Cooperation increases robustness to ecological disturbance in microbial cross-feeding networks

Generating Random Networks

Functions to calculate Entropy and Assortativity and Robustness

Entropy

Assortativity

1. Additive model

Solving the system of ODE

In[6683]:=
```
Clear[KK, cc, qq, dd, OM, nu, den]
```

In[6684]:=

$$\text{fNewMonoK}[\text{Net\_}, \text{Dh\_}] := \Big($$

$$dB_1 = B_1[t]\left(-B_1[t]\,\kappa_1 + \frac{\text{nuK}\,M_1[t]}{\text{denK} + M_1[t]} + \frac{\text{nuK}\,M_2[t]}{\text{denK} + M_2[t]} + \frac{\text{nuK}\,M_3[t]}{\text{denK} + M_3[t]} + \frac{\text{nuK}\,M_4[t]}{\text{denK} + M_4[t]} + \frac{\text{nuK}\,M_5[t]}{\text{denK} + M_5[t]}\right) - \left(c_{1,1} + c_{1,2} + c_{1,3} + c_{1,4} + c_{1,5} + \text{Dh}\right) B_1[t];$$

$$dB_2 = B_2[t]\left(-B_2[t]\,\kappa_2 + \frac{\text{nuK}\,M_1[t]}{\text{denK} + M_1[t]} + \frac{\text{nuK}\,M_2[t]}{\text{denK} + M_2[t]} + \frac{\text{nuK}\,M_3[t]}{\text{denK} + M_3[t]} + \frac{\text{nuK}\,M_4[t]}{\text{denK} + M_4[t]} + \frac{\text{nuK}\,M_5[t]}{\text{denK} + M_5[t]}\right) - \left(c_{2,1} + c_{2,2} + c_{2,3} + c_{2,4} + c_{2,5} + \text{Dh}\right) B_2[t];$$

$$dB_3 = B_3[t]\left(-B_3[t]\,\kappa_3 + \frac{\text{nuK}\,M_1[t]}{\text{denK} + M_1[t]} + \frac{\text{nuK}\,M_2[t]}{\text{denK} + M_2[t]} + \frac{\text{nuK}\,M_3[t]}{\text{denK} + M_3[t]} + \right.$$

$$\frac{\text{nuK}\,M_4[t]}{\text{denK}+M_4[t]}+\frac{\text{nuK}\,M_5[t]}{\text{denK}+M_5[t]}\Bigg)-\left(c_{3,1}+c_{3,2}+c_{3,3}+c_{3,4}+c_{3,5}+\text{Dh}\right)B_3[t];$$

$$\text{dB}_4=B_4[t]\left(-B_4[t]\,\kappa_4+\frac{\text{nuK}\,M_1[t]}{\text{denK}+M_1[t]}+\frac{\text{nuK}\,M_2[t]}{\text{denK}+M_2[t]}+\frac{\text{nuK}\,M_3[t]}{\text{denK}+M_3[t]}+\frac{\text{nuK}\,M_4[t]}{\text{denK}+M_4[t]}+\frac{\text{nuK}\,M_5[t]}{\text{denK}+M_5[t]}\right)-\left(c_{4,1}+c_{4,2}+c_{4,3}+c_{4,4}+c_{4,5}+\text{Dh}\right)B_4[t];$$

$$\text{dB}_5=B_5[t]\left(-B_5[t]\,\kappa_5+\frac{\text{nuK}\,M_1[t]}{\text{denK}+M_1[t]}+\frac{\text{nuK}\,M_2[t]}{\text{denK}+M_2[t]}+\frac{\text{nuK}\,M_3[t]}{\text{denK}+M_3[t]}+\frac{\text{nuK}\,M_4[t]}{\text{denK}+M_4[t]}+\frac{\text{nuK}\,M_5[t]}{\text{denK}+M_5[t]}\right)-\left(c_{5,1}+c_{5,2}+c_{5,3}+c_{5,4}+c_{5,5}+\text{Dh}\right)B_5[t];$$

$$\text{dM}_1=-M_1[t]\,q_1+\left(\frac{\text{nuK}\,M_1[t]}{\text{denK}+M_1[t]}+\frac{\text{nuK}\,M_2[t]}{\text{denK}+M_2[t]}+\frac{\text{nuK}\,M_3[t]}{\text{denK}+M_3[t]}+\frac{\text{nuK}\,M_4[t]}{\text{denK}+M_4[t]}+\frac{\text{nuK}\,M_5[t]}{\text{denK}+M_5[t]}\right)\left(-B_1[t]\,d_{1,1}-B_2[t]\,d_{1,2}-B_3[t]\,d_{1,3}-B_4[t]\,d_{1,4}-B_5[t]\,d_{1,5}\right)+B_1[t]\,\Omega_{1,1}+B_2[t]\,\Omega_{1,2}+B_3[t]\,\Omega_{1,3}+B_4[t]\,\Omega_{1,4}+B_5[t]\,\Omega_{1,5};$$

$$\text{dM}_2=-M_2[t]\,q_2+\left(\frac{\text{nuK}\,M_1[t]}{\text{denK}+M_1[t]}+\frac{\text{nuK}\,M_2[t]}{\text{denK}+M_2[t]}+\frac{\text{nuK}\,M_3[t]}{\text{denK}+M_3[t]}+\frac{\text{nuK}\,M_4[t]}{\text{denK}+M_4[t]}+\frac{\text{nuK}\,M_5[t]}{\text{denK}+M_5[t]}\right)\left(-B_1[t]\,d_{2,1}-B_2[t]\,d_{2,2}-B_3[t]\,d_{2,3}-B_4[t]\,d_{2,4}-B_5[t]\,d_{2,5}\right)+B_1[t]\,\Omega_{2,1}+B_2[t]\,\Omega_{2,2}+B_3[t]\,\Omega_{2,3}+B_4[t]\,\Omega_{2,4}+B_5[t]\,\Omega_{2,5};$$

$$\text{dM}_3=-M_3[t]\,q_3+\left(\frac{\text{nuK}\,M_1[t]}{\text{denK}+M_1[t]}+\frac{\text{nuK}\,M_2[t]}{\text{denK}+M_2[t]}+\frac{\text{nuK}\,M_3[t]}{\text{denK}+M_3[t]}+\frac{\text{nuK}\,M_4[t]}{\text{denK}+M_4[t]}+\frac{\text{nuK}\,M_5[t]}{\text{denK}+M_5[t]}\right)\left(-B_1[t]\,d_{3,1}-B_2[t]\,d_{3,2}-B_3[t]\,d_{3,3}-B_4[t]\,d_{3,4}-B_5[t]\,d_{3,5}\right)+B_1[t]\,\Omega_{3,1}+B_2[t]\,\Omega_{3,2}+B_3[t]\,\Omega_{3,3}+B_4[t]\,\Omega_{3,4}+B_5[t]\,\Omega_{3,5};$$

$$\text{dM}_4=-M_4[t]\,q_4+\left(\frac{\text{nuK}\,M_1[t]}{\text{denK}+M_1[t]}+\frac{\text{nuK}\,M_2[t]}{\text{denK}+M_2[t]}+\frac{\text{nuK}\,M_3[t]}{\text{denK}+M_3[t]}+\frac{\text{nuK}\,M_4[t]}{\text{denK}+M_4[t]}+\frac{\text{nuK}\,M_5[t]}{\text{denK}+M_5[t]}\right)\left(-B_1[t]\,d_{4,1}-B_2[t]\,d_{4,2}-B_3[t]\,d_{4,3}-B_4[t]\,d_{4,4}-B_5[t]\,d_{4,5}\right)+B_1[t]\,\Omega_{4,1}+B_2[t]\,\Omega_{4,2}+B_3[t]\,\Omega_{4,3}+B_4[t]\,\Omega_{4,4}+B_5[t]\,\Omega_{4,5};$$

$$\text{dM}_5=-M_5[t]\,q_5+\left(\frac{\text{nuK}\,M_1[t]}{\text{denK}+M_1[t]}+\frac{\text{nuK}\,M_2[t]}{\text{denK}+M_2[t]}+\frac{\text{nuK}\,M_3[t]}{\text{denK}+M_3[t]}+\frac{\text{nuK}\,M_4[t]}{\text{denK}+M_4[t]}+\frac{\text{nuK}\,M_5[t]}{\text{denK}+M_5[t]}\right)\left(-B_1[t]\,d_{5,1}-B_2[t]\,d_{5,2}-B_3[t]\,d_{5,3}-B_4[t]\,d_{5,4}-B_5[t]\,d_{5,5}\right)+B_1[t]\,\Omega_{5,1}+B_2[t]\,\Omega_{5,2}+B_3[t]\,\Omega_{5,3}+B_4[t]\,\Omega_{5,4}+B_5[t]\,\Omega_{5,5};$$

```
KK = 0.2;
cc = 0.05;
qq = 0.3;
dd = 0.0015;
OM = 1;
nu = 20;
den = 5;
```

```
tmax = 1000;
par = {
  κ₁ → KK, κ₂ → KK, κ₃ → KK, κ₄ → KK, κ₅ → KK,

  c₁,₁ → cc Net[[1]][[1]], c₁,₂ → cc Net[[1]][[2]],
  c₁,₃ → cc Net[[1]][[3]], c₁,₄ → cc Net[[1]][[4]], c₁,₅ → cc Net[[1]][[5]],
  c₂,₁ → cc Net[[2]][[1]], c₂,₂ → cc Net[[2]][[2]], c₂,₃ → cc Net[[2]][[3]],
  c₂,₄ → cc Net[[2]][[4]], c₂,₅ → cc Net[[2]][[5]],
  c₃,₁ → cc Net[[3]][[1]], c₃,₂ → cc Net[[3]][[2]], c₃,₃ → cc Net[[3]][[3]],
  c₃,₄ → cc Net[[3]][[4]], c₃,₅ → cc Net[[3]][[5]],
  c₄,₁ → cc Net[[4]][[1]], c₄,₂ → cc Net[[4]][[2]], c₄,₃ → cc Net[[4]][[3]],
  c₄,₄ → cc Net[[4]][[4]], c₄,₅ → cc Net[[4]][[5]],
  c₅,₁ → cc Net[[5]][[1]], c₅,₂ → cc Net[[5]][[2]], c₅,₃ → cc Net[[5]][[3]],
  c₅,₄ → cc Net[[5]][[4]], c₅,₅ → cc Net[[5]][[5]],

  q₁ → qq, q₂ → qq, q₃ → qq, q₄ → qq, q₅ → qq,

  d₁,₁ → dd, d₁,₂ → dd, d₁,₃ → dd, d₁,₄ → dd, d₁,₅ → dd,
  d₂,₁ → dd, d₂,₂ → dd, d₂,₃ → dd, d₂,₄ → dd, d₂,₅ → dd,
  d₃,₁ → dd, d₃,₂ → dd, d₃,₃ → dd, d₃,₄ → dd, d₃,₅ → dd,
  d₄,₁ → dd, d₄,₂ → dd, d₄,₃ → dd, d₄,₄ → dd, d₄,₅ → dd,
  d₅,₁ → dd, d₅,₂ → dd, d₅,₃ → dd, d₅,₄ → dd, d₅,₅ → dd,

  Ω₁,₁ → OM Net[[1]][[1]], Ω₁,₂ → OM Net[[1]][[2]],
  Ω₁,₃ → OM Net[[1]][[3]], Ω₁,₄ → OM Net[[1]][[4]], Ω₁,₅ → OM Net[[1]][[5]],
  Ω₂,₁ → OM Net[[2]][[1]], Ω₂,₂ → OM Net[[2]][[2]], Ω₂,₃ → OM Net[[2]][[3]],
  Ω₂,₄ → OM Net[[2]][[4]], Ω₂,₅ → OM Net[[2]][[5]],
  Ω₃,₁ → OM Net[[3]][[1]], Ω₃,₂ → OM Net[[3]][[2]], Ω₃,₃ → OM Net[[3]][[3]],
  Ω₃,₄ → OM Net[[3]][[4]], Ω₃,₅ → OM Net[[3]][[5]],
  Ω₄,₁ → OM Net[[4]][[1]], Ω₄,₂ → OM Net[[4]][[2]], Ω₄,₃ → OM Net[[4]][[3]],
  Ω₄,₄ → OM Net[[4]][[4]], Ω₄,₅ → OM Net[[4]][[5]],
  Ω₅,₁ → OM Net[[5]][[1]], Ω₅,₂ → OM Net[[5]][[2]], Ω₅,₃ → OM Net[[5]][[3]],
  Ω₅,₄ → OM Net[[5]][[4]], Ω₅,₅ → OM Net[[5]][[5]],
  nuK → nu,
  denK → den


 };


B10 = 1500;
B20 = 1500;
B30 = 1500;
```

```
B40 = 1500;
B50 = 1500;
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;


sol =
 NDSolve[
  {
    B1'[t] == dB1,
    B2'[t] == dB2,
    B3'[t] == dB3,
    B4'[t] == dB4,
    B5'[t] == dB5,

    M1'[t] == dM1,
    M2'[t] == dM2,
    M3'[t] == dM3,
    M4'[t] == dM4,
    M5'[t] == dM5,


    B1[0] == B10,
    B2[0] == B20,
    B3[0] == B30,
    B4[0] == B40,
    B5[0] == B50,
    M1[0] == M10,
    M2[0] == M20,
    M3[0] == M30,
    M4[0] == M40,
    M5[0] == M50

   } /. par,
  {B1, B2, B3, B4, B5, M1, M2, M3, M4, M5},
  {t, 0, tmax}];

{B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax],
   M1[tmax], M2[tmax], M3[tmax], M4[tmax], M5[tmax]} /. sol /. par
```

```
)
```

In[6685]:=
```
NetK = {
   {0, 1, 0, 1, 0},
   {1, 0, 1, 1, 0},
   {1, 0, 1, 0, 1},
   {0, 1, 0, 1, 0},
   {0, 0, 0, 0, 1}
  };
```

