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Beyond Neutrosophic Graphs

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Abstract

In this outlet, a journey amid three models are designed. Graphs, fuzzy graphs and neutrosophic graphs are three models which form main parts. Assigning one specific number with some conditions to vertices and edges of graphs make them to be titled as fuzzy graphs and assigning three specific numbers with some conditions to vertices and edges of graphs make them to be titled as neutrosophic graphs. In other viewpoint, neutrosophic graphs are 3-array fuzzy graphs which every things are triple. To make more sense, the well-known graphs are defined in new ways. For example, crisp complete, fuzzy complete and neutrosophic complete when the context is about being complete in every model. New notions are defined in the comparable structures on these three models to understand the behaviors of these models according to the notions. Different edges define new form of connections amid vertices. Thus defining new notion of coloring is possible when the connections of vertices which determine new color and it's decider whether using new color or not, have been considered if they've special edges. The tools to define specific edges are studied. One notion is to use the connectedness to have two different types of numbers which are neutrosophic chromatic number and chromatic number. Other notion is to use the idea of neutrosophic strong to get specific edges which are eligible to define new numbers. Some classes of neutrosophic graphs are studied in the the terms of different types of chromatic numbers and neutrosophic chromatic numbers. This book is based on neutrosophic graph theory which is designed to study different types of coloring in that graphs to get new ideas and new results. The results concern specific classes of neutrosophic graphs. In this book, idea of neutrosophic is applied into the setting of hypergraphs and n-SuperHyperGraphs. New setting has the name neutrosophic hypergraphs and neutrosophic n-SuperHyperGraphs. Also, idea of close numbers and super-close numbers are applied to study. The idea of closing numbers and super-closing numbers are some names for (dual) super-coloring and (dual) super-resolving alongside (dual) super-dominating which give us a set and number arising from hyper-vertices and super-vertices alongside their relations in neutrosophic hypergraphs and neutrosophic n-SuperHyperGraphs. When hyper-vertices and super-vertices are too close, idea of (dual) super-coloring and (dual) super-resolving alongside (dual) super-dominating are introduced to study the behaviors of too close hyper-vertices and super-vertices. In this book, idea of neutrosophic is applied into the setting of hypergraphs and n-SuperHyperGraphs. New setting has the name neutrosophic hypergraphs and neutrosophic n-SuperHyperGraphs. Also, idea of close numbers and super-close numbers are applied to study. The idea of closing numbers and super-closing numbers are some names for (dual) super-coloring and (dual) super-resolving alongside (dual) super-dominating which give us a set and number arising from hyper-vertices and super-vertices alongside their relations in neutrosophic hypergraphs and neutrosophic n-SuperHyperGraphs. When hyper-vertices and super-vertices are too close, idea of (dual) super-coloring and (dual) super-resolving alongside (dual) super-dominating are introduced to study the behaviors of too close hyper-vertices and super-vertices. New setting is introduced to study the alliances. Alliances are about a set of vertices which are applied into the setting of neutrosophic graphs. Neighborhood has the key role to define these notions. Also, neighborhood is defined based on the edges, strong edges and some edges which are coming from connectedness. These three types of edges get a framework as neighborhood and after that, too close vertices have key role to define offensive alliance, defensive alliance, t-offensive alliance, and t-defensive alliance based on three types of edges, common edges, strong edges and some edges which are coming from connectedness. The structure of set is studied and general results are obtained. Also, some classes of neutrosophic graphs containing complete, empty, path, cycle, bipartite, t-partite, star and wheel are investigated in the terms of set, minimal set, number, and neutrosophic number. In this study, there's an open way to extend these results into the family of these classes of neutrosophic graphs. The family of neutrosophic graphs aren't study but it seems that analogous results are determined. There's a question. How can be related to each other, two sets partitioning the vertex set of a graph? The ideas of neighborhood and neighbors based on different edges illustrate open way to get results. A set is alliance when two sets partitioning vertex set have uniform structure. All members of set have different amount of neighbors in the set and out of set. It leads us to the notion of offensive and defensive. New ideas, offensive alliance, defensive alliance, t-offensive alliance, t-defensive alliance, strong offensive alliance, strong defensive alliance, strong t-offensive alliance, strong t-defensive alliance, connected offensive alliance, connected defensive alliance, connected t-offensive alliance, and connected t-defensive alliance are introduced. Two numbers concerning cardinality and neutrosophic cardinality of alliances are introduced. A set is alliance when its complement make a relation in the terms of neighborhood. Different edges make different neighborhoods. Three types of edges are applied to define three styles of neighborhoods. General edges, strong edges and connected edges are used where connected edges are the edges arising from connectedness amid two endpoints of the edges. These notions are applied into neutrosophic graphs as individuals and family of them. Independent set as an alliance is a special set which has no neighbor inside and it implies some drawbacks for this notions. Finding special sets which are well-known, is an open way to purse this study. Special set which its members have only one neighbor inside, characterize the connected components where the cardinality of its complement is the number of connected components. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this book. New setting is introduced to study the global offensive alliance. Global offensive alliance is about a set of vertices which are applied into the setting of neutrosophic graphs. Neighborhood has the key role to define this notion. Also, neighborhood is defined based on strong edges. Strong edge gets a framework as neighborhood and after that, too close vertices have key role to define global offensive alliance based on strong edges. The structure of set is studied and general results are

obtained. Also, some classes of neutrosophic graphs containing complete, empty, path, cycle, star, and wheel are investigated in the terms of set, minimal set, number, and neutrosophic number. Neutrosophic number is defined in new way. It's first time to define this type of neutrosophic number in the way that, three values of a vertex are used and they've same share to construct this number. It's called "modified neutrosophic number". Summation of three values of vertex makes one number and applying it to a set makes neutrosophic number of set. This approach facilitates identifying minimal set and optimal set which forms minimal-global-offensive-alliance number and minimal-global-offensive-alliance-neutrosophic number. Two different types of sets namely global-offensive alliance and minimal-global-offensive alliance are defined. Global-offensive alliance identifies the sets in general vision but minimal-global-offensive alliance takes focus on the sets which deleting a vertex is impossible. Minimal-global-offensive-alliance number is about minimum cardinality amid the cardinalities of all minimal-global-offensive alliances in a given neutrosophic graph. New notions are applied in the settings both individual and family. Family of neutrosophic graphs is studied in the way that, the family only contains same classes of neutrosophic graphs. Three types of family of neutrosophic graphs including m-family of neutrosophic stars with common neutrosophic vertex set, m-family of odd complete graphs with common neutrosophic vertex set, and m-family of odd complete graphs with common neutrosophic vertex set are studied. The results are about minimal-global-offensive alliance, minimal-global-offensive-alliance number and its corresponded sets, minimal-global-offensive-alliance-neutrosophic number and its corresponded sets, and characterizing all minimal-global-offensive alliances. The connection of global-offensive-alliances with dominating set and chromatic number are obtained. The number of connected components has some relations with this new concept and it gets some results. Some classes of neutrosophic graphs behave differently when the parity of vertices are different and in this case, path, cycle, and complete illustrate these behaviors. Two applications concerning complete model as individual and family, under the titles of time table and scheduling conclude the results and they give more clarifications. In this study, there's an open way to extend these results into the family of these classes of neutrosophic graphs. The family of neutrosophic graphs aren't study deeply and with more results but it seems that analogous results are determined. Slight progress is obtained in the family of these models but there are open avenues to study family of other models as same models and different models. There's a question. How can be related to each other, two sets partitioning the vertex set of a graph? The ideas of neighborhood and neighbors based on strong edges illustrate open way to get results. A set is global offensive alliance when two sets partitioning vertex set have uniform structure. All members of set have more amount of neighbors in the set than out of set. It leads us to the notion of global offensive alliance. Different edges make different neighborhoods but it's used one style edge titled strong edge. These notions are applied into neutrosophic graphs as individuals and family of them. Independent set as an alliance is a special set which has no neighbor inside and it implies some drawbacks for these notions. Finding special sets which are well-known, is an open way to purse this study. Special set which its members have only one neighbor inside, characterize the connected components where the cardinality of its complement is the number of connected components. Some problems are proposed to pursue this study. Basic

familiarities with graph theory and neutrosophic graph theory are proposed for this book.

New notions are defined in the comparable structures on these three models to understand the behaviors of these models according to the notions. This book is based on neutrosophic graph theory which is designed to study different types of coloring in that graphs to get new ideas and new results. The results concern specific classes of neutrosophic graphs. New notions are defined in the comparable structures on these three models to understand the behaviors of these models according to the notions. This book is based on neutrosophic graph theory which is designed to study different types of coloring in that graphs to get new ideas and new results. The results concern specific classes of neutrosophic graphs.

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The words of mind and the minds of words, are too eligible to be in the stage of aknowledgements

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CHAPTER 1

Neutrosophic Graphs

Akram et al. [**1**] introduce bipolar neutrosophic graphs. He et al. [**2**] also propose operations on single-valued neutrosophic graphs. Broumi et al. [**3**] elicit an introduction to bipolar single valued neutrosophic graph theory. He et al. also introduce Interval valued neutrosophic graphs [**4**], Isolated single valued neutrosophic graphs [**5**], on bipolar single valued neutrosophic graphs [**6**], Single valued neutrosophic graphs [**7**], single valued neutrosophic graphs: degree, order and size [**8**]. Kandasamy et al. [**11**] illustrate Neutrosophic graphs: a new dimension to graph theory in 2015. In 2017, Operations on single valued neutrosophic graphs with application was introduced by Naz et al. [**12**].

1.1 Definitions

To clarify about the definitions, I use some examples and in this way, exemplifying has key role to make sense about the definitions and to introduce new ways to use on these models in the terms of new notions.

The concept of complete is used to classify specific graph in every environment. To differentiate, I use an adjective or prefix in every definition. Two adjectives "fuzzy" and "neutrosophic" are used to distinguish every graph or classes of graph or any notion on them.
The reference [**9**; **10**] is used to write the contents of this chapter.

Definition 1.1.1. $G : (V, E)$ is called a **crisp graph** where V is a set of objects and E is a subset of $V \times V$ such that this subset is symmetric.

Definition 1.1.2. A crisp graph $G : (V, E)$ is called a **fuzzy graph** $G : (\sigma, \mu)$ where $\sigma : V \to [0, 1]$ and $\mu : E \to [0, 1]$ such that $\mu(xy) \leq \sigma(x) \wedge \sigma(y)$ for all $xy \in E$.

Definition 1.1.3. A crisp graph $G : (V, E)$ is called a **neutrosophic graph** $G : (\sigma, \mu)$ where $\sigma = (\sigma_1, \sigma_2, \sigma_3) : V \to [0, 1]$ and $\mu = (\mu_1, \mu_2.\mu_3) : E \to [0, 1]$ such that $\mu(xy) \leq \sigma(x) \wedge \sigma(y)$ for all $xy \in E$.

Definition 1.1.4. A crisp graph $G : (V, E)$ is called a **crisp complete** where $\forall x \in V,\ \forall y \in V,\ xy \in E$.

Definition 1.1.5. A fuzzy graph $G : (\sigma, \mu)$ is called **fuzzy complete** where it's complete and $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $xy \in E$.

Definition 1.1.6. A neutrosophic graph $G : (\sigma, \mu)$ is called a **neutrosophic complete** where it's complete and $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $xy \in E$.

Definition 1.1.7. A crisp graph $G : (V, E)$ is called a **crisp strong**.

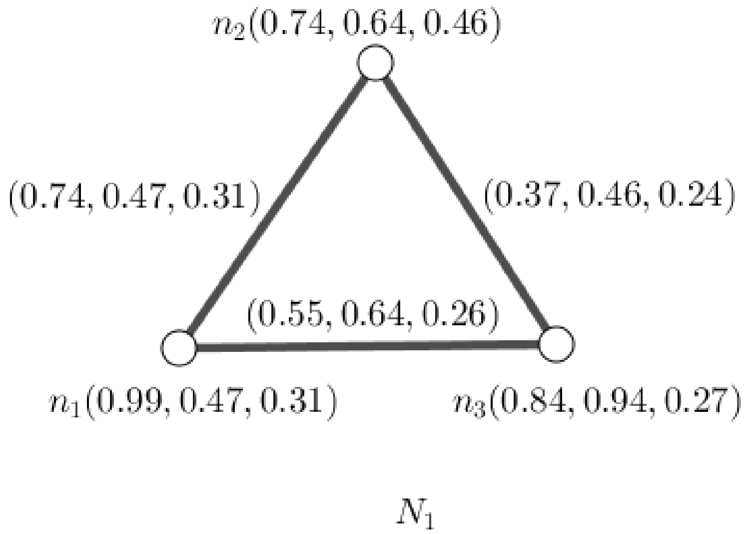


Figure 1.1: Neutrosophic Graph, N_1 nsc1

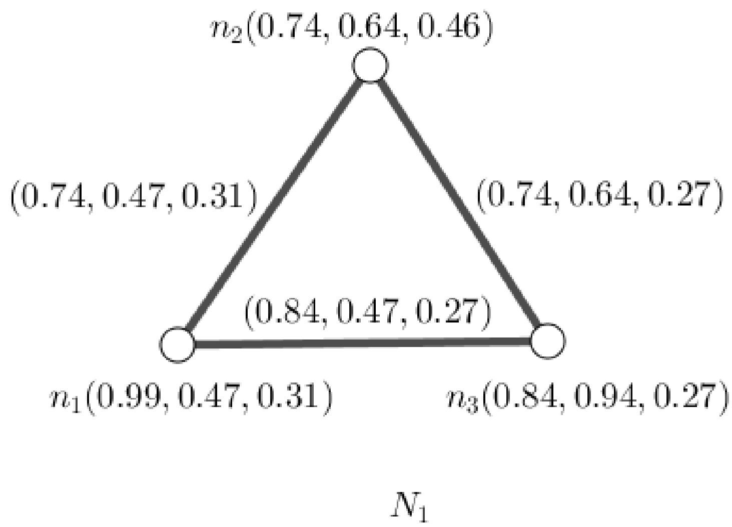


Figure 1.2: Neutrosophic Complete, N_1 nsc2

Definition 1.1.8. A fuzzy graph $G : (\sigma, \mu)$ is called **fuzzy strong** where $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $xy \in E$.

Definition 1.1.9. A neutrosophic graph $G : (\sigma, \mu)$ is called a **neutrosophic strong** where $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $xy \in E$.

Definition 1.1.10. A distinct sequence of vertices $v_0, v_1, \cdots, v_n$ in a crisp graph $G : (V, E)$ is called **crisp path** with length n from v_0 to v_n where $v_i v_{i+1} \in E,\ i = 0, 1, \cdots, n-1$.

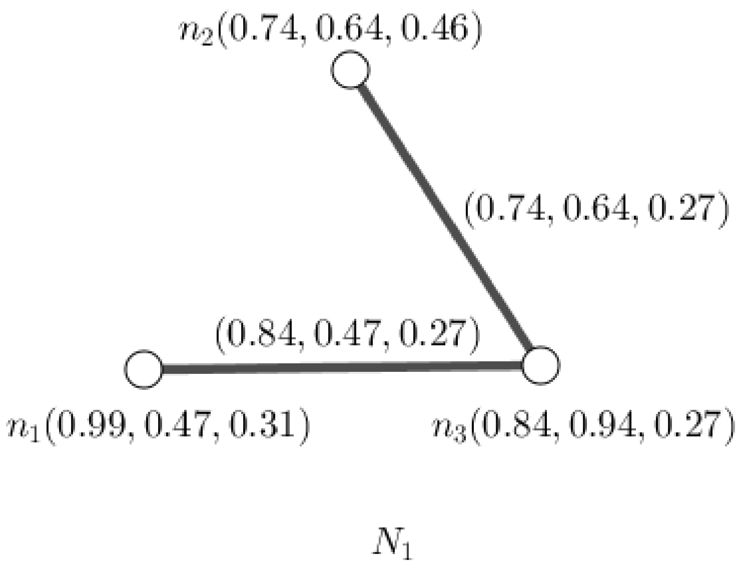


Figure 1.3: Neutrosophic Strong, N_1 nsc3

Definition 1.1.11. A path $v_0, v_1, \cdots, v_n$ is called **fuzzy path** where $\mu(v_i v_{i+1}) > 0,\ i = 0, 1, \cdots, n-1$.

Definition 1.1.12. A path $v_0, v_1, \cdots, v_n$ is called **neutrosophic path** where $\mu(v_i v_{i+1}) > 0,\ i = 0, 1, \cdots, n-1$.

Definition 1.1.13. A path $v_0, v_1, \cdots, v_n$ with exception of v_0 and v_n in a crisp graph $G : (V, E)$ is called **crisp cycle** with length n for v_0 where $v_0 = v_n$. and the order is three.

Definition 1.1.14. A crisp cycle $v_0, v_1, \cdots, v_n, v_0$ is called **fuzzy cycle** where there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$.

Definition 1.1.15. A crisp cycle $v_0, v_1, \cdots, v_n, v_0$ is called **neutrosophic cycle** where there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$.

Table 1.1: Crisp-fying, Fuzzy-fying and Neutrosophic-fying tbl1

	Crisp Graphs	Fuzzy Graphs	Neutrosophic Graphs	
	Crisp Complete	Fuzzy Complete	Neutrosophic Complete	
	Crisp Strong	Fuzzy Strong	Neutrosophic Strong	
	Crisp Path	Fuzzy Path	Neutrosophic Path	
	Crisp Cycle	Fuzzy Cycle	Neutrosophic Cycle	

New definitions are introduced in the terms of neutrosophic type. There are some questions about the relations amid these notions.
The notion of strong is too close to the notions of complete.

(1). Is neutrosophic strong, neutrosophic complete?
No.

Example 1.1.16. Consider Figure (1.3). N_1 is a neutrosophic strong which isn't also neutrosophic complete.

(2). Does neutrosophic strong imply neutrosophic complete?
Sometimes.

Example 1.1.17. Consider Figure (1.3). N_1 is a neutrosophic strong which isn't neutrosophic complete.

Example 1.1.18. Consider Figure (1.2). N_1 is a neutrosophic strong which is also neutrosophic complete.

(3). Does neutrosophic complete imply neutrosophic strong?
Yes. All neutrosophic complete from order $1, 2, 3, \cdots$ are neutrosophic strong. All neutrosophic complete from any order are neutrosophic strong.

Example 1.1.19. Consider Figure (1.2). N_1 is a neutrosophic complete which is also neutrosophic strong.

(4). When does neutrosophic complete imply neutrosophic strong?
Always.

Example 1.1.20. Consider Figure (1.2). N_1 is a neutrosophic complete which is also neutrosophic strong.

(5). When neutrosophic strong imply neutrosophic complete?
When neutrosophic graph is crisp complete.

Example 1.1.21. Consider Figure (1.2). N_1 is a neutrosophic strong which is also neutrosophic complete. Since it's neutrosophic strong and crisp complete.

(6). Which neutrosophic graphs are both neutrosophic complete and neutrosophic strong?
All neutrosophic graphs, which are neutrosophic complete, are neutrosophic strong. In other words, neutrosophic graphs, which are neutrosophic strong and crisp complete, are neutrosophic complete. Neutrosophic complete means that neutrosophic graph is neutrosophic strong and crisp complete.

Example 1.1.22. Consider Figure (1.2). N_1 is a neutrosophic strong which is also neutrosophic complete.

(7). Which neutrosophic graphs are either neutrosophic complete or neutrosophic strong?
Neutrosophic graphs, which are neutrosophic strong but not crisp complete, aren't neutrosophic complete.

Example 1.1.23. Consider Figure (1.3). N_1 is a neutrosophic strong which isn't also neutrosophic complete.

(8). Which neutrosophic graphs are neither neutrosophic complete nor neutrosophic strong?
Neutrosophic graphs, which aren't neutrosophic strong, are neithter neutrosophic complete.

Example 1.1.24. Consider Figure (2.1). N_1 is neither a neutrosophic strong nor neutrosophic complete.

The notion of cycle when the order is three, is too close to the notions of complete. Thus there are some natural questions about them.

(1). Is neutrosophic cycle, neutrosophic complete?
When the order is three and it's neutrosophic strong. For instance, there's a possibility to have neutrosophic cycle and neutrosophic complete. In these Examples, at least the neutrosophic values of two vertices have to be same and minimum to have two edges which have minimum neutrosophic values. In this case, all three edges have same neutrosophic values. Thus I represent three types neutrosophic graphs, which are neutrosophic cycle in the terms of non-isomorphic. Firstly, two vertices have same neutrosophic values and third vertex has neutrosophic value which is greater than them.

Example 1.1.25. Consider Figure (1.4). N_1 is a neutrosophic cycle and neutrosophic complete.

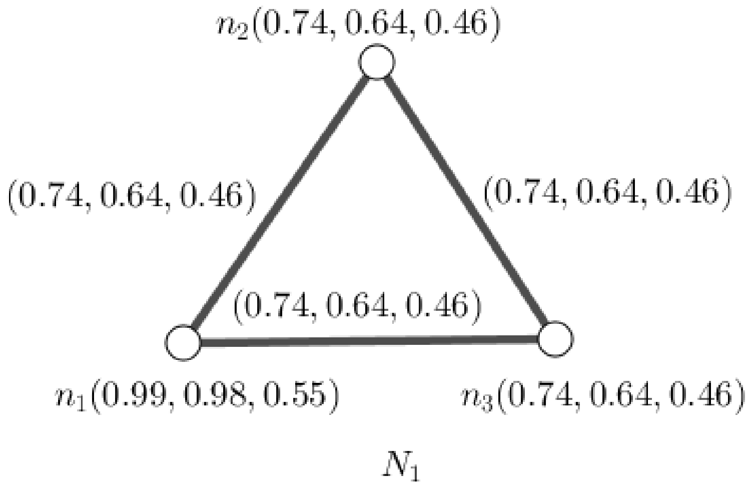


Figure 1.4: Neutrosophic Cycle, N_1, has same neutrosophic values for two vertices.

nsc4

Secondly, three vertices have same neutrosophic values.

Example 1.1.26. Consider Figure (1.4). N_1 is both a neutrosophic complete and neutrosophic cycle.

Thirdly, three vertices have different neutrosophic values.

Example 1.1.27. Consider Figure (1.2). N_1 is both a neutrosophic complete and neutrosophic cycle.

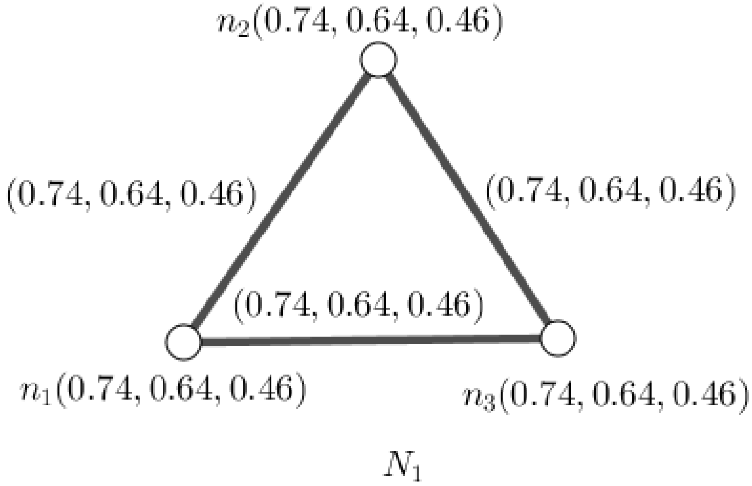


Figure 1.5: Neutrosophic Cycle, N_1, has same neutrosophic values for vertices. nsc5

(2). Does neutrosophic cycle imply neutrosophic complete?
When the order is three and it's neutrosophic strong.

(3). Does neutrosophic complete imply neutrosophic cycle?
When the order is three.

(4). When does neutrosophic complete imply neutrosophic cycle?
When the order is three.

(5). When neutrosophic cycle imply neutrosophic complete?
When the order is three and it's neutrosophic strong.

(6). Which neutrosophic graphs are both neutrosophic complete and neutrosophic cycle?
Only three types of neutrosophic graphs which are in Figures (1.2),(1.4) and (1.5). The order has to be three and it's neutrosophic strong.
Firstly, two vertices have same neutrosophic values and third vertex has neutrosophic value which is greater than them.
Secondly, three vertices have same neutrosophic values.
Thirdly, three vertices have different neutrosophic values.

(7). Which neutrosophic graphs are either neutrosophic complete or neutrosophic cycle?
Either neutrosophic complete or neutrosophic cycle which don't have the order is three for neutrosophic complete and if they have, then they aren't neutrosophic strong.

(8). Which neutrosophic graphs are neither neutrosophic cycle nor neutrosophic strong?
Neutrosophic graphs which aren't neutrosophic strong.

Proposition 1.1.28. *A neutrosophic cycle is neutrosophic complete if and only if it's neutrosophic strong and order is three.*

Proof. Let N is neutrosophic cycle.
($\Rightarrow$) If N is neutrosophic complete, then, by it's neutrosophic complete, it's neutrosophic strong. By it's crisp cycle and crisp complete, order is three. Thus N is neutrosophic strong and order is three.
($\Leftarrow$) If it's neutrosophic strong and order is three, then, by order is three and it's crisp cycle, it's crisp complete. By it's neutrosophic strong, N is neutrosophic complete. ∎

Proposition 1.1.29. *A neutrosophic complete is neutrosophic cycle if and only if it's order is three.*

Proof. Let N is neutrosophic complete.
($\Rightarrow$) If N is neutrosophic cycle, then, by it's crisp cycle and it's crisp complete, order is three.
($\Leftarrow$) If order is three, then, by order is three and it's crisp complete, it's crisp cycle. By it's neutrosophic complete, N is neutrosophic cycle. ∎

Proposition 1.1.30. *A neutrosophic path is neutrosophic complete if and only if it's neutrosophic strong and order is two.*

Proof. Let N is neutrosophic path.
($\Rightarrow$) If N is neutrosophic complete, then, by it's crisp path and it's crisp complete, order is two. By it's crisp complete, it's neutrosophic strong. Thus it's neutrosophic strong and order is two.
($\Leftarrow$) If order is two, then, by order is two, it's crisp connected and it's neutrosophic strong, N is neutrosophic complete. ∎

Proposition 1.1.31. *A neutrosophic complete is neutrosophic path if and only if it's order is two.*

Proof. Let N is neutrosophic complete.
($\Rightarrow$) Consider N is neutrosophic path. Then, by it's crisp path and it's crisp complete, order is two.
($\Leftarrow$) Suppose order is two, then, by order is two and it's crisp complete, it's crisp path. By it's neutrosophic complete, it's neutrosophic path. Thus N is neutrosophic path. ∎

Example 1.1.32. Up to isomorphic there are two neutrosophic graphs which are neutrosophic path, neutrosophic complete and neutrosophic strong.

- Firstly, two vertices have same neutrosophic values as Figure (1.6).
- Secondly, two vertices have different neutrosophic values as Figure (1.7).

Numbers are created by some tools arising from attributes concerning different models of graphs.

Definition 1.1.33. Let $G : (V, E)$ be a crisp graph. For any given subset N of V, $\Sigma_{n\in N} 1$ is called **crisp cardinality** of N and it's denoted by $|N|_c$.

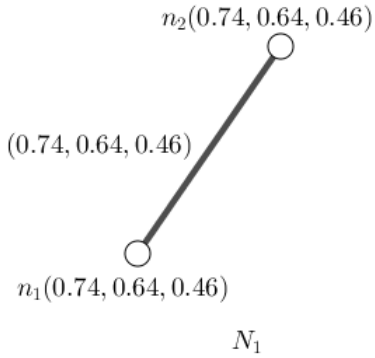


Figure 1.6: Neutrosophic path, N_1, has same neutrosophic values for vertices. It's also Neutrosophic strong and Neutrosophic complete. nsc6

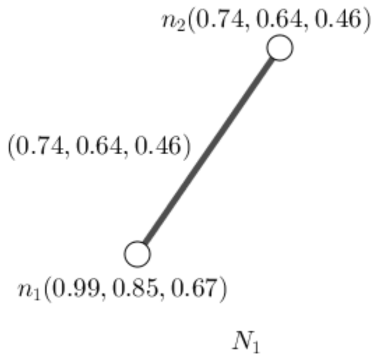


Figure 1.7: Neutrosophic Path, N_1, has same neutrosophic values for vertices. It's also Neutrosophic strong and Neutrosophic complete. nsc7

Definition 1.1.34. Let $G : (V, E)$ be a crisp graph. Crisp cardinality of V is called **crisp order** of G and it's denoted by $O_c(G)$.

Definition 1.1.35. Let $G : (\sigma, \mu)$ be a fuzzy graph. For any given subset N of V, $\Sigma_{n \in N} \sigma(n)$ is called **fuzzy cardinality** of N and it's denoted by $|N|_f$.

Definition 1.1.36. Let $G : (\sigma, \mu)$ be a fuzzy graph. Fuzzy cardinality of V is called **fuzzy order** of G and it's denoted by $O_f(G)$.

Definition 1.1.37. Let $G : (\sigma, \mu)$ be a neutrosophic graph. For any given subset N of V, $\Sigma_{n \in N} \sigma(n)$ is called **neutrosophic cardinality** of N and it's denoted by $|N|_n$.

Definition 1.1.38. Let $G : (\sigma, \mu)$ be a neutrosophic graph. Neutrosophic cardinality of V is called **neutrosophic order** of G and it's denoted by $O_n(G)$.

exm39 **Example 1.1.39.**

- Consider Figure (2.1). Neutrosophic order of N_1, $O_n(N_1)$ is $(2.57, 2.05, 1.04)$. Thus $O_n(N_1) = (2.57, 2.05, 1.04)$.

- Consider Figure (1.2). Neutrosophic order of N_1, $O_n(N_1)$ is $(2.57, 2.05, 1.04)$. Thus $O_n(N_1) = (2.57, 2.05, 1.04)$.
- Consider Figure (1.3). Neutrosophic order of N_1, $O_n(N_1)$ is $(2.57, 2.05, 1.04)$. Thus $O_n(N_1) = (2.57, 2.05, 1.04)$.
- Consider Figure (1.4). Neutrosophic order of N_1, $O_n(N_1)$ is $(2.47, 2.26, 1.47)$. Thus $O_n(N_1) = (2.47, 2.26, 1.47)$.
- Consider Figure (1.5). Neutrosophic order of N_1, $O_n(N_1)$ is $(2.22, 1.92, 1.47)$. Thus $O_n(N_1) = (2.47, 2.26, 1.38)$.
- Consider Figure (1.6). Neutrosophic order of N_1, $O_n(N_1)$ is $(1.48, 1.28, 0.92)$. Thus $O_n(N_1) = (1.48, 1.28, 0.92)$.
- Consider Figure (1.7). Neutrosophic order of N_1, $O_n(N_1)$ is $(1.73, 1.49, 1.13)$. Thus $O_n(N_1) = (1.73, 1.49, 1.13)$.

prp40

Proposition 1.1.40. $|N|_n \leq (|N|_c, |N|_c, |N|_c)$.

Proof.

$$\begin{aligned} |N|_n = \Sigma_{n \in N} \sigma(n) &= \Sigma_{n=(n_1,n_2,n_3) \in N} (\sigma(n_1), \sigma(n_2), \sigma(n_3)) \\ &\leq \Sigma_{n=(n_1,n_2,n_3) \in N} (1,1,1) = (|N|_c, |N|_c, |N|_c). \end{aligned}$$

■

cor41

Corollary 1.1.41. $O_n(N) \leq (O_c(N), O_c(N), O_c(N))$.

Proof. By Proposition (1.1.40), $O_c(N) = |V|_c$ and $O_n(N) = |V|_n$, the result is straightforward. Since

$$\begin{aligned} O_n(N) = |V|_n = \Sigma_{v \in V} \sigma(v) = \Sigma_{v=(v_1,v_2,v_3) \in V} (\sigma(v_1), \sigma(v_2), \sigma(v_3)) \\ \leq \Sigma_{n=(v_1,v_2,v_3) \in V} (1,1,1) = (|V|_c, |V|_c, |V|_c) = (O_c(N), O_c(N), O_c(N)). \end{aligned}$$

■

prp42

Proposition 1.1.42. $|N|_n = (|N|_f, |N|_f, |N|_f)$.

Proof.

$$\begin{aligned} |N|_n = \Sigma_{n \in N} \sigma(n) = \Sigma_{n=(n_1,n_2,n_3) \in N} (\sigma(n_1), \sigma(n_2), \sigma(n_3)) \\ = (|N|_f, |N|_f, |N|_f). \end{aligned}$$

■

In Example (1.1.39), the computations of this notion when they come to neutrosophic order, are done. There's same type-result with analogous to Corollary (1.1.41).

Corollary 1.1.43. $O_n(N) = (O_f(N), O_f(N), O_f(N))$.

Proof. By Proposition (1.1.42), $O_f(N) = |V|_f$ and $O_n(N) = |V|_n$, the result is straightforward. Since

$$O_n(N) = |V|_n = \Sigma_{v \in V}\sigma(v) = \Sigma_{v=(v_1,v_2,v_3)\in V}(\sigma(v_1), \sigma(v_2), \sigma(v_3))$$
$$= (|V|_f, |V|_f, |V|_f) = (O_f(N), O_f(N), O_f(N)).$$

■

prp44

Proposition 1.1.44. *Let $N = (\sigma, \mu)$ be a neutrosophic graph and $S, S' \subseteq V$. If $S \subseteq S'$, then $|S|_n \leq |S'|_n$.*

Proof.

$$|S|_n = \Sigma_{s\in S}\sigma(s) = \Sigma_{s\in S\subseteq S'}\sigma(s) \leq \Sigma_{s'\in S'}\sigma(s') = |S'|_n.$$

■

The converse of Proposition (1.1.44), doesn't hold. Since in Figure (1.6), $S = \{n_1\}, S' = \{n_2\} \subseteq V = \{n_1, n_2\}$. $|S|_n = (0.74, 0.64, 0.46) = (0.74, 0.64, 0.46) = |S'|_n$. Thus $|S|_n \leq |S'|_n$ but $S \not\subseteq S'$.

cor45

Corollary 1.1.45. *Let $N = (\sigma, \mu)$ be a neutrosophic graph. $S \subseteq V$ if and only if $|S|_n \leq |V|_n$.*

Proof. (⇒). By $S \subseteq V$ and Proposition (1.1.44), $|S|_n \leq |V|_n$. In other words,

$$|S|_n = \Sigma_{s\in S}\sigma(s) = \Sigma_{s\in S\subseteq V}\sigma(s) \leq \Sigma_{v\in V}\sigma(v) = |V|_n.$$

(⇐). This case is obvious. ■

Corollary 1.1.46. *Let $N = (\sigma, \mu)$ be a neutrosophic graph and $S \subseteq V$. $|S|_n = O_n(N)$ if and only if $S = V$.*

Proof. (⇒). Suppose $|S|_n = O_n(N)$. Hence $|S|_n = O_n(N) = |V|_n$. Thus $|S|_n = |V|_n$. By Corollary (1.1.45), we get $S = V$.
(⇐). Consider $S = V$. Thus $|V|_n = |S|_n$. By $O_n(N) = |V|_n$, $|S|_n = O_n(N)$. ■

Definition 1.1.47. Let $C = (V, E)$ be a crisp graph. It's called **crisp connected** if for every given couple of vertices, there's at least one path amid them.

Definition 1.1.48. Let $F = (\sigma, \mu)$ be a fuzzy graph. It's called **fuzzy connected** if for every given couple of vertices, there's at least one path amid them.

Definition 1.1.49. Let $N = (\sigma, \mu)$ be a neutrosophic graph. It's called **neutrosophic connected** if for every given couple of vertices, there's at least one path amid them.

Example 1.1.50. Neutrosophic complete, neutrosophic path and neutrosophic cycle, are only neutrosophic connected but neutrosophic strong could be either neutrosophic connected or not. In other words, if neutrosophic graph is neutrosophic strong, then it's neutrosophic connected or not but if neutrosophic graph is either of neutrosophic complete, neutrosophic path and neutrosophic cycle, then it's forever neutrosophic connected.

Definition 1.1.51. Let $C = (V, E)$ be a crisp graph. Suppose a path $P : v_0, v_1, \cdots, v_{n-1}, v_n$ from v_0 to v_n. $\min_{i=0,1,2,\cdots,n-1} 1$ is called **crisp strength** of P and it's denoted by $\mathcal{S}_c(P)$.

Definition 1.1.52. Let $F = (\sigma, \mu)$ be a fuzzy graph. Suppose a path $P : v_0, v_1, \cdots, v_{n-1}, v_n$ from v_0 to v_n. $\min_{i=0,1,2,\cdots,n-1} \mu(v_i v_{i+1})$ is called **fuzzy strength** of P and it's denoted by $\mathcal{S}_f(P)$.

Definition 1.1.53. Let $N = (\sigma, \mu)$ be a neutrosophic graph. Suppose a path $P : v_0, v_1, \cdots, v_{n-1}, v_n$ from v_0 to v_n. $\min_{i=0,1,2,\cdots,n-1} \mu(v_i v_{i+1})$ is called **neutrosophic strength** of P and it's denoted by $\mathcal{S}_n(P)$.

i-path is a path with i edges, it's also called **length** of path.

Example 1.1.54. In Figures (2.1), (1.2), (1.3), (1.4), (1.5), (1.6), (1.7), neutrosophic strengths are computed for all possible paths.

(a) : Consider Figure (2.1).

- (i) : An 1-path $P_1 : n_1, n_2$ has neutrosophic strength $(0.74, 0.47, 0.31)$.
- (ii) : An 1-path $P_2 : n_1, n_3$ has neutrosophic strength $(0.55, 0.64, 0.26)$.
- (iii) : An 1-path $P_3 : n_2, n_3$ has neutrosophic strength $(0.37, 0.46, 0.24)$.
- (iv) : An 2-path $P_4 : n_1, n_2, n_3$ has neutrosophic strength $(0.37, 0.46, 0.24)$.
- (v) : There are only four distinct paths.
- (vi) : There are only three neutrosophic strengths.
- (vii) : There are only two same neutrosophic strengths.

(b) : Consider Figure (1.2).

- (i) : An 1-path $P_1 : n_1, n_2$ has neutrosophic strength $(0.74, 0.47, 0.31)$.
- (ii) : An 1-path $P_2 : n_1, n_3$ has neutrosophic strength $(0.84, 0.47, 0.27)$.
- (iii) : An 1-path $P_3 : n_2, n_3$ has neutrosophic strength $(0.74, 0.64, 0.27)$.
- (iv) : An 2-path $P_4 : n_1, n_2, n_3$ has neutrosophic strength $(0.74, 0.47, 0.27)$.
- (v) : There are only four distinct paths.
- (vi) : There are only four different neutrosophic strengths.
- (vii) : There is no same neutrosophic strengths.

(c) : Consider Figure (1.3).

- (i) : An 1-path $P_1 : n_1, n_3$ has neutrosophic strength $(0.84, 0.47, 0.27)$.
- (ii) : An 1-path $P_2 : n_2, n_3$ has neutrosophic strength $(0.74, 0.64, 0.27)$.
- (iii) : An 2-path $P_3 : n_1, n_3, n_2$ has neutrosophic strength $(0.74, 0.47, 0.27)$.
- (iv) : There are only three distinct paths.
- (v) : There are only three different neutrosophic strengths.
- (vii) : There is no same neutrosophic strengths.

(d) : Consider Figure (1.4).

- (i) : An 1-path $P_1 : n_1, n_2$ has neutrosophic strength $(0.74, 0.64, 0.46)$.

(ii) : An 1-path $P_2 : n_1, n_3$ has neutrosophic strength $(0.74, 0.64, 0.46)$.

(iii) : An 1-path $P_3 : n_2, n_3$ has neutrosophic strength $(0.74, 0.64, 0.46)$.

(iv) : An 2-path $P_4 : n_1, n_2, n_3$ has neutrosophic strength $(0.74, 0.64, 0.46)$.

(v) : There are only four distinct paths.

(vi) : There are only four different neutrosophic strengths.

(vii) : There are only four same neutrosophic strengths.

(e) : Consider Figure (1.5).

(i) : An 1-path $P_1 : n_1, n_2$ has neutrosophic strength $(0.74, 0.64, 0.46)$.

(ii) : An 1-path $P_2 : n_1, n_3$ has neutrosophic strength $(0.74, 0.64, 0.46)$.

(iii) : An 1-path $P_3 : n_2, n_3$ has neutrosophic strength $(0.74, 0.64, 0.46)$.

(iv) : An 2-path $P_4 : n_1, n_2, n_3$ has neutrosophic strength $(0.74, 0.64, 0.46)$.

(v) : There are only four distinct paths.

(vi) : There are only four different neutrosophic strengths.

(vii) : There are only four same neutrosophic strengths.

(f) : Consider Figure (1.6).

(i) : An 1-path $P_1 : n_1, n_2$ has neutrosophic strength $(0.74, 0.64, 0.46)$.

(ii) : There is only one different neutrosophic strengths.

(iii) : There is no same neutrosophic strengths.

(g) : Consider Figure (1.7).

(i) : An 1-path $P_1 : n_1, n_2$ has neutrosophic strength $(0.74, 0.64, 0.46)$.

(ii) : There is only one different neutrosophic strengths.

(iii) : There is no same neutrosophic strengths.

prp55

Proposition 1.1.55. *Let $N = (\sigma, \mu)$ be a neutrosophic cycle. Then the number of distinct neutrosophic path is $2^n - n - 1$.*

Proof. The number of subsets of number n is 2^n. The vertex of 1-set couldn't be considered as path. The number of 1-set is n. Thus it remains $2^n - n$. Also, the vertex of 0-set couldn't be considered as path. The number of 0-set is 1. Thus it finally remains $2^n - n - 1$. ■

Corollary 1.1.56. *Let $N = (\sigma, \mu)$ be a neutrosophic cycle. Then the number of distinct neutrosophic path is $2^n - n - 1$.*

Proof. neutrosophic path implies having distinct vertices in a consecutive sequence of vertices. Thus neutrosophic cycle is as same case as neutrosophic path. So by applying Proposition (1.1.55), the result is straightforward. In other way, there's direct proof as follows. The number of subsets of number n is 2^n. The vertex of 1-set couldn't be considered as path. The number of 1-set is n. Thus it remains $2^n - n$. Also, the vertex of 0-set couldn't be considered as path. The number of 0-set is 1. Thus it finally remains $2^n - n - 1$. ■

Definition 1.1.57. Let $C = (V, E)$ be a crisp graph which isn't crisp path. For any given couple of vertices v_0 and v_n,

$(i):$ $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\}} \mathcal{S}_c(P)$ is denoted by $\mathcal{C}(v_0, v_n)$ and it's called **t-connectedness** amid v_0 and v_n in C.

$(ii):$ $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\}\setminus\{P:v_0v_n\}} \mathcal{S}_c(P)$ is denoted by $\mathcal{C}_\alpha(v_0, v_n)$ it's called $\alpha-$**connectedness** v_0 and v_n in C where v_0v_n is an edge, if $\mathcal{C}_\alpha(v_0, v_n) > \mu(v_0v_n)$.

$(iii):$ $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\}\setminus\{P:v_0v_n\}} \mathcal{S}_c(P)$ is denoted by $\mathcal{C}_\alpha(v_0, v_n)$ it's called $\beta-$**connectedness** v_0 and v_n in C where v_0v_n is an edge, if $\mathcal{C}_\alpha(v_0, v_n) = \mu(v_0v_n)$.

$(iv):$ $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\}\setminus\{P:v_0v_n\}} \mathcal{S}_c(P)$ is denoted by $\mathcal{C}_\alpha(v_0, v_n)$ it's called $\delta-$**connectedness** v_0 and v_n in C where v_0v_n is an edge, if $\mathcal{C}_\alpha(v_0, v_n) < \mu(v_0v_n)$.

Definition 1.1.58. Let $C = (V, E)$ be a crisp graph which isn't crisp path. For any given couple of vertices v_0 and v_n,

$(i):$ $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\}} \mathcal{S}_c(P) = c \in \mathbb{Q}$ is denoted by $\mathcal{C}_t$ and it's called **t-crisp**.

$(ii):$ $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\}\setminus\{P:v_0v_n\}} \mathcal{S}_c(P) > \mu(v_0v_n)$ is denoted by $\mathcal{C}_\alpha$ it's called $\alpha-$**crisp** where v_0v_n is an edge.

$(iii):$ $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\}\setminus\{P:v_0v_n\}} \mathcal{S}_c(P) = \mu(v_0v_n)$ is denoted by $\mathcal{C}_\beta$ it's called $\beta-$**crisp** where v_0v_n is an edge.

$(iv):$ $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\}\setminus\{P:v_0v_n\}} \mathcal{S}_c(P) < \mu(v_0v_n)$ is denoted by $\mathcal{C}_\delta$ it's called $\delta-$**crisp** where v_0v_n is an edge.

Definition 1.1.59. Let $F = (\sigma, \mu)$ be a fuzzy graph which isn't fuzzy path. For any given couple of vertices v_0 and v_n,

$(i):$ $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\}} \mathcal{S}_f(P)$ is denoted by $\mathcal{F}(v_0, v_n)$ and it's called **t-connectedness** amid v_0 and v_n in F.

$(ii):$ $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\}\setminus\{P:v_0v_n\}} \mathcal{S}_f(P)$ is denoted by $\mathcal{F}_\alpha(v_0, v_n)$ it's called $\alpha-$**connectedness** v_0 and v_n in F where v_0v_n is an edge, if $\mathcal{F}_\alpha(v_0, v_n) > \mu(v_0v_n)$.

$(iii):$ $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\}\setminus\{P:v_0v_n\}} \mathcal{S}_f(P)$ is denoted by $\mathcal{F}_\alpha(v_0, v_n)$ it's called $\beta-$**connectedness** v_0 and v_n in F where v_0v_n is an edge, if $\mathcal{F}_\alpha(v_0, v_n) = \mu(v_0v_n)$.

$(iv):$ $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\}\setminus\{P:v_0v_n\}} \mathcal{S}_f(P)$ is denoted by $\mathcal{F}_\alpha(v_0, v_n)$ it's called $\delta-$**connectedness** v_0 and v_n in F where v_0v_n is an edge, if $\mathcal{F}_\alpha(v_0, v_n) < \mu(v_0v_n)$.

Definition 1.1.60. Let $F = (\sigma, \mu)$ be a fuzzy graph which isn't fuzzy path. For any given couple of vertices v_0 and v_n,

$(i):$ $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\}} \mathcal{S}_f(P) = c \in \mathbb{Q}$ is denoted by $\mathcal{F}_t$ and it's called **t-fuzzy**.

(ii) : $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\} \setminus \{P: v_0 v_n\}} \mathcal{S}_f(P) > \mu(v_0 v_n)$ is denoted by $\mathcal{F}_\alpha$ it's called $\alpha-$**fuzzy** where $v_0 v_n$ is an edge.

(iii) : $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\} \setminus \{P: v_0 v_n\}} \mathcal{S}_f(P) = \mu(v_0 v_n)$ is denoted by $\mathcal{F}_\beta$ it's called $\beta-$**fuzzy** where $v_0 v_n$ is an edge.

(iv) : $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\} \setminus \{P: v_0 v_n\}} \mathcal{S}_f(P) < \mu(v_0 v_n)$ is denoted by $\mathcal{F}_\delta$ it's called $\delta-$**fuzzy** where $v_0 v_n$ is an edge.

Definition 1.1.61. Let $N = (\sigma, \mu)$ be a neutrosophic graph which isn't neutrosophic path. For any given couple of vertices v_0 and v_n,

(i) : $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\}} \mathcal{S}_n(P)$ is denoted by $\mathcal{N}(v_0, v_n)$ and it's called **t-connectedness** amid v_0 and v_n in N.

(ii) : $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\} \setminus \{P: v_0 v_n\}} \mathcal{S}_n(P)$ is denoted by $\mathcal{N}_\alpha(v_0, v_n)$ it's called $\alpha-$**connectedness** v_0 and v_n in N where $v_0 v_n$ is an edge, if $\mathcal{N}_\alpha(v_0, v_n) > \mu(v_0 v_n)$.

(iii) : $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\} \setminus \{P: v_0 v_n\}} \mathcal{S}_n(P)$ is denoted by $\mathcal{N}_\alpha(v_0, v_n)$ it's called $\beta-$**connectedness** v_0 and v_n in N where $v_0 v_n$ is an edge, if $\mathcal{N}_\alpha(v_0, v_n) = \mu(v_0 v_n)$.

(iv) : $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\} \setminus \{P: v_0 v_n\}} \mathcal{S}_n(P)$ is denoted by $\mathcal{N}_\alpha(v_0, v_n)$ it's called $\delta-$**connectedness** v_0 and v_n in N where $v_0 v_n$ is an edge, if $\mathcal{N}_\alpha(v_0, v_n) < \mu(v_0 v_n)$.

Definition 1.1.62. Let $N = (\sigma, \mu)$ be a neutrosophic graph which isn't neutrosophic path. For any given couple of vertices v_0 and v_n,

(i) : $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\}} \mathcal{S}_n(P) = c \in \mathbb{Q}$. Then $N = (\sigma, \mu)$ is denoted by $\mathcal{N}_t$ and it's called **t-neutrosophic**.

(ii) : $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\} \setminus \{P: v_0 v_n\}} \mathcal{S}_n(P) > \mu(v_0 v_n)$. Then $N = (\sigma, \mu)$ is denoted by $\mathcal{N}_\alpha$ it's called $\alpha-$**neutrosophic** where $v_0 v_n$ is an edge.

(iii) : $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\} \setminus \{P: v_0 v_n\}} \mathcal{S}_n(P) = \mu(v_0 v_n)$. Then $N = (\sigma, \mu)$ is denoted by $\mathcal{N}_\beta$ it's called $\beta-$**neutrosophic** where $v_0 v_n$ is an edge.

(iv) : $\max_{\{P \text{ is a path from } v_0 \text{ to } v_n\} \setminus \{P: v_0 v_n\}} \mathcal{S}_n(P) < \mu(v_0 v_n)$. Then $N = (\sigma, \mu)$ is denoted by $\mathcal{N}_\delta$ it's called $\delta-$**neutrosophic** where $v_0 v_n$ is an edge.

Example 1.1.63. In Figures (2.1), (1.2), (1.3), (1.4), (1.5), (1.6), (1.7), neutrosophic graphs and all possible edges are characterized.

(a) : Consider Figure (2.1).

  (i) : The edge $n_1 n_2$ is $\alpha-$connectedness and $\mathcal{N}_\alpha(v_0, v_n) = (0.74, 0.47, 0.31)$.

  (ii) : The edge $n_1 n_3$ is $\alpha-$connectedness and $\mathcal{N}_\alpha(v_0, v_n) = (0.55, 0.64, 0.26)$.

(iii) : The edge n_2n_3 is neither of $t-$connectedness $\alpha-$connectedness $\beta-$connectedness and $\delta-$connectedness. Since for path P : $n_2, n_1, n_3,\ \mathcal{S}_n(P)$ isn't computable. So

$$\max_{\{P \text{ is a path from } v_2 \text{ to } v_3\} \setminus \{P: v_2v_3\}} \mathcal{S}_n(P)$$

isn't computable.

(iv) : $N = (\sigma, \mu)$ is neither of $t-$neutrosophic, $\mathcal{N}_t,$ $\alpha-$neutrosophic, $\mathcal{N}_\alpha,$ $\beta-$neutrosophic, $\mathcal{N}_\beta,$ and $\delta-$connectedness, $\mathcal{N}_\delta$.

(b) : Consider Figure (1.2).

(i) : The edge n_1n_2 is neither of $t-$connectedness, $\alpha-$connectedness, $\beta-$connectedness and $\delta-$connectedness. Since for path P : $n_1, n_2, n_2,\ \mathcal{S}_n(P)$ isn't computable. So

$$\max_{\{P \text{ is a path from } v_1 \text{ to } v_2\} \setminus \{P: v_1v_2\}} \mathcal{S}_n(P)$$

isn't computable.

(ii) : The edge n_1n_3 is neither of $t-$connectedness, $\alpha-$connectedness, $\beta-$connectedness and $\delta-$connectedness. Since for path P : $n_1, n_2, n_3,\ \mathcal{S}_n(P)$ isn't computable. So

$$\max_{\{P \text{ is a path from } v_1 \text{ to } v_3\} \setminus \{P: v_1v_3\}} \mathcal{S}_n(P)$$

isn't computable.

(iii) : The edge n_2n_3 is neither of $t-$connectedness, $\alpha-$connectedness, $\beta-$connectedness and $\delta-$connectedness. Since for path P : $n_2, n_1, n_3,\ \mathcal{S}_n(P)$ isn't computable. So

$$\max_{\{P \text{ is a path from } v_2 \text{ to } v_3\} \setminus \{P: v_2v_3\}} \mathcal{S}_n(P)$$

isn't computable.

(iv) : $N = (\sigma, \mu)$ is neither of $t-$neutrosophic, $\mathcal{N}_t,$ $\alpha-$neutrosophic, $\mathcal{N}_\alpha,$ $\beta-$neutrosophic, $\mathcal{N}_\beta$ and $\delta-$connectedness, $\mathcal{N}_\delta$.

(c) : Consider Figure (1.3).

(i) : It's neutrosophic path. Thus the notion couldn't be applied.

(d) : Consider Figure (1.4).

(i) : The edge n_1n_2 is $t-$connectedness and $\alpha-$connectedness and $\mathcal{N}_\alpha(v_1, v_2) = (0.74, 0.64, 0.46)$.

(ii) : The edge n_1n_3 is $t-$connectedness and $\alpha-$connectedness and $\mathcal{N}_\alpha(v_1, v_3) = (0.74, 0.64, 0.46)$.

(iii) : The edge n_1n_3 is $t-$connectedness and $\alpha-$connectedness and $\mathcal{N}_\alpha(v_1, v_3) = (0.74, 0.64, 0.46)$.

(iv) : $N = (\sigma, \mu)$ is neither of $\alpha-$neutrosophic, $\mathcal{N}_\alpha,$ and $\delta-$connectedness, $\mathcal{N}_\delta$.

(v) : $N = (\sigma, \mu)$ is both $t-$neutrosophic, $\mathcal{N}_t$, and $\beta-$neutrosophic, $\mathcal{N}_\beta$.

(e) : Consider Figure (1.5).

(i) : The edge n_1n_2 is $t-$connectedness and $\alpha-$connectedness and $\mathcal{N}_\alpha(v_1, v_2) = (0.74, 0.64, 0.46)$.

(ii) : The edge n_1n_3 is $t-$connectedness and $\alpha-$connectedness and $\mathcal{N}_\alpha(v_1, v_3) = (0.74, 0.64, 0.46)$.

(iii) : The edge n_1n_3 is $t-$connectedness and $\alpha-$connectedness and $\mathcal{N}_\alpha(v_1, v_3) = (0.74, 0.64, 0.46)$.

(iv) : $N = (\sigma, \mu)$ is neither of $\alpha-$neutrosophic, $\mathcal{N}_\alpha$, and $\delta-$connectedness, $\mathcal{N}_\delta$.

(v) : $N = (\sigma, \mu)$ is both $t-$neutrosophic, $\mathcal{N}_t$, and $\beta-$neutrosophic, $\mathcal{N}_\beta$.

(f) : Consider Figure (1.6).

(i) : It's neutrosophic path. Thus the notion couldn't be applied.

(g) : Consider Figure (1.7).

(i) : It's neutrosophic path. Thus the notion couldn't be applied.

Proposition 1.1.64. *Let $N = (\sigma, \mu)$ be a neutrosophic complete. Then it's $\beta-$neutrosophic.*

Proof. Suppose xy is a given neutrosophic edge. For any given neutrosophic path $P : x = v_0, v_1, \cdots, v_n = y$, neutrosophic strength is $\min\{\sigma(x), \sigma(v_1), \cdots, \sigma(y)\} \leq \min\{\sigma(x), \sigma(y)\}$. It implies $\mathcal{S}_n(P) \leq \min\{\sigma(x), \sigma(y)\}$. In other hand, by xy is an edge, $P' : x, y$ is a path thus $\mathcal{S}_n(P) \geq \min\{\sigma(x), \sigma(y)\}$. Thus $\mathcal{S}_n(P) = \min\{\sigma(x), \sigma(y)\}$. It means every edge is $\beta-$neutrosophic. It induces $N = (\sigma, \mu)$ is $\beta-$neutrosophic. So $N = (\sigma, \mu)$ is $\mathcal{N}_\beta$. ■

Proposition 1.1.65. *Let $N = (\sigma, \mu)$ be a neutrosophic graph such that for every neutrosophic edges xy and uv, $\mu(xy) = \mu(uv)$. Then it's $\beta-$neutrosophic.*

Proof. Suppose xy is a given neutrosophic edge. Consider $\mu(xy) = c,\ c \in \mathbb{Q}$. For any given neutrosophic path $P : x = v_0, v_1, \cdots, v_n = y$, neutrosophic strength is $\min\{\mu(xv_1), \mu(v_1v_2), \cdots, \mu(v_{n-1}y)\} = \min\{c, c, \cdots, c\}$. It implies $\mathcal{S}_n(P) \leq c$. In other hand, by xy is an edge, $P' : x, y$ is a path thus $\mathcal{S}_n(P) \geq \mu(xy) = c$. Thus $\mathcal{S}_n(P) = c$. It means every edge is $\beta-$**neutrosophic**. It induces $N = (\sigma, \mu)$ is $\beta-$**neutrosophic**. So $N = (\sigma, \mu)$ is $\mathcal{N}_\beta$. ■

Proposition 1.1.66. *Let $N = (\sigma, \mu)$ be a neutrosophic graph. Then it's neither $\alpha-$neutrosophic nor $\delta-$neutrosophic.*

Proof. If all edges have same values, then every given edge isn't neither $\alpha-$neutrosophic nor $\delta-$neutrosophic. Otherwise, if there's an edge which has different value, then there's one edge which has minimum value so it isn't neither $\alpha-$neutrosophic nor $\delta-$neutrosophic. ■

Definition 1.1.67. Let $N = (\sigma, \mu)$ be a neutrosophic graph. **Coloring number** is minimum number of distinct colors which are used to color the vertices which are neighbors.

Example 1.1.68. In Figures (2.1), (1.2), (1.3), (1.4), (1.5), (1.6), (1.7), neutrosophic graphs and all possible edges are characterized.

(a) : Consider Figure (2.1). Coloring number is three.

(b) : Consider Figure (1.2). Coloring number is three.

(c) : Consider Figure (1.3). Coloring number is two.

(d) : Consider Figure (1.4). Coloring number is three.

(e) : Consider Figure (1.5). Coloring number is three.

(f) : Consider Figure (1.6). Coloring number is two.

(g) : Consider Figure (1.7). Coloring number is two.

Proposition 1.1.69. *In complete neutrosophic, coloring number is n.*

Proof. Every vertex has $n-1$ neighbors. Thus the number of colors are n. ∎

Proposition 1.1.70. *In path neutrosophic, coloring number is* 2.

Proof. Every vertex has two different neighbors. Thus coloring number is 2. ∎

Proposition 1.1.71. *In even cycle neutrosophic, coloring number is* 2.

Proof. Every vertex has two different neighbors. Thus coloring number is 2. ∎

Proposition 1.1.72. *In odd cycle neutrosophic, coloring number is* 3.

Proof. Every vertex has two different neighbors but one vertex has two neighbors which have different colors. Thus coloring number is 3. ∎

Definition 1.1.73. A fuzzy(neutrosophic) graph is called **fuzzy(neutrosophic) t-partite** if V is partitioned to t parts, $V_1, V_2, \cdots, V_t$ and the edge xy implies $x \in V_i$ and $y \in V_j$ where $i \neq j$. If it's fuzzy(neutrosophic) complete, then it's denoted by $K_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where σ_i is σ on V_i instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. If $t = 2$, then it's called **fuzzy(neutrosophic) complete bipartite** and it's denoted by K_{σ_1,σ_2} especially, if $|V_1| = 1$, then it's called **fuzzy(neutrosophic) star** and it's denoted by S_{1,σ_2}. In this case, the vertex in V_1 is called **center** and if a vertex joins to all vertices of fuzzy(neutrosophic), it's called **fuzzy(neutrosophic) wheel** and it's denoted by W_{1,σ_2}.

Example 1.1.74. In Figures (2.1), (1.2), (1.3), (1.4), (1.5), (1.6), (1.7), neutrosophic graphs and all possible edges are characterized.

(a) : Consider Figure (2.1).

  (i) : Neutrosophic graph is neutrosophic wheel.

(b) : Consider Figure (1.2).

Table 1.2: Crisp-fying, Fuzzy-fying and Neutrosophic-fying tbl2

Crisp Graphs	Fuzzy Graphs	Neutrosophic Graphs
Crisp Complete	Fuzzy Complete	Neutrosophic Complete
Crisp Strong	Fuzzy Strong	Neutrosophic Strong
Crisp Path	Fuzzy Path	Neutrosophic Path
Crisp Cycle	Fuzzy Cycle	Neutrosophic Cycle
Crisp t-partite	Fuzzy t-partite	Neutrosophic t-partite
Crisp Bipartite	Fuzzy Bipartite	Neutrosophic Bipartite
Crisp Star	Fuzzy Star	Neutrosophic Star
Crisp Wheel	Fuzzy Wheel	Neutrosophic Wheel

(i) : Neutrosophic graph is neutrosophic wheel.

(c) : Consider Figure (1.3).

(i) : Neutrosophic graph is neutrosophic star.
(ii) : Neutrosophic graph is neutrosophic bipartite.
(iii) : Neutrosophic graph is neutrosophic t-partite.
(iv) : Neutrosophic graph is neutrosophic complete.

(d) : Consider Figure (1.4).

(i) : Neutrosophic graph is neutrosophic wheel.

(e) : Consider Figure (1.5).

(i) : Neutrosophic graph is neutrosophic wheel.

(f) : Consider Figure (1.6).

(i) : Neutrosophic graph is neutrosophic wheel.
(ii) : Neutrosophic graph is neutrosophic star.
(iii) : Neutrosophic graph is neutrosophic bipartite.
(iv) : Neutrosophic graph is neutrosophic t-partite.
(v) : Neutrosophic graph is neutrosophic complete.

(g) : Consider Figure (1.7).

(i) : Neutrosophic graph is neutrosophic wheel.
(ii) : Neutrosophic graph is neutrosophic star.
(iii) : Neutrosophic graph is neutrosophic bipartite.
(iv) : Neutrosophic graph is neutrosophic t-partite.
(v) : Neutrosophic graph is neutrosophic complete.

Proposition 1.1.75. *In star neutrosophic, coloring number is* 2.

Proof. The center has $n-1$ different neighbors and its neighbors have no neighbor instead of center. So the neighbors have same color and center has different color. Thus coloring number is 2. ∎

Proposition 1.1.76. *In wheel neutrosophic, coloring number is* 4.

