\, d_{2,5}\right) + B_1[t]\, \Omega_{2,1} + B_2[t]\, \Omega_{2,2} + B_3[t]\, \Omega_{2,3} + B_4[t]\, \Omega_{2,4} + B_5[t]\, \Omega_{2,5};$$

$$dM_3 = v - M_3[t]\, q_3 + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) \left(-B_1[t]\, d_{3,1} - B_2[t]\, d_{3,2} - B_3[t]\, d_{3,3} - B_4[t]\, d_{3,4} - B_5[t]\, d_{3,5}\right) + B_1[t]\, \Omega_{3,1} + B_2[t]\, \Omega_{3,2} + B_3[t]\, \Omega_{3,3} + B_4[t]\, \Omega_{3,4} + B_5[t]\, \Omega_{3,5};$$

$$dM_4 = v - M_4[t]\, q_4 + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) \left(-B_1[t]\, d_{4,1} - B_2[t]\, d_{4,2} - B_3[t]\, d_{4,3} - B_4[t]\, d_{4,4} - B_5[t]\, d_{4,5}\right) + B_1[t]\, \Omega_{4,1} + B_2[t]\, \Omega_{4,2} + B_3[t]\, \Omega_{4,3} + B_4[t]\, \Omega_{4,4} + B_5[t]\, \Omega_{4,5};$$

$$dM_5 = v - M_5[t]\, q_5 + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) \left(-B_1[t]\, d_{5,1} - B_2[t]\, d_{5,2} - B_3[t]\, d_{5,3} - B_4[t]\, d_{5,4} - B_5[t]\, d_{5,5}\right) + B_1[t]\, \Omega_{5,1} + B_2[t]\, \Omega_{5,2} + B_3[t]\, \Omega_{5,3} + B_4[t]\, \Omega_{5,4} + B_5[t]\, \Omega_{5,5};$$

```
KK = 0.2;
cc = 0.05;
qq = 0.3;
```

```
dd = 0.00015;
OM = 1;
nu = 1500;
den = 2;

tmax = 1000;
par = {
  κ1 → KK, κ2 → KK, κ3 → KK, κ4 → KK, κ5 → KK,

  c1,1 → cc Net[[1]][[1]], c1,2 → cc Net[[1]][[2]],
  c1,3 → cc Net[[1]][[3]], c1,4 → cc Net[[1]][[4]], c1,5 → cc Net[[1]][[5]],
  c2,1 → cc Net[[2]][[1]], c2,2 → cc Net[[2]][[2]], c2,3 → cc Net[[2]][[3]],
  c2,4 → cc Net[[2]][[4]], c2,5 → cc Net[[2]][[5]],
  c3,1 → cc Net[[3]][[1]], c3,2 → cc Net[[3]][[2]], c3,3 → cc Net[[3]][[3]],
  c3,4 → cc Net[[3]][[4]], c3,5 → cc Net[[3]][[5]],
  c4,1 → cc Net[[4]][[1]], c4,2 → cc Net[[4]][[2]], c4,3 → cc Net[[4]][[3]],
  c4,4 → cc Net[[4]][[4]], c4,5 → cc Net[[4]][[5]],
  c5,1 → cc Net[[5]][[1]], c5,2 → cc Net[[5]][[2]], c5,3 → cc Net[[5]][[3]],
  c5,4 → cc Net[[5]][[4]], c5,5 → cc Net[[5]][[5]],

  q1 → qq, q2 → qq, q3 → qq, q4 → qq, q5 → qq,

  d1,1 → dd, d1,2 → dd, d1,3 → dd, d1,4 → dd, d1,5 → dd,
  d2,1 → dd, d2,2 → dd, d2,3 → dd, d2,4 → dd, d2,5 → dd,
  d3,1 → dd, d3,2 → dd, d3,3 → dd, d3,4 → dd, d3,5 → dd,
  d4,1 → dd, d4,2 → dd, d4,3 → dd, d4,4 → dd, d4,5 → dd,
  d5,1 → dd, d5,2 → dd, d5,3 → dd, d5,4 → dd, d5,5 → dd,

  Ω1,1 → OM Net[[1]][[1]], Ω1,2 → OM Net[[1]][[2]],
  Ω1,3 → OM Net[[1]][[3]], Ω1,4 → OM Net[[1]][[4]], Ω1,5 → OM Net[[1]][[5]],
  Ω2,1 → OM Net[[2]][[1]], Ω2,2 → OM Net[[2]][[2]], Ω2,3 → OM Net[[2]][[3]],
  Ω2,4 → OM Net[[2]][[4]], Ω2,5 → OM Net[[2]][[5]],
  Ω3,1 → OM Net[[3]][[1]], Ω3,2 → OM Net[[3]][[2]], Ω3,3 → OM Net[[3]][[3]],
  Ω3,4 → OM Net[[3]][[4]], Ω3,5 → OM Net[[3]][[5]],
  Ω4,1 → OM Net[[4]][[1]], Ω4,2 → OM Net[[4]][[2]], Ω4,3 → OM Net[[4]][[3]],
  Ω4,4 → OM Net[[4]][[4]], Ω4,5 → OM Net[[4]][[5]],
  Ω5,1 → OM Net[[5]][[1]], Ω5,2 → OM Net[[5]][[2]], Ω5,3 → OM Net[[5]][[3]],
  Ω5,4 → OM Net[[5]][[4]], Ω5,5 → OM Net[[5]][[5]],
  nuK → nu,
  denK → den


 };
```

```
B10 = 1500;
B20 = 1500;
B30 = 1500;
B40 = 1500;
B50 = 1500;
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;


sol =
 NDSolve[
  {
    B1'[t] == dB1,
    B2'[t] == dB2,
    B3'[t] == dB3,
    B4'[t] == dB4,
    B5'[t] == dB5,

    M1'[t] == dM1,
    M2'[t] == dM2,
    M3'[t] == dM3,
    M4'[t] == dM4,
    M5'[t] == dM5,



    B1[0] == B10,
    B2[0] == B20,
    B3[0] == B30,
    B4[0] == B40,
    B5[0] == B50,
    M1[0] == M10,
    M2[0] == M20,
    M3[0] == M30,
    M4[0] == M40,
    M5[0] == M50

   } /. par,
  {B1, B2, B3, B4, B5, M1, M2, M3, M4, M5},
  {t, 0, tmax}];
```

```
  {B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax],
     M1[tmax], M2[tmax], M3[tmax], M4[tmax], M5[tmax]} /. sol /. par

  (*Min[{B1[tmax],B2[tmax],B3[tmax],B4[tmax],B5[tmax]}/.sol/.par]*)

 )
```

As an example let's take the following Network

```
In[7240]:= NetK = {
    {0, 1, 0, 1, 0},
    {1, 0, 1, 1, 0},
    {1, 0, 1, 0, 1},
    {0, 1, 0, 1, 0},
    {0, 0, 0, 0, 1}
   };

In[7241]:= fNewSaitoOPsyK[NetK, 0]
           fNewSaitoOPsyK[NetK, 1]

Out[7241]= {{6662.18, 6661.93, 6661.93, 6662.18,
    6662.43, 22 221.5, 44 428.8, 44 429.6, 22 221.5, 15.9528}}

Out[7242]= {{6657.18, 6656.93, 6656.93, 6657.18,
    6657.43, 22 204.9, 44 395.5, 44 396.3, 22 204.9, 15.9528}}
```

The function "fNewSaito" solves the ODE system and gives the lowest microbial population size (this is used to calculate the Robustness). The function "fNewSaito" receives a network and a disturbance value as arguments.

```
In[7243]:= fNewSaitoOPsy[Net_, Dh_] := (
```

$$dB_1 = B_1[t]\left(-B_1[t]\,\varkappa_1 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - \left(c_{1,1} + c_{1,2} + c_{1,3} + c_{1,4} + c_{1,5} + Dh\right) B_1[t];$$

$$dB_2 = B_2[t]\left(-B_2[t]\,\varkappa_2 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - \left(c_{2,1} + c_{2,2} + c_{2,3} + c_{2,4} + c_{2,5} + Dh\right) B_2[t];$$

$$dB_3 = B_3[t]\left(-B_3[t]\,\varkappa_3 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} *\right.$$

$$\frac{M_4[t]}{\text{denK} + M_4[t]} * \frac{M_5[t]}{\text{denK} + M_5[t]}\Bigg) - \left(c_{3,1} + c_{3,2} + c_{3,3} + c_{3,4} + c_{3,5} + \text{Dh}\right) B_3[t];$$

$$dB_4 = B_4[t] \left(-B_4[t]\, \varkappa_4 + \text{nuK} * \frac{M_1[t]}{\text{denK} + M_1[t]} * \frac{M_2[t]}{\text{denK} + M_2[t]} * \frac{M_3[t]}{\text{denK} + M_3[t]} * \frac{M_4[t]}{\text{denK} + M_4[t]} * \frac{M_5[t]}{\text{denK} + M_5[t]}\right) - \left(c_{4,1} + c_{4,2} + c_{4,3} + c_{4,4} + c_{4,5} + \text{Dh}\right) B_4[t];$$

$$dB_5 = B_5[t] \left(-B_5[t]\, \varkappa_5 + \text{nuK} * \frac{M_1[t]}{\text{denK} + M_1[t]} * \frac{M_2[t]}{\text{denK} + M_2[t]} * \frac{M_3[t]}{\text{denK} + M_3[t]} * \frac{M_4[t]}{\text{denK} + M_4[t]} * \frac{M_5[t]}{\text{denK} + M_5[t]}\right) - \left(c_{5,1} + c_{5,2} + c_{5,3} + c_{5,4} + c_{5,5} + \text{Dh}\right) B_5[t];$$

$$dM_1 = v - M_1[t]\, q_1 + \left(\text{nuK} * \frac{M_1[t]}{\text{denK} + M_1[t]} * \frac{M_2[t]}{\text{denK} + M_2[t]} * \frac{M_3[t]}{\text{denK} + M_3[t]} * \frac{M_4[t]}{\text{denK} + M_4[t]} * \frac{M_5[t]}{\text{denK} + M_5[t]}\right) \left(-B_1[t]\, d_{1,1} - B_2[t]\, d_{1,2} - B_3[t]\, d_{1,3} - B_4[t]\, d_{1,4} - B_5[t]\, d_{1,5}\right) + B_1[t]\, \Omega_{1,1} + B_2[t]\, \Omega_{1,2} + B_3[t]\, \Omega_{1,3} + B_4[t]\, \Omega_{1,4} + B_5[t]\, \Omega_{1,5};$$

$$dM_2 = v - M_2[t]\, q_2 + \left(\text{nuK} * \frac{M_1[t]}{\text{denK} + M_1[t]} * \frac{M_2[t]}{\text{denK} + M_2[t]} * \frac{M_3[t]}{\text{denK} + M_3[t]} * \frac{M_4[t]}{\text{denK} + M_4[t]} * \frac{M_5[t]}{\text{denK} + M_5[t]}\right) \left(-B_1[t]\, d_{2,1} - B_2[t]\, d_{2,2} - B_3[t]\, d_{2,3} - B_4[t]\, d_{2,4} - B_5[t]\, d_{2,5}\right) + B_1[t]\, \Omega_{2,1} + B_2[t]\, \Omega_{2,2} + B_3[t]\, \Omega_{2,3} + B_4[t]\, \Omega_{2,4} + B_5[t]\, \Omega_{2,5};$$

