Cooperation increases robustness to ecological disturbance in microbial cross-feeding networks

Generating Random Networks

Functions to calculate Entropy and Assortativity

Entropy

Assortativity

1. Colimitation model

Solving the system of ODE

The function “fNewSaitoK” solves the ODE system and gives the population at steady state of the system. The function “fNewSaitoK” receives a network and a disturbance value as arguments.

In[6944]:=

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fNewSaitoOMK[Net_, Dh_] := (
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$$dB_1 = B_1[t]\left(-B_1[t]\,\kappa_1 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - (c_{1,1} + c_{1,2} + c_{1,3} + c_{1,4} + c_{1,5})\, B_1[t];$$

$$dB_2 = B_2[t]\left(-B_2[t]\,\kappa_2 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - (c_{2,1} + c_{2,2} + c_{2,3} + c_{2,4} + c_{2,5})\, B_2[t];$$

$$dB_3 = B_3[t]\left(-B_3[t]\,\kappa_3 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} *\right.$$

$$\frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\Bigg) - (c_{3,1} + c_{3,2} + c_{3,3} + c_{3,4} + c_{3,5})\ B_3[t];$$

$$dB_4 = B_4[t]\ \Bigg(-B_4[t]\ \varkappa_4 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\Bigg) - (c_{4,1} + c_{4,2} + c_{4,3} + c_{4,4} + c_{4,5})\ B_4[t];$$

$$dB_5 = B_5[t]\ \Bigg(-B_5[t]\ \varkappa_5 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\Bigg) - (c_{5,1} + c_{5,2} + c_{5,3} + c_{5,4} + c_{5,5})\ B_5[t];$$

$$dM_1 = -M_1[t]\ (Dh + q_1) + \Bigg(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\Bigg) (-B_1[t]\ d_{1,1} - B_2[t]\ d_{1,2} - B_3[t]\ d_{1,3} - B_4[t]\ d_{1,4} - B_5[t]\ d_{1,5}) + B_1[t]\ \Omega_{1,1} + B_2[t]\ \Omega_{1,2} + B_3[t]\ \Omega_{1,3} + B_4[t]\ \Omega_{1,4} + B_5[t]\ \Omega_{1,5};$$

$$dM_2 = -M_2[t]\ (Dh + q_2) + \Bigg(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\Bigg) (-B_1[t]\ d_{2,1} - B_2[t]\ d_{2,2} - B_3[t]\ d_{2,3} - B_4[t]\ d_{2,4} - B_5[t]\ d_{2,5}) + B_1[t]\ \Omega_{2,1} + B_2[t]\ \Omega_{2,2} + B_3[t]\ \Omega_{2,3} + B_4[t]\ \Omega_{2,4} + B_5[t]\ \Omega_{2,5};$$

$$dM_3 = -M_3[t]\ (Dh + q_3) + \Bigg(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\Bigg) (-B_1[t]\ d_{3,1} - B_2[t]\ d_{3,2} - B_3[t]\ d_{3,3} - B_4[t]\ d_{3,4} - B_5[t]\ d_{3,5}) + B_1[t]\ \Omega_{3,1} + B_2[t]\ \Omega_{3,2} + B_3[t]\ \Omega_{3,3} + B_4[t]\ \Omega_{3,4} + B_5[t]\ \Omega_{3,5};$$

$$dM_4 = -M_4[t]\ (Dh + q_4) + \Bigg(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\Bigg) (-B_1[t]\ d_{4,1} - B_2[t]\ d_{4,2} - B_3[t]\ d_{4,3} - B_4[t]\ d_{4,4} - B_5[t]\ d_{4,5}) + B_1[t]\ \Omega_{4,1} + B_2[t]\ \Omega_{4,2} + B_3[t]\ \Omega_{4,3} + B_4[t]\ \Omega_{4,4} + B_5[t]\ \Omega_{4,5};$$

$$dM_5 = -M_5[t]\ (Dh + q_5) + \Bigg(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\Bigg) (-B_1[t]\ d_{5,1} - B_2[t]\ d_{5,2} - B_3[t]\ d_{5,3} - B_4[t]\ d_{5,4} - B_5[t]\ d_{5,5}) + B_1[t]\ \Omega_{5,1} + B_2[t]\ \Omega_{5,2} + B_3[t]\ \Omega_{5,3} + B_4[t]\ \Omega_{5,4} + B_5[t]\ \Omega_{5,5};$$

```
KK = 0.2;
cc = 0.05;
qq = 0.3;
dd = 0.00015;
OM = 1;
nu = 1500;
den = 2;
```

```
tmax = 1000;
par = {
  κ1 → KK, κ2 → KK, κ3 → KK, κ4 → KK, κ5 → KK,

  c1,1 → cc Net[[1]][[1]], c1,2 → cc Net[[1]][[2]],
  c1,3 → cc Net[[1]][[3]], c1,4 → cc Net[[1]][[4]], c1,5 → cc Net[[1]][[5]],
  c2,1 → cc Net[[2]][[1]], c2,2 → cc Net[[2]][[2]], c2,3 → cc Net[[2]][[3]],
  c2,4 → cc Net[[2]][[4]], c2,5 → cc Net[[2]][[5]],
  c3,1 → cc Net[[3]][[1]], c3,2 → cc Net[[3]][[2]], c3,3 → cc Net[[3]][[3]],
  c3,4 → cc Net[[3]][[4]], c3,5 → cc Net[[3]][[5]],
  c4,1 → cc Net[[4]][[1]], c4,2 → cc Net[[4]][[2]], c4,3 → cc Net[[4]][[3]],
  c4,4 → cc Net[[4]][[4]], c4,5 → cc Net[[4]][[5]],
  c5,1 → cc Net[[5]][[1]], c5,2 → cc Net[[5]][[2]], c5,3 → cc Net[[5]][[3]],
  c5,4 → cc Net[[5]][[4]], c5,5 → cc Net[[5]][[5]],

  q1 → qq, q2 → qq, q3 → qq, q4 → qq, q5 → qq,

  d1,1 → dd, d1,2 → dd, d1,3 → dd, d1,4 → dd, d1,5 → dd,
  d2,1 → dd, d2,2 → dd, d2,3 → dd, d2,4 → dd, d2,5 → dd,
  d3,1 → dd, d3,2 → dd, d3,3 → dd, d3,4 → dd, d3,5 → dd,
  d4,1 → dd, d4,2 → dd, d4,3 → dd, d4,4 → dd, d4,5 → dd,
  d5,1 → dd, d5,2 → dd, d5,3 → dd, d5,4 → dd, d5,5 → dd,

  Ω1,1 → OM Net[[1]][[1]], Ω1,2 → OM Net[[1]][[2]],
  Ω1,3 → OM Net[[1]][[3]], Ω1,4 → OM Net[[1]][[4]], Ω1,5 → OM Net[[1]][[5]],
  Ω2,1 → OM Net[[2]][[1]], Ω2,2 → OM Net[[2]][[2]], Ω2,3 → OM Net[[2]][[3]],
  Ω2,4 → OM Net[[2]][[4]], Ω2,5 → OM Net[[2]][[5]],
  Ω3,1 → OM Net[[3]][[1]], Ω3,2 → OM Net[[3]][[2]], Ω3,3 → OM Net[[3]][[3]],
  Ω3,4 → OM Net[[3]][[4]], Ω3,5 → OM Net[[3]][[5]],
  Ω4,1 → OM Net[[4]][[1]], Ω4,2 → OM Net[[4]][[2]], Ω4,3 → OM Net[[4]][[3]],
  Ω4,4 → OM Net[[4]][[4]], Ω4,5 → OM Net[[4]][[5]],
  Ω5,1 → OM Net[[5]][[1]], Ω5,2 → OM Net[[5]][[2]], Ω5,3 → OM Net[[5]][[3]],
  Ω5,4 → OM Net[[5]][[4]], Ω5,5 → OM Net[[5]][[5]],
  nuK → nu,
  denK → den


 };


B10 = 1500;
B20 = 1500;
B30 = 1500;
```

```
B40 = 1500;
B50 = 1500;
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;



sol =
 NDSolve[
  {
    B1'[t] == dB1,
    B2'[t] == dB2,
    B3'[t] == dB3,
    B4'[t] == dB4,
    B5'[t] == dB5,

    M1'[t] == dM1,
    M2'[t] == dM2,
    M3'[t] == dM3,
    M4'[t] == dM4,
    M5'[t] == dM5,



    B1[0] == B10,
    B2[0] == B20,
    B3[0] == B30,
    B4[0] == B40,
    B5[0] == B50,
    M1[0] == M10,
    M2[0] == M20,
    M3[0] == M30,
    M4[0] == M40,
    M5[0] == M50

   } /. par,
  {B1, B2, B3, B4, B5, M1, M2, M3, M4, M5},
  {t, 0, tmax}];

{B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax],
   M1[tmax], M2[tmax], M3[tmax], M4[tmax], M5[tmax]} /. sol /. par

(*Min[{B1[tmax],B2[tmax],B3[tmax],B4[tmax],B5[tmax]}/.sol/.par]*)
```

```
)
```

As an example let's take the following Network

In[6945]:=
```
NetK = {
   {0, 1, 0, 1, 0},
   {1, 0, 1, 1, 0},
   {1, 0, 1, 0, 1},
   {0, 1, 0, 1, 0},
   {0, 0, 0, 0, 1}
  };
```

In[6946]:=
```
fNewSaitoOMK[NetK, 0]
fNewSaitoOMK[NetK, 1]
```

Out[6946]=
```
{{6661.68, 6661.43, 6661.43, 6661.68,
  6661.93, 22219.9, 44425.5, 44426.3, 22219.9, 15.9422}}
```

Out[6947]=
```
{{6645.95, 6645.7, 6645.7, 6645.95,
  6646.2, 5127.62, 10239.9, 10240.1, 5127.62, 15.7354}}
```

The function "fNewSaito" solves the ODE system and gives the lowest microbial population size (this is used to calculate the Robustness). The function "fNewSaito" receives a network and a disturbance value as arguments.

In[6948]:=
```
fNewSaitoOM[Net_, Dh_] := (
```

$$dB_1 = B_1[t]\left(-B_1[t]\,\kappa_1 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - (c_{1,1} + c_{1,2} + c_{1,3} + c_{1,4} + c_{1,5})\,B_1[t];$$

$$dB_2 = B_2[t]\left(-B_2[t]\,\kappa_2 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - (c_{2,1} + c_{2,2} + c_{2,3} + c_{2,4} + c_{2,5})\,B_2[t];$$

$$dB_3 = B_3[t]\left(-B_3[t]\,\kappa_3 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - (c_{3,1} + c_{3,2} + c_{3,3} + c_{3,4} + c_{3,5})\,B_3[t];$$

$$dB_4 = B_4[t]\left(-B_4[t]\,\kappa_4 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} *\right.$$

$$\frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\Bigg) - (c_{4,1} + c_{4,2} + c_{4,3} + c_{4,4} + c_{4,5})\ B_4[t];$$

$$dB_5 = B_5[t]\ \Bigg(-B_5[t]\ \kappa_5 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\Bigg) - (c_{5,1} + c_{5,2} + c_{5,3} + c_{5,4} + c_{5,5})\ B_5[t];$$

$$dM_1 = -M_1[t]\ (Dh + q_1) + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) (-B_1[t]\ d_{1,1} - B_2[t]\ d_{1,2} - B_3[t]\ d_{1,3} - B_4[t]\ d_{1,4} - B_5[t]\ d_{1,5}) + B_1[t]\ \Omega_{1,1} + B_2[t]\ \Omega_{1,2} + B_3[t]\ \Omega_{1,3} + B_4[t]\ \Omega_{1,4} + B_5[t]\ \Omega_{1,5};$$

$$dM_2 = -M_2[t]\ (Dh + q_2) + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) (-B_1[t]\ d_{2,1} - B_2[t]\ d_{2,2} - B_3[t]\ d_{2,3} - B_4[t]\ d_{2,4} - B_5[t]\ d_{2,5}) + B_1[t]\ \Omega_{2,1} + B_2[t]\ \Omega_{2,2} + B_3[t]\ \Omega_{2,3} + B_4[t]\ \Omega_{2,4} + B_5[t]\ \Omega_{2,5};$$

$$dM_3 = -M_3[t]\ (Dh + q_3) + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) (-B_1[t]\ d_{3,1} - B_2[t]\ d_{3,2} - B_3[t]\ d_{3,3} - B_4[t]\ d_{3,4} - B_5[t]\ d_{3,5}) + B_1[t]\ \Omega_{3,1} + B_2[t]\ \Omega_{3,2} + B_3[t]\ \Omega_{3,3} + B_4[t]\ \Omega_{3,4} + B_5[t]\ \Omega_{3,5};$$

$$dM_4 = -M_4[t]\ (Dh + q_4) + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) (-B_1[t]\ d_{4,1} - B_2[t]\ d_{4,2} - B_3[t]\ d_{4,3} - B_4[t]\ d_{4,4} - B_5[t]\ d_{4,5}) + B_1[t]\ \Omega_{4,1} + B_2[t]\ \Omega_{4,2} + B_3[t]\ \Omega_{4,3} + B_4[t]\ \Omega_{4,4} + B_5[t]\ \Omega_{4,5};$$