Proof. The center has $n-1$ different neighbors and its neighbors have two neighbors which are distinct from center. So the neighbors have same color and center has different color. Thus coloring number is 4. ∎

Proposition 1.1.77. *In bipartite neutrosophic such that it's neutrosophic complete, coloring number is* 2.

Proof. There are two parts and in every part, there's no neighbor. Thus coloring number is 2. ∎

Proposition 1.1.78. *In t-partite neutrosophic such that it's neutrosophic complete, coloring number is* t.

Proof. There are t parts and in every part, there's no neighbor. Thus coloring number is t. ∎

Definition 1.1.79. Let $N = (\sigma, \mu)$ be a neutrosophic graph. **Dominating number** is minimum number of vertices which has at least one edge with the vertices out of this set.

Example 1.1.80. In Figures (2.1), (1.2), (1.3), (1.4), (1.5), (1.6), (1.7), neutrosophic graphs and all possible edges are characterized.

(a) : Consider Figure (2.1). Dominating number is one.

(b) : Consider Figure (1.2). Dominating number is one.

(c) : Consider Figure (1.3). Dominating number is one.

(d) : Consider Figure (1.4). Dominating number is one.

(e) : Consider Figure (1.5). Dominating number is one.

(f) : Consider Figure (1.6). Dominating number is one.

(g) : Consider Figure (1.7). Dominating number is one.

Proposition 1.1.81. *In complete neutrosophic, dominating number is* 1.

Proof. Every vertex has $n-1$ neighbors. Thus dominating number of is 1. ∎

Proposition 1.1.82. *In path neutrosophic, dominating number is* $\lfloor \frac{n}{3} \rfloor$.

Proof. Every vertex has two different neighbors. One vertex has edge with its neighbors and the next vertex is the vertex has two vertices amid itself and the last vertex in the set. Since the minimum number is on demand, one vertex dominates its neighbors and every of these neighbors has one neighbor which is dominated by the vertex which is coming up after it. Thus dominating number is $\lfloor \frac{n}{3} \rfloor$. ∎

Proposition 1.1.83. *In cycle neutrosophic, dominating number is* $\lfloor \frac{n}{3} \rfloor$.

Proof. Every vertex has two different neighbors. One vertex has edge with its neighbors and the next vertex is the vertex has two vertices amid itself and the last vertex in the set. Since the minimum number is on demand, one vertex dominates its neighbors and every of these neighbors has one neighbor which is dominated by the vertex which is coming up after it. Thus dominating number is $\lfloor \frac{n}{3} \rfloor$. ■

Proposition 1.1.84. *In star neutrosophic, dominating number is* 1.

Proof. The center has $n-1$ different neighbors and its neighbors have no neighbor instead of center. So the neighbors are only dominated by center as singleton. Since the minimum number is on demand, center is 1-set which is on demand. Thus dominating number is 1. ■

Proposition 1.1.85. *In wheel neutrosophic, dominating number is* 1.

Proof. The center has $n-1$ different neighbors and its neighbors but neighbors have two neighbors instead of center. So the neighbors are only dominated by center as singleton. Since the minimum number is on demand, center is 1-set which is on demand. Thus dominating number is 1. ■

Proposition 1.1.86. *In bipartite neutrosophic such that it's neutrosophic complete, dominating number is* 2.

Proof. There are two parts and in every part, there's no neighbor. Every vertex from one part, dominates all vertex from different part. Thus dominating number is 2. ■

Proposition 1.1.87. *In t-partite neutrosophic such that it's neutrosophic complete, dominating number is* 2.

Proof. There are t parts and in every part, there's no neighbor. Every vertex from one part, dominates all vertices from different parts. Since minimum number is on demand, one vertex x, dominates all vertices from other parts and one vertex y, from different part, dominates all vertices which have common part with first vertex x. Thus dominating number is 2. ■

1.2 New Ideas

New ideas are applied on these models to explore behaviors of these models in the mathematical perspective. Another ways to make sense about them, are used by relatively comparable results to conclude analysis.

The reference [**9**; **10**] is used to write the contents of this chapter and next chapter.

1.3 Abstract

New notion of dimension as set, as two optimal numbers including metric number, dimension number and as optimal set are introduced in individual framework and in formation of family. Behaviors of twin and antipodal are explored in fuzzy(neutrosophic) graphs. Fuzzy(neutrosophic) graphs, under conditions, fixed-edges, fixed-vertex and strong fixed-vertex are studied. Some classes as path, cycle, complete, strong, t-partite, bipartite, star and wheel in the formation of individual case and in the case, they form a family are studied

in the term of dimension. Fuzzification (neutrosofication) of twin vertices but using crisp concept of antipodal vertices are another approaches of this study. Thus defining two notions concerning vertices which one of them is fuzzy(neutrosophic) titled twin and another is crisp titled antipodal to study the behaviors of cycles which are partitioned into even and odd, are concluded. Classes of cycles according to antipodal vertices are divided into two classes as even and odd. Parity of the number of edges in cycle causes to have two subsections under the section is entitled to antipodal vertices. In this study, the term dimension is introduced on fuzzy(neutrosophic) graphs. The locations of objects by a set of some junctions which have distinct distance from any couple of objects out of the set, are determined. Thus it's possible to have the locations of objects outside of this set by assigning partial number to any objects. The classes of these specific graphs are chosen to obtain some results based on dimension. The types of crisp notions and fuzzy(neutrosophic) notions are used to make sense about the material of this study and the outline of this study uses some new notions which are crisp and fuzzy(neutrosophic). Some questions and problems are posed concerning ways to do further studies on this topic. Basic familiarities with fuzzy(neutrosophic) graph theory and graph theory are proposed for this article.

Keywords: Fuzzy Graphs, Neutrosophic Graphs, Dimension

AMS Subject Classification: 05C17, 05C22, 05E45

1.4 Background

Fuzzy set, neutrosophic set, related definitions of other sets, graphs and new notions on them, neutrosophic graphs, studies on neutrosophic graphs, relevant definitions of other graphs based on fuzzy graphs, related definitions of other graphs based on neutrosophic graphs, are proposed.
In this section, I use two subsections to illustrate a perspective about the background of this study.

Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 1.4.1. *Is it possible to use mixed versions of ideas concerning "crisp", "fuzzy" and "neutrosophic" to define some notions which are applied to fuzzy(neutrosophic) graphs?*

It's motivation to find notions to use in any classes of fuzzy(neutrosophic) graphs. Real-world applications about locating the item, are another thoughts which lead to be considered as motivation. Distance and path amid two items have key roles to locate. Thus they're used to define new ideas which conclude to the structure of metric dimension. The concept of connectedness inspire to study the behavior of path and distance in the way that, both individual fuzzy(neutrosophic) graphs and family of them are the cases of study.
The framework of this study is as follows. In section (3.32), I introduce main definitions alongside some examples to clarify about them. In section (3.29), one idea titled fuzzy(neutrosophic) twin about specific fuzzy(neutrosophic) vertices, is used to form the results for fuzzy(neutrosophic) graphs and family of them

but in this section, there're some results concerning largest metric number since fuzzy(neutrosophic) twin forms largest metric number as possible. In section (3.30), one idea titled antipodal vertices about specific crisp vertices, is used to form the results for fuzzy(neutrosophic) graphs and family of them especially fuzzy(neutrosophic) cycles as two subsections. Fuzzy(neutrosophic) cycles form smallest metric number but In section (3.31), the results are extended and they're inclusive and especific for fuzzy(neutrosophic) graphs and family of them in the way that, the classification is done in the terms of smallest metric number and largest metric number. In section (**??**), two applications are posed for fuzzy(neutrosophic) graphs and family of them. In section (1.10), some problems and questions for further studies are proposed. In section (1.11), gentle discussion about results and applications are featured. In section (1.11), a brief overview concerning advantages and limitations of this study alongside conclusions are formed.

Preliminaries

To clarify about the models, I use some definitions and results, and in this way, results have a key role to make sense about the definitions and to introduce new ways to use on these models in the terms of new notions. For instance, the concept of complete is used to specialize a graph in every environment. To differentiate, I use an adjective or prefix in every definition. Two adjectives "fuzzy" and "neutrosophic" are used to distinguish every graph or classes of graph or any notion on them.
$G : (V, E)$ is called a **crisp graph** where V is a set of objects and E is a subset of $V \times V$ such that this subset is symmetric. A crisp graph $G : (V, E)$ is called a **fuzzy graph** $G : (\sigma, \mu)$ where $\sigma : V \to [0, 1]$ and $\mu : E \to [0, 1]$ such that $\mu(xy) \leq \sigma(x) \wedge \sigma(y)$ for all $xy \in E$. A crisp graph $G : (V, E)$ is called a **neutrosophic graph** $G : (\sigma, \mu)$ where $\sigma = (\sigma_1, \sigma_2, \sigma_3) : V \to [0, 1]$ and $\mu = (\mu_1, \mu_2, \mu_3) : E \to [0, 1]$ such that $\mu(xy) \leq \sigma(x) \wedge \sigma(y)$ for all $xy \in E$. A crisp graph $G : (V, E)$ is called a **crisp complete** where $\forall x \in V,\ \forall y \in V,\ xy \in E$. A fuzzy graph $G : (\sigma, \mu)$ is called **fuzzy complete** where it's complete and $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $xy \in E$. A neutrosophic graph $G : (\sigma, \mu)$ is called a **neutrosophic complete** where it's complete and $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $xy \in E$. An N which is a set of vertices, is called **fuzzy(neutrosophic) cardinality** and it's denoted by $|N|$ such that $|N| = \Sigma_{n \in N} \sigma(n)$. A crisp graph $G : (V, E)$ is called a **crisp strong**. A fuzzy graph $G : (\sigma, \mu)$ is called **fuzzy strong** where $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $xy \in E$. A neutrosophic graph $G : (\sigma, \mu)$ is called a **neutrosophic strong** where $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $xy \in E$. A distinct sequence of vertices $v_0, v_1, \cdots, v_n$ in a crisp graph $G : (V, E)$ is called **crisp path** with length n from v_0 to v_n where $v_i v_{i+1} \in E,\ i = 0, 1, \cdots, n-1$. If one edge is incident to a vertex, the vertex is called **leaf**. A path $v_0, v_1, \cdots, v_n$ is called **fuzzy path** where $\mu(v_i v_{i+1}) > 0,\ i = 0, 1, \cdots, n-1$. A path $v_0, v_1, \cdots, v_n$ is called **neutrosophic path** where $\mu(v_i v_{i+1}) > 0,\ i = 0, 1, \cdots, n-1$. Let $P : v_0, v_1, \cdots, v_n$ be fuzzy(neutrosophic) path from v_0 to v_n such that it has minimum number of vertices as possible, then $d(v_0, v_n)$ is defined as $\Sigma_{i=0}^{n} \mu(v_{i-1} v_i)$. A path $v_0, v_1, \cdots, v_n$ with exception of v_0 and v_n in a crisp graph $G : (V, E)$ is called **crisp cycle** with length n for v_0 where $v_0 = v_n$. A cycle $v_0, v_1, \cdots, v_0$ is called **fuzzy cycle** where there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$. A cycle $v_0, v_1, \cdots, v_0$ is

called **neutrosophic cycle** where there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$. A fuzzy(neutrosophic) cycle is called **odd** if the number of its vertices is odd. Similarly, a fuzzy(neutrosophic) cycle is called **even** if the number of its vertices is even. A fuzzy(neutrosophic) graph is called **fuzzy(neutrosophic) t-partite** if V is partitioned to t parts, $V_1, V_2, \cdots, V_t$ and the edge xy implies $x \in V_i$ and $y \in V_j$ where $i \neq j$. If it's fuzzy(neutrosophic) complete, then it's denoted by $K_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where σ_i is σ on V_i instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. If $t = 2$, then it's called **fuzzy(neutrosophic) complete bipartite** and it's denoted by K_{σ_1,σ_2} especially, if $|V_1| = 1$, then it's called **fuzzy(neutrosophic) star** and it's denoted by S_{1,σ_2}. In this case, the vertex in V_1 is called **center** and if a vertex joins to all vertices of fuzzy(neutrosophic), it's called **fuzzy(neutrosophic) wheel** and it's denoted by W_{1,σ_2}. A set is **n-set** if its cardinality is n. A

Table 1.3: Crisp-fying, Fuzzy-fying and Neutrosophic-fying

T1

Crisp Graphs	Fuzzy Graphs	Neutrosophic Graphs
Crisp Complete	Fuzzy Complete	Neutrosophic Complete
Crisp Strong	Fuzzy Strong	Neutrosophic Strong
Crisp Path	Fuzzy Path	Neutrosophic Path
Crisp Cycle	Fuzzy Cycle	Neutrosophic Cycle
Crisp t-partite	Fuzzy t-partite	Neutrosophic t-partite
Crisp Bipartite	Fuzzy Bipartite	Neutrosophic Bipartite
Crisp Star	Fuzzy Star	Neutrosophic Star
Crisp Wheel	Fuzzy Wheel	Neutrosophic Wheel

fuzzy vertex set is the subset of vertex set of (neutrosophic) fuzzy graph such that the values of these vertices are considered. A **fuzzy edge set** is the subset of edge set of (neutrosophic) fuzzy graph such that the values of these edges are considered. Let $\mathcal{G}$ be a family of fuzzy graphs or neutrosophic graphs. This family have **fuzzy(neutrosophic) common** vertex set if all graphs have same vertex set and its values but edges set is subset of fuzzy edge set. A (neutrosophic) fuzzy graph is called **fixed-edge fuzzy(neutrosophic) graph** if all edges have same values. A (neutrosophic) fuzzy graph is called **fixed-vertex fuzzy(neutrosophic) graph** if all vertices have same values. A couple of vertices x and y is called **crisp twin** vertices if either $N(x) = N(y)$ or $N[x] = N[y]$ where $\forall x \in V,\ N(x) = \{y|\ xy \in E\},\ N[x] = N(x) \cup \{x\}$. Two vertices t and t' are called **fuzzy(neutrosophic) twin** vertices if $N(t) = N(t')$ and $\mu(ts) = \mu(t's)$, for all $s \in N(t) = N(t')$. $\max_{x,y \in V(G)} |E(P(x,y))|$ is called

Table 1.4: Crisp-fying, Fuzzy-fying and Neutrosophic-fying

T1

Crisp Vertex Set	Fuzzy Vertex Set	Neutrosophic Vertex Set
Crisp Edge Set	Fuzzy Edge Set	Neutrosophic Edge Set
Crisp Common	Fuzzy Common	Neutrosophic Common
Crisp Fixed-edge	Fuzzy Fixed-edge	Neutrosophic Fixed-edge
Crisp Fixed-vertex	Fuzzy Fixed-vertex	Neutrosophic Fixed-vertex
Crisp Twin	Fuzzy Twin	Neutrosophic Twin

diameter of G and it's denoted by $D(G)$ where $|E(P(x,y))|$ is the number of edges on the path from x to y. For any given vertex x if there's exactly one vertex y such that $\min_{P(x,y)} |E(P(x,y))| = D(G)$, then a couple of vertices x and y are called **antipodal** vertices.

1.5 Definitions

sec2

I use the notion of vertex in fuzzy(neutrosophic) graphs to define new notions which state the relation amid vertices. In this way, the set of vertices are distinguished by another set of vertices.

Definition 1.5.1. Let $G = (V, \sigma, \mu)$ be a fuzzy(neutrosophic) graph. A vertex m *fuzzy(neutrosophic)-resolves* vertices f_1 and f_2 if $d(m, f_1) \neq d(m, f_2)$. A set M is *fuzzy(neutrosophic)-resolving set* if for every couple of vertices $f_1, f_2 \in V \setminus M$, there's a vertex $m \in M$ such that m fuzzy(neutrosophic)-resolves f_1 and f_2. $|M|$ is called *fuzzy(neutrosophic)-metric number* of G and

$$\min_{S \text{ is fuzzy(neutrosophic)-resolving set}} \Sigma_{s \in S} \sigma(s) = \Sigma_{m \in M} \sigma(m)$$

is called *fuzzy(neutrosophic)-metric dimension* of G and if

$$\min_{S \text{ is fuzzy(neutrosophic)-resolving set}} \Sigma_{s \in S} \sigma(s) = \Sigma_{m \in M} \sigma(m)$$

where M is fuzzy(neutrosophic)-resolving set, then M is called *fuzzy(neutrosophic)-metric set* of G.

Example 1.5.2. Let G be a fuzzy(neutrosophic) graph as figure (1.8). By applying Table (1.5), the 1-set is explored which its cardinality is minimum. $\{f_6\}$ and $\{f_4\}$ are 1-set which has minimum cardinality amid all sets of vertices but $\{f_4\}$ isn't fuzzy(neutrosophic)-resolving set and $\{f_6\}$ is fuzzy(neutrosophic)-resolving set. Thus there's no fuzzy(neutrosophic)-metric set but $\{f_6\}$. f_6 fuzzy(neutrosophic)-resolves all given couple of vertices. Therefore one is fuzzy(neutrosophic)-metric number of G and 0.13 is fuzzy(neutrosophic)-metric dimension of G. By using Table (1.5), f_4 doesn't fuzzy(neutrosophic)-resolve f_2 and f_6. f_4 doesn't fuzzy(neutrosophic)-resolve f_1 and f_5, too.

Table 1.5: Distances of Vertices from sets of vertices $\{f_6\}$ and $\{f_4\}$ in fuzzy(neutrosophic) Graph G. T1

Vertices	f_1	f_2	f_3	f_4	f_5	f_6
f_6	0.22	0.26	0.39	0.24	0.13	0
Vertices	f_1	f_2	f_3	f_4	f_5	f_6
f_4	0.11	0.24	0.37	0	0.11	0.24

Definition 1.5.3. Consider $\mathcal{G}$ as a family of fuzzy(neutrosophic) graphs on a fuzzy(neutrosophic) common vertex set V. A vertex m *simultaneously fuzzy(neutrosophic)-resolves* vertices f_1 and f_2 if $d_G(m, f_1) \neq d_G(m, f_2)$, for all $G \in \mathcal{G}$. A set M is *simultaneously fuzzy(neutrosophic)-resolving set* if for every couple of vertices $f_1, f_2 \in V \setminus M$, there's a vertex $m \in M$ such that m resolves

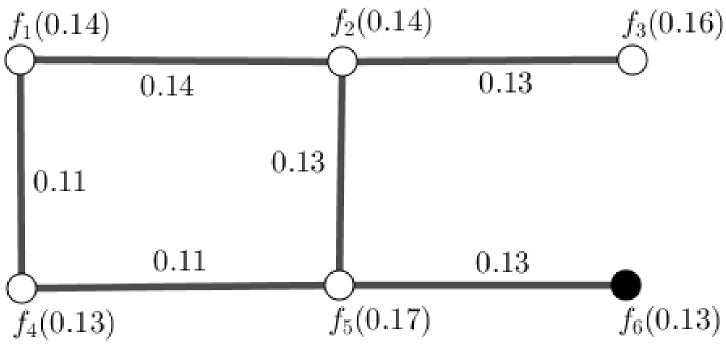


Figure 1.8: Black vertex $\{f_6\}$ is only fuzzy(neutrosophic)-metric set amid all sets of vertices for fuzzy(neutrosophic) graph G.

F1

f_1 and f_2, for all $G \in \mathcal{G}$. $|M|$ is called *simultaneously fuzzy(neutrosophic)-metric number* of $\mathcal{G}$ and

$$\min_{S \text{ is fuzzy(neutrosophic)-resolving set}} \Sigma_{s\in S}\sigma(s) = \Sigma_{m\in M}\sigma(m)$$

is called *simultaneously fuzzy(neutrosophic)-metric dimension* of $\mathcal{G}$ and if

$$\min_{S \text{ is fuzzy(neutrosophic)-resolving set}} \Sigma_{s\in S}\sigma(s) = \Sigma_{m\in M}\sigma(m)$$

where M is fuzzy(neutrosophic)-resolving set, then M is called *simultaneously fuzzy(neutrosophic)-metric set* of $\mathcal{G}$.

Example 1.5.4. Let $\mathcal{G} = \{G_1, G_2, G_3\}$ be a collection of fuzzy(neutrosophic) graphs with common fuzzy(neutrosophic) vertex set and a subset of fuzzy(neutrosophic) edge set as figure (1.9). By applying Table (1.6), the 1-set is explored which its cardinality is minimum. $\{f_2\}$ and $\{f_4\}$ are 1-set which has minimum cardinality amid all sets of vertices. $\{f_4\}$ is as fuzzy(neutrosophic)-resolving set as $\{f_6\}$ is. Thus there's no fuzzy(neutrosophic)-metric set but $\{f_4\}$ and $\{f_6\}$. f_6 as fuzzy(neutrosophic)-resolves all given couple of vertices as f_4. Therefore one is fuzzy(neutrosophic)-metric number of $\mathcal{G}$ and 0.13 is fuzzy(neutrosophic)-metric dimension of $\mathcal{G}$. By using Table (1.6), f_4 fuzzy(neutrosophic)-resolves all given couple of vertices.

Table 1.6: Distances of Vertices from set of vertices $\{f_6\}$ in Family of fuzzy(neutrosophic) Graphs $\mathcal{G}$.

T2

Vertices of G_1	f_1	f_2	f_3	f_4
f_4	0.37	0.26	0.13	0
Vertices of G_2	f_1	f_2	f_3	f_4
f_4	0.11	0.22	0.13	0
Vertices of G_3	f_1	f_2	f_3	f_4
f_4	0.24	0.26	0.13	0

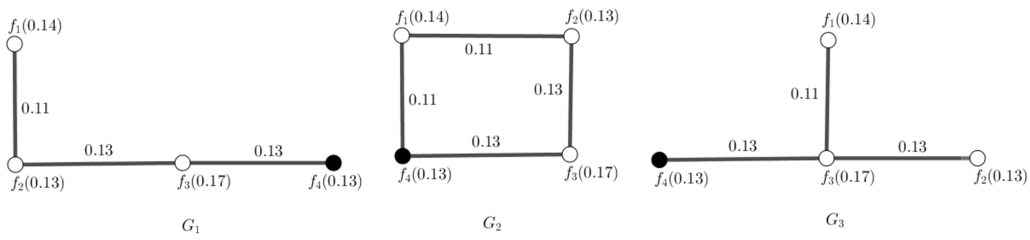


Figure 1.9: Black vertex $\{f_4\}$ and the set of vertices $\{f_2\}$ are simultaneously fuzzy(neutrosophic)-metric set amid all sets of vertices for family of fuzzy(neutrosophic) graphs $\mathcal{G}$.

1.6 Fuzzy(Neutrosophic) Twin Vertices

Proposition 1.6.1. *Let G be a fuzzy(neutrosophic) graph. An $(k-1)$-set from an k-set of fuzzy(neutrosophic) twin vertices is subset of a fuzzy(neutrosophic)-resolving set.*

Proof. If t and t' are fuzzy(neutrosophic) twin vertices, then $N(t) = N(t')$ and $\mu(ts) = \mu(t's)$, for all $s \in N(t) = N(t')$. ■

Corollary 1.6.2. *Let G be a fuzzy(neutrosophic) graph. The number of fuzzy(neutrosophic) twin vertices is $n-1$. Then fuzzy(neutrosophic)-metric number is $n-2$.*

Proof. Let f and f' be two vertices. By supposition, the cardinality of set of fuzzy(neutrosophic) twin vertices is $n-2$. Thus there are two cases. If both are fuzzy(neutrosophic) twin vertices, then $N(f) = N(f')$ and $\mu(fs) = \mu(f's')$, $\forall s \in N(f),\ \ \forall s' \in N(f')$. It implies $d(f,t) = d(f,t)$ for all $t \in V$. Thus suppose if not, then let f be a vertex which isn't fuzzy(neutrosophic) twin vertices with any given vertex and let f' be a vertex which is fuzzy(neutrosophic) twin vertices with any given vertex but not f. By supposition, it's possible and this is only case. Therefore, any given distinct vertex fuzzy(neutrosophic)-resolves f and f'. Then $V \setminus \{f, f'\}$ is fuzzy(neutrosophic)-resolving set. It implies fuzzy(neutrosophic)-metric number is $n-2$. ■

Corollary 1.6.3. *Let G be a fuzzy(neutrosophic) graph. The number of fuzzy(neutrosophic) twin vertices is n. Then G is fixed-edge fuzzy(neutrosophic) graph.*

Proof. Suppose f and f' are two given edges. By supposition, every couple of vertices are fuzzy(neutrosophic) twin vertices. It implies $\mu(f) = \mu(f')$. f and f' are arbitrary so every couple of edges have same values. It induces G is fixed-edge fuzzy(neutrosophic) graph. ■

Corollary 1.6.4. *Let G be a fixed-vertex fuzzy(neutrosophic) graph. The number of fuzzy(neutrosophic) twin vertices is $n-1$. Then fuzzy(neutrosophic)-metric number is $n-2$, fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ where m is fuzzy(neutrosophic) twin vertex with a vertex. Every $(n-2)$-set including fuzzy(neutrosophic) twin vertices is fuzzy(neutrosophic)-metric set.*

Proof. By Corollary (3.29.2), fuzzy(neutrosophic)-metric number is $n-2$. By G is a fixed-vertex fuzzy(neutrosophic) graph, fuzzy metric dimension is $(n-2)\sigma(m)$ where m is fuzzy(neutrosophic) twin vertex with a vertex. One vertex doesn't belong to set of fuzzy(neutrosophic) twin vertices and a vertex from that set, are out of fuzzy metric set. It induces every $(n-2)$-set including fuzzy(neutrosophic) twin vertices is fuzzy metric set. ∎

Proposition 1.6.5. *Let G be a fixed-vertex fuzzy(neutrosophic) graph such that it's fuzzy(neutrosophic) complete. Then fuzzy(neutrosophic)-metric number is $n-1$, fuzzy(neutrosophic)-metric dimension is $(n-1)\sigma(m)$ where m is a given vertex. Every $(n-1)$-set is fuzzy(neutrosophic)-metric set.*

Proof. In fuzzy(neutrosophic) complete, every couple of vertices are twin vertices. By G is a fixed-vertex fuzzy(neutrosophic) graph and it's fuzzy(neutrosophic) complete, every couple of vertices are fuzzy(neutrosophic) twin vertices. Thus by Proposition (3.29.1), the result follows. ∎

prp3

Proposition 1.6.6. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with fuzzy(neutrosophic) common vertex set. Then simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{G}$ is $n-1$.*

Proof. Consider $(n-1)$-set. Thus there's no couple of vertices to be fuzzy(neutrosophic)-resolved. Therefore, every $(n-1)$-set is fuzzy(neutrosophic)-resolving set for any given fuzzy(neutrosophic) graph. Then it holds for any fuzzy(neutrosophic) graph. It implies it's fuzzy(neutrosophic)-resolving set and its cardinality is fuzzy(neutrosophic)-metric number. $(n-1)$-set has the cardinality $n-1$. Then it holds for any fuzzy(neutrosophic) graph. It induces it's simultaneously fuzzy(neutrosophic)-resolving set and its cardinality is simultaneously fuzzy(neutrosophic)-metric number. ∎

prp4

Proposition 1.6.7. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with fuzzy(neutrosophic) common vertex set. Then simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{G}$ is greater than the maximum fuzzy(neutrosophic)-metric number of $G \in \mathcal{G}$.*

Proof. Suppose t and t' are simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{G}$ and fuzzy(neutrosophic)-metric number of $G \in \mathcal{G}$. Thus t is fuzzy(neutrosophic)-metric number for any $G \in \mathcal{G}$. Hence, $t \geq t'$. So simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{G}$ is greater than the maximum fuzzy(neutrosophic)-metric number of $G \in \mathcal{G}$. ∎

prp5

Proposition 1.6.8. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with fuzzy(neutrosophic) common vertex set. Then simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{G}$ is greater than simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{H} \subseteq \mathcal{G}$.*

Proof. Suppose t and t' are simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{G}$ and $\mathcal{H}$. Thus t is fuzzy(neutrosophic)-metric number for any $G \in \mathcal{G}$. It implies t is fuzzy(neutrosophic)-metric number for any $G \in \mathcal{H}$. So t is simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{H}$. By applying Definition about being the minimum number, $t \geq t'$. So simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{G}$ is greater than simultaneously fuzzy(neutrosophic)-metric number of $\mathcal{H} \subseteq \mathcal{G}$. ∎

thm1

Theorem 1.6.9. *Fuzzy(neutrosophic) twin vertices aren't fuzzy(neutrosophic)-resolved in any given fuzzy(neutrosophic) graph.*

Proof. Let t and t' be fuzzy(neutrosophic) twin vertices. Then $N(t) = N(t')$ and $\mu(ts) = \mu(t's)$, for all $s, s' \in V$ such that $ts, t's \in E$. Thus for every given vertex $s' \in V$, $d_G(s', t) = d_G(s, t)$ where G is a given fuzzy(neutrosophic) graph. It means that t and t' aren't resolved in any given fuzzy(neutrosophic) graph. t and t' are arbitrary so fuzzy(neutrosophic) twin vertices aren't resolved in any given fuzzy(neutrosophic) graph. ∎

prp6

Proposition 1.6.10. *Let G be a fixed-vertex fuzzy(neutrosophic) graph. If G is fuzzy(neutrosophic) complete, then every couple of vertices are fuzzy(neutrosophic) twin vertices.*

Proof. Let t and t' be couple of given vertices. By G is fuzzy(neutrosophic) complete, $N(t) = N(t')$. By G is a fixed-vertex fuzzy(neutrosophic) graph, $\mu(ts) = \mu(t's)$, for all edges $ts, t's \in E$. Thus t and t' are fuzzy(neutrosophic) twin vertices. t and t' are arbitrary couple of vertices, hence every couple of vertices are fuzzy(neutrosophic) twin vertices. ∎

thm17

Theorem 1.6.11. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with fuzzy(neutrosophic) common vertex set and $G \in \mathcal{G}$ is a fixed-vertex fuzzy(neutrosophic) graph such that it's fuzzy(neutrosophic) complete. Then simultaneously fuzzy(neutrosophic)-metric number is $n-1$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n-1)\sigma(m)$ where m is a given vertex. Every $(n-1)$-set is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Proof. G is fixed-vertex fuzzy(neutrosophic) graph and it's fuzzy(neutrosophic) complete. So by Theorem (3.29.9), I get every couple of vertices in fuzzy(neutrosophic) complete are fuzzy(neutrosophic) twin vertices. So every couple of vertices, by Theorem (3.29.8), aren't resolved. ∎

Corollary 1.6.12. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with fuzzy(neutrosophic) common vertex set and $G \in \mathcal{G}$ is a fuzzy(neutrosophic) complete. Then simultaneously fuzzy(neutrosophic)-metric number is $n-1$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n-1)\sigma(m)$ where m is a given vertex. Every $(n-1)$-set is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Proof. By fuzzy(neutrosophic) graphs with fuzzy(neutrosophic) common vertex set, G is fixed-vertex fuzzy(neutrosophic) graph. It's fuzzy(neutrosophic) complete. So by Theorem (3.29.10), I get intended result. ∎

Theorem 1.6.13. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with fuzzy(neutrosophic) common vertex set and for every given couple of vertices, there's a $G \in \mathcal{G}$ such that in that, they're fuzzy(neutrosophic) twin vertices. Then simultaneously fuzzy(neutrosophic)-metric number is $n-1$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n-1)\sigma(m)$ where m is a given vertex. Every $(n-1)$-set is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Proof. By Proposition (3.29.5), simultaneously fuzzy(neutrosophic)-metric number is $n-1$. By Theorem (3.29.8), simultaneously fuzzy(neutrosophic)-metric dimension is $(n-1)\sigma(m)$ where m is a given vertex. Also, every $(n-1)$-set is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$. ■

thm19

Theorem 1.6.14. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with fuzzy(neutrosophic) common vertex set. If $\mathcal{G}$ contains three fixed-vertex fuzzy(neutrosophic) stars with different center, then simultaneously fuzzy(neutrosophic)-metric number is $n-2$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ where m is a given vertex. Every $(n-2)$-set is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Proof. The cardinality of set of fuzzy(neutrosophic) twin vertices is $n-1$. Thus by Corollary (3.29.3), the result follows. ■

Corollary 1.6.15. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) graphs with fuzzy(neutrosophic) common vertex set. If $\mathcal{G}$ contains three fuzzy(neutrosophic) stars with different center, then simultaneously fuzzy(neutrosophic)-metric number is $n-2$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ where m is a given vertex. Every $(n-2)$-set is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Proof. By fuzzy(neutrosophic) graphs with fuzzy(neutrosophic) common vertex set, G is fixed-vertex fuzzy(neutrosophic) graph. It's fuzzy(neutrosophic) complete. So by Theorem (3.29.13), I get intended result. ■

1.7 Antipodal Vertices

sec5

Even Fuzzy(Neutrosophic) Cycle

prp5.1

Proposition 1.7.1. *Consider two antipodal vertices x and y in any given fixed-edge even fuzzy(neutrosophic) cycle. Let u and v be given vertices. Then $d(x,u) \neq d(x,v)$ if and only if $d(y,u) \neq d(y,v)$.*

Proof. $(\Rightarrow)$. Consider $d(x,u) \neq d(x,v)$. By $d(x,u)+d(u,y) = d(x,y) = D(G)$, $D(G)-d(x,u) \neq D(G)-d(x,v)$. It implies $d(y,u) \neq d(y,v)$.
$(\Leftarrow)$. Consider $d(y,u) \neq d(y,v)$. By $d(y,u)+d(u,x) = d(x,y) = D(G)$, $D(G)-d(y,u) \neq D(G)-d(y,v)$. It implies $d(x,u) \neq d(x,v)$. ■

Proposition 1.7.2. *Consider two antipodal vertices x and y in any given fixed-edge even fuzzy(neutrosophic) cycle. Let u and v be given vertices. Then $d(x,u) = d(x,v)$ if and only if $d(y,u) = d(y,v)$.*

Proof. $(\Rightarrow)$. Consider $d(x,u) = d(x,v)$. By $d(x,u)+d(u,y) = d(x,y) = D(G)$, $D(G)-d(x,u) = D(G)-d(x,v)$. It implies $d(y,u) = d(y,v)$.
$(\Leftarrow)$. Consider $d(y,u) = d(y,v)$. By $d(y,u)+d(u,x) = d(x,y) = D(G)$, $D(G)-d(y,u) = D(G)-d(y,v)$. It implies $d(x,u) = d(x,v)$. ■

Proposition 1.7.3. *The set contains two antipodal vertices, isn't fuzzy(neutrosophic)-metric set in any given fixed-edge even fuzzy(neutrosophic) cycle.*

Proof. Let x and y be two given antipodal vertices in any given even fuzzy(neutrosophic) cycle. By Proposition (3.30.1), $d(x,u) \neq d(x,v)$ if and only if $d(y,u) \neq d(y,v)$. It implies that if x fuzzy(neutrosophic)-resolves a couple of vertices, then y fuzzy(neutrosophic)-resolves them, too. Thus either x is in fuzzy(neutrosophic)-metric set or y is. It induces the set contains two antipodal vertices, isn't fuzzy(neutrosophic)-metric set in any given even fuzzy(neutrosophic) cycle. ■

Proposition 1.7.4. *Consider two antipodal vertices x and y in any given fixed-edge even fuzzy(neutrosophic) cycle. x fuzzy(neutrosophic)-resolves a given couple of vertices, z and z', if and only if y does.*

Proof. ($\Rightarrow$). x fuzzy(neutrosophic)-resolves a given couple of vertices, z and z', then $d(x,z) \neq d(x,z')$. By Proposition (3.30.1), $d(x,z) \neq d(x,z')$ if and only if $d(y,z) \neq d(y,z')$. Thus y fuzzy(neutrosophic)-resolves a given couple of vertices z and z'.
($\Leftarrow$). y fuzzy(neutrosophic)-resolves a given couple of vertices, z and z', then $d(y,z) \neq d(y,z')$. By Proposition (3.30.1), $d(y,z) \neq d(y,z')$ if and only if $d(x,z) \neq d(x,z')$. Thus x fuzzy(neutrosophic)-resolves a given couple of vertices z and z'. ■

Proposition 1.7.5. *There are two antipodal vertices aren't fuzzy(neutrosophic)-resolved by other two antipodal vertices in any given fixed-edge even fuzzy(neutrosophic) cycle.*

Proof. Suppose x and y are a couple of vertices. It implies $d(x,y) = D(G)$. Consider u and v are another couple of vertices such that $d(x,u) = \frac{D(G)}{2}$. It implies $d(y,u) = \frac{D(G)}{2}$. Thus $d(x,u) = d(y,u)$. Therefore, u doesn't fuzzy(neutrosophic)-resolve a given couple of vertices x and y. By $D(G) = d(u,v) = d(u,x) + d(x,v) = \frac{D(G)}{2} + d(x,v)$, $d(x,v) = \frac{D(G)}{2}$. It implies $d(y,v) = \frac{D(G)}{2}$. Thus $d(x,v) = d(y,v)$. Therefore, v doesn't fuzzy(neutrosophic)-resolve a given couple of vertices x and y. ■

Proposition 1.7.6. *For any two antipodal vertices in any given fixed-edge even fuzzy(neutrosophic) cycle, there are only two antipodal vertices don't fuzzy(neutrosophic)-resolve them*

Proof. Suppose x and y are a couple of vertices such that they're antipodal vertices. Let u be a vertex such that $d(x,u) = \frac{D(G)}{2}$. It implies $d(y,u) = \frac{D(G)}{2}$. Thus $d(x,u) = d(y,u)$. Therefore, u doesn't fuzzy(neutrosophic)-resolve a given couple of vertices x and y. Let v be a antipodal vertex for u such that u and v are antipodal vertices. Thus v $d(x,v) = \frac{D(G)}{2}$. It implies $d(y,v) = \frac{D(G)}{2}$. Therefore, v doesn't fuzzy(neutrosophic)-resolve a given couple of vertices x and y. If u is a vertex such that $d(x,u) \neq \frac{D(G)}{2}$ and v is a vertex such that u and v are antipodal vertices. Thus $d(x,v) \neq \frac{D(G)}{2}$ It induces either $d(x,u) \neq d(y,u)$ or $d(x,v) \neq d(y,v)$. It means either u fuzzy(neutrosophic)-resolves a given couple of vertices x and y or v fuzzy(neutrosophic)-resolves a given couple of vertices x and y. ■

Proposition 1.7.7. *In any given fixed-edge even fuzzy(neutrosophic) cycle, for any vertex, there's only one vertex such that they're antipodal vertices.*

Proof. If $d(x,y) = D(G)$, then x and y are antipodal vertices. ∎

prp5.8 **Proposition 1.7.8.** *Let G be a fixed-edge even fuzzy(neutrosophic) cycle. Then every couple of vertices are fuzzy(neutrosophic)-resolving set if and only if they aren't antipodal vertices.*

Proof. If x and y are antipodal vertices, then they don't fuzzy(neutrosophic)-resolve a given couple of vertices u and v such that they're antipodal vertices and $d(x,u) = \frac{D(G)}{2}$. Since $d(x,u) = d(x,v) = d(y,u) = d(y,v) = \frac{D(G)}{2}$. ∎

cor5.9 **Corollary 1.7.9.** *Let G be a fixed-edge even fuzzy(neutrosophic) cycle. Then fuzzy(neutrosophic)-metric number is two.*

Proof. A set contains one vertex x isn't fuzzy(neutrosophic)-resolving set. Since it doesn't fuzzy(neutrosophic)-resolve a given couple of vertices u and v such that $d(x,u) = d(x,v) = 1$. Thus fuzzy(neutrosophic)-metric number ≥ 2. By Proposition (3.30.8), every couple of vertices such that they aren't antipodal vertices, are fuzzy(neutrosophic)-resolving set. Therefore, fuzzy(neutrosophic)-metric number is 2. ∎

cor5.10 **Corollary 1.7.10.** *Let G be a fixed-edge even fuzzy(neutrosophic) cycle. Then fuzzy(neutrosophic)-metric set contains couple of vertices such that they aren't antipodal vertices.*

Proof. By Corollary (3.30.9), fuzzy(neutrosophic)-metric number is two. By Proposition (3.30.8), every couple of vertices such that they aren't antipodal vertices, are fuzzy(neutrosophic)-resolving set. Therefore, fuzzy(neutrosophic)-metric set contains couple of vertices such that they aren't antipodal vertices. ∎

cor4.11 **Corollary 1.7.11.** *Let $\mathcal{G}$ be a family of fixed-edge odd fuzzy(neutrosophic) cycles with fuzzy(neutrosophic) common vertex set. Then simultaneously fuzzy(neutrosophic)-metric set contains couple of vertices such that they aren't antipodal vertices and fuzzy(neutrosophic)-metric number is two.*

Odd Fuzzy(Neutrosophic) Cycle

prp5.11 **Proposition 1.7.12.** *In any given fixed-edge odd fuzzy(neutrosophic) cycle, for any vertex, there's no vertex such that they're antipodal vertices.*

Proof. Let G be a fixed-edge odd fuzzy(neutrosophic) cycle. if x is a given vertex. Then there are two vertices u and v such that $d(x,u) = d(x,v) = D(G)$. It implies they aren't antipodal vertices. ∎

prp5.12 **Proposition 1.7.13.** *Let G be a fixed-edge odd fuzzy(neutrosophic) cycle. Then every couple of vertices are fuzzy(neutrosophic)-resolving set.*

Proof. Let l and l' be couple of vertices. Thus, by Proposition (3.30.12), l and l' aren't antipodal vertices. It implies for every given couple of vertices f_i and f_j, I get either $d(l,f_i) \neq d(l,f_j)$ or $d(l',f_i) \neq d(l',f_j)$. Therefore, f_i and f_j are fuzzy(neutrosophic)-resolved by either l or l'. It induces the set $\{l,l'\}$ is fuzzy(neutrosophic)-resolving set. ∎

prp5.13

Proposition 1.7.14. *Let G be a fixed-edge odd fuzzy(neutrosophic) cycle. Then fuzzy(neutrosophic)-metric number is two.*

Proof. Let l and l' be couple of vertices. Thus, by Proposition (3.30.12), l and l' aren't antipodal vertices. It implies for every given couple of vertices f_i and f_j, I get either $d(l, f_i) \neq d(l, f_j)$ or $d(l', f_i) \neq d(l', f_j)$. Therefore, f_i and f_j are fuzzy(neutrosophic)-resolved by either l or l'. It induces the set $\{l, l'\}$ is fuzzy(neutrosophic)-resolving set. ■

Corollary 1.7.15. *Let G be a fixed-edge odd fuzzy(neutrosophic) cycle. Then fuzzy(neutrosophic)-metric set contains couple of vertices.*

Proof. By Proposition (3.30.14), fuzzy(neutrosophic)-metric number is two. By Proposition (3.30.13), every couple of vertices are fuzzy(neutrosophic)-resolving set. Therefore, fuzzy(neutrosophic)-metric set contains couple of vertices. ■

Corollary 1.7.16. *Let $\mathcal{G}$ be a family of fixed-edge odd fuzzy(neutrosophic) cycles with fuzzy(neutrosophic) common vertex set. Then simultaneously fuzzy(neutrosophic)-metric set contains couple of vertices and fuzzy(neutrosophic)-metric number is two.*

sec6

1.8 Extended Results

Smallest Metric Number

prp1

Proposition 1.8.1. *Let G be a fuzzy(neutrosophic) path. Then every leaf is fuzzy(neutrosophic)-resolving set.*

Proof. Let l be a leaf. For every given a couple of vertices f_i and f_j, I get $d(l, f_i) \neq d(l, f_j)$. Since if I reassign indexes to vertices such that every vertex f_i and l have i vertices amid themselves, then $d(l, f_i) = \Sigma_{j \leq i}\mu(f_j f_i) \leq i$. Thus $j \leq i$ implies

$$\Sigma_{t\leq j}\mu(f_t f_j)+\Sigma_{j\leq s\leq i}\mu(f_s f_i) > \Sigma_{j\leq i}\mu(f f_i) \equiv d(l, f_j)+c = d(l, f_i) \equiv d(l, f_j) < d(l, f_i).$$

Therefore, by $d(l, f_j) < d(l, f_i)$, I get $d(l, f_i) \neq d(l, f_j)$. f_i and f_j are arbitrary so l fuzzy(neutrosophic)-resolves any given couple of vertices f_i and f_j which implies $\{l\}$ is a fuzzy(neutrosophic)-resolving set. ■

Corollary 1.8.2. *Let G be a fixed-edge fuzzy(neutrosophic) path. Then every leaf is fuzzy(neutrosophic)-resolving set.*

Proof. Let l be a leaf. For every given couple of vertices, f_i and f_j, I get $d(l, f_i) = ci \neq d(l, f_j) = cj$. It implies l fuzzy(neutrosophic)-resolves any given couple of vertices f_i and f_j which implies $\{l\}$ is a fuzzy(neutrosophic)-resolving set. ■

Corollary 1.8.3. *Let G be a fixed-vertex fuzzy(neutrosophic) path. Then every leaf is fuzzy(neutrosophic)-metric set, fuzzy(neutrosophic)-metric number is one and fuzzy(neutrosophic)-metric dimension is c where $c = \sigma(f),\ f \in V$.*

Proof. By Proposition (3.31.1), every leaf is fuzzy(neutrosophic)-resolving set. By $c = \sigma(f)$, $\forall f \in V$, every leaf is fuzzy(neutrosophic)-metric set, fuzzy(neutrosophic)-metric number is one and fuzzy(neutrosophic)-metric dimension is c. ∎

prp7

Proposition 1.8.4. *Let G be a fuzzy(neutrosophic) path. Then a set including every couple of vertices is fuzzy(neutrosophic)-resolving set.*

Proof. Let f and f' be a couple of vertices. For every given a couple of vertices f_i and f_j, I get either $d(f, f_i) \neq d(f, f_j)$ or $d(f', f_i) \neq d(f', f_j)$. ∎

Corollary 1.8.5. *Let G be a fixed-edge fuzzy(neutrosophic) path. Then every set containing couple of vertices is fuzzy(neutrosophic)-resolving set.*

Proof. Consider G is a fuzzy(neutrosophic) path. Thus by Proposition (3.31.2), every set containing couple of vertices is fuzzy(neutrosophic)-resolving set. So it holds for any given fixed-edge path fuzzy(neutrosophic) graph. ∎

Proposition 1.8.6. *If I use fixed-vertex strong fuzzy(neutrosophic) cycles instead of fixed-edge fuzzy(neutrosophic) cycles, then all results of Section* (3.30) *hold.*

Proof. Let G be a fixed-vertex strong fuzzy(neutrosophic) cycles. By G is fuzzy(neutrosophic) strong and it's fixed-vertex, G is fixed-edge fuzzy(neutrosophic). ∎

prp6.2

Proposition 1.8.7. *Let G be a fixed-vertex strong fuzzy(neutrosophic) path. Then an* 1*-set contains leaf, is fuzzy(neutrosophic)-resolving set. An* 1*-set contains leaf, is fuzzy(neutrosophic)-metric set. Fuzzy(neutrosophic)-metric number is one. Fuzzy(neutrosophic)-metric dimension is $\sigma(m)$ where m is a given vertex.*

Proof. There are two leaves. Consider l is a given leaf. By G is a fixed-vertex strong fuzzy(neutrosophic) path, there's only one number to be seen. Thus if v and e are a given vertex and given edge, then $\sigma(v) = \sigma(e) = c$ where $c \in [0, 1]$. Further, for every given vertices v and v', $\sigma(v) = \sigma(v')$. With analogous, for every given edges e and e', $\sigma(e) = \sigma(e')$. With rearranging the indexes of vertices, $d(l, v_i) = ci$. Further more, $d(l, v_i) = ci \neq cj = d(l, v_j)$. Therefore, l fuzzy(neutrosophic)-resolves every given couple of vertices x and v. It induces 1-set containing leaf, is fuzzy(neutrosophic)-resolving set. By G is a fixed-vertex, for every given vertices v and v', $\sigma(v) = \sigma(v')$. It implies 1-set containing leaf, is fuzzy(neutrosophic)-metric set. Also, fuzzy(neutrosophic)-metric number is one. Hence, fuzzy(neutrosophic)-metric dimension is $\sigma(m)$ where m is a given vertex. ∎

cor6.3

Corollary 1.8.8. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) paths with fuzzy(neutrosophic) common vertex set such that they've a common leaf. Then simultaneously fuzzy(neutrosophic)-metric number is* 1*, simultaneously fuzzy(neutrosophic)-metric dimension is $\sigma(m)$ where m is a given vertex.* 1*-set contains common leaf, is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Proof. By Proposition (3.31.3), common leaf fuzzy(neutrosophic)-resolves every given couple of vertices x and v, simultaneously. Thus 1-set containing common leaf, is simultaneously fuzzy(neutrosophic)-metric set. Also, simultaneously fuzzy(neutrosophic)-metric number is one. Hence, simultaneously fuzzy(neutrosophic)-metric dimension is $\sigma(m)$ where m is a given vertex. ■

prp6.4

Proposition 1.8.9. *Let G be a fixed-vertex strong fuzzy(neutrosophic) path. Then an 2-set contains every couple of vertices, is fuzzy(neutrosophic)-resolving set. An 2-set contains every couple of vertices, is fuzzy(neutrosophic)-metric set. Fuzzy(neutrosophic)-metric number is two. Fuzzy(neutrosophic)-metric dimension is $2\sigma(m)$ where m is a given vertex.*

Proof. Suppose v is a given vertex. If there are two vertices x and y such that $d(x,v) \neq d(y,v)$, then x fuzzy(neutrosophic)-resolves x and y and the proof is done. If not, $d(x,v) = d(y,v)$, but for every given vertex v', $d(x,v') \neq d(y,v')$. ■

Corollary 1.8.10. *Let $\mathcal{G}$ be a family of fuzzy(neutrosophic) paths with fuzzy(neutrosophic) common vertex set such that they've no common leaf. Then an 2-set is simultaneously fuzzy(neutrosophic)-resolving set, simultaneously fuzzy(neutrosophic)-metric number is 2, simultaneously fuzzy(neutrosophic)-metric dimension is $\min_{m,m' \in V} \sigma(m) + \sigma(m')$. Every 2-set is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Proof. By Corollary (3.31.4), common leaf forms a simultaneously fuzzy(neutrosophic)-resolving set but in this case, there's no common leaf. Thus by Proposition (3.31.5), an 2-set is fuzzy(neutrosophic)-resolving set for any fuzzy(neutrosophic). Then an 2-set is simultaneously fuzzy(neutrosophic)-resolving set. It induces simultaneously fuzzy(neutrosophic)-metric number is 2. It also implies simultaneously fuzzy(neutrosophic)-metric dimension is $\min_{m,m' \in V} \sigma(m) + \sigma(m')$. So every 2-set is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$. ■

Largest Metric Number

Fuzzy(neutrosophic) t-partite(bipartite/star/wheel) is also studied but by adding one restriction on these models. Fuzzy(neutrosophic) t-partite gets us two results as individual and family when they're either fixed-edge or strong fixed-vertex.

prp55.11

Proposition 1.8.11. *Let G be a fixed-edge fuzzy(neutrosophic) t-partite. Then every set excluding couple of vertices in different parts whose cardinalities of them are strictly greater than one, is fuzzy(neutrosophic)-resolving set.*

Proof. Consider two vertices x and y. Suppose m has same part with either x or y. Without loosing the generality, suppose m has same part with x thus it doesn't have common part with y. Therefore, $d(m,x) = 2 \neq 1 = d(m,y)$. ■

cor55.12

Corollary 1.8.12. *Let G be a fixed-vertex strong fuzzy(neutrosophic) t-partite. Let $n \geq 3$. Then every $(n-2)$-set excludes two vertices from different parts whose cardinalities of them are strictly greater than one, is fuzzy(neutrosophic)-resolving set. Every $(n-2)$-set excludes two vertices from different parts whose* *cardinalities of them are strictly greater than one, is fuzzy(neutrosophic)-metric set. Fuzzy(neutrosophic)-metric number is $n-2$. Fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ where m is a given vertex.*

Proof. By Proposition (3.31.7), every $(n-2)$-set excludes two vertices from different parts whose cardinalities of them are strictly greater than one, is fuzzy(neutrosophic)-resolving set. It means that every $(n-2)$-set excludes two vertices from different parts whose cardinalities of them are strictly greater than one, is fuzzy(neutrosophic)-metric set. Since if x and y are either in same part or in different parts, then, by any given vertex w, $d(w,x) = d(w,y)$. Thus 1-set isn't fuzzy(neutrosophic)-resolving set. There are same arguments for a set with cardinality $\leq n-3$ when pigeonhole principle implies at least two vertices have same conditions concerning either being in same part or in different parts. ■

cor55.13

Corollary 1.8.13. *Let G be a fixed-vertex strong fuzzy(neutrosophic) bipartite. Let $n \geq 3$. Then every $(n-2)$-set excludes two vertices from different parts, is fuzzy(neutrosophic)-resolving set. Every $(n-2)$-set excludes two vertices from different parts, is fuzzy(neutrosophic)-metric set. Fuzzy(neutrosophic)-metric number is $n-2$. Fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ where m is a given vertex.*

Proof. Consider x and y are excluded by a $(n-2)$-set. Let m be a given vertex which is distinct from them. By G is bipartite, m has a common part with either x or y and not with both of them. It implies $d(x,m) \neq d(y,m)$. Since if m has a common part with x, then $d(x,m) = 1 \neq 2 = d(y,m)$. And if m has a common part with y, then $d(x,m) = 2 \neq 1 = d(y,m)$. Thus m fuzzy(neutrosophic)-resolves x and y. If w is another vertex which is distinct from them, then pigeonhole principle induces at least two vertices have same conditions concerning either being in same part or in different parts. It implies $(n-3)$-set isn't fuzzy(neutrosophic)-resolving set. Therefore, every $(n-2)$-set excludes two vertices from different parts, is fuzzy(neutrosophic)-metric set. Fuzzy(neutrosophic)-metric number is $n-2$. By G is fixed-vertex, for any given vertices m and m', $\sigma(m) = \sigma(m')$. So fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ where m is a given vertex. ■

cor55.14

Corollary 1.8.14. *Let G be a fixed-vertex strong fuzzy(neutrosophic) star. Then every $(n-2)$-set excludes center and a given vertex, is fuzzy(neutrosophic)-resolving set. An $(n-2)$-set excludes center and a given vertex, is fuzzy(neutrosophic)-metric set. Fuzzy(neutrosophic)-metric number is $(n-2)$. Fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ where m is a given vertex.*

Proof. Consider x and y are excluded by a $(n-2)$-set. Let m be a given vertex which is distinct from them. By G is star, m has a common part with either x or y and not with both of them. It implies $d(x,m) \neq d(y,m)$. Since if m has a common part with x, then $d(x,m) = 1 \neq 2 = d(y,m)$. And if m has a common part with y, then $d(x,m) = 2 \neq 1 = d(y,m)$. Thus m fuzzy(neutrosophic)-resolves x and y. If w is another vertex which is distinct from them, then pigeonhole principle induces at least two vertices have same conditions concerning either being in same part or in different parts. It implies $(n-3)$-set isn't fuzzy(neutrosophic)-resolving set. Therefore, every $(n-2)$-set excludes two vertices from different parts, is fuzzy(neutrosophic)-metric set.

Fuzzy(neutrosophic)-metric number is $n-2$. By G is fixed-vertex, for any given vertices m and m', $\sigma(m) = \sigma(m')$. So fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ where m is a given vertex. ■

cor55.15

Corollary 1.8.15. *Let G be a fixed-vertex strong fuzzy(neutrosophic) wheel. Let $n \geq 3$. Then every $(n-2)$-set excludes center and a given vertex, is fuzzy(neutrosophic)-resolving set. Every $(n-2)$-set excludes center and a given vertex, is fuzzy(neutrosophic)-metric set. Fuzzy(neutrosophic)-metric number is $n-2$. Fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ where m is a given vertex.*

Proof. Consider x and y are excluded by a $(n-2)$-set. Let m be a given vertex which is distinct from them. By G is wheel, m has a common part with either x or y and not with both of them. It implies $d(x, m) \neq d(y, m)$. Since if m has a common part with x, then $d(x, m) = 1 \neq 2 = d(y, m)$. And if m has a common part with y, then $d(x, m) = 2 \neq 1 = d(y, m)$. Thus m fuzzy(neutrosophic)-resolves x and y. If w is another vertex which is distinct from them, then pigeonhole principle induces at least two vertices have same conditions concerning either being in same part or in different parts. It implies $(n-3)$-set isn't fuzzy(neutrosophic)-resolving set. Therefore, every $(n-2)$-set excludes two vertices from different parts, is fuzzy(neutrosophic)-metric set. Fuzzy(neutrosophic)-metric number is $n-2$. By G is fixed-vertex, for any given vertices m and m', $\sigma(m) = \sigma(m')$. So fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ where m is a given vertex. ■

Fuzzy(neutrosophic) t-partite(bipartite/star/wheel) is also studied but by adding one restriction on these models. Fuzzy(neutrosophic) t-partite gets us one result involving family of them when they're either fixed-edge or strong fixed-vertex.

Corollary 1.8.16. *Let $\mathcal{G}$ be a family of fixed-vertex strong fuzzy(neutrosophic) t-partite with fuzzy(neutrosophic) common vertex set. Let $n \geq 3$. Then simultaneously fuzzy(neutrosophic)-metric number is $n-2$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$. Every $(n-2)$-set excludes two vertices from different parts, is simultaneously fuzzy(neutrosophic)-resolving set for $\mathcal{G}$. There's an $(n-2)$-set which is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Proof. By Corollary (3.31.8), every result hold for any given fixed-vertex strong fuzzy(neutrosophic) t-partite. Thus every result hold for any given fixed-vertex strong fuzzy(neutrosophic) t-partite, simultaneously. Therefore, simultaneously fuzzy(neutrosophic)-metric number is $n-2$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$. Every $(n-2)$-set excludes two vertices from different parts, is simultaneously fuzzy(neutrosophic)-resolving set for $\mathcal{G}$. There's an $(n-2)$-set which is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$. ■

Corollary 1.8.17. *Let $\mathcal{G}$ be a family of fixed-vertex strong fuzzy(neutrosophic) bipartite with fuzzy(neutrosophic) common vertex set. Let $n \geq 3$. Then simultaneously fuzzy(neutrosophic)-metric number is $n-2$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ Every $(n-2)$-set excludes two vertices from different parts, is simultaneously fuzzy(neutrosophic)-resolving* *set for $\mathcal{G}$. There's an $(n-2)$-set which is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Proof. By Corollary (3.31.9), every result hold for any given fixed-vertex strong fuzzy(neutrosophic) bipartite. Thus every result hold for any given fixed-vertex strong fuzzy(neutrosophic) bipartite, simultaneously. Therefore, simultaneously fuzzy(neutrosophic)-metric number is $n-2$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$. Every $(n-2)$-set excludes two vertices from different parts, is simultaneously fuzzy(neutrosophic)-resolving set for $\mathcal{G}$. There's an $(n-2)$-set which is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$. ■

Corollary 1.8.18. *Let $\mathcal{G}$ be a family of fixed-vertex strong fuzzy(neutrosophic) star with fuzzy(neutrosophic) common vertex set. Let $n \geq 3$. Then simultaneously fuzzy(neutrosophic)-metric number is $n-2$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ Every $(n-2)$-set excludes center and a given vertex, is simultaneously fuzzy(neutrosophic)-resolving set for $\mathcal{G}$. There's an $(n-2)$-set which is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Proof. By Corollary (3.31.10), every result hold for any given fixed-vertex strong fuzzy(neutrosophic) star. Thus every result hold for any given fixed-vertex strong fuzzy(neutrosophic) star, simultaneously. Therefore, simultaneously fuzzy(neutrosophic)-metric number is $n-2$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$. Every $(n-2)$-set excludes two vertices from different parts, is simultaneously fuzzy(neutrosophic)-resolving set for $\mathcal{G}$. There's an $(n-2)$-set which is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$. ■

Corollary 1.8.19. *Let $\mathcal{G}$ be a family of fixed-vertex strong fuzzy(neutrosophic) wheel with fuzzy(neutrosophic) common vertex set. Let $n \geq 3$. Then simultaneously fuzzy(neutrosophic)-metric number is $n-2$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$ Every $(n-2)$-set excludes center and a given vertex, is simultaneously fuzzy(neutrosophic)-resolving set for $\mathcal{G}$. There's an $(n-2)$-set which is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$.*

Proof. By Corollary (3.31.11), every result hold for any given fixed-vertex strong fuzzy(neutrosophic) wheel. Thus every result hold for any given fixed-vertex strong fuzzy(neutrosophic) wheel, simultaneously. Therefore, simultaneously fuzzy(neutrosophic)-metric number is $n-2$, simultaneously fuzzy(neutrosophic)-metric dimension is $(n-2)\sigma(m)$. Every $(n-2)$-set excludes two vertices from different parts, is simultaneously fuzzy(neutrosophic)-resolving set for $\mathcal{G}$. There's an $(n-2)$-set which is simultaneously fuzzy(neutrosophic)-metric set for $\mathcal{G}$. ■

1.9 Applications

In this chapter, I introduce some applications concerning new ideas and in this ways, the results make sense more about their impacts on different models.

Two applications are posed as follow.

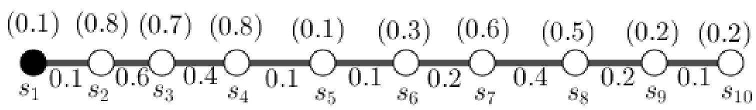
Located Places

A program is devised for a robot to locate every couple of given places, separately. The number which this program assigns to any place from a given couple of places are unique. Thus every place has an unique number when a couple of places are given. Three numbers are assigned to a place. First number is about a model concerning attributes which titled to be obstacle for locating the place, second number is about a model concerning attributes which titled to be indeterminate for locating the place and sometimes, they're obstacle but sometimes, they're determinate to locate that place. Third number is about a model concerning attributes which titled to be determinate for locating the place. For example, $(0.2, 0.5, 0.8)$ is assigned to a place v as information about its location. This is a brief outline of this application. To get it more precisely, I use some steps to clarify about them.

Step 1. (Definition) Located place is a term to categorize places into two classes. Applications for this function are too many but they've noticed to some parameters like decreasing costs, precise analysis, decreasing the ranges of analysis, restrictions on cases, low amount of selective data as possible, et cetera. Selective points as possible to distinguish about every couple of points out of them, are optimal case as possibilities allow.

Step 2. (Issue) A train has some stops which every stop has some attributes. A couple of stops are given but they're impossible to locate by their attributes.

Step 3. (Model) I use attributes of stops to get a model with three numbers chosen from real numbers amid zero and one. Every number illustrates every aspect of their attributes. The first number is obstacle means bad attributes, the second number is indeterminate and third number is determinate means good attributes. But to use sensible clarification, I use a fuzzy model as Figure (1.10). To get it more precisely, consider Table (1.7) as a fuzzy model which assigns to every stations and connections a value, separately. In fact, set of stations and set of connections are used to make fuzzy sets from them.