$$dM_3 = v - M_3[t]\, q_3 + \left(\text{nuK} * \frac{M_1[t]}{\text{denK} + M_1[t]} * \frac{M_2[t]}{\text{denK} + M_2[t]} * \frac{M_3[t]}{\text{denK} + M_3[t]} * \frac{M_4[t]}{\text{denK} + M_4[t]} * \frac{M_5[t]}{\text{denK} + M_5[t]}\right) \left(-B_1[t]\, d_{3,1} - B_2[t]\, d_{3,2} - B_3[t]\, d_{3,3} - B_4[t]\, d_{3,4} - B_5[t]\, d_{3,5}\right) + B_1[t]\, \Omega_{3,1} + B_2[t]\, \Omega_{3,2} + B_3[t]\, \Omega_{3,3} + B_4[t]\, \Omega_{3,4} + B_5[t]\, \Omega_{3,5};$$

$$dM_4 = v - M_4[t]\, q_4 + \left(\text{nuK} * \frac{M_1[t]}{\text{denK} + M_1[t]} * \frac{M_2[t]}{\text{denK} + M_2[t]} * \frac{M_3[t]}{\text{denK} + M_3[t]} * \frac{M_4[t]}{\text{denK} + M_4[t]} * \frac{M_5[t]}{\text{denK} + M_5[t]}\right) \left(-B_1[t]\, d_{4,1} - B_2[t]\, d_{4,2} - B_3[t]\, d_{4,3} - B_4[t]\, d_{4,4} - B_5[t]\, d_{4,5}\right) + B_1[t]\, \Omega_{4,1} + B_2[t]\, \Omega_{4,2} + B_3[t]\, \Omega_{4,3} + B_4[t]\, \Omega_{4,4} + B_5[t]\, \Omega_{4,5};$$

$$dM_5 = v - M_5[t]\, q_5 + \left(\text{nuK} * \frac{M_1[t]}{\text{denK} + M_1[t]} * \frac{M_2[t]}{\text{denK} + M_2[t]} * \frac{M_3[t]}{\text{denK} + M_3[t]} * \frac{M_4[t]}{\text{denK} + M_4[t]} * \frac{M_5[t]}{\text{denK} + M_5[t]}\right) \left(-B_1[t]\, d_{5,1} - B_2[t]\, d_{5,2} - B_3[t]\, d_{5,3} - B_4[t]\, d_{5,4} - B_5[t]\, d_{5,5}\right) + B_1[t]\, \Omega_{5,1} + B_2[t]\, \Omega_{5,2} + B_3[t]\, \Omega_{5,3} + B_4[t]\, \Omega_{5,4} + B_5[t]\, \Omega_{5,5};$$

```
KK = 0.2;
cc = 0.05;
qq = 0.3;
dd = 0.00015;
OM = 1;
nu = 1500;
den = 2;
```

```
tmax = 1000;
par = {
  κ1 → KK, κ2 → KK, κ3 → KK, κ4 → KK, κ5 → KK,

  c1,1 → cc Net[[1]][[1]], c1,2 → cc Net[[1]][[2]],
  c1,3 → cc Net[[1]][[3]], c1,4 → cc Net[[1]][[4]], c1,5 → cc Net[[1]][[5]],
  c2,1 → cc Net[[2]][[1]], c2,2 → cc Net[[2]][[2]], c2,3 → cc Net[[2]][[3]],
  c2,4 → cc Net[[2]][[4]], c2,5 → cc Net[[2]][[5]],
  c3,1 → cc Net[[3]][[1]], c3,2 → cc Net[[3]][[2]], c3,3 → cc Net[[3]][[3]],
  c3,4 → cc Net[[3]][[4]], c3,5 → cc Net[[3]][[5]],
  c4,1 → cc Net[[4]][[1]], c4,2 → cc Net[[4]][[2]], c4,3 → cc Net[[4]][[3]],
  c4,4 → cc Net[[4]][[4]], c4,5 → cc Net[[4]][[5]],
  c5,1 → cc Net[[5]][[1]], c5,2 → cc Net[[5]][[2]], c5,3 → cc Net[[5]][[3]],
  c5,4 → cc Net[[5]][[4]], c5,5 → cc Net[[5]][[5]],

  q1 → qq, q2 → qq, q3 → qq, q4 → qq, q5 → qq,

  d1,1 → dd, d1,2 → dd, d1,3 → dd, d1,4 → dd, d1,5 → dd,
  d2,1 → dd, d2,2 → dd, d2,3 → dd, d2,4 → dd, d2,5 → dd,
  d3,1 → dd, d3,2 → dd, d3,3 → dd, d3,4 → dd, d3,5 → dd,
  d4,1 → dd, d4,2 → dd, d4,3 → dd, d4,4 → dd, d4,5 → dd,
  d5,1 → dd, d5,2 → dd, d5,3 → dd, d5,4 → dd, d5,5 → dd,

  Ω1,1 → OM Net[[1]][[1]], Ω1,2 → OM Net[[1]][[2]],
  Ω1,3 → OM Net[[1]][[3]], Ω1,4 → OM Net[[1]][[4]], Ω1,5 → OM Net[[1]][[5]],
  Ω2,1 → OM Net[[2]][[1]], Ω2,2 → OM Net[[2]][[2]], Ω2,3 → OM Net[[2]][[3]],
  Ω2,4 → OM Net[[2]][[4]], Ω2,5 → OM Net[[2]][[5]],
  Ω3,1 → OM Net[[3]][[1]], Ω3,2 → OM Net[[3]][[2]], Ω3,3 → OM Net[[3]][[3]],
  Ω3,4 → OM Net[[3]][[4]], Ω3,5 → OM Net[[3]][[5]],
  Ω4,1 → OM Net[[4]][[1]], Ω4,2 → OM Net[[4]][[2]], Ω4,3 → OM Net[[4]][[3]],
  Ω4,4 → OM Net[[4]][[4]], Ω4,5 → OM Net[[4]][[5]],
  Ω5,1 → OM Net[[5]][[1]], Ω5,2 → OM Net[[5]][[2]], Ω5,3 → OM Net[[5]][[3]],
  Ω5,4 → OM Net[[5]][[4]], Ω5,5 → OM Net[[5]][[5]],
  nuK → nu,
  denK → den


 };


B10 = 1500;
B20 = 1500;
B30 = 1500;
```

```
B40 = 1500;
B50 = 1500;
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;


sol =
 NDSolve[
  {
    B₁'[t] == dB₁,
    B₂'[t] == dB₂,
    B₃'[t] == dB₃,
    B₄'[t] == dB₄,
    B₅'[t] == dB₅,

    M₁'[t] == dM₁,
    M₂'[t] == dM₂,
    M₃'[t] == dM₃,
    M₄'[t] == dM₄,
    M₅'[t] == dM₅,



    B₁[0] == B10,
    B₂[0] == B20,
    B₃[0] == B30,
    B₄[0] == B40,
    B₅[0] == B50,
    M₁[0] == M10,
    M₂[0] == M20,
    M₃[0] == M30,
    M₄[0] == M40,
    M₅[0] == M50

   } /. par,
  {B₁, B₂, B₃, B₄, B₅, M₁, M₂, M₃, M₄, M₅},
  {t, 0, tmax}];

{B₁[tmax], B₂[tmax], B₃[tmax], B₄[tmax], B₅[tmax],
   M₁[tmax], M₂[tmax], M₃[tmax], M₄[tmax], M₅[tmax]} /. sol /. par;

Min[{B₁[tmax], B₂[tmax], B₃[tmax], B₄[tmax], B₅[tmax]} /. sol /. par]
```

```
)
```

The function “robustnessNewSaito” uses the previous function “fNewSaito” and calculates the Robustness. The function “robustnessNewSaito” simply receives a network as an argument.

```
In[7244]:= robustnessNewSaitoOPsy[NetTop_] := (
  n1 = 1;
  n2 = 5000;
  mid = (n1 + n2) / 2;

  While[(n1 ≠ mid && n2 ≠ mid),
   (If[fNewSaitoOPsy[NetTop, mid] < 1, n2 = mid, n1 = mid];
    mid = Floor[N[(n1 + n2) / 2]];); {n1, n2, mid}]; mid

 )
```

As an example let’s take the following Network

```
In[7245]:= NetK = {
    {0, 1, 0, 1, 0},
    {1, 0, 1, 1, 0},
    {1, 0, 1, 0, 1},
    {0, 1, 0, 1, 0},
    {0, 0, 0, 0, 1}
   };
```

Using the function fNewSaito we can calculate the smallest value of a bacterial population in the community for a given disturbance vale. For example, let’s take Disturbance value 1 and 500:

```
In[7246]:= fNewSaitoOPsy[NetK, 0]
Out[7246]= 6661.93

In[7247]:= fNewSaitoOPsy[NetK, 500]
Out[7247]= 4159.54
```

Using the function fNewSaito we can calculate Robustness of the Network:

```
In[7248]:= robustnessNewSaitoOPsy[NetK]
Out[7248]= 925
```

We can calculate the (Relative) Entropy and the Assortativity:

In[7249]:=
```
RelatEntrop5[NetK]
```
Out[7249]=
```
0.960956
```

In[7250]:=
```
assortativity[NetK]
```
Out[7250]=
```
-0.113228
```

We can calculate the robustness of the previously generated random networks with different number of auxotrophies:

In[7251]:=
```
AuxoComm6OPsy = Parallelize[robustnessNewSaitoOPsy /@ hk6];
AuxoComm7OPsy = Parallelize[robustnessNewSaitoOPsy /@ hk7];
AuxoComm8OPsy = Parallelize[robustnessNewSaitoOPsy /@ hk8];
AuxoComm9OPsy = Parallelize[robustnessNewSaitoOPsy /@ hk9];
AuxoComm10OPsy = Parallelize[robustnessNewSaitoOPsy /@ hk10];
AuxoComm11OPsy = Parallelize[robustnessNewSaitoOPsy /@ hk11];
AuxoComm12OPsy = Parallelize[robustnessNewSaitoOPsy /@ hk12];
AuxoComm13OPsy = Parallelize[robustnessNewSaitoOPsy /@ hk13];
AuxoComm14OPsy = Parallelize[robustnessNewSaitoOPsy /@ hk14];
AuxoComm15OPsy = Parallelize[robustnessNewSaitoOPsy /@ hk15];
AuxoComm16OPsy = Parallelize[robustnessNewSaitoOPsy /@ hk16];
AuxoComm17OPsy = Parallelize[robustnessNewSaitoOPsy /@ hk17];
```

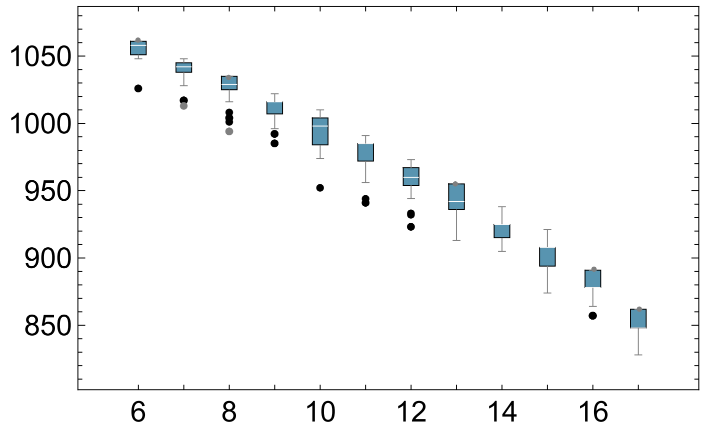
In[7263]:=
```
LikOPsy = {AuxoComm6OPsy, AuxoComm7OPsy, AuxoComm8OPsy, AuxoComm9OPsy,
   AuxoComm10OPsy, AuxoComm11OPsy, AuxoComm12OPsy, AuxoComm13OPsy,
   AuxoComm14OPsy, AuxoComm15OPsy, AuxoComm16OPsy, AuxoComm17OPsy};
```

In[∘]:=
```
coco = RGBColor[0.34509803921568627, 0.5803921568627451, 0.6901960784313725]
```
Out[∘]=