$$dM_5 = -M_5[t]\ (Dh + q_5) + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) (-B_1[t]\ d_{5,1} - B_2[t]\ d_{5,2} - B_3[t]\ d_{5,3} - B_4[t]\ d_{5,4} - B_5[t]\ d_{5,5}) + B_1[t]\ \Omega_{5,1} + B_2[t]\ \Omega_{5,2} + B_3[t]\ \Omega_{5,3} + B_4[t]\ \Omega_{5,4} + B_5[t]\ \Omega_{5,5};$$

```
KK = 0.2;
cc = 0.05;
qq = 0.3;
dd = 0.00015;
OM = 1;
nu = 1500;
den = 2;

tmax = 1000;
par = {
  κ1 → KK, κ2 → KK, κ3 → KK, κ4 → KK, κ5 → KK,
```

```
    c1,1 → cc Net[[1]][[1]], c1,2 → cc Net[[1]][[2]],
    c1,3 → cc Net[[1]][[3]], c1,4 → cc Net[[1]][[4]], c1,5 → cc Net[[1]][[5]],
    c2,1 → cc Net[[2]][[1]], c2,2 → cc Net[[2]][[2]], c2,3 → cc Net[[2]][[3]],
    c2,4 → cc Net[[2]][[4]], c2,5 → cc Net[[2]][[5]],
    c3,1 → cc Net[[3]][[1]], c3,2 → cc Net[[3]][[2]], c3,3 → cc Net[[3]][[3]],
    c3,4 → cc Net[[3]][[4]], c3,5 → cc Net[[3]][[5]],
    c4,1 → cc Net[[4]][[1]], c4,2 → cc Net[[4]][[2]], c4,3 → cc Net[[4]][[3]],
    c4,4 → cc Net[[4]][[4]], c4,5 → cc Net[[4]][[5]],
    c5,1 → cc Net[[5]][[1]], c5,2 → cc Net[[5]][[2]], c5,3 → cc Net[[5]][[3]],
    c5,4 → cc Net[[5]][[4]], c5,5 → cc Net[[5]][[5]],

    q1 → qq, q2 → qq, q3 → qq, q4 → qq, q5 → qq,

    d1,1 → dd, d1,2 → dd, d1,3 → dd, d1,4 → dd, d1,5 → dd,
    d2,1 → dd, d2,2 → dd, d2,3 → dd, d2,4 → dd, d2,5 → dd,
    d3,1 → dd, d3,2 → dd, d3,3 → dd, d3,4 → dd, d3,5 → dd,
    d4,1 → dd, d4,2 → dd, d4,3 → dd, d4,4 → dd, d4,5 → dd,
    d5,1 → dd, d5,2 → dd, d5,3 → dd, d5,4 → dd, d5,5 → dd,

    Ω1,1 → OM Net[[1]][[1]], Ω1,2 → OM Net[[1]][[2]],
    Ω1,3 → OM Net[[1]][[3]], Ω1,4 → OM Net[[1]][[4]], Ω1,5 → OM Net[[1]][[5]],
    Ω2,1 → OM Net[[2]][[1]], Ω2,2 → OM Net[[2]][[2]], Ω2,3 → OM Net[[2]][[3]],
    Ω2,4 → OM Net[[2]][[4]], Ω2,5 → OM Net[[2]][[5]],
    Ω3,1 → OM Net[[3]][[1]], Ω3,2 → OM Net[[3]][[2]], Ω3,3 → OM Net[[3]][[3]],
    Ω3,4 → OM Net[[3]][[4]], Ω3,5 → OM Net[[3]][[5]],
    Ω4,1 → OM Net[[4]][[1]], Ω4,2 → OM Net[[4]][[2]], Ω4,3 → OM Net[[4]][[3]],
    Ω4,4 → OM Net[[4]][[4]], Ω4,5 → OM Net[[4]][[5]],
    Ω5,1 → OM Net[[5]][[1]], Ω5,2 → OM Net[[5]][[2]], Ω5,3 → OM Net[[5]][[3]],
    Ω5,4 → OM Net[[5]][[4]], Ω5,5 → OM Net[[5]][[5]],
    nuK → nu,
    denK → den


  };


B10 = 1500;
B20 = 1500;
B30 = 1500;
B40 = 1500;
B50 = 1500;
M10 = 10;
M20 = 10;
```

```
M30 = 10;
M40 = 10;
M50 = 10;


sol =
 NDSolve[
  {
    B1'[t] == dB1,
    B2'[t] == dB2,
    B3'[t] == dB3,
    B4'[t] == dB4,
    B5'[t] == dB5,

    M1'[t] == dM1,
    M2'[t] == dM2,
    M3'[t] == dM3,
    M4'[t] == dM4,
    M5'[t] == dM5,


    B1[0] == B10,
    B2[0] == B20,
    B3[0] == B30,
    B4[0] == B40,
    B5[0] == B50,
    M1[0] == M10,
    M2[0] == M20,
    M3[0] == M30,
    M4[0] == M40,
    M5[0] == M50

   } /. par,
  {B1, B2, B3, B4, B5, M1, M2, M3, M4, M5},
  {t, 0, tmax}];

{B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax],
   M1[tmax], M2[tmax], M3[tmax], M4[tmax], M5[tmax]} /. sol /. par;

Min[{B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax]} /. sol /. par]

)
```

The function “robustnessNewSaito” uses the previous function “fNewSaito” and calculates the Robustness. The function “robustnessNewSaito” simply receives a network as an argument.

In[6949]:=
```
robustnessNewSaitoOM[NetTop_] := (
  n1 = 1;
  n2 = 5000;
  mid = (n1 + n2) / 2;

  While[(n1 ≠ mid && n2 ≠ mid),
   (If[fNewSaitoOM[NetTop, mid] < 1, n2 = mid, n1 = mid];
    mid = Floor[N[(n1 + n2) / 2]];); {n1, n2, mid}]; mid

 )
```

As an example let’s take the following Network

In[6950]:=
```
NetK = {
    {0, 1, 0, 1, 0},
    {1, 0, 1, 1, 0},
    {1, 0, 1, 0, 1},
    {0, 1, 0, 1, 0},
    {0, 0, 0, 0, 1}
  };
```

Using the function fNewSaito we can calculate the smallest value of a bacterial population in the community for a given disturbance vale. For example, let’s take Disturbance value 1 and 500:

In[6951]:=
```
fNewSaitoOM[NetK, 0]
```

Out[6951]= 6661.43

In[6952]:=
```
fNewSaitoOM[NetK, 500]
```

Out[6952]= -4.61275×10^{-11}

Using the function fNewSaito we can calculate Robustness of the Network:

In[6953]:=
```
robustnessNewSaitoOM[NetK]
```

Out[6953]= 473

We can calculate the (Relative) Entropy and the Assortativity:

In[∘]:=
```
RelatEntrop5[NetK]
```
Out[∘]= 0.960956

In[∘]:=
```
assortativity[NetK]
```
Out[∘]= -0.113228

We can calculate the robustness of the previously generated random networks with different number of auxotrophies:

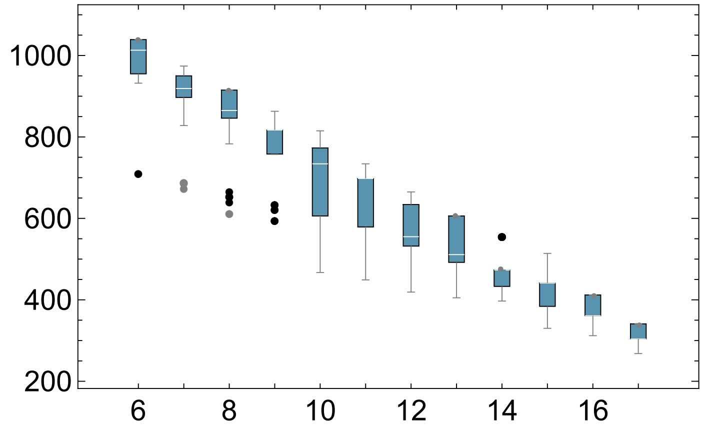
In[6954]:=
```
AuxoComm6OM = Parallelize[robustnessNewSaitoOM /@ hk6];
AuxoComm7OM = Parallelize[robustnessNewSaitoOM /@ hk7];
AuxoComm8OM = Parallelize[robustnessNewSaitoOM /@ hk8];
AuxoComm9OM = Parallelize[robustnessNewSaitoOM /@ hk9];
AuxoComm10OM = Parallelize[robustnessNewSaitoOM /@ hk10];
AuxoComm11OM = Parallelize[robustnessNewSaitoOM /@ hk11];
AuxoComm12OM = Parallelize[robustnessNewSaitoOM /@ hk12];
AuxoComm13OM = Parallelize[robustnessNewSaitoOM /@ hk13];
AuxoComm14OM = Parallelize[robustnessNewSaitoOM /@ hk14];
AuxoComm15OM = Parallelize[robustnessNewSaitoOM /@ hk15];
AuxoComm16OM = Parallelize[robustnessNewSaitoOM /@ hk16];
AuxoComm17OM = Parallelize[robustnessNewSaitoOM /@ hk17];
```

In[6966]:=
```
LikOM = {AuxoComm6OM, AuxoComm7OM, AuxoComm8OM,
   AuxoComm9OM, AuxoComm10OM, AuxoComm11OM, AuxoComm12OM, AuxoComm13OM,
   AuxoComm14OM, AuxoComm15OM, AuxoComm16OM, AuxoComm17OM};
```

In[6967]:=
```
coco = RGBColor[0.34509803921568627, 0.5803921568627451, 0.6901960784313725]
```
Out[6967]=

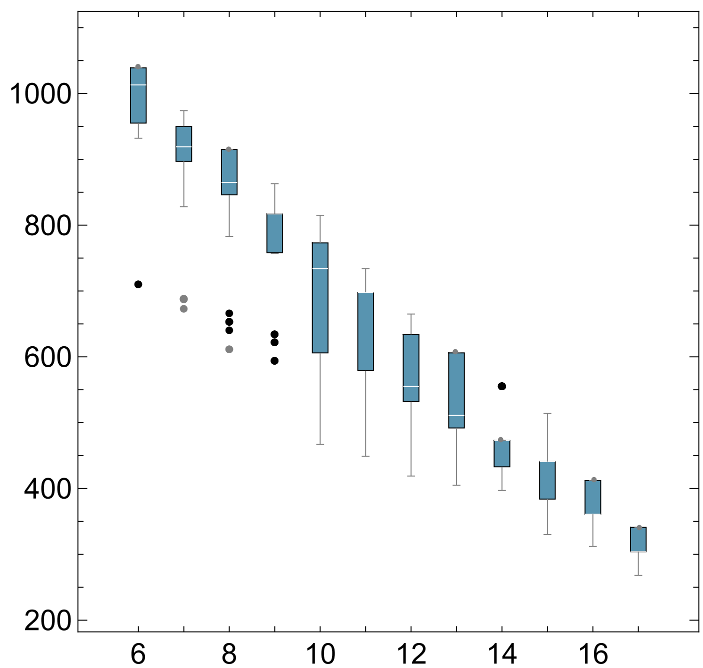
In[6968]:=
```
BoxWhiskerChart[LikOM, "Outliers",
  ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{coco}}, Frame → True,
  ChartLabels → {"6", "", "8", "", "10", "", "12", "", "14", "", "16", ""},
  BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15]]
```

Out[6968]=



In[6969]:=
```
BoxWhiskerChart[LikOM, "Outliers",
  ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{coco}}, Frame → True,
  ChartLabels → {"6", "", "8", "", "10", "", "12", "", "14", "", "16", ""},
  BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15], AspectRatio → 1]
```

Out[6969]=



In[6970]:=
```
AuxoComm7OM
```

Out[6970]=
```
{897, 897, 974, 897, 897, 897, 675, 974, 690, 689, 974, 950, 897, 876, 950, 897, 974, 974,
 950, 897, 876, 950, 950, 876, 974, 897, 950, 828, 876, 828, 897, 950, 950, 690, 974,
 974, 974, 950, 897, 950, 974, 950, 876, 950, 950, 974, 876, 974, 950, 950, 950, 974,
 897, 974, 897, 974, 950, 897, 950, 876, 897, 919, 897, 897, 974, 897, 950, 974,
 974, 974, 897, 974, 897, 974, 876, 897, 919, 950, 897, 919, 897, 897, 689, 950,
 974, 897, 897, 897, 919, 950, 950, 828, 974, 828, 897, 974, 950, 690, 897, 897}
```

We can study the correlation between Relative entropy and assortativity with Robustness for Networks with 7 auxotrophies.