T

Figure 1.10: Black vertex $\{s_1\}$ is only fuzzy(neutrosophic)-metric set amid all sets of vertices for fuzzy(neutrosophic) graph T.

F3

Step 4. (Solution) As figure (1.10) shows, I study this fuzzy model. By Proposition (3.31.1), the stop s_1 locates every given couple of stations. To get beyond this result, If I've a family of fuzzy(neutrosophic) paths excerpt from family of trains with fuzzy(neutrosophic) common and s_1 in common, then by Corollary (3.31.4), the stop s_1 locates every given

Table 1.7: A Train concerning its Stations and its Connections as a Fuzzy Graph in a Model.

Stations of T	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9, s_{10}
Values	0.1	0.8	0.7	0.8	0.1	0.3	0.6	0.5	0.2
Connections of T	s_1s_2	s_2s_3	s_3s_4	s_4s_5	s_5s_6	s_6s_7	s_7s_8	s_8s_9	s_9s_{10}
Values	0.1	0.6	0.4	0.1	0.1	0.2	0.4	0.2	0.1

couple of stations in every fuzzy(neutrosophic) graph excerpt from any trains, simultaneously.

Covid-19 and Identifying Infected People

Dark network is description for infected people who are anonymous in the matter of Covid-19. Virus and its anonymously style to transmit the virus from one person to another person, could make a dark network involving people. Consider everyone as network titled fuzzy(neutrosophic). It means that the person and his networks containing his connections make two models, fixed-edge fuzzy(neutrosophic) and fixed-vertex strong fuzzy(neutrosophic). Now, I have a family of people which everyone is a model in the terms of Covid-19.

Step 1. (Definition) Covid-19 is well-known disease which like every disease has general parameters. Parameters are intensity of symptom, decreasing impacts, relatively treatments, complete treatments and et cetera. But Covid-19 has specific ways which they transmit this disease. It's coming up with finding impressive networks of people to identify infected people. People and their connections are important cases to develop this notion.

Step 2. (Issue) A person has been infected and I try to find the connections and the people which transmit this disease.

Step 3. (Model) A person and his connections are a network which are a fuzzy model. Two numbers are assigned to a person and his connections. To do this, I need to identify a couple of people which are given in a network of this person. I proposed two fuzzy models. Firstly, as Figure (1.11), a fuzzy graph containing the people who connect to this person, is proposed in Table (1.8). Secondly, as Figure (1.11), a fuzzy model including person with his two selective connections and other people with two selective connections of them, is posed in Table (1.9). The attributes are like the iterations of connections, the intensity of infected people, serious symptom, locations of people and et cetera, are used to have couple of people who are selected. Capable for being infected and infected people are used to make these models.

Step 4. (Solution) By Corollary (3.30.10), a person i_1 and his partner i_2 identify every given couple of partners which are in Figure (1.11) as T. To get beyond this result, if a person i_1 and the partner i_2 aren't antipodal vertices in every fuzzy cycles are contained in a family of person's networks, then by Corollary (3.30.11), a person i_1 and the partner i_2 identify every given couple of partners in every fuzzy cycles, simultaneously. By Corollary

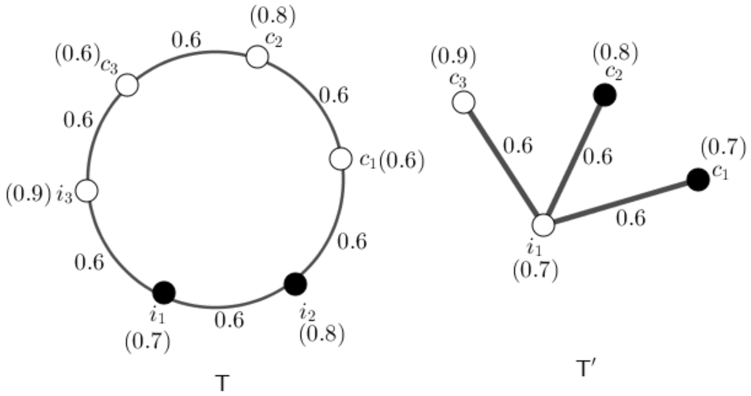


Figure 1.11: Black vertices $\{i_1, i_2\}$ are only fuzzy(neutrosophic)-metric set amid all sets of vertices for fuzzy(neutrosophic) graph T. Black vertices $V \setminus \{c_1, c_2\}$ are only fuzzy(neutrosophic)-metric set amid all sets of vertices for fuzzy(neutrosophic) graph T'. F4

Table 1.8: An Infected Person concerning his two selective Connections and his Partners With their two selective Connections as a Fuzzy Graph T in a Model. T4

People of T	i_1	i_2	c_1	c_2	c_3	i_3
Values	0.7	0.8	0.6	0.8	0.6	0.9
Connections of T	i_1i_2	i_2c_1	c_1c_2	c_2c_3	c_3i_3	i_3i_1
Values	0.6	0.6	0.6	0.6	0.6	0.6

Table 1.9: An Infected Person concerning his Connections and his Partners as a Fuzzy Graph T' in a Model. T5

People of T'	i_1	c_1	c_2	c_3
Values	0.7	0.7	0.8	0.9
Connections of T'	i_1c_1	i_1c_2	i_1c_3	c_3i_1
Values	0.6	0.6	0.6	0.6

(3.31.10), $\{c_1, c_2\}$ identify couple of person i_1 and his partner c_3, in Figure (1.11) as T' in optimal way and this set is unique.

1.10 Open Problems

sec8

The crisp notion of dimension is defined on fuzzy(neutrosophic) graphs. Thus

Question 1.10.1. *Is it possible to define fuzzy(neutrosophic) notion of dimension on fuzzy(neutrosophic) graphs?*

There are too many limitations on the classes of fuzzy(neutrosophic) graphs by using fixed-edge fuzzy(neutrosophic) graphs and fixed-vertex strong fuzzy(neutrosophic) graphs.

Question 1.10.2. *Is an approach existed to compute current dimension for specific classes of fuzzy(neutrosophic) graphs?*

Question 1.10.3. *What are basic attributes of current dimension for general classes of fuzzy(neutrosophic) graphs?*

Finding other classes of fuzzy(neutrosophic) graphs has an ordinary approach to develop this study.

Question 1.10.4. *Which new classes of fuzzy(neutrosophic) graphs are existed to develop this notion of current dimension?*

Question 1.10.5. *Which new classes of fuzzy(neutrosophic) graphs are existed to compute this notion of current dimension?*

Question 1.10.6. *Which general approaches are existed to study this notion of current dimension in fuzzy(neutrosophic) graphs?*

Question 1.10.7. *Which specific approaches are existed to study this notion of current dimension in fuzzy(neutrosophic) graphs?*

Problem 1.10.8. *Are there special crisp sets of vertices, e.g. antipodal vertices for fuzzy(neutrosophic) cycles, which have key role to study this notion of current dimension in fuzzy(neutrosophic) graphs?*

Problem 1.10.9. *Are there fuzzy(neutrosophic) special sets of vertices, e.g. fuzzy(neutrosophic) twin vertices for general classes, which have key role to study this notion of current dimension in fuzzy(neutrosophic) graphs?*

1.11 Conclusion and Closing Remarks

sec10

This study uses mixed combinations of fuzzy concepts and crisp concepts to explore new notion of crisp dimension in fuzzy(neutrosophic) graphs as individual and as family. In this way, some crisp notions like antipodal vertices are defined to use as a tool to study fuzzy(neutrosophic) cycles as individual and as family. Also, some fuzzy(neutrosophic) notions like fuzzy(neutrosophic) twin vertices are defined to use as a tool to study general classes of fuzzy(neutrosophic) graphs as individual and as family. Mixed family of fuzzy(neutrosophic) graphs are slightly studied by using fuzzy(neutrosophic) twin vertices and other ideas as individual and as family. In Table (1.10), I mention some advantages and limitations concerning this article and its proposed notions.

Table 1.10: A Brief Overview about Advantages and Limitations of this study

tbl5

Advantages	Limitations
1. Using crisp and fuzzy(neutrosophic) notions in one framework together simultaneously.	1. The most usages of fixed-edge fuzzy(neutrosophic) graphs and fixed-vertex strong fuzzy(neutrosophic) graphs.
2. Study on fuzzy(neutrosophic) as individual and as family.	
3. Involved classes as complete, strong, path, cycle, t-partite, bipartite, star, wheel.	2. Study on family of different models
4. Characterizing classes of fuzzy(neutrosophic) graphs with smallest metric number and largest metric number.	3. Characterizing classes of fuzzy(neutrosophic) graphs with smallest dimension number and largest dimension number.

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CHAPTER 2

Neutrosophic Chromatic Number

Rohini et al. [4] introduce single valued neutrosophic coloring. He et al. [5] also propose operations of single valued neutrosophic coloring. Rohini et al. [6] study on single valued neutrosophic irregular vertex coloring.

2.1 Definitions

To clarify about the definitions, I use some examples and in this way, exemplifying has key role to make sense about the definitions and to introduce new ways to use on these models in the terms of new notions.

The reference [1] is used to write the contents of this chapter.

Definition 2.1.1. $G : (V, E)$ is called a **crisp graph** where V is a set of objects and E is a subset of $V \times V$ such that this subset is symmetric.

Definition 2.1.2. A crisp graph $G : (V, E)$ is called a **neutrosophic graph** $G : (\sigma, \mu)$ where $\sigma = (\sigma_1, \sigma_2, \sigma_3) : V \to [0, 1]$ and $\mu = (\mu_1, \mu_2.\mu_3) : E \to [0, 1]$ such that $\mu(xy) \leq \sigma(x) \wedge \sigma(y)$ for all $xy \in E$.

Definition 2.1.3. A neutrosophic graph is called **neutrosophic empty** if it has no edge. It's also called **neutrosophic trivial**. A neutrosophic graph which isn't neutrosophic empty, is called **neutrosophic nontrivial**.

Definition 2.1.4. A neutrosophic graph $G : (\sigma, \mu)$ is called a **neutrosophic complete** where it's complete and $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $xy \in E$.

Definition 2.1.5. A neutrosophic graph $G : (\sigma, \mu)$ is called a **neutrosophic strong** where $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $xy \in E$.

Definition 2.1.6. A path $v_0, v_1, \cdots, v_n$ is called **neutrosophic path** where $\mu(v_i v_{i+1}) > 0,\ i = 0, 1, \cdots, n-1$. i-path is a path with i edges, it's also called **length** of path.

Definition 2.1.7. A crisp cycle $v_0, v_1, \cdots, v_n, v_0$ is called **neutrosophic cycle** where there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$.

Definition 2.1.8. A neutrosophic graph is called **neutrosophic t-partite** if V is partitioned to t parts, $V_1, V_2, \cdots, V_t$ and the edge xy implies $x \in V_i$ and $y \in V_j$ where $i \neq j$. If it's neutrosophic complete, then it's denoted by $K_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where σ_i is σ on V_i instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. If $t = 2$, then it's called **neutrosophic complete bipartite** and it's denoted by K_{σ_1,σ_2} especially, if $|V_1| = 1$, then it's called **neutrosophic star** and it's denoted by S_{1,σ_2}. In this case, the vertex in V_1 is called **center** and if a vertex joins to all vertices of neutrosophic cycle, it's called **neutrosophic wheel** and it's denoted by W_{1,σ_2}.

Definition 2.1.9. Let $G : (\sigma, \mu)$ be a neutrosophic graph. For any given subset N of V, $\Sigma_{n\in N}\sigma(n)$ is called **neutrosophic cardinality** of N and it's denoted by $|N|_n$.

Definition 2.1.10. Let $G : (\sigma, \mu)$ be a neutrosophic graph. Neutrosophic cardinality of V is called **neutrosophic order** of G and it's denoted by $O_n(G)$.

Definition 2.1.11. Let $G : (\sigma, \mu)$ be a neutrosophic graph. The number of vertices is denoted by n and the number of edges is denoted by m.

Definition 2.1.12. Let $N = (\sigma, \mu)$ be a neutrosophic graph. It's called **neutrosophic connected** if for every given couple of vertices, there's at least one neutrosophic path amid them.

Definition 2.1.13. Let $N = (\sigma, \mu)$ be a neutrosophic graph. Suppose a path $P : v_0, v_1, \cdots, v_{n-1}, v_n$ from v_0 to v_n. $\min_{i=0,1,2,\cdots,n-1} \mu(v_i v_{i+1})$ is called **neutrosophic strength** of P and it's denoted by $\mathcal{S}_n(P)$.

Definition 2.1.14. Let $N = (\sigma, \mu)$ be a neutrosophic graph. The number of maximum edges for a vertex, amid all vertices, is denoted by $\Delta(N)$.

First case for the contents is to use the article from [**1**]. The contents are used in the way that, usages of new contents are preferences and the preliminaries are passed in the beginning of this chapter.

2.2 Chromatic Number and Neutrosophic Chromatic Number

2.3 Abstract

New setting is introduced to study chromatic number. Neutrosophic chromatic number and chromatic number are proposed in this way, some results are obtained. Classes of neutrosophic graphs are used to obtains these numbers and the representatives of the colors. Using colors to assign to the vertices of neutrosophic graphs is applied. Some questions and problems are posed concerning ways to do further studies on this topic. Using strong edge to define the relation amid vertices which implies having different colors amid them and as consequences, choosing one vertex as a representative of each color to use them in a set of representatives and finally, using neutrosophic cardinality of this set to compute neutrosophic chromatic number. This specific relation amid edges is necessary to compute both chromatic number concerning the number of representative in the set of representatives and neutrosophic chromatic number concerning neutrosophic cardinality of set of representatives. If two vertices have no strong edge, then they can be assigned to same color even they've common edge. Basic familiarities with neutrosophic graph theory and graph theory are proposed for this article.
Keywords: Neutrosophic Strong, Neutrosophic Graphs, Chromatic Number

AMS Subject Classification: 05C17, 05C22, 05E45

2.4 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 2.4.1. *Is it possible to use mixed versions of ideas concerning "neutrosophic strong edges", "neutrosophic graphs" and "neutrosophic coloring" to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Connections amid two items have key roles to assign colors. Thus they're used to define new ideas which conclude to the structure of coloring. The concept of having strong edge inspires to study the behavior of strong edge in the way that, both neutrosophic chromatic number and chromatic number are the cases of study.
The framework of this study is as follows. In the beginning of chapter, I introduced basic definitions to clarify about preliminaries. In subsection "Chromatic Number and Neutrosophic Chromatic Number", new notion of coloring is applied to the vertices of neutrosophic graphs. Neutrosophic strong edge has the key role in this way. Classes of neutrosophic graphs are studied when the edges are neutrosophic strong. In subsection "Applications in Time Table and Scheduling", one application is posed for neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects. In subsection "Open Problems", some problems and questions for further studies are proposed. In subsection "Conclusion and Closing Remarks", gentle discussion about results and applications are featured. In subsection "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions are formed.

2.5 Chromatic Number and Neutrosophic Chromatic Number

sec2

Definition 2.5.1. Let $N = (\sigma, \mu)$ be a neutrosophic graph. **Chromatic number** is minimum number of distinct colors which are used to color the vertices which have neutrosophic strong edge. Neutrosophic cardinality of the set of these distinct colors when it's minimum amid all of these sets, is called **neutrosophic chromatic number** with respect with first order.

Example 2.5.2. Consider Figure (2.1). The chromatic number is three and neutrosophic chromatic number is 2.57 with respect to first order.

Neutrosophic chromatic number of some classes of neutrosophic graphs are computed.

Proposition 2.5.3. *Let $N = (\sigma, \mu)$ be a neutrosophic complete. Then chromatic number is n and neutrosophic chromatic number is neutrosophic order.*

Proof. All edges are neutrosophic strong. Every vertex has edge with $n - 1$ vertices. Thus n is chromatic number. Since any given vertex has different color in comparison to another vertex, neutrosophic cardinality of V is neutrosophic chromatic number. Therefore, neutrosophic chromatic number is neutrosophic order. ■

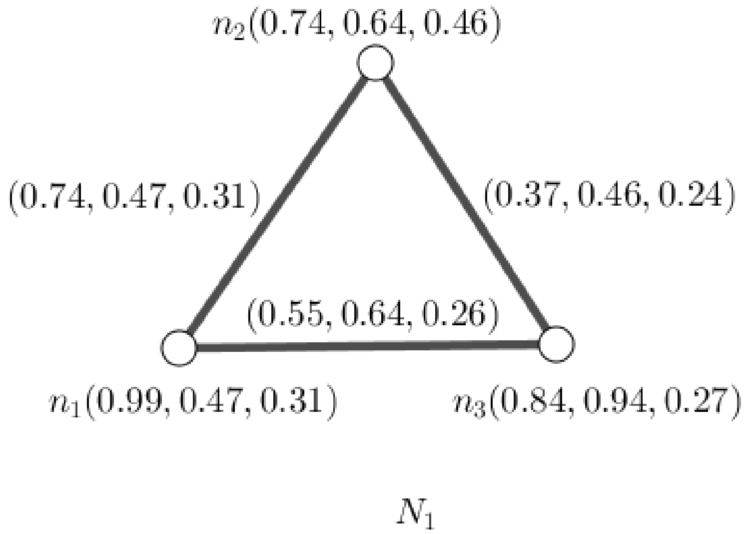

Figure 2.1: Neutrosophic Graph, N_1

nsc1

Proposition 2.5.4. *Let $N = (\sigma, \mu)$ be a neutrosophic strong path. Then chromatic number is two and neutrosophic chromatic number is*

$$\min_{x \textit{ and } y \textit{ have different colors}} \{\sigma(x) + \sigma(y)\}.$$

Proof. With alternative colors, neutrosophic strong path has distinct color for every vertices which have one edge in common. Thus if x and y are two vertices which have one edge in common, then x and y have different color. Therefore, chromatic number is two. The representative of colors are a vertex with minimum value amid all vertices which have same color with it. Thus,

$$\min_{x \text{ and } y \text{ have different colors}} \{\sigma(x) + \sigma(y)\}.$$

■

Proposition 2.5.5. *Let $N = (\sigma, \mu)$ be an even neutrosophic strong cycle. Then chromatic number is two and neutrosophic chromatic number is*

$$\min_{x \textit{ and } y \textit{ have different colors}} \{\sigma(x) + \sigma(y)\}.$$

Proof. All edges are neutrosophic strong. Since the cycle has even vertices, with alternative coloring of vertices, the vertices which have common edge, have different colors. So chromatic number is two. With every color, the vertex which has minimum value amid vertices with same color with it, is representative of that color. Thus,

$$\min_{x \text{ and } y \text{ have different colors}} \{\sigma(x) + \sigma(y)\}.$$

■

Proposition 2.5.6. *Let $N = (\sigma, \mu)$ be an odd neutrosophic strong cycle. Then chromatic number is three and neutrosophic chromatic number is*

$$\min_{x,y \text{ and } z \text{ have different colors}} \{\sigma(x) + \sigma(y) + \sigma(z)\}.$$

Proof. With alternative coloring on vertices, at end, two vertices have same color, and they've same edge. So, chromatic number is three. Since the colors are three, the vertices with minimum values in every color, are representatives. Hence,

$$\min_{x,y \text{ and } z \text{ have different colors}} \{\sigma(x) + \sigma(y) + \sigma(z)\}.$$

■

Proposition 2.5.7. *Let $N = (\sigma, \mu)$ be a neutrosophic strong star with c as center. Then chromatic number is two and neutrosophic chromatic number is*

$$\min_{x \text{ is non-center vertex}} \{\sigma(c) + \sigma(x)\}.$$

Proof. All edges are neutrosophic strong. Center vertex has common edge with every given vertex. So it has different color in comparison to other vertices. So one color has only one vertex which has that color. All non-center vertices have no common edge amid each other. Then they've same color. The representative of this color is a non-center vertex which has minimum value amid all non-center vertices. Hence,

$$\min_{x \text{ is non-center vertex}} \{\sigma(c) + \sigma(x)\}.$$

■

Proposition 2.5.8. *Let $N = (\sigma, \mu)$ be a neutrosophic strong wheel with c as center. Then chromatic number is three where neutrosophic cycle has even number as its length and neutrosophic chromatic number is*

$$\min_{x,y \text{ are non-center vertices and have different colors}} \{\sigma(c) + \sigma(x) + \sigma(y)\}.$$

Proof. Center vertex has unique color. So it's only representative of this color. Non-center vertices form a neutrosophic cycle which have distinct colors for the vertices which have common edge with each other when the number of colors is two. So a color for center vertex and two colors for non-center vertices, make neutrosophic strong wheel has distinct colors for vertices which have common edge. Hence, chromatic number is three when the non-center vertices form odd cycle. Therefore,

$$\min_{x,y \text{ are non-center vertices and have different colors}} \{\sigma(c) + \sigma(x) + \sigma(y)\}.$$

■

Proposition 2.5.9. *Let $N = (\sigma, \mu)$ be a neutrosophic strong wheel with c as center. Then chromatic number is four where neutrosophic cycle has odd number as its length and neutrosophic chromatic number is*

$$\min_{x,y,z \text{ are non-center vertices and have different colors}} \{\sigma(c) + \sigma(x) + \sigma(y) + \sigma(z)\}.$$

Proof. All edges are neutrosophic strong and non-center vertices form odd neutrosophic strong cycles. Odd neutrosophic strong cycle have chromatic number which is three. Non-center vertex has same edges with all non-center vertices. Thus non-center vertex has different colors with non-center vertices. Therefore, chromatic number is four. Four representatives of colors form neutrosophic chromatic number where one representative is center vertex and other three representatives are non-center vertices. So,

$$\min_{x,y,z \text{ are non-center vertices and have different colors}} \{\sigma(c)+\sigma(x)+\sigma(y)+\sigma(z)\}.$$

■

Proposition 2.5.10. *Let $N=(\sigma,\mu)$ be a neutrosophic complete bipartite. Then chromatic number is two and neutrosophic chromatic number is*

$$\min_{x \textit{ and } y \textit{ are in different parts}} \{\sigma(x)+\sigma(y)\}.$$

Proof. Every given vertex has neutrosophic strong edge with all vertices from another part. So the color of every vertex which is in a same part is same. Hence, two parts implies two different colors. It induces chromatic number is two. The minimum value of a vertex amid all vertices in every part, identify the representative of every color. Therefore,

$$\min_{x \text{ and } y \text{ are in different parts}} \{\sigma(x)+\sigma(y)\}.$$

■

Proposition 2.5.11. *Let $N=(\sigma,\mu)$ be a neutrosophic complete $t-$partite. Then chromatic number is t and neutrosophic chromatic number is*

$$\min_{x_1,x_2\ ,\cdots,x_t \textit{ are in different parts}} \{\sigma(x_1)+\sigma(x_2)+\cdots+\sigma(x_t)\}.$$

Proof. Every part has same color for its vertices. So chromatic number is t. Every part introduces one vertex as a representative of its color. Thus, neutrosophic chromatic number is

$$\min_{x_1,x_2\ ,\cdots,x_t \text{ are in different parts}} \{\sigma(x_1)+\sigma(x_2)+\cdots+\sigma(x_t)\}.$$

■

Proposition 2.5.12. *Let $N=(\sigma,\mu)$ be a neutrosophic strong. Then chromatic number is* 1 *if and only if $N=(\sigma,\mu)$ is neutrosophic empty.*

Proof. ($\Rightarrow$). Let chromatic number be 1. It implies there's no vertex which has same edge with a vertex. So there's no neutrosophic strong edge. Since $N=(\sigma,\mu)$ is a neutrosophic strong, $N=(\sigma,\mu)$ is a neutrosophic empty.
($\Leftarrow$). Let $N=(\sigma,\mu)$ be neutrosophic empty and neutrosophic strong. Hence there's no edge. It implies for every given vertex, there's no common neutrosophic strong edge. It induces there's only one color for vertices. Hence the representative of this color is chosen from n vertices. Thus chromatic number is 1. ■

Proposition 2.5.13. *Let $N = (\sigma, \mu)$ be a neutrosophic strong. Then chromatic number is* 2 *if and only if $N = (\sigma, \mu)$ is neutrosophic complete bipartite.*

Proof. ($\Rightarrow$). Let chromatic number be two. So every vertex has either one vertex or two vertices with a common edge. The number of colors are two so there are two sets which each set has the vertices which same color. If two vertices have same color, then they don't have a common edge. So every set is a part in that, no vertex has common edge. The number of these sets is two. Hence there are two parts in each of them, every vertex has no common edge with other vertices. Since $N = (\sigma, \mu)$ is a neutrosophic strong, $N = (\sigma, \mu)$ is neutrosophic complete bipartite.
($\Leftarrow$). Assume $N = (\sigma, \mu)$ is neutrosophic complete bipartite. Then all edges are neutrosophic strong. Every part has the vertices which have no edge in common. So they're assigned to have same color. There are two parts. Thus there are two colors to assign to the vertices in that, the vertices with common edge, have different colors. It induces chromatic number is 2. ∎

Proposition 2.5.14. *Let $N = (\sigma, \mu)$ be a neutrosophic strong. Then chromatic number is n if and only if $N = (\sigma, \mu)$ is neutrosophic complete.*

Proof. ($\Rightarrow$). Let chromatic number be n. So any given vertex has n vertices which have common edge with them and every of them have common edge with each other. It implies every vertex has n vertices which have common edge with them. Since $N = (\sigma, \mu)$ is a neutrosophic strong, $N = (\sigma, \mu)$ is neutrosophic complete.
($\Leftarrow$). Suppose $N = (\sigma, \mu)$ is neutrosophic complete. Every vertex has n vertices which have common edge with them. Since all edges are neutrosophic strong, the minimum number of colors are n. Thus chromatic number is n. ∎

General bounds for neutrosophic chromatic number are computed.

Proposition 2.5.15. *Let $N = (\sigma, \mu)$ be a neutrosophic graph. Then chromatic number is at most the number of vertices and neutrosophic chromatic number is at most neutrosophic order.*

Proof. When every vertex is a representative of each color, chromatic number is the number of vertices and it happens in chromatic number of neutrosophic complete which is n. When all vertices have distinct colors, neutrosophic chromatic number is neutrosophic order and it's sharp for neutrosophic complete. ∎

The relation amid neutrosophic chromatic number and main parameters of neutrosophic graphs is computed.

Proposition 2.5.16. *Let $N = (\sigma, \mu)$ be a neutrosophic strong. Then chromatic number is at most $\Delta + 1$ and at least* 2.

Proof. Neutrosophic strong is neutrosophic nontrivial. So it isn't neutrosophic empty which induces there's no edge. It implies chromatic number is two. Since chromatic number is one if and only if $N = (\sigma, \mu)$ is neutrosophic empty if and only if $N = (\sigma, \mu)$ is neutrosophic trivial. A vertex with degree Δ, has Δ vertices which have common edges with them. If these vertices have no edge amid each other, then chromatic number is two especially, neutrosophic star. If not, then in the case, all vertices have edge amid each other, chromatic number is $\Delta + 1$, especially, neutrosophic complete. ■

Proposition 2.5.17. *Let* $N = (\sigma, \mu)$ *be a neutrosophic* $r-$*regular. Then chromatic number is at most* $r + 1$.

Proof. $N = (\sigma, \mu)$ is a neutrosophic $r-$regular. So any of vertex has r vertices which have common edge with it. If these vertices have no common edge with each other, for instance neutrosophic star, chromatic number is two. But since the vertices have common edge with each other, chromatic number is $r + 1$, for instance, neutrosophic complete. ■

2.6 Applications in Time Table and Scheduling

sec3

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has important to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) scheduling of program has faced with difficulties to differ amid consecutive section. Beyond that, sometimes sections are not the same.

Step 3. (Model) As Figure (2.2), the situation is designed as a model. The model uses data to assign every section and to assign to relation amid section, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two section is at least the number of the relation amid them. Table (2.1), clarifies about the assigned numbers to these situation.

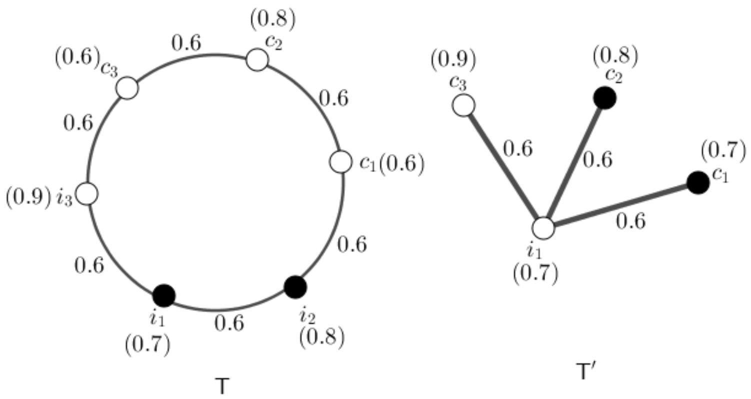

Figure 2.2: Black vertices are suspicions about choosing them. fgr1

Table 2.1: Scheduling concerns its Subjects and its Connections as a Neutrosophic Graph in a Model.

tbl1

Sections of T	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9, s_{10}
Values	0.1	0.8	0.7	0.8	0.1	0.3	0.6	0.5	0.2
Connections of T	s_1s_2	s_2s_3	s_3s_4	s_4s_5	s_5s_6	s_6s_7	s_7s_8	s_8s_9	s_9s_{10}
Values	0.1	0.6	0.4	0.1	0.1	0.2	0.4	0.2	0.1

Step 4. (Solution) As Figure (2.2) shows, neutrosophic model, propose to use chromatic number 2 in the case with is titled T'. In this case, i_1 and c_1 are representative of these two colors and neutrosophic chromatic number is 1.4. The set $\{i_1, c_1\}$ contains representatives of colors which pose chromatic number and neutrosophic chromatic number. Thus the decision amid choosing the subject c_1 an c_2 is concluded to choose c_1. To get brief overview, neutrosophic model uses one number for every array so 0.9 means $(0.9, 0.9, 0.9)$. In Figure (2.2), the neutrosophic model T introduce the common situation. The representatives of colors are i_2 and c_1. Thus chromatic number is two and neutrosophic chromatic number is 1.4. Thus suspicion about choosing i_1 and i_2 is determined to be i_2. The sets of representative for colors are $\{i_2, c_1\}$.

sec4

2.7 Open Problems

The two notions of coloring of vertices concerning neutrosophic chromatic number and chromatic number are defined on neutrosophic graphs when neutrosophic strong edges have key role to have these notions. Thus

Question 2.7.1. *Is it possible to use other types edges to define chromatic number and neutrosophic chromatic number?*

Question 2.7.2. *Is it possible to use other types of ways to make number to define chromatic number and neutrosophic chromatic number?*

Question 2.7.3. *Which classes of neutrosophic graphs have the eligibility to pursue independent study in this way?*

Question 2.7.4. *Which applications do make an independent study to define chromatic number and neutrosophic chromatic number?*

Problem 2.7.5. *Which approaches do work to construct classes of neutrosophic graphs to continue this study?*

Problem 2.7.6. *Which approaches do work to construct applications to create independent study?*

Problem 2.7.7. *Which approaches do work to construct definitions which use all three arrays and the relations amid them instead of one array of three arrays to create independent study?*

2.8 Conclusion and Closing Remarks

sec5

This study uses mixed combinations of neutrosophic chromatic number and chromatic number to study on neutrosophic graphs. The connections of vertices which are clarified by neutrosophic strong edges, differ them from each other and and put them in different categories to represent one representative for each color. Further studies could be about changes in the settings to compare this notion amid different settings of graph theory. One way is finding some relations amid array of vertices to make sensible definitions. In Table (2.2), some limitations and advantages of this study is pointed out. Second case for

Table 2.2: A Brief Overview about Advantages and Limitations of this study tbl2

Advantages	Limitations
1. Using neutrosophic strong edges	1. Using only one array of three arrays
2. Using neutrosophic cardinality	
3. Using cardinality	2. Study on a few classes
4. Characterizing smallest number	
5. Characterizing biggest number	3. Quality of Results

the contents is to use the article from [**3**]. The contents are used in the way that, usages of new contents are preferences and the preliminaries are passed in the beginning of this chapter.

2.9 Neutrosophic Chromatic Number Based on Connectedness

2.10 Abstract

New setting is introduced to study chromatic number. vital chromatic number and n-vital chromatic number are proposed in this way, some results are obtained. Classes of neutrosophic graphs are used to obtains these numbers and the representatives of the colors. Using colors to assign to the vertices of neutrosophic graphs is applied. Some questions and problems are posed concerning ways to do further studies on this topic. Using vital edge from connectedness to define the relation amid vertices which implies having different colors amid them and as consequences, choosing one vertex as a representative of each color to use them in a set of representatives and finally, using neutrosophic cardinality of this set to compute vital chromatic number. This specific relation amid edges is necessary to compute both vital chromatic number concerning the number of representative in the set of representatives and n-vital chromatic number concerning neutrosophic cardinality of set of representatives. If two vertices have no vital edge, then they can be assigned to same color even they've common edge. Basic familiarities with neutrosophic graph theory and graph theory are proposed for this article.

Keywords: Neutrosophic Connctedness, Neutrosophic Graphs, Chromatic Number
AMS Subject Classification: 05C17, 05C22, 05E45

2.11 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 2.11.1. *Is it possible to use mixed versions of ideas concerning "connectedness", "neutrosophic graphs" and "neutrosophic coloring" to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Connections amid two items have key roles to assign colors. Thus they're used to define new ideas which conclude to the structure of coloring. The concept of having vital edge from connectedness inspires me to study the behavior of vital edge in the way that, both vital chromatic number and n-vital number are the cases of study.
The framework of this study is as follows. In the beginning of chapter, I introduced basic definitions to clarify about preliminaries. In subsection "Definitions and Clarification", new notion of coloring is applied to the vertices of neutrosophic graphs. Vital edge from connectedness has the key role in this way. Classes of neutrosophic graphs are studied in the terms of vital edges. In subsection "Applications in Time Table and Scheduling", one application is posed for neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects. In subsection "Open Problems", some problems and questions for further studies are proposed. In subsection "Conclusion and Closing Remarks", gentle discussion about results and applications are featured. In subsection "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions are formed.

2.12 Definitions and Clarification

Definition 2.12.1. Let $N = (\sigma, \mu)$ be a neutrosophic graph. A neutrosophic edge xy is called **vital** if deletion of xy has no change on its **connectedness** which is a maximum strength of paths amid them.

Definition 2.12.2. Let $N = (\sigma, \mu)$ be a neutrosophic graph. A vertex which has common vital edge with another vertex, has assigned different color from that vertex. The number of different colors, is called **vital chromatic number** and its neutrosophic cardinality is called **n-vital chromatic number**.

Example 2.12.3. Assume Figure (2.3) with respect to first order.

(i) : Only vital edge is n_2n_3. Other edges aren't vital.

(ii) : The vertices n_2 and n_3 have different colors.

(iii) : The vertex n_1 could get any color.

(iv) : The vertex n_1 has no vital edge with any given vertex.

(v) : The set of representatives of colors is $\{n_1, n_2\}$.

(vi) : Amid n_2 and n_3, n_2 has minimum value.

(vii) : Deletion of edge n_1n_2 has no change in the connectedness of obtained neutrosophic graph.

$(viii)$: The vital number is two.

(ix) : n-vital chromatic number is 2.57.

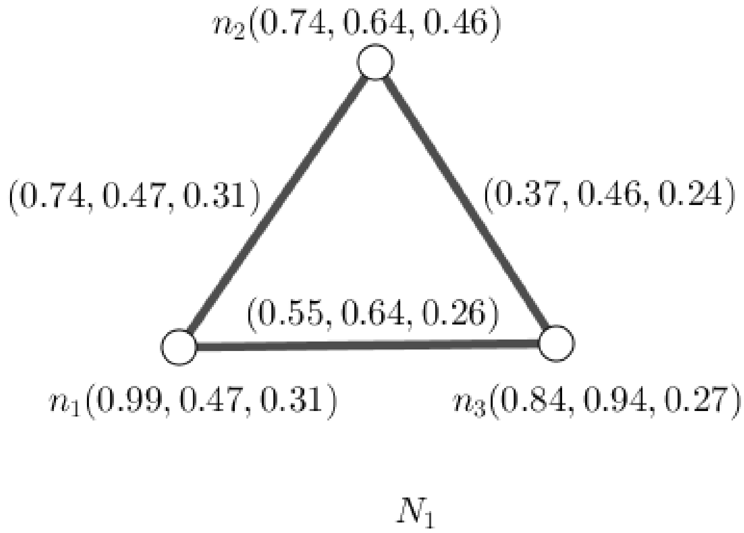

Figure 2.3: Neutrosophic graph N_1 is considered with respect to first order. It's complete but it isn't neutrosophic complete. It's cycle but it isn't neutrosophic cycle. It's neutrosophic 3-partite but it isn't neutrosophic complete 3-partite. nsc1b

2.13 Basic Properties

prp5b **Proposition 2.13.1.** *Let $N = (\sigma, \mu)$ be a neutrosophic cycle. Then all edges are vital.*

Proof. Consider $N = (\sigma, \mu)$ be a neutrosophic cycle. Hence, there are at least two edges which are weakest, it means there are $xy, uv \in E$ such that

$$\mu(uv) = \mu(xy) = \min_{e \in E} \mu(e).$$

In other hand, for every given vertices x and y, there are two paths from x to y. So for every given path,

$$S(P) = \min_{e \in E} \mu(e).$$

Thus for every $x, y \in V, xy \in E$, the value $\mu(xy)$ forms the connectedness amid x to y. Therefore connectedness amid any given couple of vertices, doesn't change when they form an edge and they're deleted. It induces every edge is vital. ■

prp6b

Proposition 2.13.2. *Let $N = (\sigma, \mu)$ be a neutrosophic complete which is neither neutrosophic empty nor neutrosophic path. Then all edges are vital.*

Proof. Suppose $N = (\sigma, \mu)$ is a neutrosophic complete which is neither neutrosophic empty nor neutrosophic path. If $x, y \in V$, then $xy \in E$. Thus $P : x, y$ is a path for every given couple of vertices. Hence

$$S(P) = \mu(xy).$$

Therefore, connectedness $\geq \mu(xy)$. In other hands, assume $P' : x, \cdots, y$ is an arbitrary path from x to y. By $N = (\sigma, \mu)$ is a neutrosophic complete, $N = (\sigma, \mu)$ is a neutrosophic strong. By $N = (\sigma, \mu)$ is a neutrosophic strong,

$$S(P') \leq \mu(xy).$$

Then connectedness $\leq S(P)$. It implies connectedness $\leq \mu(xy)$. To sum it up, connectedness $= \mu(xy)$. It induces xy is vital. ■

Proposition 2.13.3. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is fixed-edge and which is neither neutrosophic empty nor neutrosophic path. Then all edges are vital.*

Proof. Assume $N = (\sigma, \mu)$ is a neutrosophic graph which is fixed-edge and which is neither neutrosophic empty nor neutrosophic path. By$N = (\sigma, \mu)$ is a fixed-edge,

$$\forall e, e' \in E, \ \mu(e) = \mu(e').$$

It induces for every given edge e and every given paths P, P'

$$S(P) = S(P') = \mu(e).$$

It implies connectedness is fixed and it equals to $\mu(e)$ where $e \in E$. Therefore, the deletion of e has no change on connectedness amid every couple of vertices. It means every edge is vital. ■

prp8b

Proposition 2.13.4. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is neither neutrosophic empty nor neutrosophic path. Then there's at least one vital edge.*

Proof. Consider $N = (\sigma, \mu)$ is a neutrosophic graph which is neither neutrosophic empty nor neutrosophic path. Assume $N = (\sigma, \mu)$ is a neutrosophic graph which is either fixed-edge or fixed-vertex and neutrosophic strong. Hence, all edges have same value. It means

$$\forall e, e' \in E, \ \mu(e) = \mu(e').$$

It induces for every given edge e and every given paths P, P'

$$S(P) = S(P') = \mu(e).$$

It implies connectedness is fixed and it equals to $\mu(e)$ where $e \in E$. Therefore, the deletion of e has no change on connectedness amid every couple of vertices. It means every edge is vital. In other hand, suppose otherwise. So by $|E| > 2$, there's one edge e such that for every edge $e' \neq e$,

$$\mu(e) > \mu(e').$$

Let a number $\mu(e')$ be

$$\min_{e \in E} \mu(e).$$

Then connectedness is $\geq \mu(e')$. But there's a cycle which implies $|E| > 3$. It induces there are at least two paths corresponded to e'. By $\mu(e) > \mu(e')$, connectedness $\geq \mu(e')$. It implies corresponded connectedness to e' isn't changed when the deletion of e' is done. Thus the edge $e' \in E$ is vital. ■

prp9b

Proposition 2.13.5. *Let $N = (\sigma, \mu)$ be a neutrosophic strong which is fixed-vertex and which is neither neutrosophic empty nor neutrosophic path. Then all edges are vital.*

Proof. Assume $N = (\sigma, \mu)$ is a neutrosophic strong which is fixed-vertex and which is neither neutrosophic empty nor neutrosophic path. Thus by $N = (\sigma, \mu)$ is a neutrosophic fixed-vertex, for all $v, v' \in V$,

$$\sigma(v) = \sigma(v').$$

By $N = (\sigma, \mu)$ is a neutrosophic strong, for all $e, e' \in V$,

$$\mu(e) = \mu(e').$$

It induces for every couple of vertices which form an edge, connectedness amid them is same and equals $\mu(e)$ where e is a given edge. It implies at least there are two paths with strength $\mu(e)$. Thus deletion of every edge has no change on connectedness amid its vertices. Therefore, every edge is vital. ■

Proposition 2.13.6. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is fixed-vertex and complete. Then all edges are vital.*

Proof. By $N = (\sigma, \mu)$ is neutrosophic complete, $N = (\sigma, \mu)$ is neutrosophic strong. By $N = (\sigma, \mu)$ is a neutrosophic graph which is fixed-vertex, complete and applying Proposition (2.13.5), all edges are vital. ■

prp11b

Proposition 2.13.7. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is fixed-edge. Then all edges are vital.*

Proof. Suppose $N = (\sigma, \mu)$ is a neutrosophic graph which is fixed-edge. Then for every edges e and e',

$$\mu(e) = \mu(e').$$

It means all paths has same strength which is the value of an edge since all edges have same values. It means the connectedness amid all given couple of vertices is the same. There are at least two paths. So deletion any edge has no change on the connectedness amid all given couple of vertices. ■

2.14 Vital Chromatic Number

Proposition 2.14.1. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is neither neutrosophic empty nor neutrosophic path. Then vital chromatic number is at most n and at least* 1.

Proof. These bounds are sharp and tight as they'll be shown in upcoming results. If there's no edge, then vital chromatic number is 1 but if the number of vertices are n and they're connected to each other, then vital chromatic number is n. ■

2.15 Largest Vital Chromatic Number

Proposition 2.15.1. *Let $N = (\sigma, \mu)$ be a neutrosophic complete which is neither neutrosophic empty nor neutrosophic path. Then vital chromatic number is n.*

Proof. Consider $N = (\sigma, \mu)$ is a neutrosophic complete which is neither neutrosophic empty nor neutrosophic path. By Proposition (2.13.2), all edges are vital. By $N = (\sigma, \mu)$ isn't a neutrosophic path, there are at least two path amid two given edges. In other words, there is at least one cycle. By $N = (\sigma, \mu)$ is a neutrosophic complete, all vertices are connected to each other. It implies,

$$\forall v, v' \in V, vv' \in E.$$

It induces all vertices have different colors. The number of vertices are n. So vital chromatic number is n. ■

Proposition 2.15.2. *Let $N = (\sigma, \mu)$ be a neutrosophic path. Then vital chromatic number aren't computable.*

Proof. Assume $N = (\sigma, \mu)$ is a neutrosophic path. Then there's only one path amid two given vertices. So deletion of an edge makes the connectedness amid its vertices, to be incomputable. ■

Proposition 2.15.3. *Let $N = (\sigma, \mu)$ be a neutrosophic star. Then vital chromatic number aren't computable.*

Proof. Consider $N = (\sigma, \mu)$ is a neutrosophic star. Hence there's only one path amid two given vertices. Thus deletion of an edge makes the connectedness amid its vertices, to be incomputable. ■

2.16 Smallest Vital Chromatic Number

Proposition 2.16.1. *Let $N = (\sigma, \mu)$ be a neutrosophic empty. Then vital chromatic number is* 1.

Proof. Let $N = (\sigma, \mu)$ be a neutrosophic empty. Then there's no edge. It implies all vertices have same colors where the minimum number of colors are applied. Thus vital chromatic number is 1. ■

prp17b **Proposition 2.16.2.** *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is neither neutrosophic empty nor neutrosophic path. Then vital chromatic number isn't* 1.

Proof. Assume $N = (\sigma, \mu)$ is a neutrosophic graph which is neither neutrosophic empty nor neutrosophic path. By Proposition (2.13.4), there's at least one vital edge. ∎

Proposition 2.16.3. *Let $N = (\sigma, \mu)$ be a neutrosophic cycle. Then vital chromatic number is at least* 2 *and at most* 3.

Proof. Suppose $N = (\sigma, \mu)$ is a neutrosophic cycle. There's at least amid two vertices. By Proposition (2.13.1), all edges are vital. So at least the colors of two vertices are different. It implies vital chromatic number is at least 2. By applying colors on vertices in alternative ways, at most two vertices have common edges with same color. Hence vital chromatic number is at most 3. ∎

Proposition 2.16.4. *Let $N = (\sigma, \mu)$ be an even neutrosophic cycle. Then vital chromatic number is* 2.

Proof. Assume $N = (\sigma, \mu)$ is an even neutrosophic cycle. By Proposition (2.16.2), vital chromatic number is at least 2. By applying coloring on vertices in alternative ways, two vertices with common edge, has different colors. Since the cycle has even number of edges. Thus vital chromatic number is 2. ∎

Proposition 2.16.5. *Let $N = (\sigma, \mu)$ be an odd neutrosophic cycle. Then vital chromatic number is* 3.

Proof. Consider $N = (\sigma, \mu)$ is an odd neutrosophic cycle. By Proposition (2.13.1), all edges are vital. So by using coloring in alternative way, there are two vertices which have common edge and have same color. Thus vital chromatic number is 3. ∎

prp21b

Proposition 2.16.6. *Let $N = (\sigma, \mu)$ be a neutrosophic bipartite which is fixed-edge and complete. Then vital chromatic number is* 2.

Proof. Suppose $N = (\sigma, \mu)$ is a neutrosophic bipartite which is fixed-edge and complete. Thus strength of every path is as same as connectedness amid two vertices is. Thus all edges are vital. By $N = (\sigma, \mu)$ is complete, all vertices from one part are connected to all vertices of another part. Every part has no connection amid its vertices so all vertices from every part, have same color. There are two parts. Thus vital chromatic number is 2. ∎

Proposition 2.16.7. *Let $N = (\sigma, \mu)$ be a neutrosophic bipartite which is fixed-vertex and complete. Then vital chromatic number is* 2.

Proof. By $N = (\sigma, \mu)$ is fixed-vertex and complete, $N = (\sigma, \mu)$ is fixed-edge and complete. Therefore, by Proposition (2.16.6), vital chromatic number is 2. ∎

prp23b

Proposition 2.16.8. *Let $N = (\sigma, \mu)$ be a neutrosophic $t-$partite which is fixed-edge and complete. Then vital chromatic number is t.*

Proof. By $N = (\sigma, \mu)$ is fixed-edge, all edges have same value. Thus all paths have same strength. So connectedness amid two given vertices are same. Therefore all edges are vital. Inside every part, there's no edge amid two vertices. It induces the vertices of every part have same color. There are t parts. It implies t different colors are applied. Therefore vital chromatic number is t. ∎

Proposition 2.16.9. *Let $N = (\sigma, \mu)$ be a neutrosophic $t-$partite which is fixed-vertex and complete. Then vital chromatic number is t.*

Proof. It's fixed-vertex and complete. So It's fixed-edge and complete. By Proposition (2.16.8), vital chromatic number is t. ■

Proposition 2.16.10. *Let $N = (\sigma, \mu)$ be a neutrosophic wheel which is fixed-vertex and neutrosophic strong. Then vital chromatic number is 3 or 4.*

Proof. Consider $N = (\sigma, \mu)$ is a neutrosophic wheel which is fixed-vertex and neutrosophic strong. By it's fixed-vertex and neutrosophic strong, it's fixed-edge. Every edges have same value. So strength of paths and connectedness are same and equal to each other. Thus all edges are vital. Then the center has one color and since it's connected to all other vertices, the color of center is unique. Therefore, vital chromatic number is at least 2. Non-center vertices form a path which are colored by two colors when applying colors are in alternative ways. Thus vital chromatic number is 3 if the non-center vertices form even color and vital chromatic number is 4 if the non-center vertices form odd color. ■

Proposition 2.16.11. *Let $N = (\sigma, \mu)$ be a neutrosophic wheel which is fixed-edge and neutrosophic strong. Then vital chromatic number is 3 or 4.*

Proof. Consider $N = (\sigma, \mu)$ is a neutrosophic wheel which is fixed-vertex and neutrosophic strong. It's fixed-edge. Every edges have same value. So strength of paths and connectedness are same and equal to each other. Thus all edges are vital. Then the center has one color and since it's connected to all other vertices, the color of center is unique. Therefore, vital chromatic number is at least 2. Non-center vertices form a path which are colored by two colors when applying colors are in alternative ways. Thus vital chromatic number is 3 if the non-center vertices form even color and vital chromatic number is 4 if the non-center vertices form odd color. ■

2.17 n-Vital Chromatic Number

Proposition 2.17.1. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is neither neutrosophic empty nor neutrosophic path. Then n-vital chromatic number is at most order of N which is neutrosophic cardinality of V.*

Proof. Assume $N = (\sigma, \mu)$ is a neutrosophic graph which is neither neutrosophic empty nor neutrosophic path. If all edges are vital and all vertices are connected to each other, then vital chromatic number is n. Thus n-vital chromatic number is at most order of N which is neutrosophic cardinality of V. ■

2.18 Largest n-Vital Chromatic Number

Proposition 2.18.1. *Let $N = (\sigma, \mu)$ be a neutrosophic complete which is neither neutrosophic empty nor neutrosophic path. Then n-vital chromatic number is order of N which is neutrosophic cardinality of V.*

Proof. Suppose $N = (\sigma, \mu)$ is a neutrosophic complete which is neither neutrosophic empty nor neutrosophic path. By it's complete, then all vertices are connected to each other and all edges are vital. Thus n colors are used. It means n-vital chromatic number is order of N which is neutrosophic cardinality of V. ■

Proposition 2.18.2. *Let $N = (\sigma, \mu)$ be a neutrosophic path. Then n-vital chromatic number aren't computable.*

Proof. Deletion of one edge, make $N = (\sigma, \mu)$ be in the situation where n-vital chromatic number aren't computable. Since there's need to have at least two paths to compute n-vital chromatic number. In other words, this notion is computable in neutrosophic graph which has at least one cycle. ■

Proposition 2.18.3. *Let $N = (\sigma, \mu)$ be a neutrosophic star. Then n-vital chromatic number aren't computable.*

Proof. Assume $N = (\sigma, \mu)$ is a neutrosophic star. Then there's only one path amid two given vertices. Deletion one edge causes the connectedness to be incomputable. Thus n-vital chromatic number aren't computable. ■

2.19 Smallest n-Vital Chromatic Number

Proposition 2.19.1. *Let $N = (\sigma, \mu)$ be a neutrosophic empty. Then n-vital chromatic number is*

$$\min_{x \in V} \sigma(x).$$

Proof. Suppose $N = (\sigma, \mu)$ is a neutrosophic empty. Then there's no edge. It induces there's no vital edge. So all vertices are colored by one color. Hence all vertices have same color. It means the number of color is one. It induces the cardinality of set includes the representative of color is one. To find the representative of color, we have 1 choice from n options. Thus n-vital chromatic number is

$$\min_{x \in V} \sigma(x).$$

■

Proposition 2.19.2. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is neither neutrosophic empty nor neutrosophic path. Then n-vital chromatic number isn't*

$$\min_{x \in V} \sigma(x).$$

Proof. Consider $N = (\sigma, \mu)$ is a neutrosophic graph which is neither neutrosophic empty nor neutrosophic path. Then there's at least one edge. By Proposition (2.13.4), there's at least one vital edge. It induces the number of color is at least two. Therefore, the cardinality of set of representative is at least two. It implies n-vital chromatic number isn't

$$\min_{x \in V} \sigma(x).$$

■

Proposition 2.19.3. *Let $N = (\sigma, \mu)$ be a neutrosophic cycle. Then n-vital chromatic number is at least*

$$\min_{x,y \in V,\ xy \in E} \sigma(x) + \sigma(y).$$

And at most

$$\min_{x,y,z \in V, xy,yz,xz \in E} \sigma(x) + \sigma(y) + \sigma(z).$$

Proof. Suppose $N = (\sigma, \mu)$ is a neutrosophic cycle. By using alternative coloring of vertices, two or three numbers of colors are used. So the cardinality of set of representative is two or three. There are only these possibilities. Therefore n-vital chromatic number is at least

$$\min_{x,y \in V,\ xy \in E} \sigma(x) + \sigma(y).$$

And at most

$$\min_{x,y,z \in V, xy,yz,xz \in E} \sigma(x) + \sigma(y) + \sigma(z).$$

■

Proposition 2.19.4. *Let $N = (\sigma, \mu)$ be an even neutrosophic cycle. Then n-vital chromatic number is*

$$\min_{x,y \in V,\ xy \in E} \sigma(x) + \sigma(y).$$

Proof. Assume $N = (\sigma, \mu)$ is an even neutrosophic cycle. If colors are applied on vertices in alternative ways which cause two vertices with a common edge, have different colors, then by it's even neutrosophic cycle, the representatives of colors are two. Since there are even edges which by Proposition (2.13.1), all are vital. It induces the cardinality of set of representatives is two. Thus n-vital chromatic number is n-vital chromatic number is

$$\min_{x,y \in V,\ xy \in E} \sigma(x) + \sigma(y).$$

■

Proposition 2.19.5. *Let $N = (\sigma, \mu)$ be an odd neutrosophic cycle. Then n-vital chromatic number is*

$$\min_{x,y,z \in V,\ xy \in E} \sigma(x) + \sigma(y) + \sigma(z).$$

Proof. Consider $N = (\sigma, \mu)$ is an odd neutrosophic cycle. Then number of edges are odd. By Proposition (2.13.1), all edges are vital. Using different colors on the vertices which have common edges, implies usage of three colors. Hence the set of representatives has the cardinality three. To choose, the representatives, in every color, minimum value of vertices, introduces the representative of specific color. Then n-vital chromatic number is

$$\min_{x,y,z \in V,\ xy \in E} \sigma(x) + \sigma(y) + \sigma(z).$$

■

prp36b

Proposition 2.19.6. *Let $N = (\sigma, \mu)$ be neutrosophic bipartite which is fixed-edge and complete. Then n-vital chromatic number is*

$$\min_{x,y\in V,\ xy\in E} \sigma(x) + \sigma(y).$$

Proof. Assume $N = (\sigma, \mu)$ is neutrosophic bipartite which is fixed-edge and complete. It's fixed-edge so all edges have same value and as its consequences, all paths have same strength and all connectedness are same. Hence all edges are vital. By it's complete, all vertices from one part are connected to all vertices from another part. By it's bipartite, there are two colors to use on vertices such that every part has same color. So the set of representatives has the cardinality two which implies n-vital chromatic number is

$$\min_{x,y\in V,\ xy\in E} \sigma(x) + \sigma(y).$$

■

Proposition 2.19.7. *Let $N = (\sigma, \mu)$ be neutrosophic bipartite which is fixed-vertex and complete. Then n-vital chromatic number is*

$$\min_{x,y\in V,\ xy\in E} \sigma(x) + \sigma(y).$$

Proof. Assume $N = (\sigma, \mu)$ is neutrosophic bipartite which is fixed-vertex and complete. By it's fixed-vertex and complete, it's fixed-edge and complete. By Proposition (2.19.6), n-vital chromatic number is

$$\min_{x,y\in V,\ xy\in E} \sigma(x) + \sigma(y).$$

■

prp38b

Proposition 2.19.8. *Let $N = (\sigma, \mu)$ be neutrosophic $t-$partite which is fixed-edge and complete. Then n-vital chromatic number is*

$$\min_{x_1,x_2,\cdots,x_t\in V,\ x_ix_j\in E} \sigma(x_1) + \sigma(x_2) + \cdots + \sigma(x_t).$$

Proof. Assume $N = (\sigma, \mu)$ is neutrosophic $t-$partite which is fixed-edge and complete. All parts have same color on their vertices. By it's fixed-edge and applying Proposition (2.13.7), all edges are vital. Thus minimum number of colors is t. And the set of representatives has the cardinality t. It means n-vital chromatic number is

$$\min_{x_1,x_2,\cdots,x_t\in V,\ x_ix_j\in E} \sigma(x_1) + \sigma(x_2) + \cdots + \sigma(x_t).$$

■

Proposition 2.19.9. *Let $N = (\sigma, \mu)$ be neutrosophic $t-$partite which is fixed-vertex and complete. Then n-vital chromatic number is*

$$\min_{x_1,x_2,\cdots,x_t\in V,\ x_ix_j\in E} \sigma(x_1) + \sigma(x_2) + \cdots + \sigma(x_t).$$

Proof. Assume $N = (\sigma, \mu)$ is neutrosophic $t-$partite which is fixed-vertex and complete. Then by it's fixed-vertex and complete, it's it's fixed-edge and complete. By Proposition (2.19.8), n-vital chromatic number is

$$\min_{x_1, x_2, \cdots, x_t \in V,\ x_i x_j \in E} \sigma(x_1) + \sigma(x_2) + \cdots + \sigma(x_t).$$

∎

prp40b

Proposition 2.19.10. *Let $N = (\sigma, \mu)$ be neutrosophic wheel which is fixed-vertex and neutrosophic strong. Then n-vital chromatic number is*

$$\min_{y,z \in V, yz \in E} \sigma(c) + \sigma(y) + \sigma(z).$$

Or

$$\min_{y,z \in V, yz, zt \in E} \sigma(c) + \sigma(y) + \sigma(z) + \sigma(t).$$

Proof. Consider $N = (\sigma, \mu)$ is neutrosophic wheel which is fixed-vertex and neutrosophic strong. By fixed-vertex and neutrosophic strong, it's fixed-edge. By it's fixed-edge and applying Proposition (2.13.7), all edges are vital. Center is connected to non-center vertices. So center uses unique color. Non-center vertices form a cycle. If the cycle is even, then n-vital chromatic number is

$$\min_{y,z \in V, yz \in E} \sigma(c) + \sigma(y) + \sigma(z).$$

If it's odd, then n-vital chromatic number is

$$\min_{y,z \in V, yz \in E} \sigma(c) + \sigma(y) + \sigma(z).$$

Or

$$\min_{y,z \in V, yz, zt \in E} \sigma(c) + \sigma(y) + \sigma(z) + \sigma(t).$$

∎

Proposition 2.19.11. *Let $N = (\sigma, \mu)$ be neutrosophic wheel which is fixed-edge and neutrosophic strong. Then n-vital chromatic number is*

$$\min_{y,z \in V, yz \in E} \sigma(c) + \sigma(y) + \sigma(z).$$

Proof. Assume $N = (\sigma, \mu)$ is neutrosophic wheel which is fixed-edge and neutrosophic strong. By it's fixed-edge and neutrosophic strong, it's fixed-vertex and neutrosophic strong. By Proposition (2.19.10),

$$\min_{y,z \in V, yz \in E} \sigma(c) + \sigma(y) + \sigma(z).$$

Or

$$\min_{y,z \in V, yz, zt \in E} \sigma(c) + \sigma(y) + \sigma(z) + \sigma(t).$$

∎

The relation amid neutrosophic chromatic number and main parameters of neutrosophic graphs is computed.

Proposition 2.19.12. *Let $N = (\sigma, \mu)$ be a neutrosophic strong. Then vital chromatic number is at most $\Delta + 1$ and at least 2.*

Proof. Neutrosophic strong is neutrosophic nontrivial. So it isn't neutrosophic empty which induces there's no edge. It implies chromatic number is two. Since chromatic number is one if and only if $N = (\sigma, \mu)$ is neutrosophic empty if and only if $N = (\sigma, \mu)$ is neutrosophic trivial. A vertex with degree Δ, has Δ vertices which have common edges with them. If these vertices have no edge amid each other, then chromatic number is two especially, neutrosophic star. If not, then in the case, all vertices have edge amid each other, chromatic number is $\Delta + 1$, especially, neutrosophic complete. ■

Proposition 2.19.13. *Let $N = (\sigma, \mu)$ be a neutrosophic $r-$regular. Then vital chromatic number is at most $r + 1$.*

Proof. $N = (\sigma, \mu)$ is a neutrosophic $r-$regular. So any of vertex has r vertices which have common edge with it. If these vertices have no common edge with each other, for instance neutrosophic star, chromatic number is two. But since the vertices have common edge with each other, chromatic number is $r + 1$, for instance, neutrosophic complete. ■

2.20 Applications in Time Table and Scheduling

sec3b

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has important to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive section. Beyond that, sometimes sections are not the same.

Step 3. (Model) As Figure (2.4), the situation is designed as a model. The model uses data to assign every section and to assign to relation amid section, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relation amid them. Table (2.3), clarifies about the assigned numbers to these situation.

Table 2.3: Scheduling concerns its Subjects and its Connections as a Neutrosophic Graph in a Model. tbl1b

Sections of T	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9, s_{10}
Values	0.1	0.8	0.7	0.8	0.1	0.3	0.6	0.5	0.2
Connections of T	s_1s_2	s_2s_3	s_3s_4	s_4s_5	s_5s_6	s_6s_7	s_7s_8	s_8s_9	s_9s_{10}
Values	0.1	0.6	0.4	0.1	0.1	0.2	0.4	0.2	0.1

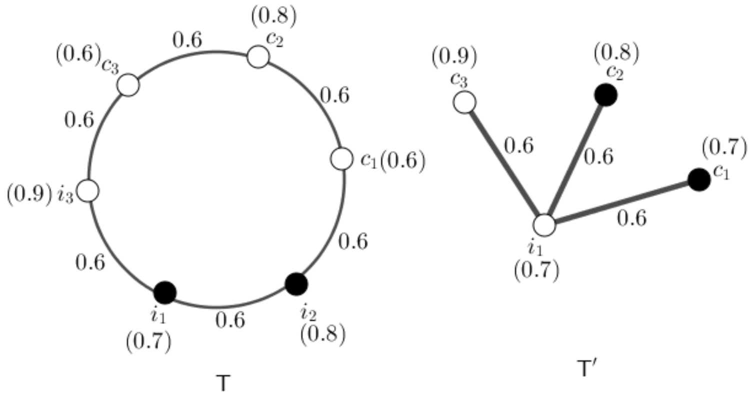

Figure 2.4: Black vertices are suspicions about choosing them.

fgr1b

Step 4. (Solution) As Figure (2.4) shows, neutrosophic model, proposes to use vital chromatic number which is incomputable in the case which is titled T'. In this case, i_1 and c_1 aren't representative of these two colors and n-vital chromatic number is incomputable. The set $\{i_1, c_1\}$ doesn't contain representatives of colors which pose vital chromatic number and n-vital chromatic number. Thus the decision amid choosing the subject c_1 an c_2 isn't concluded to choose c_1. To get brief overview, neutrosophic model uses one number for every array so 0.9 means $(0.9, 0.9, 0.9)$. In Figure (2.4), the neutrosophic model T introduces the common situation. The representatives of colors are i_2 and c_1. Thus vital chromatic number is two and n-vital chromatic number is 1.4. Thus suspicion about choosing i_1 and i_2 is determined to be i_2. The sets of representative for colors are $\{i_2, c_1\}$.