In[7265]:=

```
BoxWhiskerChart[LikOPsy, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{coco}}, Frame → True,
 ChartLabels → {"6", "", "8", "", "10", "", "12", "", "14", "", "16", ""},
 BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15]]
```

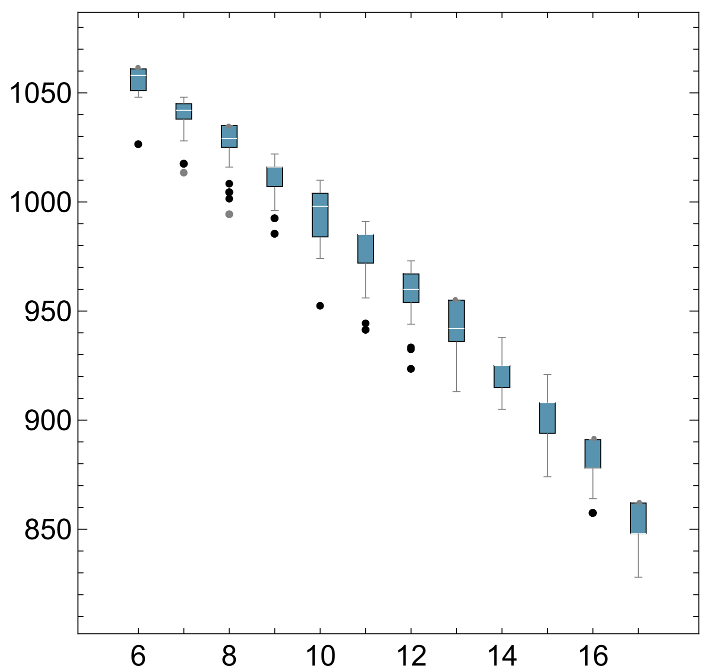
Out[7265]=



In[7266]:=

```
BoxWhiskerChart[LikOPsy, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{coco}}, Frame → True,
 ChartLabels → {"6", "", "8", "", "10", "", "12", "", "14", "", "16", ""},
 BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15], AspectRatio → 1]
```

Out[7266]=



In[7267]:=
```
AuxoComm7OPsy
```

Out[7267]=
```
{1038, 1038, 1048, 1038, 1038, 1038, 1014, 1048, 1018, 1018, 1048, 1045,
 1038, 1035, 1045, 1038, 1048, 1048, 1045, 1038, 1035, 1045, 1045, 1035,
 1048, 1038, 1045, 1028, 1035, 1028, 1038, 1045, 1045, 1018, 1048, 1048,
 1048, 1045, 1038, 1045, 1048, 1045, 1035, 1045, 1045, 1048, 1035, 1048,
 1045, 1045, 1045, 1048, 1038, 1048, 1038, 1048, 1045, 1038, 1045, 1035, 1038,
 1042, 1038, 1038, 1048, 1038, 1045, 1048, 1048, 1048, 1038, 1048, 1038, 1048,
 1035, 1038, 1042, 1045, 1038, 1042, 1038, 1038, 1018, 1045, 1048, 1038, 1038,
 1038, 1042, 1045, 1045, 1028, 1048, 1028, 1038, 1048, 1045, 1018, 1038, 1038}
```

We can study the correlation between Relative entropy and assortativity with Robustness for Networks with 7 auxotrophies.

In[∘]:=
```
Entropy7 = RelatEntrop5 /@ hk7;
```

In[∘]:=
```
Assort7 = assortativity /@ hk7;
```

In[7268]:=
```
RobustNewSaito7bOPsy = AuxoComm7OPsy;
```

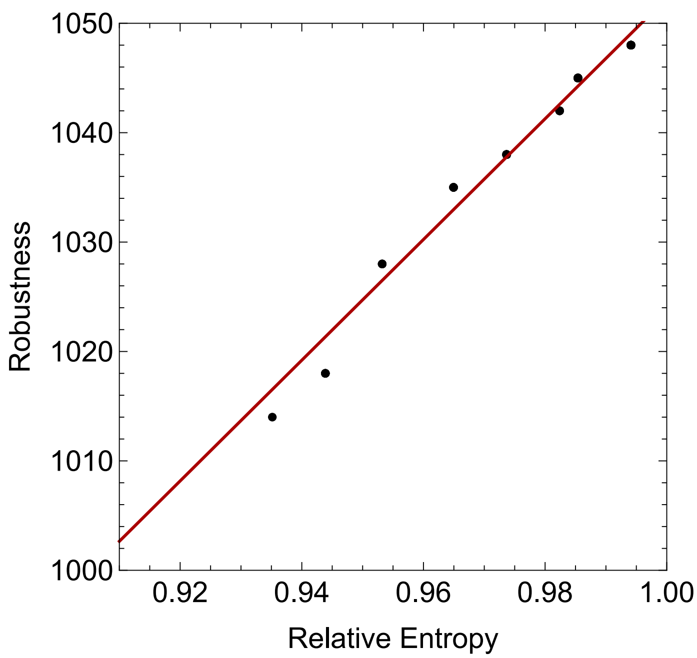
```
Length[Entropy7]
Length[Assort7]
Length[RobustNewSaito7bOPsy]
```

Out[∘]= 100

Out[∘]= 100

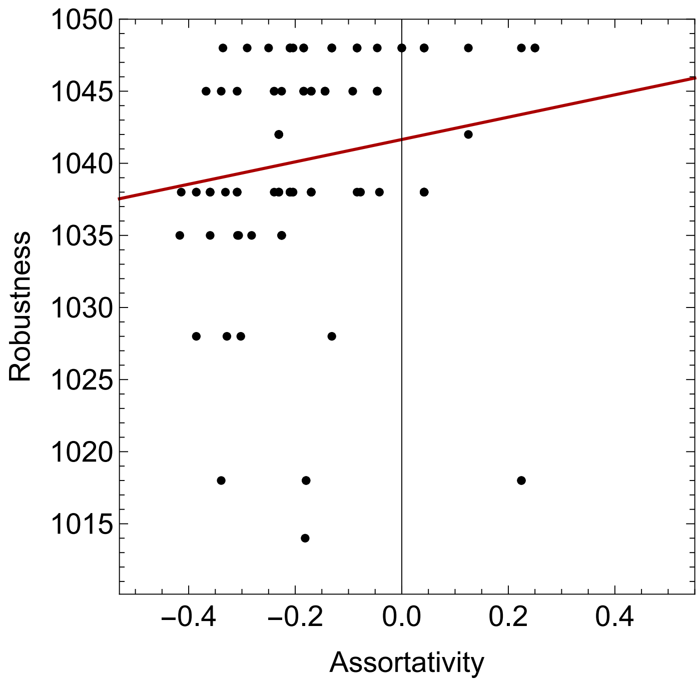
Out[∘]= 100

In[•]:=
```
{Min[Entropy7], Max[Entropy7]}
{Min[Assort7], Max[Assort7]}
```
Out[•]=
```
{0.935154, 0.994118}
```
Out[•]=
```
{-0.416667, 0.25}
```
In[•]:=
```
Position[Entropy7, Min[Entropy7]]
```
Out[•]=
```
{{7}}
```
In[7269]:=
```
RobustNewSaito7bOPsy[[#]] & /@ {1, 2, 24}
```
Out[7269]=
```
{1038, 1038, 1035}
```
In[7270]:=
```
{Min[RobustNewSaito7bOPsy], {Max[RobustNewSaito7bOPsy]}}
```
Out[7270]=
```
{1014, {1048}}
```
In[7271]:=
```
linerobustnessNewSaito25OPsy =
  Fit[Partition[Riffle[Entropy7, RobustNewSaito7bOPsy], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Entropy7, RobustNewSaito7bOPsy], {2}],
  Frame → True, FrameLabel → {"Relative Entropy", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]}, PlotRange → {{0.91, 1}, {1000, 1050}},
  AspectRatio → 0.5], Plot[linerobustnessNewSaito25OPsy, {x, 0.91, 1},
  AspectRatio → 0.5, PlotStyle → Darker[Red]], AspectRatio → 1]
```
Out[7272]=



In[7538]:=
```
lineAssoRobrobustnessNewSaito25OPsy =
  Fit[Partition[Riffle[Assort7, RobustNewSaito7bOPsy], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Assort7, RobustNewSaito7bOPsy], {2}],
  Frame → True, FrameLabel → {"Assortativity", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]},
  PlotRange → {{-0.53, 0.55}, Automatic}, AspectRatio → 0.5],
 Plot[lineAssoRobrobustnessNewSaito25OPsy, {x, -0.53, 0.55},
  AspectRatio → 0.5, PlotStyle → Darker[Red]], AspectRatio → 1]
```

Out[7539]=



In[7275]:=
```
SpearmanRankTest[Entropy7, RobustNewSaito7bOPsy, "TestDataTable"]
```

Out[7275]=

	Statistic	P-Value
Spearman Rank	1.	0.

In[7276]:=
```
SpearmanRankTest[Assort7, RobustNewSaito7bOPsy, "TestDataTable"]
```

Out[7276]=

	Statistic	P-Value
Spearman Rank	0.351095	0.000341587

Solving the system of ODE with Overproduction

In[7277]:=

$$\text{fNewSaitoOVxOPsy[Net\_, Dh\_, coop\_]} := \Big($$

$$dB_1 = B_1[t]\left(-B_1[t]\,\kappa_1 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - \left(c_{1,1} + c_{1,2} + c_{1,3} + c_{1,4} + c_{1,5} + Dh\right) B_1[t];$$

$$dB_2 = B_2[t]\left(-B_2[t]\,\kappa_2 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - \left(c_{2,1} + c_{2,2} + c_{2,3} + c_{2,4} + c_{2,5} + Dh\right) B_2[t];$$

$$dB_3 = B_3[t]\left(-B_3[t]\,\kappa_3 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - \left(c_{3,1} + c_{3,2} + c_{3,3} + c_{3,4} + c_{3,5} + Dh\right) B_3[t];$$

$$dB_4 = B_4[t]\left(-B_4[t]\,\kappa_4 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - \left(c_{4,1} + c_{4,2} + c_{4,3} + c_{4,4} + c_{4,5} + Dh\right) B_4[t];$$

$$dB_5 = B_5[t]\left(-B_5[t]\,\kappa_5 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - \left(c_{5,1} + c_{5,2} + c_{5,3} + c_{5,4} + c_{5,5} + Dh\right) B_5[t];$$

$$dM_1 = v - M_1[t]\,q_1 + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) \left(-B_1[t]\,d_{1,1} - B_2[t]\,d_{1,2} - B_3[t]\,d_{1,3} - B_4[t]\,d_{1,4} - B_5[t]\,d_{1,5}\right) + B_1[t]\,\Omega_{1,1} + B_2[t]\,\Omega_{1,2} + B_3[t]\,\Omega_{1,3} + B_4[t]\,\Omega_{1,4} + B_5[t]\,\Omega_{1,5};$$

$$dM_2 = v - M_2[t]\,q_2 + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) \left(-B_1[t]\,d_{2,1} - B_2[t]\,d_{2,2} - B_3[t]\,d_{2,3} - B_4[t]\,d_{2,4} - B_5[t]\,d_{2,5}\right) + B_1[t]\,\Omega_{2,1} + B_2[t]\,\Omega_{2,2} + B_3[t]\,\Omega_{2,3} + B_4[t]\,\Omega_{2,4} + B_5[t]\,\Omega_{2,5};$$