In[∘]:=
```
Entropy7 = RelatEntrop5 /@ hk7;
```

In[∘]:=
```
Assort7 = assortativity /@ hk7;
```

In[6971]:=
```
RobustNewSaito7bOM = AuxoComm7OM;
```

In[6972]:=
```
Length[Entropy7]
Length[Assort7]
Length[RobustNewSaito7bOM]
```

Out[6972]= 100

Out[6973]= 100

Out[6974]= 100

In[∘]:=
```
{Min[Entropy7], Max[Entropy7]}
{Min[Assort7], Max[Assort7]}
```
Out[∘]= {0.935154, 0.994118}

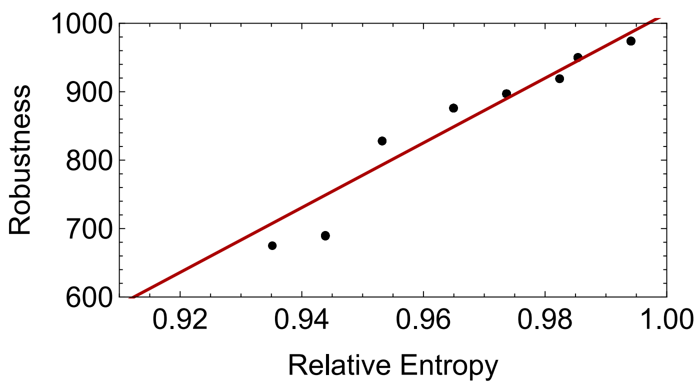
Out[∘]= {-0.416667, 0.25}

In[∘]:=
```
Position[Entropy7, Min[Entropy7]]
```
Out[∘]= {{7}}

In[6975]:=
```
RobustNewSaito7bOM[[#]] & /@ {1, 2, 24}
```
Out[6975]= {897, 897, 876}

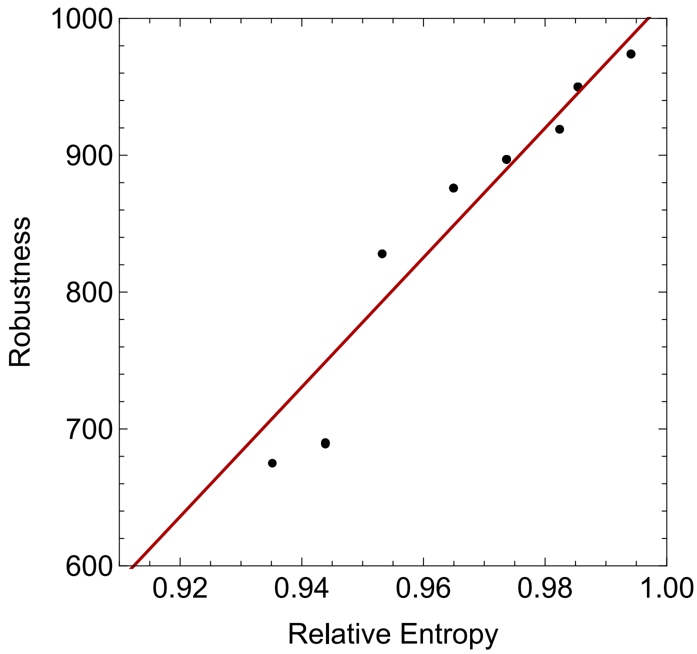
In[6976]:=
```
{Min[RobustNewSaito7bOM], {Max[RobustNewSaito7bOM]}}
```
Out[6976]= {675, {974}}

In[6977]:=
```
linerobustnessNewSaito25OM =
  Fit[Partition[Riffle[Entropy7, RobustNewSaito7bOM], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Entropy7, RobustNewSaito7bOM], {2}],
  Frame → True, FrameLabel → {"Relative Entropy", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]}, PlotRange → {{0.91, 1}, {600, 1000}},
  AspectRatio → 0.5], Plot[linerobustnessNewSaito25OM,
  {x, 0.91, 1}, AspectRatio → 0.5, PlotStyle → Darker[Red]]]
```
Out[6978]=



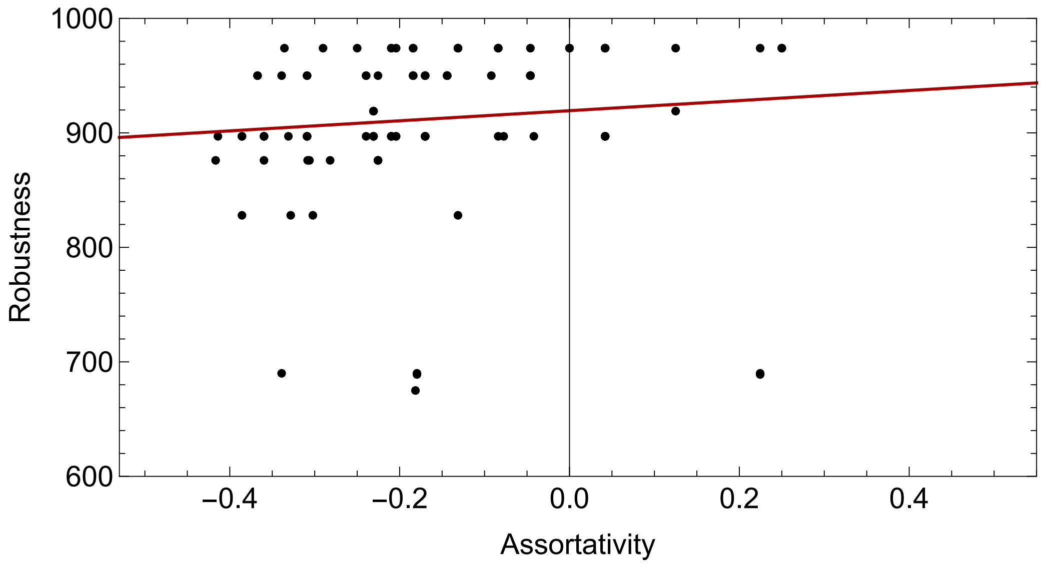
In[6979]:=
```
linerobustnessNewSaito25OM =
  Fit[Partition[Riffle[Entropy7, RobustNewSaito7bOM], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Entropy7, RobustNewSaito7bOM], {2}],
  Frame → True, FrameLabel → {"Relative Entropy", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]}, PlotRange → {{0.91, 1}, {600, 1000}},
  AspectRatio → 1], Plot[linerobustnessNewSaito25OM,
  {x, 0.91, 1}, AspectRatio → 1, PlotStyle → Darker[Red]]]
```

Out[6980]=



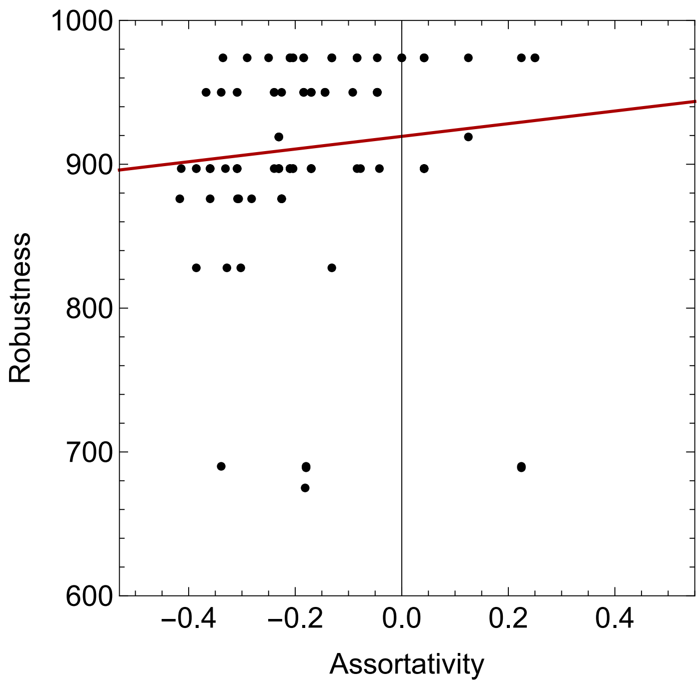
In[6981]:=
```
lineAssoRobrobustnessNewSaito25OM =
  Fit[Partition[Riffle[Assort7, RobustNewSaito7bOM], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Assort7, RobustNewSaito7bOM], {2}],
  Frame → True, FrameLabel → {"Assortativity", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]}, PlotRange → {{-0.53, 0.55}, {600, 1000}},
  AspectRatio → 0.5], Plot[lineAssoRobrobustnessNewSaito25OM,
  {x, -0.53, 0.55}, AspectRatio → 0.5, PlotStyle → Darker[Red]]]
```

Out[6982]=



In[6983]:=
```
lineAssoRobrobustnessNewSaito25OM =
  Fit[Partition[Riffle[Assort7, RobustNewSaito7bOM], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Assort7, RobustNewSaito7bOM], {2}],
  Frame → True, FrameLabel → {"Assortativity", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]}, PlotRange → {{-0.53, 0.55}, {600, 1000}},
  AspectRatio → 1], Plot[lineAssoRobrobustnessNewSaito25OM,
  {x, -0.53, 0.55}, AspectRatio → 1, PlotStyle → Darker[Red]]]
```

Out[6984]=



In[6985]:=
```
SpearmanRankTest[Entropy7, RobustNewSaito7bOM, "TestDataTable"]
```

Out[6985]=

	Statistic	P-Value
Spearman Rank	0.999952	7.4862×10^{-199}

In[6986]:=
```
SpearmanRankTest[Assort7, RobustNewSaito7bOM, "TestDataTable"]
```

Out[6986]=

	Statistic	P-Value
Spearman Rank	0.35029	0.000353237

Solving the system of ODE with Overproduction

In[6987]:=
```
fNewSaitoOVxOM[Net_, Dh_, coop_] := (
```

$$dB_1 = B_1[t]\left(-B_1[t]\,\kappa_1 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - (c_{1,1} + c_{1,2} + c_{1,3} + c_{1,4} + c_{1,5})\,B_1[t];$$

$$dB_2 = B_2[t]\left(-B_2[t]\,\kappa_2 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - (c_{2,1} + c_{2,2} + c_{2,3} + c_{2,4} + c_{2,5})\,B_2[t];$$

$$dB_3 = B_3[t]\left(-B_3[t]\,\kappa_3 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - (c_{3,1} + c_{3,2} + c_{3,3} + c_{3,4} + c_{3,5})\,B_3[t];$$

$$dB_4 = B_4[t]\left(-B_4[t]\,\kappa_4 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - (c_{4,1} + c_{4,2} + c_{4,3} + c_{4,4} + c_{4,5})\,B_4[t];$$

$$dB_5 = B_5[t]\left(-B_5[t]\,\kappa_5 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - (c_{5,1} + c_{5,2} + c_{5,3} + c_{5,4} + c_{5,5})\,B_5[t];$$

$$dM_1 = -M_1[t]\,(Dh + q_1) + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right)\left(-B_1[t]\,d_{1,1} - B_2[t]\,d_{1,2} - B_3[t]\,d_{1,3} - B_4[t]\,d_{1,4} - B_5[t]\,d_{1,5}\right) + B_1[t]\,\Omega_{1,1} + B_2[t]\,\Omega_{1,2} + B_3[t]\,\Omega_{1,3} + B_4[t]\,\Omega_{1,4} + B_5[t]\,\Omega_{1,5};$$

$$dM_2 = -M_2[t]\,(Dh + q_2) + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right)\left(-B_1[t]\,d_{2,1} - B_2[t]\,d_{2,2} - B_3[t]\,d_{2,3} - B_4[t]\,d_{2,4} - B_5[t]\,d_{2,5}\right) + B_1[t]\,\Omega_{2,1} + B_2[t]\,\Omega_{2,2} + B_3[t]\,\Omega_{2,3} + B_4[t]\,\Omega_{2,4} + B_5[t]\,\Omega_{2,5};$$

$$dM_3 = -M_3[t]\,(Dh + q_3) + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right)\left(-B_1[t]\,d_{3,1} - B_2[t]\,d_{3,2} - B_3[t]\,d_{3,3} - B_4[t]\,d_{3,4} - B_5[t]\,d_{3,5}\right) + B_1[t]\,\Omega_{3,1} + B_2[t]\,\Omega_{3,2} + B_3[t]\,\Omega_{3,3} + B_4[t]\,\Omega_{3,4} + B_5[t]\,\Omega_{3,5};$$