2.21 Open Problems

sec4b

The two notions of coloring of vertices concerning vital chromatic number and n-vital chromatic number are defined on neutrosophic graphs when connectedness and as its consequences, vital edges have key role to have these notions. Thus

Question 2.21.1. *Is it possible to use other types edges via connectedness to define vital chromatic number and n-vital chromatic number?*

Question 2.21.2. *Are existed some connections amid the coloring from connectedness inside this concept and external connections with other types of coloring from other notions?*

Question 2.21.3. *Is it possible to construct some classes neutrosophic graphs which have "nice" behavior?*

Question 2.21.4. *Which applications do make an independent study to apply vital chromatic number and n-vital chromatic number?*

Problem 2.21.5. *Which parameters are related to this parameter?*

Problem 2.21.6. *Which approaches do work to construct applications to create independent study?*

Problem 2.21.7. *Which approaches do work to construct definitions which use all three arrays and the relations amid them instead of one array of three arrays to create independent study?*

2.22 Conclusion and Closing Remarks

sec5b

This study uses mixed combinations of vital chromatic number and n-vital chromatic number to study on neutrosophic graphs. The connections of vertices which are clarified by vital edges from connectedness, differ them from each other and and put them in different categories to represent one representative for each color. Further studies could be about changes in the settings to compare this notion amid different settings of graph theory. One way is finding some relations amid array of vertices to make sensible definitions. In Table (2.4), some limitations and advantages of this study is pointed out.

Table 2.4: A Brief Overview about Advantages and Limitations of this study

tbl2b

Advantages	Limitations
1. Using connectedness for vital edges	1. Acyclic neutrosophic graphs
2. Using neutrosophic cardinality	
3. Using cardinality	2. Connections with parameters
4. Characterizing smallest number	
5. Characterizing biggest number	3. Star and path

2.23 New Ideas

New ideas are applied on this model to explore behaviors of these models in the mathematical perspective. Another ways to make sense about them, are used by relatively comparable results to conclude analysis.

Having different colors when two vertices have common "connection". Common connection can only be an edge. An edge with special attribute can be common "connection". Using neutrosophic attributes are expected to make sense about the study in this framework. In what follows, some definitions are introduced to be in the form of common "connection".

2.24 Different Types of Neutrosophic Chromatic Number

Third case for the contents is to use the article from [**2**]. The contents are used in the way that, usages of new contents are preferences and the preliminaries are passed in the beginning of this chapter.

2.25 Abstract

New setting is introduced to study chromatic number. Different types of chromatic numbers and neutrosophic chromatic number are proposed in this way, some results are obtained. Classes of neutrosophic graphs are used to obtains these numbers and the representatives of the colors. Using colors to assign to the vertices of neutrosophic graphs is applied. Some questions and problems are posed concerning ways to do further studies on this topic. Using different types of edges from connectedness in same neutrosophic graphs and in modified neutrosophic graphs to define the relation amid vertices which implies having different colors amid them and as consequences, choosing one vertex as a representative of each color to use them in a set of representatives and finally, using neutrosophic cardinality of this set to compute types of chromatic numbers. This specific relation amid edges is necessary to compute both types of chromatic number concerning the number of representative in the set of representatives and types of neutrosophic chromatic number concerning neutrosophic cardinality of set of representatives. If two vertices have no intended edge, then they can be assigned to same color even they've common edge. Basic familiarities with neutrosophic graph theory and graph theory are proposed for this article.

Keywords: Neutrosophic Connctedness, Neutrosophic Graphs, Chromatic

Number

AMS Subject Classification: 05C17, 05C22, 05E45

2.26 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 2.26.1. *Is it possible to use mixed versions of ideas concerning "connectedness", "neutrosophic graphs" and "neutrosophic coloring" to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Connections amid two items have key roles to assign colors. Thus they're used to define new ideas which conclude to the structure of coloring. The concept of having specific edge from connectedness inspires me to study the behavior of specific edge in the way that, both types of chromatic numbers and types of neutrosophic chromatic numbers are the cases of study.

The framework of this study is as follows. In the beginning, I introduced basic definitions to clarify about preliminaries. In section "New Ideas", new notion of coloring is applied to the vertices of neutrosophic graphs. Specific edge from connectedness has the key role in this way. Classes of neutrosophic graphs are studied in the terms of different types of edges in section "New Results". In section "Applications in Time Table and Scheduling", one application is posed for neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications are featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions are formed.

2.27 New Ideas

Question 2.27.1. *What-if the common "connection" is beyond having one common edge?*

The first step is the definition of common "connection".

Definition 2.27.2. Let $N = (\sigma, \mu)$ be a neutrosophic graph. A neutrosophic edge xy is called **type-I** if value of xy is **connectedness** which is a maximum strength of paths amid them.

Example 2.27.3. Consider Figure (2.5).

(i) : From n_1 to n_2, there's no edge which is type-I but n_2n_3.

(ii) : From n_2 to n_3, there's no edge which is type-I but n_2n_3.

(iii) : From n_1 to n_3, there's no edge which is type-I but n_1n_3.

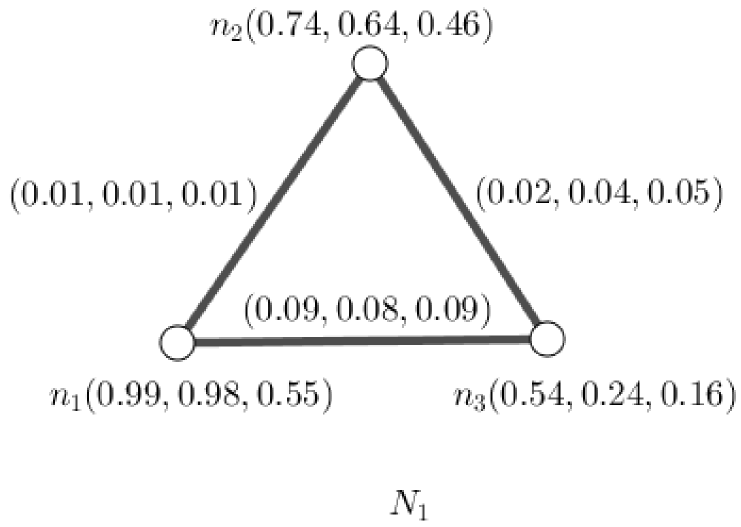

Figure 2.5: Two edges aren't type-I.

ncs1c2

There's a curious question.

Question 2.27.4. *Is there a neutrosophic graph whose edges are type-I?*

Yes but only one class. Two upcoming Propositions give simple answers about a class of neutrosophic graphs. Other classes of neutrosophic graphs have at least one edge which isn't type-I.

Proposition 2.27.5. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is fixed-edge. Then all edges are type-I.*

Proposition 2.27.6. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is strong fixed-vertex. Then $N = (\sigma, \mu)$ is fixed-edge.*

Proposition 2.27.7. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is strong fixed-vertex. Then all edges are type-I.*

Example 2.27.8. Consider Figure (2.6). All edges are type-I.

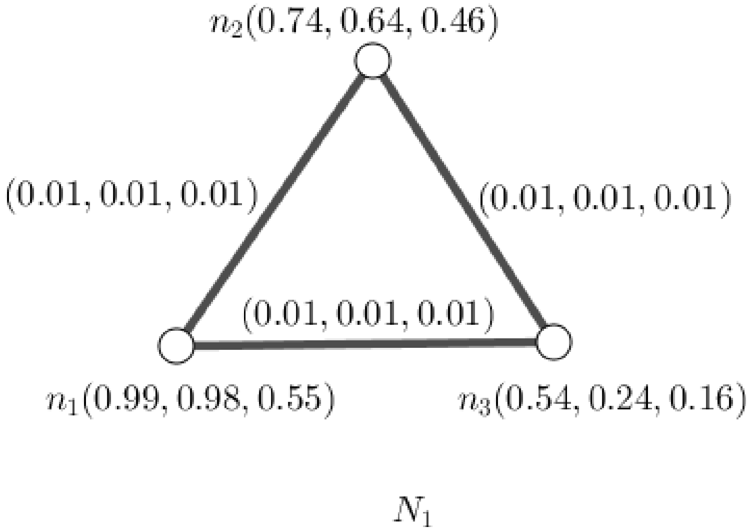

Figure 2.6: Neutrosophic graph which is fixed-edge but not strong fixed-vertex. ncs2c2

Definition 2.27.9. Let $N = (\sigma, \mu)$ be a neutrosophic graph. A neutrosophic edge xy is called **type-II** if value of xy is lower than **connectedness** which is a maximum strength of paths amid them.

Example 2.27.10. The comparison amid the variant of edges which are either type-I or type-II, is possible when common neutrosophic graphs are studied.

(a) : Consider Figure (2.5).

- (i) : From n_1 to n_2, there's no edge which is type-II but n_1n_2.
- (ii) : From n_2 to n_3, there's no edge which is type-II but n_1n_2.
- (iii) : From n_1 to n_3, there's no edge which is type-II but n_1n_2 and n_2n_3.

(b) : Consider Figure (2.6). There's no edge which is type-II.

Definition 2.27.11. Let $N = (\sigma, \mu)$ be a neutrosophic graph. A neutrosophic edge xy is called **type-III** if value of xy is the only value which is **connectedness** which is a maximum strength of paths amid them.

Example 2.27.12. The comparison amid the variant of edges which are either type-I or type-II or type-III, is possible when common neutrosophic graphs are studied.

(a) : Consider Figure (2.5).

(i) : From n_1 to n_2, there's no edge which is type-III but n_2n_3.
(ii) : From n_2 to n_3, there's no edge which is type-III but n_2n_3.
(iii) : From n_1 to n_3, there's no edge which is type-III but n_1n_3 and n_2n_3.

(b) : Consider Figure (2.6). There's no edge which is type-III.

Definition 2.27.13. Let $N = (\sigma, \mu)$ be a neutrosophic graph. A neutrosophic edge xy is called **type-IV** if value of xy is **connectedness** which is a maximum strength of paths amid them but in $N = (\sigma, \mu)$ doesn't have xy.

Example 2.27.14. The comparison amid the variant of edges which are either type-I or...or type-IV, is possible when common neutrosophic graphs are studied.

(a) : Consider Figure (2.5).

(i) : From n_1 to n_2, there's no edge which is type-IV.
(ii) : From n_2 to n_3, there's no edge which is type-IV.
(iii) : From n_1 to n_3, there's no edge which is type-IV.

(b) : Consider Figure (2.6). All edges are type-IV.

Definition 2.27.15. Let $N = (\sigma, \mu)$ be a neutrosophic graph. A neutrosophic edge xy is called **type-V** if value of xy is lower than **connectedness** which is a maximum strength of paths amid them but in $N = (\sigma, \mu)$ doesn't have xy.

Example 2.27.16. The comparison amid the variant of edges which are either type-I or...or type-V, is possible when common neutrosophic graphs are studied.

(a) : Consider Figure (2.5).

(i) : From n_1 to n_2, edge n_1n_2 is type-V.
(ii) : From n_2 to n_3, there's no edge which is type-V.
(iii) : From n_1 to n_3, there's no edge which is type-V.

(b) : Consider Figure (2.6). There's no edge which is type-V.

Definition 2.27.17. Let $N = (\sigma, \mu)$ be a neutrosophic graph. A neutrosophic edge xy is called **type-VI** if value of xy is greater than **connectedness** which is a maximum strength of paths amid them but in $N = (\sigma, \mu)$ doesn't have xy.

Example 2.27.18. The comparison amid the variant of edges which are either type-I or...or type-VI, is possible when common neutrosophic graphs are studied.

(a) : Consider Figure (2.5).

(i) : From n_1 to n_2, there's no edge which is type-VI.
(ii) : From n_2 to n_3, edges n_2n_3 and n_1n_3 are type-VI.
(iii) : From n_1 to n_3, edges n_2n_3 and n_1n_3 are type-VI.

(b) : Consider Figure (2.6). There's no edge which is type-VI.

Definition 2.27.19. Let $N = (\sigma, \mu)$ be a neutrosophic graph. A neutrosophic edge xy is called **type-VII** if value of xy is the only value which is **connectedness** which is a maximum strength of paths amid them but in $N = (\sigma, \mu)$ doesn't have xy.

Example 2.27.20. The comparison amid the variant of edges which are either type-I or...or type-VII, is possible when common neutrosophic graphs are studied.

(a) : Consider Figure (2.5).

- (i) : From n_1 to n_2, there's no edge which is type-VII.
- (ii) : From n_2 to n_3, there's no edge which is type-VII.
- (iii) : From n_1 to n_3, there's no edge which is type-VII.

(b) : Consider Figure (2.6). There's no edge which is type-VII.

Common way to define the number, could be twofold. One is about the cardinality and another is about neutrosophic cardinality.

Definition 2.27.21. Let $N = (\sigma, \mu)$ be a neutrosophic graph. A vertex which has common type edge with another vertex, has assigned different color from that vertex. The cardinality of the set of representatives of colors, is called **type chromatic number** and its neutrosophic cardinality concerning the set of representatives of colors is called **n-type chromatic number**.

Definition 2.27.22. It's worthy to note that there are two types of definitions. One is about the comparison amid edges and connectedness. Another is about one edge when it's deleted, new connectedness is compared to deleted edge. Thus in first type, all edges are compared to connectedness but in second type, for every edge, there's a computation to have connectedness. So in first type, connectedness is unique and there's one number for all edges as connectedness but in second type, for every edge, there's a new connectedness to decide about the edge whether has intended attribute or not. To avoid confusion, chromatic number is computed with respect to n_1 and n_2 where second style is used and all edges are labelled even they're not deleted edges so **third type** is introduced when deletion of one edge, is enough to label all edges. Also first order is used to have these concepts.

In following example, third type of definitions which are except from type-IV,V,VI,VII, are studied.

Example 2.27.23. The comparison amid the variant of numbers which are either type-I or...or type-VII, is possible when common neutrosophic graphs are studied. Chromatic number is computed with respect to n_1 and n_2. Also first order is used to have these concepts.

(a) : Consider Figure (2.5).

- (i) : The set of representatives of colors is $\{n_1, n_2\}$. Thus type-I chromatic number is 2 and n-type-I chromatic number is 1.73.
- (ii) : The set of representatives of colors is $\{n_1, n_2\}$. Thus type-II chromatic number is 2 and n-type-II chromatic number is 1.73.
- (iii) : The set of representatives of colors is $\{n_2, n_3\}$. Thus type-III chromatic number is 2 and n-type-III chromatic number is 1.28.
- (iv) : The set of representatives of colors is $\{n_2, n_3\}$. Thus type-IV chromatic number is 2 and n-type-IV chromatic number is 1.28.

- (v) : The set of representatives of colors is $\{n_1, n_2\}$. Thus type-V chromatic number is 2 and n-type-V chromatic number is 1.73.
- (vi) : The set of representatives of colors is $\{n_2, n_3\}$. Thus type-VI chromatic number is 2 and n-type-VI chromatic number is 1.28.
- (vii) : The set of representatives of colors is $\{n_2, n_3\}$. Thus type-VII chromatic number is 2 and n-type-VII chromatic number is 1.28.

(b) : Consider Figure (2.6).

- (i) : The set of representatives of colors is $\{n_1, n_2, n_3\}$. Thus type-I chromatic number is 3 and n-type-I chromatic number is 3.01.
- (ii) : The set of representatives of colors is $\{\}$. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.
- (iii) : The set of representatives of colors is $\{\}$. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.
- (iv) : The set of representatives of colors is $\{\}$. Thus type-IV chromatic number is 0 and n-type-IV chromatic number is 0.
- (v) : The set of representatives of colors is $\{\}$. Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.
- (vi) : The set of representatives of colors is $\{\}$. Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.
- (vii) : The set of representatives of colors is $\{\}$. Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

2.28 New Results

In this chapter, I introduce some results concerning new ideas and in this ways, the results make sense more about their impacts on different models.

2.29 Different Types of Neutrosophic Chromatic Number

Third case for the contents is to use the article from [**Ref12**]. The contents are used in the way that, usages of new contents are preferences and the preliminaries are passed in the beginning of this chapter.

2.30 New Results

Proposition 2.30.1. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is complete. If it's fixed-edge, then*

- (i) : *The set of representatives of colors is $\{v_1, v_2, \cdots, v_n\}$. Thus type-I chromatic number is n and n-type-I chromatic number is neutrosophic cardinality of V.*
- (ii) : *The set of representatives of colors is $\{\}$. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.*
- (iii) : *The set of representatives of colors is $\{\}$. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.*
- (iv) : *The set of representatives of colors is $\{v_1, v_2, \cdots, v_n\}$. Thus type-IV chromatic number is n and n-type-IV chromatic number is neutrosophic cardinality of V.*

(v) : *The set of representatives of colors is* $\{\}$. *Thus type-V chromatic number is* 0 *and n-type-V chromatic number is* 0.

(vi) : *The set of representatives of colors is* $\{\}$. *Thus type-VI chromatic number is* 0 *and n-type-VI chromatic number is* 0.

(vii) : *The set of representatives of colors is* $\{\}$. *Thus type-VII chromatic number is* 0 *and n-type-VII chromatic number is* 0.

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it's neutrosophic complete, every vertex has $n-1$ vertices which have common edges which are type-I. Thus the set of representatives of colors is $\{v_1, v_2, \cdots, v_n\}$. The type-I chromatic number is n and n-type-I chromatic number is neutrosophic cardinality of V.
(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-II. By it's neutrosophic complete, every vertex has $n-1$ vertices which have common edges which aren't type-II. Thus the set of representatives of colors is $\{\}$. The type-II chromatic number is 0 and n-type-II chromatic number is 0.
(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's neutrosophic complete, every vertex has $n-1$ vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it's neutrosophic complete, every vertex has $n-1$ vertices which have common edges which are type-IV. Thus the set of representatives of colors is $\{v_1, v_2, \cdots, v_n\}$. The type-IV chromatic number is n and n-type-IV chromatic number is neutrosophic cardinality of V.
(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-V. By it's neutrosophic complete, every vertex has $n-1$ vertices which have common edges which aren't type-V. Thus the set of representatives of colors is $\{\}$. The type-V chromatic number is 0 and n-type-V chromatic number is 0.
(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VI. By it's neutrosophic complete, every vertex has $n-1$ vertices which have common edges which aren't type-VI. Thus the set of representatives of colors is $\{\}$. The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.
(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VII. By it's neutrosophic complete, every vertex has $n-1$ vertices which have common edges which aren't type-VII. Thus the set of representatives of colors is $\{\}$. The type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

■

Proposition 2.30.2. *Let* $N = (\sigma, \mu)$ *be a neutrosophic graph which is complete. If it's fixed-vertex, then*

(i) : *The set of representatives of colors is* $\{v_1, v_2, \cdots, v_n\}$. *Thus type-I chromatic number is* n *and n-type-I chromatic number is* $n\sigma(v_i)$.

(*ii*) : *The set of representatives of colors is* $\{\}$. *Thus type-II chromatic number is* 0 *and n-type-II chromatic number is* 0.

(*iii*) : *The set of representatives of colors is* $\{\}$. *Thus type-III chromatic number is* 0 *and n-type-III chromatic number is* 0.

(*iv*) : *The set of representatives of colors is* $\{v_1, v_2, \cdots, v_n\}$. *Thus type-IV chromatic number is* n *and n-type-IV chromatic number is* $n\sigma(v_i)$.

(*v*) : *The set of representatives of colors is* $\{\}$. *Thus type-V chromatic number is* 0 *and n-type-V chromatic number is* 0.

(*vi*) : *The set of representatives of colors is* $\{\}$. *Thus type-VI chromatic number is* 0 *and n-type-VI chromatic number is* 0.

(*vii*) : *The set of representatives of colors is* $\{\}$. *Thus type-VII chromatic number is* 0 *and n-type-VII chromatic number is* 0.

Proof. (i). By it's fixed-vertex and it's neutrosophic complete, all edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it's neutrosophic complete, every vertex has $n-1$ vertices which have common edges which are type-I. Thus the set of representatives of colors is $\{v_1, v_2, \cdots, v_n\}$. The type-I chromatic number is n and n-type-I chromatic number is $n\sigma(v_i)$.
(ii). By it's fixed-vertex and it's neutrosophic complete, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-II. By it's neutrosophic complete, every vertex has $n-1$ vertices which have common edges which aren't type-II. Thus the set of representatives of colors is $\{\}$. The type-II chromatic number is 0 and n-type-II chromatic number is 0.
(iii). By it's fixed-vertex and it's neutrosophic complete, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's neutrosophic complete, every vertex has $n-1$ vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(iv). By it's fixed-vertex and it's neutrosophic complete, all edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it's neutrosophic complete, every vertex has $n-1$ vertices which have common edges which are type-IV. Thus the set of representatives of colors is $\{v_1, v_2, \cdots, v_n\}$. The type-IV chromatic number is n and n-type-IV chromatic number is $n\sigma(v_i)$.
(v). By it's fixed-vertex and it's neutrosophic complete, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-V. By it's neutrosophic complete, every vertex has $n-1$ vertices which have common edges which aren't type-V. Thus the set of representatives of colors is $\{\}$. The type-V chromatic number is 0 and n-type-V chromatic number is 0.
(vi). By it's fixed-vertex and it's neutrosophic complete, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VI. By it's neutrosophic complete, every vertex has $n-1$ vertices which have common edges which aren't type-VI. Thus the set of representatives

of colors is $\{\}$. The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.
(vii). By it's fixed-vertex and it's neutrosophic complete, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VII. By it's neutrosophic complete, every vertex has $n-1$ vertices which have common edges which aren't type-VII. Thus the set of representatives of colors is $\{\}$. The type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

■

Proposition 2.30.3. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is strong. If it's fixed-edge, then*

(i) : *The set of representatives of colors is $\{v_1, v_2, \cdots, v_t\}$ where $t = \Delta(N)$. Thus type-I chromatic number is t and n-type-I chromatic number is neutrosophic cardinality of $\{v_1, v_2, \cdots, v_t\}$.*

(ii) : *The set of representatives of colors is $\{\}$. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.*

(iii) : *The set of representatives of colors is $\{\}$. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.*

(iv) : *The set of representatives of colors is $\{v_1, v_2, \cdots, v_t\}$ where $t = \Delta(N)$. Thus type-IV chromatic number is t and n-type-IV chromatic number is neutrosophic cardinality of $\{v_1, v_2, \cdots, v_t\}$.*

(v) : *The set of representatives of colors is $\{\}$. Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.*

(vi) : *The set of representatives of colors is $\{\}$. Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.*

(vii) : *The set of representatives of colors is $\{\}$. Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.*

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it's neutrosophic strong, there's a vertex has $t = \Delta(N)$ vertices which have common edges which are type-I. Thus the set of representatives of colors is $\{v_1, v_2, \cdots, v_t\}$. The type-I chromatic number is t and n-type-I chromatic number is neutrosophic cardinality of $\{v_1, v_2, \cdots, v_t\}$.
(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-II. By it's neutrosophic strong, there's a vertex has $t = \Delta(N)$ vertices which have common edges which aren't type-II. Thus the set of representatives of colors is $\{\}$. The type-II chromatic number is 0 and n-type-II chromatic number is 0.
(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's neutrosophic strong, there's a vertex has $t = \Delta(N)$ vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(iv). All edges have same amount so the connectedness amid two given edges is

the same. All edges are type-IV. By it's neutrosophic strong, there's a vertex has $t = \Delta(N)$ vertices which have common edges which are type-IV. Thus the set of representatives of colors is $\{v_1, v_2, \cdots, v_t\}$. The type-IV chromatic number is t and n-type-IV chromatic number is neutrosophic cardinality of $\{v_1, v_2, \cdots, v_t\}$.
(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-V. By it's neutrosophic strong, there's a vertex has $t = \Delta(N)$ vertices which have common edges which aren't type-V. Thus the set of representatives of colors is $\{\}$. The type-V chromatic number is 0 and n-type-V chromatic number is 0.
(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VI. By it's neutrosophic strong, there's a vertex has $t = \Delta(N)$ vertices which have common edges which aren't type-VI. Thus the set of representatives of colors is $\{\}$. The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.
(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VII. By it's neutrosophic strong, there's a vertex has $t = \Delta(N)$ vertices which have common edges which aren't type-VII. Thus the set of representatives of colors is $\{\}$. The type-VII chromatic number is 0 and n-type-VII chromatic number is 0. ■

Proposition 2.30.4. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is strong. If it's fixed-vertex, then*

(i) : *The set of representatives of colors is $\{v_1, v_2, \cdots, v_t\}$ where $t = \Delta(N)$. Thus type-I chromatic number is t and n-type-I chromatic number is $t\sigma(v_i)$.*

(ii) : *The set of representatives of colors is $\{\}$. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.*

(iii) : *The set of representatives of colors is $\{\}$. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.*

(iv) : *The set of representatives of colors is $\{v_1, v_2, \cdots, v_t\}$ where $t = \Delta(N)$. Thus type-IV chromatic number is t and n-type-IV chromatic number is $t\sigma(v_i)$.*

(v) : *The set of representatives of colors is $\{\}$. Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.*

(vi) : *The set of representatives of colors is $\{\}$. Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.*

(vii) : *The set of representatives of colors is $\{\}$. Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.*

Proof. (i). By it's fixed-vertex and it's neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it's neutrosophic strong, there's a vertex has $t = \Delta(N)$ vertices which have common edges which are type-I. Thus the set of representatives of colors is $\{v_1, v_2, \cdots, v_t\}$. The type-I chromatic number is t and n-type-I chromatic number is $t\sigma(v_i)$.
(ii). By it's fixed-vertex and it's neutrosophic strong, all edges have same amount

so the connectedness amid two given edges is the same. All edges aren't type-II. By it's neutrosophic strong, there's a vertex has $t = \Delta(N)$ vertices which have common edges which aren't type-II. Thus the set of representatives of colors is $\{\}$. The type-II chromatic number is 0 and n-type-II chromatic number is 0.
(iii). By it's fixed-vertex and it's neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's neutrosophic strong, there's a vertex has $t = \Delta(N)$ vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(iv). By it's fixed-vertex and it's neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it's neutrosophic strong, there's a vertex has $t = \Delta(N)$ vertices which have common edges which are type-IV. Thus the set of representatives of colors is $\{v_1, v_2, \cdots, v_t\}$. The type-IV chromatic number is t and n-type-IV chromatic number is $t\sigma(v_i)$.
(v). By it's fixed-vertex and it's neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-V. By it's neutrosophic strong, there's a vertex has $t = \Delta(N)$ vertices which have common edges which aren't type-V. Thus the set of representatives of colors is $\{\}$. The type-V chromatic number is 0 and n-type-V chromatic number is 0.
(vi). By it's fixed-vertex and it's neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VI. By it's neutrosophic strong, there's a vertex has $t = \Delta(N)$ vertices which have common edges which aren't type-VI. Thus the set of representatives of colors is $\{\}$. The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.
(vii). By it's fixed-vertex and it's neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VII. By it's neutrosophic strong, there's a vertex has $t = \Delta(N)$ vertices which have common edges which aren't type-VII. Thus the set of representatives of colors is $\{\}$. The type-VII chromatic number is 0 and n-type-VII chromatic number is 0. ■

Proposition 2.30.5. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is strong and path. If it's fixed-edge, then*

(i) : *The set of representatives of colors is $\{v_i, v_j\}$. Thus type-I chromatic number is 2 and n-type-I chromatic number is $\sigma(v_i) + \sigma(v_j)$.*

(ii) : *The set of representatives of colors is $\{\}$. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.*

(iii) : *The set of representatives of colors is $\{\}$. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.*

(iv) : *The set of representatives of colors, type-IV chromatic number and n-type-IV chromatic number aren't defined.*

(v) : *The set of representatives of colors, type-V chromatic number and n-type-V chromatic number aren't defined.*

(vi) : *The set of representatives of colors, type-VI chromatic number and n-type-VI chromatic number aren't defined.*

(vii) : *The set of representatives of colors, type-VII chromatic number and n-type-VII chromatic number aren't defined.*

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it's neutrosophic strong, there's a vertex has 2 vertices which have common edges which are type-I. Thus the set of representatives of colors is $\{v_i, v_j\}$. The type-I chromatic number is 2 and n-type-I chromatic number is neutrosophic cardinality of $\{v_i, v_j\}$.
(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-II. By it's neutrosophic strong, there's a vertex has 2 vertices which have common edges which aren't type-II. Thus the set of representatives of colors is $\{\}$. The type-II chromatic number is 0 and n-type-II chromatic number is 0.
(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's neutrosophic strong, there's a vertex has 2 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-IV. Since it's impossible to define when there's no cycle in neutrosophic graph.
(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-V. Since it's impossible to define when there's no cycle in neutrosophic graph.
(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VI. Since it's impossible to define when there's no cycle in neutrosophic graph.
(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VII. Since it's impossible to define when there's no cycle in neutrosophic graph. ■

Proposition 2.30.6. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is strong and path. If it's fixed-vertex, then*

(i) : *The set of representatives of colors is $\{v_i, v_j\}$. Thus type-I chromatic number is 2 and n-type-I chromatic number is $2\sigma(v_i)$.*

(ii) : *The set of representatives of colors is $\{\}$. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.*

(iii) : *The set of representatives of colors is $\{\}$. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.*

(iv) : *The set of representatives of colors, type-IV chromatic number and n-type-IV chromatic number aren't defined.*

(v) : *The set of representatives of colors, type-V chromatic number and n-type-V chromatic number aren't defined.*

(vi) : *The set of representatives of colors, type-VI chromatic number and n-type-VI chromatic number aren't defined.*

(vii) : *The set of representatives of colors, type-VII chromatic number and n-type-VII chromatic number aren't defined.*

Proof. (i). By it's fixed-vertex and it's neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it's neutrosophic strong, there's a vertex has 2 vertices which have common edges which are type-I. Thus the set of representatives of colors is $\{v_i, v_j\}$. The type-I chromatic number is 2 and n-type-I chromatic number is $2\sigma(v_i)$.
(ii). By it's fixed-vertex and it's neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-II. By it's neutrosophic strong, there's a vertex has 2 vertices which have common edges which aren't type-II. Thus the set of representatives of colors is $\{\}$. The type-II chromatic number is 0 and n-type-II chromatic number is 0.
(iii). By it's fixed-vertex and it's neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's neutrosophic strong, there's a vertex has 2 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(iv). By it's fixed-vertex and it's neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-IV. Since it's impossible to define when there's no cycle in neutrosophic graph.
(v). By it's fixed-vertex and it's neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-V. Since it's impossible to define when there's no cycle in neutrosophic graph.
(vi). By it's fixed-vertex and it's neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VI. Since it's impossible to define when there's no cycle in neutrosophic graph.
(vii). By it's fixed-vertex and it's neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VII. Since it's impossible to define when there's no cycle in neutrosophic graph. ■

Proposition 2.30.7. *Let $N = (\sigma, \mu)$ be an even cycle. If it's fixed-edge, then*

(i) : *The set of representatives of colors is $\{v_i, v_j\}$. Thus type-I chromatic number is 2 and n-type-I chromatic number is $\sigma(v_i) + \sigma(v_j)$.*

(ii) : *The set of representatives of colors is $\{\}$. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.*

(iii) : *The set of representatives of colors is $\{\}$. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.*

(iv) : *The set of representatives of colors is $\{v_i, v_j\}$. Thus type-IV chromatic number is 2 and n-type-IV chromatic number is $\sigma(v_i) + \sigma(v_j)$.*

(v) : *The set of representatives of colors is* $\{\}$. *Thus type-V chromatic number is* 0 *and n-type-V chromatic number is* 0.

(vi) : *The set of representatives of colors is* $\{\}$. *Thus type-VI chromatic number is* 0 *and n-type-VI chromatic number is* 0.

(vii) : *The set of representatives of colors is* $\{\}$. *Thus type-VII chromatic number is* 0 *and n-type-VII chromatic number is* 0.

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it's cycle, all vertices have 2 vertices which have common edges which are type-I. Thus the set of representatives of colors is $\{v_i, v_j\}$. The type-I chromatic number is 2 and n-type-I chromatic number is neutrosophic cardinality of $\{v_i, v_j\}$.
(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-II. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-II. Thus the set of representatives of colors is $\{\}$. The type-II chromatic number is 0 and n-type-II chromatic number is 0.
(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it's cycle, all vertices have 2 vertices which have common edges which are type-IV. By deletion of one edge, it's possible to compute connectedness. Thus the set of representatives of colors is $\{v_i, v_j\}$. The type-IV chromatic number is 2 and n-type-IV chromatic number is neutrosophic cardinality of $\{v_i, v_j\}$.
(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0. ■

Proposition 2.30.8. *Let* $N = (\sigma, \mu)$ *be a neutrosophic graph which is strong and even cycle. If it's fixed-vertex, then*

(i) : *The set of representatives of colors is* $\{v_i, v_j\}$. *Thus type-I chromatic number is* 2 *and n-type-I chromatic number is* $2\sigma(v_i)$.

(ii) : *The set of representatives of colors is* $\{\}$. *Thus type-II chromatic number is* 0 *and n-type-II chromatic number is* 0.

(iii) : *The set of representatives of colors is* $\{\}$. *Thus type-III chromatic number is* 0 *and n-type-III chromatic number is* 0.

(iv) : *The set of representatives of colors is* $\{v_i, v_j\}$. *Thus type-IV chromatic number is* 2 *and n-type-IV chromatic number is* $2\sigma(v_i)$.

(v) : *The set of representatives of colors is* $\{\}$. *Thus type-V chromatic number is* 0 *and n-type-V chromatic number is* 0.

(vi) : *The set of representatives of colors is* $\{\}$. *Thus type-VI chromatic number is* 0 *and n-type-VI chromatic number is* 0.

(vii) : *The set of representatives of colors is* $\{\}$. *Thus type-VII chromatic number is* 0 *and n-type-VII chromatic number is* 0.

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it's cycle, all vertices have 2 vertices which have common edges which are type-I. Thus the set of representatives of colors is $\{v_i, v_j\}$. The type-I chromatic number is 2 and n-type-I chromatic number is neutrosophic cardinality of $\{v_i, v_j\}$ which is $2\sigma(v_i)$.
(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-II. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-II. Thus the set of representatives of colors is $\{\}$. The type-II chromatic number is 0 and n-type-II chromatic number is 0.
(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it's cycle, all vertices have 2 vertices which have common edges which are type-IV. By deletion of one edge, it's possible to compute connectedness. Thus the set of representatives of colors is $\{v_i, v_j\}$. The type-IV chromatic number is 2 and n-type-IV chromatic number is neutrosophic cardinality of $\{v_i, v_j\}$ which is $2\sigma(v_i)$.
(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-III. Thus the set of representatives

of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0. ■

Proposition 2.30.9. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is an odd cycle. If it's fixed-edge, then*

(i) : *The set of representatives of colors is $\{v_i, v_j, v_k\}$. Thus type-I chromatic number is 2 and n-type-I chromatic number is $\sigma(v_i) + \sigma(v_j) + \sigma(v_k)$.*

(ii) : *The set of representatives of colors is $\{\}$. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.*

(iii) : *The set of representatives of colors is $\{\}$. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.*

(iv) : *The set of representatives of colors is $\{v_i, v_j, v_k\}$. Thus type-IV chromatic number is 2 and n-type-IV chromatic number is $\sigma(v_i) + \sigma(v_j) + \sigma(v_k)$.*

(v) : *The set of representatives of colors is $\{\}$. Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.*

(vi) : *The set of representatives of colors is $\{\}$. Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.*

(vii) : *The set of representatives of colors is $\{\}$. Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.*

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it's cycle, all vertices have 2 vertices which have common edges which are type-I. Thus the set of representatives of colors is $\{v_i, v_j\}$. The type-I chromatic number is 2 and n-type-I chromatic number is neutrosophic cardinality of $\{v_i, v_j, v_k\}$ which is $\sigma(v_i) + \sigma(v_j) + \sigma(v_k)$.
(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-II. By it's neutrosophic strong, there's a vertex has 2 vertices which have common edges which aren't type-II. Thus the set of representatives of colors is $\{\}$. The type-II chromatic number is 0 and n-type-II chromatic number is 0.
(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's neutrosophic strong, there's a vertex has 2 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it's cycle, all vertices have 2 vertices which have common edges which are type-IV. Thus the set of representatives of colors is $\{v_i, v_j\}$. The type-IV chromatic number is 2 and n-type-IV chromatic number is neutrosophic cardinality of $\{v_i, v_j, v_k\}$ which is $\sigma(v_i) + \sigma(v_j) + \sigma(v_k)$.
(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-V. By it's neutrosophic strong, there's a vertex has 2 vertices which have common edges which aren't type-V. Thus the set of representatives of colors is $\{\}$. The type-V chromatic number is 0 and n-type-V chromatic number is 0.
(vi). All edges have same amount so the connectedness amid two given edges is

the same. All edges aren't type-VI. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-VI. Thus the set of representatives of colors is $\{\}$. The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.
(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VII. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-VII. Thus the set of representatives of colors is $\{\}$. The type-VII chromatic number is 0 and n-type-VII chromatic number is 0. ■

Proposition 2.30.10. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is strong and odd cycle. If it's fixed-vertex, then*

(i) : *The set of representatives of colors is $\{v_i, v_j, v_k\}$. Thus type-I chromatic number is 2 and n-type-I chromatic number is $3\sigma(v_i)$.*

(ii) : *The set of representatives of colors is $\{\}$. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.*

(iii) : *The set of representatives of colors is $\{\}$. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.*

(iv) : *The set of representatives of colors is $\{v_i, v_j, v_k\}$. Thus type-IV chromatic number is 2 and n-type-IV chromatic number is $3\sigma(v_i)$.*

(v) : *The set of representatives of colors is $\{\}$. Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.*

(vi) : *The set of representatives of colors is $\{\}$. Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.*

(vii) : *The set of representatives of colors is $\{\}$. Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.*

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it's cycle, all vertices have 2 vertices which have common edges which are type-I. Thus the set of representatives of colors is $\{v_i, v_j\}$. The type-I chromatic number is 2 and n-type-I chromatic number is neutrosophic cardinality of $\{v_i, v_j, v_k\}$ which is $3\sigma(v_i)$.
(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-II. By it's neutrosophic strong, there's a vertex has 2 vertices which have common edges which aren't type-II. Thus the set of representatives of colors is $\{\}$. The type-II chromatic number is 0 and n-type-II chromatic number is 0.
(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's neutrosophic strong, there's a vertex has 2 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it's cycle, all vertices have 2 vertices which have common edges which are type-IV. Thus the set of representatives of colors is $\{v_i, v_j\}$. The type-IV chromatic number is 2 and n-type-IV chromatic

number is neutrosophic cardinality of $\{v_i, v_j, v_k\}$ which is $3\sigma(v_i)$.
(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-V. By it's neutrosophic strong, there's a vertex has 2 vertices which have common edges which aren't type-V. Thus the set of representatives of colors is $\{\}$. The type-V chromatic number is 0 and n-type-V chromatic number is 0.
(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VI. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-VI. Thus the set of representatives of colors is $\{\}$. The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.
(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VII. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-VII. Thus the set of representatives of colors is $\{\}$. The type-VII chromatic number is 0 and n-type-VII chromatic number is 0. ■

Proposition 2.30.11. *Let $N = (\sigma, \mu)$ be an even wheel. If it's fixed-edge, then*

(i) : *The set of representatives of colors is $\{v_i, v_j, v_k\}$. Thus type-I chromatic number is 2 and n-type-I chromatic number is $\sigma(v_i) + \sigma(v_j) + \sigma(v_k)$.*

(ii) : *The set of representatives of colors is $\{\}$. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.*

(iii) : *The set of representatives of colors is $\{\}$. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.*

(iv) : *The set of representatives of colors is $\{v_i, v_j, v_k\}$. Thus type-IV chromatic number is 3 and n-type-IV chromatic number is $\sigma(v_i) + \sigma(v_j) + \sigma(v_k)$.*

(v) : *The set of representatives of colors is $\{\}$. Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.*

(vi) : *The set of representatives of colors is $\{\}$. Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.*

(vii) : *The set of representatives of colors is $\{\}$. Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.*

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it's cycle, all vertices have 2 vertices which have common edges which are type-I. Thus the set of representatives of colors is $\{v_i, v_j, v_k\}$. The type-I chromatic number is 3 and n-type-I chromatic number is neutrosophic cardinality of $\{v_i, v_j, v_k\}$ which is $\sigma(v_i)+\sigma(v_j)+\sigma(v_k)$.
(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-II. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-II. Thus the set of representatives of colors is $\{\}$. The type-II chromatic number is 0 and n-type-II chromatic number is 0.
(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-III. Thus the set of representatives

of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it's cycle, all vertices have 2 vertices which have common edges which are type-IV. By deletion of one edge, it's possible to compute connectedness. Thus the set of representatives of colors is $\{v_i, v_j, v_k\}$. The type-IV chromatic number is 3 and n-type-IV chromatic number is neutrosophic cardinality of $\{v_i, v_j, v_k\}$ which is $\sigma(v_i) + \sigma(v_j) + \sigma(v_k)$.
(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0. ■

Proposition 2.30.12. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is strong and even wheel. If it's fixed-vertex, then*

(i) : *The set of representatives of colors is $\{v_i, v_j, v_k\}$. Thus type-I chromatic number is 3 and n-type-I chromatic number is $3\sigma(v_i)$.*

(ii) : *The set of representatives of colors is $\{\}$. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.*

(iii) : *The set of representatives of colors is $\{\}$. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.*

(iv) : *The set of representatives of colors is $\{v_i, v_j, v_k\}$. Thus type-IV chromatic number is 3 and n-type-IV chromatic number is $3\sigma(v_i)$.*

(v) : *The set of representatives of colors is $\{\}$. Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.*

(vi) : *The set of representatives of colors is $\{\}$. Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.*

(vii) : *The set of representatives of colors is $\{\}$. Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.*

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it's cycle, all vertices have 2 vertices which have common edges which are type-I. Thus the set of representatives of colors is $\{v_i, v_j, v_k\}$. The type-I chromatic number is 3 and n-type-I chromatic number is neutrosophic cardinality of $\{v_i, v_j, v_k\}$ which is $3\sigma(v_i)$.

(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-II. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-II. Thus the set of representatives of colors is $\{\}$. The type-II chromatic number is 0 and n-type-II chromatic number is 0.
(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it's cycle, all vertices have 3 vertices which have common edges which are type-IV. By deletion of one edge, it's possible to compute connectedness. Thus the set of representatives of colors is $\{v_i, v_j, v_k\}$. The type-IV chromatic number is 3 and n-type-IV chromatic number is neutrosophic cardinality of $\{v_i, v_j, v_k\}$ which is $3\sigma(v_i)$.
(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0. ■

Proposition 2.30.13. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is an odd wheel. If it's fixed-edge, then*

(i) : *The set of representatives of colors is $\{v_i, v_j, v_k, v_s\}$. Thus type-I chromatic number is 4 and n-type-I chromatic number is $\sigma(v_i)+\sigma(v_j)+\sigma(v_k)+\sigma(v_s)$.*

(ii) : *The set of representatives of colors is $\{\}$. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.*

(iii) : *The set of representatives of colors is $\{\}$. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.*

(iv) : *The set of representatives of colors is $\{v_i, v_j, v_k, v_s\}$. Thus type-IV chromatic number is 2 and n-type-IV chromatic number is $\sigma(v_i) + \sigma(v_j) + \sigma(v_k) + \sigma(v_s)$.*

(v) : *The set of representatives of colors is $\{\}$. Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.*

(vi) : *The set of representatives of colors is $\{\}$. Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.*

(vii) : *The set of representatives of colors is* $\{\}$. *Thus type-VII chromatic number is* 0 *and n-type-VII chromatic number is* 0.

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it's cycle, all vertices have 2 vertices which have common edges which are type-I. Thus the set of representatives of colors is $\{v_i, v_j, v_k, v_s\}$. The type-I chromatic number is 4 and n-type-I chromatic number is neutrosophic cardinality of $\{v_i, v_j, v_k, v_s\}$ which is $\sigma(v_i) + \sigma(v_j) + \sigma(v_k) + \sigma(v_s)$.
(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-II. By it's neutrosophic strong, there's a vertex has 2 vertices which have common edges which aren't type-II. Thus the set of representatives of colors is $\{\}$. The type-II chromatic number is 0 and n-type-II chromatic number is 0.
(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's neutrosophic strong, there's a vertex has 2 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it's cycle, all vertices have 2 vertices which have common edges which are type-IV. Thus the set of representatives of colors is $\{v_i, v_j, v_k, v_s\}$. The type-IV chromatic number is 4 and n-type-IV chromatic number is neutrosophic cardinality of $\{v_i, v_j, v_k, v_s\}$ which is $\sigma(v_i) + \sigma(v_j) + \sigma(v_k) + \sigma(v_s)$.
(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-V. By it's neutrosophic strong, there's a vertex has 2 vertices which have common edges which aren't type-V. Thus the set of representatives of colors is $\{\}$. The type-V chromatic number is 0 and n-type-V chromatic number is 0.
(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VI. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-VI. Thus the set of representatives of colors is $\{\}$. The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.
(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VII. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-VII. Thus the set of representatives of colors is $\{\}$. The type-VII chromatic number is 0 and n-type-VII chromatic number is 0. ∎

Proposition 2.30.14. *Let* $N = (\sigma, \mu)$ *be a neutrosophic graph which is strong and odd wheel. If it's fixed-vertex, then*

(i) : *The set of representatives of colors is* $\{v_i, v_j, v_k, v_s\}$. *Thus type-I chromatic number is* 4 *and n-type-I chromatic number is* $4\sigma(v_i)$.

(ii) : *The set of representatives of colors is* $\{\}$. *Thus type-II chromatic number is* 0 *and n-type-II chromatic number is* 0.

(iii) : *The set of representatives of colors is* $\{\}$. *Thus type-III chromatic number is* 0 *and n-type-III chromatic number is* 0.

(iv) : *The set of representatives of colors is* $\{v_i, v_j, v_k, v_s\}$. *Thus type-IV chromatic number is* 4 *and n-type-IV chromatic number is* $4\sigma(v_i)$.

(v) : *The set of representatives of colors is* $\{\}$. *Thus type-V chromatic number is* 0 *and n-type-V chromatic number is* 0.

(vi) : *The set of representatives of colors is* $\{\}$. *Thus type-VI chromatic number is* 0 *and n-type-VI chromatic number is* 0.

(vii) : *The set of representatives of colors is* $\{\}$. *Thus type-VII chromatic number is* 0 *and n-type-VII chromatic number is* 0.

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it's cycle, all vertices have 2 vertices which have common edges which are type-I. Thus the set of representatives of colors is $\{v_i, v_j, v_k, v_s\}$. The type-I chromatic number is 4 and n-type-I chromatic number is neutrosophic cardinality of $\{v_i, v_j, v_k, v_s\}$ which is $4\sigma(v_i)$.
(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-II. By it's neutrosophic strong, there's a vertex has 2 vertices which have common edges which aren't type-II. Thus the set of representatives of colors is $\{\}$. The type-II chromatic number is 0 and n-type-II chromatic number is 0.
(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's neutrosophic strong, there's a vertex has 2 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it's cycle, all vertices have 2 vertices which have common edges which are type-IV. Thus the set of representatives of colors is $\{v_i, v_j, v_k, v_s\}$. The type-IV chromatic number is 4 and n-type-IV chromatic number is neutrosophic cardinality of $\{v_i, v_j, v_k, v_s\}$ which is $4\sigma(v_i)$.
(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-V. By it's neutrosophic strong, there's a vertex has 2 vertices which have common edges which aren't type-V. Thus the set of representatives of colors is $\{\}$. The type-V chromatic number is 0 and n-type-V chromatic number is 0.
(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VI. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-VI. Thus the set of representatives of colors is $\{\}$. The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.
(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VII. By it's cycle, all vertices have 2 vertices which have common edges which aren't type-VII. Thus the set of representatives of colors is $\{\}$. The type-VII chromatic number is 0 and n-type-VII chromatic number is 0. ∎

Proposition 2.30.15. *Let* $N = (\sigma, \mu)$ *be a neutrosophic graph which is complete* $t-$*partite. If it's fixed-edge, then* (*i*) : *The set of representatives of colors is* $\{v_1, v_2, \cdots, v_t\}$. *Thus type-I chromatic number is* t *and n-type-I chromatic number is* $\sigma(v_1) + \sigma(v_2) + \cdots + \sigma(v_t)$.

(*ii*) : *The set of representatives of colors is* $\{\}$. *Thus type-II chromatic number is* 0 *and n-type-II chromatic number is* 0.

(*iii*) : *The set of representatives of colors is* $\{\}$. *Thus type-III chromatic number is* 0 *and n-type-III chromatic number is* 0.

(*iv*) : *The set of representatives of colors is* $\{v_1, v_2, \cdots, v_t\}$. *Thus type-IV chromatic number is* t *and n-type-IV chromatic number is* $\sigma(v_1) + \sigma(v_2) + \cdots + \sigma(v_t)$.

(*v*) : *The set of representatives of colors is* $\{\}$. *Thus type-V chromatic number is* 0 *and n-type-V chromatic number is* 0.

(*vi*) : *The set of representatives of colors is* $\{\}$. *Thus type-VI chromatic number is* 0 *and n-type-VI chromatic number is* 0.

(*vii*) : *The set of representatives of colors is* $\{\}$. *Thus type-VII chromatic number is* 0 *and n-type-VII chromatic number is* 0.

Proof. (*i*). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it's neutrosophic complete, there's a vertex has $t-1$ which have common edges which are type-I. Thus the set of representatives of colors is $\{v_1, v_2, \cdots, v_t\}$. The type-I chromatic number is t and n-type-I chromatic number is neutrosophic cardinality of $\{v_1, v_2, \cdots, v_t\}$ which is $\sigma(v_1) + \sigma(v_2) + \cdots + \sigma(v_t)$.
(*ii*). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-II. By it's neutrosophic complete, there's a vertex has $t-1$ vertices which have common edges which aren't type-II. Thus the set of representatives of colors is $\{\}$. The type-II chromatic number is 0 and n-type-II chromatic number is 0.
(*iii*). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's neutrosophic complete, there's a vertex has $t-1$ vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(*iv*). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it's neutrosophic complete, there's a vertex has $t-1$ vertices which have common edges which are type-IV. Thus the set of representatives of colors is $\{v_1, v_2, \cdots, v_t\}$. The type-IV chromatic number is t and n-type-IV chromatic number is neutrosophic cardinality of $\{v_1, v_2, \cdots, v_t\}$ which is $\sigma(v_1) + \sigma(v_2) + \cdots + \sigma(v_t)$.
(*v*). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-V. By it's neutrosophic complete, there's a vertex has $t-1$ vertices which have common edges which aren't type-V. Thus the set of representatives of colors is $\{\}$. The type-V chromatic number is 0 and n-type-V chromatic number is 0.
(*vi*). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VI. By it's neutrosophic complete, there's a

vertex has $t-1$ vertices which have common edges which aren't type-VI. Thus the set of representatives of colors is $\{\}$. The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.
(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VII. By it's neutrosophic complete, there's a vertex has $t-1$ vertices which have common edges which aren't type-VII. Thus the set of representatives of colors is $\{\}$. The type-VII chromatic number is 0 and n-type-VII chromatic number is 0. ■

Proposition 2.30.16. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is complete $t-$partite. If it's fixed-vertex, then*

(i) : *The set of representatives of colors is $\{v_1, v_2, \cdots, v_t\}$. Thus type-I chromatic number is t and n-type-I chromatic number is $t\sigma(v_i)$.*

(ii) : *The set of representatives of colors is $\{\}$. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.*

(iii) : *The set of representatives of colors is $\{\}$. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.*

(iv) : *The set of representatives of colors is $\{v_1, v_2, \cdots, v_t\}$. Thus type-IV chromatic number is t and n-type-IV chromatic number is $t\sigma(v_i)$.*

(v) : *The set of representatives of colors is $\{\}$. Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.*

(vi) : *The set of representatives of colors is $\{\}$. Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.*

(vii) : *The set of representatives of colors is $\{\}$. Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.*

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it's neutrosophic complete, there's a vertex has $t-1$ vertices which have common edges which are type-I. Thus the set of representatives of colors is $\{v_1, v_2, \cdots, v_t\}$. The type-I chromatic number is t and n-type-I chromatic number is neutrosophic cardinality of $\{v_1, v_2, \cdots, v_t\}$. which is $t\sigma(v_i)$.
(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-II. By it's neutrosophic complete, there's a vertex has $t-1$ vertices which have common edges which aren't type-II. Thus the set of representatives of colors is $\{\}$. The type-II chromatic number is 0 and n-type-II chromatic number is 0.
(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's neutrosophic complete, there's a vertex has $t-1$ vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it's neutrosophic complete, there's a vertex has $t-1$ vertices which have common edges which are type-IV. Thus the set of representatives of colors is $\{v_1, v_2, \cdots, v_t\}$. The type-IV chromatic number is t

and n-type-IV chromatic number is neutrosophic c$\{v_1, v_2, \cdots, v_t\}.\{v_i, v_j, v_k, v_s\}$ which is $t\sigma(v_i)$.
(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-V. By it's neutrosophic complete, there's a vertex has $t-1$ vertices which have common edges which aren't type-V. Thus the set of representatives of colors is $\{\}$. The type-V chromatic number is 0 and n-type-V chromatic number is 0.
(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VI. By it's neutrosophic complete, there's a vertex has $t-1$ vertices which have common edges which aren't type-VI. Thus the set of representatives of colors is $\{\}$. The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.
(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VII. By it's neutrosophic complete, there's a vertex has $t-1$ vertices which have common edges which aren't type-VII. Thus the set of representatives of colors is $\{\}$. The type-VII chromatic number is 0 and n-type-VII chromatic number is 0. ∎

Corollary 2.30.17. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is complete bipartite. If it's fixed-edge, then*

(i) : *The set of representatives of colors is $\{v_1, v_2\}$. Thus type-I chromatic number is 2 and n-type-I chromatic number is $\sigma(v_1) + \sigma(v_2)$.*

(ii) : *The set of representatives of colors is $\{\}$. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.*

(iii) : *The set of representatives of colors is $\{\}$. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.*

(iv) : *The set of representatives of colors is $\{v_1, v_2\}$. Thus type-IV chromatic number is 2 and n-type-IV chromatic number is $\sigma(v_1) + \sigma(v_2)$.*

(v) : *The set of representatives of colors is $\{\}$. Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.*

(vi) : *The set of representatives of colors is $\{\}$. Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.*

(vii) : *The set of representatives of colors is $\{\}$. Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.*

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it's neutrosophic complete, there's a vertex has 1 which have common edges which are type-I. Thus the set of representatives of colors is $\{v_1, v_2\}$. The type-I chromatic number is 2 and n-type-I chromatic number is neutrosophic cardinality of $\{v_1, v_2\}$ which is $\sigma(v_1) + \sigma(v_2)$.
(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-II. By it's neutrosophic complete, there's a vertex has 1 vertices which have common edges which aren't type-II. Thus the set of representatives of colors is $\{\}$. The type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's neutrosophic complete, there's a vertex has 1 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it's neutrosophic complete, there's a vertex has 1 vertices which have common edges which are type-IV. Thus the set of representatives of colors is $\{v_1, v_2\}$. The type-IV chromatic number is 2 and n-type-IV chromatic number is neutrosophic cardinality of $\{v_1, v_2\}$ which is $\sigma(v_1) + \sigma(v_2)$.
(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-V. By it's neutrosophic complete, there's a vertex has 1 vertices which have common edges which aren't type-V. Thus the set of representatives of colors is $\{\}$. The type-V chromatic number is 0 and n-type-V chromatic number is 0.
(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VI. By it's neutrosophic complete, there's a vertex has 1 vertices which have common edges which aren't type-VI. Thus the set of representatives of colors is $\{\}$. The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.
(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VII. By it's neutrosophic complete, there's a vertex has 1 vertices which have common edges which aren't type-VII. Thus the set of representatives of colors is $\{\}$. The type-VII chromatic number is 0 and n-type-VII chromatic number is 0. ■

Corollary 2.30.18. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is complete bipartite. If it's fixed-vertex, then*

(i) : *The set of representatives of colors is $\{v_1, v_2\}$. Thus type-I chromatic number is 2 and n-type-I chromatic number is $2\sigma(v_i)$.*

(ii) : *The set of representatives of colors is $\{\}$. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.*

(iii) : *The set of representatives of colors is $\{\}$. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.*

(iv) : *The set of representatives of colors is $\{v_1, v_2\}$. Thus type-IV chromatic number is t and n-type-IV chromatic number is $2\sigma(v_i)$.*

(v) : *The set of representatives of colors is $\{\}$. Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.*

(vi) : *The set of representatives of colors is $\{\}$. Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.*

(vii) : *The set of representatives of colors is $\{\}$. Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.*

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it's neutrosophic complete, there's

a vertex has 1 vertices which have common edges which are type-I. Thus the set of representatives of colors is $\{v_1, v_2\}$. The type-I chromatic number is 2 and n-type-I chromatic number is neutrosophic cardinality of $\{v_1, v_2\}$. which is $2\sigma(v_i)$.
(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-II. By it's neutrosophic complete, there's a vertex has 1 vertices which have common edges which aren't type-II. Thus the set of representatives of colors is $\{\}$. The type-II chromatic number is 0 and n-type-II chromatic number is 0.
(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's neutrosophic complete, there's a vertex has 1 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it's neutrosophic complete, there's a vertex has 1 vertices which have common edges which are type-IV. Thus the set of representatives of colors is $\{v_1, v_2\}$. The type-IV chromatic number is 2 and n-type-IV chromatic number is neutrosophic $\{v_1, v_2\}$ which is $2\sigma(v_i)$.
(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-V. By it's neutrosophic complete, there's a vertex has 1 vertices which have common edges which aren't type-V. Thus the set of representatives of colors is $\{\}$. The type-V chromatic number is 0 and n-type-V chromatic number is 0.
(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VI. By it's neutrosophic complete, there's a vertex has 1 vertices which have common edges which aren't type-VI. Thus the set of representatives of colors is $\{\}$. The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.
(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VII. By it's neutrosophic complete, there's a vertex has 1 vertices which have common edges which aren't type-VII. Thus the set of representatives of colors is $\{\}$. The type-VII chromatic number is 0 and n-type-VII chromatic number is 0. ∎

Corollary 2.30.19. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is star. If it's fixed-edge, then*

(i) : *The set of representatives of colors is $\{c, v_2\}$. Thus type-I chromatic number is 2 and n-type-I chromatic number is $\sigma(c) + \sigma(v_2)$.*

(ii) : *The set of representatives of colors is $\{\}$. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.*

(iii) : *The set of representatives of colors is $\{\}$. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.*

(iv) : *The set of representatives of colors, type-IV chromatic number and n-type-IV chromatic number aren't defined.*

(v) : *The set of representatives of colors, type-V chromatic number and n-type-V chromatic number aren't defined.*

(*vi*) : *The set of representatives of colors, type-VI chromatic number and n-type-VI chromatic number aren't defined.*

(*vii*) : *The set of representatives of colors, type-VII chromatic number and n-type-VII chromatic number aren't defined.*

Proof. (*i*). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it's neutrosophic complete, there's a vertex has 1 which have common edges which are type-I. Thus the set of representatives of colors is $\{v_1, v_2\}$. The type-I chromatic number is 2 and n-type-I chromatic number is neutrosophic cardinality of $\{v_1, v_2\}$ which is $\sigma(v_1) + \sigma(v_2)$.
(*ii*). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-II. By it's neutrosophic complete, there's a vertex has 1 vertices which have common edges which aren't type-II. Thus the set of representatives of colors is $\{\}$. The type-II chromatic number is 0 and n-type-II chromatic number is 0.
(*iii*). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's neutrosophic complete, there's a vertex has 1 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(*iv*). By it's fixed-vertex and it's neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-IV. Since it's impossible to define when there's no cycle in neutrosophic graph.
(*v*). By it's fixed-vertex and it's neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-V. Since it's impossible to define when there's no cycle in neutrosophic graph.
(*vi*). By it's fixed-vertex and it's neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VI. Since it's impossible to define when there's no cycle in neutrosophic graph.
(*vii*). By it's fixed-vertex and it's neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VII. Since it's impossible to define when there's no cycle in neutrosophic graph.

■

Corollary 2.30.20. *Let $N = (\sigma, \mu)$ be a neutrosophic graph which is star. If it's fixed-vertex, then*

(*i*) : *The set of representatives of colors is $\{v_1, c\}$. Thus type-I chromatic number is 2 and n-type-I chromatic number is $2\sigma(c)$.*

(*ii*) : *The set of representatives of colors is $\{\}$. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.*

(*iii*) : *The set of representatives of colors is $\{\}$. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.*

(iv) : *The set of representatives of colors, type-IV chromatic number and n-type-IV chromatic number aren't defined.*

(v) : *The set of representatives of colors, type-V chromatic number and n-type-V chromatic number aren't defined.*

(vi) : *The set of representatives of colors, type-VI chromatic number and n-type-VI chromatic number aren't defined.*

(vii) : *The set of representatives of colors, type-VII chromatic number and n-type-VII chromatic number aren't defined.*

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it's neutrosophic complete, there's a vertex has 1 vertices which have common edges which are type-I. Thus the set of representatives of colors is $\{c, v_2\}$. The type-I chromatic number is 2 and n-type-I chromatic number is neutrosophic cardinality of $\{c, v_2\}$. which is $2\sigma(c)$.
(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-II. By it's neutrosophic complete, there's a vertex has 1 vertices which have common edges which aren't type-II. Thus the set of representatives of colors is $\{\}$. The type-II chromatic number is 0 and n-type-II chromatic number is 0.
(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-III. By it's neutrosophic complete, there's a vertex has 1 vertices which have common edges which aren't type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.
(iv). By it's fixed-vertex and it's neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-IV. Since it's impossible to define when there's no cycle in neutrosophic graph.
(v). By it's fixed-vertex and it's neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-V. Since it's impossible to define when there's no cycle in neutrosophic graph.
(vi). By it's fixed-vertex and it's neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VI. Since it's impossible to define when there's no cycle in neutrosophic graph.
(vii). By it's fixed-vertex and it's neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren't type-VII. Since it's impossible to define when there's no cycle in neutrosophic graph. ■

2.31 Applications in Time Table and Scheduling

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has important to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive section. Beyond that, sometimes sections are not the same.