In[6686]:=
```
fNewMonoK[NetK, 0]
```

Out[6686]=
```
{{497.577, 497.327, 497.327, 497.577,
  497.827, 2077.31, 3735.9, 3736.74, 2077.31, 420.389}}
```

In[6687]:=
```
fNewMono[Net_, Dh_] := (
```

$$dB_1 = B_1[t]\left(-B_1[t]\,\kappa_1 + \frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} + \frac{nuK\,M_5[t]}{denK + M_5[t]}\right) - \left(c_{1,1} + c_{1,2} + c_{1,3} + c_{1,4} + c_{1,5} + Dh\right) B_1[t];$$

$$dB_2 = B_2[t]\left(-B_2[t]\,\kappa_2 + \frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} + \frac{nuK\,M_5[t]}{denK + M_5[t]}\right) - \left(c_{2,1} + c_{2,2} + c_{2,3} + c_{2,4} + c_{2,5} + Dh\right) B_2[t];$$

$$dB_3 = B_3[t]\left(-B_3[t]\,\kappa_3 + \frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} + \frac{nuK\,M_5[t]}{denK + M_5[t]}\right) - \left(c_{3,1} + c_{3,2} + c_{3,3} + c_{3,4} + c_{3,5} + Dh\right) B_3[t];$$

$$dB_4 = B_4[t]\left(-B_4[t]\,\kappa_4 + \frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} + \frac{nuK\,M_5[t]}{denK + M_5[t]}\right) - \left(c_{4,1} + c_{4,2} + c_{4,3} + c_{4,4} + c_{4,5} + Dh\right) B_4[t];$$

$$dB_5 = B_5[t]\left(-B_5[t]\,\kappa_5 + \frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} + \frac{nuK\,M_5[t]}{denK + M_5[t]}\right) - \left(c_{5,1} + c_{5,2} + c_{5,3} + c_{5,4} + c_{5,5} + Dh\right) B_5[t];$$

$$dM_1 = -M_1[t]\,q_1 + \left(\frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} + \frac{nuK\,M_5[t]}{denK + M_5[t]}\right)$$

```
    (-B1[t] d1,1 - B2[t] d1,2 - B3[t] d1,3 - B4[t] d1,4 - B5[t] d1,5) +
   B1[t] Ω1,1 + B2[t] Ω1,2 + B3[t] Ω1,3 + B4[t] Ω1,4 + B5[t] Ω1,5;
dM2 = -M2[t] q2 + (nuK M1[t]/(denK + M1[t]) + nuK M2[t]/(denK + M2[t]) + nuK M3[t]/(denK + M3[t]) + nuK M4[t]/(denK + M4[t]) +
      nuK M5[t]/(denK + M5[t])) (-B1[t] d2,1 - B2[t] d2,2 - B3[t] d2,3 - B4[t] d2,4 - B5[t] d2,5) +
   B1[t] Ω2,1 + B2[t] Ω2,2 + B3[t] Ω2,3 + B4[t] Ω2,4 + B5[t] Ω2,5;
dM3 = -M3[t] q3 + (nuK M1[t]/(denK + M1[t]) + nuK M2[t]/(denK + M2[t]) + nuK M3[t]/(denK + M3[t]) + nuK M4[t]/(denK + M4[t]) +
      nuK M5[t]/(denK + M5[t])) (-B1[t] d3,1 - B2[t] d3,2 - B3[t] d3,3 - B4[t] d3,4 - B5[t] d3,5) +
   B1[t] Ω3,1 + B2[t] Ω3,2 + B3[t] Ω3,3 + B4[t] Ω3,4 + B5[t] Ω3,5;
dM4 = -M4[t] q4 + (nuK M1[t]/(denK + M1[t]) + nuK M2[t]/(denK + M2[t]) + nuK M3[t]/(denK + M3[t]) + nuK M4[t]/(denK + M4[t]) +
      nuK M5[t]/(denK + M5[t])) (-B1[t] d4,1 - B2[t] d4,2 - B3[t] d4,3 - B4[t] d4,4 - B5[t] d4,5) +
   B1[t] Ω4,1 + B2[t] Ω4,2 + B3[t] Ω4,3 + B4[t] Ω4,4 + B5[t] Ω4,5;
dM5 = -M5[t] q5 + (nuK M1[t]/(denK + M1[t]) + nuK M2[t]/(denK + M2[t]) + nuK M3[t]/(denK + M3[t]) + nuK M4[t]/(denK + M4[t]) +
      nuK M5[t]/(denK + M5[t])) (-B1[t] d5,1 - B2[t] d5,2 - B3[t] d5,3 - B4[t] d5,4 - B5[t] d5,5) +
   B1[t] Ω5,1 + B2[t] Ω5,2 + B3[t] Ω5,3 + B4[t] Ω5,4 + B5[t] Ω5,5;

KK = 0.2;
cc = 0.05;
qq = 0.3;
dd = 0.0015;
OM = 1;
nu = 20;
den = 5;

tmax = 1000;
par = {
   κ1 → KK, κ2 → KK, κ3 → KK, κ4 → KK, κ5 → KK,

   c1,1 → cc Net[[1]][[1]], c1,2 → cc Net[[1]][[2]],
   c1,3 → cc Net[[1]][[3]], c1,4 → cc Net[[1]][[4]], c1,5 → cc Net[[1]][[5]],
   c2,1 → cc Net[[2]][[1]], c2,2 → cc Net[[2]][[2]], c2,3 → cc Net[[2]][[3]],
   c2,4 → cc Net[[2]][[4]], c2,5 → cc Net[[2]][[5]],
   c3,1 → cc Net[[3]][[1]], c3,2 → cc Net[[3]][[2]], c3,3 → cc Net[[3]][[3]],
   c3,4 → cc Net[[3]][[4]], c3,5 → cc Net[[3]][[5]],
   c4,1 → cc Net[[4]][[1]], c4,2 → cc Net[[4]][[2]], c4,3 → cc Net[[4]][[3]],
   c4,4 → cc Net[[4]][[4]], c4,5 → cc Net[[4]][[5]],
   c5,1 → cc Net[[5]][[1]], c5,2 → cc Net[[5]][[2]], c5,3 → cc Net[[5]][[3]],
```

```
  c5,4 → cc Net[[5]][[4]], c5,5 → cc Net[[5]][[5]],

  q1 → qq, q2 → qq, q3 → qq, q4 → qq, q5 → qq,

  d1,1 → dd, d1,2 → dd, d1,3 → dd, d1,4 → dd, d1,5 → dd,
  d2,1 → dd, d2,2 → dd, d2,3 → dd, d2,4 → dd, d2,5 → dd,
  d3,1 → dd, d3,2 → dd, d3,3 → dd, d3,4 → dd, d3,5 → dd,
  d4,1 → dd, d4,2 → dd, d4,3 → dd, d4,4 → dd, d4,5 → dd,
  d5,1 → dd, d5,2 → dd, d5,3 → dd, d5,4 → dd, d5,5 → dd,

  Ω1,1 → OM Net[[1]][[1]], Ω1,2 → OM Net[[1]][[2]],
  Ω1,3 → OM Net[[1]][[3]], Ω1,4 → OM Net[[1]][[4]], Ω1,5 → OM Net[[1]][[5]],
  Ω2,1 → OM Net[[2]][[1]], Ω2,2 → OM Net[[2]][[2]], Ω2,3 → OM Net[[2]][[3]],
  Ω2,4 → OM Net[[2]][[4]], Ω2,5 → OM Net[[2]][[5]],
  Ω3,1 → OM Net[[3]][[1]], Ω3,2 → OM Net[[3]][[2]], Ω3,3 → OM Net[[3]][[3]],
  Ω3,4 → OM Net[[3]][[4]], Ω3,5 → OM Net[[3]][[5]],
  Ω4,1 → OM Net[[4]][[1]], Ω4,2 → OM Net[[4]][[2]], Ω4,3 → OM Net[[4]][[3]],
  Ω4,4 → OM Net[[4]][[4]], Ω4,5 → OM Net[[4]][[5]],
  Ω5,1 → OM Net[[5]][[1]], Ω5,2 → OM Net[[5]][[2]], Ω5,3 → OM Net[[5]][[3]],
  Ω5,4 → OM Net[[5]][[4]], Ω5,5 → OM Net[[5]][[5]],
  nuK → nu,
  denK → den


 };


B10 = 1500;
B20 = 1500;
B30 = 1500;
B40 = 1500;
B50 = 1500;
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;


sol =
 NDSolve[
  {
    B1'[t] == dB1,
    B2'[t] == dB2,
```

```
      B₃'[t] == dB₃,
      B₄'[t] == dB₄,
      B₅'[t] == dB₅,

      M₁'[t] == dM₁,
      M₂'[t] == dM₂,
      M₃'[t] == dM₃,
      M₄'[t] == dM₄,
      M₅'[t] == dM₅,


      B₁[0] == B10,
      B₂[0] == B20,
      B₃[0] == B30,
      B₄[0] == B40,
      B₅[0] == B50,
      M₁[0] == M10,
      M₂[0] == M20,
      M₃[0] == M30,
      M₄[0] == M40,
      M₅[0] == M50

     } /. par,
    {B₁, B₂, B₃, B₄, B₅, M₁, M₂, M₃, M₄, M₅},
    {t, 0, tmax}];

  {B₁[tmax], B₂[tmax], B₃[tmax], B₄[tmax], B₅[tmax],
     M₁[tmax], M₂[tmax], M₃[tmax], M₄[tmax], M₅[tmax]} /. sol /. par;

  Min[{B₁[tmax], B₂[tmax], B₃[tmax], B₄[tmax], B₅[tmax]} /. sol /. par]

)
```

In[6688]:=
```
robustnessNewMono[NetTop_] := (
  n1 = 1;
  n2 = 5000;
  mid = (n1 + n2) / 2;

  While[(n1 ≠ mid && n2 ≠ mid ),
   (If[fNewMono[NetTop, mid] < 1, n2 = mid, n1 = mid];
    mid = Floor[N[(n1 + n2) / 2]];); {n1, n2, mid}]; mid

 )
```

As an example let's take the following Network

In[6689]:=
```
NetK = {
    {0, 1, 0, 1, 0},
    {1, 0, 1, 1, 0},
    {1, 0, 1, 0, 1},
    {0, 1, 0, 1, 0},
    {0, 0, 0, 0, 1}
  };
```

Using the function f we can calculate the smallest value of a bacterial population in the community for a given disturbance vale. For example, let's take Disturbance value 1 and 70:

In[6690]:=
```
fNewMono[NetK, 1]
```
Out[6690]= 492.308

In[6691]:=
```
fNewMono[NetK, 4]
```
Out[6691]= 477.248

Using the function g3 we can calculate Robustness of the Network:

In[6692]:=
```
robustnessNewMono[NetK]
```
Out[6692]= 86

We can calculate the (Relative) Entropy and the Assortativity:

In[6693]:=
```
RelatEntrop5[NetK]
```
Out[6693]= 0.960956

In[6694]:=
```
assortativity[NetK]
```
Out[6694]= -0.113228

We can calculate the robustness of the previously generated random networks with different number of auxotrophies:

In[6695]:=
```
AuxoComm6Additive = Parallelize[robustnessNewMono /@ hk6];
AuxoComm7Additive = Parallelize[robustnessNewMono /@ hk7];
AuxoComm8Additive = Parallelize[robustnessNewMono /@ hk8];
AuxoComm9Additive = Parallelize[robustnessNewMono /@ hk9];
AuxoComm10Additive = Parallelize[robustnessNewMono /@ hk10];
AuxoComm11Additive = Parallelize[robustnessNewMono /@ hk11];
AuxoComm12Additive = Parallelize[robustnessNewMono /@ hk12];
AuxoComm13Additive = Parallelize[robustnessNewMono /@ hk13];
AuxoComm14Additive = Parallelize[robustnessNewMono /@ hk14];
AuxoComm15Additive = Parallelize[robustnessNewMono /@ hk15];
AuxoComm16Additive = Parallelize[robustnessNewMono /@ hk16];
AuxoComm17Additive = Parallelize[robustnessNewMono /@ hk17];
```

In[6707]:=
```
Timing[robustnessNewMono /@ hk6[[1 ;; 3]];]
```

Out[6707]=
```
{0.59554, Null}
```

In[6708]:=
```
Timing[Parallelize[robustnessNewMono /@ hk6[[1 ;; 3]];]]
```

Out[6708]=
```
{0.032704, Null}
```

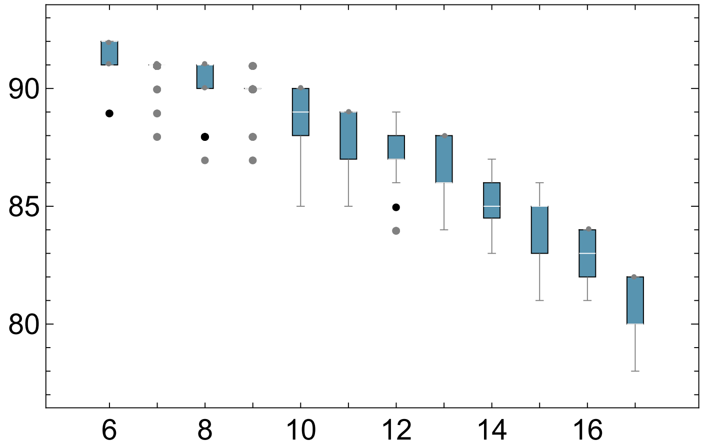
In[6709]:=
```
LikAdditive =
  {AuxoComm6Additive, AuxoComm7Additive, AuxoComm8Additive, AuxoComm9Additive,
   AuxoComm10Additive, AuxoComm11Additive, AuxoComm12Additive, AuxoComm13Additive,
   AuxoComm14Additive, AuxoComm15Additive, AuxoComm16Additive, AuxoComm17Additive};
```

In[6710]:=
```
coco = RGBColor[0.34509803921568627, 0.5803921568627451, 0.6901960784313725]
```