$$dM_4 = -M_4[t]\,(Dh + q_4) + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right)\left(-B_1[t]\,d_{4,1} - B_2[t]\,d_{4,2} - B_3[t]\,d_{4,3} - B_4[t]\,d_{4,4} - B_5[t]\,d_{4,5}\right) + B_1[t]\,\Omega_{4,1} + B_2[t]\,\Omega_{4,2} + B_3[t]\,\Omega_{4,3} + B_4[t]\,\Omega_{4,4} + B_5[t]\,\Omega_{4,5};$$

```
dM5 = -M5[t] (Dh + q5) + (nuK * M1[t]/(denK + M1[t]) * M2[t]/(denK + M2[t]) * M3[t]/(denK + M3[t]) *
      M4[t]/(denK + M4[t]) * M5[t]/(denK + M5[t])) (-B1[t] d5,1 - B2[t] d5,2 - B3[t] d5,3 - B4[t] d5,4 -
      B5[t] d5,5) + B1[t] Ω5,1 + B2[t] Ω5,2 + B3[t] Ω5,3 + B4[t] Ω5,4 + B5[t] Ω5,5;

KK = 0.2;
cc = 0.05;
qq = 0.3;
dd = 0.00015;
OM = 1;
nu = 1500;
den = 2;


op = coop; (*Number of links with overExpression*)
posNe = Position[Net, 1];
(*Positions in the matrix where there are links (=1)*)
RaN = RandomSample[posNe, op];
(*Random sample of op links that will be overproduced*)

costincr = 1.3; (*Term multiplying the cost link*)
overprodincr = 1.15;
(*Term multiplying the overproduction link*)


NewNetCost = Net cc;
Table[NewNetCost[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] =
  NewNetCost[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] * costincr, {i, Length[RaN]}];

NewNetOvProd = Net OM;
Table[NewNetOvProd[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] =
  NewNetOvProd[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] * overprodincr, {i,
  Length[RaN]}];


tmax = 1000;
par = {
  κ1 → KK, κ2 → KK, κ3 → KK, κ4 → KK, κ5 → KK,

  c1,1 → NewNetCost[[1]][[1]],
  c1,2 → NewNetCost[[1]][[2]], c1,3 → NewNetCost[[1]][[3]],
  c1,4 → NewNetCost[[1]][[4]], c1,5 → NewNetCost[[1]][[5]],
  c2,1 → NewNetCost[[2]][[1]], c2,2 → NewNetCost[[2]][[2]],
```

```
   c_{2,3} → NewNetCost[[2]][[3]], c_{2,4} → NewNetCost[[2]][[4]],
   c_{2,5} → NewNetCost[[2]][[5]],
   c_{3,1} → NewNetCost[[3]][[1]], c_{3,2} → NewNetCost[[3]][[2]],
   c_{3,3} → NewNetCost[[3]][[3]], c_{3,4} → NewNetCost[[3]][[4]],
   c_{3,5} → NewNetCost[[3]][[5]],
   c_{4,1} → NewNetCost[[4]][[1]], c_{4,2} → NewNetCost[[4]][[2]],
   c_{4,3} → NewNetCost[[4]][[3]], c_{4,4} → NewNetCost[[4]][[4]],
   c_{4,5} → NewNetCost[[4]][[5]],
   c_{5,1} → NewNetCost[[5]][[1]], c_{5,2} → NewNetCost[[5]][[2]],
   c_{5,3} → NewNetCost[[5]][[3]], c_{5,4} → NewNetCost[[5]][[4]],
   c_{5,5} → NewNetCost[[5]][[5]],

   q_{1} → qq, q_{2} → qq, q_{3} → qq, q_{4} → qq, q_{5} → qq,

   d_{1,1} → dd, d_{1,2} → dd, d_{1,3} → dd, d_{1,4} → dd, d_{1,5} → dd,
   d_{2,1} → dd, d_{2,2} → dd, d_{2,3} → dd, d_{2,4} → dd, d_{2,5} → dd,
   d_{3,1} → dd, d_{3,2} → dd, d_{3,3} → dd, d_{3,4} → dd, d_{3,5} → dd,
   d_{4,1} → dd, d_{4,2} → dd, d_{4,3} → dd, d_{4,4} → dd, d_{4,5} → dd,
   d_{5,1} → dd, d_{5,2} → dd, d_{5,3} → dd, d_{5,4} → dd, d_{5,5} → dd,

   Ω_{1,1} → NewNetOvProd[[1]][[1]],
   Ω_{1,2} → NewNetOvProd[[1]][[2]], Ω_{1,3} → NewNetOvProd[[1]][[3]],
   Ω_{1,4} → NewNetOvProd[[1]][[4]], Ω_{1,5} → NewNetOvProd[[1]][[5]],
   Ω_{2,1} → NewNetOvProd[[2]][[1]], Ω_{2,2} → NewNetOvProd[[2]][[2]],
   Ω_{2,3} → NewNetOvProd[[2]][[3]], Ω_{2,4} → NewNetOvProd[[2]][[4]],
   Ω_{2,5} → NewNetOvProd[[2]][[5]],
   Ω_{3,1} → NewNetOvProd[[3]][[1]], Ω_{3,2} → NewNetOvProd[[3]][[2]],
   Ω_{3,3} → NewNetOvProd[[3]][[3]], Ω_{3,4} → NewNetOvProd[[3]][[4]],
   Ω_{3,5} → NewNetOvProd[[3]][[5]],
   Ω_{4,1} → NewNetOvProd[[4]][[1]], Ω_{4,2} → NewNetOvProd[[4]][[2]],
   Ω_{4,3} → NewNetOvProd[[4]][[3]], Ω_{4,4} → NewNetOvProd[[4]][[4]],
   Ω_{4,5} → NewNetOvProd[[4]][[5]],
   Ω_{5,1} → NewNetOvProd[[5]][[1]], Ω_{5,2} → NewNetOvProd[[5]][[2]],
   Ω_{5,3} → NewNetOvProd[[5]][[3]], Ω_{5,4} → NewNetOvProd[[5]][[4]],
   Ω_{5,5} → NewNetOvProd[[5]][[5]],
   nuK → nu,
   denK → den


  };


B10 = 1500;
B20 = 1500;
```

```
B30 = 1500;
B40 = 1500;
B50 = 1500;
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;


sol =
 NDSolve[
  {
    B_1'[t] == dB_1,
    B_2'[t] == dB_2,
    B_3'[t] == dB_3,
    B_4'[t] == dB_4,
    B_5'[t] == dB_5,

    M_1'[t] == dM_1,
    M_2'[t] == dM_2,
    M_3'[t] == dM_3,
    M_4'[t] == dM_4,
    M_5'[t] == dM_5,



    B_1[0] == B10,
    B_2[0] == B20,
    B_3[0] == B30,
    B_4[0] == B40,
    B_5[0] == B50,
    M_1[0] == M10,
    M_2[0] == M20,
    M_3[0] == M30,
    M_4[0] == M40,
    M_5[0] == M50

   } /. par,
  {B_1, B_2, B_3, B_4, B_5, M_1, M_2, M_3, M_4, M_5},
  {t, 0, tmax}];

{B_1[tmax], B_2[tmax], B_3[tmax], B_4[tmax], B_5[tmax],
   M_1[tmax], M_2[tmax], M_3[tmax], M_4[tmax], M_5[tmax]} /. sol /. par;
```

```
  Min[{B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax]} /. sol /. par]

 )
```

In[6988]:=
```
robustnessNewSaitoOVxOM[NetTop_, coop_] := (
  n1 = 1;
  n2 = 5000;
  mid = (n1 + n2) / 2;

  While[(n1 ≠ mid && n2 ≠ mid ),
   (If[fNewSaitoOVxOM[NetTop, mid, coop] < 1, n2 = mid, n1 = mid];
    mid = Floor[N[(n1 + n2) / 2]];); {n1, n2, mid}]; mid

 )
```

In[•]:=
```
NetK = {
   {0, 1, 0, 1, 0},
   {1, 0, 1, 1, 0},
   {1, 0, 1, 0, 1},
   {0, 1, 0, 1, 0},
   {0, 0, 0, 0, 1}
  };
```

In[6991]:=
```
fNewSaitoOM[NetK, 0]
```

Out[6991]= 6661.43

In[6993]:=
```
fNewSaitoOVxOM[NetK, 0, 10]
```

Out[6993]= 7476.33

In[6995]:=
```
robustnessNewSaitoOM[NetK]
```

Out[6995]= 473

In[6994]:=
```
robustnessNewSaitoOVxOM[NetK, 10]
```
Out[6994]=
```
553
```

In[6996]:=
```
AuxoComm8OM
```
Out[6996]=
```
{846, 800, 846, 915, 783, 800, 655, 915, 915, 846, 865, 865, 783, 865, 894, 846, 846, 800,
 865, 668, 846, 846, 865, 846, 846, 915, 865, 655, 915, 865, 865, 655, 865, 915, 915,
 915, 846, 894, 915, 915, 846, 915, 894, 915, 915, 865, 846, 846, 915, 846, 846, 865,
 846, 613, 915, 865, 642, 915, 915, 865, 846, 846, 915, 915, 865, 846, 846, 915,
 915, 894, 865, 846, 915, 915, 915, 915, 865, 800, 846, 915, 800, 915, 865, 846,
 846, 915, 894, 846, 846, 846, 613, 915, 915, 894, 915, 865, 846, 846, 865, 915}
```

In[6997]:=
```
coop5to15OM = {Table[robustnessNewSaitoOVxOM[#, 5], {20}],
    Table[robustnessNewSaitoOVxOM[#, 10], {20}],
    Table[robustnessNewSaitoOVxOM[#, 15], {20}]} &;
```

In[6998]:=
```
wf8OM = Parallelize[coop5to15OM /@ hk8];
```

In[6999]:=
```
wf8NormalizedOM = N[wf8OM[[#]] / AuxoComm8OM[[#]]] & /@ Range[100]
```

In[7000]:=
```
wf8NormalizedWith5CoopOM = wf8NormalizedOM[[#]][[1]] & /@ Range[100]
```

In[7001]:=
```
wf8NormalizedWith10CoopOM = wf8NormalizedOM[[#]][[2]] & /@ Range[100]
```

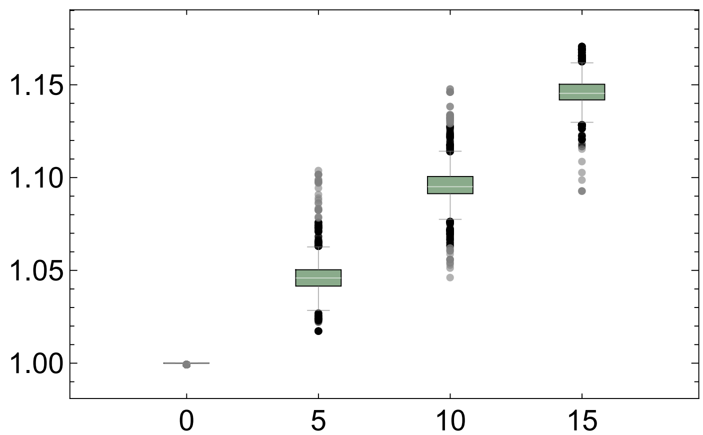
In[7002]:=
```
wf8NormalizedWith15CoopOM = wf8NormalizedOM[[#]][[3]] & /@ Range[100]
```

In[7003]:=
```
allcoopWith8AuxoOM = {Flatten[wf8NormalizedWith5CoopOM],
  Flatten[wf8NormalizedWith10CoopOM], Flatten[wf8NormalizedWith15CoopOM]}
```

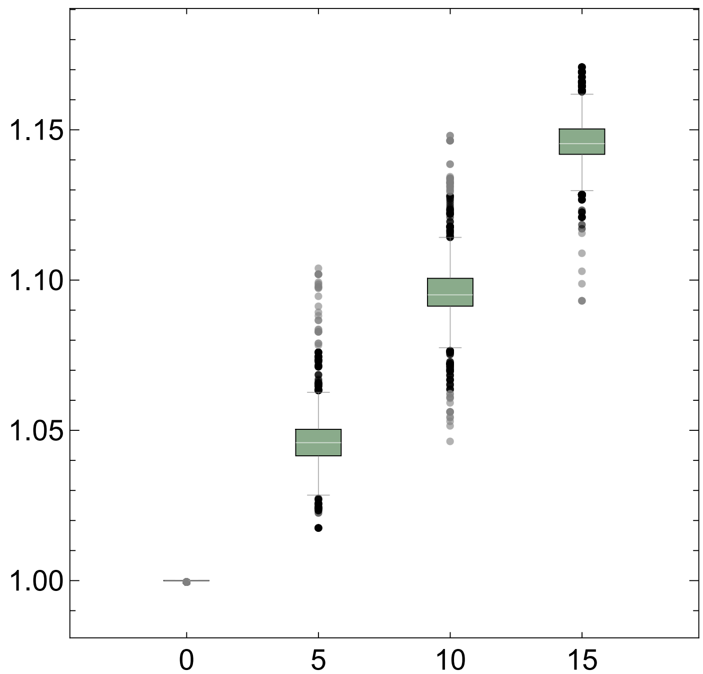
In[7004]:=
```
allcoopWith8AuxoPlusAuxoOM = Join[{ConstantArray[1, {2000}]}, allcoopWith8AuxoOM]
```

In[7005]:=
```
BoxWhiskerChart[allcoopWith8AuxoPlusAuxoOM, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{gree1}},
 Frame → True, ChartLabels → {"0", "5", "10", "15"},
 BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15]]
```

Out[7005]=



In[7006]:=
```
BoxWhiskerChart[allcoopWith8AuxoPlusAuxoOM, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{gree1}},
 Frame → True, ChartLabels → {"0", "5", "10", "15"}, BarSpacing → 1.9,
 FrameStyle → Directive[Black, FontSize → 15], AspectRatio → 1]
```

Out[7006]=


In[7007]:=
```
allcoopWith8AuxoPlusAuxoOM // Length
```

Out[7007]= 4

In[7008]:=
```
SignedRankTest[allcoopWith8AuxoPlusAuxoOM[[2]], 1]
SignedRankTest[allcoopWith8AuxoPlusAuxoOM[[3]], 1]
SignedRankTest[allcoopWith8AuxoPlusAuxoOM[[4]], 1]
```

Out[7008]= 0.

Out[7009]= 0.

Out[7010]= 0.