$$dM_3 = v - M_3[t]\,q_3 + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) \left(-B_1[t]\,d_{3,1} - B_2[t]\,d_{3,2} - B_3[t]\,d_{3,3} - B_4[t]\,d_{3,4} - B_5[t]\,d_{3,5}\right) + B_1[t]\,\Omega_{3,1} + B_2[t]\,\Omega_{3,2} + B_3[t]\,\Omega_{3,3} + B_4[t]\,\Omega_{3,4} + B_5[t]\,\Omega_{3,5};$$

```
dM4 = v - M4[t] q4 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) (-B1[t] d4,1 - B2[t] d4,2 - B3[t] d4,3 - B4[t] d4,4 - B5[t] d4,5) +
   B1[t] Ω4,1 + B2[t] Ω4,2 + B3[t] Ω4,3 + B4[t] Ω4,4 + B5[t] Ω4,5;
dM5 = v - M5[t] q5 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) (-B1[t] d5,1 - B2[t] d5,2 - B3[t] d5,3 - B4[t] d5,4 - B5[t] d5,5) +
   B1[t] Ω5,1 + B2[t] Ω5,2 + B3[t] Ω5,3 + B4[t] Ω5,4 + B5[t] Ω5,5;

KK = 0.2;
cc = 0.05;
qq = 0.3;
dd = 0.00015;
OM = 1;
nu = 1500;
den = 2;



op = coop; (*Number of links with overExpression*)
posNe = Position[Net, 1];
(*Positions in the matrix where there are links (=1)*)
RaN = RandomSample[posNe, op];
(*Random sample of op links that will be overproduced*)

costincr = 1.3; (*Term multiplying the cost link*)
overprodincr = 1.15;
(*Term multiplying the overproduction link*)



NewNetCost = Net cc;
Table[NewNetCost[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] =
   NewNetCost[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] * costincr, {i, Length[RaN]}];

NewNetOvProd = Net OM;
Table[NewNetOvProd[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] =
   NewNetOvProd[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] * overprodincr, {i,
   Length[RaN]}];



tmax = 1000;
par = {
   κ1 → KK, κ2 → KK, κ3 → KK, κ4 → KK, κ5 → KK,
```

```
c[1,1] → NewNetCost[[1]][[1]],
c[1,2] → NewNetCost[[1]][[2]], c[1,3] → NewNetCost[[1]][[3]],
c[1,4] → NewNetCost[[1]][[4]], c[1,5] → NewNetCost[[1]][[5]],
c[2,1] → NewNetCost[[2]][[1]], c[2,2] → NewNetCost[[2]][[2]],
c[2,3] → NewNetCost[[2]][[3]], c[2,4] → NewNetCost[[2]][[4]],
c[2,5] → NewNetCost[[2]][[5]],
c[3,1] → NewNetCost[[3]][[1]], c[3,2] → NewNetCost[[3]][[2]],
c[3,3] → NewNetCost[[3]][[3]], c[3,4] → NewNetCost[[3]][[4]],
c[3,5] → NewNetCost[[3]][[5]],
c[4,1] → NewNetCost[[4]][[1]], c[4,2] → NewNetCost[[4]][[2]],
c[4,3] → NewNetCost[[4]][[3]], c[4,4] → NewNetCost[[4]][[4]],
c[4,5] → NewNetCost[[4]][[5]],
c[5,1] → NewNetCost[[5]][[1]], c[5,2] → NewNetCost[[5]][[2]],
c[5,3] → NewNetCost[[5]][[3]], c[5,4] → NewNetCost[[5]][[4]],
c[5,5] → NewNetCost[[5]][[5]],

q1 → qq, q2 → qq, q3 → qq, q4 → qq, q5 → qq,

d[1,1] → dd, d[1,2] → dd, d[1,3] → dd, d[1,4] → dd, d[1,5] → dd,
d[2,1] → dd, d[2,2] → dd, d[2,3] → dd, d[2,4] → dd, d[2,5] → dd,
d[3,1] → dd, d[3,2] → dd, d[3,3] → dd, d[3,4] → dd, d[3,5] → dd,
d[4,1] → dd, d[4,2] → dd, d[4,3] → dd, d[4,4] → dd, d[4,5] → dd,
d[5,1] → dd, d[5,2] → dd, d[5,3] → dd, d[5,4] → dd, d[5,5] → dd,

Ω[1,1] → NewNetOvProd[[1]][[1]],
Ω[1,2] → NewNetOvProd[[1]][[2]], Ω[1,3] → NewNetOvProd[[1]][[3]],
Ω[1,4] → NewNetOvProd[[1]][[4]], Ω[1,5] → NewNetOvProd[[1]][[5]],
Ω[2,1] → NewNetOvProd[[2]][[1]], Ω[2,2] → NewNetOvProd[[2]][[2]],
Ω[2,3] → NewNetOvProd[[2]][[3]], Ω[2,4] → NewNetOvProd[[2]][[4]],
Ω[2,5] → NewNetOvProd[[2]][[5]],
Ω[3,1] → NewNetOvProd[[3]][[1]], Ω[3,2] → NewNetOvProd[[3]][[2]],
Ω[3,3] → NewNetOvProd[[3]][[3]], Ω[3,4] → NewNetOvProd[[3]][[4]],
Ω[3,5] → NewNetOvProd[[3]][[5]],
Ω[4,1] → NewNetOvProd[[4]][[1]], Ω[4,2] → NewNetOvProd[[4]][[2]],
Ω[4,3] → NewNetOvProd[[4]][[3]], Ω[4,4] → NewNetOvProd[[4]][[4]],
Ω[4,5] → NewNetOvProd[[4]][[5]],
Ω[5,1] → NewNetOvProd[[5]][[1]], Ω[5,2] → NewNetOvProd[[5]][[2]],
Ω[5,3] → NewNetOvProd[[5]][[3]], Ω[5,4] → NewNetOvProd[[5]][[4]],
Ω[5,5] → NewNetOvProd[[5]][[5]],
nuK → nu,
denK → den
```

```
 };


B10 = 1500;
B20 = 1500;
B30 = 1500;
B40 = 1500;
B50 = 1500;
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;


sol =
 NDSolve[
  {
    B1'[t] == dB1,
    B2'[t] == dB2,
    B3'[t] == dB3,
    B4'[t] == dB4,
    B5'[t] == dB5,

    M1'[t] == dM1,
    M2'[t] == dM2,
    M3'[t] == dM3,
    M4'[t] == dM4,
    M5'[t] == dM5,


    B1[0] == B10,
    B2[0] == B20,
    B3[0] == B30,
    B4[0] == B40,
    B5[0] == B50,
    M1[0] == M10,
    M2[0] == M20,
    M3[0] == M30,
    M4[0] == M40,
    M5[0] == M50

   } /. par,
  {B1, B2, B3, B4, B5, M1, M2, M3, M4, M5},
```

```
    {t, 0, tmax}];

  {B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax],
     M1[tmax], M2[tmax], M3[tmax], M4[tmax], M5[tmax]} /. sol /. par;

  Min[{B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax]} /. sol /. par]

 )
```

In[7278]:=
```
robustnessNewSaitoOVxOPsy[NetTop_, coop_] := (
  n1 = 1;
  n2 = 5000;
  mid = (n1 + n2) / 2;

  While[(n1 ≠ mid && n2 ≠ mid),
   (If[fNewSaitoOVxOPsy[NetTop, mid, coop] < 1, n2 = mid, n1 = mid];
    mid = Floor[N[(n1 + n2) / 2]];); {n1, n2, mid}]; mid

 )
```

In[∘]:=
```
NetK = {
   {0, 1, 0, 1, 0},
   {1, 0, 1, 1, 0},
   {1, 0, 1, 0, 1},
   {0, 1, 0, 1, 0},
   {0, 0, 0, 0, 1}
  };
```

In[7279]:=
```
fNewSaitoOPsy[NetK, 0]
```

Out[7279]= 6661.93

In[7280]:=
```
fNewSaitoOVxOPsy[NetK, 0, 10]
```

Out[7280]= 7476.43

In[7281]:=
```
robustnessNewSaitoOPsy[NetK]
```
Out[7281]=
```
925
```

In[7282]:=
```
robustnessNewSaitoOVxOPsy[NetK, 10]
```
Out[7282]=
```
951
```

In[7283]:=
```
AuxoComm8OPsy
```
Out[7283]=
```
{1025, 1019, 1025, 1035, 1016, 1019, 1005, 1035, 1035, 1025, 1029, 1029,
 1016, 1029, 1032, 1025, 1025, 1019, 1029, 1009, 1025, 1025, 1029, 1025,
 1025, 1035, 1029, 1005, 1035, 1029, 1029, 1005, 1029, 1035, 1035, 1035,
 1025, 1032, 1035, 1035, 1025, 1035, 1032, 1035, 1035, 1029, 1025, 1025,
 1035, 1025, 1025, 1029, 1025, 995, 1035, 1029, 1002, 1035, 1035, 1029, 1025,
 1025, 1035, 1035, 1029, 1025, 1025, 1035, 1035, 1032, 1029, 1025, 1035, 1035,
 1035, 1035, 1029, 1019, 1025, 1035, 1019, 1035, 1029, 1025, 1025, 1035, 1032,
 1025, 1025, 1025, 995, 1035, 1035, 1032, 1035, 1029, 1025, 1025, 1029, 1035}
```

In[7284]:=
```
coop5to15OPsy = {Table[robustnessNewSaitoOVxOPsy[#, 5], {20}],
    Table[robustnessNewSaitoOVxOPsy[#, 10], {20}],
    Table[robustnessNewSaitoOVxOPsy[#, 15], {20}]} &;
```

In[7285]:=
```
wf8OPsy = Parallelize[coop5to15OPsy /@ hk8];
```

In[7286]:=
```
wf8NormalizedOPsy = N[wf8OPsy[[#]] / AuxoComm8OPsy[[#]]] & /@ Range[100]
```

In[7287]:=
```
wf8NormalizedWith5CoopOPsy = wf8NormalizedOPsy[[#]][[1]] & /@ Range[100]
```

In[7288]:=
```
wf8NormalizedWith10CoopOPsy = wf8NormalizedOPsy[[#]][[2]] & /@ Range[100]
```

In[7289]:=
```
wf8NormalizedWith15CoopOPsy = wf8NormalizedOPsy[[#]][[3]] & /@ Range[100]
```

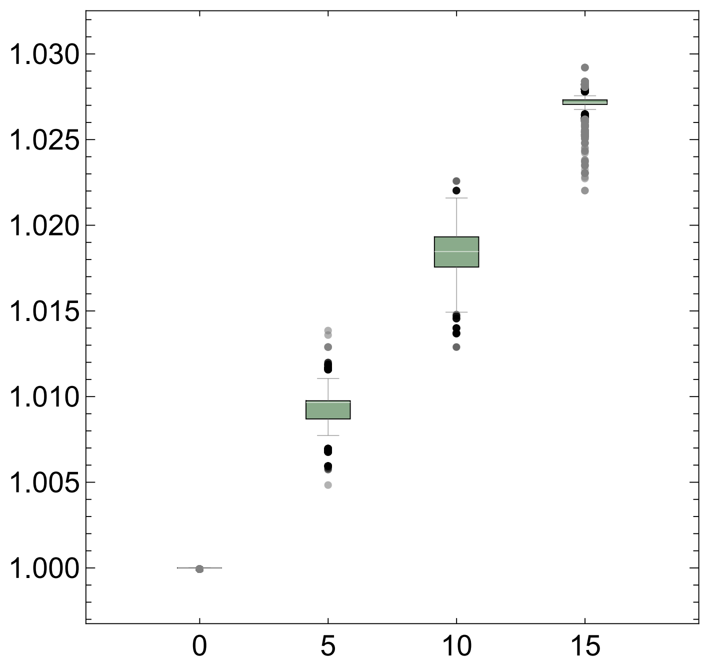
```
(*For 8 auxotrophies networks*)
```

In[7290]:=
```
allcoopWith8AuxoOPsy = {Flatten[wf8NormalizedWith5CoopOPsy],
  Flatten[wf8NormalizedWith10CoopOPsy], Flatten[wf8NormalizedWith15CoopOPsy]}
```