Out[6710]=

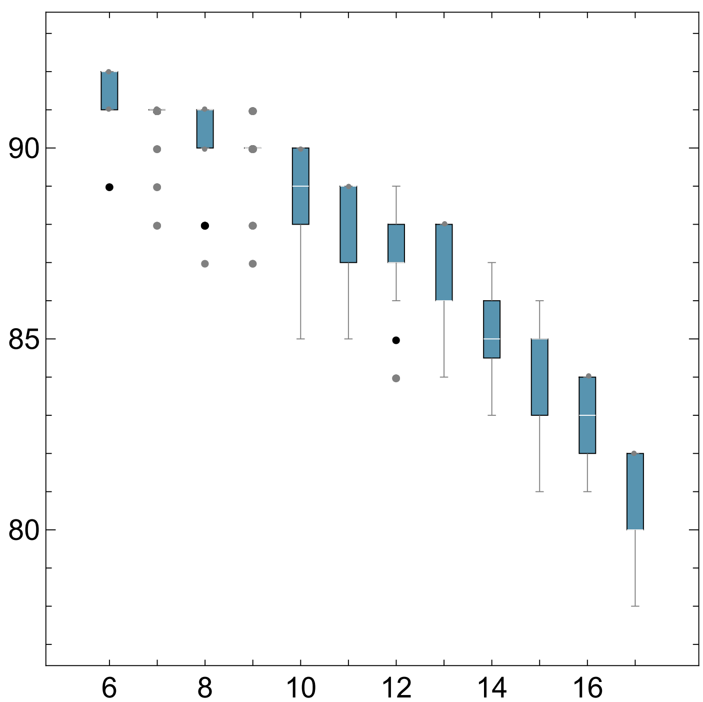
In[6711]:=
```
BoxWhiskerChart[LikAdditive, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{coco}}, Frame → True,
 ChartLabels → {"6", "", "8", "", "10", "", "12", "", "14", "", "16", ""},
 BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15]]
```

Out[6711]=



In[6712]:=
```
BoxWhiskerChart[LikAdditive, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{coco}}, Frame → True,
 ChartLabels → {"6", "", "8", "", "10", "", "12", "", "14", "", "16", ""},
 BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15], AspectRatio → 1]
```

Out[6712]=



In[6713]:=
```
AuxoComm8Additive
```

Out[6713]=
```
{90, 90, 91, 91, 90, 90, 88, 91, 91, 90, 91, 91, 90, 91, 91, 90, 90, 90, 91, 88,
 90, 90, 91, 90, 90, 91, 91, 88, 91, 91, 91, 88, 91, 91, 91, 91, 90, 91, 91, 91,
 90, 91, 91, 91, 91, 91, 90, 90, 91, 90, 90, 91, 90, 88, 91, 91, 88, 91, 91, 91,
 90, 90, 91, 91, 91, 90, 90, 91, 91, 91, 91, 90, 91, 91, 91, 91, 91, 90, 90, 91,
 90, 91, 91, 90, 90, 91, 91, 91, 90, 90, 87, 91, 91, 91, 91, 91, 90, 90, 91, 91}
```

We can study the correlation between Relative entropy and assortativity with Robustness for Networks with 7 auxotrophies.

In[∘]:=
```
Entropy13 = RelatEntrop5 /@ hk8;
```

In[∘]:=
```
Assort13 = assortativity /@ hk8;
```

In[6714]:=
```
RobustAdditive8 = AuxoComm8Additive;
```

In[6715]:=
```
Length[Entropy8]
Length[Assort8]
Length[RobustAdditive8]
```

Out[6715]= 100

Out[6716]= 100

Out[6717]= 100

In[∘]:=
```
{Min[Entropy8], Max[Entropy8]}
{Min[Assort8], Max[Assort8]}
```

Out[∘]= {0.9188, 0.993652}

Out[∘]= {-0.520325, 0.235702}

In[∘]:=
```
Position[Entropy8, Min[Entropy8]]
```

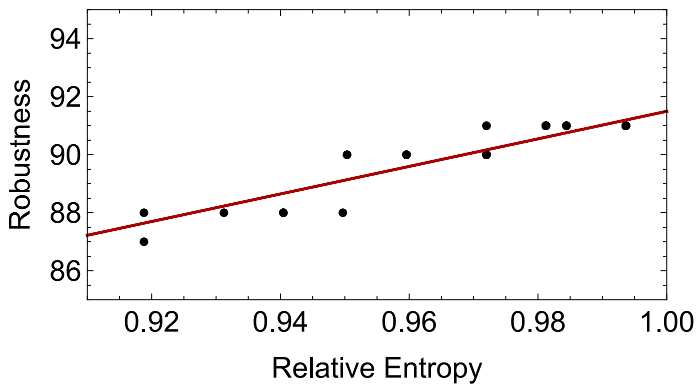
Out[∘]= {{54}, {91}}

```
RobustAdditive8[[#]] & /@ {1, 2, 24}
```

Out[∘]= {181, 181, 182}

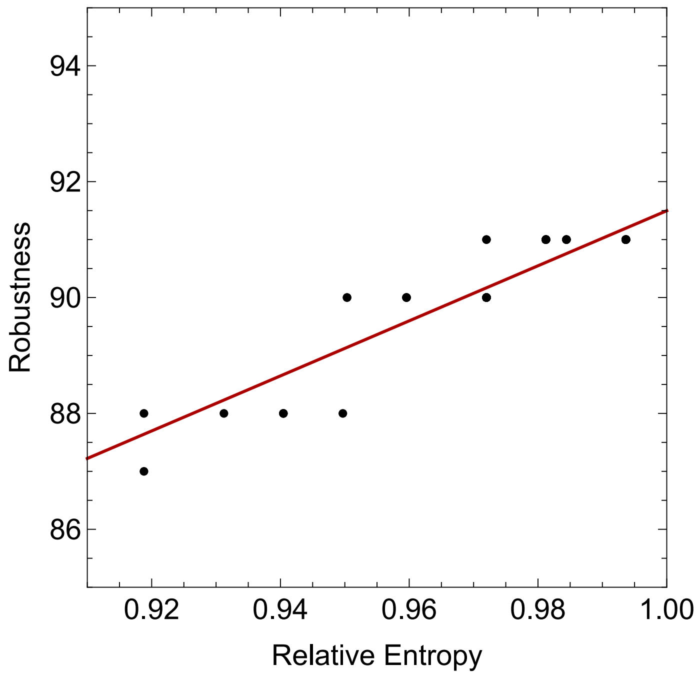
In[6718]:=
```
{Min[RobustAdditive8], {Max[RobustAdditive8]}}
```
Out[6718]=
```
{87, {91}}
```

In[6719]:=
```
linerobustnessAdditive25 =
  Fit[Partition[Riffle[Entropy8, RobustAdditive8], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Entropy8, RobustAdditive8], {2}],
  Frame → True, FrameLabel → {"Relative Entropy", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]}, PlotRange → {{0.91, 1}, {85, 95}},
  AspectRatio → 0.5], Plot[linerobustnessAdditive25,
  {x, 0.91, 1}, AspectRatio → 0.5, PlotStyle → Darker[Red]]]
```
Out[6720]=



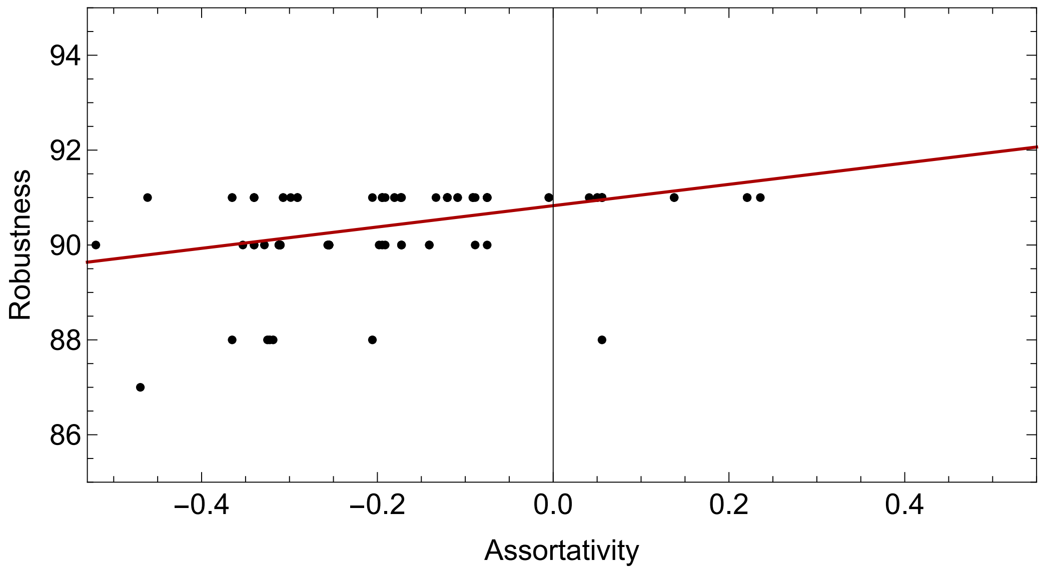
In[6723]:=
```
linerobustnessAdditive25 =
  Fit[Partition[Riffle[Entropy8, RobustAdditive8], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Entropy8, RobustAdditive8], {2}],
  Frame → True, FrameLabel → {"Relative Entropy", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]}, PlotRange → {{0.91, 1}, {85, 95}},
  AspectRatio → 1], Plot[linerobustnessAdditive25,
  {x, 0.91, 1}, PlotStyle → Darker[Red], AspectRatio → 1]]
```

Out[6724]=



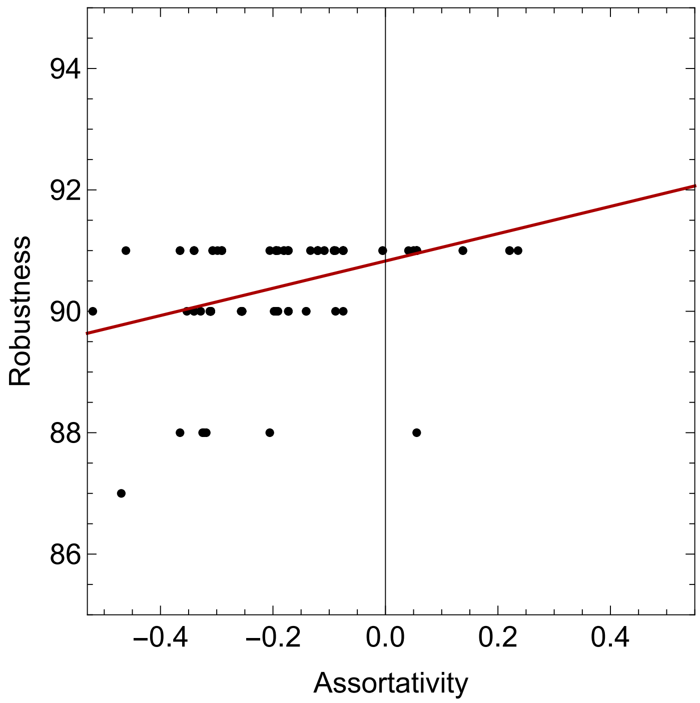
In[6727]:=
```
lineAssoRobrobustnessAdditive25 =
  Fit[Partition[Riffle[Assort8, RobustAdditive8], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Assort8, RobustAdditive8], {2}],
  Frame → True, FrameLabel → {"Assortativity", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]}, PlotRange → {{-0.53, 0.55}, {85, 95}},
  AspectRatio → 0.5], Plot[lineAssoRobrobustnessAdditive25,
  {x, -0.53, 0.55}, AspectRatio → 0.5, PlotStyle → Darker[Red]]]
```

Out[6728]=



In[6729]:=
```
lineAssoRobrobustnessAdditive25 =
  Fit[Partition[Riffle[Assort8, RobustAdditive8], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Assort8, RobustAdditive8], {2}],
  Frame → True, FrameLabel → {"Assortativity", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]}, PlotRange → {{-0.53, 0.55}, {85, 95}},
  AspectRatio → 1], Plot[lineAssoRobrobustnessAdditive25,
  {x, -0.53, 0.55}, AspectRatio → 1, PlotStyle → Darker[Red]]]
```