Solving the system of ODE Random parametrization

In[7011]:=

```
Knum = 0.2;
ccrnum = 0.05;
qqrnum = 0.3;
ddrnum = 0.00015;
OMrnum = 1;
nurum = 1500;
den2rum = 2;

corrpar0 = 10^3;
corrpar1 = 10^4;
corrpar2 = 10^6;

KKr := RandomVariate[
    GammaDistribution[corrpar0 Sqrt[Knum], (1/corrpar0) Sqrt[Knum]], 1][[1]];
ccr := RandomVariate[GammaDistribution[corrpar1 Sqrt[ccrnum],
     (1/corrpar1) Sqrt[ccrnum]], 1][[1]];
qqr := RandomVariate[GammaDistribution[corrpar0 Sqrt[qqrnum],
     (1/corrpar0) Sqrt[qqrnum]], 1][[1]];
ddr := RandomVariate[GammaDistribution[corrpar1 Sqrt[ddrnum],
     (1/corrpar1) Sqrt[ddrnum]], 1][[1]];
OMr := RandomVariate[GammaDistribution[corrpar0 Sqrt[OMrnum],
     (1/corrpar0) Sqrt[OMrnum]], 1][[1]];
nur := (*nurum*) RandomVariate[GammaDistribution[
     corrpar2 Sqrt[nurum], (1/corrpar2) Sqrt[nurum]], 1][[1]] ;
denr2 := (*den2rum*) RandomVariate[GammaDistribution[
     corrpar2 Sqrt[den2rum], (1/corrpar2) Sqrt[den2rum]], 1][[1]] ;


parR = Join[Table[KKr, {5}], Table[ccr, {25}],
  Table[qqr, {5}], Table[ddr, {25}], Table[OMr, {25}], {nur}, {denr2}]
```

Out[7028]=
```
{0.188815, 0.195082, 0.196854, 0.19052, 0.20302, 0.0502137, 0.0496925, 0.0501363,
 0.0498408, 0.0505404, 0.0499921, 0.0494163, 0.0496893, 0.049898, 0.0513354,
 0.0503837, 0.049978, 0.0500244, 0.054186, 0.0500949, 0.0501257, 0.0499547,
 0.0501146, 0.0482454, 0.0492807, 0.0510178, 0.0494115, 0.0503012, 0.0497217,
 0.0501424, 0.295728, 0.315882, 0.319463, 0.303337, 0.307523, 0.0001423, 0.000141181,
 0.000144834, 0.000150668, 0.000162101, 0.000129145, 0.000160026, 0.000144877,
 0.000165359, 0.000151779, 0.000140578, 0.000147872, 0.000144543, 0.000136926,
 0.000163379, 0.0001566, 0.000140889, 0.000159583, 0.000142451, 0.000156462,
 0.000148262, 0.000146895, 0.000141387, 0.000142278, 0.0001342, 0.931435,
 0.990468, 1.06573, 0.999991, 0.981819, 1.0199, 1.02194, 1.00908, 0.988543,
 0.9723, 0.987644, 1.03431, 1.03561, 1.01106, 1.01296, 1.02621, 1.02569, 0.978797,
 0.952987, 1.03563, 1.02647, 1.02245, 1.00566, 1.00163, 0.945659, 1499.57, 2.00109}
```

In[7029]:=
```
parR = %
```

Out[7029]=
```
{0.188815, 0.195082, 0.196854, 0.19052, 0.20302, 0.0502137, 0.0496925, 0.0501363,
 0.0498408, 0.0505404, 0.0499921, 0.0494163, 0.0496893, 0.049898, 0.0513354,
 0.0503837, 0.049978, 0.0500244, 0.054186, 0.0500949, 0.0501257, 0.0499547,
 0.0501146, 0.0482454, 0.0492807, 0.0510178, 0.0494115, 0.0503012, 0.0497217,
 0.0501424, 0.295728, 0.315882, 0.319463, 0.303337, 0.307523, 0.0001423, 0.000141181,
 0.000144834, 0.000150668, 0.000162101, 0.000129145, 0.000160026, 0.000144877,
 0.000165359, 0.000151779, 0.000140578, 0.000147872, 0.000144543, 0.000136926,
 0.000163379, 0.0001566, 0.000140889, 0.000159583, 0.000142451, 0.000156462,
 0.000148262, 0.000146895, 0.000141387, 0.000142278, 0.0001342, 0.931435,
 0.990468, 1.06573, 0.999991, 0.981819, 1.0199, 1.02194, 1.00908, 0.988543,
 0.9723, 0.987644, 1.03431, 1.03561, 1.01106, 1.01296, 1.02621, 1.02569, 0.978797,
 0.952987, 1.03563, 1.02647, 1.02245, 1.00566, 1.00163, 0.945659, 1499.57, 2.00109}
```

In[7030]:=

fNewSaitoROM[Net_, Dh_] := (

$$dB_1 = B_1[t]\left(-B_1[t]\,\kappa_1 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - (c_{1,1} + c_{1,2} + c_{1,3} + c_{1,4} + c_{1,5})\,B_1[t];$$

$$dB_2 = B_2[t]\left(-B_2[t]\,\kappa_2 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - (c_{2,1} + c_{2,2} + c_{2,3} + c_{2,4} + c_{2,5})\,B_2[t];$$

$$dB_3 = B_3[t]\left(-B_3[t]\,\kappa_3 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - (c_{3,1} + c_{3,2} + c_{3,3} + c_{3,4} + c_{3,5})\,B_3[t];$$

$$dB_4 = B_4[t]\left(-B_4[t]\,\kappa_4 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} *\right.$$

$$\frac{M_4[t]}{denK+M_4[t]} * \frac{M_5[t]}{denK+M_5[t]}\Bigg) - (c_{4,1}+c_{4,2}+c_{4,3}+c_{4,4}+c_{4,5})\ B_4[t];$$

$$dB_5 = B_5[t]\ \Bigg(-B_5[t]\ \kappa_5 + nuK * \frac{M_1[t]}{denK+M_1[t]} * \frac{M_2[t]}{denK+M_2[t]} * \frac{M_3[t]}{denK+M_3[t]} * \frac{M_4[t]}{denK+M_4[t]} * \frac{M_5[t]}{denK+M_5[t]}\Bigg) - (c_{5,1}+c_{5,2}+c_{5,3}+c_{5,4}+c_{5,5})\ B_5[t];$$

$$dM_1 = -M_1[t]\ (Dh+q_1) + \left(nuK * \frac{M_1[t]}{denK+M_1[t]} * \frac{M_2[t]}{denK+M_2[t]} * \frac{M_3[t]}{denK+M_3[t]} * \frac{M_4[t]}{denK+M_4[t]} * \frac{M_5[t]}{denK+M_5[t]}\right) (-B_1[t]\ d_{1,1} - B_2[t]\ d_{1,2} - B_3[t]\ d_{1,3} - B_4[t]\ d_{1,4} - B_5[t]\ d_{1,5}) + B_1[t]\ \Omega_{1,1} + B_2[t]\ \Omega_{1,2} + B_3[t]\ \Omega_{1,3} + B_4[t]\ \Omega_{1,4} + B_5[t]\ \Omega_{1,5};$$

$$dM_2 = -M_2[t]\ (Dh+q_2) + \left(nuK * \frac{M_1[t]}{denK+M_1[t]} * \frac{M_2[t]}{denK+M_2[t]} * \frac{M_3[t]}{denK+M_3[t]} * \frac{M_4[t]}{denK+M_4[t]} * \frac{M_5[t]}{denK+M_5[t]}\right) (-B_1[t]\ d_{2,1} - B_2[t]\ d_{2,2} - B_3[t]\ d_{2,3} - B_4[t]\ d_{2,4} - B_5[t]\ d_{2,5}) + B_1[t]\ \Omega_{2,1} + B_2[t]\ \Omega_{2,2} + B_3[t]\ \Omega_{2,3} + B_4[t]\ \Omega_{2,4} + B_5[t]\ \Omega_{2,5};$$

$$dM_3 = -M_3[t]\ (Dh+q_3) + \left(nuK * \frac{M_1[t]}{denK+M_1[t]} * \frac{M_2[t]}{denK+M_2[t]} * \frac{M_3[t]}{denK+M_3[t]} * \frac{M_4[t]}{denK+M_4[t]} * \frac{M_5[t]}{denK+M_5[t]}\right) (-B_1[t]\ d_{3,1} - B_2[t]\ d_{3,2} - B_3[t]\ d_{3,3} - B_4[t]\ d_{3,4} - B_5[t]\ d_{3,5}) + B_1[t]\ \Omega_{3,1} + B_2[t]\ \Omega_{3,2} + B_3[t]\ \Omega_{3,3} + B_4[t]\ \Omega_{3,4} + B_5[t]\ \Omega_{3,5};$$

$$dM_4 = -M_4[t]\ (Dh+q_4) + \left(nuK * \frac{M_1[t]}{denK+M_1[t]} * \frac{M_2[t]}{denK+M_2[t]} * \frac{M_3[t]}{denK+M_3[t]} * \frac{M_4[t]}{denK+M_4[t]} * \frac{M_5[t]}{denK+M_5[t]}\right) (-B_1[t]\ d_{4,1} - B_2[t]\ d_{4,2} - B_3[t]\ d_{4,3} - B_4[t]\ d_{4,4} - B_5[t]\ d_{4,5}) + B_1[t]\ \Omega_{4,1} + B_2[t]\ \Omega_{4,2} + B_3[t]\ \Omega_{4,3} + B_4[t]\ \Omega_{4,4} + B_5[t]\ \Omega_{4,5};$$

$$dM_5 = -M_5[t]\ (Dh+q_5) + \left(nuK * \frac{M_1[t]}{denK+M_1[t]} * \frac{M_2[t]}{denK+M_2[t]} * \frac{M_3[t]}{denK+M_3[t]} * \frac{M_4[t]}{denK+M_4[t]} * \frac{M_5[t]}{denK+M_5[t]}\right) (-B_1[t]\ d_{5,1} - B_2[t]\ d_{5,2} - B_3[t]\ d_{5,3} - B_4[t]\ d_{5,4} - B_5[t]\ d_{5,5}) + B_1[t]\ \Omega_{5,1} + B_2[t]\ \Omega_{5,2} + B_3[t]\ \Omega_{5,3} + B_4[t]\ \Omega_{5,4} + B_5[t]\ \Omega_{5,5};$$

tmax = 1000;

par = {

$\kappa_1 \to$ parR[[1]], $\kappa_2 \to$ parR[[2]], $\kappa_3 \to$ parR[[3]], $\kappa_4 \to$ parR[[4]], $\kappa_5 \to$ parR[[5]],