In[7291]:=
```
allcoopWith8AuxoPlusAuxoOPsy =
 Join[{ConstantArray[1, {2000}]}, allcoopWith8AuxoOPsy]
```

In[7292]:=
```
BoxWhiskerChart[allcoopWith8AuxoPlusAuxoOPsy, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{gree1}},
 Frame → True, ChartLabels → {"0", "5", "10", "15"}, BarSpacing → 1.9,
 FrameStyle → Directive[Black, FontSize → 15], AspectRatio → 1]
```

Out[7292]=



Solving the system of ODE Random parametrization

In[7293]:=

```
Knum = 0.2;
ccrnum = 0.05;
qqrnum = 0.3;
ddrnum = 0.00015;
OMrnum = 1;
nurum = 1500;
den2rum = 2;

corrpar0 = 10^3;
corrpar1 = 10^4;
corrpar2 = 10^6;

KKr := RandomVariate[
   GammaDistribution[ corrpar0 Sqrt[Knum], (1/corrpar0) Sqrt[Knum]], 1][[1]];
ccr := RandomVariate[GammaDistribution[ corrpar1 Sqrt[ccrnum],
     (1/corrpar1) Sqrt[ccrnum]], 1][[1]];
qqr := RandomVariate[GammaDistribution[ corrpar0 Sqrt[qqrnum],
     (1/corrpar0) Sqrt[qqrnum]], 1][[1]];
ddr := RandomVariate[GammaDistribution[ corrpar1 Sqrt[ddrnum],
     (1/corrpar1) Sqrt[ddrnum]], 1][[1]];
OMr := RandomVariate[GammaDistribution[ corrpar0 Sqrt[OMrnum],
     (1/corrpar0) Sqrt[OMrnum]], 1][[1]];
nur := (*nurum*) RandomVariate[GammaDistribution[
     corrpar2 Sqrt[nurum], (1/corrpar2) Sqrt[nurum]], 1][[1]] ;
denr2 := (*den2rum*) RandomVariate[GammaDistribution[
     corrpar2 Sqrt[den2rum], (1/corrpar2) Sqrt[den2rum]], 1][[1]] ;


parR = Join[Table[KKr, {5}], Table[ccr, {25}],
  Table[qqr, {5}], Table[ddr, {25}], Table[OMr, {25}], {nur}, {denr2}]
```

Out[7310]= {0.190762, 0.180908, 0.208167, 0.193426, 0.21115, 0.0496362, 0.050543, 0.0487452, 0.0481617, 0.0491492, 0.0486716, 0.0508299, 0.0504686, 0.0525196, 0.0496127, 0.0511452, 0.050238, 0.0504502, 0.0503739, 0.0499003, 0.0516985, 0.0503392, 0.0512505, 0.0503843, 0.0498629, 0.0500577, 0.0492161, 0.0509623, 0.0500082, 0.0487811, 0.279547, 0.303377, 0.307555, 0.288028, 0.2928, 0.000155559, 0.000147002, 0.000143951, 0.000145656, 0.000154679, 0.000148017, 0.000175507, 0.000164252, 0.000162771, 0.00015076, 0.000167575, 0.000114509, 0.000145536, 0.000129764, 0.000152956, 0.00015606, 0.000134837, 0.000161698, 0.000128809, 0.00014301, 0.000157054, 0.000190466, 0.0001472, 0.000175848, 0.00013194, 0.974799, 0.983474, 1.02283, 0.987779, 1.04031, 1.01185, 0.986802, 1.00578, 1.06007, 0.965155, 1.08087, 0.975913, 1.06851, 1.01916, 0.99345, 1.00106, 0.979573, 1.0306, 1.00041, 1.01442, 0.988162, 0.99272, 0.998908, 0.990389, 0.962861, 1500.19, 1.99844}

In[7311]:= parR = %

Out[7311]= {0.190762, 0.180908, 0.208167, 0.193426, 0.21115, 0.0496362, 0.050543, 0.0487452, 0.0481617, 0.0491492, 0.0486716, 0.0508299, 0.0504686, 0.0525196, 0.0496127, 0.0511452, 0.050238, 0.0504502, 0.0503739, 0.0499003, 0.0516985, 0.0503392, 0.0512505, 0.0503843, 0.0498629, 0.0500577, 0.0492161, 0.0509623, 0.0500082, 0.0487811, 0.279547, 0.303377, 0.307555, 0.288028, 0.2928, 0.000155559, 0.000147002, 0.000143951, 0.000145656, 0.000154679, 0.000148017, 0.000175507, 0.000164252, 0.000162771, 0.00015076, 0.000167575, 0.000114509, 0.000145536, 0.000129764, 0.000152956, 0.00015606, 0.000134837, 0.000161698, 0.000128809, 0.00014301, 0.000157054, 0.000190466, 0.0001472, 0.000175848, 0.00013194, 0.974799, 0.983474, 1.02283, 0.987779, 1.04031, 1.01185, 0.986802, 1.00578, 1.06007, 0.965155, 1.08087, 0.975913, 1.06851, 1.01916, 0.99345, 1.00106, 0.979573, 1.0306, 1.00041, 1.01442, 0.988162, 0.99272, 0.998908, 0.990389, 0.962861, 1500.19, 1.99844}

In[7312]:= fNewSaitoROPsy[Net_, Dh_] := (

$$dB_1 = B_1[t]\left(-B_1[t]\,\kappa_1 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - \left(c_{1,1} + c_{1,2} + c_{1,3} + c_{1,4} + c_{1,5} + Dh\right) B_1[t];$$

$$dB_2 = B_2[t]\left(-B_2[t]\,\kappa_2 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - \left(c_{2,1} + c_{2,2} + c_{2,3} + c_{2,4} + c_{2,5} + Dh\right) B_2[t];$$

$$dB_3 = B_3[t]\left(-B_3[t]\,\kappa_3 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - \left(c_{3,1} + c_{3,2} + c_{3,3} + c_{3,4} + c_{3,5} + Dh\right) B_3[t];$$

$$dB_4 = B_4[t]\left(-B_4[t]\,\kappa_4 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} *\right.$$

$$\frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\Big) - (c_{4,1} + c_{4,2} + c_{4,3} + c_{4,4} + c_{4,5} + Dh)\, B_4[t];$$

$$dB_5 = B_5[t]\Big(-B_5[t]\,\kappa_5 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\Big) - (c_{5,1} + c_{5,2} + c_{5,3} + c_{5,4} + c_{5,5} + Dh)\, B_5[t];$$

$$dM_1 = v - M_1[t]\,q_1 + \Big(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\Big)(-B_1[t]\,d_{1,1} - B_2[t]\,d_{1,2} - B_3[t]\,d_{1,3} - B_4[t]\,d_{1,4} - B_5[t]\,d_{1,5}) + B_1[t]\,\Omega_{1,1} + B_2[t]\,\Omega_{1,2} + B_3[t]\,\Omega_{1,3} + B_4[t]\,\Omega_{1,4} + B_5[t]\,\Omega_{1,5};$$

$$dM_2 = v - M_2[t]\,q_2 + \Big(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\Big)(-B_1[t]\,d_{2,1} - B_2[t]\,d_{2,2} - B_3[t]\,d_{2,3} - B_4[t]\,d_{2,4} - B_5[t]\,d_{2,5}) + B_1[t]\,\Omega_{2,1} + B_2[t]\,\Omega_{2,2} + B_3[t]\,\Omega_{2,3} + B_4[t]\,\Omega_{2,4} + B_5[t]\,\Omega_{2,5};$$

$$dM_3 = v - M_3[t]\,q_3 + \Big(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\Big)(-B_1[t]\,d_{3,1} - B_2[t]\,d_{3,2} - B_3[t]\,d_{3,3} - B_4[t]\,d_{3,4} - B_5[t]\,d_{3,5}) + B_1[t]\,\Omega_{3,1} + B_2[t]\,\Omega_{3,2} + B_3[t]\,\Omega_{3,3} + B_4[t]\,\Omega_{3,4} + B_5[t]\,\Omega_{3,5};$$

$$dM_4 = v - M_4[t]\,q_4 + \Big(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\Big)(-B_1[t]\,d_{4,1} - B_2[t]\,d_{4,2} - B_3[t]\,d_{4,3} - B_4[t]\,d_{4,4} - B_5[t]\,d_{4,5}) + B_1[t]\,\Omega_{4,1} + B_2[t]\,\Omega_{4,2} + B_3[t]\,\Omega_{4,3} + B_4[t]\,\Omega_{4,4} + B_5[t]\,\Omega_{4,5};$$

$$dM_5 = v - M_5[t]\,q_5 + \Big(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\Big)(-B_1[t]\,d_{5,1} - B_2[t]\,d_{5,2} - B_3[t]\,d_{5,3} - B_4[t]\,d_{5,4} - B_5[t]\,d_{5,5}) + B_1[t]\,\Omega_{5,1} + B_2[t]\,\Omega_{5,2} + B_3[t]\,\Omega_{5,3} + B_4[t]\,\Omega_{5,4} + B_5[t]\,\Omega_{5,5};$$

tmax = 1000;

par = {

$\kappa_1 \to$ parR[[1]], $\kappa_2 \to$ parR[[2]], $\kappa_3 \to$ parR[[3]], $\kappa_4 \to$ parR[[4]], $\kappa_5 \to$ parR[[5]],

$c_{1,1} \to$ parR[[6]] × Net[[1]][[1]],
$c_{1,2} \to$ parR[[7]] × Net[[1]][[2]], $c_{1,3} \to$ parR[[8]] × Net[[1]][[3]],
$c_{1,4} \to$ parR[[9]] × Net[[1]][[4]], $c_{1,5} \to$ parR[[10]] × Net[[1]][[5]],
$c_{2,1} \to$ parR[[11]] × Net[[2]][[1]], $c_{2,2} \to$ parR[[12]] × Net[[2]][[2]],
$c_{2,3} \to$ parR[[13]] × Net[[2]][[3]], $c_{2,4} \to$ parR[[14]] × Net[[2]][[4]],
$c_{2,5} \to$ parR[[15]] × Net[[2]][[5]],

$c_{3,1} \to$ parR[[16]] × Net[[3]][[1]], $c_{3,2} \to$ parR[[17]] × Net[[3]][[2]],
$c_{3,3} \to$ parR[[18]] × Net[[3]][[3]], $c_{3,4} \to$ parR[[19]] × Net[[3]][[4]],
$c_{3,5} \to$ parR[[20]] × Net[[3]][[5]],
$c_{4,1} \to$ parR[[21]] × Net[[4]][[1]], $c_{4,2} \to$ parR[[22]] × Net[[4]][[2]],
$c_{4,3} \to$ parR[[23]] × Net[[4]][[3]], $c_{4,4} \to$ parR[[24]] × Net[[4]][[4]],
$c_{4,5} \to$ parR[[25]] × Net[[4]][[5]],
$c_{5,1} \to$ parR[[26]] × Net[[5]][[1]], $c_{5,2} \to$ parR[[27]] × Net[[5]][[2]],
$c_{5,3} \to$ parR[[28]] × Net[[5]][[3]], $c_{5,4} \to$ parR[[29]] × Net[[5]][[4]],
$c_{5,5} \to$ parR[[30]] × Net[[5]][[5]],

$q_1 \to$ parR[[31]], $q_2 \to$ parR[[32]],
$q_3 \to$ parR[[33]], $q_4 \to$ parR[[34]], $q_5 \to$ parR[[35]],