Step 3. (Model) As Figure (2.7), the situation is designed as a model. The model uses data to assign every section and to assign to relation amid section, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relation amid them. Table (4.5), clarifies about the assigned numbers to these situation.

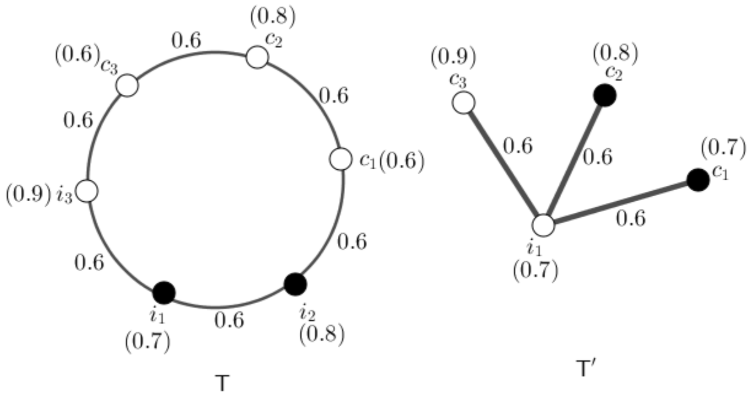

Figure 2.7: Black vertices are suspicions about choosing them. fgr1c

Table 2.5: Scheduling concerns its Subjects and its Connections as a Neutrosophic Graph in a Model. tbl1c

Sections of T	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9, s_{10}
Values	0.1	0.8	0.7	0.8	0.1	0.3	0.6	0.5	0.2
Connections of T	s_1s_2	s_2s_3	s_3s_4	s_4s_5	s_5s_6	s_6s_7	s_7s_8	s_8s_9	s_9s_{10}
Values	0.1	0.6	0.4	0.1	0.1	0.2	0.4	0.2	0.1

Step 4. (Solution) As Figure (2.7) shows, neutrosophic model, proposes to use different types of chromatic number which is incomputable for types IV,V,VI,VII in the case which is titled T'. In this case, i_1 and c_1 aren't representative of these two colors and different types of chromatic number is incomputable for types IV,V,VI,VII. The set $\{i_1, c_1\}$ doesn't contain representatives of colors which pose different types of chromatic number and different types of chromatic number for types IV,V,VI,VII. Thus the decision amid choosing the subject c_1 an c_2 isn't concluded to choose c_1 for types IV,V,VI,VII. To get brief overview, neutrosophic model uses one number for every array so 0.9 means $(0.9, 0.9, 0.9)$. In Figure (2.7), the neutrosophic model T introduces the common situation. The

representatives of colors are i_2 and c_1. Thus different types of chromatic numbers is two for types I and IV and different types of neutrosophic chromatic number is 1.4 for types I and IV. Thus suspicion about choosing i_1 and i_2 is determined to be i_2. The sets of representative for colors are $\{i_2, c_1\}$ for types I and IV. Thus the comparative studies based on different types of chromatic number and neutrosophic chromatic number are concluded.

2.32 Open Problems

The two notions of coloring of vertices concerning different types of chromatic number and different types of neutrosophic chromatic number are defined on neutrosophic graphs when connectedness and as its consequences, different types of edges have key role to have these notions. Thus

Question 2.32.1. *Is it possible to use other types edges via connectedness to define different types of chromatic number and different types of neutrosophic chromatic number?*

Question 2.32.2. *Are existed some connections amid the coloring from connectedness inside this concept and external connections with other types of coloring from other notions?*

Question 2.32.3. *Is it possible to construct some classes neutrosophic graphs which have "nice" behavior?*

Question 2.32.4. *Which applications do make an independent study to apply different types of chromatic number and different types of neutrosophic chromatic number?*

Problem 2.32.5. *Which parameters are related to this parameter?*

Problem 2.32.6. *Which approaches do work to construct applications to create independent study?*

Problem 2.32.7. *Which approaches do work to construct definitions which use all three arrays and the relations amid them instead of one array of three arrays to create independent study?*

2.33 Conclusion and Closing Remarks

This study uses mixed combinations of different types of chromatic number and different types of neutrosophic chromatic number to study on neutrosophic graphs. The connections of vertices which are clarified by special edges and different edges from connectedness, differ them from each other and and put them in different categories to represent one representative for each color. Further studies could be about changes in the settings to compare this notion amid different settings of graph theory. One way is finding some relations amid array of vertices to make sensible definitions. In Table (2.4), some limitations and advantages of this study is pointed out.

Table 2.6: A Brief Overview about Advantages and Limitations of this study tbl2c

Advantages	Limitations
1. Using connectedness for labelling edges	1. General Results
2. Using neutrosophic cardinality	
3. Using cardinality	2. Connections with parameters
4. Applying Different Types of Edges	
5. Different Types of Chromatic Notions	3. Connections of Results

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CHAPTER 3

Neutrosophic Hypergraphs

Akram et al. introduce A new decision-making method based on bipolar neutrosophic directed hypergraphs [**1**], Bipolar neutrosophic hypergraphs with applications [**2**], Certain networks models using single-valued neutrosophic directed hypergraphs [**3**]. Also, Akram et al. in Fuzzy hypergraphs and related extensions [**4**], get some directions to this topic. Hamidi et al. [**8**] propose single-valued neutrosophic directed (hyper) graphs and applications [**9**] poses generalized neutrosophic hypergraphs. Luqman et al. [**10**] introduce complex neutrosophic hypergraphs: new social network models. Malik et al propose Isomorphism of single valued neutrosophic hypergraphs [**11**] and regular single valued neutrosophic hypergraphs [**12**].

3.1 Numbers and Sets

Based on some ideas, numbers and sets are defined in the ways that, some results are obtained. Thus it's an open way to have well-understandable structures and well-defined ideas from neutrosophic hypergraphs.

3.2 Preliminaries For Setting of Neutrosophic n-SuperHyperGraph and Setting of Neutrosophic Hypergraphs

Definition 3.2.1. (Graph).
$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Definition 3.2.2. (Hypergraph).
$H = (V, E)$ is called a **hypergraph** if V is a set of objects and for every nonnegative integer $t \leq n$, E is a set of $t-$subsets of V where V is called **vertex set** and E is called **hyperedge set**.

Definition 3.2.3. (Neutrosophic Hypergraph).
$NHG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic hypergraph** if it's hypergraph, $\sigma_i : V \to [0, 1]$, $\mu_i : E \to [0, 1]$, and for every $v_1 v_2 \cdots v_t \in E$,

$$\mu(v_1 v_2 \cdots v_t) \leq \sigma(v_1) \wedge \sigma(v_2) \wedge \cdots \sigma(v_t).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic hyperedge set**.

(iii) : $|V|$ is called **order** of NHG and it's denoted by $\mathcal{O}(NHG)$.

(iv) : $\Sigma_{v\in V}\sigma(v)$ is called **neutrosophic order** of NHG and it's denoted by $\mathcal{O}_n(NHG)$.

(vi) : $|E|$ is called **size** of NHG and it's denoted by $\mathcal{S}(NHG)$.

(vii) : $\Sigma_{e\in E}\mu(e)$ is called **neutrosophic size** of NHG and it's denoted by $\mathcal{S}_n(NHG)$.

Example 3.2.4. Assume Figure (3.11).

(i) : Neutrosophic hyperedge $n_1n_2n_3$ has three neutrosophic vertices.

(ii) : Neutrosophic hyperedge $n_3n_4n_5n_6$ has four neutrosophic vertices.

(iii) : Neutrosophic hyperedge $n_1n_7n_8n_9n_5n_6$ has six neutrosophic vertices.

(iv) : $\sigma = \{(n_1,(0.99,0.98,0.55)),(n_2,(0.74,0.64,0.46)),(n_3,(0.99,0.98,0.55)),$ $(n_4,(0.54,0.24,0.16)),(n_5,(0.99,0.98,0.55)),(n_6,(0.99,0.98,0.55)),$ $(n_7,(0.99,0.98,0.55)),(n_8,(0.99,0.98,0.55)),(n_9,(0.99,0.98,0.55))\})$ is neutrosophic vertex set.

(v) : $\mu = \{(e_1,(0.01,0.01,0.01)),(e_2,(0.01,0.01,0.01)),(e_3,(0.01,0.01,0.01))\})$ is neutrosophic hyperedge set.

(vi) : $\mathcal{O}(NHG) = 9$.

(vii) : $\mathcal{O}_n(NHG) = (8.21, 7.74, 4.47)$.

$(viii)$: $\mathcal{S}(NHG) = 3$.

(ix) : $\mathcal{S}_n(NHG) = (0.03, 0.03, 0.03)$.

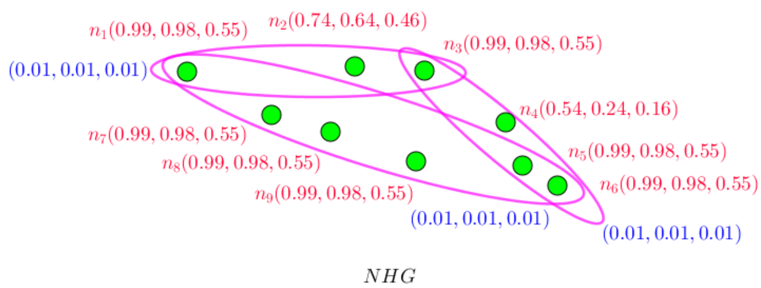


Figure 3.1: There are three neutrosophic hyperedges and two neutrosophic vertices.

nhg1

Definition 3.2.5. (Neutrosophic Edge $t-$Regular Hypergraph).
A neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ is called a **neutrosophic edge $t-$regular hypergraph** if every neutrosophic hyperedge has only t neutrosophic vertices.

Question 3.2.6. *What-if all neutrosophic hypergraphs are either edge $t-$regular or not?*

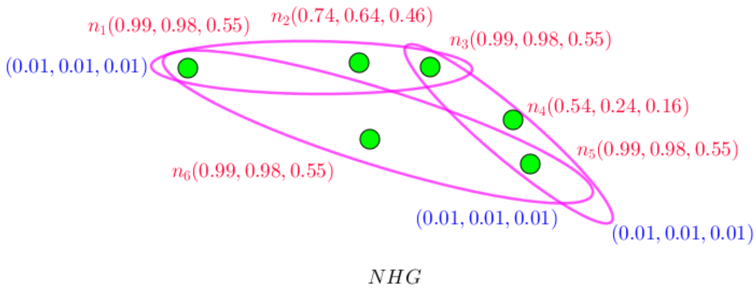


Figure 3.2: $NHG = (V, E, \sigma, \mu)$ is neutrosophic edge 3−regular hypergraph

nhg2

In the following, there are some directions which clarify the existence of some neutrosophic hypergraphs which are either edge $t-$regular or not.

Example 3.2.7. Two neutrosophic hypergraphs are presented such that one of them is edge $t-$regular and another isn't.

(i) : Assume Figure (3.11). It isn't neutrosophic edge $t-$regular hypergraph.

(ii) : Suppose Figure (3.2). It's neutrosophic edge 3−regular hypergraph.

Definition 3.2.8. (Neutrosophic vertex $t-$Regular Hypergraph).
A neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ is called a **neutrosophic vertex $t-$regular hypergraph** if every neutrosophic vertex is incident to only t neutrosophic hyperedges.

Example 3.2.9. Three neutrosophic hypergraphs are presented such that one of them is vertex $t-$regular and anothers aren't.

(i) : Consider Figure (3.11). It isn't neutrosophic edge $t-$regular hypergraph.

(ii) : Suppose Figure (3.2). It's neutrosophic edge 3−regular hypergraph but It isn't neutrosophic vertex 3−regular hypergraph.

(iii) : Assume Figure (3.3). It's neutrosophic vertex 2−regular hypergraph but It isn't neutrosophic edge $t-$regular hypergraph.

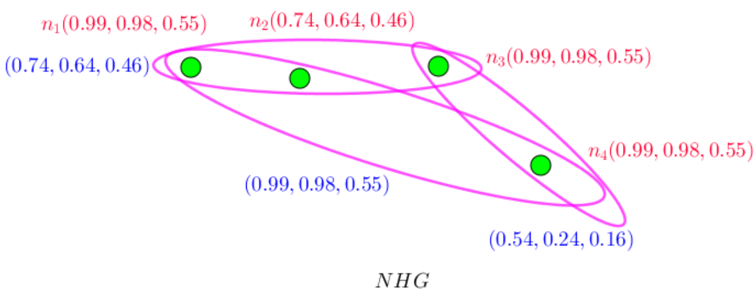


Figure 3.3: $NHG = (V, E, \sigma, \mu)$ is neutrosophic strong hypergraph.

nhg3

Definition 3.2.10. (Neutrosophic Strong Hypergraph).
A neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ is called a **neutrosophic strong hypergraph** if it's hypergraph and for every $v_1 v_2 \cdots v_t \in E$,

$$\mu(v_1 v_2 \cdots v_t) = \sigma(v_1) \wedge \sigma(v_2) \wedge \cdots \sigma(v_t).$$

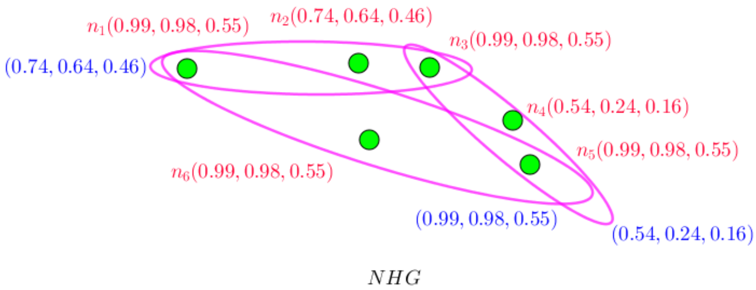


Figure 3.4: $NHG = (V, E, \sigma, \mu)$ is neutrosophic strong hypergraph. nhg4

Example 3.2.11. Three neutrosophic hypergraphs are presented such that one of them is neutrosophic strong hypergraph and others aren't.

(i) : Consider Figure (3.11). It isn't neutrosophic strong hypergraph.

(ii) : Assume Figure (3.2). It isn't neutrosophic strong hypergraph.

(iii) : Suppose Figure (3.3). It isn't neutrosophic strong hypergraph.

(iv) : Assume Figure (3.4). It's neutrosophic strong hypergraph. It's also neutrosophic edge $3-$regular hypergraph but it isn't neutrosophic vertex $t-$regular hypergraph.

Definition 3.2.12. (Neutrosophic Strong Hypergraph).
Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu.)$ A neutrosophic hyperedge $v_1 v_2 \cdots v_t \in E$ is called a **neutrosophic strong hyperedge** if

$$\mu(v_1 v_2 \cdots v_t) = \sigma(v_1) \wedge \sigma(v_2) \wedge \cdots \sigma(v_t).$$

Proposition 3.2.13. *Assume neutrosophic strong hypergraph $NHG = (V, E, \sigma, \mu.)$ Then all neutrosophic hyperedges are neutrosophic strong.*

Definition 3.2.14. (Neutrosophic Hyperpath).
A path $v_0, E_0, v_1, v_1, E_1, v_2, \cdots, v_{t-1}, E_{t-1}, v_t$, is called **neutrosophic hyperpath** such that v_{i-1} and v_i have incident to E_{i-1} for all nonnegative integers $0 \leq i \leq t$. In this case, $t-1$ is called **length** of neutrosophic hyperpath. Also, if x and y are two neutrosophic vertices, then maximum length of neutrosophic hyperpaths from x to y, is called **neutrosophic hyperdistance** and it's denoted by $d(x, y)$. If $v_0 = v_t$, then it's called **neutrosophic hypercycle**.

Example 3.2.15. Assume Figure (3.11).

(i) : $n_1, E_1, n_3, E_2, n_6, E_3, n_1$ is a neutrosophic hypercycle.

(ii) : $n_1, E_1, n_n, E_2, n_6, E_3, n_1$ isn't neither neutrosophic hypercycle nor neutrosophic hyperpath.

(iii) : $n_1E_1n_3E_2n_6E_3n_1$ isn't neither neutrosophic hypercycle nor neutrosophic hyperpath.

(iv) : n_1, n_3, n_6, n_1 isn't neither neutrosophic hypercycle nor neutrosophic hyperpath.

(v) : $n_1E_1, n_3, E_2, n_6, E_3, n_1$ isn't neither neutrosophic hypercycle nor neutrosophic hyperpath.

(vi) : $n_1, E_1, n_3, E_2, n_6, E_3, n_7$ is a neutrosophic hyperpath.

(vii) : Neutrosophic hyperdistance amid n_1 and n_4 is two.

$(viii)$: Neutrosophic hyperdistance amid n_1 and n_7 is one.

(ix) : Neutrosophic hyperdistance amid n_1 and n_2 is one.

(x) : Neutrosophic hyperdistance amid two given neutrosophic vertices is either one or two.

First case for the contents is to use the article from [**7**]. The contents are used in the way that, usages of new contents are preferences and the preliminaries are passed in the beginning of this chapter.

3.3 Dimension and Coloring alongside Domination in Neutrosophic Hypergraphs

3.4 Abstract

New setting is introduced to study resolving number and chromatic number alongside dominating number. Different types of procedures including set, optimal set, and optimal number alongside study on the family of neutrosophic hypergraphs are proposed in this way, some results are obtained. General classes of neutrosophic hypergraphs are used to obtains these numbers and the representatives of the colors, dominating sets and resolving sets. Using colors to assign to the vertices of neutrosophic hypergraphs and characterizing resolving sets and dominating sets are applied. Some questions and problems are posed concerning ways to do further studies on this topic. Using different ways of study on neutrosophic hypergraphs to get new results about numbers and sets in the way that some numbers get understandable perspective. Family of neutrosophic hypergraphs are studied to investigate about the notions, dimension and coloring alongside domination in neutrosophic hypergraphs. In this way, sets of representatives of colors, resolving sets and dominating sets have key role. Optimal sets and optimal numbers have key points to get new results but in some cases, there are usages of sets and numbers instead of optimal ones. Simultaneously, three notions are applied into neutrosophic hypergraphs to get sensible results about their structures. Basic familiarities with neutrosophic hypergraphs theory and hypergraph theory are proposed for this article.

Keywords: Dimension, Coloring, Domination

AMS Subject Classification: 05C17, 05C22, 05E45

3.5 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 3.5.1. *Is it possible to use mixed versions of ideas concerning "neutrosophic domination", "neutrosophic dimension" and "neutrosophic coloring" to define some notions which are applied to neutrosophic hypergraphs?*

It's motivation to find notions to use in any classes of neutrosophic hypergraphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Connections amid two items have key roles to assign colors, dominating and domination. Thus they're used to define new ideas which conclude to the structure of coloring, dominating and domination. The concept of having general neutrosophic hyperedge inspires me to study the behavior of general neutrosophic hyperedge in the way that, three types of coloring numbers, dominating number and resolving set are the cases of study in individuals and families.
The framework of this study is as follows. In the beginning, I introduced basic definitions to clarify about preliminaries. In section "New Ideas For Neutrosophic Hypergraphs", new notions of coloring, dominating and domination are applied to neutrosophic vertices of neutrosophic graphs as individuals. In section "Optimal Numbers For Neutrosophic Hypergraphs", specific numbers have the key role in this way. Classes of neutrosophic graphs are studied in the terms of different numbers in section "Optimal Numbers For Neutrosophic Hypergraphs" as individuals. In the section "Optimal Sets For Neutrosophic Hypergraphs", usages of general neutrosophic sets and special neutrosophic sets have key role in this study as individuals. In section "Optimal Sets and Numbers For Family of Neutrosophic Hypergraphs", both sets and numbers have applied into the family of neutrosophic hypergraphs. In section "Applications in Time Table and Scheduling", one application is posed for neutrosophic hypergraphs concerning time table and scheduling when the suspicions are about choosing some subjects. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications are featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions are formed.

3.6 New Ideas For Neutrosophic Hypergraphs

Definition 3.6.1. (Dominating, Resolving and Coloring).
Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$.

(a) : Neutrosophic-dominating set and number are defined as follows.

(i) : A neutrosophic vertex x **neutrosophic-dominates** a vertex y if there's at least one neutrosophic strong hyperedge which have them.

(ii) : A set S is called **neutrosophic-dominating set** if for every $y \in V \setminus S$, there's at least one vertex x which neutrosophic-dominates vertex y.

(iii) : If $\mathcal{S}$ is set of all neutrosophic-dominating sets, then

$$\Sigma_{x \in X} \sigma(x) = \min_{S \in \mathcal{S}} \Sigma_{x \in S} \sigma(x)$$

is called **optimal-neutrosophic-dominating number** and X is called **optimal-neutrosophic-dominating set.**

(b) : Neutrosophic-resolving set and number are defined as follows.

(i) : A neutrosophic vertex x **neutrosophic-resolves** vertices y, w if

$$d(x, y) \neq d(x, w).$$

.

(ii) : A set S is called **neutrosophic-resolving set** if for every $y \in V \setminus S$, there's at least one vertex x which neutrosophic-resolves vertices y, w.

(iii) : If $\mathcal{S}$ is set of all neutrosophic-resolving sets, then

$$\Sigma_{x \in X} \sigma(x) = \min_{S \in \mathcal{S}} \Sigma_{x \in S} \sigma(x)$$

is called **optimal-neutrosophic-resolving number** and X is called **optimal-neutrosophic-resolving set.**

(c) : Neutrosophic-coloring set and number are defined as follows.

(i) : A neutrosophic vertex x **neutrosophic-colors** a vertex y differently with itself if there's at least one neutrosophic strong hyperedge which have them.

(ii) : A set S is called **neutrosophic-coloring set** if for every $y \in V \setminus S$, there's at least one vertex x which neutrosophic-colors vertex y.

(iii) : If $\mathcal{S}$ is set of all neutrosophic-coloring sets, then

$$\Sigma_{x \in X} \sigma(x) = \min_{S \in \mathcal{S}} \Sigma_{x \in S} \sigma(x)$$

is called **optimal-neutrosophic-coloring number** and X is called **optimal-neutrosophic-coloring set.**

Example 3.6.2. Consider Figure (3.11) where the improvements on its hyperedges to have neutrosophic strong hypergraph.

(a) : The notions of dominating are clarified.

(i) : n_1 neutrosophic-dominates every vertex from the set of vertices $\{n_7, n_8, n_9, n_2, n_3\}$. n_4 neutrosophic-dominates every vertex from the set of vertices $\{n_6, n_5, n_3\}$. n_4 doesn't neutrosophic-dominate every vertex from the set of vertices $\{n_1, n_2, n_7, n_8, n_9\}$.

(ii) : $\{n_1, n_3\}$ is neutrosophic-coloring set but $\{n_1, n_4\}$ is optimal-neutrosophic-dominating set.

(iii) : $(1.53, 1.22, 0.71)$ is optimal-neutrosophic-dominating number.

(b) : The notions of resolving are clarified.

(i) : n_1 neutrosophic-resolves two vertices n_4 and n_6.

(ii) : $V \backslash \{n_1, n_4\}$ is neutrosophic-resolves set but $V \backslash \{n_2, n_4, n_9\}$ is optimal-neutrosophic-resolving set.

(iii) : $(5, 94, 6.36, 3.3)$ is optimal-neutrosophic-resolving number.

(c) : The notions of coloring are clarified.

(i) : n_1 neutrosophic-colors every vertex from the set of vertices $\{n_7, n_8, n_9, n_2, n_3\}$. n_4 neutrosophic-colors every vertex from the set of vertices $\{n_6, n_5, n_3\}$. n_4 doesn't neutrosophic-dominate every vertex from the set of vertices $\{n_1, n_2, n_7, n_8, n_9\}$.

(ii) : $\{n_1, n_5, n_7, n_8, n_9, n_6, n_4\}$ is neutrosophic-coloring set but $\{n_1, n_5, n_7, n_8, n_2, n_4\}$ is optimal-neutrosophic-coloring set.

(iii) : $(5.24, 4.8, 2.82)$ is optimal-neutrosophic-coloring number.

Example 3.6.3. Consider Figure (3.3).

(a) : The notions of dominating are clarified.

(i) : n_1 neutrosophic-dominates every vertex from the set of vertices $\{n_5, n_6, n_2, n_3\}$. n_4 neutrosophic-dominates every vertex from the set of vertices $\{n_5, n_3\}$. n_4 doesn't neutrosophic-dominate every vertex from the set of vertices $\{n_1, n_2, n_6\}$.

(ii) : $\{n_1, n_3\}$ is neutrosophic-dominating set but $\{n_1, n_4\}$ is optimal-neutrosophic-dominating set.

(iii) : $(1.53, 1.22, 0.71)$ is optimal-neutrosophic-dominating number.

(b) : The notions of resolving are clarified.

(i) : n_1 neutrosophic-resolves two vertices n_4 and n_6.

(ii) : $V \backslash \{n_1, n_4\}$ is neutrosophic-resolves set but $V \backslash \{n_2, n_4, n_6\}$ is optimal-neutrosophic-resolving set.

(iii) : $(5, 94, 6.36, 3.3)$ is optimal-neutrosophic-resolving number.

(c) : The notions of coloring are clarified.

(i) : n_1 neutrosophic-colors every vertex from the set of vertices $\{n_5, n_6, n_2, n_3\}$. n_4 neutrosophic-colors every vertex from the set of vertices $\{n_5, n_3\}$. n_4 doesn't neutrosophic-dominate every vertex from the set of vertices $\{n_1, n_2, n_6\}$.

(ii) : $\{n_1, n_5, n_6\}$ is neutrosophic-coloring set but $\{n_5, n_2, n_4\}$ is optimal-neutrosophic-coloring set.

(iii) : $(2.27, 1.86, 1.17)$ is optimal-neutrosophic-coloring number.

3.7 Optimal Numbers For Neutrosophic Hypergraphs

Proposition 3.7.1. *Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. S is maximum set of vertices which form a hyperedge. Then optimal-neutrosophic-coloring set has as cardinality as S has.*

Proof. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Every neutrosophic hyperedge has neutrosophic vertices which have common neutrosophic hyperedge. Thus every neutrosophic vertex has different color with other neutrosophic vertices which are incident with a neutrosophic hyperedge. It induces a neutrosophic hyperedge with the most number of neutrosophic vertices determines optimal-neutrosophic-coloring set. S is maximum set of vertices which form a hyperedge. Thus optimal-neutrosophic-coloring set has as cardinality as S has. ∎

Proposition 3.7.2. *Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. S is maximum set of vertices which form a hyperedge. Then optimal-neutrosophic-coloring number is*

$$\Sigma_{s\in S}\sigma(s).$$

Proof. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Every neutrosophic hyperedge has neutrosophic vertices which have common neutrosophic hyperedge. Thus every neutrosophic vertex has different color with other neutrosophic vertices which are incident with a neutrosophic hyperedge. It induces a neutrosophic hyperedge with the most number of neutrosophic vertices determines optimal-neutrosophic-coloring set. S is maximum set of vertices which form a hyperedge. Thus optimal-neutrosophic-coloring number is

$$\Sigma_{s\in S}\sigma(s).$$

∎

Proposition 3.7.3. *Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. If optimal-neutrosophic-coloring number is*

$$\Sigma_{v\in V}\sigma(v),$$

then there's at least one hyperedge which contains n vertices where n is the cardinality of the set V.

Proof. Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Consider optimal-neutrosophic-coloring number is

$$\Sigma_{v\in V}\sigma(v).$$

It implies there's one neutrosophic hyperedge which has all neutrosophic vertices. Since if all neutrosophic vertices are incident to a neutrosophic hyperedge, then all have different colors. ∎

Proposition 3.7.4. *Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. If there's at least one hyperedge which contains n vertices where n is the cardinality of the set V, then optimal-neutrosophic-coloring number is*

$$\Sigma_{v\in V}\sigma(v).$$

Proof. Consider neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Suppose there's at least one hyperedge which contains n vertices where n is the cardinality of the set V. It implies there's one neutrosophic hyperedge which has all neutrosophic vertices. If all neutrosophic vertices are incident to a neutrosophic hyperedge, then all have different colors. So V is optimal-neutrosophic-coloring set. It induces optimal-neutrosophic-coloring number is

$$\Sigma_{v \in V} \sigma(v).$$

■

Proposition 3.7.5. *Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. If optimal-neutrosophic-dominating number is*

$$\Sigma_{v \in V} \sigma(v),$$

then there's at least one neutrosophic vertex which doesn't have incident to any neutrosophic hyperedge.

Proof. Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Consider optimal-neutrosophic-dominating number is

$$\Sigma_{v \in V} \sigma(v).$$

If for all given neutrosophic vertex, there's at least one neutrosophic hyperedge which the neutrosophic vertex has incident to it, then there's a neutrosophic vertex x such that optimal-neutrosophic-dominating number is

$$\Sigma_{v \in V - \{x\}} \sigma(v).$$

It induces contradiction with hypothesis. It implies there's at least one neutrosophic vertex which doesn't have incident to any neutrosophic hyperedge. ■

Proposition 3.7.6. *Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Then optimal-neutrosophic-dominating number is $<$*

$$\Sigma_{v \in V} \sigma(v).$$

Proof. Consider neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Thus $V - \{x\}$ is a neutrosophic-dominating set. Since if not, x isn't incident to any given neutrosophic hyperedge. This is contradiction with supposition. It induces that x belongs to a neutrosophic hyperedge which has another vertex s. It implies s neutrosophic-dominates x. Thus $V - \{x\}$ is a neutrosophic-dominating set. It induces optimal-neutrosophic-dominating number is $<$

$$\Sigma_{v \in V} \sigma(v).$$

■

Proposition 3.7.7. *Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. If optimal-neutrosophic-resolving number is*

$$\Sigma_{v \in V} \sigma(v),$$

then every given vertex doesn't have incident to any hyperedge.

Proof. Consider neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Let optimal-neutrosophic-resolving number be

$$\Sigma_{v \in V} \sigma(v).$$

It implies every neutrosophic vertex isn't neutrosophic-resolved by a neutrosophic vertex. It's contradiction with hypothesis. So every given vertex doesn't have incident to any hyperedge. ■

Proposition 3.7.8. *Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Then optimal-neutrosophic-resolving number is <*

$$\Sigma_{v \in V} \sigma(v).$$

Proof. Consider neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. If optimal-neutrosophic-resolving number is

$$\Sigma_{v \in V} \sigma(v),$$

then there's a contradiction to hypothesis. Since the set $V \setminus \{x\}$ is neutrosophic-resolving set. It implies optimal-neutrosophic-resolving number is <

$$\Sigma_{v \in V} \sigma(v).$$

■

Proposition 3.7.9. *Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. If optimal-neutrosophic-coloring number is*

$$\Sigma_{v \in V} \sigma(v),$$

then all neutrosophic verties which have incident to at least one neutrosophic hyperedge.

Proof. Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Consider optimal-neutrosophic-coloring number is

$$\Sigma_{v \in V} \sigma(v).$$

If for all given neutrosophic vertices, there's no neutrosophic hyperedge which the neutrosophic vertices have incident to it, then there's neutrosophic vertex x such that optimal-neutrosophic-coloring number is

$$\Sigma_{v \in V - \{x\}} \sigma(v).$$

It induces contradiction with hypothesis. It implies all neutrosophic vertices have incident to at least one neutrosophic hyperedge. ■

Proposition 3.7.10. *Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Then optimal-neutrosophic-coloring number isn't <*

$$\Sigma_{v \in V} \sigma(v).$$

Proof. Consider neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Thus $V - \{x\}$ isn't a neutrosophic-coloring set. Since if not, x isn't incident to any given neutrosophic hyperedge. This is contradiction with supposition. It induces that x belongs to a neutrosophic hyperedge which has another vertex s. It implies s neutrosophic-colors x. Thus $V - \{x\}$ isn't a neutrosophic-coloring set. It induces optimal-neutrosophic-coloring number isn't $<$

$$\Sigma_{v \in V} \sigma(v).$$

■

Proposition 3.7.11. *Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Then optimal-neutrosophic-dominating set has cardinality which is greater than $n - 1$ where n is is the cardinality of the set V.*

Proof. Consider neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. The set V is neutrosophic-dominating set. So optimal-neutrosophic-dominating set has cardinality which is greater than n where n is is the cardinality of the set V. But the set $V \setminus \{x\}$, for every given neutrosophic vertex is optimal-neutrosophic-dominating set has cardinality which is greater than $n - 1$ where n is is the cardinality of the set V. The result is obtained. ■

Proposition 3.7.12. *Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. S is maximum set of vertices which form a hyperedge. Then S is optimal-neutrosophic-coloring set and*

$$\Sigma_{s \in S} \sigma(S)$$

is optimal-neutrosophic-coloring number.

Proof. Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Consider S is maximum set of vertices which form a hyperedge. Thus all vertices of S have incident to hyperedge. It implies the number of different colors equals to cardinality of S. Therefore, optimal-neutrosophic-coloring number $\geq$

$$\Sigma_{s \in S} \sigma(S).$$

In other hand, S is maximum set of vertices which form a hyperedge. It induces optimal-neutrosophic-coloring number $\leq$

$$\Sigma_{s \in S} \sigma(S).$$

So S is neutrosophic-coloring set. Hence S is optimal-neutrosophic-coloring set and

$$\Sigma_{s \in S} \sigma(S)$$

is optimal-neutrosophic-coloring number. ■

3.8 Optimal Sets For Neutrosophic Hypergraphs

Proposition 3.8.1. *Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. If S is neutrosophic-dominating set, then D contains S is neutrosophic-dominating set.*

Proof. Consider neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Suppose S is neutrosophic-dominating set. Then all neutrosophic vertices are neutrosophic-dominated. Thus D contains S is neutrosophic-dominating set. ∎

Proposition 3.8.2. *Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. If S is neutrosophic-resolving set, then D contains S is neutrosophic-resolving set.*

Proof. Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Consider S is neutrosophic-resolving set. Hence All two given neutrosophic vertices are neutrosophic-resolved by at least one neutrosophic vertex of S. It induces D contains S is neutrosophic-resolving set. ∎

Proposition 3.8.3. *Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. If S is neutrosophic-coloring set, then D contains S is neutrosophic-coloring set.*

Proof. Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Consider S is neutrosophic-coloring set. So all neutrosophic vertices which have a common neutrosophic hyperedge have different colors. Thus every neutrosophic vertex neutrosophic-colored by a neutrosophic vertex of S. It induces every neutrosophic vertex which has a common neutrosophic hyperedge has different colors with other neutrosophic vertices belong to that neutrosophic hyperedge. then D contains S is neutrosophic-coloring set. ∎

Proposition 3.8.4. *Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Then V is neutrosophic-dominating set.*

Proof. Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Since $V \setminus \{x\}$ is neutrosophic-dominating set. Then V contains $V \setminus \{x\}$ is neutrosophic-dominating set. ∎

Proposition 3.8.5. *Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Then V is neutrosophic-resolving set.*

Proof. Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. If there's no neutrosophic vertex, then all neutrosophic vertices are neutrosophic-resolved. Hence if I choose V, then there's no neutrosophic vertex such that neutrosophic vertex is neutrosophic-resolved. It implies V is neutrosophic-resolving set but V isn't optimal-neutrosophic-resolving set. Since if I construct one set from V such that only one neutrosophic vertex is out of S, then S is neutrosophic-resolving set. It implies V isn't optimal-neutrosophic-resolving set. Thus V is neutrosophic-resolving set. ∎

Proposition 3.8.6. *Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Then V is neutrosophic-coloring set.*

Proof. Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. All neutrosophic vertices belong to a neutrosophic hyperedge have to color differently. If V is chosen, then all neutrosophic vertices have different colors. It induces that n colors are used where n is the number of neutrosophic vertices. Every neutrosophic vertex has unique color. Thus V is neutrosophic-coloring set. ∎

3.9 Optimal Sets and Numbers For Family of Neutrosophic Hypergraphs

Proposition 3.9.1. *Assume $\mathcal{G}$ is a family of neutrosophic hypergraphs. Then V is neutrosophic-dominating set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of neutrosophic hypergraphs. Thus V is neutrosophic-dominating set for every given neutrosophic hypergraph of $\mathcal{G}$. It implies V is neutrosophic-dominating set for all members of $\mathcal{G}$, simultaneously. ■

Proposition 3.9.2. *Assume $\mathcal{G}$ is a family of neutrosophic hypergraphs. Then V is neutrosophic-resolving set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of neutrosophic hypergraphs. Thus V is neutrosophic-resolving set for every given neutrosophic hypergraph of $\mathcal{G}$. It implies V is neutrosophic-resolving set for all members of $\mathcal{G}$, simultaneously. ■

Proposition 3.9.3. *Assume $\mathcal{G}$ is a family of neutrosophic hypergraphs. Then V is neutrosophic-coloring set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of neutrosophic hypergraphs. Thus V is neutrosophic-coloring set for every given neutrosophic hypergraph of $\mathcal{G}$. It implies V is neutrosophic-coloring set for all members of $\mathcal{G}$, simultaneously. ■

Proposition 3.9.4. *Assume $\mathcal{G}$ is a family of neutrosophic hypergraphs. Then $V \setminus \{x\}$ is neutrosophic-dominating set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of neutrosophic hypergraphs. Thus $V \setminus \{x\}$ is neutrosophic-dominating set for every given neutrosophic hypergraph of $\mathcal{G}$. One neutrosophic vertex is out of $V \setminus \{x\}$. It's neutrosophic-dominated from any neutrosophic vertex in $V \setminus \{x\}$. Hence every given two neutrosophic vertices are neutrosophic-dominated from any neutrosophic vertex in $V \setminus \{x\}$. It implies $V \setminus \{x\}$ is neutrosophic-dominating set for all members of $\mathcal{G}$, simultaneously. ■

Proposition 3.9.5. *Assume $\mathcal{G}$ is a family of neutrosophic hypergraphs. Then $V \setminus \{x\}$ is neutrosophic-resolving set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of neutrosophic hypergraphs. Thus $V \setminus \{x\}$ is neutrosophic-resolving set for every given neutrosophic hypergraph of $\mathcal{G}$. One neutrosophic vertex is out of $V \setminus \{x\}$. It's neutrosophic-resolved from any neutrosophic vertex in $V \setminus \{x\}$. Hence every given two neutrosophic vertices are neutrosophic-resolved from any neutrosophic vertex in $V \setminus \{x\}$. It implies $V \setminus \{x\}$ is neutrosophic-resolving set for all members of $\mathcal{G}$, simultaneously. ■

Proposition 3.9.6. *Assume $\mathcal{G}$ is a family of neutrosophic hypergraphs. Then $V \setminus \{x\}$ isn't neutrosophic-coloring set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of neutrosophic hypergraphs. Thus $V \setminus \{x\}$ isn't neutrosophic-coloring set for every given neutrosophic hypergraph of $\mathcal{G}$. One neutrosophic vertex is out of $V \setminus \{x\}$. It isn't neutrosophic-colored from any neutrosophic vertex in $V \setminus \{x\}$. Hence every given two neutrosophic vertices aren't neutrosophic-colored from any neutrosophic vertex in $V \setminus \{x\}$. It implies $V \setminus \{x\}$ isn't neutrosophic-coloring set for all members of $\mathcal{G}$, simultaneously. ■

Proposition 3.9.7. *Assume $\mathcal{G}$ is a family of neutrosophic hypergraphs. Then union of neutrosophic-dominating sets from each member of $\mathcal{G}$ is neutrosophic-dominating set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of neutrosophic hypergraphs. For every chosen neutrosophic hypergraph, there's one neutrosophic-dominating set in the union of neutrosophic-dominating sets from each member of $\mathcal{G}$. Thus union of neutrosophic-dominating sets from each member of $\mathcal{G}$ is neutrosophic-dominating set for every given neutrosophic hypergraph of $\mathcal{G}$. Even one neutrosophic vertex isn't out of the union. It's neutrosophic-dominated from any neutrosophic vertex in the union. Hence every given two neutrosophic vertices are neutrosophic-dominated from any neutrosophic vertex in union of neutrosophic-coloring sets. It implies union of neutrosophic-coloring sets is neutrosophic-dominating set for all members of $\mathcal{G}$, simultaneously. ■

Proposition 3.9.8. *Assume $\mathcal{G}$ is a family of neutrosophic hypergraphs. Then union of neutrosophic-resolving sets from each member of $\mathcal{G}$ is neutrosophic-resolving set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of neutrosophic hypergraphs. For every chosen neutrosophic hypergraph, there's one neutrosophic-resolving set in the union of neutrosophic-resolving sets from each member of $\mathcal{G}$. Thus union of neutrosophic-resolving sets from each member of $\mathcal{G}$ is neutrosophic-resolving set for every given neutrosophic hypergraph of $\mathcal{G}$. Even one neutrosophic vertex isn't out of the union. It's neutrosophic-resolved from any neutrosophic vertex in the union. Hence every given two neutrosophic vertices are neutrosophic-resolved from any neutrosophic vertex in union of neutrosophic-coloring sets. It implies union of neutrosophic-coloring sets is neutrosophic-resolved set for all members of $\mathcal{G}$, simultaneously. ■

Proposition 3.9.9. *Assume $\mathcal{G}$ is a family of neutrosophic hypergraphs. Then union of neutrosophic-coloring sets from each member of $\mathcal{G}$ is neutrosophic-coloring set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of neutrosophic hypergraphs. For every chosen neutrosophic hypergraph, there's one neutrosophic-coloring set in the union of neutrosophic-coloring sets from each member of $\mathcal{G}$. Thus union of neutrosophic-coloring sets from each member of $\mathcal{G}$ is neutrosophic-coloring set for every given neutrosophic hypergraph of $\mathcal{G}$. Even one neutrosophic vertex isn't out of the union. It's neutrosophic-colored from any neutrosophic vertex in the union. Hence every given two neutrosophic vertices are neutrosophic-colored from any neutrosophic vertex in union of neutrosophic-coloring sets. It implies union of neutrosophic-coloring sets is neutrosophic-colored set for all members of $\mathcal{G}$, simultaneously. ■

Proposition 3.9.10. *Assume $\mathcal{G}$ is a family of neutrosophic hypergraphs. For every given neutrosophic vertex, there's one neutrosophic hypergraph such that the vertex has another neutrosophic vertex which are incident to a neutrosophic hyperedge. If for given neutrosophic vertex, all neutrosophic vertices have a common neutrosophic hyperedge in this way, then $V \setminus \{x\}$ is optimal-neutrosophic-dominating set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of neutrosophic hypergraphs. For all neutrosophic hypergraphs, there's no neutrosophic-dominating set from any of member of $\mathcal{G}$. Thus $V \setminus \{x\}$ is neutrosophic-dominating set for every given neutrosophic hypergraph of $\mathcal{G}$. For every given neutrosophic vertex, there's one neutrosophic hypergraph such that the vertex has another neutrosophic vertex which are incident to a neutrosophic hyperedge. Only one neutrosophic vertex is out of $V \setminus \{x\}$. It's neutrosophic-dominated from any neutrosophic vertex in the $V \setminus \{x\}$. Hence every given two neutrosophic vertices are neutrosophic-dominated from any neutrosophic vertex in $V \setminus \{x\}$. It implies $V \setminus \{x\}$ is neutrosophic-dominating set for all members of $\mathcal{G}$, simultaneously. If for given neutrosophic vertex, all neutrosophic vertices have a common neutrosophic hyperedge in this way, then $V \setminus \{x\}$ is optimal-neutrosophic-dominating set for all members of $\mathcal{G}$, simultaneously. ∎

Proposition 3.9.11. *Assume $\mathcal{G}$ is a family of neutrosophic hypergraphs. For every given neutrosophic vertex, there's one neutrosophic hypergraph such that the neutrosophic vertex has another neutrosophic vertex which are incident to a neutrosophic hyperedge. If for given neutrosophic vertex, all neutrosophic vertices have a common neutrosophic hyperedge in this way, then $V \setminus \{x\}$ is optimal-neutrosophic-resolving set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of neutrosophic hypergraphs. For all neutrosophic hypergraphs, there's no neutrosophic-resolving set from any of member of $\mathcal{G}$. Thus $V \setminus \{x\}$ is neutrosophic-resolving set for every given neutrosophic hypergraph of $\mathcal{G}$. For every given neutrosophic vertex, there's one neutrosophic hypergraph such that the vertex has another neutrosophic vertex which are incident to a neutrosophic hyperedge. Only one neutrosophic vertex is out of $V \setminus \{x\}$. It's neutrosophic-resolved from any neutrosophic vertex in the $V \setminus \{x\}$. Hence every given two neutrosophic vertices are neutrosophic-resolving from any neutrosophic vertex in $V \setminus \{x\}$. It implies $V \setminus \{x\}$ is neutrosophic-resolved set for all members of $\mathcal{G}$, simultaneously. If for given neutrosophic vertex, all neutrosophic vertices have a common neutrosophic hyperedge in this way, then $V \setminus \{x\}$ is optimal-neutrosophic-resolving set for all members of $\mathcal{G}$, simultaneously. ∎

Proposition 3.9.12. *Assume $\mathcal{G}$ is a family of neutrosophic hypergraphs. For every given neutrosophic vertex, there's one neutrosophic hypergraph such that the neutrosophic vertex has another neutrosophic vertex which are incident to a neutrosophic hyperedge. If for given neutrosophic vertex, all neutrosophic vertices have a common neutrosophic hyperedge in this way, then V is optimal-neutrosophic-coloring set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of neutrosophic hypergraphs. For all neutrosophic hypergraphs, there's no neutrosophic-coloring set from any of member of $\mathcal{G}$. Thus V is neutrosophic-coloring set for every given neutrosophic hypergraphs of $\mathcal{G}$. For every given neutrosophic vertex, there's one neutrosophic hypergraph such that the vertex has another neutrosophic vertex which are incident to a neutrosophic hyperedge. No neutrosophic vertex is out of V. It's neutrosophic-colored from any neutrosophic vertex in the V. Hence every given two neutrosophic vertices are neutrosophic-colored from any neutrosophic vertex in V. It implies V is neutrosophic-coloring set for all members of $\mathcal{G}$, simultaneously. If for given neutrosophic vertex, all neutrosophic vertices have a common neutrosophic hyperedge in this way, then V is optimal-neutrosophic-coloring set for all members of $\mathcal{G}$, simultaneously. ■

3.10 Applications in Time Table and Scheduling

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has important to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive section. Beyond that, sometimes sections are not the same.

Step 3. (Model) As Figure (3.11), the situation is designed as a model. The model uses data to assign every section and to assign to relation amid section, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relation amid them. Table (4.5), clarifies about the assigned numbers to these situation.

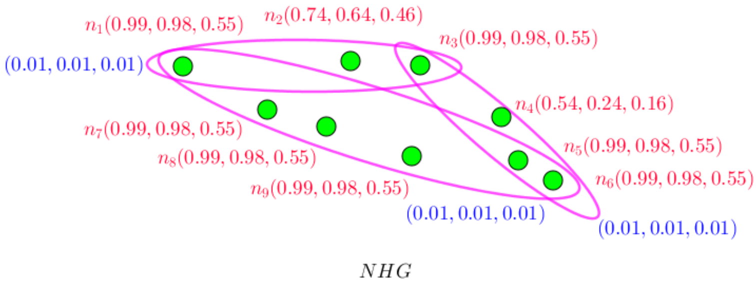


Figure 3.5: Vertices are suspicions about choosing them. nhg1

Table 3.1: Scheduling concerns its Subjects and its Connections as a Neutrosophic Hypergraph in a Model. tbl1c

Sections of NHG	n_1	$n_2 \cdots$	n_9
Values	$(0.99, 0.98, 0.55)$	$(0.74, 0.64, 0.46) \cdots$	$(0.99, 0.98, 0.55)$
Connections of NHG	E_1	E_2	E_3
Values	$(0.01, 0.01, 0.01)$	$(0.01, 0.01, 0.01)$	$(0.01, 0.01, 0.01)$

Step 4. (Solution) As Figure (3.11) shows, neutrosophic hyper graph as model, proposes to use different types of coloring, resolving and dominating as numbers, sets, optimal numbers, optimal sets and et cetera.

(a) : The notions of dominating are applied.

(i) : n_1 neutrosophic-dominates every vertex from the set of vertices $\{n_7, n_8, n_9, n_2, n_3\}$. n_4 neutrosophic-dominates every vertex from the set of vertices $\{n_6, n_5, n_3\}$. n_4 doesn't neutrosophic-dominate every vertex from the set of vertices $\{n_1, n_2, n_7, n_8, n_9\}$.

(ii) : $\{n_1, n_3\}$ is neutrosophic-coloring set but $\{n_1, n_4\}$ is optimal-neutrosophic-dominating set.

(iii) : $(1.53, 1.22, 0.71)$ is optimal-neutrosophic-dominating number.

(b) : The notions of resolving are applied.

(i) : n_1 neutrosophic-resolves two vertices n_4 and n_6.

(ii) : $V \setminus \{n_1, n_4\}$ is neutrosophic-resolves set but $V \setminus \{n_2, n_4, n_9\}$ is optimal-neutrosophic-resolving set.

(iii) : $(5, 94, 6.36, 3.3)$ is optimal-neutrosophic-resolving number.

(c) : The notions of coloring are applied.

(i) : n_1 neutrosophic-colors every vertex from the set of vertices $\{n_7, n_8, n_9, n_2, n_3\}$. n_4 neutrosophic-colors every vertex from the set of vertices $\{n_6, n_5, n_3\}$. n_4 doesn't neutrosophic-dominate every vertex from the set of vertices $\{n_1, n_2, n_7, n_8, n_9\}$.

(ii) : $\{n_1, n_5, n_7, n_8, n_9, n_6, n_4\}$ is neutrosophic-coloring set but $\{n_1, n_5, n_7, n_8, n_2, n_4\}$ is optimal-neutrosophic-coloring set.

(iii) : $(5.24, 4.8, 2.82)$ is optimal-neutrosophic-coloring number.

3.11 Open Problems

The three notions of coloring, resolving and dominating are introduced on neutrosophic hypergraphs. Thus,

Question 3.11.1. *Is it possible to use other types neutrosophic hyperedges to define different types of coloring, resolving and dominating on neutrosophic hypergraphs?*

Question 3.11.2. *Are existed some connections amid the coloring, resolving and dominating inside this concept and external connections with other types of coloring, resolving and dominating on neutrosophic hypergraphs?*

Question 3.11.3. *Is it possible to construct some classes on neutrosophic hypergraphs which have "nice" behavior?*

Question 3.11.4. *Which applications do make an independent study to apply these three types coloring, resolving and dominating on neutrosophic hypergraphs?*

Problem 3.11.5. *Which parameters are related to this parameter?*

Problem 3.11.6. *Which approaches do work to construct applications to create independent study?*

Problem 3.11.7. *Which approaches do work to construct definitions which use all three definitions and the relations amid them instead of separate definitions to create independent study?*

Table 3.2: A Brief Overview about Advantages and Limitations of this study tbl2c

Advantages	Limitations
1. Defining Dimension	1. General Results
2. Defining Domination	
3. Defining Coloring	2. Connections Amid New Notions
4. Applying on Individuals	
5. Applying on Family	3. Connections of Results

3.12 Conclusion and Closing Remarks

This study uses mixed combinations of different types of definitions, including coloring, resolving and dominating to study on neutrosophic hypergraphs. The connections of neutrosophic vertices which are clarified by general hyperedges differ them from each other and and put them in different categories to represent one representative for each color, resolver and dominator. Further studies could be about changes in the settings to compare this notion amid different settings of neutrosophic hypergraphs theory. One way is finding some relations amid three definitions of notions to make sensible definitions. In Table (4.6), some limitations and advantages of this study are pointed out.

3.13 Classes Of Neutrosophic Hypergraphs

Second case for the contents is to use the article from [**6**]. The contents are used in the way that, usages of new contents are preferences and the preliminaries are passed in the beginning of first chapter.

3.14 Co-degree and Degree of classes of Neutrosophic Hypergraphs

3.15 Abstract

New setting is introduced to study types of coloring numbers, degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic degree of vertices, neutrosophic degree of hyperedges, neutrosophic co-degree of vertices, neutrosophic co-degree of hyperedges, neutrosophic number of vertices, neutrosophic number of hyperedges in neutrosophic hypergraphs. Different types of procedures including neutrosophic $(r, n)-$regular hypergraphs and neutrosophic complete $r-$partite hypergraphs are proposed in this way, some results are obtained. General classes of neutrosophic hypergraphs are used to obtain chromatic number, the representatives of the colors, degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic degree of vertices, neutrosophic degree of hyperedges, neutrosophic co-degree of vertices, neutrosophic co-degree of hyperedges, neutrosophic num-

ber of vertices, neutrosophic number of hyperedges in neutrosophic hypergraphs. Using colors to assign to the vertices of neutrosophic hypergraphs and characterizing representatives of the colors are applied in neutrosophic $(r, n)-$regular hypergraphs and neutrosophic complete $r-$partite hypergraphs. Some questions and problems are posed concerning ways to do further studies on this topic. Using different ways of study on neutrosophic hypergraphs to get new results about number, degree and co-degree in the way that some number, degree and co-degree get understandable perspective. Neutrosophic $(r, n)-$regular hypergraphs and neutrosophic complete $r-$partite hypergraphs are studied to investigate about the notions, coloring, the representatives of the colors, degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic degree of vertices, neutrosophic degree of hyperedges, neutrosophic co-degree of vertices, neutrosophic co-degree of hyperedges, neutrosophic number of vertices, neutrosophic number of hyperedges in neutrosophic $(r, n)-$regular hypergraphs and neutrosophic complete $r-$partite hypergraphs. In this way, sets of representatives of colors, degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic degree of vertices, neutrosophic degree of hyperedges, neutrosophic co-degree of vertices, neutrosophic co-degree of hyperedges, neutrosophic number of vertices, neutrosophic number of hyperedges have key points to get new results but in some cases, there are usages of sets and numbers instead of optimal ones. Simultaneously, notions chromatic number, the representatives of the colors, degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic degree of vertices, neutrosophic degree of hyperedges, neutrosophic co-degree of vertices, neutrosophic co-degree of hyperedges, neutrosophic number of vertices, neutrosophic number of hyperedges are applied into neutrosophic hypergraphs, especially, neutrosophic $(r, n)-$regular hypergraphs and neutrosophic complete $r-$partite hypergraphs to get sensible results about their structures. Basic familiarities with neutrosophic hypergraphs theory and hypergraph theory are proposed for this article.

Keywords: Degree, Coloring, Co-degree

AMS Subject Classification: 05C17, 05C22, 05E45

3.16 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 3.16.1. *Is it possible to use mixed versions of ideas concerning "neutrosophic degree", "neutrosophic co-degree" and "neutrosophic coloring" to define some notions which are applied to neutrosophic hypergraphs?*

It's motivation to find notions to use in any classes of neutrosophic hypergraphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Connections amid two items have key roles to assign colors and introducing different types of degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic degree of vertices, neutrosophic degree of hyperedges, neutrosophic co-degree of vertices, neutrosophic co-degree of hyperedges, neutrosophic number of vertices, neutrosophic number of hyperedges in neutrosophic hypergraphs. Thus they're used to define new ideas which conclude to the structure of coloring, degree and co-degree. The concept of having general neutrosophic hyperedge inspires me to study the behavior of general neutrosophic hyperedge in the way that, types of coloring numbers, degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic degree of vertices, neutrosophic degree of hyperedges, neutrosophic co-degree of vertices, neutrosophic co-degree of hyperedges, neutrosophic number of vertices, neutrosophic number of hyperedges in neutrosophic hypergraphs are introduced. The framework of this study is as follows. In the beginning, I introduced basic definitions to clarify about preliminaries. In section "New Ideas For Neutrosophic Hypergraphs", new notions of coloring, degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic degree of vertices, neutrosophic degree of hyperedges, neutrosophic co-degree of vertices, neutrosophic co-degree of hyperedges, neutrosophic number of vertices, neutrosophic number of hyperedges in neutrosophic hypergraphs are introduced. In section "Applications in Time Table and Scheduling", one application is posed for neutrosophic hypergraphs concerning time table and scheduling when the suspicions are about choosing some subjects. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications are featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions are formed.

3.17 New Ideas For Neutrosophic Hypergraphs

Question 3.17.1. *What-if the notion of complete proposes some classes of neutrosophic hypergraphs?*

In the setting of neutrosophic hypergraphs, the notion of complete have introduced some classes. Since the vertex could have any number of arbitrary hyperedges. This notion is too close to the notion of regularity. Thus the idea of complete has an obvious structure in that, every hyperedge has n vertices so there's only one hyperedge.

Definition 3.17.2. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. It's denoted by NHG^r_n and it's $(r, n)-$ **regular** if every hyperedge has exactly r vertices in the way that, all $r-$subsets of the vertices have an unique hyperedge where $r \leq n$ and $|V| = n$.

Example 3.17.3. In Figure (3.9), NHG^3_4 is shown.

Definition 3.17.4. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$.

(i) : **Maximum number** is maximum number of hyperedges which are incident to a vertex and it's denoted by $\Delta(NHG)$;

(ii) : **Minimum number** is minimum number of hyperedges which are incident to a vertex and it's denoted by $\delta(NHG)$;

(iii) : **Maximum value** is maximum value of vertices and it's denoted by $\Delta_n(NHG)$;

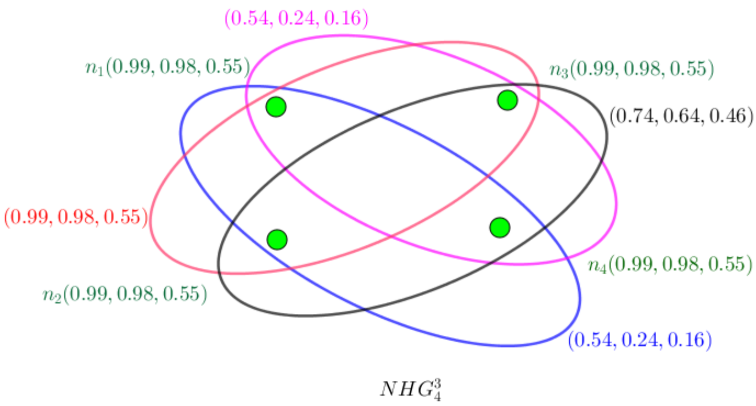


Figure 3.6: $NHG_4^3 = (V, E, \sigma, \mu)$ is neutrosophic $(3, 4)-$ regular hypergraph.

nhg6

$(iv):$ **Minimum value** is minimum value of vertices and it's denoted by $\delta_n(NHG)$.

Example 3.17.5. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ as Figure (3.9).

$(i):$ $\Delta(NHG) = 3$;

$(ii):$ $\delta(NHG) = 3$;

$(iii):$ $\Delta_n(NHG) = (0.99, 0.98, 0.55)$;

$(iv):$ $\delta_n(NHG) = (0.99, 0.98, 0.55)$.

Proposition 3.17.6. *Assume neutrosophic hypergraph $NHG_n^r = (V, E, \sigma, \mu)$ which is $(r, n)-$ regular. Then $\Delta(NHG) = \delta(NHG)$.*

Proof. Consider neutrosophic hypergraph $NHG_n^r = (V, E, \sigma, \mu)$ which is $(r, n)-$ regular. Every hyperedge has same number of vertices. Hyperedges are distinct. It implies the number of hyperedges which are incident to every vertex is the same. ■

Proposition 3.17.7. *Assume neutrosophic hypergraph $NHG_n^r = (V, E, \sigma, \mu)$ which is $(r, n)-$ regular. Then the number of hyperedges equals to n choose r.*

Proof. Suppose neutrosophic hypergraph $NHG_n^r = (V, E, \sigma, \mu)$ which is $(r, n)-$ regular. Every hyperedge has r vertices. Thus $r-$subsets of n form hyperedges. It induces n choose r. ■

Proposition 3.17.8. *Assume neutrosophic hypergraph $NHG_n^r = (V, E, \sigma, \mu)$ which is $(r, n)-$ regular. Then*

$(i):$ *Chromatic number is at least r;*

$(ii):$ *Chromatic number is at most Δr;*

$(iii):$ *Neutrosophic chromatic number is at most $\Delta_n r$.*

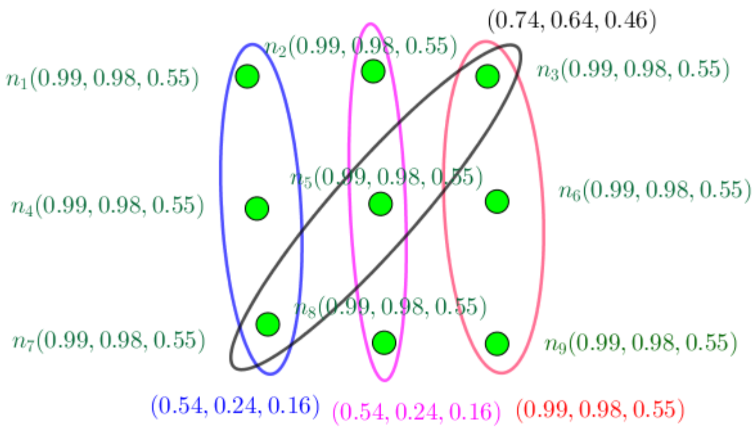
Proof. (i). Suppose $NHG_n^r = (V, E, \sigma, \mu)$. Every hyperedge has r vertices. It implies the set of representatives has at least r members. Hence chromatic number is at least r.
(ii). Suppose $NHG_n^r = (V, E, \sigma, \mu)$. Every hyperedge has r vertices. It implies the set of representatives has at least r members. If all vertices have at least one common hyperedge, then chromatic number is at most Δr. Thus chromatic number is at most Δr.
(iii). Consider $NHG_n^r = (V, E, \sigma, \mu)$. Every hyperedge has r vertices. It implies the set of representatives has at least r members. If all vertices have at least one common hyperedge, then neutrosophic chromatic number is at most $\Delta_n r$. Thus neutrosophic chromatic number is at most $\Delta_n r$. ∎

Question 3.17.9. *What-if the notion of complete proposes some classes of neutrosophic hypergraphs with some parts?*

In the setting of neutrosophic hypergraphs, when every part has specific attribute inside and outside, the notion of complete is applied to parts to form the idea of completeness.

Definition 3.17.10. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. It's denoted by $NHG^r_{n_1, n_2, \cdots, n_r}$ and it's **complete** $r-$**partite** if V can be partitioned into r non-empty parts, V_i, and every hyperedge has only one vertex from each part where n_i is the number of vertices in part V_i.