Out[6730]=



In[6731]:=
```
SpearmanRankTest[Entropy8, RobustAdditive8, "TestDataTable"]
```

Out[6731]=

	Statistic	P-Value
Spearman Rank	0.880425	1.59281×10^{-33}

In[6732]:=
```
SpearmanRankTest[Assort8, RobustAdditive8, "TestDataTable"]
```

Out[6732]=

	Statistic	P-Value
Spearman Rank	0.500922	1.10949×10^{-7}

Solving the system of ODE with Overproduction

In[6733]:=

$$\text{fNewMonoOV[Net\_, Dh\_, coop\_] := }\Bigg($$

$$dB_1 = B_1[t]\left(-B_1[t]\,\varkappa_1 + \frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} + \frac{nuK\,M_5[t]}{denK + M_5[t]}\right) - \left(c_{1,1} + c_{1,2} + c_{1,3} + c_{1,4} + c_{1,5} + Dh\right) B_1[t];$$

$$dB_2 = B_2[t]\left(-B_2[t]\,\varkappa_2 + \frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} + \frac{nuK\,M_5[t]}{denK + M_5[t]}\right) - \left(c_{2,1} + c_{2,2} + c_{2,3} + c_{2,4} + c_{2,5} + Dh\right) B_2[t];$$

$$dB_3 = B_3[t]\left(-B_3[t]\,\varkappa_3 + \frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} + \frac{nuK\,M_5[t]}{denK + M_5[t]}\right) - \left(c_{3,1} + c_{3,2} + c_{3,3} + c_{3,4} + c_{3,5} + Dh\right) B_3[t];$$

$$dB_4 = B_4[t]\left(-B_4[t]\,\varkappa_4 + \frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} + \frac{nuK\,M_5[t]}{denK + M_5[t]}\right) - \left(c_{4,1} + c_{4,2} + c_{4,3} + c_{4,4} + c_{4,5} + Dh\right) B_4[t];$$

$$dB_5 = B_5[t]\left(-B_5[t]\,\varkappa_5 + \frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} + \frac{nuK\,M_5[t]}{denK + M_5[t]}\right) - \left(c_{5,1} + c_{5,2} + c_{5,3} + c_{5,4} + c_{5,5} + Dh\right) B_5[t];$$

$$dM_1 = -M_1[t]\,q_1 + \left(\frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} + \frac{nuK\,M_5[t]}{denK + M_5[t]}\right) \left(-B_1[t]\,d_{1,1} - B_2[t]\,d_{1,2} - B_3[t]\,d_{1,3} - B_4[t]\,d_{1,4} - B_5[t]\,d_{1,5}\right) + B_1[t]\,\Omega_{1,1} + B_2[t]\,\Omega_{1,2} + B_3[t]\,\Omega_{1,3} + B_4[t]\,\Omega_{1,4} + B_5[t]\,\Omega_{1,5};$$

$$dM_2 = -M_2[t]\,q_2 + \left(\frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} + \frac{nuK\,M_5[t]}{denK + M_5[t]}\right) \left(-B_1[t]\,d_{2,1} - B_2[t]\,d_{2,2} - B_3[t]\,d_{2,3} - B_4[t]\,d_{2,4} - B_5[t]\,d_{2,5}\right) + B_1[t]\,\Omega_{2,1} + B_2[t]\,\Omega_{2,2} + B_3[t]\,\Omega_{2,3} + B_4[t]\,\Omega_{2,4} + B_5[t]\,\Omega_{2,5};$$

$$dM_3 = -M_3[t]\,q_3 + \left(\frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} + \frac{nuK\,M_5[t]}{denK + M_5[t]}\right) \left(-B_1[t]\,d_{3,1} - B_2[t]\,d_{3,2} - B_3[t]\,d_{3,3} - B_4[t]\,d_{3,4} - B_5[t]\,d_{3,5}\right) + B_1[t]\,\Omega_{3,1} + B_2[t]\,\Omega_{3,2} + B_3[t]\,\Omega_{3,3} + B_4[t]\,\Omega_{3,4} + B_5[t]\,\Omega_{3,5};$$

$$dM_4 = -M_4[t]\,q_4 + \Bigg(\frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} +$$

$$\frac{nuK\,M_5[t]}{denK+M_5[t]}\Big)\left(-B_1[t]\,d_{4,1}-B_2[t]\,d_{4,2}-B_3[t]\,d_{4,3}-B_4[t]\,d_{4,4}-B_5[t]\,d_{4,5}\right)+$$

$$B_1[t]\,\Omega_{4,1}+B_2[t]\,\Omega_{4,2}+B_3[t]\,\Omega_{4,3}+B_4[t]\,\Omega_{4,4}+B_5[t]\,\Omega_{4,5};$$

$$dM_5=-M_5[t]\,q_5+\left(\frac{nuK\,M_1[t]}{denK+M_1[t]}+\frac{nuK\,M_2[t]}{denK+M_2[t]}+\frac{nuK\,M_3[t]}{denK+M_3[t]}+\frac{nuK\,M_4[t]}{denK+M_4[t]}+\frac{nuK\,M_5[t]}{denK+M_5[t]}\right)\left(-B_1[t]\,d_{5,1}-B_2[t]\,d_{5,2}-B_3[t]\,d_{5,3}-B_4[t]\,d_{5,4}-B_5[t]\,d_{5,5}\right)+$$

$$B_1[t]\,\Omega_{5,1}+B_2[t]\,\Omega_{5,2}+B_3[t]\,\Omega_{5,3}+B_4[t]\,\Omega_{5,4}+B_5[t]\,\Omega_{5,5};$$

```
KK = 0.2;
cc = 0.05;
qq = 0.3;
dd = 0.0015;
OM = 1;
nu = 20;
den = 5;


op = coop; (*Number of links with overExpression*)
posNe = Position[Net, 1];
(*Positions in the matrix where there are links (=1)*)
RaN = RandomSample[posNe, op];
(*Random sample of op links that will be overproduced*)

costincr = 1.3; (*Term multiplying the cost link*)
overprodincr = 1.15;
(*Term multiplying the overproduction link*)


NewNetCost = Net cc;
Table[NewNetCost[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] =
  NewNetCost[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] * costincr, {i, Length[RaN]}];

NewNetOvProd = Net OM;
Table[NewNetOvProd[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] =
  NewNetOvProd[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] * overprodincr, {i,
  Length[RaN]}];


tmax = 1000;
par = {
  κ_1 → KK, κ_2 → KK, κ_3 → KK, κ_4 → KK, κ_5 → KK,

  c_{1,1} → NewNetCost[[1]][[1]],
```

```
c_{1,2} -> NewNetCost[[1]][[2]], c_{1,3} -> NewNetCost[[1]][[3]],
c_{1,4} -> NewNetCost[[1]][[4]], c_{1,5} -> NewNetCost[[1]][[5]],
c_{2,1} -> NewNetCost[[2]][[1]], c_{2,2} -> NewNetCost[[2]][[2]],
c_{2,3} -> NewNetCost[[2]][[3]], c_{2,4} -> NewNetCost[[2]][[4]],
c_{2,5} -> NewNetCost[[2]][[5]],
c_{3,1} -> NewNetCost[[3]][[1]], c_{3,2} -> NewNetCost[[3]][[2]],
c_{3,3} -> NewNetCost[[3]][[3]], c_{3,4} -> NewNetCost[[3]][[4]],
c_{3,5} -> NewNetCost[[3]][[5]],
c_{4,1} -> NewNetCost[[4]][[1]], c_{4,2} -> NewNetCost[[4]][[2]],
c_{4,3} -> NewNetCost[[4]][[3]], c_{4,4} -> NewNetCost[[4]][[4]],
c_{4,5} -> NewNetCost[[4]][[5]],
c_{5,1} -> NewNetCost[[5]][[1]], c_{5,2} -> NewNetCost[[5]][[2]],
c_{5,3} -> NewNetCost[[5]][[3]], c_{5,4} -> NewNetCost[[5]][[4]],
c_{5,5} -> NewNetCost[[5]][[5]],

q_1 -> qq, q_2 -> qq, q_3 -> qq, q_4 -> qq, q_5 -> qq,

d_{1,1} -> dd, d_{1,2} -> dd, d_{1,3} -> dd, d_{1,4} -> dd, d_{1,5} -> dd,
d_{2,1} -> dd, d_{2,2} -> dd, d_{2,3} -> dd, d_{2,4} -> dd, d_{2,5} -> dd,
d_{3,1} -> dd, d_{3,2} -> dd, d_{3,3} -> dd, d_{3,4} -> dd, d_{3,5} -> dd,
d_{4,1} -> dd, d_{4,2} -> dd, d_{4,3} -> dd, d_{4,4} -> dd, d_{4,5} -> dd,
d_{5,1} -> dd, d_{5,2} -> dd, d_{5,3} -> dd, d_{5,4} -> dd, d_{5,5} -> dd,

Ω_{1,1} -> NewNetOvProd[[1]][[1]],
Ω_{1,2} -> NewNetOvProd[[1]][[2]], Ω_{1,3} -> NewNetOvProd[[1]][[3]],
Ω_{1,4} -> NewNetOvProd[[1]][[4]], Ω_{1,5} -> NewNetOvProd[[1]][[5]],
Ω_{2,1} -> NewNetOvProd[[2]][[1]], Ω_{2,2} -> NewNetOvProd[[2]][[2]],
Ω_{2,3} -> NewNetOvProd[[2]][[3]], Ω_{2,4} -> NewNetOvProd[[2]][[4]],
Ω_{2,5} -> NewNetOvProd[[2]][[5]],
Ω_{3,1} -> NewNetOvProd[[3]][[1]], Ω_{3,2} -> NewNetOvProd[[3]][[2]],
Ω_{3,3} -> NewNetOvProd[[3]][[3]], Ω_{3,4} -> NewNetOvProd[[3]][[4]],
Ω_{3,5} -> NewNetOvProd[[3]][[5]],
Ω_{4,1} -> NewNetOvProd[[4]][[1]], Ω_{4,2} -> NewNetOvProd[[4]][[2]],
Ω_{4,3} -> NewNetOvProd[[4]][[3]], Ω_{4,4} -> NewNetOvProd[[4]][[4]],
Ω_{4,5} -> NewNetOvProd[[4]][[5]],
Ω_{5,1} -> NewNetOvProd[[5]][[1]], Ω_{5,2} -> NewNetOvProd[[5]][[2]],
Ω_{5,3} -> NewNetOvProd[[5]][[3]], Ω_{5,4} -> NewNetOvProd[[5]][[4]],
Ω_{5,5} -> NewNetOvProd[[5]][[5]],
nuK -> nu,
denK -> den


};
```

```
B10 = 1500;
B20 = 1500;
B30 = 1500;
B40 = 1500;
B50 = 1500;
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;


sol =
 NDSolve[
  {
    B1'[t] == dB1,
    B2'[t] == dB2,
    B3'[t] == dB3,
    B4'[t] == dB4,
    B5'[t] == dB5,

    M1'[t] == dM1,
    M2'[t] == dM2,
    M3'[t] == dM3,
    M4'[t] == dM4,
    M5'[t] == dM5,


    B1[0] == B10,
    B2[0] == B20,
    B3[0] == B30,
    B4[0] == B40,
    B5[0] == B50,
    M1[0] == M10,
    M2[0] == M20,
    M3[0] == M30,
    M4[0] == M40,
    M5[0] == M50

   } /. par,
  {B1, B2, B3, B4, B5, M1, M2, M3, M4, M5},
  {t, 0, tmax}];
```

```
  {B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax],
     M1[tmax], M2[tmax], M3[tmax], M4[tmax], M5[tmax]} /. sol /. par;

  Min[{B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax]} /. sol /. par]

 )
```

In[6734]:=
```
robustnessNewMonoOV[NetTop_, coop_] := (
  n1 = 1;
  n2 = 5000;
  mid = (n1 + n2) / 2;

  While[(n1 ≠ mid && n2 ≠ mid),
   (If[fNewMonoOV[NetTop, mid, coop] < 1, n2 = mid, n1 = mid];
    mid = Floor[N[(n1 + n2) / 2]];); {n1, n2, mid}]; mid

 )
```

In[6735]:=
```
NetK = {
   {0, 1, 0, 1, 0},
   {1, 0, 1, 1, 0},
   {1, 0, 1, 0, 1},
   {0, 1, 0, 1, 0},
   {0, 0, 0, 0, 1}
  };
```

Compare the Robustness with and without (5 links) overproduction (ratio cost/production = 1.3/1.15)

In[6736]:=
```
fNewMono[NetK, 1]
```

Out[6736]= 492.308

In[6737]:=
```
fNewMono[NetK, 20]
```

Out[6737]= 396.851

In[6738]:=
```
fNewMonoOV[NetK, 1, 5]
```
Out[6738]=
```
492.322
```

In[6739]:=
```
fNewMonoOV[NetK, 20, 5]
```
Out[6739]=
```
397.301
```

In[6740]:=
```
AuxoComm8Additive
```
Out[6740]=
```
{90, 90, 91, 91, 90, 90, 88, 91, 91, 90, 91, 91, 90, 91, 91, 90, 90, 90, 91, 88,
 90, 90, 91, 90, 90, 91, 91, 88, 91, 91, 91, 88, 91, 91, 91, 91, 90, 91, 91, 91,
 90, 91, 91, 91, 91, 91, 90, 90, 91, 90, 90, 91, 90, 88, 91, 91, 88, 91, 91, 91,
 90, 90, 91, 91, 91, 90, 90, 91, 91, 91, 91, 90, 91, 91, 91, 91, 91, 90, 90, 91,
 90, 91, 91, 90, 90, 91, 91, 91, 90, 90, 87, 91, 91, 91, 91, 91, 90, 90, 91, 91}
```

In[6741]:=
```
coop5to15Add =
  {Table[robustnessNewMonoOV[#, 5], {20}], Table[robustnessNewMonoOV[#, 10],
     {20}], Table[robustnessNewMonoOV[#, 15], {20}]} &;
```

In[6742]:=
```
wf8Add = Parallelize[coop5to15Add /@ hk8];
```

In[6743]:=
```
wf8NormalizedAdd = N[wf8Add[[#]] / AuxoComm8Additive[[#]]] & /@ Range[100]
```

In[6744]:=
```
wf8NormalizedWith5CoopAdd = wf8NormalizedAdd[[#]][[1]] & /@ Range[100]
```

In[6745]:=
```
wf8NormalizedWith10CoopAdd = wf8NormalizedAdd[[#]][[2]] & /@ Range[100]
```

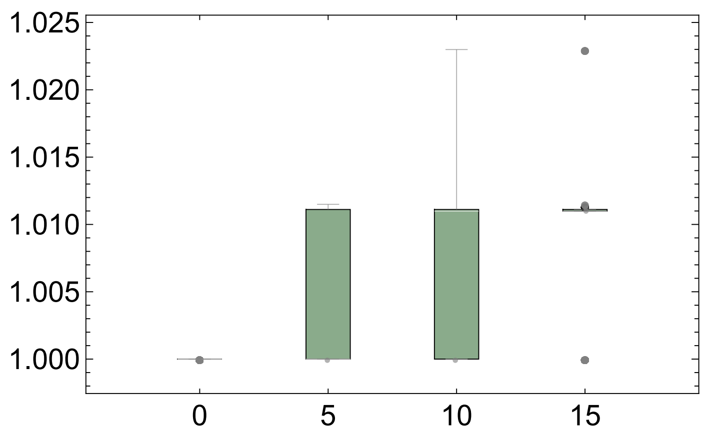
In[6746]:=
```
wf8NormalizedWith15CoopAdd = wf8NormalizedAdd[[#]][[3]] & /@ Range[100]
```

In[6747]:=
```
allcoopWith8AuxoAdd = {Flatten[wf8NormalizedWith5CoopAdd],
  Flatten[wf8NormalizedWith10CoopAdd], Flatten[wf8NormalizedWith15CoopAdd]}
```