$d_{1,1} \to$ parR[[36]], $d_{1,2} \to$ parR[[37]],
$d_{1,3} \to$ parR[[38]], $d_{1,4} \to$ parR[[39]], $d_{1,5} \to$ parR[[40]],
$d_{2,1} \to$ parR[[41]], $d_{2,2} \to$ parR[[42]], $d_{2,3} \to$ parR[[43]],
$d_{2,4} \to$ parR[[44]], $d_{2,5} \to$ parR[[45]],
$d_{3,1} \to$ parR[[46]], $d_{3,2} \to$ parR[[47]], $d_{3,3} \to$ parR[[48]],
$d_{3,4} \to$ parR[[49]], $d_{3,5} \to$ parR[[50]],
$d_{4,1} \to$ parR[[51]], $d_{4,2} \to$ parR[[52]], $d_{4,3} \to$ parR[[53]],
$d_{4,4} \to$ parR[[54]], $d_{4,5} \to$ parR[[55]],
$d_{5,1} \to$ parR[[56]], $d_{5,2} \to$ parR[[57]], $d_{5,3} \to$ parR[[58]],
$d_{5,4} \to$ parR[[59]], $d_{5,5} \to$ parR[[60]],

$\Omega_{1,1} \to$ parR[[61]] × Net[[1]][[1]],
$\Omega_{1,2} \to$ parR[[62]] × Net[[1]][[2]], $\Omega_{1,3} \to$ parR[[63]] × Net[[1]][[3]],
$\Omega_{1,4} \to$ parR[[64]] × Net[[1]][[4]], $\Omega_{1,5} \to$ parR[[65]] × Net[[1]][[5]],
$\Omega_{2,1} \to$ parR[[66]] × Net[[2]][[1]], $\Omega_{2,2} \to$ parR[[67]] × Net[[2]][[2]],
$\Omega_{2,3} \to$ parR[[68]] × Net[[2]][[3]], $\Omega_{2,4} \to$ parR[[69]] × Net[[2]][[4]],
$\Omega_{2,5} \to$ parR[[70]] × Net[[2]][[5]],
$\Omega_{3,1} \to$ parR[[71]] × Net[[3]][[1]], $\Omega_{3,2} \to$ parR[[72]] × Net[[3]][[2]],
$\Omega_{3,3} \to$ parR[[73]] × Net[[3]][[3]], $\Omega_{3,4} \to$ parR[[74]] × Net[[3]][[4]],
$\Omega_{3,5} \to$ parR[[75]] × Net[[3]][[5]],
$\Omega_{4,1} \to$ parR[[76]] × Net[[4]][[1]], $\Omega_{4,2} \to$ parR[[77]] × Net[[4]][[2]],
$\Omega_{4,3} \to$ parR[[78]] × Net[[4]][[3]], $\Omega_{4,4} \to$ parR[[79]] × Net[[4]][[4]],
$\Omega_{4,5} \to$ parR[[80]] × Net[[4]][[5]],
$\Omega_{5,1} \to$ parR[[81]] × Net[[5]][[1]], $\Omega_{5,2} \to$ parR[[82]] × Net[[5]][[2]],
$\Omega_{5,3} \to$ parR[[83]] × Net[[5]][[3]], $\Omega_{5,4} \to$ parR[[84]] × Net[[5]][[4]],
$\Omega_{5,5} \to$ parR[[85]] × Net[[5]][[5]],
nuK $\to$ parR[[86]],
denK $\to$ parR[[87]]

```
 };



B10 = 1500;
B20 = 1500;
B30 = 1500;
B40 = 1500;
B50 = 1500;
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;


sol =
 NDSolve[
  {
    B1'[t] == dB1,
    B2'[t] == dB2,
    B3'[t] == dB3,
    B4'[t] == dB4,
    B5'[t] == dB5,

    M1'[t] == dM1,
    M2'[t] == dM2,
    M3'[t] == dM3,
    M4'[t] == dM4,
    M5'[t] == dM5,


    B1[0] == B10,
    B2[0] == B20,
    B3[0] == B30,
    B4[0] == B40,
    B5[0] == B50,
    M1[0] == M10,
    M2[0] == M20,
    M3[0] == M30,
    M4[0] == M40,
    M5[0] == M50
```

```
    } /. par,
   {B1, B2, B3, B4, B5, M1, M2, M3, M4, M5},
   {t, 0, tmax}];

  {B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax],
    M1[tmax], M2[tmax], M3[tmax], M4[tmax], M5[tmax]} /. sol /. par;

  Min[{B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax]} /. sol /. par]

 )
```

In[7313]:=
```
robustnessNewSaitoROPsy[NetTop_] := (
  n1 = 1;
  n2 = 5000;
  mid = (n1 + n2) / 2;

  While[(n1 ≠ mid && n2 ≠ mid),
   (If[fNewSaitoROPsy[NetTop, mid] < 1, n2 = mid, n1 = mid];
    mid = Floor[N[(n1 + n2) / 2]];); {n1, n2, mid}]; mid

 )
```

As an example let’s take the following Network

In[7314]:=
```
NetK = {
   {0, 1, 0, 1, 0},
   {1, 0, 1, 1, 0},
   {1, 0, 1, 0, 1},
   {0, 1, 0, 1, 0},
   {0, 0, 0, 0, 1}
  };
```

Using the function fNewSaito we can calculate the smallest value of a bacterial population in the community for a given disturbance vale. For example, let’s take Disturbance value 1 and 500:

In[7315]:=
```
fNewSaitoOPsy[NetK, 0]
```

Out[7315]= 6661.93

In[7316]:=
```
fNewSaitoOPsy[NetK, 500]
```

Out[7316]= 4159.54

In[7317]:=
```
fNewSaitoROPsy[NetK, 0]
```
Out[7317]=
```
5244.24
```
In[7318]:=
```
fNewSaitoROPsy[NetK, 500]
```
Out[7318]=
```
2875.52
```

Using the function fNewSaito we can calculate Robustness of the Network:

In[7319]:=
```
robustnessNewSaitoOPsy[NetK]
```
Out[7319]=
```
925
```
In[7320]:=
```
robustnessNewSaitoROPsy[NetK]
```
Out[7320]=
```
925
```

We can calculate the (Relative) Entropy and the Assortativity:

In[∘]:=
```
RelatEntrop5[NetK]
```
Out[∘]=
```
0.960956
```
In[∘]:=
```
assortativity[NetK]
```
Out[∘]=
```
-0.113228
```

We can calculate the robustness of the previously generated random networks with different number of auxotrophies:

In[7321]:=
```
AuxoComm6ROPsy = Parallelize[robustnessNewSaitoROPsy /@ hk6];
AuxoComm7ROPsy = Parallelize[robustnessNewSaitoROPsy /@ hk7];
AuxoComm8ROPsy = Parallelize[robustnessNewSaitoROPsy /@ hk8];
AuxoComm9ROPsy = Parallelize[robustnessNewSaitoROPsy /@ hk9];
AuxoComm10ROPsy = Parallelize[robustnessNewSaitoROPsy /@ hk10];
AuxoComm11ROPsy = Parallelize[robustnessNewSaitoROPsy /@ hk11];
AuxoComm12ROPsy = Parallelize[robustnessNewSaitoROPsy /@ hk12];
AuxoComm13ROPsy = Parallelize[robustnessNewSaitoROPsy /@ hk13];
AuxoComm14ROPsy = Parallelize[robustnessNewSaitoROPsy /@ hk14];
AuxoComm15ROPsy = Parallelize[robustnessNewSaitoROPsy /@ hk15];
AuxoComm16ROPsy = Parallelize[robustnessNewSaitoROPsy /@ hk16];
AuxoComm17ROPsy = Parallelize[robustnessNewSaitoROPsy /@ hk17];
```

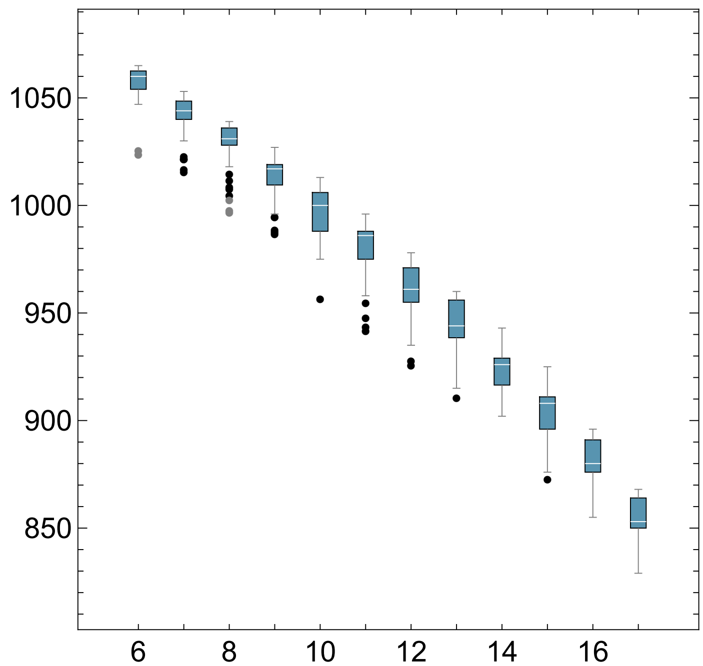
In[7333]:=
```
LikROPsy = {AuxoComm6ROPsy, AuxoComm7ROPsy, AuxoComm8ROPsy, AuxoComm9ROPsy,
   AuxoComm10ROPsy, AuxoComm11ROPsy, AuxoComm12ROPsy, AuxoComm13ROPsy,
   AuxoComm14ROPsy, AuxoComm15ROPsy, AuxoComm16ROPsy, AuxoComm17ROPsy};
```

In[∘]:=
```
coco = RGBColor[0.34509803921568627, 0.5803921568627451, 0.6901960784313725]
```

Out[∘]=

In[7334]:=
```
BoxWhiskerChart[LikROPsy, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{coco}}, Frame → True,
 ChartLabels → {"6", "", "8", "", "10", "", "12", "", "14", "", "16", ""},
 BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15], AspectRatio → 1]
```

Out[7334]=



In[7335]:=
```
AuxoComm7ROPsy
```

Out[7335]=
```
{1041, 1038, 1050, 1041, 1039, 1041, 1017, 1050, 1023, 1022, 1051, 1046,
 1040, 1036, 1047, 1041, 1050, 1052, 1047, 1041, 1034, 1046, 1046, 1039,
 1052, 1042, 1049, 1030, 1037, 1031, 1040, 1047, 1047, 1022, 1051, 1053,
 1050, 1046, 1040, 1046, 1051, 1047, 1038, 1046, 1046, 1051, 1035, 1051,
 1048, 1047, 1045, 1049, 1041, 1051, 1042, 1048, 1046, 1042, 1047, 1038, 1041,
 1045, 1042, 1041, 1049, 1042, 1047, 1051, 1051, 1052, 1040, 1050, 1040, 1051,
 1039, 1041, 1041, 1045, 1041, 1044, 1039, 1041, 1016, 1047, 1049, 1040, 1041,
 1044, 1041, 1047, 1049, 1031, 1051, 1032, 1041, 1050, 1047, 1022, 1041, 1039}
```

We can study the correlation between Relative entropy and assortativity with Robustness for Networks with 7 auxotrophies.