Example 3.17.11. In Figure (3.10), $NHG^3_{3,3,3} = (V, E, \sigma, \mu)$ is shown.



$NHG^3_{3,3,3}$

Figure 3.7: $NHG^3_{3,3,3} = (V, E, \sigma, \mu)$ is neutrosophic complete $3-$partite hypergraph.

nhg7

Proposition 3.17.12. *For any given r, the number of neutrosophic complete $r-$partite hypergraph $NHG^r_{p_1, p_2, \cdots, p_r} = (V, E, \sigma, \mu)$ is at most*

$$p_1 \times p_2, \times \cdots \times p_r.$$

Proof. Assume r is given. Consider $NHG^r_{p_1,p_2,\cdots,p_r} = (V, E, \sigma, \mu)$ is neutrosophic complete $r-$partite hypergraph. Any possible hyperedge has to choose exactly one vertex from every part. First part has p_1 vertices. Thus there are p_1 choices. Second part has p_2 vertices and et cetera. Thus for any given r, the number of neutrosophic complete $r-$partite hypergraph $NHG^r_{p_1,p_2,\cdots,p_r} = (V, E, \sigma, \mu)$ is at most

$$p_1 \times p_2 \times \cdots \times p_r.$$

■

Proposition 3.17.13. *Assume neutrosophic complete $r-$partite hypergraph $NHG^r_{n_1,n_2,\cdots,n_r} = (V, E, \sigma, \mu)$. Then*

(i) : *Chromatic number is at least r;*

(ii) : *Neutrosophic chromatic number is at least*

$$\min_{X \subseteq V and\ X\ is\ r\text{-}subset} \Sigma_{x\in X}\sigma(x).$$

Proof. (i). Suppose neutrosophic complete $r-$partite hypergraph $NHG^r_{n_1,n_2,\cdots,n_r}$. Every hyperedge has r vertices. It implies the set of representatives has r members. Hence chromatic number is least r.
(ii). Consider neutrosophic complete $r-$partite hypergraph $NHG^r_{n_1,n_2,\cdots,n_r}$. Every hyperedge has r vertices. It implies the set of representatives has r members. If all vertices have at least one common hyperedge, then neutrosophic chromatic number is at least $\min_{X\subseteq V \text{and } X \text{ is r-subset}} \Sigma_{x\in X}\sigma(x)$. ■

Definition 3.17.14. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$.

(i) : A **neutrosophic number** of vertices $x_1, x_2, \cdots, x_n$ is

$$\Sigma^n_{i=1}\sigma(x_i).$$

(ii) : A **neutrosophic number** of hyperedges $e_1, e_2, \cdots, e_n$ is

$$\Sigma^n_{i=1}\mu(e_i).$$

Example 3.17.15. I get some clarifications about new definitions.

(i) : In Figure (3.9), NHG^3_4 is shown.

  (a) : A neutrosophic number of vertices n_1, n_2, n_3 is

$$\Sigma^3_{i=1}\sigma(n_i) = (2.97, 2.94, 1.65).$$

  (b) : A neutrosophic number of hyperedges e_1, e_2, e_3 is

$$\Sigma^3_{i=1}\sigma(e_i) = (1.82, 1.12, 0.78).$$

  where $e_1 = (0.54, 0.24, 0.16), e_2 = (0.74, 0.64, 0.46), e_3 = (0.54, 0.24, 0.16)$.

(ii) : In Figure (3.10), $NHG^3_{3,3,3} = (V, E, \sigma, \mu)$ is shown.

(a) : A neutrosophic number of vertices n_1, n_2, n_3 is

$$\Sigma_{i=1}^{3}\sigma(n_i) = (2.97, 2.94, 1.65).$$

(b) : A neutrosophic number of hyperedges e_1, e_2, e_3 is

$$\Sigma_{i=1}^{3}\sigma(e_i) = (1.82, 1.12, 0.78).$$

where $e_1 = (0.54, 0.24, 0.16), e_2 = (0.74, 0.64, 0.46), e_3 = (0.54, 0.24, 0.16)$.

Proposition 3.17.16. *Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. A neutrosophic number of vertices is at least δ_n and at most $\mathcal{O}_n$.*

Proof. Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Let v be a given vertex. Then $\sigma(v) \geq \min_{v \in V} \sigma(v)$. Thus $\sigma(v) \geq \delta_n$. So a neutrosophic number of vertices is at least δ_n. $\sigma(v) \leq \Sigma_{v \in V}\sigma(v)$. Thus $\sigma(v) \leq \mathcal{O}_n$. So a neutrosophic number of vertices is at most $\mathcal{O}_n$. Hence a neutrosophic number of vertices is at least δ_n and at most $\mathcal{O}_n$. ■

Proposition 3.17.17. *Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. A neutrosophic number of hyperedges is at least δ_n^e and at most $\mathcal{S}_n$ where $\delta_n^e = \min_{e \in E} \mu(e)$.*

Proof. Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. Let e be a given hyperedge. Then $\mu(e) \geq \min_{e \in E} \mu(e)$. Thus $\mu(v) \geq \delta_n^e$. So a neutrosophic number of hyperedges is at least δ_n^e. $\mu(e) \leq \Sigma_{e \in E}\mu(e)$. Thus $\mu(e) \leq \mathcal{S}_n$. So a neutrosophic number of hyperedges is at most $\mathcal{S}_n$. Hence a neutrosophic number of hyperedges is at least δ_n^e and at most $\mathcal{S}_n$. ■

Definition 3.17.18. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$.

(i) : A **degree** of vertex x is the number of hyperedges which are incident to x.

(ii) : A **neutrosophic degree** of vertex x is the neutrosophic number of hyperedges which are incident to x.

(iii) : A **degree** of hyperedge e is the number of vertices which e is incident to them.

(iv) : A **neutrosophic degree** of hyperedge e is the neutrosophic number of vertices which e is incident to them.

(v) : A **co-degree** of vertices $x_1, x_2, \cdots, x_n$ is the number of hyperedges which are incident to $x_1, x_2, \cdots, x_n$.

(vi) : A **neutrosophic co-degree** of vertices $x_1, x_2, \cdots, x_n$ is the neutrosophic number of hyperedges which are incident to $x_1, x_2, \cdots, x_n$.

(vii) : A **co-degree** of hyperedges $e_1, e_2, \cdots, e_n$ is the number of vertices which $e_1, e_2, \cdots, e_n$ are incident to them.

$(viii)$: A **neutrosophic co-degree** of hyperedges $e_1, e_2, \cdots, e_n$ is the neutrosophic number of vertices which $e_1, e_2, \cdots, e_n$ are incident to them.

Example 3.17.19. I get some clarifications about new definitions.

(i) : In Figure (3.9), NHG_4^3 is shown.

- (a) : A degree of any vertex is 3.
- (b) : A neutrosophic degree of vertex n_1 is $(2.07, 1.46, 0.87)$.
- (c) : A degree of hyperedge e where $\mu(e) = (0.99, 0.98, 0.55)$ is 3.
- (d) : A neutrosophic degree of hyperedge e where $\mu(e) = (0.99, 0.98, 0.55)$ is $(2.97, 2.94, 1.65)$.
- (e) : A co-degree of vertices n_1, n_3 is 2.
- (f) : A neutrosophic co-degree of vertices n_1, n_3 is $(1.53, 1.22, 0.71)$.
- (g) : A co-degree of hyperedges e_1, e_2 where $\mu(e_1) = (0.99, 0.98, 0.55)$ and $\mu(e_2) = (0.54, 0.24, 0.16)$ is 2.
- (h) : A neutrosophic co-degree of hyperedges e_1, e_2 where $\mu(e_1) = (0.99, 0.98, 0.55)$ and $\mu(e_2) = (0.54, 0.24, 0.16)$ is $(1.98, 1.96, 1.1)$.

(ii) : In Figure (3.10), $NHG_{3,3,3}^3 = (V, E, \sigma, \mu)$ is shown.

- (a) : A degree of any vertex $n_1, n_2, n_4, n_6, n_8, n_9$ is 1 and degree of any vertex n_3, n_5, n_7 is 2.
- (b) : A neutrosophic degree of vertex $n_1, n_2, n_4, n_6, n_8, n_9$ is $(0.99, 0.98, 0.55)$ and degree of any vertex n_3, n_5, n_7 is $(1.98, 1.96, 1.1)$.
- (c) : A degree of any hyperedge is 3.
- (d) : A neutrosophic degree of hyperedge is $(2.97, 2.94, 1.65)$.
- (e) : A co-degree of vertices n_1, n_4 is 1.
- (f) : A neutrosophic co-degree of vertices n_1, n_4 is $(0.54, 0.24, 0.16)$.
- (g) : A co-degree of hyperedges e_1, e_2 where $\mu(e_1) = (0.99, 0.98, 0.55)$ and $\mu(e_2) = (0.54, 0.24, 0.16)$ is 1.
- (h) : A neutrosophic co-degree of hyperedges e_1, e_2 where $\mu(e_1) = (0.99, 0.98, 0.55)$ and $\mu(e_2) = (0.54, 0.24, 0.16)$ is $(0.99, 0.98, 0.55)$.

Proposition 3.17.20. *Assume neutrosophic complete $r-$partite hypergraph $NHG_{p_1, p_2, \cdots, p_r}^r = (V, E, \sigma, \mu)$.*

(i) : *A degree of vertex x is at most*

$$p_2 \times \cdots \times p_r.$$

(ii) : *A degree of hyperedge e is r.*

(iii) : *A co-degree of vertices $x_1, x_2, \cdots, x_t$ is at most*

$$p_{t+1} \times \cdots \times p_r.$$

(iv) : *A co-degree of hyperedges $e_1, e_2, \cdots, e_t$ is $r - t$.*

Proof. (i). Suppose neutrosophic complete $r-$partite hypergraph $NHG^r_{p_1,p_2,\cdots,p_r} = (V,E,\sigma,\mu)$. Vertex x belongs to part first part. x is chosen so for second part, there are p_2 choices and et cetera. By it's neutrosophic complete $r-$partite hypergraph $NHG^r_{p_1,p_2,\cdots,p_r} = (V,E,\sigma,\mu)$, possible choice from every part is exactly one vertex. It induces for second part, one vertex has to be chosen and et cetera. Therefore the number of neutrosophic complete $r-$partite hypergraph $NHG^r_{p_1,p_2,\cdots,p_r} = (V,E,\sigma,\mu)$, when x is chosen, introduces biggest possible number of degree of x which is $p_2 \times \cdots \times p_r$. Hence a degree of vertex x is at most

$$p_2 \times \cdots \times p_r.$$

(ii). Consider neutrosophic complete $r-$partite hypergraph $NHG^r_{p_1,p_2,\cdots,p_r} = (V,E,\sigma,\mu)$. Vertex x belongs to part first part. x is chosen so for second part, there is one choice and et cetera. By it's neutrosophic complete $r-$partite hypergraph $NHG^r_{p_1,p_2,\cdots,p_r} = (V,E,\sigma,\mu)$, possible choice from every part is exactly one vertex. It induces for second part, one vertex has to be chosen and et cetera. Therefore neutrosophic complete $r-$partite hypergraph $NHG^r_{p_1,p_2,\cdots,p_r} = (V,E,\sigma,\mu)$ introduces exact number of degree of e which is r. Hence a degree of hyperedge e is

$$r.$$

(iii). Suppose neutrosophic complete $r-$partite hypergraph $NHG^r_{p_1,p_2,\cdots,p_r} = (V,E,\sigma,\mu)$. Vertices $x_1,x_2,\cdots,x_t$ belong to part first part, second part,...., and part t. $x_1,x_2,\cdots,x_t$ are chosen so for part $t+1$, there are p_{t+1} choices and et cetera. By it's neutrosophic complete $r-$partite hypergraph $NHG^r_{p_1,p_2,\cdots,p_r} = (V,E,\sigma,\mu)$, possible choice from every part is exactly one vertex. It induces for part $t+1$, one vertex has to be chosen and et cetera. Therefore the number of neutrosophic complete $r-$partite hypergraph $NHG^r_{p_1,p_2,\cdots,p_r} = (V,E,\sigma,\mu)$, when $x_1,x_2,\cdots,x_t$ are chosen, introduces biggest possible number of co-degree of $x_1,x_2,\cdots,x_t$ which is $p_{t+1} \times \cdots \times p_r$. Hence a co-degree of vertices $x_1,x_2,\cdots,x_t$ is at most

$$p_{t+1} \times \cdots \times p_r.$$

(iv). Consider neutrosophic complete $r-$partite hypergraph $NHG^r_{p_1,p_2,\cdots,p_r} = (V,E,\sigma,\mu)$. Vertex x belongs to part first part. x is chosen so for second part, there is one choice and et cetera. By it's neutrosophic complete $r-$partite hypergraph $NHG^r_{p_1,p_2,\cdots,p_r} = (V,E,\sigma,\mu)$, possible choice from every part is exactly one vertex. It induces for second part, one vertex has to be chosen and et cetera. Therefore neutrosophic complete $r-$partite hypergraph $NHG^r_{p_1,p_2,\cdots,p_r} = (V,E,\sigma,\mu)$ introduces exact number of co-degree of $e_1,e_2,\cdots,e_t$ which is $r-t$. Hence a co-degree of hyperedges $e_1,e_2,\cdots,e_t$ is

$$r-t.$$

∎

Proposition 3.17.21. *Assume neutrosophic hypergraph $NHG = (V,E,\sigma,\mu)$ where E is power set of V. Then the number of hyperedges is*

$$2^n.$$

Proof. Consider neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ where E is power set of V. The cardinality of E is 2^n. The number of hyperedges is

$$2^n.$$

■

Proposition 3.17.22. *Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ where E is power set of V. Then*

(i) : *A degree of vertex x is*
$$2^{n-1}.$$

(ii) : *A degree of hyperedge e is at most*
$$\mathcal{O}$$
and at least
$$0.$$

(iii) : *A co-degree of vertices $x_1, x_2, \cdots, x_t$ is at most*
$$2^{n-t}.$$

(iv) : *A co-degree of hyperedges $e_1, e_2, \cdots, e_t$ is at most*
$$\mathcal{O} - t$$
and at least
$$0.$$

Proof. (i). Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ where E is power set of V. Vertex x is chosen. Thus all hyperedges have to have x. It induces E' is power set of $V \setminus \{x\}$. The cardinality of E' is 2^{n-1}. So the number of hyperedges which are incident to x, is 2^{n-1}. It implies a degree of vertex x is

$$2^{n-1}.$$

(ii). Consider neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ where E is power set of V. Hyperedge e is chosen. Thus a hyperedge has either all vertices or no vertex. It induces for hyperedge e, the number of vertices is either $\mathcal{O}$ or 0. Then a degree of hyperedge e is at most

$$\mathcal{O}$$

and at least

$$0.$$

(iii). Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ where E is power set of V. Vertices $x_1, x_2, \cdots, x_t$ are chosen. Thus all hyperedges have to have $x_1, x_2, \cdots, x_t$. It induces E' is power set of $V \setminus \{x_1, x_2, \cdots, x_t\}$. The cardinality of E' is 2^{n-t}. So the number of hyperedges which are incident to $x_1, x_2, \cdots, x_t$, is 2^{n-t}. It implies a co-degree of vertices $x_1, x_2, \cdots, x_t$ is

$$2^{n-t}.$$

(iv). Consider neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ where E is power set of V. Hyperedges $e_1, e_2, \cdots, e_t$ are chosen. Thus hyperedges $e_1, e_2, \cdots, e_t$ don't have all vertices. Since one edge is incident to all vertices and there's no second edge to be incident to all vertices. It implies hyperedges $e_1, e_2, \cdots, e_t$ have all vertices excluding only t vertices or no vertex. It induces for hyperedges $e_1, e_2, \cdots, e_t$, the number of vertices is either $\mathcal{O} - t$ or 0. Hence a co-degree of hyperedges $e_1, e_2, \cdots, e_t$ is at most

$$\mathcal{O} - t$$

and at least

$$0.$$

■

Proposition 3.17.23. *Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ where E is power set of V. Then*

(i) : *Chromatic number is*

$$\mathcal{O};$$

(ii) : *Neutrosophic chromatic number is*

$$\mathcal{O}_n.$$

Proof. (i). Suppose neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ where E is power set of V. Every hyperedge has either of $0, 1, 2, \cdots, \mathcal{O}$ vertices but for any of two vertices, there's at least one hyperedge which is incident to them. Furthermore, all vertices have at least one common hyperedge which is V.. Since $V \in E$ and V is also a hyperedge. It implies the set of representatives has $\mathcal{O}$ members. Hence chromatic number is

$$\mathcal{O}.$$

(ii). Consider neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ where E is power set of V. Every hyperedge has either of $0, 1, 2, \cdots, \mathcal{O}$ vertices but for any of two vertices, there's at least one hyperedge which is incident to them. Furthermore, all vertices have at least one common hyperedge which is V.. Since $V \in E$ and V is also a hyperedge. It implies the set of representatives has $\mathcal{O}$ members. Hence neutrosophic chromatic number is

$$\mathcal{O}_n.$$

■

3.18 Applications in Time Table and Scheduling

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has important to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive section. Beyond that, sometimes sections are not the same.

Step 3. (Model) As Figure (3.12), the situation is designed as a model. The model uses data to assign every section and to assign to relation amid section, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relation amid them. Table (4.5), clarifies about the assigned numbers to these situation.

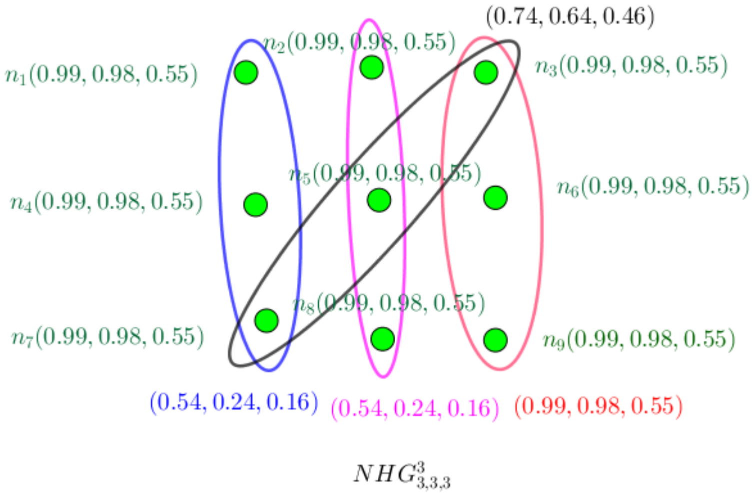


Figure 3.8: Vertices are suspicions about choosing them.

nhg8

Table 3.3: Scheduling concerns its Subjects and its Connections as a Neutrosophic Hypergraph in a Model.

tbl1c

Sections of NHG	n_1	$n_2 \cdots$	n_9
Values	$(0.99, 0.98, 0.55)$	$(0.74, 0.64, 0.46) \cdots$	$(0.99, 0.98, 0.55)$
Connections of NHG	E_1, E_2	E_3	E_4
Values	$(0.54, 0.24, 0.16)$	$(0.99, 0.98, 0.55)$	$(0.74, 0.64, 0.46)$

Step 4. (Solution) As Figure (3.12) shows, $NHG^3_{3,3,3} = (V, E, \sigma, \mu)$ is neutrosophic complete 3−partite hypergraph as model, proposes to use different types of degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic number of vertices, neutrosophic number of hyperedges and et cetera.

(i) : The notions of neutrosophic number are applied on vertices and hyperedges.

(a) : A neutrosophic number of vertices n_1, n_2, n_3 is

$$\Sigma_{i=1}^3 \sigma(n_i) = (2.97, 2.94, 1.65).$$

(b) : A neutrosophic number of hyperedges e_1, e_2, e_3 is

$$\Sigma_{i=1}^3 \sigma(e_i) = (1.82, 1.12, 0.78).$$

where $e_1 = (0.54, 0.24, 0.16), e_2 = (0.74, 0.64, 0.46), e_3 = (0.54, 0.24, 0.16)$.

(ii) : The notions of degree, co-degree, neutrosophic degree and neutrosophic co-degree are applied on vertices and hyperedges.

(a) : A degree of any vertex $n_1, n_2, n_4, n_6, n_8, n_9$ is 1 and degree of any vertex n_3, n_5, n_7 is 2.

(b) : A neutrosophic degree of vertex $n_1, n_2, n_4, n_6, n_8, n_9$ is $(0.99, 0.98, 0.55)$ and degree of any vertex n_3, n_5, n_7 is $(1.98, 1.96, 1.1)$.

(c) : A degree of any hyperedge is 3.

(d) : A neutrosophic degree of hyperedge is $(2.97, 2.94, 1.65)$.

(e) : A co-degree of vertices n_1, n_4 is 1.

(f) : A neutrosophic co-degree of vertices n_1, n_4 is $(0.54, 0.24, 0.16)$.

(g) : A co-degree of hyperedges e_1, e_2 where $\mu(e_1) = (0.99, 0.98, 0.55)$ and $\mu(e_2) = (0.54, 0.24, 0.16)$ is 1.

(h) : A neutrosophic co-degree of hyperedges e_1, e_2 where $\mu(e_1) = (0.99, 0.98, 0.55)$ and $\mu(e_2) = (0.54, 0.24, 0.16)$ is $(0.99, 0.98, 0.55)$.

3.19 Open Problems

The different types of degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic number of vertices, neutrosophic number of hyperedges are introduced on neutrosophic hypergraphs. Thus,

Question 3.19.1. *Is it possible to use other types neutrosophic hyperedges to define different types of degree and co-degree in neutrosophic hypergraphs?*

Question 3.19.2. *Are existed some connections amid degree and co-degree inside this concept and external connections with other types of neutrosophic degree and neutrosophic co-degree in neutrosophic hypergraphs?*

Question 3.19.3. *Is it possible to construct some classes on neutrosophic hypergraphs which have "nice" behavior?*

Question 3.19.4. *Which applications do make an independent study to apply these types degree, co-degree, neutrosophic degree and neutrosophic co-degree in neutrosophic hypergraphs?*

Problem 3.19.5. *Which parameters are related to this parameter?*

Problem 3.19.6. *Which approaches do work to construct applications to create independent study?*

Problem 3.19.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

3.20 Conclusion and Closing Remarks

This study introduces different types of degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic degree of vertices, neutrosophic degree of hyperedges, neutrosophic co-degree of vertices, neutrosophic co-degree of hyperedges, neutrosophic number of vertices, neutrosophic number of hyperedges in neutrosophic hypergraphs. The connections of neutrosophic vertices which are clarified by general hyperedges differ them from each other and and put them in different categories to represent one representative for each color. Further studies could be about changes in the settings to compare this notion amid different settings of neutrosophic hypergraphs theory. One way is finding some relations amid these definitions of notions to make sensible definitions. In Table (4.6), some limitations and some advantages of this study are pointed out.

Table 3.4: A Brief Overview about Advantages and Limitations of this study

tbl2c

Advantages	Limitations
1. Defining degree	1. General Results
2. Defining co-degree	
3. Defining neutrosophic degree	2. Connections With Parameters
4. Applying colortring	
5. Defining neutrosophic co-degree	3. Connections of Results

3.21 Beyond Neutrosophic Hypergraphs

Extended settings are used to apply Neutrosophic ideas.

Third case for the contents is to use the article from [**5**]. The contents are used in the way that, usages of new contents are preferences and the preliminaries are passed in the beginning of first chapter.

3.22 Closing Numbers and Super-Closing Numbers as (Dual)Resolving and (Dual)Coloring alongside (Dual)Dominating in (Neutrosophic)n-SuperHyperGraph

3.23 Abstract

New setting is introduced to study "closing numbers" and "super-closing numbers" as optimal-super-resolving number, optimal-super-coloring number and optimal-super-dominating number. In this way, some approaches are applied to get some sets from (Neutrosophic)n-SuperHyperGraph and after that, some ideas are applied to get different types of super-closing numbers which are called by optimal-super-resolving number, optimal-super-coloring number and optimal-super-dominating number. The notion of dual is another new idea which is covered by these notions and results. In the setting of dual, the set of super-vertices is exchanged with the set of super-edges. Thus these results and definitions hold in the setting of dual. Setting of neutrosophic n-SuperHyperGraph is used to get some examples and solutions for two applications which are proposed. Both setting of SuperHyperGraph and neutrosophic n-SuperHyperGraph are simultaneously studied but the results are about the setting of n-SuperHyperGraphs. Setting of neutrosophic n-SuperHyperGraph get some examples where neutrosophic hypergraphs as special case of neutrosophic n-SuperHyperGraph are used. The clarifications use neutrosophic n-SuperHyperGraph and theoretical study is to use n-SuperHyperGraph but these results are also applicable into neutrosophic n-SuperHyperGraph. Special usage from different attributes of neutrosophic n-SuperHyperGraph are appropriate to have open ways to pursue this study. Different types of procedures including optimal-super-set, and optimal-super-number alongside study on the family of (neutrosophic)n-SuperHyperGraph are proposed in this way, some results are obtained. General classes of (neutrosophic)n-SuperHyperGraph are used to obtains these closing numbers and super-closing numbers and the representatives of the optimal-super-coloring sets, optimal-super-dominating sets and optimal-super-resolving sets. Using colors to assign to the super-vertices of n-SuperHyperGraph and characterizing optimal-super-resolving sets and optimal-super-dominating sets are applied. Some questions and problems are posed concerning ways to do further studies on this topic. Using different ways of study on n-SuperHyperGraph to get new results about closing numbers and super-closing numbers alongside sets in the way that some closing numbers super-closing numbers get understandable perspective. Family of n-SuperHyperGraph are studied to investigate about the notions, super-resolving and super-coloring alongside super-dominating in n-SuperHyperGraph. In this way, sets of representatives of optimal-super-colors, optimal-super-resolving sets and optimal-super-dominating sets have key role. Optimal-super sets and optimal-super numbers have key points to get new results but in some cases, there are usages of sets and numbers instead of optimal-super ones. Simultaneously, three notions are applied into (neutrosophic)n-SuperHyperGraph to get sensible results about their structures. Basic familiarities with n-SuperHyperGraph theory and neutrosophic n-SuperHyperGraph theory are proposed for this article.

Keywords: Coloring Numbers, Resolving Numbers, Dominating Numbers

AMS Subject Classification: 05C17, 05C22, 05E45

Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 3.23.1. *Is it possible to use mixed versions of ideas concerning "super-domination", "super-dimension" and "super-coloring" to define some super-notions which are applied to n-SuperHyperGraph?*

It's motivation to find notions to use in any classes of n-SuperHyperGraph. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Connections amid two items have key roles to assign super-colors, super-domination and super-dimension. Thus they're used to define new super-ideas which conclude to the structure of super-coloring, super-dominating and super-resolving. The concept of having general super-edge inspires me to study the behavior of general super-edge in the way that, three types of "**super-closing**" numbers, e.g., super-coloring numbers, super-dominating numbers and super-resolving numbers are the cases of study in the settings of individuals and in settings of families.
The framework of this study is as follows. In the beginning, I introduced basic definitions to clarify about preliminaries. In section "New Ideas For n-SuperHyperGraph", new notions of super-coloring, super-dominating and super-resolving are applied to super-vertices of SuperHyperGraph as individuals. In section "Optimal Numbers For n-SuperHyperGraph", specific closing numbers have the key role in this way. Classes of n-SuperHyperGraph are studied in the terms of different closing numbers in section "Optimal Numbers For n-SuperHyperGraph" as individuals. In the section "Optimal Sets For n-SuperHyperGraph", usages of general sets and special sets have key role in this study as individuals. In section "Optimal Sets and Numbers For Family of n-SuperHyperGraph", both sets and closing numbers have applied into the family of n-SuperHyperGraph. In section "Applications in Time Table and Scheduling", two applications are posed for n-SuperHyperGraph concerning time table and scheduling when the suspicions are about choosing some subjects. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications are featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions are formed.

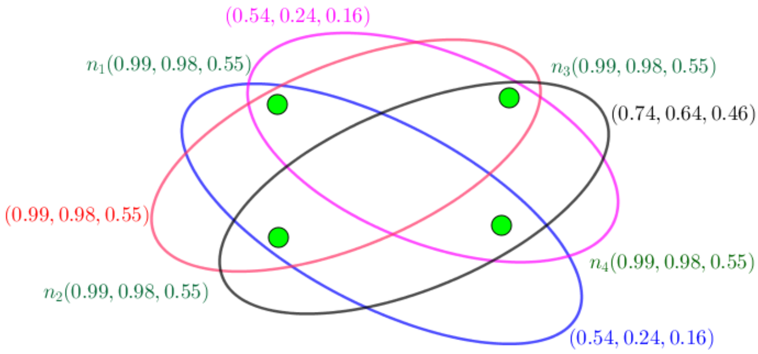
3.24 New Ideas For Setting of Neutrosophic n-SuperHyperGraph

Question 3.24.1. *What-if the notion of complete proposes some classes of neutrosophic hypergraphs?*

In the setting of neutrosophic hypergraphs, the notion of complete have introduced some classes. Since the vertex could have any number of arbitrary hyperedges. This notion is too close to the notion of regularity. Thus the idea of complete has an obvious structure in that, every hyperedge has n vertices so there's only one hyperedge.

Definition 3.24.2. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. It's denoted by NHG^r_n and it's $(r, n)-$ **regular** if every hyperedge has exactly r vertices in the way that, all $r-$subsets of the vertices have an unique hyperedge where $r \leq n$ and $|V| = n$.

Example 3.24.3. In Figure (3.9), NHG^3_4 is shown.



NHG^3_4

Figure 3.9: $NHG^3_4 = (V, E, \sigma, \mu)$ is neutrosophic $(3, 4)-$ regular hypergraph.

nhg6

Definition 3.24.4. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$.

(i) : **Maximum number** is maximum number of hyperedges which are incident to a vertex and it's denoted by $\Delta(NHG)$;

(ii) : **Minimum number** is minimum number of hyperedges which are incident to a vertex and it's denoted by $\delta(NHG)$;

(iii) : **Maximum value** is maximum value of vertices and it's denoted by $\Delta_n(NHG)$;

(iv) : **Minimum value** is minimum value of vertices and it's denoted by $\delta_n(NHG)$.

Example 3.24.5. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$ as Figure (3.9).

(i) : $\Delta(NHG) = 3$;

(ii) : $\delta(NHG) = 3$;

(iii) : $\Delta_n(NHG) = (0.99, 0.98, 0.55)$;

(iv) : $\delta_n(NHG) = (0.99, 0.98, 0.55)$.

Question 3.24.6. *What-if the notion of complete proposes some classes of neutrosophic hypergraphs with some parts?*

In the setting of neutrosophic hypergraphs, when every part has specific attribute inside and outside, the notion of complete is applied to parts to form the idea of completeness.

Definition 3.24.7. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$. It's denoted by $NHG^{r}_{n_1,n_2,\cdots,n_r}$ and it's **complete** $r-$**partite** if V can be partitioned into r non-empty parts, V_i, and every hyperedge has only one vertex from each part where n_i is the number of vertices in part V_i.

Example 3.24.8. In Figure (3.10), $NHG^{3}_{3,3,3} = (V, E, \sigma, \mu)$ is shown.

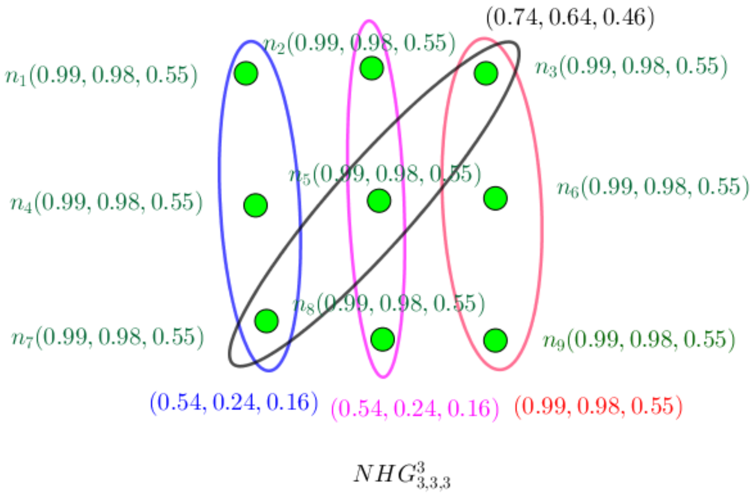


Figure 3.10: $NHG^{3}_{3,3,3} = (V, E, \sigma, \mu)$ is neutrosophic complete $3-$partite hypergraph.

nhg7

Definition 3.24.9. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$.

(i) : A **neutrosophic number** of vertices $x_1, x_2, \cdots, x_n$ is

$$\Sigma_{i=1}^{n}\sigma(x_i).$$

(ii) : A **neutrosophic number** of hyperedges $e_1, e_2, \cdots, e_n$ is

$$\Sigma_{i=1}^{n}\mu(e_i).$$

Example 3.24.10. I get some clarifications about new definitions.

(i) : In Figure (3.9), NHG^{3}_{4} is shown.

(a) : A neutrosophic number of vertices n_1, n_2, n_3 is

$$\Sigma_{i=1}^{3}\sigma(n_i) = (2.97, 2.94, 1.65).$$

(b) : A neutrosophic number of hyperedges e_1, e_2, e_3 is

$$\Sigma_{i=1}^{3}\sigma(e_i) = (1.82, 1.12, 0.78).$$

where $e_1 = (0.54, 0.24, 0.16), e_2 = (0.74, 0.64, 0.46), e_3 = (0.54, 0.24, 0.16)$.

(ii) : In Figure (3.10), $NHG^3_{3,3,3} = (V, E, \sigma, \mu)$ is shown.

(a) : A neutrosophic number of vertices n_1, n_2, n_3 is

$$\Sigma^3_{i=1}\sigma(n_i) = (2.97, 2.94, 1.65).$$

(b) : A neutrosophic number of hyperedges e_1, e_2, e_3 is

$$\Sigma^3_{i=1}\sigma(e_i) = (1.82, 1.12, 0.78).$$

where $e_1 = (0.54, 0.24, 0.16), e_2 = (0.74, 0.64, 0.46), e_3 = (0.54, 0.24, 0.16)$.

Definition 3.24.11. Assume neutrosophic hypergraph $NHG = (V, E, \sigma, \mu)$.

(i) : A **degree** of vertex x is the number of hyperedges which are incident to x.

(ii) : A **neutrosophic degree** of vertex x is the neutrosophic number of hyperedges which are incident to x.

(iii) : A **degree** of hyperedge e is the number of vertices which e is incident to them.

(iv) : A **neutrosophic degree** of hyperedge e is the neutrosophic number of vertices which e is incident to them.

(v) : A **co-degree** of vertices $x_1, x_2, \cdots, x_n$ is the number of hyperedges which are incident to $x_1, x_2, \cdots, x_n$.

(vi) : A **neutrosophic co-degree** of vertices $x_1, x_2, \cdots, x_n$ is the neutrosophic number of hyperedges which are incident to $x_1, x_2, \cdots, x_n$.

(vii) : A **co-degree** of hyperedges $e_1, e_2, \cdots, e_n$ is the number of vertices which $e_1, e_2, \cdots, e_n$ are incident to them.

$(viii)$: A **neutrosophic co-degree** of hyperedges $e_1, e_2, \cdots, e_n$ is the neutrosophic number of vertices which $e_1, e_2, \cdots, e_n$ are incident to them.

Example 3.24.12. I get some clarifications about new definitions.

(i) : In Figure (3.9), NHG^3_4 is shown.

(a) : A degree of any vertex is 3.

(b) : A neutrosophic degree of vertex n_1 is $(2.07, 1.46, 0.87)$.

(c) : A degree of hyperedge e where $\mu(e) = (0.99, 0.98, 0.55)$ is 3.

(d) : A neutrosophic degree of hyperedge e where $\mu(e) = (0.99, 0.98, 0.55)$ is $(2.97, 2.94, 1.65)$.

(e) : A co-degree of vertices n_1, n_3 is 2.

(f) : A neutrosophic co-degree of vertices n_1, n_3 is $(1.53, 1.22, 0.71)$.

(g) : A co-degree of hyperedges e_1, e_2 where $\mu(e_1) = (0.99, 0.98, 0.55)$ and $\mu(e_2) = (0.54, 0.24, 0.16)$ is 2.

(h) : A neutrosophic co-degree of hyperedges e_1, e_2 where $\mu(e_1) = (0.99, 0.98, 0.55)$ and $\mu(e_2) = (0.54, 0.24, 0.16)$ is $(1.98, 1.96, 1.1)$.

(ii) : In Figure (3.10), $NHG^3_{3,3,3} = (V, E, \sigma, \mu)$ is shown.

(a) : A degree of any vertex $n_1, n_2, n_4, n_6, n_8, n_9$ is 1 and degree of any vertex n_3, n_5, n_7 is 2.

(b) : A neutrosophic degree of vertex $n_1, n_2, n_4, n_6, n_8, n_9$ is $(0.99, 0.98, 0.55)$ and degree of any vertex n_3, n_5, n_7 is $(1.98, 1.96, 1.1)$.

(c) : A degree of any hyperedge is 3.

(d) : A neutrosophic degree of hyperedge is $(2.97, 2.94, 1.65)$.

(e) : A co-degree of vertices n_1, n_4 is 1.

(f) : A neutrosophic co-degree of vertices n_1, n_4 is $(0.54, 0.24, 0.16)$.

(g) : A co-degree of hyperedges e_1, e_2 where $\mu(e_1) = (0.99, 0.98, 0.55)$ and $\mu(e_2) = (0.54, 0.24, 0.16)$ is 1.

(h) : A neutrosophic co-degree of hyperedges e_1, e_2 where $\mu(e_1) = (0.99, 0.98, 0.55)$ and $\mu(e_2) = (0.54, 0.24, 0.16)$ is $(0.99, 0.98, 0.55)$.

Example 3.24.13. Consider Figure (3.11) where the improvements on its super-edges to have super strong hypergraph.

(a) : The notions of dominating are clarified.

(i) : n_1 super-dominates every super-vertex from the set of super-vertices $\{n_7, n_8, n_9, n_2, n_3\}$. n_4 super-dominates every super-vertex from the set of super-vertices $\{n_6, n_5, n_3\}$. n_4 doesn't super-dominate every super-vertex from the set of super-vertices $\{n_1, n_2, n_7, n_8, n_9\}$.

(ii) : $\{n_1, n_3\}$ is super-coloring set but $\{n_1, n_4\}$ is optimal-super-dominating set.

(iii) : $(1.53, 1.22, 0.71)$ is optimal-super-dominating number.

(b) : The notions of resolving are clarified.

(i) : n_1 super-resolves two super-vertices n_4 and n_6.

(ii) : $V \setminus \{n_1, n_4\}$ is super-resolves set but $V \setminus \{n_2, n_4, n_9\}$ is optimal-super-resolving set.

(iii) : $(5, 94, 6.36, 3.3)$ is optimal-super-resolving number.

(c) : The notions of coloring are clarified.

(i) : n_1 super-colors every super-vertex from the set of super-vertices $\{n_7, n_8, n_9, n_2, n_3\}$. n_4 super-colors every super-vertex from the set of super-vertices $\{n_6, n_5, n_3\}$. n_4 doesn't super-dominate every super-vertex from the set of super-vertices $\{n_1, n_2, n_7, n_8, n_9\}$.

(ii) : $\{n_1, n_5, n_7, n_8, n_9, n_6, n_4\}$ is super-coloring set but $\{n_1, n_5, n_7, n_8, n_2, n_4\}$ is optimal-super-coloring set.

(iii) : $(5.24, 4.8, 2.82)$ is optimal-super-coloring number.

Example 3.24.14. Consider Figure (3.3).

(a) : The notions of dominating are clarified.

(i) : n_1 super-dominates every super-vertex from the set of super-vertices $\{n_5, n_6, n_2, n_3\}$. n_4 super-dominates every super-vertex from the set of super-vertices $\{n_5, n_3\}$. n_4 doesn't super-dominate every super-vertex from the set of super-vertices $\{n_1, n_2, n_6\}$.

(ii) : $\{n_1, n_3\}$ is super-dominating set but $\{n_1, n_4\}$ is optimal-super-dominating set.

(iii) : $(1.53, 1.22, 0.71)$ is optimal-super-dominating number.

(b) : The notions of resolving are clarified.

(i) : n_1 super-resolves two super-vertices n_4 and n_6.

(ii) : $V \setminus \{n_1, n_4\}$ is super-resolves set but $V \setminus \{n_2, n_4, n_6\}$ is optimal-super-resolving set.

(iii) : $(5, 94, 6.36, 3.3)$ is optimal-super-resolving number.

(c) : The notions of coloring are clarified.

(i) : n_1 super-colors every super-vertex from the set of super-vertices $\{n_5, n_6, n_2, n_3\}$. n_4 super-colors every super-vertex from the set of super-vertices $\{n_5, n_3\}$. n_4 doesn't super-dominate every super-vertex from the set of super-vertices $\{n_1, n_2, n_6\}$.

(ii) : $\{n_1, n_5, n_6\}$ is super-coloring set but $\{n_5, n_2, n_4\}$ is optimal-super-coloring set.

(iii) : $(2.27, 1.86, 1.17)$ is optimal-super-coloring number.

Preliminaries For Setting of n-SuperHyperGraph

Definition 3.24.15. (n-SuperHyperGraph).
A graph $(G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ is called by **n-SuperHyperGraph** and it's denoted by n-SHG.

3.25 New Ideas For n-SuperHyperGraph

Definition 3.25.1. (Dominating, Resolving and Coloring).
Assume n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$.

(a) : Super-dominating set and number are defined as follows.

(i) : A super-vertex X_n **super-dominates** a super-vertex Y_n if there's at least one super-edge which have them.

(ii) : A set S is called **super-dominating set** if for every $Y_n \in G_n \setminus S$, there's at least one super-vertex X_n which super-dominates super-vertex Y_n.

(iii) : If $\mathcal{S}$ is set of all sets of super-dominating sets, then

$$|X| = \min_{S \in \mathcal{S}} |\{\cup X_n | X_n \in S\}|$$

is called **optimal-super-dominating number** and X is called **optimal-super-dominating set.**

(b) : Super-resolving set and number are defined as follows.

(i) : A super-vertex x **super-resolves** super-vertices y, w if

$$d(x, y) \neq d(x, w).$$

.

(ii) : A set S is called **super-resolving set** if for every $Y_n \in G_n \setminus S$, there's at least one super-vertex X_n which super-resolves super-vertices Y_n, W_n.

(iii) : If $\mathcal{S}$ is set of all sets of super-resolving sets, then

$$|X| = \min_{S \in \mathcal{S}} |\{\cup X_n | X_n \in S\}|$$

is called **optimal-super-resolving number** and X is called **optimal-super-resolving set.**

(c) : Super-coloring set and number are defined as follows.

(i) : A super-vertex X_n **super-colors** a super-vertex Y_n differently with itself if there's at least one super-edge which is incident to them.

(ii) : A set S_n is called **super-coloring set** if for every $y \in G_n \setminus S_n$, there's at least one super-vertex X_n which super-colors super-vertex Y_n.

(iii) : If $\mathcal{S}_n$ is set of all sets of super-coloring sets, then

$$|X| = \min_{S_n \in \mathcal{S}_n} |\{\cup X_n | X_n \in S_n\}|$$

is called **optimal-super-coloring number** and X is called **optimal-super-coloring set.**

3.26 Optimal Numbers For n-SuperHyperGraph

Proposition 3.26.1. *Assume n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. S is maximum set of super-vertices which form a super-edge. Then optimal-super-coloring set has as cardinality as S has.*

Proof. Assume n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. Every super-edge has super-vertices which have common super-edge. Thus every super-vertex has different color with other super-vertices which are incident with a super-edge. It induces a super-edge with the most number of super-vertices determines optimal-super-coloring set. S is maximum set of super-vertices which form a super-edge. Thus optimal-super-coloring set has as cardinality as S has. ■

Proposition 3.26.2. *Assume n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. If optimal-super-coloring number is*

$$|V|,$$

then for every super-vertex there's at least one super-edge which contains has all members of V.

Proof. Suppose n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. Consider optimal-super-coloring number is

$$|V|.$$

It implies there's one super-edge which has all members of V. Since if all members of V are incident to a super-edge via a super-vertex, then all have different colors. ■

Proposition 3.26.3. *Assume n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. If there's at least one super-edge which has all members of* V, *then optimal-super-coloring number is*

$$|V|.$$

Proof. Consider n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. Suppose there's at least one super-edge which has all members of V. It implies there's one super-edge which has some super-vertices but all members of V. If all super-vertices are incident to a super-edge, then all have different colors. It means if some super-vertices have all members of V, in the way that, for every member of V, there's a distinct super-vertex which has it and all such these super-vertices are incident to a super-edge, then all have different colors. So the set of these super-vertices are V, is optimal-super-coloring set. It induces optimal-super-coloring number is

$$|V|.$$

■

Proposition 3.26.4. *Assume n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. If optimal-super-dominating number is*

$$|V|,$$

then there's one member of V, *is contained in, at least one super-vertex which doesn't have incident to any super-edge.*

Proof. Suppose n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$.Consider optimal-super-dominating number is

$$|V|.$$

If for all given super-vertex and all members of V, there's at least one super-edge, which the super-vertex has incident to it, then there's a super-vertex X_n such that optimal-super-dominating number is

$$|V| - |X_n|.$$

It induces contradiction with hypothesis. It implies there's one member of V, is contained in, at least one super-vertex which doesn't have incident to any super-edge. ■

Proposition 3.26.5. *Assume n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. Then optimal-super-dominating number is* $<$

$$|V|.$$

Proof. Consider n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. Thus $G_n - \{X_n\}$, is a super-dominating set where $X_n \in G_n$. Since if not, X_n isn't incident to any given super-edge. This is contradiction with supposition. It induces that X_n belongs to a super-edge which has another super-vertex X'_n. It implies X'_n super-dominates X_n. Thus $G_n - \{X_n\}$ is a super-dominating set. It induces optimal-super-dominating number is <

$$|V|.$$

■

Proposition 3.26.6. *Assume n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. If optimal-super-resolving number is*

$$|V|.$$

then every given super-vertex doesn't have incident to any super-edge.

Proof. Consider n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. Let optimal-super-resolving number be

$$|V|.$$

If it implies there's a super-vertex is super-resolved by a super-vertex, then it's contradiction with hypothesis. So every given super-vertex doesn't have incident to any super-edge. ■

Proposition 3.26.7. *Assume n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. Then optimal-super-resolving number is <*

$$|V|.$$

Proof. Consider n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. If optimal-super-resolving number is

$$|V|,$$

then there's a contradiction to hypothesis. Since the set $G_n - \{X_n\}$, is super-resolving set. It implies optimal-super-resolving number is <

$$|V|.$$

■

Proposition 3.26.8. *Assume n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. If optimal-super-coloring number is*

$$|V|,$$

then all super-vertices which have incident to at least one super-edge.

Proof. Suppose n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. Consider optimal-super-coloring number is

$$|V| - |X_n|.$$

If for all given super-vertices, there's no super-edge which the super-vertices have incident to it, then there's super-vertex X_n such that optimal-super-coloring number is

$$|V| - |X_n|.$$

It induces contradiction with hypothesis. It implies all super-vertices have incident to at least one super-edge. ■

Proposition 3.26.9. *Assume n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. Then optimal-super-coloring number isn't* $<$

$$|V|.$$

Proof. Consider n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. Thus $G_n - \{X_n\}$ isn't a super-coloring set. Since if not, X_n isn't incident to any given super-edge. This is contradiction with supposition. It induces that X_n belongs to a super-edge which has another super-vertex S_n. It implies S_n super-colors X_n. Thus $G_n - \{X_n\}$ isn't a super-coloring set. It induces optimal-super-coloring number isn't $<$

$$|V|.$$

■

Proposition 3.26.10. *Assume n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. Then optimal-super-dominating set has cardinality which is greater than $n-1$ where n is the cardinality of the set V.*

Proof. Consider n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. The set G_n is super-dominating set. So optimal-super-dominating set has cardinality which is greater than $n-1$ where n is the cardinality of the set V. But the set $G_n \setminus \{X_n\}$, for every given super-vertex X_n is optimal-super-dominating set has cardinality which is greater than $n-1$ where n is is the cardinality of the set V. The result is obtained. ■

Proposition 3.26.11. *Assume n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. S is maximum set of super-vertices which form a super-edge. Then S is optimal-super-coloring set and*

$$|\{\cup X_n \mid X_n \in S\}|$$

is optimal-super-coloring number.

Proof. Suppose n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. Consider S is maximum set of super-vertices which form a super-edge. Thus all super-vertices of S have incident to super-edge. It implies the number of different colors equals to a cardinality based on S. Therefore, optimal-super-coloring number $\geq$

$$|\{\cup X_n \mid X_n \in S\}|$$

In other hand, S is maximum set of super-vertices which form a super-edge. It induces optimal-super-coloring number $\leq$

$$|\{\cup X_n \mid X_n \in S\}|$$

So S is super-coloring set. Hence S is optimal-super-coloring set and

$$|\{\cup X_n \mid X_n \in S\}|$$

is optimal-super-coloring number. ■

3.27 Optimal Sets For n-SuperHyperGraph

Proposition 3.27.1. *Assume n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. If S is super-dominating set, then D contains S is super-dominating set.*

Proof. Consider n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. Suppose S is super-dominating set. Then all super-vertices are super-dominated. Thus D contains S is super-dominating set. ■

Proposition 3.27.2. *Assume n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. If S is super-resolving set, then D contains S is super-resolving set.*

Proof. Suppose n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. Consider S is super-resolving set. Hence All two given super-vertices are super-resolved by at least one super-vertex of S. It induces D contains S is super-resolving set. ■

Proposition 3.27.3. *Assume n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. If S is super-coloring set, then D contains S is super-coloring set.*

Proof. Suppose n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. Consider S is super-coloring set. So all super-vertices which have a common super-edge have different colors. Thus every super-vertex super-colored by a super-vertex of S. It induces every super-vertex which has a common super-edge has different colors with other super-vertices belong to that super-edge. Then D contains S is super-coloring set. ■

Proposition 3.27.4. *Assume n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. Then G_n is super-dominating set.*

Proof. Suppose n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. Since $G_n \setminus \{X_n\}$ is super-dominating set. Then G_n contains $G_n \setminus \{X_n\}$ is super-dominating set. ■

Proposition 3.27.5. *Assume n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$.Then G_n is super-resolving set.*

Proof. Suppose n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. If there's no super-vertex, then all super-vertices are super-resolved. Hence if I choose G_n, then there's no super-vertex such that super-vertex is super-resolved. It implies G_n is super-resolving set but G_n isn't optimal-super-resolving set. Since if I construct one set from G_n such that only one super-vertex is out of S, then S is super-resolving set. It implies G_n isn't optimal-super-resolving set. Thus G_n is super-resolving set. ■

Proposition 3.27.6. *Assume n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. Then G_n is super-coloring set.*

Proof. Suppose n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$. All super-vertices belong to a super-edge have to color differently. If G_n is chosen, then all super-vertices have different colors. It induces that t colors are used where t is the number of super-vertices. Every super-vertex has unique color. Thus G_n is super-coloring set. ■

3.28 Optimal Sets and Numbers For Family of n-SuperHyperGraph

Proposition 3.28.1. *Assume $\mathcal{G}$ is a family of n-SuperHyperGraph. Then G_n is super-dominating set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of n-SuperHyperGraph. Thus G_n is super-dominating set for every given n-SuperHyperGraph of $\mathcal{G}$. It implies G_n is super-dominating set for all members of $\mathcal{G}$, simultaneously. ■

Proposition 3.28.2. *Assume $\mathcal{G}$ is a family of n-SuperHyperGraph. Then G_n is super-resolving set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of n-SuperHyperGraph. Thus G_n is super-resolving set for every given n-SuperHyperGraph of $\mathcal{G}$. It implies G_n is super-resolving set for all members of $\mathcal{G}$, simultaneously. ■

Proposition 3.28.3. *Assume $\mathcal{G}$ is a family of n-SuperHyperGraph. Then G_n is super-coloring set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of n-SuperHyperGraph. Thus G_n is super-coloring set for every given n-SuperHyperGraph of $\mathcal{G}$. It implies G_n is super-coloring set for all members of $\mathcal{G}$, simultaneously. ■

Proposition 3.28.4. *Assume $\mathcal{G}$ is a family of n-SuperHyperGraph. Then $G_n \setminus \{X_n\}$ is super-dominating set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of n-SuperHyperGraph. Thus $G_n \setminus \{X_n\}$ is super-dominating set for every given n-SuperHyperGraph of $\mathcal{G}$. One super-vertex is out of $G_n \setminus \{X_n\}$. It's super-dominated from any super-vertex in $G_n \setminus \{X_n\}$. Hence every given two super-vertices are super-dominated from any super-vertex in $G_n \setminus \{X_n\}$. It implies $G_n \setminus \{X_n\}$ is super-dominating set for all members of $\mathcal{G}$, simultaneously. ■

Proposition 3.28.5. *Assume $\mathcal{G}$ is a family of n-SuperHyperGraph. Then $G_n \setminus \{X_n\}$ is super-resolving set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of n-SuperHyperGraph. Thus $G_n \setminus \{X_n\}$ is super-resolving set for every given n-SuperHyperGraph of $\mathcal{G}$. One super-vertex is out of $G_n \setminus \{X_n\}$. It's super-resolved from any super-vertex in $G_n \setminus \{X_n\}$. Hence every given two super-vertices are super-resolved from any super-vertex in $G_n \setminus \{X_n\}$. It implies $G_n \setminus \{X_n\}$ is super-resolving set for all members of $\mathcal{G}$, simultaneously. ■

Proposition 3.28.6. *Assume $\mathcal{G}$ is a family of n-SuperHyperGraph. Then $G_n \setminus \{X_n\}$ isn't super-coloring set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of n-SuperHyperGraph. Thus $G_n \setminus \{X_n\}$ isn't super-coloring set for every given n-SuperHyperGraph of $\mathcal{G}$. One super-vertex is out of $G_n \setminus \{X_n\}$. It isn't super-colored from any super-vertex in $G_n \setminus \{X_n\}$. Hence every given two super-vertices aren't super-colored from any super-vertex in $G_n \setminus \{X_n\}$. It implies $G_n \setminus \{X_n\}$ isn't super-coloring set for all members of $\mathcal{G}$, simultaneously. ■

Proposition 3.28.7. *Assume $\mathcal{G}$ is a family of n-SuperHyperGraph. Then union of super-dominating sets from each member of $\mathcal{G}$ is super-dominating set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of n-SuperHyperGraph. For every chosen n-SuperHyperGraph, there's one super-dominating set in the union of super-dominating sets from each member of $\mathcal{G}$. Thus union of super-dominating sets from each member of $\mathcal{G}$ is super-dominating set for every given n-SuperHyperGraph of $\mathcal{G}$. Even one super-vertex isn't out of the union. It's super-dominated from any super-vertex in the union. Hence every given two super-vertices are super-dominated from any super-vertex in union of super-coloring sets. It implies union of super-coloring sets is super-dominating set for all members of $\mathcal{G}$, simultaneously. ■

Proposition 3.28.8. *Assume $\mathcal{G}$ is a family of n-SuperHyperGraph. Then union of super-resolving sets from each member of $\mathcal{G}$ is super-resolving set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of n-SuperHyperGraph. For every chosen n-SuperHyperGraph, there's one super-resolving set in the union of super-resolving sets from each member of $\mathcal{G}$. Thus union of super-resolving sets from each member of $\mathcal{G}$ is super-resolving set for every given n-SuperHyperGraph of $\mathcal{G}$. Even one super-vertex isn't out of the union. It's super-resolved from any super-vertex in the union. Hence every given two super-vertices are super-resolved from any super-vertex in union of super-coloring sets. It implies union of super-coloring sets is super-resolved set for all members of $\mathcal{G}$, simultaneously. ■

Proposition 3.28.9. *Assume $\mathcal{G}$ is a family of n-SuperHyperGraph. Then union of super-coloring sets from each member of $\mathcal{G}$ is super-coloring set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of n-SuperHyperGraph. For every chosen n-SuperHyperGraph, there's one super-coloring set in the union of super-coloring sets from each member of $\mathcal{G}$. Thus union of super-coloring sets from each member of $\mathcal{G}$ is super-coloring set for every given n-SuperHyperGraph of $\mathcal{G}$. Even one super-vertex isn't out of the union. It's super-colored from any super-vertex in the union. Hence every given two super-vertices are super-colored from any super-vertex in union of super-coloring sets. It implies union of super-coloring sets is super-colored set for all members of $\mathcal{G}$, simultaneously. ∎

Proposition 3.28.10. *Assume $\mathcal{G}$ is a family of n-SuperHyperGraph. For every given super-vertex, there's one n-SuperHyperGraph such that the super-vertex has another super-vertex which are incident to a super-edge. If for given super-vertex, all super-vertices have a common super-edge in this way, then $G_n \setminus \{X_n\}$ is optimal-super-dominating set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of n-SuperHyperGraph. For all n-SuperHyperGraph, there's no super-dominating set from any of member of $\mathcal{G}$. Thus $G_n \setminus \{X_n\}$ is super-dominating set for every given n-SuperHyperGraph of $\mathcal{G}$. For every given super-vertex, there's one n-SuperHyperGraph such that the super-vertex has another super-vertex which are incident to a super-edge. Only one super-vertex is out of $V \setminus \{x\}$. It's super-dominated from any super-vertex in the $V \setminus \{x\}$. Hence every given two super-vertices are super-dominated from any super-vertex in $G_n \setminus \{X_n\}$ It implies $G_n \setminus \{X_n\}$ is super-dominating set for all members of $\mathcal{G}$, simultaneously. If for given super-vertex, all super-vertices have a common super-edge in this way, then $G_n \setminus \{X_n\}$ is optimal-super-dominating set for all members of $\mathcal{G}$, simultaneously. ∎

Proposition 3.28.11. *Assume $\mathcal{G}$ is a family of n-SuperHyperGraph. For every given super-vertex, there's one n-SuperHyperGraph such that the super-vertex has another super-vertex which are incident to a super-edge. If for given super-vertex, all super-vertices have a common super-edge in this way, then $G_n \setminus \{X_n\}$ is optimal-super-resolving set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of n-SuperHyperGraph. For all n-SuperHyperGraph, there's no super-resolving set from any of member of $\mathcal{G}$. Thus $G_n \setminus \{X_n\}$ is super-resolving set for every given n-SuperHyperGraph of $\mathcal{G}$. For every given super-vertex, there's one n-SuperHyperGraph such that the super-vertex has another super-vertex which are incident to a super-edge. Only one super-vertex is out of $G_n \setminus \{X_n\}$. It's super-resolved from any super-vertex in the $G_n \setminus \{X_n\}$. Hence every given two super-vertices are super-resolving from any super-vertex in $G_n \setminus \{X_n\}$. It implies $G_n \setminus \{X_n\}$ is super-resolved set for all members of $\mathcal{G}$, simultaneously. If for given super-vertex, all super-vertices have a common super-edge in this way, then $G_n \setminus \{X_n\}$ is optimal-super-resolving set for all members of $\mathcal{G}$, simultaneously. ∎

Proposition 3.28.12. *Assume $\mathcal{G}$ is a family of n-SuperHyperGraph. For every given super-vertex, there's one n-SuperHyperGraph such that the super-vertex has another super-vertex which are incident to a super-edge. If for given super-vertex, all super-vertices have a common super-edge in this way, then G_n is optimal-super-coloring set for all members of $\mathcal{G}$, simultaneously.*

Proof. Suppose $\mathcal{G}$ is a family of n-SuperHyperGraph. For all n-SuperHyperGraph, there's no super-coloring set from any of member of $\mathcal{G}$. Thus G_n is super-coloring set for every given n-SuperHyperGraph of $\mathcal{G}$. For every given super-vertex, there's one n-SuperHyperGraph such that the super-vertex has another super-vertex which are incident to a super-edge. No super-vertex is out of G_n. It's super-colored from any super-vertex in the G_n. Hence every given two super-vertices are super-colored from any super-vertex in G_n. It implies G_n is super-coloring set for all members of $\mathcal{G}$, simultaneously. If for given super-vertex, all super-vertices have a common super-edge in this way, then G_n is optimal-super-coloring set for all members of $\mathcal{G}$, simultaneously. ■

3.29 Twin Super-vertices in n-SuperHyperGraph

sec4

prp2

Proposition 3.29.1. *Let n-SHG be a n-SuperHyperGraph. An $(k-1)$-set from an k-set of twin super-vertices is subset of a super-resolving set.*

Proof. If X_n and X'_n are twin super-vertices, then $N(X_n) = N(X'_n)$. It implies $d(X_n, T_n) = d(X'_n, T_n)$ for all $T_n \in G_n$. ■

cor2

Corollary 3.29.2. *Let n-SHG be a n-SuperHyperGraph. The number of twin super-vertices is $n-1$. Then super-resolving number is $n-2$.*

Proof. Let X_n and X'_n be two super-vertices. By supposition, the cardinality of set of twin super-vertices is $n-2$. Thus there are two cases. If both are twin super-vertices, then $N(X_n) = N(X'_n)$. It implies $d(X_n, T_n) = d(X'_n, T_n)$ for all $T_n \in G_n$. Thus suppose if not, then let X_n be a super-vertex which isn't twin super-vertices with any given super-vertex and let X'_n be a super-vertex which is twin super-vertices with any given super-vertex but not X_n. By supposition, it's possible and this is only case. Therefore, any given distinct super-vertex super-resolves X_n and X'_n. Then $G_n \setminus \{X_n, X'_n\}$ is super-resolving set. It implies -super-resolving number is $n-2$. ■

cor1

Corollary 3.29.3. *Let n-SHG be n-SuperHyperGraph. The number of twin super-vertices is $n-1$. Then super-resolving number is $n-2$. Every $(n-2)$-set including twin super-vertices is super-resolving set.*

Proof. By Corollary (3.29.2), super-resolving number is $n-2$. By n-SHG is n-SuperHyperGraph, one super-vertex doesn't belong to set of twin super-vertices and a vertex from that set, are out of super-resolving set. It induces every $(n-2)$-set including twin super-vertices is super-resolving set. ■

Proposition 3.29.4. *Let n-SHG be n-SuperHyperGraph such that it's complete. Then super-resolving number is $n-1$. Every $(n-1)$-set is super-resolving set.*

Proof. In complete, every couple of super-vertices are twin super-vertices. By n-SHG is complete, every couple of super-vertices are twin super-vertices. Thus by Proposition (3.29.1), the result follows. ■

prp3

Proposition 3.29.5. *Let $\mathcal{G}$ be a family of n-SuperHyperGraphs with common super vertex set G_n. Then simultaneously super-resolving number of $\mathcal{G}$ is $|V|-1$*