$c_{1,1} \to$ parR[[6]] × Net[[1]][[1]],
$c_{1,2} \to$ parR[[7]] × Net[[1]][[2]], $c_{1,3} \to$ parR[[8]] × Net[[1]][[3]],
$c_{1,4} \to$ parR[[9]] × Net[[1]][[4]], $c_{1,5} \to$ parR[[10]] × Net[[1]][[5]],
$c_{2,1} \to$ parR[[11]] × Net[[2]][[1]], $c_{2,2} \to$ parR[[12]] × Net[[2]][[2]],
$c_{2,3} \to$ parR[[13]] × Net[[2]][[3]], $c_{2,4} \to$ parR[[14]] × Net[[2]][[4]],
$c_{2,5} \to$ parR[[15]] × Net[[2]][[5]],

```
c_{3,1} → parR[[16]] × Net[[3]][[1]], c_{3,2} → parR[[17]] × Net[[3]][[2]],
c_{3,3} → parR[[18]] × Net[[3]][[3]], c_{3,4} → parR[[19]] × Net[[3]][[4]],
c_{3,5} → parR[[20]] × Net[[3]][[5]],
c_{4,1} → parR[[21]] × Net[[4]][[1]], c_{4,2} → parR[[22]] × Net[[4]][[2]],
c_{4,3} → parR[[23]] × Net[[4]][[3]], c_{4,4} → parR[[24]] × Net[[4]][[4]],
c_{4,5} → parR[[25]] × Net[[4]][[5]],
c_{5,1} → parR[[26]] × Net[[5]][[1]], c_{5,2} → parR[[27]] × Net[[5]][[2]],
c_{5,3} → parR[[28]] × Net[[5]][[3]], c_{5,4} → parR[[29]] × Net[[5]][[4]],
c_{5,5} → parR[[30]] × Net[[5]][[5]],

q_1 → parR[[31]], q_2 → parR[[32]],
q_3 → parR[[33]], q_4 → parR[[34]], q_5 → parR[[35]],

d_{1,1} → parR[[36]], d_{1,2} → parR[[37]],
d_{1,3} → parR[[38]], d_{1,4} → parR[[39]], d_{1,5} → parR[[40]],
d_{2,1} → parR[[41]], d_{2,2} → parR[[42]], d_{2,3} → parR[[43]],
d_{2,4} → parR[[44]], d_{2,5} → parR[[45]],
d_{3,1} → parR[[46]], d_{3,2} → parR[[47]], d_{3,3} → parR[[48]],
d_{3,4} → parR[[49]], d_{3,5} → parR[[50]],
d_{4,1} → parR[[51]], d_{4,2} → parR[[52]], d_{4,3} → parR[[53]],
d_{4,4} → parR[[54]], d_{4,5} → parR[[55]],
d_{5,1} → parR[[56]], d_{5,2} → parR[[57]], d_{5,3} → parR[[58]],
d_{5,4} → parR[[59]], d_{5,5} → parR[[60]],

Ω_{1,1} → parR[[61]] × Net[[1]][[1]],
Ω_{1,2} → parR[[62]] × Net[[1]][[2]], Ω_{1,3} → parR[[63]] × Net[[1]][[3]],
Ω_{1,4} → parR[[64]] × Net[[1]][[4]], Ω_{1,5} → parR[[65]] × Net[[1]][[5]],
Ω_{2,1} → parR[[66]] × Net[[2]][[1]], Ω_{2,2} → parR[[67]] × Net[[2]][[2]],
Ω_{2,3} → parR[[68]] × Net[[2]][[3]], Ω_{2,4} → parR[[69]] × Net[[2]][[4]],
Ω_{2,5} → parR[[70]] × Net[[2]][[5]],
Ω_{3,1} → parR[[71]] × Net[[3]][[1]], Ω_{3,2} → parR[[72]] × Net[[3]][[2]],
Ω_{3,3} → parR[[73]] × Net[[3]][[3]], Ω_{3,4} → parR[[74]] × Net[[3]][[4]],
Ω_{3,5} → parR[[75]] × Net[[3]][[5]],
Ω_{4,1} → parR[[76]] × Net[[4]][[1]], Ω_{4,2} → parR[[77]] × Net[[4]][[2]],
Ω_{4,3} → parR[[78]] × Net[[4]][[3]], Ω_{4,4} → parR[[79]] × Net[[4]][[4]],
Ω_{4,5} → parR[[80]] × Net[[4]][[5]],
Ω_{5,1} → parR[[81]] × Net[[5]][[1]], Ω_{5,2} → parR[[82]] × Net[[5]][[2]],
Ω_{5,3} → parR[[83]] × Net[[5]][[3]], Ω_{5,4} → parR[[84]] × Net[[5]][[4]],
Ω_{5,5} → parR[[85]] × Net[[5]][[5]],
nuK → parR[[86]],
denK → parR[[87]]
```

```
 };



B10 = 1500;
B20 = 1500;
B30 = 1500;
B40 = 1500;
B50 = 1500;
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;


sol =
 NDSolve[
  {
    B1'[t] == dB1,
    B2'[t] == dB2,
    B3'[t] == dB3,
    B4'[t] == dB4,
    B5'[t] == dB5,

    M1'[t] == dM1,
    M2'[t] == dM2,
    M3'[t] == dM3,
    M4'[t] == dM4,
    M5'[t] == dM5,


    B1[0] == B10,
    B2[0] == B20,
    B3[0] == B30,
    B4[0] == B40,
    B5[0] == B50,
    M1[0] == M10,
    M2[0] == M20,
    M3[0] == M30,
    M4[0] == M40,
    M5[0] == M50
```

```
    } /. par,
   {B₁, B₂, B₃, B₄, B₅, M₁, M₂, M₃, M₄, M₅},
   {t, 0, tmax}];

  {B₁[tmax], B₂[tmax], B₃[tmax], B₄[tmax], B₅[tmax],
    M₁[tmax], M₂[tmax], M₃[tmax], M₄[tmax], M₅[tmax]} /. sol /. par;

  Min[{B₁[tmax], B₂[tmax], B₃[tmax], B₄[tmax], B₅[tmax]} /. sol /. par]

 )
```

In[7031]:=
```
robustnessNewSaitoROM[NetTop_] := (
  n1 = 1;
  n2 = 5000;
  mid = (n1 + n2) / 2;

  While[(n1 ≠ mid && n2 ≠ mid),
   (If[fNewSaitoROM[NetTop, mid] < 1, n2 = mid, n1 = mid];
    mid = Floor[N[(n1 + n2) / 2]];); {n1, n2, mid}]; mid

 )
```

As an example let's take the following Network

In[7032]:=
```
NetK = {
   {0, 1, 0, 1, 0},
   {1, 0, 1, 1, 0},
   {1, 0, 1, 0, 1},
   {0, 1, 0, 1, 0},
   {0, 0, 0, 0, 1}
  };
```

Using the function fNewSaito we can calculate the smallest value of a bacterial population in the community for a given disturbance vale. For example, let's take Disturbance value 1 and 500:

In[7033]:= `fNewSaitoOM[NetK, 0]`

Out[7033]= 6661.43

In[7034]:= `fNewSaitoOM[NetK, 500]`

Out[7034]= -4.61275×10^{-11}

In[7035]:=
```
fNewSaitoROM[NetK, 0]
```
Out[7035]= 6257.85

In[7036]:=
```
fNewSaitoROM[NetK, 500]
```
Out[7036]= -2.31312×10^{-15}

Using the function fNewSaito we can calculate Robustness of the Network:

In[7037]:=
```
robustnessNewSaitoOM[NetK]
```
Out[7037]= 473

In[7038]:=
```
robustnessNewSaitoROM[NetK]
```
Out[7038]= 469

We can calculate the (Relative) Entropy and the Assortativity:

In[•]:=
```
RelatEntrop5[NetK]
```
Out[•]= 0.960956

In[•]:=
```
assortativity[NetK]
```
Out[•]= -0.113228

We can calculate the robustness of the previously generated random networks with different number of auxotrophies:

In[7039]:=
```
AuxoComm6ROM = Parallelize[robustnessNewSaitoROM /@ hk6];
AuxoComm7ROM = Parallelize[robustnessNewSaitoROM /@ hk7];
AuxoComm8ROM = Parallelize[robustnessNewSaitoROM /@ hk8];
AuxoComm9ROM = Parallelize[robustnessNewSaitoROM /@ hk9];
AuxoComm10ROM = Parallelize[robustnessNewSaitoROM /@ hk10];
AuxoComm11ROM = Parallelize[robustnessNewSaitoROM /@ hk11];
AuxoComm12ROM = Parallelize[robustnessNewSaitoROM /@ hk12];
AuxoComm13ROM = Parallelize[robustnessNewSaitoROM /@ hk13];
AuxoComm14ROM = Parallelize[robustnessNewSaitoROM /@ hk14];
AuxoComm15ROM = Parallelize[robustnessNewSaitoROM /@ hk15];
AuxoComm16ROM = Parallelize[robustnessNewSaitoROM /@ hk16];
AuxoComm17ROM = Parallelize[robustnessNewSaitoROM /@ hk17];
```

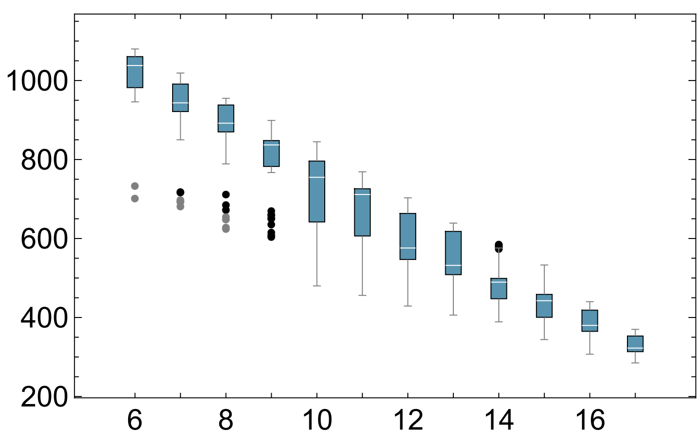
In[7051]:=
```
LikROM = {AuxoComm6ROM, AuxoComm7ROM, AuxoComm8ROM, AuxoComm9ROM,
   AuxoComm10ROM, AuxoComm11ROM, AuxoComm12ROM, AuxoComm13ROM,
   AuxoComm14ROM, AuxoComm15ROM, AuxoComm16ROM, AuxoComm17ROM};
```

In[∘]:=
```
coco = RGBColor[0.34509803921568627, 0.5803921568627451, 0.6901960784313725]
```

Out[∘]=

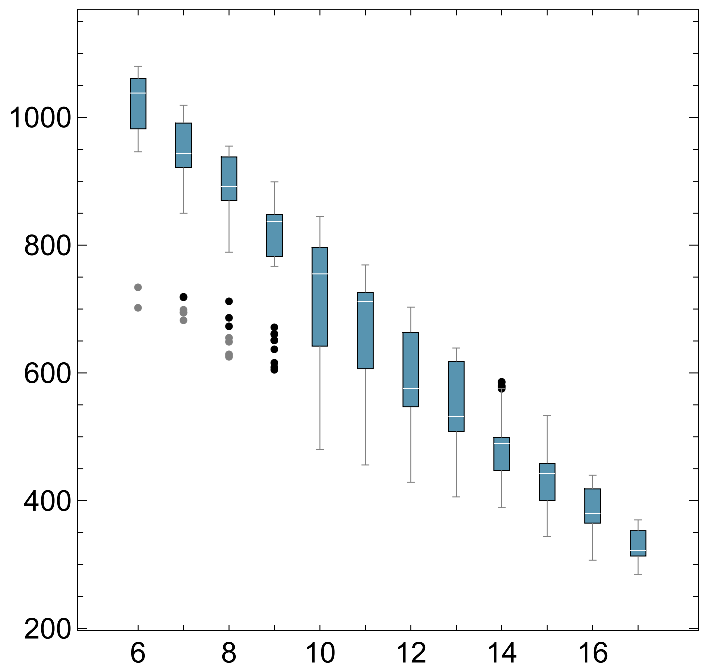
In[7052]:=
```
BoxWhiskerChart[LikROM, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{coco}}, Frame → True,
 ChartLabels → {"6", "", "8", "", "10", "", "12", "", "14", "", "16", ""},
 BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15]]
```

Out[7052]=



In[7053]:=
```
BoxWhiskerChart[LikROM, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{coco}}, Frame → True,
 ChartLabels → {"6", "", "8", "", "10", "", "12", "", "14", "", "16", ""},
 BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15], AspectRatio → 1]
```

Out[7053]=



In[7054]:=
```
AuxoComm7ROM
```

Out[7054]=
```
{936, 925, 991, 943, 933, 928, 720, 1003, 721, 696, 1019, 984, 930, 894,
 978, 935, 1005, 1007, 991, 929, 896, 973, 980, 908, 1000, 924, 976, 858,
 903, 865, 936, 978, 969, 701, 997, 1014, 1005, 970, 916, 979, 1009, 978,
 916, 973, 969, 1004, 895, 1005, 989, 978, 970, 1001, 929, 1007, 927, 993,
 978, 934, 979, 916, 934, 960, 924, 927, 998, 923, 971, 998, 1007, 1000,
 928, 999, 911, 994, 906, 927, 942, 971, 934, 942, 924, 920, 684, 979, 999,
 914, 918, 930, 944, 974, 991, 860, 1001, 850, 931, 998, 974, 697, 917, 912}
```

We can study the correlation between Relative entropy and assortativity with Robustness for Networks with 7 auxotrophies.

In[•]:=
```
Entropy7 = RelatEntrop5 /@ hk7;
```

In[•]:=
```
Assort7 = assortativity /@ hk7;
```

In[7055]:=
```
RobustNewSaito7bROM = AuxoComm7ROM;
```

In[7056]:=
```
Length[Entropy7]
Length[Assort7]
Length[RobustNewSaito7bROM]
```

Out[7056]= 100

Out[7057]= 100

Out[7058]= 100

In[•]:=
```
{Min[Entropy7], Max[Entropy7]}
{Min[Assort7], Max[Assort7]}
```

Out[•]= {0.935154, 0.994118}

Out[•]= {-0.416667, 0.25}

In[•]:=
```
Position[Entropy7, Min[Entropy7]]
```

Out[•]= {{7}}

In[7059]:=
```
RobustNewSaito7bROM[[#]] & /@ {1, 2, 24}
```

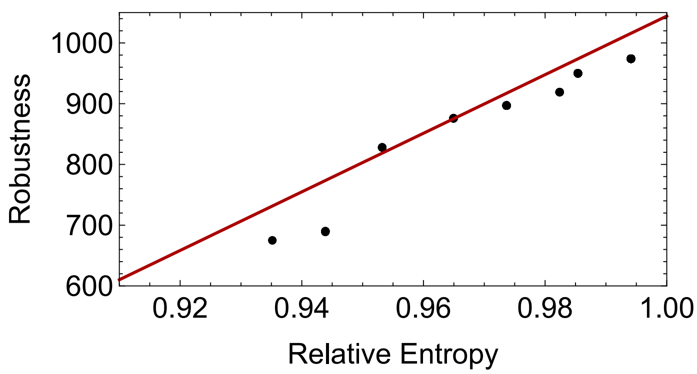
Out[7059]= {936, 925, 908}

In[7060]:=
```
{Min[RobustNewSaito7bROM], {Max[RobustNewSaito7bROM]}}
```

Out[7060]= {684, {1019}}

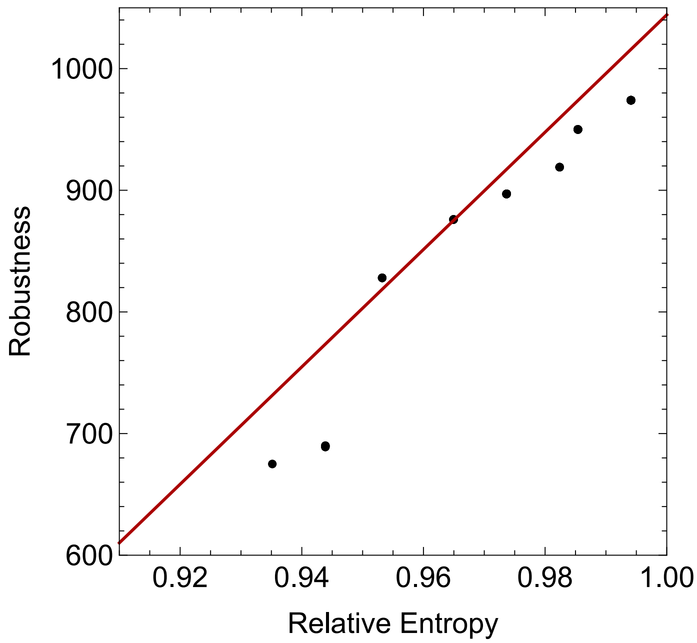
In[7061]:=
```
linerobustnessNewSaito25ROM =
  Fit[Partition[Riffle[Entropy7, RobustNewSaito7bROM], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Entropy7, RobustNewSaito7bOM], {2}],
  Frame → True, FrameLabel → {"Relative Entropy", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]}, PlotRange → {{0.91, 1}, {600, 1050}},
  AspectRatio → 0.5], Plot[linerobustnessNewSaito25ROM,
  {x, 0.91, 1}, AspectRatio → 0.5, PlotStyle → Darker[Red]]]
```