In[6748]:=
```
allcoopWith8AuxoPlusAuxoAdd =
 Join[{ConstantArray[1, {2000}]}, allcoopWith8AuxoAdd]
```

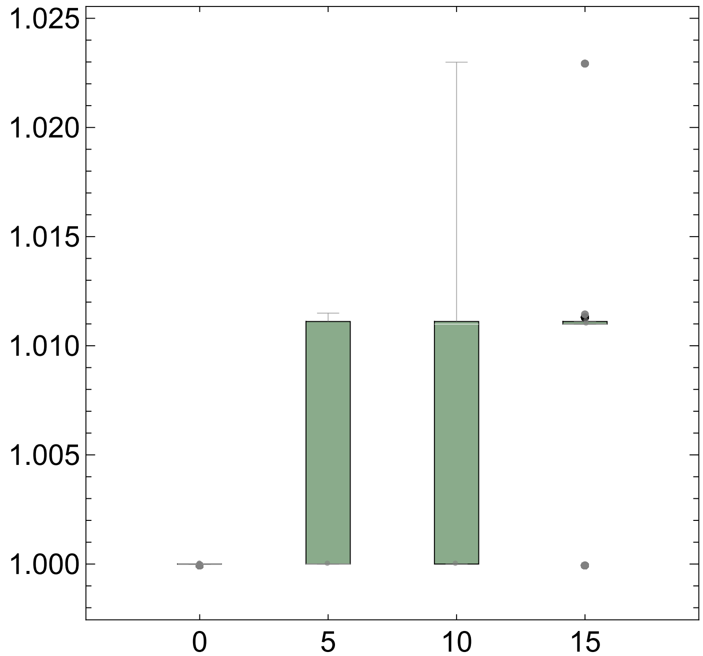
In[6749]:=
```
BoxWhiskerChart[allcoopWith8AuxoPlusAuxoAdd, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{gree1}},
 Frame → True, ChartLabels → {"0", "5", "10", "15"},
 BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15]]
```

Out[6749]=



In[6750]:=
```
BoxWhiskerChart[allcoopWith8AuxoPlusAuxoAdd, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{gree1}},
 Frame → True, ChartLabels → {"0", "5", "10", "15"}, BarSpacing → 1.9,
 FrameStyle → Directive[Black, FontSize → 15], AspectRatio → 1]
```

Out[6750]=



In[6751]:=
```
allcoopWith8AuxoPlusAuxoAdd // Length
```

Out[6751]= 4

In[6752]:=
```
SignedRankTest[allcoopWith8AuxoPlusAuxoAdd[[2]], 1]
SignedRankTest[allcoopWith8AuxoPlusAuxoAdd[[3]], 1]
SignedRankTest[allcoopWith8AuxoPlusAuxoAdd[[4]], 1]
```

Out[6752]= 1.24036×10^{-122}

Out[6753]= 9.45081×10^{-250}

Out[6754]= 1.01108×10^{-307}

Solving the system of ODE Random parametrization

In[6763]:=

```
Knum = 0.2;
ccrnum = 0.05;
qqrnum = 0.3;
ddrnum = 0.0015;
OMrnum = 1;
nurum = 20;
den2rum = 5;

corrpar0 = 10^3;
corrpar1 = 10^4;
corrpar2 = 10^6;

KKr := RandomVariate[
    GammaDistribution[ corrpar0 Sqrt[Knum], (1/corrpar0) Sqrt[Knum]], 1][[1]];
ccr := RandomVariate[GammaDistribution[ corrpar1 Sqrt[ccrnum],
     (1 / corrpar1) Sqrt[ccrnum]], 1][[1]];
qqr := RandomVariate[GammaDistribution[ corrpar0 Sqrt[qqrnum],
     (1/corrpar0) Sqrt[qqrnum]], 1][[1]];
ddr := RandomVariate[GammaDistribution[ corrpar1 Sqrt[ddrnum],
     (1 / corrpar1) Sqrt[ddrnum]], 1][[1]];
OMr := RandomVariate[GammaDistribution[ corrpar0 Sqrt[OMrnum],
     (1/corrpar0) Sqrt[OMrnum]], 1][[1]];
nur := RandomVariate[GammaDistribution[ corrpar2 Sqrt[nurum],
     (1/corrpar2) Sqrt[nurum]], 1][[1]] ;
denr2 := RandomVariate[GammaDistribution[ corrpar2 Sqrt[den2rum],
     (1/corrpar2) Sqrt[den2rum]], 1][[1]] ;


parR = Join[Table[KKr, {5}], Table[ccr, {25}],
  Table[qqr, {5}], Table[ddr, {25}], Table[OMr, {25}], {nur}, {denr2}]
```

Out[6780]= {0.189848, 0.204247, 0.194102, 0.201048, 0.190936, 0.0517625, 0.0500403, 0.0478304, 0.050131, 0.0507092, 0.0476753, 0.0489554, 0.0496724, 0.0496858, 0.0506048, 0.0500907, 0.0502637, 0.0500698, 0.0510821, 0.0517472, 0.0482529, 0.0488776, 0.049362, 0.0487134, 0.051578, 0.0514061, 0.0474186, 0.0509663, 0.0507707, 0.0504926, 0.293164, 0.314404, 0.28339, 0.31854, 0.296673, 0.00137794, 0.00150021, 0.00158888, 0.00147558, 0.00156729, 0.00146327, 0.00146845, 0.00150834, 0.00151975, 0.00158622, 0.00165019, 0.00141246, 0.00147366, 0.00152076, 0.00145163, 0.00153074, 0.00140519, 0.00152421, 0.00157961, 0.00145304, 0.00146812, 0.00143831, 0.00142943, 0.00155001, 0.00156199, 1.00557, 0.98471, 1.03208, 1.0013, 0.982572, 0.931254, 0.981258, 1.01304, 1.06027, 0.993983, 1.02517, 0.954885, 1.02495, 0.964104, 1.02774, 0.956126, 0.944756, 0.974591, 0.976226, 1.03274, 1.03533, 0.983036, 1.01541, 1.04861, 0.944022, 20.0106, 4.99684}

```
In[6898]:= parR = {0.1898476755696357`, 0.20424706235026566`, 0.19410227191526325`,
   0.20104807174115624`, 0.19093630227178493`, 0.05176251606275553`,
   0.05004032350382594`, 0.047830430237250074`, 0.05013100576544391`,
   0.05070917784465817`, 0.04767532536036458`, 0.04895540363002236`,
   0.04967241386807649`, 0.049685841900388036`, 0.050604764350212356`,
   0.050090730107261935`, 0.050263693261385085`, 0.05006978757055582`,
   0.05108206567821288`, 0.05174722507993957`, 0.048252892958383885`,
   0.04887756964147701`, 0.04936200927513005`, 0.0487134396092386`,
   0.05157797631620972`, 0.05140614789276238`, 0.04741856591330015`,
   0.050966311865779136`, 0.05077068055624562`, 0.05049259695307121`,
   0.2931641318539354`, 0.31440353971246704`, 0.2833895200095432`,
   0.31853992275975973`, 0.29667348015598083`, 0.0013779426741653384`,
   0.0015002143828080283`, 0.0015888826744496415`, 0.0014755819298235984`,
   0.0015672858453788427`, 0.0014632722572765372`, 0.0014684535214811048`,
   0.0015083368512277295`, 0.0015197473246003675`, 0.0015862226097913175`,
   0.0016501912086286483`, 0.0014124566225262093`, 0.0014736602902984694`,
   0.001520758018508533`, 0.0014516281582254362`, 0.0015307365023040265`,
   0.0014051921331468674`, 0.0015242113856328854`, 0.0015796105475089693`,
   0.0014530378821914804`, 0.0014681214329693572`, 0.0014383124765171293`,
   0.0014294281413476115`, 0.0015500121202824597`, 0.001561994088615431`,
   1.0055712427818038`, 0.9847096966231175`, 1.032084475338937`,
   1.0013005798981616`, 0.9825717085752652`, 0.9312542093314393`,
   0.981258209094866`, 1.0130393306419623`, 1.060266596062302`,
   0.9939828279325185`, 1.0251666620417295`, 0.9548845859950983`,
   1.024952894164548`, 0.964103960656699`, 1.0277406165759646`,
   0.9561263513840252`, 0.9447564527887095`, 0.9745906981501513`,
   0.9762259958043418`, 1.0327406339460024`, 1.0353308884931731`,
   0.9830359718342756`, 1.0154109730699672`, 1.0486101235760301`,
   0.9440217011604666`, 20.010641145892475`, 4.9968443726640395`};

In[6897]:= Clear[parR]
```

In[6869]:=

$$\text{fNewMonoR[Net\_, Dh\_]} := \Big($$

$$dB_1 = B_1[t]\left(-B_1[t]\,\kappa_1 + \frac{\text{nuK}\,M_1[t]}{\text{denK}+M_1[t]} + \frac{\text{nuK}\,M_2[t]}{\text{denK}+M_2[t]} + \frac{\text{nuK}\,M_3[t]}{\text{denK}+M_3[t]} + \frac{\text{nuK}\,M_4[t]}{\text{denK}+M_4[t]} + \frac{\text{nuK}\,M_5[t]}{\text{denK}+M_5[t]}\right) - \left(c_{1,1}+c_{1,2}+c_{1,3}+c_{1,4}+c_{1,5}+\text{Dh}\right) B_1[t];$$

$$dB_2 = B_2[t]\left(-B_2[t]\,\kappa_2 + \frac{\text{nuK}\,M_1[t]}{\text{denK}+M_1[t]} + \frac{\text{nuK}\,M_2[t]}{\text{denK}+M_2[t]} + \frac{\text{nuK}\,M_3[t]}{\text{denK}+M_3[t]} + \frac{\text{nuK}\,M_4[t]}{\text{denK}+M_4[t]} + \frac{\text{nuK}\,M_5[t]}{\text{denK}+M_5[t]}\right) - \left(c_{2,1}+c_{2,2}+c_{2,3}+c_{2,4}+c_{2,5}+\text{Dh}\right) B_2[t];$$

$$dB_3 = B_3[t]\left(-B_3[t]\,\kappa_3 + \frac{\text{nuK}\,M_1[t]}{\text{denK}+M_1[t]} + \frac{\text{nuK}\,M_2[t]}{\text{denK}+M_2[t]} + \frac{\text{nuK}\,M_3[t]}{\text{denK}+M_3[t]} + \frac{\text{nuK}\,M_4[t]}{\text{denK}+M_4[t]} + \frac{\text{nuK}\,M_5[t]}{\text{denK}+M_5[t]}\right) - \left(c_{3,1}+c_{3,2}+c_{3,3}+c_{3,4}+c_{3,5}+\text{Dh}\right) B_3[t];$$

$$dB_4 = B_4[t]\left(-B_4[t]\,\kappa_4 + \frac{\text{nuK}\,M_1[t]}{\text{denK}+M_1[t]} + \frac{\text{nuK}\,M_2[t]}{\text{denK}+M_2[t]} + \frac{\text{nuK}\,M_3[t]}{\text{denK}+M_3[t]} + \frac{\text{nuK}\,M_4[t]}{\text{denK}+M_4[t]} + \frac{\text{nuK}\,M_5[t]}{\text{denK}+M_5[t]}\right) - \left(c_{4,1}+c_{4,2}+c_{4,3}+c_{4,4}+c_{4,5}+\text{Dh}\right) B_4[t];$$

$$dB_5 = B_5[t]\left(-B_5[t]\,\kappa_5 + \frac{\text{nuK}\,M_1[t]}{\text{denK}+M_1[t]} + \frac{\text{nuK}\,M_2[t]}{\text{denK}+M_2[t]} + \frac{\text{nuK}\,M_3[t]}{\text{denK}+M_3[t]} + \frac{\text{nuK}\,M_4[t]}{\text{denK}+M_4[t]} + \frac{\text{nuK}\,M_5[t]}{\text{denK}+M_5[t]}\right) - \left(c_{5,1}+c_{5,2}+c_{5,3}+c_{5,4}+c_{5,5}+\text{Dh}\right) B_5[t];$$

$$dM_1 = -M_1[t]\,q_1 + \left(\frac{\text{nuK}\,M_1[t]}{\text{denK}+M_1[t]} + \frac{\text{nuK}\,M_2[t]}{\text{denK}+M_2[t]} + \frac{\text{nuK}\,M_3[t]}{\text{denK}+M_3[t]} + \frac{\text{nuK}\,M_4[t]}{\text{denK}+M_4[t]} + \frac{\text{nuK}\,M_5[t]}{\text{denK}+M_5[t]}\right) \left(-B_1[t]\,d_{1,1} - B_2[t]\,d_{1,2} - B_3[t]\,d_{1,3} - B_4[t]\,d_{1,4} - B_5[t]\,d_{1,5}\right) + B_1[t]\,\Omega_{1,1} + B_2[t]\,\Omega_{1,2} + B_3[t]\,\Omega_{1,3} + B_4[t]\,\Omega_{1,4} + B_5[t]\,\Omega_{1,5};$$

$$dM_2 = -M_2[t]\,q_2 + \left(\frac{\text{nuK}\,M_1[t]}{\text{denK}+M_1[t]} + \frac{\text{nuK}\,M_2[t]}{\text{denK}+M_2[t]} + \frac{\text{nuK}\,M_3[t]}{\text{denK}+M_3[t]} + \frac{\text{nuK}\,M_4[t]}{\text{denK}+M_4[t]} + \frac{\text{nuK}\,M_5[t]}{\text{denK}+M_5[t]}\right) \left(-B_1[t]\,d_{2,1} - B_2[t]\,d_{2,2} - B_3[t]\,d_{2,3} - B_4[t]\,d_{2,4} - B_5[t]\,d_{2,5}\right) + B_1[t]\,\Omega_{2,1} + B_2[t]\,\Omega_{2,2} + B_3[t]\,\Omega_{2,3} + B_4[t]\,\Omega_{2,4} + B_5[t]\,\Omega_{2,5};$$