Proof. Consider $(|V|-1)$-set. Thus there's no couple of super-vertices to be super-resolved. Therefore, every $(|V|-1)$-set is super-resolving set for any given n-SuperHyperGraph. Then it holds for any n-SuperHyperGraph. It implies it's super-resolving set and its cardinality is super-resolving number. $(|V|-1)$-set has the cardinality$|V|-1$. Then it holds for any n-SuperHyperGraph. It induces it's simultaneously super-resolving set and its cardinality is simultaneously super-resolving number. ∎

prp4

Proposition 3.29.6. *Let $\mathcal{G}$ be a family of n-SuperHyperGraphs with common super-vertex set G_n. Then simultaneously super-resolving number of $\mathcal{G}$ is greater than the maximum super-resolving number of n-$SHG \in \mathcal{G}$.*

Proof. Suppose t and t' are simultaneously super-resolving number of $\mathcal{G}$ and super-resolving number of n-$SHG \in \mathcal{G}$. Thus t is super-resolving number for any n-$SHG \in \mathcal{G}$. Hence, $t \geq t'$. So simultaneously super-resolving number of $\mathcal{G}$ is greater than the maximum super-resolving number of n-$SHG \in \mathcal{G}$. ∎

prp5

Proposition 3.29.7. *Let $\mathcal{G}$ be a family of n-SuperHyperGraphs with common super-vertex set G_n. Then simultaneously super-resolving number of $\mathcal{G}$ is greater than simultaneously super-resolving number of $\mathcal{H} \subseteq \mathcal{G}$.*

Proof. Suppose t and t' are simultaneously super-resolving number of $\mathcal{G}$ and $\mathcal{H}$. Thus t is -super-resolving number for any n-$SHG \in \mathcal{G}$. It implies t is super-resolving number for any n-$SHG \in \mathcal{H}$. So t is simultaneously super-resolving number of $\mathcal{H}$. By applying Definition about being the minimum number, $t \geq t'$. So simultaneously super-resolving number of $\mathcal{G}$ is greater than simultaneously super-resolving number of $\mathcal{H} \subseteq \mathcal{G}$. ∎

thm1

Theorem 3.29.8. *Twin super-vertices aren't super-resolved in any given n-SuperHyperGraph.*

Proof. Let X_n and X'_n be twin super-vertices. Then $N(X_n) = N(X'_n)$. Thus for every given super-vertex $S'_n \in G_n,\ d_{\text{n-}SHG}(s', t) = d_{\text{n-}SHG}(s, t)$ where n-SHG is a given n-SuperHyperGraph. It means that t and t' aren't super-resolved in any given n-SuperHyperGraph. t and t' are arbitrary so twin super-vertices aren't super-resolved in any given n-SuperHyperGraph. ∎

prp6

Proposition 3.29.9. *Let n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be a n-SuperHyperGraph. If n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ is complete, then every couple of super-vertices are twin super-vertices.*

Proof. Let X_n and X'_n be couple of given super-vertices. By n-SHG is complete, $N(X_n) = N(X'_n)$. Thus X_n and X'_n are twin super-vertices. X_n and X'_n are arbitrary couple of super-vertices, hence every couple of super-vertices are twin super-vertices. ∎

thm17

Theorem 3.29.10. *Let $\mathcal{G}$ be a family of n-SuperHyperGraphs n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ with super-vertex set G_n and n-$SHG \in \mathcal{G}$ is complete. Then simultaneously super-resolving number is $|V|-1$. Every $(n-1)$-set is simultaneously super-resolving set for $\mathcal{G}$.*

Proof. Suppose n-$SHG \in \mathcal{G}$ is SuperHyperGraph and it's complete. So by Theorem (3.29.9), I get every couple of super-vertices in complete SuperHyperGraph are twin super-vertices. So every couple of super-vertices, by Theorem (3.29.8), aren't super-resolved. ■

Corollary 3.29.11. *Let $\mathcal{G}$ be a family of n-SuperHyperGraphs n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ with super-vertex set G_n and n-$SHG \in \mathcal{G}$ is complete. Then simultaneously super-resolving number is $|V| - 1$. Every $(|V| - 1)$-set is simultaneously super-resolving set for $\mathcal{G}$.*

Proof. It's complete. So by Theorem (3.29.10), I get intended result. ■

Theorem 3.29.12. *Let $\mathcal{G}$ be a family of n-SuperHyperGraphs n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ with super-vertex set G_n and for every given couple of super-vertices, there's a n-$SHG \in \mathcal{G}$ such that in that, they're twin super-vertices. Then simultaneously super-resolving number is $|V| - 1$. Every $(|V| - 1)$-set is simultaneously super-resolving set for $\mathcal{G}$.*

Proof. By Proposition (3.29.5), simultaneously super-resolving number is $|V|-1$. Also, every $(|V| - 1)$-set is simultaneously super-resolving set for $\mathcal{G}$. ■

thm19

Theorem 3.29.13. *Let $\mathcal{G}$ be a family of n-SuperHyperGraphs n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ with super-vertex set G_n. If $\mathcal{G}$ contains three super-stars with different super-centers, then simultaneously super-resolving number is $|V| - 2$. Every $(|V| - 2)$-set is simultaneously super-resolving set for $\mathcal{G}$.*

Proof. The cardinality of set of twin super-vertices is $|V| - 1$. Thus by Corollary (3.29.3), the result follows. ■

Corollary 3.29.14. *Let $\mathcal{G}$ be a family of n-SuperHyperGraphs n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ with super-vertex set G_n. If $\mathcal{G}$ contains three super-stars with different super-centers, then simultaneously super-resolving number is $|V| - 2$. Every $(|V| - 2)$-set is simultaneously super-resolving set for $\mathcal{G}$.*

Proof. $\mathcal{G}$ be a family of n-SuperHyperGraphs n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ with super-vertex set G_n. It's complete. So by Theorem (3.29.13), I get intended result. ■

3.30 Antipodal super-vertices in n-SuperHyperGraph

sec5

Even super-cycle

prp5.1

Proposition 3.30.1. *Consider two antipodal super-vertices X_n and Y_n in any given even super-cycle. Let U_n and V_n be given super-vertices. Then $d(X_n, U_n) \neq d(X_n, V_n)$ if and only if $d(Y_n, U_n) \neq d(Y_n, V_n)$.*

Proof. $(\Rightarrow)$. Consider $d(X_n, U_n) \neq d(X_n, V_n)$. By $d(X_n, U_n) + d(U_n, Y_n) = d(X_n, Y_n) = D(\text{n-}SHG)$, $D(\text{n-}SHG) - d(X_n, U_n) \neq D(\text{n-}SHG) - d(X_n, V_n)$. It implies $d(Y_n, U_n) \neq d(Y_n, V_n)$.
$(\Leftarrow)$. Consider $d(Y_n, U_n) \neq d(Y_n, V_n)$. By $d(Y_n, U_n) + d(U_n, X_n) = d(X_n, Y_n) = D(\text{n-}SHG)$, $D(\text{n-}SHG) - d(Y_n, U_n) \neq D(\text{n-}SHG) - d(Y_n, V_n)$. It implies $d(X_n, U_n) \neq d(X_n, V_n)$. ■

Proposition 3.30.2. *Consider two antipodal super-vertices X_n and Y_n in any given even cycle. Let U_n and V_n be given super-vertices. Then $d(X_n, U_n) = d(X_n, V_n)$ if and only if $d(Y_n, U_n) = d(Y_n, V_n)$.*

Proof. ($\Rightarrow$). Consider $d(X_n, U_n) = d(X_n, V_n)$. By $d(X_n, U_n) + d(U_n, Y_n) = d(X_n, Y_n) = D(\text{n-}SHG)$, $D(\text{n-}SHG) - d(X_n, U_n) = D(\text{n-}SHG) - d(X_n, V_n)$. It implies $d(Y_n, U_n) = d(Y_n, V_n)$.
($\Leftarrow$). Consider $d(Y_n, U_n) = d(Y_n, V_n)$. By $d(Y_n, U_n) + d(U_n, X_n) = d(X_n, Y_n) = D(\text{n-}SHG)$, $D(\text{n-}SHG) - d(Y_n, U_n) = D(\text{n-}SHG) - d(Y_n, V_n)$. It implies $d(X_n, U_n) = d(X_n, V_n)$. ∎

Proposition 3.30.3. *The set contains two antipodal super-vertices, isn't super-resolving set in any given even super-cycle.*

Proof. Let X_n and Y_n be two given antipodal super-vertices in any given even super-cycle. By Proposition (3.30.1), $d(X_n, U_n) \neq d(X_n, V_n)$ if and only if $d(Y_n, U_n) \neq d(Y_n, V_n)$. It implies that if X_n super-resolves a couple of super-vertices, then Y_n super-resolves them, too. Thus either X_n is in a super-resolving set or Y_n is in. It induces the set contains two antipodal super-vertices, isn't super-resolving set in any given even super-cycle. ∎

Proposition 3.30.4. *Consider two antipodal super-vertices X_n and Y_n in any given even super-cycle. X_n super-resolves a given couple of super-vertices, Z_n and Z'_n, if and only if Y_n does.*

Proof. ($\Rightarrow$). X_n super-resolves a given couple of super-vertices, Z_n and Z'_n, then $d(X_n, Z_n) \neq d(X_n, Z'_n)$. By Proposition (3.30.1), $d(X_n, Z_n) \neq d(X_n, Z'_n)$ if and only if $d(Y_n, Z_n) \neq d(Y_n, Z'_n)$. Thus Y_n super-resolves a given couple of super-vertices Z_n and Z'_n.
($\Leftarrow$). Y_n super-resolves a given couple of super-vertices, Z_n and Z'_n, then $d(Y_n, Z_n) \neq d(Y_n, Z'_n)$. By Proposition (3.30.1), $d(Y_n, Z_n) \neq d(Y_n, Z'_n)$ if and only if $d(X_n, Z_n) \neq d(X_n, Z'_n)$. Thus X_n super-resolves a given couple of super-vertices Z_n and Z'_n. ∎

Proposition 3.30.5. *There are two antipodal super-vertices aren't super-resolved by other two antipodal super-vertices in any given even super-cycle.*

Proof. Suppose X_n and Y_n are a couple of super-vertices. It implies $d(X_n, Y_n) = D(\text{n-}SHG)$. Consider U_n and V_n are another couple of super-vertices such that $d(X_n, U_n) = \frac{D(\text{n-}SHG)}{2}$. It implies $d(Y_n, U_n) = \frac{D(\text{n-}SHG)}{2}$. Thus $d(X_n, U_n) = d(Y_n, U_n)$. Therefore, U_n doesn't super-resolve a given couple of super-vertices X_n and Y_n. By $D(\text{n-}SHGG) = d(U_n, V_n) = d(U_n, X_n) + d(X_n, V_n) = \frac{D(\text{n-}SHG)}{2} + d(X_n, V_n)$, $d(X_n, V_n) = \frac{D(\text{n-}SHG)}{2}$. It implies $d(Y_n, V_n) = \frac{D(\text{n-}SHG)}{2}$. Thus $d(X_n, V_n) = d(Y_n, V_n)$. Therefore, V_n doesn't super-resolve a given couple of super-vertices X_n and Y_n. ∎

Proposition 3.30.6. *For any two antipodal super-vertices in any given even super-cycle, there are only two antipodal super-vertices don't super-resolve them.*

Proof. Suppose X_n and Y_n are a couple of super-vertices such that they're antipodal super-vertices. Let U_n be a super-vertex such that $d(X_n, U_n) = \frac{D(\text{n-}SHG)}{2}$. It implies $d(Y_n, U_n) = \frac{D(\text{n-}SHG)}{2}$. Thus $d(X_n, U_n) = d(Y_n, U_n)$. Therefore, U_n doesn't super-resolve a given couple of super-vertices X_n and Y_n. Let V_n be a antipodal vertex for U_n such that U_n and V_n are antipodal super-vertices. Thus V_n $d(X_n, V_n) = \frac{D(\text{n-}SHG)}{2}$. It implies $d(Y_n, V_n) = \frac{D(\text{n-}SHG)}{2}$. Therefore, V_n doesn't super-resolve a given couple of super-vertices X_n and Y_n. If U_n is a super-vertex such that $d(X_n, U_n) \neq \frac{D(\text{n-}SHG)}{2}$ and V_n is a super-vertex such that U_n and V_n are antipodal super-vertices. Thus $d(X_n, V_n) \neq \frac{D(\text{n-}SHG)}{2}$ It induces either $d(X_n, U_n) \neq d(Y_n, U_n)$ or $d(X_n, V_n) \neq d(Y_n, V_n)$. It means either U_n super-resolves a given couple of super-vertices X_n and Y_n or V_n super-resolves a given couple of super-vertices X_n and Y_n. ■

Proposition 3.30.7. *In any given even super-cycle, for any super-vertex, there's only one super-vertex such that they're antipodal super-vertices.*

Proof. If $d(X_n, Y_n) = D(\text{n-}SHG)$, then X_n and Y_n are antipodal super-vertices. ■

prp5.8

Proposition 3.30.8. *Let n-SuperHyperGraphs n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be an even super-cycle. Then every couple of super-vertices are super-resolving set if and only if they aren't antipodal super-vertices.*

Proof. If X_n and Y_n are antipodal super-vertices, then they don't super-resolve a given couple of super-vertices U_n and V_n such that they're antipodal super-vertices and $d(X_n, U_n) = \frac{D(\text{n-}SHG)}{2}$. Since $d(X_n, U_n) = d(X_n, V_n) = d(Y_n, u) = d(Y_n, V_n) = \frac{D(\text{n-}SHG)}{2}$. ■

cor5.9

Corollary 3.30.9. *Let n-SuperHyperGraphs n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be an even super-cycle. Then super-resolving number is two.*

Proof. A set contains one super-vertex X_n isn't super-resolving set. Since it doesn't super-resolve a given couple of super-vertices U_n and V_n such that $d(X_n, U_n) = d(X_n, V_n) = 1$. Thus super-resolving number ≥ 2. By Proposition (3.30.8), every couple of super-vertices such that they aren't antipodal super-vertices, are super-resolving set. Therefore, super-resolving number is 2. ■

cor5.10

Corollary 3.30.10. *Let n-SuperHyperGraphs n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be an even super-cycle. Then super-resolving set contains couple of super-vertices such that they aren't antipodal super-vertices.*

Proof. By Corollary (3.30.9), super-resolving number is two. By Proposition (3.30.8), every couple of super-vertices such that they aren't antipodal super-vertices, form super-resolving set. Therefore, super-resolving set contains couple of super-vertices such that they aren't antipodal super-vertices. ■

cor4.11

Corollary 3.30.11. *Let $\mathcal{G}$ be a family n-SuperHyperGraphs n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be an odd super-cycle with common super-vertex set G_n. Then simultaneously super-resolving set contains couple of super-vertices such that they aren't antipodal super-vertices and super-resolving number is two.*

Odd super-cycle

prp5.11

Proposition 3.30.12. *In any given n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ which is odd super-cycle, for any super-vertex, there's no super-vertex such that they're antipodal super-vertices.*

Proof. if X_n is a given super-vertex. Then there are two super-vertices U_n and V_n such that $d(X_n, U_n) = d(X_n, V_n) = D(\text{n-}SHG)$. It implies they aren't antipodal super-vertices. ■

prp5.12

Proposition 3.30.13. *Let n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be an odd super-cycle. Then every couple of super-vertices are super-resolving set.*

Proof. Let X_n and X'_n be couple of super-vertices. Thus, by Proposition (3.30.12), X_n and X'_n aren't antipodal super-vertices. It implies for every given couple of super-vertices T_n and T'_n, I get either $d(X_n, T_n) \neq d(X_n, T'_n)$ or $d(X'_n, T_n) \neq d(X'_n, T'_n)$. Therefore, T_n and T'_n are super-resolved by either X_n or X'_n. It induces the set $\{X_n, X'_n\}$ is super-resolving set. ■

prp5.13

Proposition 3.30.14. *Let n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be an odd cycle. Then super-resolving number is two.*

Proof. Let X_n and X'_n be couple of super-vertices. Thus, by Proposition (3.30.12), X_n and X'_n aren't antipodal super-vertices. It implies for every given couple of super-vertices T_n and T'_n, I get either $d(X_n, T_n) \neq d(X_n, T'_n)$ or $d(X'_n, T_n) \neq d(X'_n, T'_n)$. Therefore, T_n and T'_n are super-resolved by either X_n or X'_n. It induces the set $\{X_n, X'_n\}$ is super-resolving set. ■

Corollary 3.30.15. *Let n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be an odd cycle. Then super-resolving set contains couple of super-vertices.*

Proof. By Proposition (3.30.14), super-resolving number is two. By Proposition (3.30.13), every couple of super-vertices form super-resolving set. Therefore, super-resolving set contains couple of super-vertices. ■

Corollary 3.30.16. *Let $\mathcal{G}$ be a family of n-SuperHyperGraphs n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ which are odd super-cycles with common super-vertex set G_n. Then simultaneously super-resolving set contains couple of super-vertices and super-resolving number is two.*

3.31 Extended Results For n-SuperHyperGraph

sec6

Smallest Super-resolving Number

prp1

Proposition 3.31.1. *Let n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be a super-path. Then every super-leaf forms super-resolving set.*

Proof. Let L_n be a super-leaf. For every given a couple of super-vertices X_n and X'_n, I get $d(L_n, X_n) \neq d(L_n, X'_n)$. Since if I reassign indexes to super-vertices such that every super-vertex X'_n and L_n have i super-vertices amid themselves. Thus $j \leq i$ implies

$$d(L_n, X_n) + c = d(L_n, X'_n) \equiv d(L_n, X_n) < d(L_n, X'_n).$$

Therefore, by $d(L_n, X_n) < d(L_n, X'_n)$, I get $d(L_n, X_n) \neq d(L_n, X'_n)$. X_n and X'_n are arbitrary so L_n super-resolves any given couple of super-vertices X_n and X'_n which implies $\{L_n\}$ is a super-resolving set. ■

prp7

Proposition 3.31.2. *Let n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be a super-path. Then a set including every couple of super-vertices is super-resolving set.*

Proof. Let X_n and X'_n be a couple of super-vertices. For every given a couple of super-vertices Y_n and Y'_n, I get either

$$d(X_n, Y_n) \neq d(X_n, Y'_n)$$

or

$$d(X'_n, Y_n) \neq d(X'_n, Y'_n).$$

■

prp6.2

Proposition 3.31.3. *Let n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be a super-path. Then an 1-set contains leaf is super-resolving set and super-resolving number is one.*

Proof. There are two super-leaves. Consider L_n is a given super-leaf. By n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ is a super-path, there's only one number to be seen. With rearranging the indexes of super-vertices, $d(L_n, V_n) = i$. Further more, $d(L_n, V_n) = i \neq j = d(L_n, V'_n)$. Therefore, L_n super-resolves every given couple of super-vertices V_n and V'_n. It induces 1-set containing leaf is super-resolving set. Also, super-resolving number is one. ■

cor6.3

Corollary 3.31.4. *Let $\mathcal{G}$ be a family of n-SuperHyperGraphs n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ are super-paths with common super-vertex set G_n such that they've a common super-leaf. Then simultaneously super-resolving number is 1, 1-set contains common leaf, is simultaneously super-resolving set for $\mathcal{G}$.*

Proof. By Proposition (3.31.3), common super-leaf super-resolves every given couple of super-vertices X_n and X'_n, simultaneously. Thus 1-set containing common super-leaf, is simultaneously super-resolving set. Also, simultaneously super-resolving number is one. ■

prp6.4

Proposition 3.31.5. *Let $\mathcal{G}$ be a family of n-SuperHyperGraphs n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ are super-paths with common super-vertex set G_n such that for every super-leaf L_n from n-SHG, there's another n-$SHG \in \mathcal{G}$ such that L_n isn't super-leaf. Then an 2-set contains every couple of super-vertices, is super-resolving set. An 2-set contains every couple of super-vertices, is optimal-super-resolving set. Optimal-super-resolving number is two.*

Proof. Suppose V_n is a given super-vertex. If there are two super-vertices X_n and Y_n such that $d(X_n, V_n) \neq d(Y_n, V_n)$, then X_n super-resolves X_n and Y_n and the proof is done. If not, $d(X_n, V_n) = d(Y_n, V_n)$, but for every given super-vertex V'_n,

$$d(X_n, V'_n) \neq d(Y_n, V'_n).$$

■

Corollary 3.31.6. *Let $\mathcal{G}$ be a family of n-SuperHyperGraphs n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ are super-paths with common super-vertex set G_n such that they've no common super-leaf. Then an 2-set is simultaneously optimal-super-resolving set and simultaneously optimal-super-resolving number is* 2.

Proof. By Corollary (3.31.4), common super-leaf forms a simultaneously optimal-super-resolving set but in this case, there's no common super-leaf. Thus by Proposition (3.31.5), an 2-set is optimal-super-resolving set for any n-$SHG \in \mathcal{G}$. Then an 2-set is simultaneously optimal-super-resolving set. It induces simultaneously optimal-super-resolving number is 2. So every 2-set is simultaneously optimal-super-resolving set for $\mathcal{G}$. ■

Largest Optimal-super-resolving Number

Super-t-partite, super-bipartite, super-star, super-wheel are also studied and they get us two type-results as individual and family.

prp55.11

Proposition 3.31.7. *Let n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be a super-t-partite. Then every set excluding couple of super-vertices in different parts whose cardinalities of them are strictly greater than one, is optimal-super-resolving set.*

Proof. Consider two super-vertices X_n and Y_n. Suppose super-vertex M_n has same part with either X_n or Y_n. Without loosing the generality, suppose M_n has same part with X_n thus it doesn't have common part with Y_n. Therefore,

$$d(M_n, X_n) = 2 \neq 1 = d(M_n, Y_n).$$

■

cor55.12

Corollary 3.31.8. *Let n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be a super-t-partite. Let $|V| \geq 3$. Then every $(|V| - 2)$-set excludes two super-vertices from different parts whose cardinalities of them are strictly greater than one, is optimal-super-resolving set and optimal-super-resolving number is* $|V| - 2$.

Proof. By Proposition (3.31.7), every $(|V| - 2)$-set excludes two super-vertices from different parts whose cardinalities of them are strictly greater than one, is optimal-super-resolving set. Since if X_n and Y_n are either in same part or in different parts, then, by any given super-vertex W_n, $d(W_n, X_n) = d(W_n, Y_n)$. Thus 1-set isn't super-resolving set. There are same arguments for a set with cardinality $\leq |V| - 3$ when pigeonhole principle implies at least two super-vertices have same conditions concerning either being in same part or in different parts. ■

cor55.13

Corollary 3.31.9. *Let n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be a super-bipartite. Let $|V| \geq 3$. Then every $(|V|-2)$-set excludes two super-vertices from different parts, is optimal-super-resolving set and optimal-super-resolving number is $|V|-2$.*

Proof. Consider X_n and Y_n are excluded by a $(|V|-2)$-set. Let M_n be a given super-vertex which is distinct from them. By n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ is a super-bipartite, M_n has a common part with either X_n or Y_n and not with both of them. It implies $d(X_n, M_n) \neq d(Y_n, M_n)$. Since if M_n has a common part with X_n, then $d(X_n, M_n) = 2 \neq 1 = d(Y_n, M_n)$. And if M_n has a common part with Y_n, then $d(X_n, M_n) = 1 \neq 2 = d(Y_n, M_n)$. Thus M_n super-resolves X_n and Y_n. If W_n is another super-vertex which is distinct from them, then pigeonhole principle induces at least two super-vertices have same conditions concerning either being in same part or in different parts. It implies $(|V|-3)$-set isn't super-resolving set. It implies $(|V|-2)$-set excludes two super-vertices from different parts, is optimal-super-resolving set and optimal-super-resolving number is $|V|-2$. ■

cor55.14

Corollary 3.31.10. *Let n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be a super-star. Then every $(|V|-2)$-set excludes super-center and a given super-vertex, is optimal-super-resolving set and optimal-super-resolving number is* $(|V|-2)$.

Proof. Consider X_n and Y_n are excluded by a $(|V|-2)$-set. Let M_n be a given super-vertex which is distinct from them. By n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ is super-star, M_n has a common part with either X_n or Y_n and not with both of them. It implies $d(X_n, M_n) \neq d(Y_n, M_n)$. Since if M_n has a common part with X_n, then $d(X_n, M_n) = 2 \neq 1 = d(Y_n, M_n)$. And if M_n has a common part with Y_n, then $d(X_n, M_n) = 1 \neq 2 = d(Y_n, M_n)$. Thus M_n -resolves X_n and Y_n. If W_n is another super-vertex which is distinct from them, then pigeonhole principle induces at least two super-vertices have same conditions concerning either being in same part or in different parts. It implies $(|V|-3)$-set isn't super-resolving set. Therefore, every $(|V|-2)$-set excludes two super-vertices from different parts, is optimal-super-resolving set and optimal-super-resolving number is $|V|-2$. ■

cor55.15

Corollary 3.31.11. *Let n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be a super-wheel. Let $|V| \geq 3$. Then every $(|V|-2)$-set excludes super-center and a given super-vertex, is optimal-super-resolving set and optimal-super-resolving number is $|V|-2$.*

Proof. Consider X_n and Y_n are excluded by a $(|V|-2)$-set. Let M_n be a given super-vertex which is distinct from them. By n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ is a super-wheel., M_n has a common part with either X_n or Y_n and not with both of them. It implies $d(X_n, M_n) \neq d(Y_n, M_n)$. Since if X_n is super-center, then $d(X_n, M_n) = 1 \neq 2 = d(Y_n, M_n)$. And if Y_n is super-center, then $d(X_n, M_n) = 2 \neq 1 = d(Y_n, M_n)$. Thus M_n super-resolves X_n and Y_n. If W_n is another super-vertex which is distinct from them, then pigeonhole principle induces at least two super-vertices have same conditions concerning either being in same part (non-center super-vertices) or in different parts. It implies $(|V|-3)$-set isn't super-resolving set. Therefore, every $(|V|-2)$-set super-center and a given super-vertex, is optimal-super-resolving set and optimal-super-resolving number is $|V|-2$. ■

Super-t-partite, super-bipartite, super-star, super-wheel are also studied but they get us one type-result involving family of them.

Corollary 3.31.12. *Let $\mathcal{G}$ be a family of n-SuperHyperGraphs n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ which are super-t-partite with common super-vertex set G_n. Let $|V| \geq 3$. Then simultaneously optimal-super-resolving number is $|V|-2$ and every $(|V|-2)$-set excludes two super-vertices from different parts, is simultaneously optimal-super-resolving set for $\mathcal{G}$.*

Proof. By Corollary (3.31.8), every result hold for any given n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ which are super-t-partite. Thus every result hold for any given n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ which are super-t-partite, simultaneously. Therefore, simultaneously super-resolving number is $|V|-2$ and every $(|V|-2)$-set excludes two super-vertices from different parts, is simultaneously optimal-super-resolving set for $\mathcal{G}$. ■

Corollary 3.31.13. *Let $\mathcal{G}$ be a family of n-SuperHyperGraphs n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ which are super-bipartite with common super-vertex set G_n. Let $|V| \geq 3$. Then simultaneously optimal-super-resolving number is $|V|-2$ and every $(|V|-2)$-set excludes two super-vertices from different parts, is simultaneously optimal-super-resolving set for $\mathcal{G}$.*

Proof. By Corollary (3.31.9), every result hold for any given n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ which are super-bipartite. Thus every result hold for any given n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ which are super-bipartite, simultaneously. Therefore, simultaneously super-resolving number is $|V|-2$ and every $(|V|-2)$-set excludes two super-vertices from different parts, is simultaneously optimal-super-resolving set for $\mathcal{G}$. ■

Corollary 3.31.14. *Let $\mathcal{G}$ be a family of n-SuperHyperGraphs n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ which are super-star with common super-vertex set G_n. Let $|V| \geq 3$. Then simultaneously optimal-super-resolving number is $|V|-2$ and every $(|V|-2)$-set excludes super-center and a given super-vertex, is simultaneously optimal-super-resolving set for $\mathcal{G}$.*

Proof. By Corollary (3.31.10), every result hold for any given n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ which are super-star. Thus every result hold for any given n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ which are super-star, simultaneously. Therefore, simultaneously super-resolving number is $|V|-2$ and every $(|V|-2)$-set excludes super-center and a given super-vertex, is simultaneously optimal-super-resolving set for $\mathcal{G}$. ■

Corollary 3.31.15. *Let $\mathcal{G}$ be a family of n-SuperHyperGraphs n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ which are super-wheel with common super-vertex set G_n. Let $|V| \geq 3$. Then simultaneously optimal-super-resolving number is $|V|-2$*

and every $(|V| - 2)$*-set excludes super-center and a given super-vertex, is simultaneously optimal-super-resolving set for* $\mathcal{G}$.

Proof. By Corollary (3.31.11), every result hold for any given n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ which are super-star. Thus every result hold for any given n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ which are super-star, simultaneously. Therefore, simultaneously super-resolving number is $|V| - 2$ and every $(|V| - 2)$-set excludes super-center and a given super-vertex, is simultaneously optimal-super-resolving set for $\mathcal{G}$. ■

3.32 Optimal-super-coloring Number in n-SuperHyperGraph

sec2

Proposition 3.32.1. *Let n-SuperHyperGraphs n-*$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ *be a super-complete. Then optimal-super-coloring number is* $|V|$.

Proof. It's complete. It means for any two members of V, there's at least two distinct super-vertices contain them. Every super-vertex has edge with at least $|V| - 1$ super-vertices. Thus $|V|$ is optimal-super-coloring number. Since any given member of V has different color in comparison to another member of V. Then optimal-super-coloring number is $|V|$. ■

Proposition 3.32.2. *Let n-SuperHyperGraphs n-*$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ *be a super-path. Then optimal-super-coloring number is two.*

Proof. With alternative colors, super-path has distinct color for every super-vertices which have one super-edge in common. Thus if X_n and Y_n are two super-vertices which have one super-edge in common, then X_n and Y_n have different color. Therefore, optimal-super-coloring number is two. The representative of colors are two given super-vertices which have at least one super-edge in common. ■

Proposition 3.32.3. *Let n-SuperHyperGraphs n-*$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ *be an even super-cycle. Then optimal-super-coloring number is two.*

Proof. Since even super-cycle has even super-vertices, with alternative coloring of super-vertices, the super-vertices which have common super-edge, have different colors. So optimal-super-coloring number is two. ■

Proposition 3.32.4. *Let n-SuperHyperGraphs n-*$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ *be an odd super-cycle. Then optimal-super-coloring number is three.*

Proof. With alternative coloring on super-vertices, at end, two super-vertices have same color, and they've same super-edge. So, optimal-super-coloring number is three. ■

Proposition 3.32.5. *Let n-SuperHyperGraphs n-*$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ *be a super-star. Then optimal-super-coloring number is two.*

Proof. Super-center has common super-edge with every other super-vertex. So it has different color in comparison to other super-vertices. So one color has only one super-vertex which has that color. All other super-vertices have no common super-edge amid each other. Then they've same color. The representative of this color is a super-vertex which is distinct from super-center. The set of representative of colors has two representatives which are super-center and a given super-vertex which isn't super-center. Optimal-super-coloring number is two. ■

Proposition 3.32.6. *Let n-SuperHyperGraphs n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be a super-wheel such that it has even super-cycle. Then optimal-super-coloring number is Three.*

Proof. Super-center has unique color. So it's only representative of this color. Other super-vertices form a super-cycle which assigns distinct colors to the super-vertices which have common super-edge with each other when the number of colors is two. So a color for super-center and two colors for other super-vertices, make super-wheel has distinct colors for super-vertices which have common super-edge. Hence, optimal-super-coloring number is Three. ■

Proposition 3.32.7. *Let n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be a super-wheel such that it has odd super-cycle. Then optimal-super-coloring number is four.*

Proof. Without super-center, other super-vertices form odd super-cycle. Odd super-cycle has optimal-super-coloring number which is three. Super-center has common super-edges with all other super-vertices. Thus super-center has different colors with all other super-vertices. Therefore, optimal-super-coloring number is four. Four representatives of colors form optimal-super-coloring number where one representative is super-center and other three representatives are from all other super-vertices. So, optimal-super-coloring number is four. ■

Proposition 3.32.8. *Let n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be a super-complete and super-bipartite. Then optimal-super-coloring number is two.*

Proof. Every given super-vertex has super-edge with all super-vertices from another part. So the color of every super-vertex which is in a same part is same. Hence, two parts implies two different colors. It induces optimal-super-coloring number is two. The any of all super-vertices in every part, identify the representative of every color. ■

Proposition 3.32.9. *Let n-SuperHyperGraph n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be a super-complete and super-t-partite. Then optimal-super-coloring number is t.*

Proof. Every part has same color for its super-vertices. Optimal-super-coloring number is t. Every part introduces one super-vertex as a representative of its color. ■

Proposition 3.32.10. *Let n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be n-SuperHyperGraph. Then optimal-super-coloring number is 1 if and only if n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ is super-empty.*

Proof. ($\Rightarrow$). Let optimal-super-coloring number be 1. It implies there's no super-vertex which has same edge with a vertex. So there's no super-edge. Since n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ is n-SuperHyperGraph and n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ is super-empty.
($\Leftarrow$). Let n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be n-SuperHyperGraph and super-empty. Hence there's no super-edge. It implies for every given super-vertex, there's no common super-edge. It induces there's only one color for super-vertices. Hence the representative of this color is chosen from $|G_n|$ super-vertices. Thus optimal-super-coloring number is 1. ■

Proposition 3.32.11. *Let n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be n-SuperHyperGraph. Then optimal-super-coloring number is 2 if and only if n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ is both super-complete and super-bipartite.*

Proof. ($\Rightarrow$). Let optimal-super-coloring number be two. So every super-vertex has either one super-vertex or two super-vertices with a common super-edge. The number of colors are two so there are two sets which each set has the super-vertices which have same color. If two super-vertices have same color, then they don't have a common edge. So every set is a part in that, no super-vertex has common super-edge. The number of these sets is two. Hence there are two parts in each of them, every super-vertex has no common super-edge with other super-vertices. Since n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ is n-SuperHyperGraph, n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ is both super-complete and super-bipartite.
($\Leftarrow$). Assume n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ is n-SuperHyperGraph, n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ is both super-complete and super-bipartite. Then all super edges are amid two parts. Every part has the super-vertices which have no super-edge in common. So they're assigned to have same color. There are two parts. Thus there are two colors to assign to the super-vertices in that, the super-vertices with common super-edge, have different colors. It induces optimal-super-coloring number is 2. ■

Proposition 3.32.12. *Let n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be n-SuperHyperGraph. Then optimal-super-coloring number is $|V|$ if and only if n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ is super-complete.*

Proof. ($\Rightarrow$). Let optimal-super-coloring number be $|V|$. Thus $|G_n|$ colors are available. So any given super-vertex has $|G_n|$ super-vertices which have common super-edge with them and every of them have common super-edge with each other. It implies every super-vertex has $|G_n|$ super-vertices which have common super-edge with them. Since n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ is n-SuperHyperGraph, $SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ is super-complete.
($\Leftarrow$). Suppose n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ is super-complete. Every vertex has $|G_n|$ super-vertices which have common super-edge with them. Since all possible super-edges are available, the minimum number of colors are $|G_n|$. Thus optimal-super-coloring number is $|V|$. ■

General bounds for optimal-super-coloring number are computed.

Proposition 3.32.13. *Let n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be n-SuperHyperGraph. Then optimal-super-coloring number is obtained from the number of super-vertices which is $|G_n|$ and optimal-super-coloring number is at most $|V|$.*

Proof. When every super-vertex is a representative of each color, optimal-super-coloring number is the union of number of members of all super-vertices and it happens in optimal-super-coloring number of super-complete which is $|V|$. When all super-vertices have distinct colors, optimal-super-coloring number is $|V|$ and it's sharp for super-complete. ∎

The relation amid optimal-super-coloring number and main parameters of n-SuperHyperGraph is computed.

Proposition 3.32.14. *Let n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be n-SuperHyperGraph. Then optimal-super-coloring number is at most $\Delta + 1$ and at least 2.*

Proof. n-SuperHyperGraph is super-nontrivial. So it isn't super-empty which induces there's no super-edge. It implies optimal-super-coloring number is two. Since optimal-super-coloring number is one if and only if n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ is super-empty if and only if n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ is super-trivial. A super-vertex with degree Δ, has Δ super-vertices which have common super-edges with them. If these super-vertices have no super-edge amid each other, then optimal-super-coloring number is two especially, super-star. If not, then in the case, all super-vertices have super-edge amid each other, optimal-super-coloring number is $\Delta + 1$, especially, super-complete. ∎

Proposition 3.32.15. *Let n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ be n-SuperHyperGraph and super-r-regular. Then optimal-super-coloring number is at most $r + 1$.*

Proof. n-$SHG = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$ is super-r-regular. So any of super-vertex has r super-vertices which have common super-edge with it. If these super-vertices have no common super-edge with each other, for instance super-star, optimal-super-coloring number is two. But since the super-vertices have common super-edge with each other, optimal-super-coloring number is $r + 1$, for instance, super-complete. ∎

3.33 Applications in Time Table and Scheduling in Neutrosophic n-SuperHyperGraph

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has important to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive section. Beyond that, sometimes sections are not the same.

First Case

Step 3. (Model) As Figure (3.11), the situation is designed as a model. The model uses data to assign every section and to assign to relation amid section, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relation amid them. Table (4.5), clarifies about the assigned numbers to these situation.

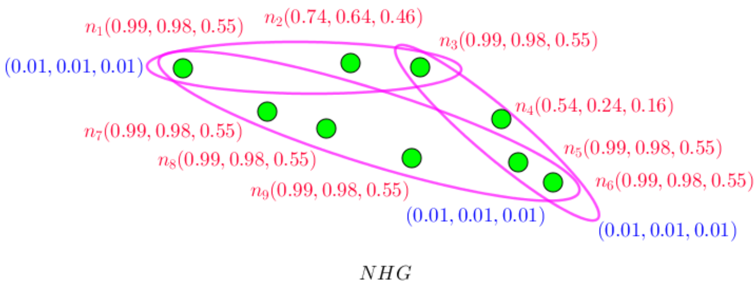


Figure 3.11: super-vertices are suspicions about choosing them. nhg1

Table 3.5: Scheduling concerns its Subjects and its Connections as a n-SuperHyperGraph in a Model. tbl1c

Sections of NHG	n_1	$n_2 \cdots$	n_9
Values	$(0.99, 0.98, 0.55)$	$(0.74, 0.64, 0.46) \cdots$	$(0.99, 0.98, 0.55)$
Connections of NHG	E_1	E_2	E_3
Values	$(0.01, 0.01, 0.01)$	$(0.01, 0.01, 0.01)$	$(0.01, 0.01, 0.01)$

Step 4. (Solution) As Figure (3.11) shows, super hyper graph as model, proposes to use different types of coloring, resolving and dominating as numbers, sets, optimal numbers, optimal sets and et cetera.

(a) : The notions of dominating are applied.

(i) : n_1 super-dominates every super-vertex from the set of super-vertices $\{n_7, n_8, n_9, n_2, n_3\}$. n_4 super-dominates every super-vertex from the set of super-vertices $\{n_6, n_5, n_3\}$. n_4 doesn't super-dominate every super-vertex from the set of super-vertices $\{n_1, n_2, n_7, n_8, n_9\}$.

(ii) : $\{n_1, n_3\}$ is super-coloring set but $\{n_1, n_4\}$ is optimal-super-dominating set.

(iii) : $(1.53, 1.22, 0.71)$ is optimal-super-dominating number.

(b) : The notions of resolving are applied.

(i) : n_1 super-resolves two super-vertices n_4 and n_6.

(ii) : $V \setminus \{n_1, n_4\}$ is super-resolves set but $V \setminus \{n_2, n_4, n_9\}$ is optimal-super-resolving set.

(iii) : $(5, 94, 6.36, 3.3)$ is optimal-super-resolving number.

(c) : The notions of coloring are applied.

(i) : n_1 super-colors every super-vertex from the set of super-vertices $\{n_7, n_8, n_9, n_2, n_3\}$. n_4 super-colors every super-vertex from the set of super-vertices $\{n_6, n_5, n_3\}$. n_4 doesn't super-dominate every super-vertex from the set of super-vertices $\{n_1, n_2, n_7, n_8, n_9\}$.

(ii) : $\{n_1, n_5, n_7, n_8, n_9, n_6, n_4\}$ is super-coloring set but $\{n_1, n_5, n_7, n_8, n_2, n_4\}$ is optimal-super-coloring set.

(iii) : $(5.24, 4.8, 2.82)$ is optimal-super-coloring number.

Second Case

Step 3. (Model) As Figure (3.12), the situation is designed as a model. The model uses data to assign every section and to assign to relation amid section, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relation amid them. Table (4.5), clarifies about the assigned numbers to these situation.

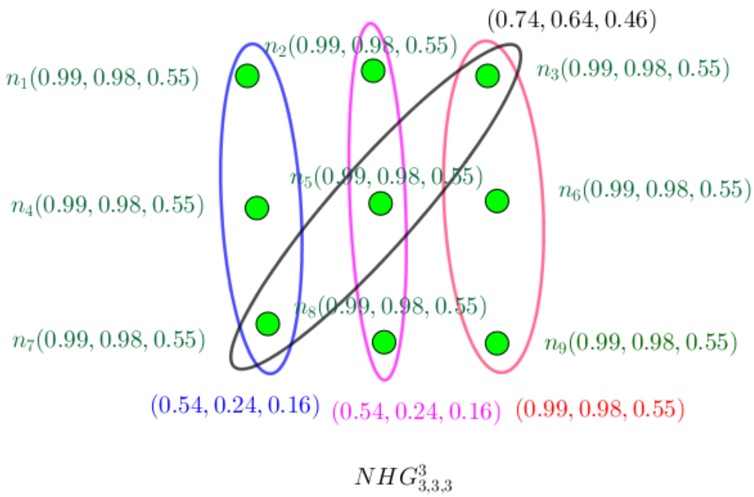


Figure 3.12: Vertices are suspicions about choosing them.

nhg8

Step 4. (Solution) As Figure (3.12) shows, $NHG^3_{3,3,3} = (V, E, \sigma, \mu)$ is neutrosophic complete 3−partite hypergraph as model, proposes to use different types of degree of vertices, degree of hyperedges, co-degree of vertices, co-degree of hyperedges, neutrosophic number of vertices, neutrosophic number of hyperedges and et cetera.

(i) : The notions of neutrosophic number are applied on vertices and hyperedges.

Table 3.6: Scheduling concerns its Subjects and its Connections as a Neutrosophic Hypergraph in a Model.

tbl1c

Sections of NHG	n_1	$n_2 \cdots$	n_9
Values	$(0.99, 0.98, 0.55)$	$(0.74, 0.64, 0.46) \cdots$	$(0.99, 0.98, 0.55)$
Connections of NHG	E_1, E_2	E_3	E_4
Values	$(0.54, 0.24, 0.16)$	$(0.99, 0.98, 0.55)$	$(0.74, 0.64, 0.46)$

(a) : A neutrosophic number of vertices n_1, n_2, n_3 is

$$\Sigma_{i=1}^{3} \sigma(n_i) = (2.97, 2.94, 1.65).$$

(b) : A neutrosophic number of hyperedges e_1, e_2, e_3 is

$$\Sigma_{i=1}^{3} \sigma(e_i) = (1.82, 1.12, 0.78).$$

where $e_1 = (0.54, 0.24, 0.16), e_2 = (0.74, 0.64, 0.46), e_3 = (0.54, 0.24, 0.16)$.

(ii) : The notions of degree, co-degree, neutrosophic degree and neutrosophic co-degree are applied on vertices and hyperedges.

(a) : A degree of any vertex $n_1, n_2, n_4, n_6, n_8, n_9$ is 1 and degree of any vertex n_3, n_5, n_7 is 2.

(b) : A neutrosophic degree of vertex $n_1, n_2, n_4, n_6, n_8, n_9$ is $(0.99, 0.98, 0.55)$ and degree of any vertex n_3, n_5, n_7 is $(1.98, 1.96, 1.1)$.

(c) : A degree of any hyperedge is 3.

(d) : A neutrosophic degree of hyperedge is $(2.97, 2.94, 1.65)$.

(e) : A co-degree of vertices n_1, n_4 is 1.

(f) : A neutrosophic co-degree of vertices n_1, n_4 is $(0.54, 0.24, 0.16)$.

(g) : A co-degree of hyperedges e_1, e_2 where $\mu(e_1) = (0.99, 0.98, 0.55)$ and $\mu(e_2) = (0.54, 0.24, 0.16)$ is 1.

(h) : A neutrosophic co-degree of hyperedges e_1, e_2 where $\mu(e_1) = (0.99, 0.98, 0.55)$ and $\mu(e_2) = (0.54, 0.24, 0.16)$ is $(0.99, 0.98, 0.55)$.

3.34 Open Problems

The three notions of coloring, resolving and dominating are introduced on n-SuperHyperGraph. Thus,

Question 3.34.1. *Is it possible to use other types super-edges to define different types of coloring, resolving and dominating on n-SuperHyperGraph?*

Question 3.34.2. *Are existed some connections amid the coloring, resolving and dominating inside this concept and external connections with other types of coloring, resolving and dominating on n-SuperHyperGraph?*

Question 3.34.3. *Is it possible to construct some classes on n-SuperHyperGraph which have "nice" behavior?*

Question 3.34.4. *Which applications do make an independent study to apply these three types coloring, resolving and dominating on n-SuperHyperGraph?*

Problem 3.34.5. *Which parameters are related to this parameter?*

Problem 3.34.6. *Which approaches do work to construct applications to create independent study?*

Problem 3.34.7. *Which approaches do work to construct definitions which use all three definitions and the relations amid them instead of separate definitions to create independent study?*

3.35 Conclusion and Closing Remarks

This study uses mixed combinations of different types of definitions, including coloring, resolving and dominating to study on n-SuperHyperGraph. The connections of super-vertices which are clarified by general super-edges differ them from each other and put them in different categories to represent one representative for each color, resolver and dominator. Further studies could be about changes in the settings to compare this notion amid different settings of n-SuperHyperGraph theory. One way is finding some relations amid three definitions of notions to make sensible definitions. In Table (4.6), some limitations and advantages of this study is pointed out.

Table 3.7: A Brief Overview about Advantages and Limitations of this study tbl2c

Advantages	Limitations
1. Defining (Dual) Dimension	1. General Results
2. Defining (Dual) Domination	
3. Defining (Dual) Coloring	2. Connections Amid New Notions
4. Applying on Individuals	
5. Applying on Family	3. Connections of Results

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CHAPTER 4

Neutrosophic Alliances

4.1 Different Neutrosophic Alliances

The following sections are cited as [**3**].

4.2 Three Types of Neutrosophic Alliances based on Connectedness and (Strong) Edges

4.3 Abstract

New setting is introduced to study the alliances. Alliances are about a set of vertices which are applied into the setting of neutrosophic graphs. Neighborhood has the key role to define these notions. Also, neighborhood is defined based on the edges, strong edges and some edges which are coming from connectedness. These three types of edges get a framework as neighborhood and after that, too close vertices have key role to define offensive alliance, defensive alliance, t-offensive alliance, and t-defensive alliance based on three types of edges, common edges, strong edges and some edges which are coming from connectedness. The structure of set is studied and general results are obtained. Also, some classes of neutrosophic graphs containing complete, empty, path, cycle, bipartite, t-partite, star and wheel are investigated in the terms of set, minimal set, number, and neutrosophic number. In this study, there's an open way to extend these results into the family of these classes of neutrosophic graphs. The family of neutrosophic graphs aren't study but it seems that analogous results are determined. There's a question. How can be related to each other, two sets partitioning the vertex set of a graph? The ideas of neighborhood and neighbors based on different edges illustrate open way to get results. A set is alliance when two sets partitioning vertex set have uniform structure. All members of set have different amount of neighbors in the set and out of set. It leads us to the notion of offensive and defensive. New ideas, offensive alliance, defensive alliance, t-offensive alliance, t-defensive alliance, strong offensive alliance, strong defensive alliance, strong t-offensive alliance, strong t-defensive alliance, connected offensive alliance, connected defensive alliance, connected t-offensive alliance, and connected t-defensive alliance are introduced. Two numbers concerning cardinality and neutrosophic cardinality of alliances are introduced. A set is alliance when its complement make a relation in the terms of neighborhood. Different edges make different neighborhoods. Three types of

edges are applied to define three styles of neighborhoods. General edges, strong edges and connected edges are used where connected edges are the edges arising from connectedness amid two endpoints of the edges. These notions are applied into neutrosophic graphs as individuals and family of them. Independent set as an alliance is a special set which has no neighbor inside and it implies some drawbacks for this notions. Finding special sets which are well-known, is an open way to purse this study. Special set which its members have only one neighbor inside, characterize the connected components where the cardinality of its complement is the number of connected components. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Keywords: Alliance, Offensive Alliance, Defensive Alliance

AMS Subject Classification: 05C17, 05C22, 05E45 In this section, I use two subsections to illustrate a perspective about the background of this study.

4.4 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 4.4.1. *Is it possible to use mixed versions of ideas concerning "alliance", "offensive" and "defensive" to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Connections amid two vertices have key roles to assign alliances, defensive alliances and offensive alliances. Thus they're used to define new ideas which conclude to the structure alliances, defensive alliances and offensive alliances. The concept of having general edge inspires me to study the behavior of general, strong edges and connected edge in the way that, three types of numbers and set, e.g., alliances, defensive alliances and offensive alliances are the cases of study in the settings of individuals and in settings of families. Also, there are some extensions into alliances, t-defensive alliances and t-offensive alliances.

The framework of this study is as follows. In the beginning, I introduced basic definitions to clarify about preliminaries. In subsection "Preliminaries", new notions of (strong/connected)alliances, (strong/connected)t-defensive alliances and (strong/connected)t-offensive alliances are applied to set of vertices of neutrosophic graphs as individuals. In section "In the Setting of Set", specific sets have the key role in this way. Classes of neutrosophic graphs are studied in the terms of different sets in section "Classes of Neutrosophic Graphs" as individuals. In the section "In the Setting of Number", usages of general numbers have key role in this study as individuals. In section "Classes of Neutrosophic Graphs", both numbers have applied into individuals. And as a concluding result, there's one statement about the family of neutrosophic graphs in this section. In section "Applications in Time Table and Scheduling", some applications are posed for alliances concerning time table and scheduling when the suspicions are about choosing some subjects. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications are featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions are formed.

4.5 Preliminaries

Definition 4.5.1. (Graph).
$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Definition 4.5.2. (Neutrosophic Graph).
$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, $\mu_i : E \to [0, 1]$, and for every $v_iv_j \in E$,

$$\mu(v_iv_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic edge set**.

(iii) : $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

(iv) : $\Sigma_{v\in V}\sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

(v) : $|E|$ is called **size** of NTG and it's denoted by $\mathcal{S}(NTG)$.

(vi) : $\Sigma_{e\in E}\mu(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$.

Definition 4.5.3. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) : a sequence of vertices $P : x_0, x_1, \cdots, x_n$ is called a **path** where $x_ix_{i+1} \in E$, $i = 0, 1, \cdots, n-1$;

(ii) : **strength** of path $P : x_0, x_1, \cdots, x_n$ is $\bigwedge_{i=0,\cdots,n-1} \mu(x_ix_{i+1})$;

(iii) : **connectedness** amid vertices x_0 and x_n is

$$\mu^{\infty}(x, y) = \bigwedge_{P:x_0,x_1,\cdots,x_n} \bigwedge_{i=0,\cdots,n-1} \mu(x_ix_{i+1}).$$

(iv) : a sequence of vertices $P : x_0, x_1, \cdots, x_n$ is called a **path** where $x_ix_{i+1} \in E$, $i = 0, 1, \cdots, n-1$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_iv_{i+1})$;

(v) : it's a **t-partite** where V is partitioned to t parts, $V_1, V_2, \cdots, V_t$ and the edge xy implies $x \in V_i$ and $y \in V_j$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where σ_i is σ on V_i instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$.

(v) : an t-partite is **complete bipartite** If $t = 2$, and it's denoted by K_{σ_1,σ_2}.

(vi) : a complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1,σ_2}.

(vii) : a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1,σ_2}.

$(viii)$: it's a **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

(ix) : it's a **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

Based on different edges, it's possible to define different neighbors as follows.

Definition 4.5.4. (Different Neighbors).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Suppose $x \in V$.

(i) : $N(x) = \{y \in V \mid xy \in E\}$;

(ii) : $N_s(x) = \{y \in N(x) \mid \mu(xy) = \sigma(x) \wedge \sigma(y)\}$;

(iii) : $N_c(x) = \{y \in N(x) \mid \mu(xy) = \mu^{\infty}(x, y)\}$.

Definition 4.5.5. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. A set S is called

(i) : **offensive alliance** if $\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)|$;

(ii) : **defensive alliance** if $\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)|$;

(iii) : **t-offensive alliance** if $\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V \setminus S))| > t$;

(iv) : **t-defensive alliance** if $\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V \setminus S))| < t$.

Definition 4.5.6. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. A set S is called

(i) : **strong offensive alliance** if $\forall a \in S,\ |N_s(a) \cap S| > |N_s(a) \cap (V \setminus S)|$;

(ii) : **strong defensive alliance** if $\forall a \in S,\ |N_s(a) \cap S| < |N_s(a) \cap (V \setminus S)|$;

(iii) : **strong t-offensive alliance** if $\forall a \in S,\ |(N_s(a) \cap S) - (N_s(a) \cap (V \setminus S))| > t$;

(iv) : **strong t-defensive alliance** if $\forall a \in S,\ |(N_s(a) \cap S) - (N_s(a) \cap (V \setminus S))| < t$.

Definition 4.5.7. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. A set S is called

(i) : **connected offensive alliance** if $\forall a \in S,\ |N_c(a) \cap S| > |N_c(a) \cap (V \setminus S)|$;

(ii) : **connected defensive alliance** if $\forall a \in S,\ |N_c(a) \cap S| < |N_c(a) \cap (V \setminus S)|$;

(iii) : **connected t-offensive alliance** if $\forall a \in S,\ |(N_c(a) \cap S) - (N_c(a) \cap (V \setminus S))| > t$;

(iv) : **connected t-defensive alliance** if $\forall a \in S,\ |(N_c(a) \cap S) - (N_c(a) \cap (V \setminus S))| < t$.

Definition 4.5.8. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) : **number** of NTG is $\bigwedge_{S \text{ is alliance}} |S|$;

(ii) : **neutrosophic number** of NTG is $\bigwedge_{S \text{ is alliance}} \Sigma_{s \in S} \sigma(s)$.

4.6 In the Setting of Set

Proposition 4.6.1. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then V is*

(i) : *offensive alliance;*

(ii) : *strong offensive alliance;*

(iii) : *connected offensive alliance;*

(iv) : *δ-offensive alliance;*

(v) : *strong δ-offensive alliance;*

(vi) : *connected δ-offensive alliance.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider V. All members of V have at least one neighbor inside the set more than neighbor out of set. Thus,
(i). V is offensive alliance since the following statements are equivalent.

$$\begin{gathered}\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv\\ \forall a \in V,\ |N(a) \cap V| > |N(a) \cap (V \setminus V)| \equiv\\ \forall a \in V,\ |N(a) \cap V| > |N(a) \cap \emptyset| \equiv\\ \forall a \in V,\ |N(a) \cap V| > |\emptyset| \equiv\\ \forall a \in V,\ |N(a) \cap V| > 0 \equiv\\ \forall a \in V,\ \delta > 0.\end{gathered}$$

(ii). V is strong offensive alliance since the following statements are equivalent.

$$\begin{gathered}\forall a \in S,\ |N_s(a) \cap S| > |N_s(a) \cap (V \setminus S)| \equiv\\ \forall a \in V,\ |N_s(a) \cap V| > |N_s(a) \cap (V \setminus V)| \equiv\\ \forall a \in V,\ |N_s(a) \cap V| > |N_s(a) \cap \emptyset| \equiv\\ \forall a \in V,\ |N_s(a) \cap V| > |\emptyset| \equiv\\ \forall a \in V,\ |N_s(a) \cap V| > 0 \equiv\\ \forall a \in V,\ \delta > 0.\end{gathered}$$

(iii). V is connected offensive alliance since the following statements are equivalent.

$$\begin{gathered}\forall a \in S,\ |N_c(a) \cap S| > |N_c(a) \cap (V \setminus S)| \equiv\\ \forall a \in V,\ |N_c(a) \cap V| > |N_c(a) \cap (V \setminus V)| \equiv\\ \forall a \in V,\ |N_c(a) \cap V| > |N_c(a) \cap \emptyset| \equiv\\ \forall a \in V,\ |N_c(a) \cap V| > |\emptyset| \equiv\\ \forall a \in V,\ |N_c(a) \cap V| > 0 \equiv\\ \forall a \in V,\ \delta > 0.\end{gathered}$$

(iv). V is offensive alliance since the following statements are equivalent.

$$\begin{gathered}\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V \setminus S))| > \delta \equiv\\ \forall a \in V,\ |(N(a) \cap V) - (N(a) \cap (V \setminus V))| > \delta \equiv\\ \forall a \in V,\ |(N(a) \cap V) - (N(a) \cap (\emptyset))| > \delta \equiv\\ \forall a \in V,\ |(N(a) \cap V) - (\emptyset)| > \delta \equiv\\ \forall a \in V,\ |(N(a) \cap V)| > \delta.\end{gathered}$$

(v). V is strong offensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N_s(a) \cap S) - (N_s(a) \cap (V \setminus S))| > \delta \equiv$$
$$\forall a \in V,\ |(N_s(a) \cap V) - (N_s(a) \cap (V \setminus V))| > \delta \equiv$$
$$\forall a \in V,\ |(N_s(a) \cap V) - (N_s(a) \cap (\emptyset))| > \delta \equiv$$
$$\forall a \in V,\ |(N_s(a) \cap V) - (\emptyset)| > \delta \equiv$$
$$\forall a \in V,\ |(N_s(a) \cap V)| > \delta.$$

(vi). V is connected offensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N_c(a) \cap S) - (N_c(a) \cap (V \setminus S))| > \delta \equiv$$
$$\forall a \in V,\ |(N_c(a) \cap V) - (N_c(a) \cap (V \setminus V))| > \delta \equiv$$
$$\forall a \in V,\ |(N_c(a) \cap V) - (N_c(a) \cap (\emptyset))| > \delta \equiv$$
$$\forall a \in V,\ |(N_c(a) \cap V) - (\emptyset)| > \delta \equiv$$
$$\forall a \in V,\ |(N_c(a) \cap V)| > \delta.$$

■

Proposition 4.6.2. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then $\emptyset$ is*

(i) : *defensive alliance;*

(ii) : *strong defensive alliance;*

(iii) : *connected defensive alliance;*

(iv) : *δ-defensive alliance;*

(v) : *strong δ-defensive alliance;*

(vi) : *connected δ-defensive alliance.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider $\emptyset$. All members of $\emptyset$ have no neighbor inside the set less than neighbor out of set. Thus,
(i). $\emptyset$ is defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in \emptyset,\ |N(a) \cap \emptyset| < |N(a) \cap (V \setminus \emptyset)| \equiv$$
$$\forall a \in \emptyset,\ |\emptyset| < |N(a) \cap (V \setminus \emptyset)| \equiv$$
$$\forall a \in \emptyset,\ 0 < |N(a) \cap V| \equiv$$
$$\forall a \in \emptyset,\ 0 < |N(a) \cap V| \equiv$$
$$\forall a \in V,\ \delta > 0.$$

(ii). $\emptyset$ is strong defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N_s(a) \cap S| < |N_s(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in \emptyset,\ |N_s(a) \cap \emptyset| < |N_s(a) \cap (V \setminus \emptyset)| \equiv$$
$$\forall a \in \emptyset,\ |\emptyset| < |N_s(a) \cap (V \setminus \emptyset)| \equiv$$
$$\forall a \in \emptyset,\ 0 < |N_s(a) \cap V| \equiv$$
$$\forall a \in \emptyset,\ 0 < |N_s(a) \cap V| \equiv$$
$$\forall a \in V,\ \delta > 0.$$

(iii). $\emptyset$ is connected defensive alliance since the following statements are equivalent.

$$\begin{gathered}\forall a \in S,\ |N_c(a) \cap S| < |N_c(a) \cap (V \setminus S)| \equiv \\ \forall a \in \emptyset,\ |N_c(a) \cap \emptyset| < |N_c(a) \cap (V \setminus \emptyset)| \equiv \\ \forall a \in \emptyset,\ |\emptyset| < |N_c(a) \cap (V \setminus \emptyset)| \equiv \\ \forall a \in \emptyset,\ 0 < |N_c(a) \cap V| \equiv \\ \forall a \in \emptyset,\ 0 < |N_c(a) \cap V| \equiv \\ \forall a \in V,\ \delta > 0.\end{gathered}$$

(iv). $\emptyset$ is defensive alliance since the following statements are equivalent.

$$\begin{gathered}\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V \setminus S))| < \delta \equiv \\ \forall a \in \emptyset,\ |(N(a) \cap \emptyset) - (N(a) \cap (V \setminus \emptyset))| < \delta \equiv \\ \forall a \in \emptyset,\ |(N(a) \cap \emptyset) - (N(a) \cap (V))| < \delta \equiv \\ \forall a \in \emptyset,\ |\emptyset| < \delta \equiv \\ \forall a \in V,\ 0 < \delta.\end{gathered}$$

(v). $\emptyset$ is strong defensive alliance since the following statements are equivalent.

$$\begin{gathered}\forall a \in S,\ |(N_s(a) \cap S) - (N_s(a) \cap (V \setminus S))| < \delta \equiv \\ \forall a \in \emptyset,\ |(N_s(a) \cap \emptyset) - (N_s(a) \cap (V \setminus \emptyset))| < \delta \equiv \\ \forall a \in \emptyset,\ |(N_s(a) \cap \emptyset) - (N_s(a) \cap (V))| < \delta \equiv \\ \forall a \in \emptyset,\ |\emptyset| < \delta \equiv \\ \forall a \in V,\ 0 < \delta.\end{gathered}$$

(vi). $\emptyset$ is connected defensive alliance since the following statements are equivalent.

$$\begin{gathered}\forall a \in S,\ |(N_c(a) \cap S) - (N_c(a) \cap (V \setminus S))| < \delta \equiv \\ \forall a \in \emptyset,\ |(N_c(a) \cap \emptyset) - (N_c(a) \cap (V \setminus \emptyset))| < \delta \equiv \\ \forall a \in \emptyset,\ |(N_c(a) \cap \emptyset) - (N_c(a) \cap (V))| < \delta \equiv \\ \forall a \in \emptyset,\ |\emptyset| < \delta \equiv \\ \forall a \in V,\ 0 < \delta.\end{gathered}$$

■

Proposition 4.6.3. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then an independent set is*

(i) : *defensive alliance;*

(ii) : *strong defensive alliance;*

(iii) : *connected defensive alliance;*

(iv) : *δ-defensive alliance;*

(v) : *strong δ-defensive alliance;*

(vi) : *connected δ-defensive alliance.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider $\emptyset$. All members of $\emptyset$ have no neighbor inside the set less than neighbor out of set. Thus,
(i). An independent set is defensive alliance since the following statements are equivalent.

$$\begin{gathered}
\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
\forall a \in S,\ |\emptyset| < |N(a) \cap (V \setminus S)| \equiv \\
\forall a \in S,\ 0 < |N(a) \cap V| \equiv \\
\forall a \in S,\ 0 < |N(a)| \equiv \\
\forall a \in V,\ \delta > 0.
\end{gathered}$$

(ii). An independent set is strong defensive alliance since the following statements are equivalent.

$$\begin{gathered}
\forall a \in S,\ |N_s(a) \cap S| < |N_s(a) \cap (V \setminus S)| \equiv \\
\forall a \in S,\ |N_s(a) \cap S| < |N_s(a) \cap (V \setminus S)| \equiv \\
\forall a \in S,\ |\emptyset| < |N_s(a) \cap (V \setminus S)| \equiv \\
\forall a \in S,\ 0 < |N_s(a) \cap V| \equiv \\
\forall a \in S,\ 0 < |N_s(a)| \equiv \\
\forall a \in V,\ \delta > 0.
\end{gathered}$$

(iii). An independent set is connected defensive alliance since the following statements are equivalent.

$$\begin{gathered}
\forall a \in S,\ |N_c(a) \cap S| < |N_c(a) \cap (V \setminus S)| \equiv \\
\forall a \in S,\ |N_c(a) \cap S| < |N_c(a) \cap (V \setminus S)| \equiv \\
\forall a \in S,\ |\emptyset| < |N_c(a) \cap (V \setminus S)| \equiv \\
\forall a \in S,\ 0 < |N_c(a) \cap V| \equiv \\
\forall a \in S,\ 0 < |N_c(a)| \equiv \\
\forall a \in V,\ \delta > 0.
\end{gathered}$$

(iv). An independent set is defensive alliance since the following statements are equivalent.

$$\begin{gathered}
\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V \setminus S))| < \delta \equiv \\
\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V \setminus S))| < \delta \equiv \\
\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V))| < \delta \equiv \\
\forall a \in S,\ |\emptyset| < \delta \equiv \\
\forall a \in V,\ 0 < \delta.
\end{gathered}$$

(v). An independent set is strong defensive alliance since the following statements are equivalent.

$$\begin{gathered}
\forall a \in S,\ |(N_s(a) \cap S) - (N_s(a) \cap (V \setminus S))| < \delta \equiv \\
\forall a \in S,\ |(N_s(a) \cap S) - (N_s(a) \cap (V \setminus S))| < \delta \equiv \\
\forall a \in S,\ |(N_s(a) \cap S) - (N_s(a) \cap (V))| < \delta \equiv \\
\forall a \in S,\ |\emptyset| < \delta \equiv \\
\forall a \in V,\ 0 < \delta.
\end{gathered}$$

(vi). An independent set is connected defensive alliance since the following statements are equivalent.

$$\begin{gathered}
\forall a \in S,\ |(N_c(a) \cap S) - (N_c(a) \cap (V \setminus S))| < \delta \equiv \\
\forall a \in S,\ |(N_c(a) \cap S) - (N_c(a) \cap (V \setminus S))| < \delta \equiv \\
\forall a \in S,\ |(N_c(a) \cap S) - (N_c(a) \cap (V))| < \delta \equiv \\
\forall a \in S,\ |\emptyset| < \delta \equiv \\
\forall a \in V,\ 0 < \delta.
\end{gathered}$$

■

4.7 Classes of Neutrosophic Graphs

Proposition 4.7.1. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph which is cycle/path/wheel. Then V is minimal*

(i) : *offensive alliance;*

(ii) : *strong offensive alliance;*

(iii) : *connected offensive alliance;*

(iv) : *$\mathcal{O}(NTG)$-offensive alliance;*

(v) : *strong $\mathcal{O}(NTG)$-offensive alliance;*

(vi) : *connected $\mathcal{O}(NTG)$-offensive alliance.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph which is cycle/path//wheel.
(i). Consider one vertex is out of S which is alliance. This vertex has one neighbor in S, i.e, Suppose $x \in V \setminus S$ such that $y, z \in N(x)$. By it's cycle, $|N(x)| = |N(y)| = |N(z)| = 2$. Thus

$$\begin{gathered}
\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap (V \setminus (V \setminus \{x\}))| \equiv \\
\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap \{x\})| \equiv \\
\exists y \in V \setminus \{x\},\ |\{z\}| < |\{x\})| \equiv \\
\exists y \in S,\ 1 < 1.
\end{gathered}$$

Thus it's contradiction. It implies every $V \setminus \{x\}$ isn't offensive alliance in a given cycle.
Consider one vertex is out of S which is alliance. This vertex has one neighbor in S, i.e, Suppose $x \in V \setminus S$ such that $y, z \in N(x)$. By it's path, $|N(x)| = |N(y)| = |N(z)| = 2$. Thus

$$\begin{gathered}
\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap (V \setminus (V \setminus \{x\}))| \equiv \\
\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap \{x\})| \equiv \\
\exists y \in V \setminus \{x\},\ |\{z\}| < |\{x\})| \equiv \\
\exists y \in S,\ 1 < 1.
\end{gathered}$$

Thus it's contradiction. It implies every $V \setminus \{x\}$ isn't offensive alliance in a given path.
Consider one vertex is out of S which is alliance. This vertex has one neighbor in S, i.e, Suppose $x \in V \setminus S$ such that $y, z \in N(x)$. By it's wheel, $|N(x)| = |N(y)| = |N(z)| = 2$. Thus

$$\begin{gathered}
\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap (V \setminus (V \setminus \{x\}))| \equiv \\
\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap \{x\})| \equiv \\
\exists y \in V \setminus \{x\},\ |\{z\}| < |\{x\})| \equiv \\
\exists y \in S,\ 1 < 1.
\end{gathered}$$

Thus it's contradiction. It implies every $V \setminus \{x\}$ isn't offensive alliance in a given wheel.