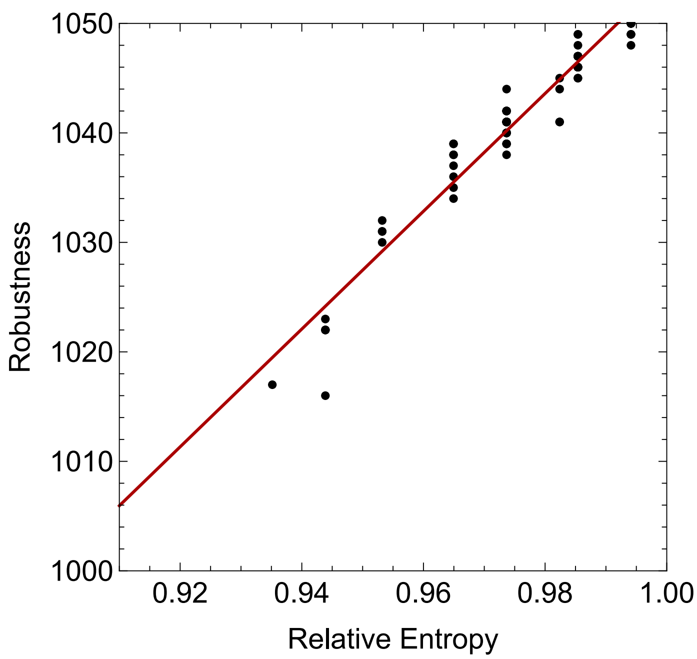
```
In[•]:= Entropy7 = RelatEntrop5 /@ hk7;

In[•]:= Assort7 = assortativity /@ hk7;

In[7336]:= RobustNewSaito7bROPsy = AuxoComm7ROPsy;
```

```
Length[Entropy7]
Length[Assort7]
Length[RobustNewSaito7bROPsy]

Out[•]= 100

Out[•]= 100

Out[•]= 100
```

```
In[•]:= {Min[Entropy7], Max[Entropy7]}
        {Min[Assort7], Max[Assort7]}

Out[•]= {0.935154, 0.994118}

Out[•]= {-0.416667, 0.25}

In[•]:= Position[Entropy7, Min[Entropy7]]

Out[•]= {{7}}

In[7337]:= RobustNewSaito7bROPsy[[#]] & /@ {1, 2, 24}

Out[7337]= {1041, 1038, 1039}

In[7338]:= {Min[RobustNewSaito7bROPsy], {Max[RobustNewSaito7bROPsy]}}

Out[7338]= {1016, {1053}}
```

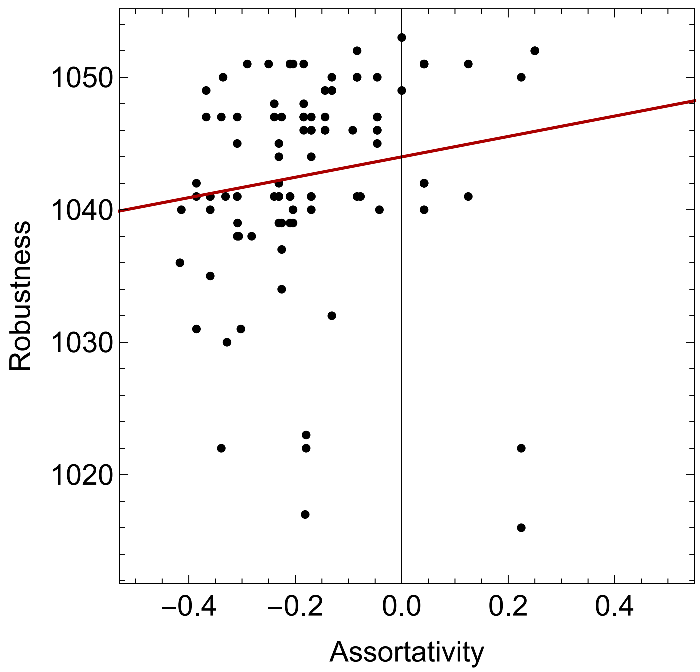
In[7339]:=
```
linerobustnessNewSaito25ROPsy =
  Fit[Partition[Riffle[Entropy7, RobustNewSaito7bROPsy], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Entropy7, RobustNewSaito7bROPsy], {2}],
  Frame → True, FrameLabel → {"Relative Entropy", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]}, PlotRange → {{0.91, 1}, {1000, 1050}},
  AspectRatio → 0.5], Plot[linerobustnessNewSaito25ROPsy, {x, 0.91, 1},
  AspectRatio → 0.5, PlotStyle → Darker[Red]], AspectRatio → 1]
```

Out[7340]=



In[7542]:=
```
lineAssoRobrobustnessNewSaito25ROPsy =
  Fit[Partition[Riffle[Assort7, RobustNewSaito7bROPsy], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Assort7, RobustNewSaito7bROPsy], {2}],
  Frame → True, FrameLabel → {"Assortativity", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]},
  PlotRange → {{-0.53, 0.55}, Automatic}, AspectRatio → 0.5],
 Plot[lineAssoRobrobustnessNewSaito25ROPsy, {x, -0.53, 0.55},
  AspectRatio → 0.5, PlotStyle → Darker[Red]], AspectRatio → 1]
```

Out[7543]=



In[7343]:=
```
SpearmanRankTest[Entropy7, RobustNewSaito7bROPsy, "TestDataTable"]
```

Out[7343]=

	Statistic	P-Value
Spearman Rank	0.96691	6.03051×10^{-60}

In[7344]:=
```
SpearmanRankTest[Assort7, RobustNewSaito7bROPsy, "TestDataTable"]
```

Out[7344]=

	Statistic	P-Value
Spearman Rank	0.339022	0.000559819

Solving the system of ODE with Overproduction Random parametrization

In[7345]:=

$$\text{fNewSaitoOVROPsy[Net\_, Dh\_, coop\_] := }\Big($$

$$dB_1 = B_1[t]\left(-B_1[t]\,\varkappa_1 + \text{nuK} * \frac{M_1[t]}{\text{denK} + M_1[t]} * \frac{M_2[t]}{\text{denK} + M_2[t]} * \frac{M_3[t]}{\text{denK} + M_3[t]} * \frac{M_4[t]}{\text{denK} + M_4[t]} * \frac{M_5[t]}{\text{denK} + M_5[t]}\right) - \left(c_{1,1} + c_{1,2} + c_{1,3} + c_{1,4} + c_{1,5} + \text{Dh}\right) B_1[t];$$

$$dB_2 = B_2[t]\left(-B_2[t]\,\varkappa_2 + \text{nuK} * \frac{M_1[t]}{\text{denK} + M_1[t]} * \frac{M_2[t]}{\text{denK} + M_2[t]} * \frac{M_3[t]}{\text{denK} + M_3[t]} * \frac{M_4[t]}{\text{denK} + M_4[t]} * \frac{M_5[t]}{\text{denK} + M_5[t]}\right) - \left(c_{2,1} + c_{2,2} + c_{2,3} + c_{2,4} + c_{2,5} + \text{Dh}\right) B_2[t];$$

$$dB_3 = B_3[t]\left(-B_3[t]\,\varkappa_3 + \text{nuK} * \frac{M_1[t]}{\text{denK} + M_1[t]} * \frac{M_2[t]}{\text{denK} + M_2[t]} * \frac{M_3[t]}{\text{denK} + M_3[t]} * \frac{M_4[t]}{\text{denK} + M_4[t]} * \frac{M_5[t]}{\text{denK} + M_5[t]}\right) - \left(c_{3,1} + c_{3,2} + c_{3,3} + c_{3,4} + c_{3,5} + \text{Dh}\right) B_3[t];$$

$$dB_4 = B_4[t]\left(-B_4[t]\,\varkappa_4 + \text{nuK} * \frac{M_1[t]}{\text{denK} + M_1[t]} * \frac{M_2[t]}{\text{denK} + M_2[t]} * \frac{M_3[t]}{\text{denK} + M_3[t]} * \frac{M_4[t]}{\text{denK} + M_4[t]} * \frac{M_5[t]}{\text{denK} + M_5[t]}\right) - \left(c_{4,1} + c_{4,2} + c_{4,3} + c_{4,4} + c_{4,5} + \text{Dh}\right) B_4[t];$$

$$dB_5 = B_5[t]\left(-B_5[t]\,\varkappa_5 + \text{nuK} * \frac{M_1[t]}{\text{denK} + M_1[t]} * \frac{M_2[t]}{\text{denK} + M_2[t]} * \frac{M_3[t]}{\text{denK} + M_3[t]} * \frac{M_4[t]}{\text{denK} + M_4[t]} * \frac{M_5[t]}{\text{denK} + M_5[t]}\right) - \left(c_{5,1} + c_{5,2} + c_{5,3} + c_{5,4} + c_{5,5} + \text{Dh}\right) B_5[t];$$

$$dM_1 = v - M_1[t]\,q_1 + \left(\text{nuK} * \frac{M_1[t]}{\text{denK} + M_1[t]} * \frac{M_2[t]}{\text{denK} + M_2[t]} * \frac{M_3[t]}{\text{denK} + M_3[t]} * \frac{M_4[t]}{\text{denK} + M_4[t]} * \frac{M_5[t]}{\text{denK} + M_5[t]}\right) \left(-B_1[t]\,d_{1,1} - B_2[t]\,d_{1,2} - B_3[t]\,d_{1,3} - B_4[t]\,d_{1,4} - B_5[t]\,d_{1,5}\right) + B_1[t]\,\Omega_{1,1} + B_2[t]\,\Omega_{1,2} + B_3[t]\,\Omega_{1,3} + B_4[t]\,\Omega_{1,4} + B_5[t]\,\Omega_{1,5};$$

$$dM_2 = v - M_2[t]\,q_2 + \left(\text{nuK} * \frac{M_1[t]}{\text{denK} + M_1[t]} * \frac{M_2[t]}{\text{denK} + M_2[t]} * \frac{M_3[t]}{\text{denK} + M_3[t]} * \frac{M_4[t]}{\text{denK} + M_4[t]} * \frac{M_5[t]}{\text{denK} + M_5[t]}\right) \left(-B_1[t]\,d_{2,1} - B_2[t]\,d_{2,2} - B_3[t]\,d_{2,3} - B_4[t]\,d_{2,4} - B_5[t]\,d_{2,5}\right) + B_1[t]\,\Omega_{2,1} + B_2[t]\,\Omega_{2,2} + B_3[t]\,\Omega_{2,3} + B_4[t]\,\Omega_{2,4} + B_5[t]\,\Omega_{2,5};$$

$$dM_3 = v - M_3[t]\,q_3 + \left(\text{nuK} * \frac{M_1[t]}{\text{denK} + M_1[t]} * \frac{M_2[t]}{\text{denK} + M_2[t]} * \frac{M_3[t]}{\text{denK} + M_3[t]} * \frac{M_4[t]}{\text{denK} + M_4[t]} * \frac{M_5[t]}{\text{denK} + M_5[t]}\right) \left(-B_1[t]\,d_{3,1} - B_2[t]\,d_{3,2} - B_3[t]\,d_{3,3} - B_4[t]\,d_{3,4} - B_5[t]\,d_{3,5}\right) + B_1[t]\,\Omega_{3,1} + B_2[t]\,\Omega_{3,2} + B_3[t]\,\Omega_{3,3} + B_4[t]\,\Omega_{3,4} + B_5[t]\,\Omega_{3,5};$$

```
dM4 = v - M4[t] q4 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) (-B1[t] d4,1 - B2[t] d4,2 - B3[t] d4,3 - B4[t] d4,4 - B5[t] d4,5) +
   B1[t] Ω4,1 + B2[t] Ω4,2 + B3[t] Ω4,3 + B4[t] Ω4,4 + B5[t] Ω4,5;
dM5 = v - M5[t] q5 + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) * M4[t]/(denK + M4[t]) *
      M5[t]/(denK + M5[t])) (-B1[t] d5,1 - B2[t] d5,2 - B3[t] d5,3 - B4[t] d5,4 - B5[t] d5,5) +
   B1[t] Ω5,1 + B2[t] Ω5,2 + B3[t] Ω5,3 + B4[t] Ω5,4 + B5[t] Ω5,5;


op = coop; (*Number of links with overExpression*)
posNe = Position[Net, 1];
(*Positions in the matrix where there are links (=1)*)
RaN = RandomSample[posNe, op];
(*Random sample of op links that will be overproduced*)

costincr = 1.3; (*Term multiplying the cost link*)
overprodincr = 1.15;
(*Term multiplying the overproduction link*)


NewNetCost = Partition[Flatten[Net] × parR[[6 ;; 30]], {5}];
Table[NewNetCost[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] =
   NewNetCost[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] * costincr, {i, Length[RaN]}];

NewNetOvProd = Partition[Flatten[Net] × parR[[61 ;; 85]], {5}];
Table[NewNetOvProd[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] =
   NewNetOvProd[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] * overprodincr, {i,
   Length[RaN]}];


tmax = 1000;
par = {
   κ1 → parR[[1]], κ2 → parR[[2]], κ3 → parR[[3]], κ4 → parR[[4]], κ5 → parR[[5]],

   c1,1 → NewNetCost[[1]][[1]],
   c1,2 → NewNetCost[[1]][[2]], c1,3 → NewNetCost[[1]][[3]],
   c1,4 → NewNetCost[[1]][[4]], c1,5 → NewNetCost[[1]][[5]],
   c2,1 → NewNetCost[[2]][[1]], c2,2 → NewNetCost[[2]][[2]],
   c2,3 → NewNetCost[[2]][[3]], c2,4 → NewNetCost[[2]][[4]],
   c2,5 → NewNetCost[[2]][[5]],
   c3,1 → NewNetCost[[3]][[1]], c3,2 → NewNetCost[[3]][[2]],
```