$$dM_3 = -M_3[t]\,q_3 + \left(\frac{\text{nuK}\,M_1[t]}{\text{denK}+M_1[t]} + \frac{\text{nuK}\,M_2[t]}{\text{denK}+M_2[t]} + \frac{\text{nuK}\,M_3[t]}{\text{denK}+M_3[t]} + \frac{\text{nuK}\,M_4[t]}{\text{denK}+M_4[t]} + \frac{\text{nuK}\,M_5[t]}{\text{denK}+M_5[t]}\right) \left(-B_1[t]\,d_{3,1} - B_2[t]\,d_{3,2} - B_3[t]\,d_{3,3} - B_4[t]\,d_{3,4} - B_5[t]\,d_{3,5}\right) + B_1[t]\,\Omega_{3,1} + B_2[t]\,\Omega_{3,2} + B_3[t]\,\Omega_{3,3} + B_4[t]\,\Omega_{3,4} + B_5[t]\,\Omega_{3,5};$$

$$dM_4 = -M_4[t]\,q_4 + \left(\frac{\text{nuK}\,M_1[t]}{\text{denK}+M_1[t]} + \frac{\text{nuK}\,M_2[t]}{\text{denK}+M_2[t]} + \frac{\text{nuK}\,M_3[t]}{\text{denK}+M_3[t]} + \frac{\text{nuK}\,M_4[t]}{\text{denK}+M_4[t]} + \right.$$

```
        nuK M5[t]
      ----------- ) (-B1[t] d4,1 - B2[t] d4,2 - B3[t] d4,3 - B4[t] d4,4 - B5[t] d4,5) +
      denK + M5[t]
   B1[t] Ω4,1 + B2[t] Ω4,2 + B3[t] Ω4,3 + B4[t] Ω4,4 + B5[t] Ω4,5;
dM5 = -M5[t] q5 + ( nuK M1[t]/(denK + M1[t]) + nuK M2[t]/(denK + M2[t]) + nuK M3[t]/(denK + M3[t]) + nuK M4[t]/(denK + M4[t]) +
      nuK M5[t]/(denK + M5[t]) ) (-B1[t] d5,1 - B2[t] d5,2 - B3[t] d5,3 - B4[t] d5,4 - B5[t] d5,5) +
   B1[t] Ω5,1 + B2[t] Ω5,2 + B3[t] Ω5,3 + B4[t] Ω5,4 + B5[t] Ω5,5;



tmax = 1000;
par = {
   κ1 → parR[[1]], κ2 → parR[[2]], κ3 → parR[[3]], κ4 → parR[[4]], κ5 → parR[[5]],

   c1,1 → parR[[6]] × Net[[1]][[1]],
   c1,2 → parR[[7]] × Net[[1]][[2]], c1,3 → parR[[8]] × Net[[1]][[3]],
   c1,4 → parR[[9]] × Net[[1]][[4]], c1,5 → parR[[10]] × Net[[1]][[5]],
   c2,1 → parR[[11]] × Net[[2]][[1]], c2,2 → parR[[12]] × Net[[2]][[2]],
   c2,3 → parR[[13]] × Net[[2]][[3]], c2,4 → parR[[14]] × Net[[2]][[4]],
   c2,5 → parR[[15]] × Net[[2]][[5]],
   c3,1 → parR[[16]] × Net[[3]][[1]], c3,2 → parR[[17]] × Net[[3]][[2]],
   c3,3 → parR[[18]] × Net[[3]][[3]], c3,4 → parR[[19]] × Net[[3]][[4]],
   c3,5 → parR[[20]] × Net[[3]][[5]],
   c4,1 → parR[[21]] × Net[[4]][[1]], c4,2 → parR[[22]] × Net[[4]][[2]],
   c4,3 → parR[[23]] × Net[[4]][[3]], c4,4 → parR[[24]] × Net[[4]][[4]],
   c4,5 → parR[[25]] × Net[[4]][[5]],
   c5,1 → parR[[26]] × Net[[5]][[1]], c5,2 → parR[[27]] × Net[[5]][[2]],
   c5,3 → parR[[28]] × Net[[5]][[3]], c5,4 → parR[[29]] × Net[[5]][[4]],
   c5,5 → parR[[30]] × Net[[5]][[5]],

   q1 → parR[[31]], q2 → parR[[32]],
   q3 → parR[[33]], q4 → parR[[34]], q5 → parR[[35]],

   d1,1 → parR[[36]], d1,2 → parR[[37]],
   d1,3 → parR[[38]], d1,4 → parR[[39]], d1,5 → parR[[40]],
   d2,1 → parR[[41]], d2,2 → parR[[42]], d2,3 → parR[[43]],
   d2,4 → parR[[44]], d2,5 → parR[[45]],
   d3,1 → parR[[46]], d3,2 → parR[[47]], d3,3 → parR[[48]],
   d3,4 → parR[[49]], d3,5 → parR[[50]],
   d4,1 → parR[[51]], d4,2 → parR[[52]], d4,3 → parR[[53]],
   d4,4 → parR[[54]], d4,5 → parR[[55]],
   d5,1 → parR[[56]], d5,2 → parR[[57]], d5,3 → parR[[58]],
   d5,4 → parR[[59]], d5,5 → parR[[60]],
```

```
  Ω1,1 → parR[[61]] × Net[[1]][[1]],
  Ω1,2 → parR[[62]] × Net[[1]][[2]], Ω1,3 → parR[[63]] × Net[[1]][[3]],
  Ω1,4 → parR[[64]] × Net[[1]][[4]], Ω1,5 → parR[[65]] × Net[[1]][[5]],
  Ω2,1 → parR[[66]] × Net[[2]][[1]], Ω2,2 → parR[[67]] × Net[[2]][[2]],
  Ω2,3 → parR[[68]] × Net[[2]][[3]], Ω2,4 → parR[[69]] × Net[[2]][[4]],
  Ω2,5 → parR[[70]] × Net[[2]][[5]],
  Ω3,1 → parR[[71]] × Net[[3]][[1]], Ω3,2 → parR[[72]] × Net[[3]][[2]],
  Ω3,3 → parR[[73]] × Net[[3]][[3]], Ω3,4 → parR[[74]] × Net[[3]][[4]],
  Ω3,5 → parR[[75]] × Net[[3]][[5]],
  Ω4,1 → parR[[76]] × Net[[4]][[1]], Ω4,2 → parR[[77]] × Net[[4]][[2]],
  Ω4,3 → parR[[78]] × Net[[4]][[3]], Ω4,4 → parR[[79]] × Net[[4]][[4]],
  Ω4,5 → parR[[80]] × Net[[4]][[5]],
  Ω5,1 → parR[[81]] × Net[[5]][[1]], Ω5,2 → parR[[82]] × Net[[5]][[2]],
  Ω5,3 → parR[[83]] × Net[[5]][[3]], Ω5,4 → parR[[84]] × Net[[5]][[4]],
  Ω5,5 → parR[[85]] × Net[[5]][[5]],
  nuK → parR[[86]],
  denK → parR[[87]]


 };


B10 = 1500;
B20 = 1500;
B30 = 1500;
B40 = 1500;
B50 = 1500;
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;


sol =
 NDSolve[
  {
    B1'[t] == dB1,
    B2'[t] == dB2,
    B3'[t] == dB3,
    B4'[t] == dB4,
    B5'[t] == dB5,
```

```
      M1'[t] == dM1,
      M2'[t] == dM2,
      M3'[t] == dM3,
      M4'[t] == dM4,
      M5'[t] == dM5,



      B1[0] == B10,
      B2[0] == B20,
      B3[0] == B30,
      B4[0] == B40,
      B5[0] == B50,
      M1[0] == M10,
      M2[0] == M20,
      M3[0] == M30,
      M4[0] == M40,
      M5[0] == M50

     } /. par,
    {B1, B2, B3, B4, B5, M1, M2, M3, M4, M5},
    {t, 0, tmax}];

  {B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax],
     M1[tmax], M2[tmax], M3[tmax], M4[tmax], M5[tmax]} /. sol /. par;

  Min[{B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax]} /. sol /. par]

 )
```

In[6870]:=

```
robustnessNewMonoR[NetTop_] := (
  n1 = 1;
  n2 = 5000;
  mid = (n1 + n2) / 2;

  While[(n1 ≠ mid && n2 ≠ mid),
   (If[fNewMonoR[NetTop, mid] < 1, n2 = mid, n1 = mid];
    mid = Floor[N[(n1 + n2) / 2]];); {n1, n2, mid}]; mid

 )
```

As an example let's take the following Network

In[6783]:=
```
NetK = {
    {0, 1, 0, 1, 0},
    {1, 0, 1, 1, 0},
    {1, 0, 1, 0, 1},
    {0, 1, 0, 1, 0},
    {0, 0, 0, 0, 1}
   };
```

Using the function fNewSaito we can calculate the smallest value of a bacterial population in the community for a given disturbance vale. For example, let's take Disturbance value 1 and 500:

In[6759]:=
```
fNewMono[NetK, 0]
```

Out[6759]= 497.327

In[6760]:=
```
fNewMono[NetK, 500]
```

Out[6760]= 1.95776×10^{-49}

In[6784]:=
```
fNewMonoR[NetK, 0]
```

Out[6784]= 487.136

In[6785]:=
```
fNewMonoR[NetK, 500]
```

Out[6785]= -3.31846×10^{-67}

Using the function fNewSaito we can calculate Robustness of the Network:

In[6787]:=
```
robustnessNewMono[NetK]
```

Out[6787]= 86

In[6788]:=
```
robustnessNewMonoR[NetK]
```

Out[6788]= 85

We can calculate the (Relative) Entropy and the Assortativity:

In[ ]:=
```
RelatEntrop5[NetK]
```

Out[ ]= 0.960956

In[•]:=
```
assortativity[NetK]
```
Out[•]=
```
-0.113228
```

We can calculate the robustness of the previously generated random networks with different number of auxotrophies:

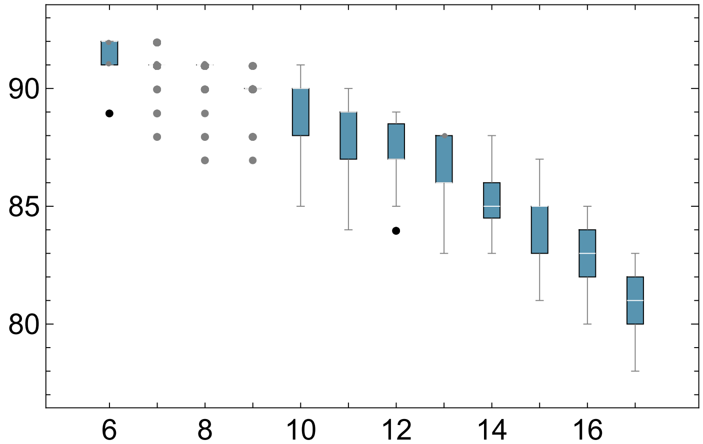
In[6789]:=
```
AuxoComm6RAdd = Parallelize[robustnessNewMonoR /@ hk6];
AuxoComm7RAdd = Parallelize[robustnessNewMonoR /@ hk7];
AuxoComm8RAdd = Parallelize[robustnessNewMonoR /@ hk8];
AuxoComm9RAdd = Parallelize[robustnessNewMonoR /@ hk9];
AuxoComm10RAdd = Parallelize[robustnessNewMonoR /@ hk10];
AuxoComm11RAdd = Parallelize[robustnessNewMonoR /@ hk11];
AuxoComm12RAdd = Parallelize[robustnessNewMonoR /@ hk12];
AuxoComm13RAdd = Parallelize[robustnessNewMonoR /@ hk13];
AuxoComm14RAdd = Parallelize[robustnessNewMonoR /@ hk14];
AuxoComm15RAdd = Parallelize[robustnessNewMonoR /@ hk15];
AuxoComm16RAdd = Parallelize[robustnessNewMonoR /@ hk16];
AuxoComm17RAdd = Parallelize[robustnessNewMonoR /@ hk17];
```

In[6801]:=
```
LikRAdd = {AuxoComm6RAdd, AuxoComm7RAdd, AuxoComm8RAdd, AuxoComm9RAdd,
   AuxoComm10RAdd, AuxoComm11RAdd, AuxoComm12RAdd, AuxoComm13RAdd,
   AuxoComm14RAdd, AuxoComm15RAdd, AuxoComm16RAdd, AuxoComm17RAdd};
```

In[6802]:=
```
coco = RGBColor[0.34509803921568627, 0.5803921568627451, 0.6901960784313725]
```
Out[6802]= 

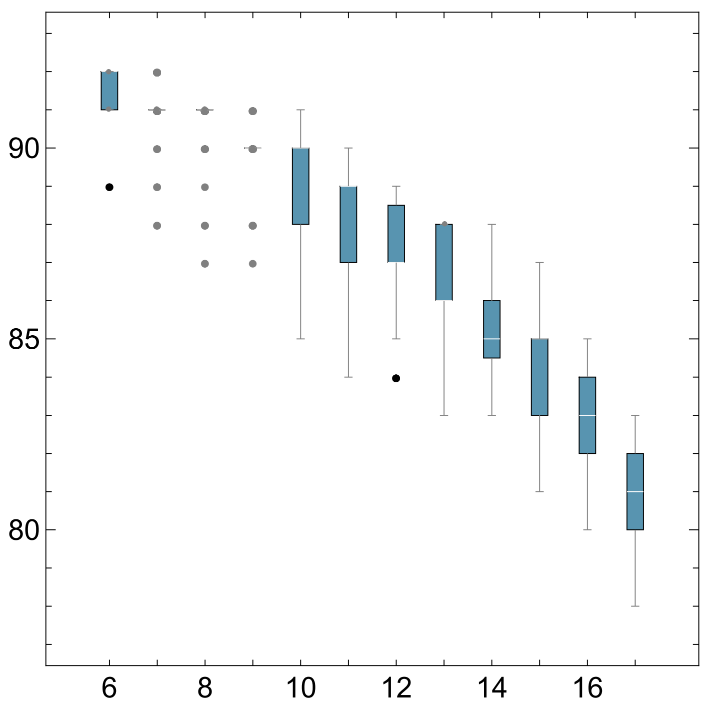
In[6803]:=
```
BoxWhiskerChart[LikRAdd, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{coco}}, Frame → True,
 ChartLabels → {"6", "", "8", "", "10", "", "12", "", "14", "", "16", ""},
 BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15]]
```

Out[6803]=



In[6804]:=
```
BoxWhiskerChart[LikRAdd, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{coco}}, Frame → True,
 ChartLabels → {"6", "", "8", "", "10", "", "12", "", "14", "", "16", ""},
 BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15], AspectRatio → 1]
```

Out[6804]=



In[6805]:=
```
AuxoComm8RAdd
```

Out[6805]=
```
{90, 90, 90, 91, 90, 90, 88, 91, 91, 90, 91, 91, 90, 91, 91, 91, 91, 90, 91, 89,
 91, 91, 91, 90, 91, 91, 91, 88, 91, 91, 91, 88, 91, 91, 91, 91, 91, 91, 91, 91,
 91, 91, 91, 91, 91, 91, 91, 91, 91, 91, 91, 91, 91, 87, 91, 91, 88, 91, 91, 91,
 91, 91, 91, 91, 91, 91, 91, 91, 91, 91, 91, 90, 91, 91, 91, 91, 91, 90, 90, 91,
 90, 91, 91, 91, 90, 91, 91, 91, 91, 91, 87, 91, 91, 91, 91, 91, 91, 91, 91, 91}
```

We can study the correlation between Relative entropy and assortativity with Robustness for Networks with 7 auxotrophies.