Out[7062]=



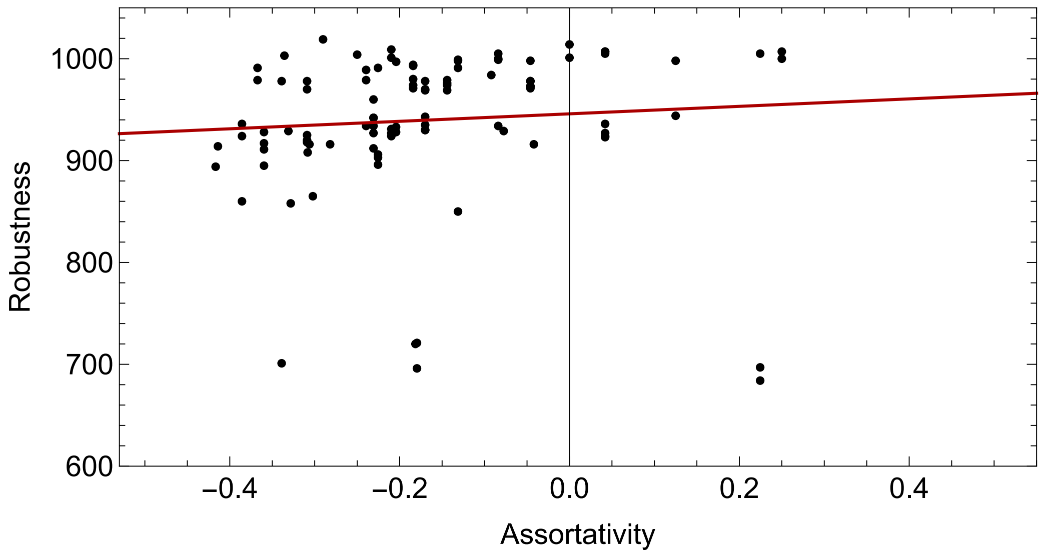
In[7063]:=
```
linerobustnessNewSaito25ROM =
  Fit[Partition[Riffle[Entropy7, RobustNewSaito7bROM], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Entropy7, RobustNewSaito7bOM], {2}],
  Frame → True, FrameLabel → {"Relative Entropy", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]}, PlotRange → {{0.91, 1}, {600, 1050}},
  AspectRatio → 1], Plot[linerobustnessNewSaito25ROM,
  {x, 0.91, 1}, AspectRatio → 1, PlotStyle → Darker[Red]]]
```

Out[7064]=



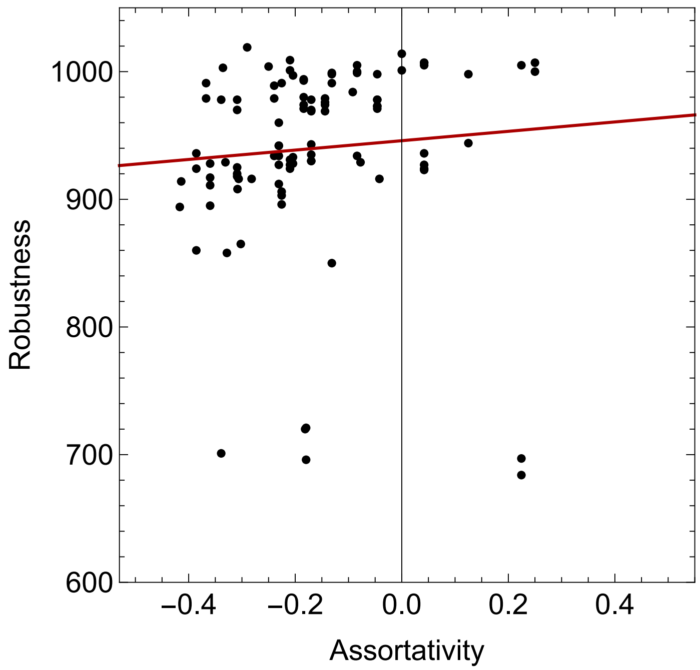
In[7065]:=
```
lineAssoRobrobustnessNewSaito25ROM =
  Fit[Partition[Riffle[Assort7, RobustNewSaito7bROM], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Assort7, RobustNewSaito7bROM], {2}],
  Frame → True, FrameLabel → {"Assortativity", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]}, PlotRange → {{-0.53, 0.55}, {600, 1050}},
  AspectRatio → 0.5], Plot[lineAssoRobrobustnessNewSaito25ROM,
  {x, -0.53, 0.55}, AspectRatio → 0.5, PlotStyle → Darker[Red]]]
```

Out[7066]=



In[7067]:=
```
lineAssoRobrobustnessNewSaito25ROM =
  Fit[Partition[Riffle[Assort7, RobustNewSaito7bROM], {2}], {1, x}, x];
Show[ListPlot[Partition[Riffle[Assort7, RobustNewSaito7bROM], {2}],
  Frame → True, FrameLabel → {"Assortativity", "Robustness"},
  FrameStyle → Directive[Black, FontSize → 15],
  PlotStyle → {Black, PointSize[Medium]}, PlotRange → {{-0.53, 0.55}, {600, 1050}},
  AspectRatio → 1], Plot[lineAssoRobrobustnessNewSaito25ROM,
  {x, -0.53, 0.55}, AspectRatio → 1, PlotStyle → Darker[Red]]]
```

Out[7068]=



In[7069]:=
```
SpearmanRankTest[Entropy7, RobustNewSaito7bROM, "TestDataTable"]
```

Out[7069]=

	Statistic	P-Value
Spearman Rank	0.969641	9.44992×10^{-62}

In[7070]:=
```
SpearmanRankTest[Assort7, RobustNewSaito7bROM, "TestDataTable"]
```

Out[7070]=

	Statistic	P-Value
Spearman Rank	0.346221	0.000417967

In[∘]:=
```
parR // Length
```
Out[∘]=
```
87
```

Solving the system of ODE with Overproduction Random parametrization

```
parR = {0.18881474022448952`, 0.19508178281710964`, 0.1968539228607387`,
   0.19052049238361846`, 0.20301954571213984`, 0.050213731449091366`,
   0.0496924896595411 5`, 0.05013628101955873 4`, 0.04984079123800136`,
   0.050540447228012216`, 0.049992115535220226`, 0.04941627274756244`,
   0.0496892725666025 8`, 0.04989803982219731`, 0.05133537329561034`,
   0.05038366999823772`, 0.0499779728385533`, 0.05002439557598579`,
   0.05418604859101766`, 0.05009489175918319`, 0.05012571272789905`,
   0.0499547166219531 5`, 0.05011457221527254 6`, 0.048245426992293`,
   0.049280716939504726`, 0.051017821360219505`, 0.04941146898497556`,
   0.050301218166985066`, 0.0497217192711843 7`, 0.05014242727618258`,
   0.295728128190498`, 0.31588233870403454`, 0.3194632266110821`,
   0.30333719069277426`, 0.3075229030951657`, 0.00014230049305341368`,
   0.0001411814297762849`, 0.00014483417101476278`, 0.00015066767171940484`,
   0.00016210105683768248`, 0.00012914510104881525`, 0.00016002598558042391`,
   0.00014487661374062148`, 0.00016535855908876148`, 0.0001517790675392051`,
   0.00014057789580658995`, 0.0001478720988920888`, 0.00014454343247977588`,
   0.00013692619442326774`, 0.00016337855467125987`, 0.00015659972968470304`,
   0.00014088870886351968`, 0.00015958329727487234`, 0.00014245104899135682`,
   0.0001564624915155431`, 0.00014826242326205358`, 0.00014689479127941404`,
   0.00014138680294648796`, 0.00014227806092187683`, 0.0001342001191167191`,
   0.9314346835762725`, 0.9904679662458319`, 1.0657290493694118`,
   0.9999909325739111`, 0.9818192099101152`, 1.0199006915561644`,
   1.0219374029647506`, 1.0090781476761679`, 0.9885432391956566`,
   0.9723000210098254`, 0.9876435931298978`, 1.0343099936133262`,
   1.0356079294870986`, 1.0110635157023833`, 1.01296115427104`,
   1.0262063701975213`, 1.0256893697169214`, 0.9787968716825342`,
   0.952986939730553`, 1.0356276632371169`, 1.0264704739169899`,
   1.0224541463822874`, 1.0056590000473318`, 1.0016335837946535`,
   0.9456589032225742`, 1499.5672120867405`, 2.0010876894398955`};
```

In[7071]:=
```
fNewSaitoOVROM[Net_, Dh_, coop_] := (
```

$$dB_1 = B_1[t] \left(-B_1[t]\,\varkappa_1 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - \left(c_{1,1} + c_{1,2} + c_{1,3} + c_{1,4} + c_{1,5}\right) B_1[t];$$

$$dB_2 = B_2[t] \left(-B_2[t]\,\kappa_2 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - (c_{2,1} + c_{2,2} + c_{2,3} + c_{2,4} + c_{2,5})\,B_2[t];$$

$$dB_3 = B_3[t] \left(-B_3[t]\,\kappa_3 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - (c_{3,1} + c_{3,2} + c_{3,3} + c_{3,4} + c_{3,5})\,B_3[t];$$

$$dB_4 = B_4[t] \left(-B_4[t]\,\kappa_4 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - (c_{4,1} + c_{4,2} + c_{4,3} + c_{4,4} + c_{4,5})\,B_4[t];$$

$$dB_5 = B_5[t] \left(-B_5[t]\,\kappa_5 + nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) - (c_{5,1} + c_{5,2} + c_{5,3} + c_{5,4} + c_{5,5})\,B_5[t];$$

$$dM_1 = -M_1[t]\,(Dh + q_1) + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) (-B_1[t]\,d_{1,1} - B_2[t]\,d_{1,2} - B_3[t]\,d_{1,3} - B_4[t]\,d_{1,4} - B_5[t]\,d_{1,5}) + B_1[t]\,\Omega_{1,1} + B_2[t]\,\Omega_{1,2} + B_3[t]\,\Omega_{1,3} + B_4[t]\,\Omega_{1,4} + B_5[t]\,\Omega_{1,5};$$

$$dM_2 = -M_2[t]\,(Dh + q_2) + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) (-B_1[t]\,d_{2,1} - B_2[t]\,d_{2,2} - B_3[t]\,d_{2,3} - B_4[t]\,d_{2,4} - B_5[t]\,d_{2,5}) + B_1[t]\,\Omega_{2,1} + B_2[t]\,\Omega_{2,2} + B_3[t]\,\Omega_{2,3} + B_4[t]\,\Omega_{2,4} + B_5[t]\,\Omega_{2,5};$$

$$dM_3 = -M_3[t]\,(Dh + q_3) + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) (-B_1[t]\,d_{3,1} - B_2[t]\,d_{3,2} - B_3[t]\,d_{3,3} - B_4[t]\,d_{3,4} - B_5[t]\,d_{3,5}) + B_1[t]\,\Omega_{3,1} + B_2[t]\,\Omega_{3,2} + B_3[t]\,\Omega_{3,3} + B_4[t]\,\Omega_{3,4} + B_5[t]\,\Omega_{3,5};$$

$$dM_4 = -M_4[t]\,(Dh + q_4) + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) (-B_1[t]\,d_{4,1} - B_2[t]\,d_{4,2} - B_3[t]\,d_{4,3} - B_4[t]\,d_{4,4} - B_5[t]\,d_{4,5}) + B_1[t]\,\Omega_{4,1} + B_2[t]\,\Omega_{4,2} + B_3[t]\,\Omega_{4,3} + B_4[t]\,\Omega_{4,4} + B_5[t]\,\Omega_{4,5};$$