$c_{3,3}$ → NewNetCost[[3]][[3]], $c_{3,4}$ → NewNetCost[[3]][[4]],
$c_{3,5}$ → NewNetCost[[3]][[5]],
$c_{4,1}$ → NewNetCost[[4]][[1]], $c_{4,2}$ → NewNetCost[[4]][[2]],
$c_{4,3}$ → NewNetCost[[4]][[3]], $c_{4,4}$ → NewNetCost[[4]][[4]],
$c_{4,5}$ → NewNetCost[[4]][[5]],
$c_{5,1}$ → NewNetCost[[5]][[1]], $c_{5,2}$ → NewNetCost[[5]][[2]],
$c_{5,3}$ → NewNetCost[[5]][[3]], $c_{5,4}$ → NewNetCost[[5]][[4]],
$c_{5,5}$ → NewNetCost[[5]][[5]],

$r_{1,1}$ → parR[[31]], $r_{1,2}$ → parR[[32]],
$r_{1,3}$ → parR[[33]], $r_{1,4}$ → parR[[34]], $r_{1,5}$ → parR[[35]],
$r_{2,1}$ → parR[[36]], $r_{2,2}$ → parR[[37]], $r_{2,3}$ → parR[[38]],
$r_{2,4}$ → parR[[39]], $r_{2,5}$ → parR[[40]],
$r_{3,1}$ → parR[[41]], $r_{3,2}$ → parR[[42]], $r_{3,3}$ → parR[[43]],
$r_{3,4}$ → parR[[44]], $r_{3,5}$ → parR[[45]],
$r_{4,1}$ → parR[[46]], $r_{4,2}$ → parR[[47]], $r_{4,3}$ → parR[[48]],
$r_{4,4}$ → parR[[49]], $r_{4,5}$ → parR[[50]],
$r_{5,1}$ → parR[[51]], $r_{5,2}$ → parR[[52]], $r_{5,3}$ → parR[[53]],
$r_{5,4}$ → parR[[54]], $r_{5,5}$ → parR[[55]],

q_1 → parR[[31]], q_2 → parR[[32]],
q_3 → parR[[33]], q_4 → parR[[34]], q_5 → parR[[35]],

$d_{1,1}$ → parR[[36]], $d_{1,2}$ → parR[[37]],
$d_{1,3}$ → parR[[38]], $d_{1,4}$ → parR[[39]], $d_{1,5}$ → parR[[40]],
$d_{2,1}$ → parR[[41]], $d_{2,2}$ → parR[[42]], $d_{2,3}$ → parR[[43]],
$d_{2,4}$ → parR[[44]], $d_{2,5}$ → parR[[45]],
$d_{3,1}$ → parR[[46]], $d_{3,2}$ → parR[[47]], $d_{3,3}$ → parR[[48]],
$d_{3,4}$ → parR[[49]], $d_{3,5}$ → parR[[50]],
$d_{4,1}$ → parR[[51]], $d_{4,2}$ → parR[[52]], $d_{4,3}$ → parR[[53]],
$d_{4,4}$ → parR[[54]], $d_{4,5}$ → parR[[55]],
$d_{5,1}$ → parR[[56]], $d_{5,2}$ → parR[[57]], $d_{5,3}$ → parR[[58]],
$d_{5,4}$ → parR[[59]], $d_{5,5}$ → parR[[60]],

$\Omega_{1,1}$ → NewNetOvProd[[1]][[1]],
$\Omega_{1,2}$ → NewNetOvProd[[1]][[2]], $\Omega_{1,3}$ → NewNetOvProd[[1]][[3]],
$\Omega_{1,4}$ → NewNetOvProd[[1]][[4]], $\Omega_{1,5}$ → NewNetOvProd[[1]][[5]],
$\Omega_{2,1}$ → NewNetOvProd[[2]][[1]], $\Omega_{2,2}$ → NewNetOvProd[[2]][[2]],
$\Omega_{2,3}$ → NewNetOvProd[[2]][[3]], $\Omega_{2,4}$ → NewNetOvProd[[2]][[4]],
$\Omega_{2,5}$ → NewNetOvProd[[2]][[5]],
$\Omega_{3,1}$ → NewNetOvProd[[3]][[1]], $\Omega_{3,2}$ → NewNetOvProd[[3]][[2]],
$\Omega_{3,3}$ → NewNetOvProd[[3]][[3]], $\Omega_{3,4}$ → NewNetOvProd[[3]][[4]],
$\Omega_{3,5}$ → NewNetOvProd[[3]][[5]],
$\Omega_{4,1}$ → NewNetOvProd[[4]][[1]], $\Omega_{4,2}$ → NewNetOvProd[[4]][[2]],

```
  Ω4,3 → NewNetOvProd[[4]][[3]], Ω4,4 → NewNetOvProd[[4]][[4]],
  Ω4,5 → NewNetOvProd[[4]][[5]],
  Ω5,1 → NewNetOvProd[[5]][[1]], Ω5,2 → NewNetOvProd[[5]][[2]],
  Ω5,3 → NewNetOvProd[[5]][[3]], Ω5,4 → NewNetOvProd[[5]][[4]],
  Ω5,5 → NewNetOvProd[[5]][[5]],
  nuK → parR[[86]],
  denK → parR[[87]]


 };


B10 = 1500;
B20 = 1500;
B30 = 1500;
B40 = 1500;
B50 = 1500;
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;


sol =
 NDSolve[
  {
    B1'[t] == dB1,
    B2'[t] == dB2,
    B3'[t] == dB3,
    B4'[t] == dB4,
    B5'[t] == dB5,

    M1'[t] == dM1,
    M2'[t] == dM2,
    M3'[t] == dM3,
    M4'[t] == dM4,
    M5'[t] == dM5,


    B1[0] == B10,
    B2[0] == B20,
    B3[0] == B30,
    B4[0] == B40,
```

```
      B5[0] == B50,
      M1[0] == M10,
      M2[0] == M20,
      M3[0] == M30,
      M4[0] == M40,
      M5[0] == M50

     } /. par,
    {B1, B2, B3, B4, B5, M1, M2, M3, M4, M5},
    {t, 0, tmax}];

  {B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax],
     M1[tmax], M2[tmax], M3[tmax], M4[tmax], M5[tmax]} /. sol /. par;

  Min[{B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax]} /. sol /. par]

 )
```

In[7349]:=
```
robustnessNewSaitoOVROPsy[NetTop_, coop_] := (
  n1 = 1;
  n2 = 5000;
  mid = (n1 + n2) / 2;

  While[(n1 ≠ mid && n2 ≠ mid),
   (If[fNewSaitoOVROPsy[NetTop, mid, coop] < 1, n2 = mid, n1 = mid];
    mid = Floor[N[(n1 + n2) / 2]];); {n1, n2, mid}]; mid

 )
```

In[7346]:=
```
NetK = {
   {0, 1, 0, 1, 0},
   {1, 0, 1, 1, 0},
   {1, 0, 1, 0, 1},
   {0, 1, 0, 1, 0},
   {0, 0, 0, 0, 1}
  };
```

Compare the Robustness with and without (n links) overproduction (ratio cost/production = 1.3/1.15)

In[7350]:=
```
fNewSaitoROPsy[NetK, 0]
```
Out[7350]= 5244.24

In[7348]:=
```
fNewSaitoOVROPsy[NetK, 0, 5]
```
Out[7348]= 5244.32

In[7351]:=
```
robustnessNewSaitoROPsy[NetK]
```
Out[7351]= 925

In[7352]:=
```
robustnessNewSaitoOVROPsy[NetK, 5]
```
Out[7352]= 938

In[7353]:=
```
robustnessNewSaitoOVROPsy[NetK, 10]
```
Out[7353]= 953

In[7354]:=
```
AuxoComm8ROPsy
```
Out[7354]=
```
{1029, 1021, 1029, 1038, 1018, 1023, 1005, 1038, 1038, 1028, 1033, 1031,
 1015, 1030, 1034, 1028, 1029, 1020, 1030, 1012, 1028, 1029, 1031, 1028,
 1028, 1037, 1030, 1008, 1036, 1033, 1032, 1009, 1030, 1039, 1038, 1036,
 1031, 1032, 1037, 1038, 1030, 1037, 1034, 1039, 1039, 1032, 1027, 1028,
 1038, 1023, 1029, 1031, 1026, 998, 1037, 1032, 1003, 1036, 1036, 1032, 1029,
 1026, 1038, 1038, 1032, 1028, 1027, 1035, 1037, 1033, 1031, 1027, 1036, 1035,
 1037, 1039, 1031, 1022, 1029, 1039, 1020, 1038, 1031, 1029, 1028, 1037, 1033,
 1028, 1029, 1027, 997, 1037, 1037, 1033, 1037, 1031, 1028, 1027, 1031, 1035}
```

In[7355]:=
```
coop5to15ROPsy = {Table[robustnessNewSaitoOVROPsy[#, 5], {20}],
    Table[robustnessNewSaitoOVROPsy[#, 10], {20}],
    Table[robustnessNewSaitoOVROPsy[#, 15], {20}]} &;
```

In[7356]:=
```
wf8ROPsy = Parallelize[coop5to15ROPsy /@ hk8];
```

In[7357]:=
```
wf8NormalizedROPsy = N[wf8ROPsy[[#]] / AuxoComm8ROPsy[[#]]] & /@ Range[100]
```

In[7358]:=
```
wf8NormalizedWith5CoopROPsy = wf8NormalizedROPsy[[#]][[1]] & /@ Range[100]
```

In[7359]:=
```
wf8NormalizedWith10CoopROPsy = wf8NormalizedROPsy[[#]][[2]] & /@ Range[100]
```

In[7360]:=
```
wf8NormalizedWith15CoopROPsy = wf8NormalizedROPsy[[#]][[3]] & /@ Range[100]
```

```
(*For 8 auxotrophies networks*)
```

In[7361]:=
```
allcoopWith8AuxoROPsy = {Flatten[wf8NormalizedWith5CoopROPsy],
  Flatten[wf8NormalizedWith10CoopROPsy], Flatten[wf8NormalizedWith15CoopROPsy]}
```

In[7362]:=
```
allcoopWith8AuxoPlusAuxoROPsy =
 Join[{ConstantArray[1, {2000}]}, allcoopWith8AuxoROPsy]
```

In[7363]:=

```
BoxWhiskerChart[allcoopWith8AuxoPlusAuxoROPsy, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{gree1}},
 Frame → True, ChartLabels → {"0", "5", "10", "15"}, BarSpacing → 1.9,
 FrameStyle → Directive[Black, FontSize → 15], AspectRatio → 1]
```

Out[7363]=