In[•]:=
```
Entropy8 = RelatEntrop5 /@ hk8;
```

In[•]:=
```
Assort8 = assortativity /@ hk8;
```

In[6806]:=
```
RobustAdd8R = AuxoComm8RAdd;
```

In[6807]:=
```
Length[Entropy8]
Length[Assort8]
Length[RobustAdd8R]
```

Out[6807]= 100

Out[6808]= 100

Out[6809]= 100

In[•]:=
```
{Min[Entropy8], Max[Entropy8]}
{Min[Assort8], Max[Assort8]}
```
Out[•]= {0.9188, 0.993652}

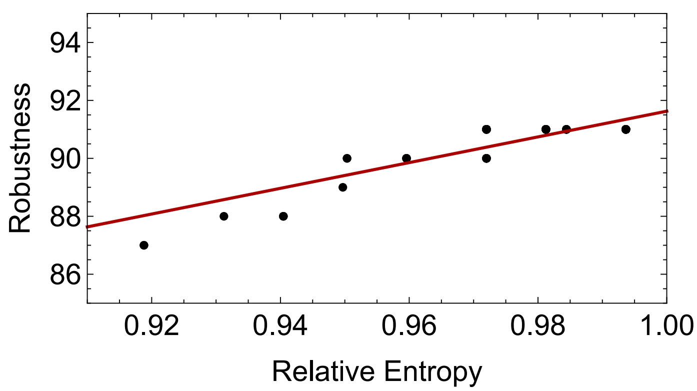
Out[•]= {-0.520325, 0.235702}

In[•]:=
```
Position[Entropy8, Min[Entropy8]]
```
Out[•]= {{54}, {91}}

In[6810]:=
```
RobustAdd8R[[#]] & /@ {1, 2, 24}
```
Out[6810]= {90, 90, 90}

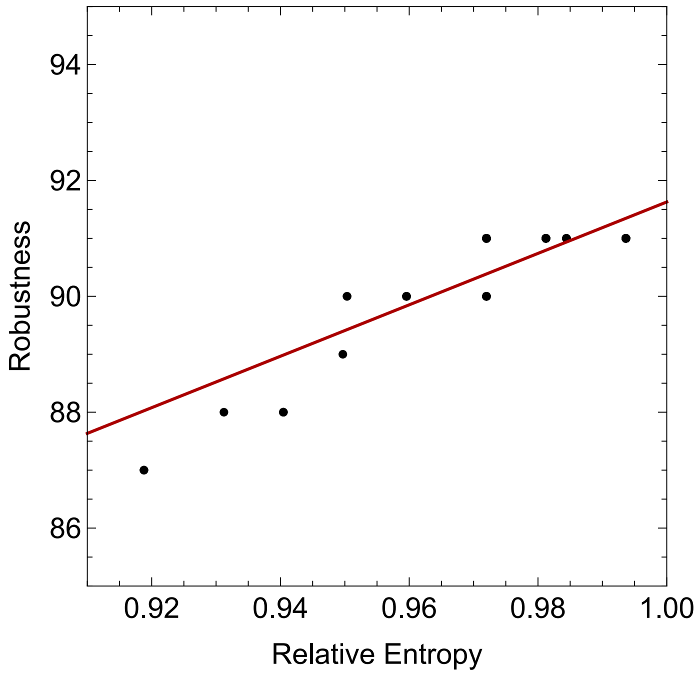
In[6811]:=
```
{Min[RobustAdd8R], {Max[RobustAdd8R]}}
```
Out[6811]= {87, {91}}

In[6812]:=
```
linerobustnessAdd8R = Fit[Partition[Riffle[Entropy8, RobustAdd8R], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Entropy8, RobustAdd8R], {2}],
  Frame → True, FrameLabel → {"Relative Entropy", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]},
  PlotRange → {{0.91, 1}, {85, 95}}, AspectRatio → 0.5],
 Plot[linerobustnessAdd8R, {x, 0.91, 1}, AspectRatio → 0.5, PlotStyle → Darker[Red]]]
```
Out[6813]=



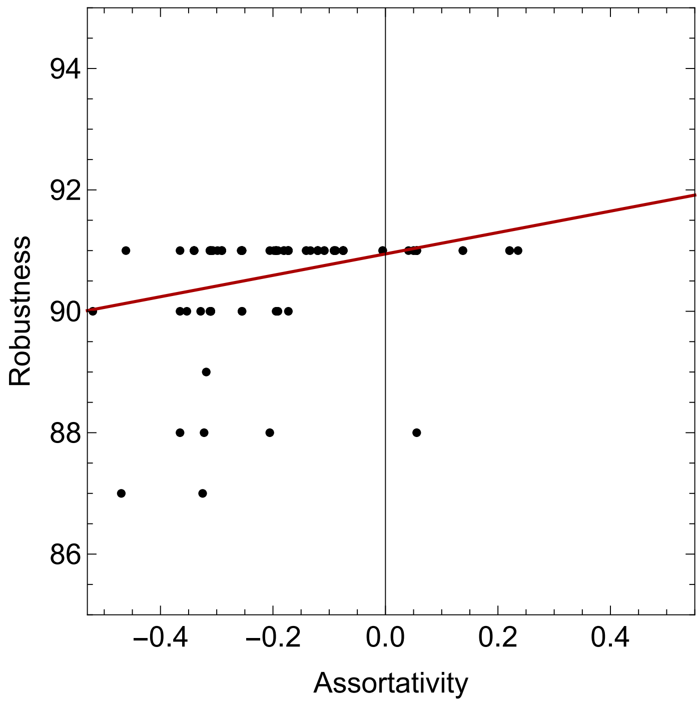
In[6814]:=
```
linerobustnessAdd8R = Fit[Partition[Riffle[Entropy8, RobustAdd8R], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Entropy8, RobustAdd8R], {2}],
  Frame → True, FrameLabel → {"Relative Entropy", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]},
  PlotRange → {{0.91, 1}, {85, 95}}, AspectRatio → 1],
 Plot[linerobustnessAdd8R, {x, 0.91, 1}, AspectRatio → 1, PlotStyle → Darker[Red]]]
```

Out[6815]=



In[7534]:=
```
lineAssoRobrobustnessAdd8R =
  Fit[Partition[Riffle[Assort8, RobustAdd8R], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Assort8, RobustAdd8R], {2}],
  Frame → True, FrameLabel → {"Assortativity", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]}, PlotRange → {{-0.53, 0.55}, {85, 95}},
  AspectRatio → 1], Plot[lineAssoRobrobustnessAdd8R,
  {x, -0.53, 0.55}, AspectRatio → 1, PlotStyle → Darker[Red]]]
```

Out[7535]=



In[6818]:=
```
SpearmanRankTest[Entropy8, RobustAdd8R, "TestDataTable"]
```

Out[6818]=

	Statistic	P-Value
Spearman Rank	0.671501	2.06742×10^{-14}

In[6819]:=
```
SpearmanRankTest[Assort8, RobustAdd8R, "TestDataTable"]
```

Out[6819]=

	Statistic	P-Value
Spearman Rank	0.43009	7.9719×10^{-6}

Solving the system of ODE with Overproduction Random parametrization

In[6899]:=

```
fNewMonoOVR[Net_, Dh_, coop_] := (
```

$$dB_1 = B_1[t]\left(-B_1[t]\,\kappa_1 + \frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} + \frac{nuK\,M_5[t]}{denK + M_5[t]}\right) - \left(c_{1,1} + c_{1,2} + c_{1,3} + c_{1,4} + c_{1,5} + Dh\right) B_1[t];$$

$$dB_2 = B_2[t]\left(-B_2[t]\,\kappa_2 + \frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} + \frac{nuK\,M_5[t]}{denK + M_5[t]}\right) - \left(c_{2,1} + c_{2,2} + c_{2,3} + c_{2,4} + c_{2,5} + Dh\right) B_2[t];$$

$$dB_3 = B_3[t]\left(-B_3[t]\,\kappa_3 + \frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} + \frac{nuK\,M_5[t]}{denK + M_5[t]}\right) - \left(c_{3,1} + c_{3,2} + c_{3,3} + c_{3,4} + c_{3,5} + Dh\right) B_3[t];$$

$$dB_4 = B_4[t]\left(-B_4[t]\,\kappa_4 + \frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} + \frac{nuK\,M_5[t]}{denK + M_5[t]}\right) - \left(c_{4,1} + c_{4,2} + c_{4,3} + c_{4,4} + c_{4,5} + Dh\right) B_4[t];$$

$$dB_5 = B_5[t]\left(-B_5[t]\,\kappa_5 + \frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} + \frac{nuK\,M_5[t]}{denK + M_5[t]}\right) - \left(c_{5,1} + c_{5,2} + c_{5,3} + c_{5,4} + c_{5,5} + Dh\right) B_5[t];$$

$$dM_1 = -M_1[t]\,q_1 + \left(\frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} + \frac{nuK\,M_5[t]}{denK + M_5[t]}\right) \left(-B_1[t]\,d_{1,1} - B_2[t]\,d_{1,2} - B_3[t]\,d_{1,3} - B_4[t]\,d_{1,4} - B_5[t]\,d_{1,5}\right) + B_1[t]\,\Omega_{1,1} + B_2[t]\,\Omega_{1,2} + B_3[t]\,\Omega_{1,3} + B_4[t]\,\Omega_{1,4} + B_5[t]\,\Omega_{1,5};$$

$$dM_2 = -M_2[t]\,q_2 + \left(\frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} + \frac{nuK\,M_5[t]}{denK + M_5[t]}\right) \left(-B_1[t]\,d_{2,1} - B_2[t]\,d_{2,2} - B_3[t]\,d_{2,3} - B_4[t]\,d_{2,4} - B_5[t]\,d_{2,5}\right) + B_1[t]\,\Omega_{2,1} + B_2[t]\,\Omega_{2,2} + B_3[t]\,\Omega_{2,3} + B_4[t]\,\Omega_{2,4} + B_5[t]\,\Omega_{2,5};$$