$$dM_5 = -M_5[t]\,(Dh + q_5) + \left(nuK * \frac{M_1[t]}{denK + M_1[t]} * \frac{M_2[t]}{denK + M_2[t]} * \frac{M_3[t]}{denK + M_3[t]} * \frac{M_4[t]}{denK + M_4[t]} * \frac{M_5[t]}{denK + M_5[t]}\right) (-B_1[t]\,d_{5,1} - B_2[t]\,d_{5,2} - B_3[t]\,d_{5,3} - B_4[t]\,d_{5,4} - B_5[t]\,d_{5,5}) + B_1[t]\,\Omega_{5,1} + B_2[t]\,\Omega_{5,2} + B_3[t]\,\Omega_{5,3} + B_4[t]\,\Omega_{5,4} + B_5[t]\,\Omega_{5,5};$$

```
op = coop; (*Number of links with overExpression*)
posNe = Position[Net, 1];
(*Positions in the matrix where there are links (=1)*)
RaN = RandomSample[posNe, op];
(*Random sample of op links that will be overproduced*)

costincr = 1.3; (*Term multiplying the cost link*)
overprodincr = 1.15;
(*Term multiplying the overproduction link*)


NewNetCost = Partition[Flatten[Net] × parR[[6 ;; 30]], {5}];
Table[NewNetCost[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] =
  NewNetCost[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] * costincr, {i, Length[RaN]}];

NewNetOvProd = Partition[Flatten[Net] × parR[[61 ;; 85]], {5}];
Table[NewNetOvProd[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] =
  NewNetOvProd[[RaN[[i]][[1]]]][[RaN[[i]][[2]]]] * overprodincr, {i,
  Length[RaN]}];


tmax = 1000;
par = {
  κ1 → parR[[1]], κ2 → parR[[2]], κ3 → parR[[3]], κ4 → parR[[4]], κ5 → parR[[5]],

  c1,1 → NewNetCost[[1]][[1]],
  c1,2 → NewNetCost[[1]][[2]], c1,3 → NewNetCost[[1]][[3]],
  c1,4 → NewNetCost[[1]][[4]], c1,5 → NewNetCost[[1]][[5]],
  c2,1 → NewNetCost[[2]][[1]], c2,2 → NewNetCost[[2]][[2]],
  c2,3 → NewNetCost[[2]][[3]], c2,4 → NewNetCost[[2]][[4]],
  c2,5 → NewNetCost[[2]][[5]],
  c3,1 → NewNetCost[[3]][[1]], c3,2 → NewNetCost[[3]][[2]],
  c3,3 → NewNetCost[[3]][[3]], c3,4 → NewNetCost[[3]][[4]],
  c3,5 → NewNetCost[[3]][[5]],
  c4,1 → NewNetCost[[4]][[1]], c4,2 → NewNetCost[[4]][[2]],
  c4,3 → NewNetCost[[4]][[3]], c4,4 → NewNetCost[[4]][[4]],
  c4,5 → NewNetCost[[4]][[5]],
  c5,1 → NewNetCost[[5]][[1]], c5,2 → NewNetCost[[5]][[2]],
  c5,3 → NewNetCost[[5]][[3]], c5,4 → NewNetCost[[5]][[4]],
  c5,5 → NewNetCost[[5]][[5]],

  r1,1 → parR[[31]], r1,2 → parR[[32]],
  r1,3 → parR[[33]], r1,4 → parR[[34]], r1,5 → parR[[35]],
  r2,1 → parR[[36]], r2,2 → parR[[37]], r2,3 → parR[[38]],
```

```
r_{2,4} → parR[[39]], r_{2,5} → parR[[40]],
r_{3,1} → parR[[41]], r_{3,2} → parR[[42]], r_{3,3} → parR[[43]],
r_{3,4} → parR[[44]], r_{3,5} → parR[[45]],
r_{4,1} → parR[[46]], r_{4,2} → parR[[47]], r_{4,3} → parR[[48]],
r_{4,4} → parR[[49]], r_{4,5} → parR[[50]],
r_{5,1} → parR[[51]], r_{5,2} → parR[[52]], r_{5,3} → parR[[53]],
r_{5,4} → parR[[54]], r_{5,5} → parR[[55]],

q_1 → parR[[31]], q_2 → parR[[32]],
q_3 → parR[[33]], q_4 → parR[[34]], q_5 → parR[[35]],

d_{1,1} → parR[[36]], d_{1,2} → parR[[37]],
d_{1,3} → parR[[38]], d_{1,4} → parR[[39]], d_{1,5} → parR[[40]],
d_{2,1} → parR[[41]], d_{2,2} → parR[[42]], d_{2,3} → parR[[43]],
d_{2,4} → parR[[44]], d_{2,5} → parR[[45]],
d_{3,1} → parR[[46]], d_{3,2} → parR[[47]], d_{3,3} → parR[[48]],
d_{3,4} → parR[[49]], d_{3,5} → parR[[50]],
d_{4,1} → parR[[51]], d_{4,2} → parR[[52]], d_{4,3} → parR[[53]],
d_{4,4} → parR[[54]], d_{4,5} → parR[[55]],
d_{5,1} → parR[[56]], d_{5,2} → parR[[57]], d_{5,3} → parR[[58]],
d_{5,4} → parR[[59]], d_{5,5} → parR[[60]],

Ω_{1,1} → NewNetOvProd[[1]][[1]],
Ω_{1,2} → NewNetOvProd[[1]][[2]], Ω_{1,3} → NewNetOvProd[[1]][[3]],
Ω_{1,4} → NewNetOvProd[[1]][[4]], Ω_{1,5} → NewNetOvProd[[1]][[5]],
Ω_{2,1} → NewNetOvProd[[2]][[1]], Ω_{2,2} → NewNetOvProd[[2]][[2]],
Ω_{2,3} → NewNetOvProd[[2]][[3]], Ω_{2,4} → NewNetOvProd[[2]][[4]],
Ω_{2,5} → NewNetOvProd[[2]][[5]],
Ω_{3,1} → NewNetOvProd[[3]][[1]], Ω_{3,2} → NewNetOvProd[[3]][[2]],
Ω_{3,3} → NewNetOvProd[[3]][[3]], Ω_{3,4} → NewNetOvProd[[3]][[4]],
Ω_{3,5} → NewNetOvProd[[3]][[5]],
Ω_{4,1} → NewNetOvProd[[4]][[1]], Ω_{4,2} → NewNetOvProd[[4]][[2]],
Ω_{4,3} → NewNetOvProd[[4]][[3]], Ω_{4,4} → NewNetOvProd[[4]][[4]],
Ω_{4,5} → NewNetOvProd[[4]][[5]],
Ω_{5,1} → NewNetOvProd[[5]][[1]], Ω_{5,2} → NewNetOvProd[[5]][[2]],
Ω_{5,3} → NewNetOvProd[[5]][[3]], Ω_{5,4} → NewNetOvProd[[5]][[4]],
Ω_{5,5} → NewNetOvProd[[5]][[5]],
nuK → parR[[86]],
denK → parR[[87]]


};
```

```
B10 = 1500;
B20 = 1500;
B30 = 1500;
B40 = 1500;
B50 = 1500;
M10 = 10;
M20 = 10;
M30 = 10;
M40 = 10;
M50 = 10;


sol =
 NDSolve[
  {
    B1'[t] == dB1,
    B2'[t] == dB2,
    B3'[t] == dB3,
    B4'[t] == dB4,
    B5'[t] == dB5,

    M1'[t] == dM1,
    M2'[t] == dM2,
    M3'[t] == dM3,
    M4'[t] == dM4,
    M5'[t] == dM5,


    B1[0] == B10,
    B2[0] == B20,
    B3[0] == B30,
    B4[0] == B40,
    B5[0] == B50,
    M1[0] == M10,
    M2[0] == M20,
    M3[0] == M30,
    M4[0] == M40,
    M5[0] == M50

   } /. par,
  {B1, B2, B3, B4, B5, M1, M2, M3, M4, M5},
  {t, 0, tmax}];

{B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax],
```

```
    M1[tmax], M2[tmax], M3[tmax], M4[tmax], M5[tmax]} /. sol /. par;

  Min[{B1[tmax], B2[tmax], B3[tmax], B4[tmax], B5[tmax]} /. sol /. par]

 )
```

In[7072]:=
```
robustnessNewSaitoOVROM[NetTop_, coop_] := (
  n1 = 1;
  n2 = 5000;
  mid = (n1 + n2) / 2;

  While[(n1 ≠ mid && n2 ≠ mid),
   (If[fNewSaitoOVROM[NetTop, mid, coop] < 1, n2 = mid, n1 = mid];
    mid = Floor[N[(n1 + n2) / 2]];); {n1, n2, mid}]; mid

 )
```

In[7073]:=
```
NetK = {
   {0, 1, 0, 1, 0},
   {1, 0, 1, 1, 0},
   {1, 0, 1, 0, 1},
   {0, 1, 0, 1, 0},
   {0, 0, 0, 0, 1}
  };
```

Compare the Robustness with and without (n links) overproduction (ratio cost/production = 1.3/1.15)

In[7074]:=
```
fNewSaitoROM[NetK, 0]
```

Out[7074]=
```
6257.85
```

In[7075]:=
```
fNewLiebigOVR[NetK, 0, 5]
```
Out[7075]=
```
7180.76
```

In[7076]:=
```
robustnessNewSaitoROM[NetK]
```
Out[7076]=
```
469
```

In[7077]:=
```
robustnessNewSaitoOVROM[NetK, 5]
```
Out[7077]=
```
505
```

In[7078]:=
```
robustnessNewSaitoOVROM[NetK, 10]
```
Out[7078]=
```
548
```

In[7079]:=
```
AuxoComm8ROM
```
Out[7079]=
```
{887, 822, 866, 939, 789, 827, 675, 955, 947, 866, 903, 886, 791, 886, 914, 866, 866, 817,
 902, 714, 884, 884, 892, 866, 882, 936, 882, 688, 924, 898, 904, 657, 898, 937, 946,
 940, 886, 916, 946, 945, 890, 955, 928, 953, 947, 905, 870, 876, 945, 859, 883, 892,
 855, 627, 934, 905, 651, 941, 942, 894, 870, 870, 949, 948, 889, 879, 870, 941,
 942, 917, 905, 876, 939, 944, 930, 944, 895, 823, 882, 945, 826, 947, 905, 880,
 865, 947, 931, 874, 862, 865, 631, 947, 946, 907, 935, 891, 887, 870, 882, 931}
```

In[7080]:=
```
coop5to15ROM = {Table[robustnessNewSaitoOVROM[#, 5], {20}],
    Table[robustnessNewSaitoOVROM[#, 10], {20}],
    Table[robustnessNewSaitoOVROM[#, 15], {20}]} &;
```

In[7081]:=
```
wf8ROM = Parallelize[coop5to15ROM /@ hk8];
```

In[7082]:=
```
wf8NormalizedROM = N[wf8ROM[[#]] / AuxoComm8ROM[[#]]] & /@ Range[100]
```

In[7083]:=
```
wf8NormalizedWith5CoopROM = wf8NormalizedROM[[#]][[1]] & /@ Range[100]
```

In[7084]:=
```
wf8NormalizedWith10CoopROM = wf8NormalizedROM[[#]][[2]] & /@ Range[100]
```

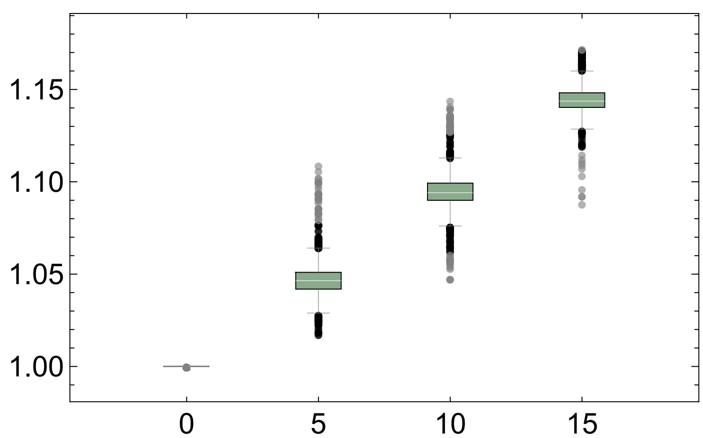
In[7085]:=
```
wf8NormalizedWith15CoopROM = wf8NormalizedROM[[#]][[3]] & /@ Range[100]
```

In[7086]:=
```
allcoopWith8AuxoROM = {Flatten[wf8NormalizedWith5CoopROM],
   Flatten[wf8NormalizedWith10CoopROM], Flatten[wf8NormalizedWith15CoopROM]}
```

In[7087]:=
```
allcoopWith8AuxoPlusAuxoROM =
  Join[{ConstantArray[1, {2000}]}, allcoopWith8AuxoROM]
```

In[7088]:=
```
BoxWhiskerChart[allcoopWith8AuxoPlusAuxoROM, "Outliers",
  ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{gree1}},
  Frame → True, ChartLabels → {"0", "5", "10", "15"},
  BarSpacing → 1.9, FrameStyle → Directive[Black, FontSize → 15]]
```

Out[7088]=



In[7089]:=

```
BoxWhiskerChart[allcoopWith8AuxoPlusAuxoROM, "Outliers",
 ChartBaseStyle → EdgeForm[Dashing[0.99]], ChartStyle → {{gree1}},
 Frame → True, ChartLabels → {"0", "5", "10", "15"}, BarSpacing → 1.9,
 FrameStyle → Directive[Black, FontSize → 15], AspectRatio → 1]
```

Out[7089]=