$$dM_3 = -M_3[t]\,q_3 + \left(\frac{nuK\,M_1[t]}{denK + M_1[t]} + \frac{nuK\,M_2[t]}{denK + M_2[t]} + \frac{nuK\,M_3[t]}{denK + M_3[t]} + \frac{nuK\,M_4[t]}{denK + M_4[t]} + \right.$$

```
        nuK M5[t]
       ----------- ) (-B1[t] d3,1 - B2[t] d3,2 - B3[t] d3,3 - B4[t] d3,4 - B5[t] d3,5) +
       denK + M5[t]
   B1[t] Ω3,1 + B2[t] Ω3,2 + B3[t] Ω3,3 + B4[t] Ω3,4 + B5[t] Ω3,5;
dM4 = -M4[t] q4 + ( nuK M1[t]/(denK + M1[t]) + nuK M2[t]/(denK + M2[t]) + nuK M3[t]/(denK + M3[t]) + nuK M4[t]/(denK + M4[t]) +
       nuK M5[t]/(denK + M5[t]) ) (-B1[t] d4,1 - B2[t] d4,2 - B3[t] d4,3 - B4[t] d4,4 - B5[t] d4,5) +
   B1[t] Ω4,1 + B2[t] Ω4,2 + B3[t] Ω4,3 + B4[t] Ω4,4 + B5[t] Ω4,5;
dM5 = -M5[t] q5 + ( nuK M1[t]/(denK + M1[t]) + nuK M2[t]/(denK + M2[t]) + nuK M3[t]/(denK + M3[t]) + nuK M4[t]/(denK + M4[t]) +
       nuK M5[t]/(denK + M5[t]) ) (-B1[t] d5,1 - B2[t] d5,2 - B3[t] d5,3 - B4[t] d5,4 - B5[t] d5,5) +
   B1[t] Ω5,1 + B2[t] Ω5,2 + B3[t] Ω5,3 + B4[t] Ω5,4 + B5[t] Ω5,5;



op = coop; (*Number of links with overExpression*)
posNe = Position[Net, 1];
(*Positions in the matrix where there are links (=1)*)
RaN = RandomSample[posNe, op];
(*Random sample of op links that will be overproduced*)

costincr = 1.3; (*Term multiplying the cost link*)
overprodincr = 1.15;
(*Term multiplying the overproduction link*)


NewNetCost = Partition[Flatten[Net] × parR[[6 ;; 30]], {5}];
Table[NewNetCost[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] =
  NewNetCost[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] * costincr, {i, Length[RaN]}];

NewNetOvProd = Partition[Flatten[Net] × parR[[61 ;; 85]], {5}];
Table[NewNetOvProd[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] =
  NewNetOvProd[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] * overprodincr, {i,
  Length[RaN]}];


tmax = 1000;
par = {
  κ1 → parR[[1]], κ2 → parR[[2]], κ3 → parR[[3]], κ4 → parR[[4]], κ5 → parR[[5]],

  c1,1 → NewNetCost[[1]][[1]],
  c1,2 → NewNetCost[[1]][[2]], c1,3 → NewNetCost[[1]][[3]],
  c1,4 → NewNetCost[[1]][[4]], c1,5 → NewNetCost[[1]][[5]],
```

```
c_{2,1} → NewNetCost[[2]][[1]], c_{2,2} → NewNetCost[[2]][[2]],
c_{2,3} → NewNetCost[[2]][[3]], c_{2,4} → NewNetCost[[2]][[4]],
c_{2,5} → NewNetCost[[2]][[5]],
c_{3,1} → NewNetCost[[3]][[1]], c_{3,2} → NewNetCost[[3]][[2]],
c_{3,3} → NewNetCost[[3]][[3]], c_{3,4} → NewNetCost[[3]][[4]],
c_{3,5} → NewNetCost[[3]][[5]],
c_{4,1} → NewNetCost[[4]][[1]], c_{4,2} → NewNetCost[[4]][[2]],
c_{4,3} → NewNetCost[[4]][[3]], c_{4,4} → NewNetCost[[4]][[4]],
c_{4,5} → NewNetCost[[4]][[5]],
c_{5,1} → NewNetCost[[5]][[1]], c_{5,2} → NewNetCost[[5]][[2]],
c_{5,3} → NewNetCost[[5]][[3]], c_{5,4} → NewNetCost[[5]][[4]],
c_{5,5} → NewNetCost[[5]][[5]],

r_{1,1} → parR[[31]], r_{1,2} → parR[[32]],
r_{1,3} → parR[[33]], r_{1,4} → parR[[34]], r_{1,5} → parR[[35]],
r_{2,1} → parR[[36]], r_{2,2} → parR[[37]], r_{2,3} → parR[[38]],
r_{2,4} → parR[[39]], r_{2,5} → parR[[40]],
r_{3,1} → parR[[41]], r_{3,2} → parR[[42]], r_{3,3} → parR[[43]],
r_{3,4} → parR[[44]], r_{3,5} → parR[[45]],
r_{4,1} → parR[[46]], r_{4,2} → parR[[47]], r_{4,3} → parR[[48]],
r_{4,4} → parR[[49]], r_{4,5} → parR[[50]],
r_{5,1} → parR[[51]], r_{5,2} → parR[[52]], r_{5,3} → parR[[53]],
r_{5,4} → parR[[54]], r_{5,5} → parR[[55]],

q_{1} → parR[[31]], q_{2} → parR[[32]],
q_{3} → parR[[33]], q_{4} → parR[[34]], q_{5} → parR[[35]],

d_{1,1} → parR[[36]], d_{1,2} → parR[[37]],
d_{1,3} → parR[[38]], d_{1,4} → parR[[39]], d_{1,5} → parR[[40]],
d_{2,1} → parR[[41]], d_{2,2} → parR[[42]], d_{2,3} → parR[[43]],
d_{2,4} → parR[[44]], d_{2,5} → parR[[45]],
d_{3,1} → parR[[46]], d_{3,2} → parR[[47]], d_{3,3} → parR[[48]],
d_{3,4} → parR[[49]], d_{3,5} → parR[[50]],
d_{4,1} → parR[[51]], d_{4,2} → parR[[52]], d_{4,3} → parR[[53]],
d_{4,4} → parR[[54]], d_{4,5} → parR[[55]],
d_{5,1} → parR[[56]], d_{5,2} → parR[[57]], d_{5,3} → parR[[58]],
d_{5,4} → parR[[59]], d_{5,5} → parR[[60]],

Ω_{1,1} → NewNetOvProd[[1]][[1]],
Ω_{1,2} → NewNetOvProd[[1]][[2]], Ω_{1,3} → NewNetOvProd[[1]][[3]],
Ω_{1,4} → NewNetOvProd[[1]][[4]], Ω_{1,5} → NewNetOvProd[[1]][[5]],
Ω_{2,1} → NewNetOvProd[[2]][[1]], Ω_{2,2} → NewNetOvProd[[2]][[2]],
Ω_{2,3} → NewNetOvProd[[2]][[3]], Ω_{2,4} → NewNetOvProd[[2]][[4]],
Ω_{2,5} → NewNetOvProd[[2]][[5]],
```

```
  Ω3,1 → NewNetOvProd[[3]][[1]], Ω3,2 → NewNetOvProd[[3]][[2]],
  Ω3,3 → NewNetOvProd[[3]][[3]], Ω3,4 → NewNetOvProd[[3]][[4]],
  Ω3,5 → NewNetOvProd[[3]][[5]],
  Ω4,1 → NewNetOvProd[[4]][[1]], Ω4,2 → NewNetOvProd[[4]][[2]],
  Ω4,3 → NewNetOvProd[[4]][[3]], Ω4,4 → NewNetOvProd[[4]][[4]],
  Ω4,5 → NewNetOvProd[[4]][[5]],
  Ω5,1 → NewNetOvProd[[5]][[1]], Ω5,2 → NewNetOvProd[[5]][[2]],
  Ω5,3 → NewNetOvProd[[5]][[3]], Ω5,4 → NewNetOvProd[[5]][[4]],
  Ω5,5 → NewNetOvProd[[5]][[5]],
  nuK → parR[[86]],
  denK → parR[[87]]


 };


B10 = 1500;
B20 = 1500;
B30 = 1500;
B40 = 1500;
B50 = 1500;
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;


sol =
 NDSolve[
  {
    B1'[t] == dB1,
    B2'[t] == dB2,
    B3'[t] == dB3,
    B4'[t] == dB4,
    B5'[t] == dB5,

    M1'[t] == dM1,
    M2'[t] == dM2,
    M3'[t] == dM3,
    M4'[t] == dM4,
    M5'[t] == dM5,
```

```
      B1[0] == B10,
      B2[0] == B20,
      B3[0] == B30,
      B4[0] == B40,
      B5[0] == B50,
      M1[0] == M10,
      M2[0] == M20,
      M3[0] == M30,
      M4[0] == M40,
      M5[0] == M50

     } /. par,
    {B1, B2, B3, B4, B5, M1, M2, M3, M4, M5},
    {t, 0, tmax}];

  {B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax],
     M1[tmax], M2[tmax], M3[tmax], M4[tmax], M5[tmax]} /. sol /. par;

  Min[{B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax]} /. sol /. par]

 )
```

In[6900]:=

```
robustnessNewMonoOVR[NetTop_, coop_] := (
  n1 = 1;
  n2 = 5000;
  mid = (n1 + n2) / 2;

  While[(n1 ≠ mid && n2 ≠ mid ),
   (If[fNewMonoOVR[NetTop, mid, coop] < 1, n2 = mid, n1 = mid];
    mid = Floor[N[(n1 + n2) / 2]];); {n1, n2, mid}]; mid

 )
```

In[6873]:=
```
NetK = {
   {0, 1, 0, 1, 0},
   {1, 0, 1, 1, 0},
   {1, 0, 1, 0, 1},
   {0, 1, 0, 1, 0},
   {0, 0, 0, 0, 1}
  };
```

Compare the Robustness with and without (5 links) overproduction (ratio cost/production = 1.3/1.15)

In[6901]:=
```
fNewMonoR[NetK, 0]
```
Out[6901]= 487.136

In[6902]:=
```
fNewMonoOVR[NetK, 0, 5]
```
Out[6902]= 487.528

In[6903]:=
```
robustnessNewMonoR[NetK]
```
Out[6903]= 85

In[6904]:=
```
robustnessNewMonoOVR[NetK, 5]
```
Out[6904]= 86

In[6905]:=
```
robustnessNewMonoOVR[NetK, 10]
```
Out[6905]= 87

In[6906]:=
```
AuxoComm8Additive
```

Out[6906]=
```
{90, 90, 91, 91, 90, 90, 88, 91, 91, 90, 91, 91, 90, 91, 91, 90, 90, 90, 91, 88,
 90, 90, 91, 90, 90, 91, 91, 88, 91, 91, 91, 88, 91, 91, 91, 91, 90, 91, 91, 91,
 90, 91, 91, 91, 91, 91, 90, 90, 91, 90, 90, 91, 90, 88, 91, 91, 88, 91, 91, 91,
 90, 90, 91, 91, 91, 90, 90, 91, 91, 91, 91, 90, 91, 91, 91, 91, 91, 90, 90, 91,
 90, 91, 91, 90, 90, 91, 91, 91, 90, 90, 87, 91, 91, 91, 91, 91, 90, 90, 91, 91}
```

In[6907]:=
```
coop5to15AddR =
  {Table[robustnessNewMonoOVR[#, 5], {20}], Table[robustnessNewMonoOVR[#, 10],
     {20}], Table[robustnessNewMonoOVR[#, 15], {20}]} &;
```

In[6908]:=
```
wf8AddR = Parallelize[coop5to15AddR /@ hk8];
```

In[6909]:=
```
wf8NormalizedAddR = N[wf8AddR[[#]] / AuxoComm8Additive[[#]]] & /@ Range[100]
```

In[6910]:=
```
wf8NormalizedWith5CoopAddR = wf8NormalizedAddR[[#]][[1]] & /@ Range[100]
```

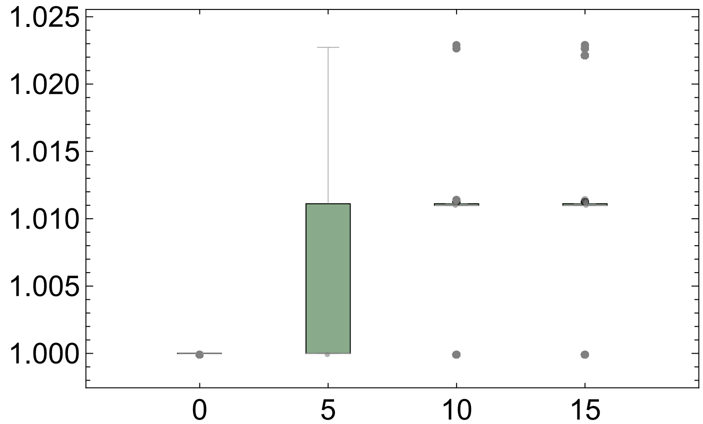
In[6911]:=
```
wf8NormalizedWith10CoopAddR = wf8NormalizedAddR[[#]][[2]] & /@ Range[100]
```

In[6912]:=
```
wf8NormalizedWith15CoopAddR = wf8NormalizedAddR[[#]][[3]] & /@ Range[100]
```

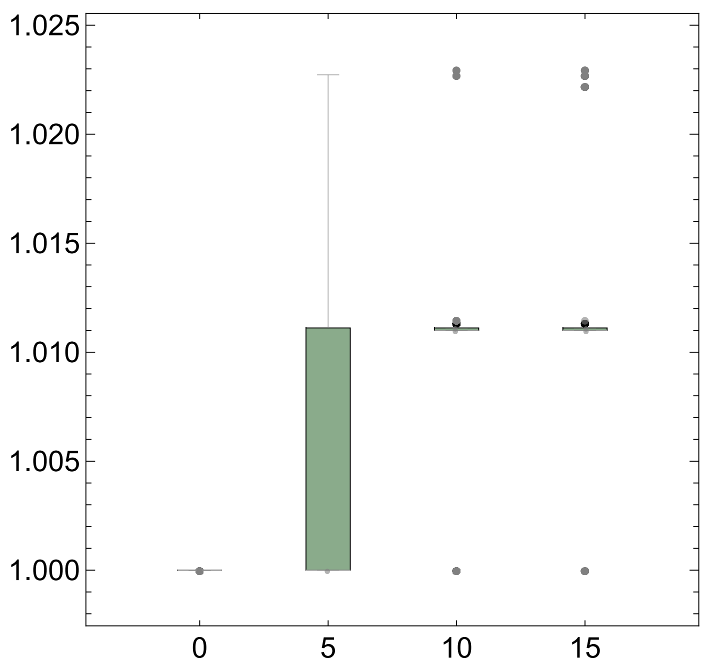
In[6913]:=
```
allcoopWith8AuxoAddR = {Flatten[wf8NormalizedWith5CoopAddR],
  Flatten[wf8NormalizedWith10CoopAddR], Flatten[wf8NormalizedWith15CoopAddR]}
```

In[6914]:=
```
allcoopWith8AuxoPlusAuxoAddR =
 Join[{ConstantArray[1, {2000}]}, allcoopWith8AuxoAddR]
```

In[6915]:=
```
BoxWhiskerChart[allcoopWith8AuxoPlusAuxoAddR, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{gree1}},
 Frame → True, ChartLabels → {"0", "5", "10", "15"},
 BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15]]
```

Out[6915]=


In[6916]:=
```
BoxWhiskerChart[allcoopWith8AuxoPlusAuxoAddR, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{gree1}},
 Frame → True, ChartLabels → {"0", "5", "10", "15"}, BarSpacing → 1.9,
 FrameStyle → Directive[Black, FontSize → 15], AspectRatio → 1]
```

Out[6916]=


In[6917]:=
```
allcoopWith8AuxoPlusAuxoAddR // Length
```
Out[6917]= 4

In[6918]:=
```
SignedRankTest[allcoopWith8AuxoPlusAuxoAddR[[2]], 1]
SignedRankTest[allcoopWith8AuxoPlusAuxoAddR[[3]], 1]
SignedRankTest[allcoopWith8AuxoPlusAuxoAddR[[4]], 1]
```
Out[6918]= 2.77928×10^{-145}

Out[6919]= 5.66892×10^{-269}

Out[6920]= 0.