(ii), (iii) are obvious by (i).

(iv). By (i), $|V|$ is minimal and it's offensive alliance. Thus it's $|V|$-offensive alliance.

$(v), (vi)$ are obvious by (iv). ■

Proposition 4.7.2. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph which is cycle/path/wheel. Then V is only*

(i) : *offensive alliance;*

(ii) : *strong offensive alliance;*

(iii) : *connected offensive alliance;*

(iv) : *$\mathcal{O}(NTG)$-offensive alliance;*

(v) : *strong $\mathcal{O}(NTG)$-offensive alliance;*

(vi) : *connected $\mathcal{O}(NTG)$-offensive alliance.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph which is cycle/path//wheel.
(i). Consider one vertex is out of S which is alliance. This vertex has one neighbor in S, i.e, Suppose $x \in V \setminus S$ such that $y, z \in N(x)$. By it's cycle, $|N(x)| = |N(y)| = |N(z)| = 2$. Thus

$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap (V \setminus (V \setminus \{x\}))| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap \{x\})| \equiv$$
$$\exists y \in V \setminus \{x\},\ |\{z\}| < |\{x\})| \equiv$$
$$\exists y \in S,\ 1 < 1.$$

Thus it's contradiction. It implies every $V \setminus \{x\}$ isn't offensive alliance in a given cycle.

Consider one vertex is out of S which is alliance. This vertex has one neighbor in S, i.e, Suppose $x \in V \setminus S$ such that $y, z \in N(x)$. By it's path, $|N(x)| = |N(y)| = |N(z)| = 2$. Thus

$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap (V \setminus (V \setminus \{x\}))| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap \{x\})| \equiv$$
$$\exists y \in V \setminus \{x\},\ |\{z\}| < |\{x\})| \equiv$$
$$\exists y \in S,\ 1 < 1.$$

Thus it's contradiction. It implies every $V \setminus \{x\}$ isn't offensive alliance in a given path.

Consider one vertex is out of S which is alliance. This vertex has one neighbor in S, i.e, Suppose $x \in V \setminus S$ such that $y, z \in N(x)$. By it's wheel, $|N(x)| = |N(y)| = |N(z)| = 2$. Thus

$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap (V \setminus (V \setminus \{x\}))| \equiv$$
$$\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap \{x\})| \equiv$$
$$\exists y \in V \setminus \{x\},\ |\{z\}| < |\{x\})| \equiv$$
$$\exists y \in S,\ 1 < 1.$$

Thus it's contradiction. It implies every $V \setminus \{x\}$ isn't offensive alliance in a given wheel.

(ii), (iii) are obvious by (i).

(iv). By (i), V is minimal and it's offensive alliance. Thus it's $\mathcal{O}(NTG)$-offensive alliance.

$(v), (vi)$ are obvious by (iv). ■

Proposition 4.7.3. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph which is star/complete bipartite/complete t-partite. Then center and n half +1 vertices is minimal*

(i) : *offensive alliance;*

(ii) : *strong offensive alliance;*

(iii) : *connected offensive alliance;*

(iv) : $\frac{\mathcal{O}(NTG)}{2} + 1$*-offensive alliance;*

(v) : *strong* $\frac{\mathcal{O}(NTG)}{2} + 1$*-offensive alliance;*

(vi) : *connected* $\frac{\mathcal{O}(NTG)}{2} + 1$*-offensive alliance.*

Proof. (i). Consider n half +1 vertices are out of S which is alliance. This vertex has n half neighbor in S. If the vertex is non-center, then

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ 1 > 0.$$

If the vertex is center, then

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is offensive alliance in a given star.

Consider n half +1 vertices are out of S which is alliance. This vertex has n half neighbor in S.

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is offensive alliance in a given complete bipartite which isn't a star.

Consider n half +1 vertices are out of S which is alliance and they are chosen from different parts, equally or almost equally as possible. This vertex has n half neighbor in S.

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is offensive alliance in a given complete t-partite which isn't neither a star nor complete bipartite.

(ii), (iii) are obvious by (i).

(iv). By (i), $\{x_i\}_{i=1}^{\frac{\mathcal{O}(NTG)}{2}+1}$ is minimal and it's offensive alliance. Thus it's $\frac{\mathcal{O}(NTG)}{2}+1$-offensive alliance.

$(v), (vi)$ are obvious by (iv). ■

Proposition 4.7.4. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph which is star/complete bipartite/complete t-partite. Then center and n half $+1$ vertices is only*

(i) : *offensive alliance;*

(ii) : *strong offensive alliance;*

(iii) : *connected offensive alliance;*

(iv) : *δ-offensive alliance;*

(v) : *strong δ-offensive alliance;*

(vi) : *connected δ-offensive alliance.*

Proof. (i). Consider n half $+1$ vertices are out of S which is alliance. This vertex has n half neighbor in S. If the vertex is non-center, then

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ 1 > 0.$$

If the vertex is center, then

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is offensive alliance in a given star.

Consider n half $+1$ vertices are out of S which is alliance. This vertex has n half neighbor in S.

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is offensive alliance in a given complete bipartite which isn't a star.

Consider n half $+1$ vertices are out of S which is alliance and they are chosen from different parts, equally or almost equally as possible. This vertex has n half neighbor in S.

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is offensive alliance in a given complete t-partite which isn't neither a star nor complete bipartite.

(ii), (iii) are obvious by (i).

(iv). By (i), $\{x_i\}_{i=1}^{\frac{\mathcal{O}(NTG)}{2}+1}$ is minimal and it's offensive alliance. Thus it's $\frac{\mathcal{O}(NTG)}{2}+1$-offensive alliance.

$(v), (vi)$ are obvious by (iv). ■

4.8 In the Setting of Number

Proposition 4.8.1. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. The number of connected component is $|V - S|$ if there's a set which is*

(i) : *offensive alliance;*

(ii) : *strong offensive alliance;*

(iii) : *connected offensive alliance;*

(iv) : *1-offensive alliance;*

(v) : *strong 1-offensive alliance;*

(vi) : *connected 1-offensive alliance.*

Proof. (i). Consider some vertices are out of S which is alliance. This vertex has n half neighbor in S but no vertex out of S. Thus

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ 1 > 0.$$

Thus it's proved. It implies every S is offensive alliance and number of connected component is $|V - S|$.

Consider some vertices are out of S which is alliance. This vertex has n half neighbor in S but no vertex out of S. Thus

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ 1 > 0.$$

Thus it's proved. It implies every S is offensive alliance and number of connected component is $|V - S|$.

Consider some vertices are out of S which is alliance. This vertex has n half neighbor in S but no vertex out of S. Thus

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ 1 > 0.$$

Thus it's proved. It implies every S is offensive alliance and number of connected component is $|V - S|$.

(ii), (iii) are obvious by (i).

(iv). By (i), $\{x\}$ is minimal and it's offensive alliance. Thus it's 1-offensive alliance.

$(v), (vi)$ are obvious by (iv). ■

Proposition 4.8.2. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then the number is at most $\mathcal{O}(NTG)$ and the neutrosophic number is at most $\mathcal{O}_n(NTG)$.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph. Consider V. All members of V have at least one neighbor inside the set more than neighbor out of set. Thus,
V is offensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in V,\ |N(a) \cap V| > |N(a) \cap (V \setminus V)| \equiv$$
$$\forall a \in V,\ |N(a) \cap V| > |N(a) \cap \emptyset| \equiv$$
$$\forall a \in V,\ |N(a) \cap V| > |\emptyset| \equiv$$
$$\forall a \in V,\ |N(a) \cap V| > 0 \equiv$$
$$\forall a \in V,\ \delta > 0.$$

V is strong offensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N_s(a) \cap S| > |N_s(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in V,\ |N_s(a) \cap V| > |N_s(a) \cap (V \setminus V)| \equiv$$
$$\forall a \in V,\ |N_s(a) \cap V| > |N_s(a) \cap \emptyset| \equiv$$
$$\forall a \in V,\ |N_s(a) \cap V| > |\emptyset| \equiv$$
$$\forall a \in V,\ |N_s(a) \cap V| > 0 \equiv$$
$$\forall a \in V,\ \delta > 0.$$

V is connected offensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N_c(a) \cap S| > |N_c(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in V,\ |N_c(a) \cap V| > |N_c(a) \cap (V \setminus V)| \equiv$$
$$\forall a \in V,\ |N_c(a) \cap V| > |N_c(a) \cap \emptyset| \equiv$$
$$\forall a \in V,\ |N_c(a) \cap V| > |\emptyset| \equiv$$
$$\forall a \in V,\ |N_c(a) \cap V| > 0 \equiv$$
$$\forall a \in V,\ \delta > 0.$$

V is offensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V \setminus S))| > \delta \equiv$$
$$\forall a \in V,\ |(N(a) \cap V) - (N(a) \cap (V \setminus V))| > \delta \equiv$$
$$\forall a \in V,\ |(N(a) \cap V) - (N(a) \cap (\emptyset))| > \delta \equiv$$
$$\forall a \in V,\ |(N(a) \cap V) - (\emptyset)| > \delta \equiv$$
$$\forall a \in V,\ |(N(a) \cap V)| > \delta.$$

V is strong offensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N_s(a) \cap S) - (N_s(a) \cap (V \setminus S))| > \delta \equiv$$
$$\forall a \in V,\ |(N_s(a) \cap V) - (N_s(a) \cap (V \setminus V))| > \delta \equiv$$
$$\forall a \in V,\ |(N_s(a) \cap V) - (N_s(a) \cap (\emptyset))| > \delta \equiv$$
$$\forall a \in V,\ |(N_s(a) \cap V) - (\emptyset)| > \delta \equiv$$
$$\forall a \in V,\ |(N_s(a) \cap V)| > \delta.$$

V is connected offensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N_c(a) \cap S) - (N_c(a) \cap (V \setminus S))| > \delta \equiv$$
$$\forall a \in V,\ |(N_c(a) \cap V) - (N_c(a) \cap (V \setminus V))| > \delta \equiv$$
$$\forall a \in V,\ |(N_c(a) \cap V) - (N_c(a) \cap (\emptyset))| > \delta \equiv$$
$$\forall a \in V,\ |(N_c(a) \cap V) - (\emptyset)| > \delta \equiv$$
$$\forall a \in V,\ |(N_c(a) \cap V)| > \delta.$$

Thus V is alliance and V is the biggest set in NTG. Then the number is at most $\mathcal{O}(NTG)$ and the neutrosophic number is at most $\mathcal{O}_n(NTG)$. ∎

Proposition 4.8.3. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph which is complete. The number is $\frac{\mathcal{O}(NTG)}{2} + 1$ and the neutrosophic number is $\min \Sigma_{v \in \{v_1, v_2, \cdots, v_t\}_{t > \frac{\mathcal{O}(NTG)}{2}} \subseteq V} \sigma(v)$, in the setting of*

(i) : *offensive alliance;*

(ii) : *strong offensive alliance;*

(iii) : *connected offensive alliance;*

(iv) : $(\frac{\mathcal{O}(NTG)}{2}+1)$*-offensive alliance;*

(v) : *strong* $(\frac{\mathcal{O}(NTG)}{2}+1)$*-offensive alliance;*

(vi) : *connected* $(\frac{\mathcal{O}(NTG)}{2}+1)$*-offensive alliance.*

Proof. (i). Consider n half $+1$ vertices are out of S which is alliance. This vertex has n half neighbor in S.

$$\forall a \in S,\ |N(a)\cap S| > |N(a)\cap (V\setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2}-1.$$

Thus it's proved. It implies every S is offensive alliance in a given complete graph. Thus the number is $\frac{\mathcal{O}(NTG)}{2}+1$ and the neutrosophic number is $\min \Sigma_{v\in\{v_1,v_2,\cdots,v_t\}_{t>\frac{\mathcal{O}(NTG)}{2}}\subseteq V}\sigma(v)$, in the setting of offensive alliance.

(ii). Consider n half $+1$ vertices are out of S which is alliance. This vertex has n half neighbor in S.

$$\forall a \in S,\ |N_s(a)\cap S| > |N_s(a)\cap (V\setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2}-1.$$

Thus it's proved. It implies every S is strong offensive alliance in a given complete graph. Thus the number is $\frac{\mathcal{O}(NTG)}{2}+1$ and the neutrosophic number is $\min \Sigma_{v\in\{v_1,v_2,\cdots,v_t\}_{t>\frac{\mathcal{O}(NTG)}{2}}\subseteq V}\sigma(v)$, in the setting of strong offensive alliance.

(iii). Consider n half $+1$ vertices are out of S which is alliance. This vertex has n half neighbor in S.

$$\forall a \in S,\ |N_c(a)\cap S| > |N_c(a)\cap (V\setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2}-1.$$

Thus it's proved. It implies every S is connected offensive alliance in a given complete graph. Thus the number is $\frac{\mathcal{O}(NTG)}{2}+1$ and the neutrosophic number is $\min \Sigma_{v\in\{v_1,v_2,\cdots,v_t\}_{t>\frac{\mathcal{O}(NTG)}{2}}\subseteq V}\sigma(v)$, in the setting of connected offensive alliance.

(iv). Consider n half $+1$ vertices are out of S which is alliance. This vertex has n half neighbor in S.

$$\forall a \in S,\ |N(a)\cap S| > |N(a)\cap (V\setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2}-1.$$

Thus it's proved. It implies every S is $(\frac{\mathcal{O}(NTG)}{2}+1)$-offensive alliance in a given complete graph. Thus the number is $\frac{\mathcal{O}(NTG)}{2}+1$ and the neutrosophic number is $\min \Sigma_{v\in\{v_1,v_2,\cdots,v_t\}_{t>\frac{\mathcal{O}(NTG)}{2}}\subseteq V}\sigma(v)$, in the setting of $(\frac{\mathcal{O}(NTG)}{2}+1)$-offensive alliance.

(v). Consider n half $+1$ vertices are out of S which is alliance. This vertex has n half neighbor in S.

$$\forall a \in S,\ |N_s(a)\cap S| > |N_s(a)\cap (V\setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2}-1.$$

Thus it's proved. It implies every S is strong $(\frac{\mathcal{O}(NTG)}{2}+1)$-offensive alliance in a given complete graph. Thus the number is $\frac{\mathcal{O}(NTG)}{2}+1$ and the neutrosophic number is $\min \Sigma_{v\in\{v_1,v_2,\cdots,v_t\}_{t>\frac{\mathcal{O}(NTG)}{2}}\subseteq V}\sigma(v)$, in the setting of strong $(\frac{\mathcal{O}(NTG)}{2}+1)$-offensive alliance.

(vi). Consider n half $+1$ vertices are out of S which is alliance. This vertex has n half neighbor in S.

$$\forall a \in S,\ |N_c(a)\cap S| > |N_c(a)\cap (V\setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is connected $(\frac{\mathcal{O}(NTG)}{2}+1)$-offensive alliance in a given complete graph. Thus the number is $\frac{\mathcal{O}(NTG)}{2}+1$ and the neutrosophic number is $\min \Sigma_{v\in\{v_1,v_2,\cdots,v_t\}_{t>\frac{\mathcal{O}(NTG)}{2}}\subseteq V}\sigma(v)$, in the setting of connected $(\frac{\mathcal{O}(NTG)}{2}+1)$-offensive alliance. ■

Proposition 4.8.4. *Let $NTG:(V,E,\sigma,\mu)$ be a neutrosophic graph which is $\emptyset$. The number is 0 and the neutrosophic number is 0, for an independent set in the setting of*

(i) : *offensive alliance;*

(ii) : *strong offensive alliance;*

(iii) : *connected offensive alliance;*

(iv) : 0-*offensive alliance;*

(v) : *strong* 0-*offensive alliance;*

(vi) : *connected* 0-*offensive alliance.*

Proof. Suppose $NTG:(V,E,\sigma,\mu)$ is a neutrosophic graph. Consider $\emptyset$. All members of $\emptyset$ have no neighbor inside the set less than neighbor out of set. Thus,

(i). $\emptyset$ is defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N(a)\cap S| < |N(a)\cap (V\setminus S)| \equiv$$
$$\forall a \in \emptyset,\ |N(a)\cap \emptyset| < |N(a)\cap (V\setminus \emptyset)| \equiv$$
$$\forall a \in \emptyset,\ |\emptyset| < |N(a)\cap (V\setminus \emptyset)| \equiv$$
$$\forall a \in \emptyset,\ 0 < |N(a)\cap V| \equiv$$
$$\forall a \in \emptyset,\ 0 < |N(a)\cap V| \equiv$$
$$\forall a \in V,\ \delta > 0.$$

The number is 0 and the neutrosophic number is 0, for an independent set in the setting of offensive alliance.

(ii). $\emptyset$ is strong defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N_s(a)\cap S| < |N_s(a)\cap (V\setminus S)| \equiv$$
$$\forall a \in \emptyset,\ |N_s(a)\cap \emptyset| < |N_s(a)\cap (V\setminus \emptyset)| \equiv$$
$$\forall a \in \emptyset,\ |\emptyset| < |N_s(a)\cap (V\setminus \emptyset)| \equiv$$
$$\forall a \in \emptyset,\ 0 < |N_s(a)\cap V| \equiv$$
$$\forall a \in \emptyset,\ 0 < |N_s(a)\cap V| \equiv$$
$$\forall a \in V,\ \delta > 0.$$

The number is 0 and the neutrosophic number is 0, for an independent set in the setting of strong offensive alliance.

(iii). $\emptyset$ is connected defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |N_c(a) \cap S| < |N_c(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in \emptyset,\ |N_c(a) \cap \emptyset| < |N_c(a) \cap (V \setminus \emptyset)| \equiv$$
$$\forall a \in \emptyset,\ |\emptyset| < |N_c(a) \cap (V \setminus \emptyset)| \equiv$$
$$\forall a \in \emptyset,\ 0 < |N_c(a) \cap V| \equiv$$
$$\forall a \in \emptyset,\ 0 < |N_c(a) \cap V| \equiv$$
$$\forall a \in V,\ \delta > 0.$$

The number is 0 and the neutrosophic number is 0, for an independent set in the setting of connected offensive alliance.

(iv). $\emptyset$ is defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V \setminus S))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |(N(a) \cap \emptyset) - (N(a) \cap (V \setminus \emptyset))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |(N(a) \cap \emptyset) - (N(a) \cap (V))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |\emptyset| < \delta \equiv$$
$$\forall a \in V,\ 0 < \delta.$$

The number is 0 and the neutrosophic number is 0, for an independent set in the setting of 0-offensive alliance.

(v). $\emptyset$ is strong defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N_s(a) \cap S) - (N_s(a) \cap (V \setminus S))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |(N_s(a) \cap \emptyset) - (N_s(a) \cap (V \setminus \emptyset))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |(N_s(a) \cap \emptyset) - (N_s(a) \cap (V))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |\emptyset| < \delta \equiv$$
$$\forall a \in V,\ 0 < \delta.$$

The number is 0 and the neutrosophic number is 0, for an independent set in the setting of strong 0-offensive alliance.

(vi). $\emptyset$ is connected defensive alliance since the following statements are equivalent.

$$\forall a \in S,\ |(N_c(a) \cap S) - (N_c(a) \cap (V \setminus S))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |(N_c(a) \cap \emptyset) - (N_c(a) \cap (V \setminus \emptyset))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |(N_c(a) \cap \emptyset) - (N_c(a) \cap (V))| < \delta \equiv$$
$$\forall a \in \emptyset,\ |\emptyset| < \delta \equiv$$
$$\forall a \in V,\ 0 < \delta.$$

The number is 0 and the neutrosophic number is 0, for an independent set in the setting of connected 0-offensive alliance. ■

Proposition 4.8.5. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph which is complete. Then there's no independent set.*

4.9 Classes of Neutrosophic Graphs

Proposition 4.9.1. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph which is cycle/path/wheel. The number is $\mathcal{O}(NTG)$ and the neutrosophic number is $\mathcal{O}_n(NTG)$, in the setting of*

(i) : *offensive alliance;*

(ii) : *strong offensive alliance;*

(iii) : *connected offensive alliance;*

(iv) : *$\mathcal{O}(NTG)$-offensive alliance;*

(v) : *strong $\mathcal{O}(NTG)$-offensive alliance;*

(vi) : *connected $\mathcal{O}(NTG)$-offensive alliance.*

Proof. Suppose $NTG : (V, E, \sigma, \mu)$ is a neutrosophic graph which is cycle/path/wheel.
(i). Consider one vertex is out of S which is alliance. This vertex has one neighbor in S, i.e, Suppose $x \in V \setminus S$ such that $y, z \in N(x)$. By it's cycle, $|N(x)| = |N(y)| = |N(z)| = 2$. Thus

$$\begin{gathered}\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\ \forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\ \exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap (V \setminus (V \setminus \{x\}))| \equiv \\ \exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap \{x\})| \equiv \\ \exists y \in V \setminus \{x\},\ |\{z\}| < |\{x\})| \equiv \\ \exists y \in S,\ 1 < 1.\end{gathered}$$

Thus it's contradiction. It implies every $V \setminus \{x\}$ isn't offensive alliance in a given cycle.
Consider one vertex is out of S which is alliance. This vertex has one neighbor in S, i.e, Suppose $x \in V \setminus S$ such that $y, z \in N(x)$. By it's path, $|N(x)| = |N(y)| = |N(z)| = 2$. Thus

$$\begin{gathered}\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\ \forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\ \exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap (V \setminus (V \setminus \{x\}))| \equiv \\ \exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap \{x\})| \equiv \\ \exists y \in V \setminus \{x\},\ |\{z\}| < |\{x\})| \equiv \\ \exists y \in S,\ 1 < 1.\end{gathered}$$

Thus it's contradiction. It implies every $V \setminus \{x\}$ isn't offensive alliance in a given path.
Consider one vertex is out of S which is alliance. This vertex has one neighbor in S, i.e, Suppose $x \in V \setminus S$ such that $y, z \in N(x)$. By it's wheel, $|N(x)| = |N(y)| = |N(z)| = 2$. Thus

$$\begin{gathered}\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\ \forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\ \exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap (V \setminus (V \setminus \{x\}))| \equiv \\ \exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap \{x\})| \equiv \\ \exists y \in V \setminus \{x\},\ |\{z\}| < |\{x\})| \equiv \\ \exists y \in S,\ 1 < 1.\end{gathered}$$

Thus it's contradiction. It implies every $V \setminus \{x\}$ isn't offensive alliance in a given wheel.
(ii), (iii) are obvious by (i).
(iv). By (i), V is minimal and it's offensive alliance. Thus it's $\mathcal{O}(NTG)$-offensive alliance.
$(v), (vi)$ are obvious by (iv).
Thus the number is $\mathcal{O}(NTG)$ and the neutrosophic number is $\mathcal{O}_n(NTG)$, in the setting of all types of alliance. ∎

Proposition 4.9.2. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph which is star/complete bipartite/complete t-partite. The number is $\frac{\mathcal{O}(NTG)}{2} + 1$ and the neutrosophic number is $\min \Sigma_{v \in \{v_1, v_2, \cdots, v_t\}_{t > \frac{\mathcal{O}(NTG)}{2}} \subseteq V} \sigma(v)$, in the setting of*

(i) : *offensive alliance;*

(ii) : *strong offensive alliance;*

(iii) : *connected offensive alliance;*

(iv) : $(\frac{\mathcal{O}(NTG)}{2}+1)$*-offensive alliance;*

(v) : *strong* $(\frac{\mathcal{O}(NTG)}{2}+1)$*-offensive alliance;*

(vi) : *connected* $(\frac{\mathcal{O}(NTG)}{2}+1)$*-offensive alliance.*

Proof. (i). Consider n half $+1$ vertices are out of S which is alliance. This vertex has n half neighbor in S. If the vertex is non-center, then

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ 1 > 0.$$

If the vertex is center, then

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is offensive alliance in a given star.
Consider n half $+1$ vertices are out of S which is alliance. This vertex has n half neighbor in S.

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is offensive alliance in a given complete bipartite which isn't a star.
Consider n half $+1$ vertices are out of S which is alliance and they are chosen from different parts, equally or almost equally as possible. This vertex has n half neighbor in S.

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is offensive alliance in a given complete t-partite which isn't neither a star nor complete bipartite.
(ii), (iii) are obvious by (i).
(iv). By (i), $\{x_i\}_{i=1}^{\frac{\mathcal{O}(NTG)}{2}+1}$ is minimal and it's offensive alliance. Thus it's $\frac{\mathcal{O}(NTG)}{2}+1$-offensive alliance.
$(v), (vi)$ are obvious by (iv).
Thus the number is $\frac{\mathcal{O}(NTG)}{2}+1$ and the neutrosophic number is $\min \Sigma_{v \in \{v_1,v_2,\cdots,v_t\}_{t > \frac{\mathcal{O}(NTG)}{2}} \subseteq V} \sigma(v)$, in the setting of all alliances. ∎

Proposition 4.9.3. *Let $\mathcal{G}$ be a family of $NTGs : (V, E, \sigma, \mu)$ neutrosophic graphs which are from one-type class which the result is obtained for individual. Then results also hold for family $\mathcal{G}$ of these specific classes of neutrosophic graphs.*

Proof. There are neither conditions nor restrictions on the vertices. Thus the result on individual is extended to the result on family. ∎

4.10 Applications in Time Table and Scheduling

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has important to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive section. Beyond that, sometimes sections are not the same.

Step 3. (Model) The situation is designed as a model. The model uses data to assign every section and to assign to relation amid section, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relation amid them. Table (4.5), clarifies about the assigned numbers to these situation.

Table 4.1: Scheduling concerns its Subjects and its Connections as a neutrosophic graphs and its alliances in a Model.

tbl1c

Sections of NTG	n_1	$n_2 \cdots$	n_9
Values	$(0.99, 0.98, 0.55)$	$(0.74, 0.64, 0.46) \cdots$	$(0.99, 0.98, 0.55)$
Connections of NTG	E_1	E_2	E_3
Values	$(0.01, 0.01, 0.01)$	$(0.01, 0.01, 0.01)$	$(0.01, 0.01, 0.01)$

Step 4. (Solution) The neutrosophic graphs and its alliances as model, propose to use different types of sets. If the configuration makes complete, the set is different. Also, it holds for other types such that star, wheel, path, and cycle.

4.11 Open Problems

14 notions concerning alliances are defined in neutrosophic graphs. Thus,

Question 4.11.1. *Is it possible to use other types neighborhood arising from different types of edges to define new alliances?*

Question 4.11.2. *Are existed some connections amid different types of alliances in neutrosophic graphs?*

Question 4.11.3. *Is it possible to construct some classes of which have "nice" behavior?*

Question 4.11.4. *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

Problem 4.11.5. *Which parameters are related to this parameter?*

Problem 4.11.6. *Which approaches do work to construct applications to create independent study?*

Problem 4.11.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

4.12 Conclusion and Closing Remarks

This study uses mixed combinations of different types of definitions concerning alliances to study neutrosophic graphs. The connections of vertices which are clarified by general edges differ them from each other and put them in different categories to represent a set which is called. Further studies could be about changes in the settings to compare this notion amid different settings of neutrosophic graphs theory. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (4.6), some limitations and advantages of this study are pointed out.

Table 4.2: A Brief Overview about Advantages and Limitations of this study tbl2c

Advantages	Limitations
1. Defining Alliances	1. Specific Results
2. Defining Strong Alliances	
3. Defining Connected Alliances	2. Specific Connections
4. Applying on Individuals	
5. Applying on Family	3. Connections of Results

4.13 Global Offensive Alliances

The following sections are cited as [**1**].

4.14 Global Offensive Alliance in Strong Neutrosophic Graphs

4.15 Abstract

New setting is introduced to study the global offensive alliance. Global offensive alliance is about a set of vertices which are applied into the setting of neutrosophic graphs. Neighborhood has the key role to define this notion. Also, neighborhood is defined based on strong edges. Strong edge gets a framework as neighborhood and after that, too close vertices have key role to define global offensive alliance based on strong edges. The structure of set is studied and general results are obtained. Also, some classes of neutrosophic graphs containing complete, empty, path, cycle, star, and wheel are investigated in the

terms of set, minimal set, number, and neutrosophic number. Neutrosophic number is defined in new way. It's first time to define this type of neutrosophic number in the way that, three values of a vertex are used and they've same share to construct this number. It's called "modified neutrosophic number". Summation of three values of vertex makes one number and applying it to a set makes neutrosophic number of set. This approach facilitates identifying minimal set and optimal set which forms minimal-global-offensive-alliance number and minimal-global-offensive-alliance-neutrosophic number. Two different types of sets namely global-offensive alliance and minimal-global-offensive alliance are defined. Global-offensive alliance identifies the sets in general vision but minimal-global-offensive alliance takes focus on the sets which deleting a vertex is impossible. Minimal-global-offensive-alliance number is about minimum cardinality amid the cardinalities of all minimal-global-offensive alliances in a given neutrosophic graph. New notions are applied in the settings both individual and family. Family of neutrosophic graphs is studied in the way that, the family only contains same classes of neutrosophic graphs. Three types of family of neutrosophic graphs including m-family of neutrosophic stars with common neutrosophic vertex set, m-family of odd complete graphs with common neutrosophic vertex set, and m-family of odd complete graphs with common neutrosophic vertex set are studied. The results are about minimal-global-offensive alliance, minimal-global-offensive-alliance number and its corresponded sets, minimal-global-offensive-alliance-neutrosophic number and its corresponded sets, and characterizing all minimal-global-offensive alliances. The connection of global-offensive-alliances with dominating set and chromatic number are obtained. The number of connected components has some relations with this new concept and it gets some results. Some classes of neutrosophic graphs behave differently when the parity of vertices are different and in this case, path, cycle, and complete illustrate these behaviors. Two applications concerning complete model as individual and family, under the titles of time table and scheduling conclude the results and they give more clarifications. In this study, there's an open way to extend these results into the family of these classes of neutrosophic graphs. The family of neutrosophic graphs aren't study deeply and with more results but it seems that analogous results are determined. Slight progress is obtained in the family of these models but there are open avenues to study family of other models as same models and different models. There's a question. How can be related to each other, two sets partitioning the vertex set of a graph? The ideas of neighborhood and neighbors based on strong edges illustrate open way to get results. A set is global offensive alliance when two sets partitioning vertex set have uniform structure. All members of set have more amount of neighbors in the set than out of set. It leads us to the notion of global offensive alliance. Different edges make different neighborhoods but it's used one style edge titled strong edge. These notions are applied into neutrosophic graphs as individuals and family of them. Independent set as an alliance is a special set which has no neighbor inside and it implies some drawbacks for these notions. Finding special sets which are well-known, is an open way to purse this study. Special set which its members have only one neighbor inside, characterize the connected components where the cardinality of its complement is the number of connected components. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Keywords: Modified Neutrosophic Number, Global Offensive Alliance,

Complete Neutrosophic Graph
AMS Subject Classification: 05C17, 05C22, 05E45

4.16 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 4.16.1. *Is it possible to use mixed versions of ideas concerning "Global Offensive Alliance", "Modified Neutrosophic Number" and "Complete Neutrosophic Graph" to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Connections amid two vertices have key roles to assign global-offensive alliance, minimal-global-offensive alliance, minimal-global-offensive-alliance number, and minimal-global-offensive-alliance-neutrosophic number. Thus they're used to define new ideas which conclude to the structure global offensive alliance. The concept of having strong edge inspires me to study the behavior of strong edges in the way that, two types of numbers and set, e.g., global-offensive alliance, minimal-global-offensive alliance, minimal-global-offensive-alliance number, and minimal-global-offensive-alliance-neutrosophic number are the cases of study in the settings of individuals and in settings of families. Also, there are some avenues to extend these notions.
The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In subsection "Preliminaries", new notions of global- offensive alliance, minimal-global-offensive alliance, minimal-global-offensive-alliance number, and minimal-global-offensive-alliance-neutrosophic number are introduced and are clarified as individuals. In section "General Results For Neutrosophic Graphs", general sets have the key role in this way. General results are obtained and also, the results about the connections between dominating set and chromatic number with the notion of global-offensive alliance are elicited. Classes of neutrosophic graphs are studied in the terms of global-offensive alliance, minimal-global-offensive alliance, minimal-global-offensive-alliance number, and minimal-global-offensive-alliance-neutrosophic number in section "Classes of Neutrosophic Graphs" as individuals. In section "Classes of Neutrosophic Graphs", both numbers have applied into individuals. As a concluding result, there are three statements about the family of neutrosophic graphs as m-family of neutrosophic stars with common neutrosophic vertex set, m-family of odd complete graphs with common neutrosophic vertex set, and m-family of even complete graphs with common neutrosophic vertex set in section "Family of Neutrosophic Graphs." The clarifications are also presented in section "Family of Neutrosophic Graphs" for introduced results. In section "Applications in Time Table and Scheduling", two applications are posed for global-offensive alliance concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are complete as individual and uniform family. In section

"Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications is featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

4.17 Preliminaries

In this subsection, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.
Basic idea is about the model which is used. First definition introduces basic model.

Definition 4.17.1. (Graph).
$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

Definition 4.17.2. (Neutrosophic Graph).
$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, $\mu_i : E \to [0, 1]$, and for every $v_i v_j \in E$,

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic edge set**.

(iii) : $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

(iv) : $\Sigma_{v \in V} \sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

(v) : $|E|$ is called **size** of NTG and it's denoted by $\mathcal{S}(NTG)$.

(vi) : $\Sigma_{e \in E} \Sigma_{i=1}^{3} \mu_i(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$.

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

Definition 4.17.3. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) : a sequence of vertices $P : x_0, x_1, \cdots, x_n$ is called **path** where $x_i x_{i+1} \in E$, $i = 0, 1, \cdots, n-1$;

(ii) : **strength** of path $P : x_0, x_1, \cdots, x_n$ is $\bigwedge_{i=0,\cdots,n-1} \mu(x_i x_{i+1})$;

(iii) : **connectedness** amid vertices x_0 and x_n is

$$\mu^{\infty}(x, y) = \bigwedge_{P: x_0, x_1, \cdots, x_n} \bigwedge_{i=0,\cdots,n-1} \mu(x_i x_{i+1});$$

(iv) : a sequence of vertices $P : x_0, x_1, \cdots, x_n$ is called **cycle** where $x_i x_{i+1} \in E,\ i = 0, 1, \cdots, n-1$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_i v_{i+1})$;

(v) : it's **t-partite** where V is partitioned to t parts, $V_1, V_2, \cdots, V_t$ and the edge xy implies $x \in V_i$ and $y \in V_j$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where σ_i is σ on V_i instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$;

(vi) : t-partite is **complete bipartite** if $t = 2$, and it's denoted by K_{σ_1,σ_2};

(vii) : complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1,σ_2};

$(viii)$: a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1,σ_2};

(ix) : it's **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$;

(x) : it's **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

The notions of neighbor and neighborhood are about some vertices which have one edge with a fixed vertex. These notions presents vertices which are close to a fixed vertex as possible. Based on strong edge, it's possible to define different neighborhood as follows.

Definition 4.17.4. (Strong Neighborhood).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Suppose $x \in V$. Then

$$N_s(x) = \{y \in N(x) \mid \mu(xy) = \sigma(x) \wedge \sigma(y)\}.$$

New notion is defined between two types of neighborhoods for a fixed vertex. A minimal set and some numbers are introduced in this way. The next definition has main role in every results which are given in this essay.

Definition 4.17.5. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) a set S is called **global-offensive alliance** if

$$\forall a \in V \setminus S,\ |N_s(a) \cap S| > |N_s(a) \cap (V \setminus S)|;$$

(ii) $\forall S' \subseteq S$, S is global offensive alliance but S' isn't global offensive alliance. Then S is called **minimal-global-offensive alliance**;

(iii) **minimal-global-offensive-alliance number** of NTG is

$$\bigwedge_{S \text{ is a minimal-global-offensive alliance.}} |S|$$

and it's denoted by Γ;

(iv) **minimal-global-offensive-alliance-neutrosophic number** of NTG is

$$\bigwedge_{S \text{ is a minimal-global-offensive alliance.}} \Sigma_{s \in S} \Sigma_{i=1}^{3} \sigma_i(s)$$

and it's denoted by Γ_s.

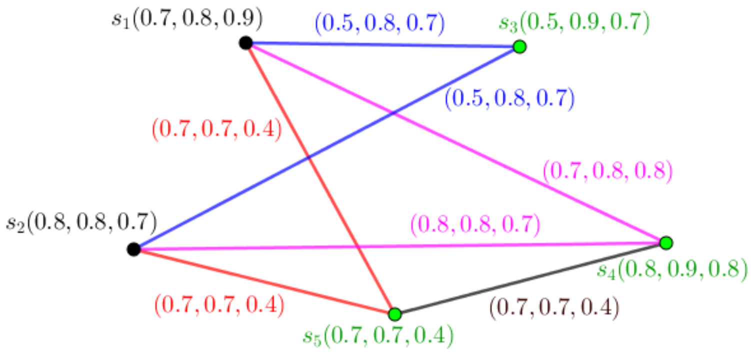

Figure 4.1: The set of black circles is minimal-global-offensive alliance. NTG1

Some clarifications are given for new definition which is presented in the paper as first time. Using new notions to make familiarity with main part of this article.

Example 4.17.6. Consider Figure (4.1).

(*i*) $S_1 = \{s_1, s_2\}, S_2 = \{s_3, s_5\}, S_3 = \{s_3, s_4\}, S_4 = \{s_4, s_5\}$ are only minimal-global-offensive alliances but only $S_3 = \{s_3, s_4\}$ is optimal such that forms minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number;

(*ii*) $N = \{s_2, s_5\}$ isn't global-offensive alliance. Since

$$\exists s_1 \in V \setminus N,\ |N_s(s_1) \cap N| = 1 < 2 = |N_s(s_1) \cap (V \setminus N)|$$
$$\exists s_1 \in V \setminus N,\ |N_s(s_1) \cap N| = 1 \not> 2 = |N_s(s_1) \cap (V \setminus N)|$$
$$\exists s_1 \in V \setminus N,\ |N_s(s_1) \cap N| \not> |N_s(s_1) \cap (V \setminus N)|;$$

(*iii*) $\Gamma_s = 4.6$;

(*iv*) $\Gamma = 2$.

4.18 General Results For Neutrosophic Graphs

In this section, general results are given based on new definition. Some relations between new definition with dominating set and chromatic number are provided. The relation amid these two types of new numbers with fundamental numbers of neutrosophic graphs as order and neutrosophic order are clarified in the terms of vertices.

Proposition 4.18.1. *Let $NTG : (V, E, \sigma, \mu)$ be a strong neutrosophic graph. If S is global-offensive alliance, then $\forall v \in V \setminus S,\ \exists x \in S$ such that*

(*i*) $v \in N_s(x)$;

(*ii*) $vx \in E$.

Proof. (*i*). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Consider $v \in V \setminus S$. Since S is global-offensive alliance,

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus S,\ |N_s(v) \cap S| > |N_s(v) \cap (V \setminus S)|$$
$$v \in V \setminus S, \exists x \in S,\ v \in N_s(x).$$

(ii). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Consider $v \in V \setminus S$. Since S is global-offensive alliance,

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus S,\ |N_s(v) \cap S| > |N_s(v) \cap (V \setminus S)|$$
$$v \in V \setminus S, \exists x \in S :\ v \in N_s(x)$$
$$v \in V \setminus S, \exists x \in S : vx \in E,\ \mu(vx) = \sigma(v) \wedge \sigma(x).$$
$$v \in V \setminus S, \exists x \in S : vx \in E.$$

■

Definition 4.18.2. Let $NTG : (V, E, \sigma, \mu)$ be a strong neutrosophic graph. Suppose S is a set of vertices. Then

(i) S is called **dominating set** if $\forall v \in V \setminus S,\ \exists s \in S$ such that either $v \in N_s(s)$ or $vs \in E$;

(ii) $|S|$ is called **chromatic number** if $\forall v \in V,\ \exists s \in S$ such that either $v \in N_s(s)$ or $vs \in E$ implies s and v have different colors.

Example 4.18.3. Consider Figure (4.1).

(i) $S = \{s_3, s_4\}$ is minimal dominating set;

(ii) $S = \{s_3, s_4\}$ is minimal-global-offensive alliance;

(iii) chromatic number is three.

Proposition 4.18.4. *Let $NTG : (V, E, \sigma, \mu)$ be a strong neutrosophic graph. If S is global-offensive alliance, then*

(i) *S is dominating set;*

(ii) *there's $S \subseteq S'$ such that $|S'|$ is chromatic number.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Consider $v \in V \setminus S$. Since S is global-offensive alliance, either

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus S,\ |N_s(v) \cap S| > |N_s(v) \cap (V \setminus S)|$$
$$v \in V \setminus S, \exists x \in S,\ v \in N_s(x)$$

or

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus S,\ |N_s(v) \cap S| > |N_s(v) \cap (V \setminus S)|$$
$$v \in V \setminus S, \exists x \in S :\ v \in N_s(x)$$
$$v \in V \setminus S, \exists x \in S : vx \in E,\ \mu(vx) = \sigma(v) \wedge \sigma(x)$$
$$v \in V \setminus S, \exists x \in S : vx \in E.$$

It implies S is dominating set.

(ii). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Consider $v \in V \setminus S$. Since S is global-offensive alliance, either

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus S,\ |N_s(v) \cap S| > |N_s(v) \cap (V \setminus S)|$$
$$v \in V \setminus S, \exists x \in S,\ v \in N_s(x)$$

or

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus S,\ |N_s(v) \cap S| > |N_s(v) \cap (V \setminus S)|$$
$$v \in V \setminus S, \exists x \in S :\ v \in N_s(x)$$
$$v \in V \setminus S, \exists x \in S : vx \in E,\ \mu(vx) = \sigma(v) \wedge \sigma(x)$$
$$v \in V \setminus S, \exists x \in S : vx \in E.$$

Thus every vertex $v \in V \setminus S$, has at least one neighbor in S. The only case is about the relation amid vertices in S in the terms of neighbors. It implies there's $S \subseteq S'$ such that $|S'|$ is chromatic number. ∎

Proposition 4.18.5. *Let $NTG : (V, E, \sigma, \mu)$ be a strong neutrosophic graph. Then*

(i) $\Gamma \leq \mathcal{O}$;

(ii) $\Gamma_s \leq \mathcal{O}_n$.

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Let $S = V$.

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus V,\ |N_s(v) \cap V| > |N_s(v) \cap (V \setminus V)|$$
$$v \in \emptyset,\ |N_s(v) \cap V| > |N_s(v) \cap \emptyset|$$
$$v \in \emptyset,\ |N_s(v) \cap V| > |\emptyset|$$
$$v \in \emptyset,\ |N_s(v) \cap V| > 0$$

It implies V is global-offensive alliance. For all set of vertices $S,\ S \subseteq V$. Thus for all set of vertices $S,\ |S| \leq |V|$. It implies for all set of vertices $S,\ |S| \leq \mathcal{O}$. So for all set of vertices $S,\ \Gamma \leq \mathcal{O}$.
(ii). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Let $S = V$.

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus V,\ |N_s(v) \cap V| > |N_s(v) \cap (V \setminus V)|$$
$$v \in \emptyset,\ |N_s(v) \cap V| > |N_s(v) \cap \emptyset|$$
$$v \in \emptyset,\ |N_s(v) \cap V| > |\emptyset|$$
$$v \in \emptyset,\ |N_s(v) \cap V| > 0$$

It implies V is global-offensive alliance. For all set of neutrosophic vertices $S,\ S \subseteq V$. Thus for all set of neutrosophic vertices $S,\ \Sigma_{s \in S} \Sigma_{i=1}^{3} \sigma_i(s) \leq \Sigma_{v \in V} \Sigma_{i=1}^{3} \sigma_i(v)$. It implies for all set of neutrosophic vertices $S,\ \Sigma_{s \in S} \Sigma_{i=1}^{3} \sigma_i(s) \leq \mathcal{O}_n$. So for all set of neutrosophic vertices $S,\ \Gamma_s \leq \mathcal{O}_n$. ∎

Proposition 4.18.6. *Let $NTG : (V, E, \sigma, \mu)$ be a strong neutrosophic graph which is connected. Then*

(i) $\Gamma \leq \mathcal{O} - 1$;

(ii) $\Gamma_s \leq \mathcal{O}_n - \Sigma_{i=1}^{3} \sigma_i(x)$.

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Let $S = V - \{x\}$ where x is arbitrary and $x \in V$.

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus V - \{x\},\ |N_s(v) \cap (V - \{x\})| > |N_s(v) \cap (V \setminus (V - \{x\}))|$$
$$|N_s(x) \cap (V - \{x\})| > |N_s(x) \cap \{x\}|$$
$$|N_s(x) \cap (V - \{x\})| > |\emptyset|$$
$$|N_s(x) \cap (V - \{x\})| > 0$$

It implies $V - \{x\}$ is global-offensive alliance. For all set of vertices $S \neq V,\ S \subseteq V - \{x\}$. Thus for all set of vertices $S \neq V,\ |S| \leq |V - \{x\}|$. It implies for all set of vertices $S \neq V,\ |S| \leq \mathcal{O} - 1$. So for all set of vertices $S,\ \Gamma \leq \mathcal{O} - 1$.

(ii). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Let $S = V - \{x\}$ where x is arbitrary and $x \in V$.

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus V - \{x\},\ |N_s(v) \cap (V - \{x\})| > |N_s(v) \cap (V \setminus (V - \{x\}))|$$
$$|N_s(x) \cap (V - \{x\})| > |N_s(x) \cap \{x\}|$$
$$|N_s(x) \cap (V - \{x\})| > |\emptyset|$$
$$|N_s(x) \cap (V - \{x\})| > 0$$

It implies $V - \{x\}$ is global-offensive alliance. For all set of neutrosophic vertices $S \neq V,\ S \subseteq V - \{x\}$. Thus for all set of neutrosophic vertices $S \neq V,\ \Sigma_{s \in S} \Sigma_{i=1}^{3} \sigma_i(s) \leq \Sigma_{v \in V - \{x\}} \Sigma_{i=1}^{3} \sigma_i(v)$. It implies for all set of neutrosophic vertices $S \neq V,\ \Sigma_{s \in S} \Sigma_{i=1}^{3} \sigma_i(s) \leq \mathcal{O}_n - \Sigma_{i=1}^{3} \sigma_i(x)$. So for all set of neutrosophic vertices $S,\ \Gamma_s \leq \mathcal{O}_n - \Sigma_{i=1}^{3} \sigma_i(x)$. ■

4.19 Classes of Neutrosophic Graphs

In this section, behaviors of some classes of neutrosophic graphs are analyzed when new definition is applied. In this way, the parity of number of vertices differentiate the results about some classes of neutrosophic graphs. Paths, cycles and complete are some classes of neutrosophic graphs which the parity of number of vertices get different results.

Proposition 4.19.1. *Let $NTG : (V, E, \sigma, \mu)$ be an odd path. Then*

(i) *the set $S = \{v_2, v_4, \cdots, v_{n-1}\}$ is minimal-global-offensive alliance;*

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor + 1$ *and corresponded set is $S = \{v_2, v_4, \cdots, v_{n-1}\}$;*

(iii) $\Gamma_s = \min\{\Sigma_{s \in S = \{v_2, v_4, \cdots, v_{n-1}\}} \Sigma_{i=1}^{3} \sigma_i(s), \Sigma_{s \in S = \{v_1, v_3, \cdots, v_{n-1}\}} \Sigma_{i=1}^{3} \sigma_i(s)\}$;

(iv) *the sets $S_1 = \{v_2, v_4, \cdots, v_{n-1}\}$ and $S_2 = \{v_1, v_3, \cdots, v_{n-1}\}$ are only minimal-global-offensive alliances.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is an odd path. Let $S = \{v_2, v_4, \cdots, v_{n-1}\}$ where for all $v_i, v_j \in \{v_2, v_4, \cdots, v_{n-1}\},\ v_i v_j \notin E$ and $v_i, v_j \in V$.

$$v \in \{v_1, v_3, \cdots, v_n\},\ |N_s(v) \cap \{v_2, v_4, \cdots .v_{n-1}\}| = 2 > 0 =$$
$$|N_s(v) \cap \{v_1, v_3, \cdots, v_n\}|\ \forall z \in V \setminus S,\ |N_s(z) \cap S| = 2 > 0 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus \{v_2, v_4, \cdots, v_{n-1}\},\ |N_s(v) \cap \{v_2, v_4, \cdots .v_{n-1}\}| >$$
$$|N_s(v) \cap (V \setminus \{v_2, v_4, \cdots .v_{n-1}\})|$$

It implies $S = \{v_2, v_4, \cdots, v_{n-1}\}$ is global-offensive alliance. If $S = \{v_2, v_4, \cdots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_2, v_4, \cdots, v_{n-1}\}$, then

$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 = 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 \not> 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|.$$

So $\{v_2, v_4, \cdots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_2, v_4, \cdots, v_{n-1}\}$ isn't global-offensive alliance. It induces $S = \{v_2, v_4, \cdots, v_{n-1}\}$ is minimal-global-offensive alliance.
(ii) and (iii) are trivial.
(iv). By (i), $S_1 = \{v_2, v_4, \cdots, v_{n-1}\}$ is minimal-global-offensive alliance. Thus it's enough to show that $S_2 = \{v_1, v_3, \cdots, v_{n-1}\}$ is minimal-global-offensive alliance. Suppose $NTG : (V, E, \sigma, \mu)$ is an odd path. Let $S = \{v_1, v_3, \cdots, v_{n-1}\}$ where for all $v_i, v_j \in \{v_1, v_3, \cdots, v_{n-1}\},\ v_iv_j \notin E$ and $v_i, v_j \in V$.

$$v \in \{v_2, v_4, \cdots, v_n\},\ |N_s(v) \cap \{v_1, v_3, \cdots .v_{n-1}\}| = 2 > 0 =$$
$$|N_s(v) \cap \{v_2, v_4, \cdots, v_n\}|\ \forall z \in V \setminus S,\ |N_s(z) \cap S| = 2 > 0 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus \{v_1, v_3, \cdots, v_{n-1}\},\ |N_s(v) \cap \{v_1, v_3, \cdots .v_{n-1}\}| >$$
$$|N_s(v) \cap (V \setminus \{v_1, v_3, \cdots .v_{n-1}\})|$$

It implies $S = \{v_1, v_3, \cdots, v_{n-1}\}$ is global-offensive alliance. If $S = \{v_1, v_3, \cdots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_1, v_3, \cdots, v_{n-1}\}$, then

$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 = 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 \not> 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|.$$

So $\{v_1, v_3, \cdots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_1, v_3, \cdots, v_{n-1}\}$ isn't global-offensive alliance. It induces $S = \{v_1, v_3, \cdots, v_{n-1}\}$ is minimal-global-offensive alliance. ■

Example 4.19.2. Consider Figure (4.2).

(i) $S_1 = \{s_1, s_3, s_4\}$ and $S_2 = \{s_2, s_4\}$ are only minimal-global-offensive alliances;

(ii) $S_1 = \{s_1, s_3, s_4\}$ is optimal such that only forms minimal-global-offensive-alliance-neutrosophic number but not minimal-global-offensive-alliance number;

(iii) $S_2 = \{s_2, s_4\}$ is optimal such that only forms minimal-global-offensive-alliance number but not minimal-global-offensive-alliance-neutrosophic number;

(iv) $N = \{s_1, s_3\}$ isn't global-offensive alliance. Since there are two instances but only one of them is enough;

(a) First counterexample for the statement "$N = \{s_1, s_3\}$ is global-offensive alliance.";

$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 1 = 1 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 1 \not> 1 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| \not> |N_s(s_4) \cap (V \setminus N)|;$$

(b) second counterexample for the statement "$N = \{s_1, s_3\}$ is global-offensive alliance.".

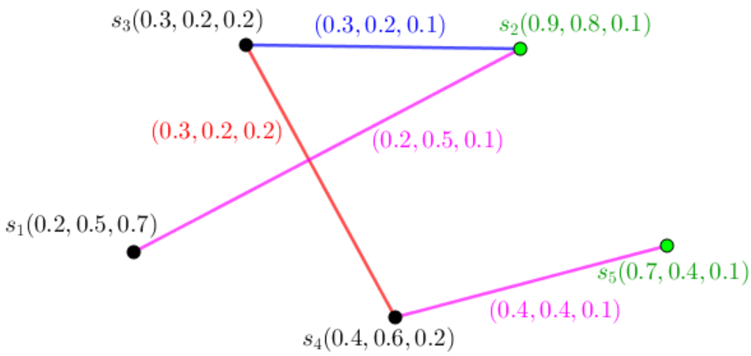

Figure 4.2: The set of black circles is minimal-global-offensive alliance. NTG2

$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| = 0 < 1 = |N_s(s_5) \cap (V \setminus N)|$$
$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| = 0 \not> 1 = |N_s(s_5) \cap (V \setminus N)|$$
$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| \not> |N_s(s_5) \cap (V \setminus N)|.$$

(v) $\Gamma_s = 3.1$ and corresponded set is $S_1 = \{s_1, s_3, s_4\}$;

(vi) $\Gamma = 2$ and corresponded set is $S_2 = \{s_2, s_4\}$.

Proposition 4.19.3. *Let $NTG : (V, E, \sigma, \mu)$ be an even path. Then*

(i) *the set $S = \{v_2, v_4, \cdots .v_n\}$ is minimal-global-offensive alliance;*

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor$ *and corresponded sets are $\{v_2, v_4, \cdots .v_n\}$ and $\{v_1, v_3, \cdots .v_{n-1}\}$;*

(iii) $\Gamma_s = \min\{\Sigma_{s \in S=\{v_2,v_4,\cdots,v_n\}} \Sigma_{i=1}^3 \sigma_i(s), \Sigma_{s \in S=\{v_1,v_3,\cdots .v_{n-1}\}} \Sigma_{i=1}^3 \sigma_i(s)\}$;

(iv) *the sets $S_1 = \{v_2, v_4, \cdots .v_n\}$ and $S_2 = \{v_1, v_3, \cdots .v_{n-1}\}$ are only minimal-global-offensive alliances.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is an even path. Let $S = \{v_2, v_4, \cdots, v_n\}$ where for all $v_i, v_j \in \{v_2, v_4, \cdots, v_n\}$, $v_i v_j \notin E$ and $v_i, v_j \in V$.

$$v \in \{v_1, v_3, \cdots, v_{n-1}\},\ |N_s(v) \cap \{v_2, v_4, \cdots .v_n\}| = 2 > 0 =$$
$$|N_s(v) \cap \{v_1, v_3, \cdots, v_{n-1}\}|\ \forall z \in V \setminus S,\ |N_s(z) \cap S| = 2 > 0 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus \{v_2, v_4, \cdots, v_n\},\ |N_s(v) \cap \{v_2, v_4, \cdots .v_n\}| >$$
$$|N_s(v) \cap (V \setminus \{v_2, v_4, \cdots .v_n\})|$$

It implies $S = \{v_2, v_4, \cdots, v_n\}$ is global-offensive alliance. If $S = \{v_2, v_4, \cdots, v_n\} - \{v_i\}$ where $v_i \in \{v_2, v_4, \cdots, v_n\}$, then

$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 = 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 \not> 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|.$$

So $\{v_2, v_4, \cdots, v_n\} - \{v_i\}$ where $v_i \in \{v_2, v_4, \cdots, v_n\}$ isn't global-offensive alliance. It induces $S = \{v_2, v_4, \cdots, v_n\}$ is minimal-global-offensive alliance.
(ii) and (iii) are trivial.
(iv). By (i), $S_1 = \{v_2, v_4, \cdots, v_n\}$ is minimal-global-offensive alliance. Thus it's enough to show that $S_2 = \{v_1, v_3, \cdots, v_{n-1}\}$ is minimal-global-offensive alliance.

Suppose $NTG : (V, E, \sigma, \mu)$ is an even path. Let $S = \{v_1, v_3, \cdots, v_{n-1}\}$ where for all $v_i, v_j \in \{v_1, v_3, \cdots, v_{n-1}\},\ v_iv_j \notin E$ and $v_i, v_j \in V$.

$$v \in \{v_2, v_4, \cdots, v_n\},\ |N_s(v) \cap \{v_1, v_3, \cdots .v_{n-1}\}| = 2 > 0 =$$
$$|N_s(v) \cap \{v_2, v_4, \cdots, v_n\}|\ \forall z \in V \setminus S,\ |N_s(z) \cap S| = 2 > 0 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus \{v_1, v_3, \cdots, v_{n-1}\},\ |N_s(v) \cap \{v_1, v_3, \cdots .v_{n-1}\}| >$$
$$|N_s(v) \cap (V \setminus \{v_1, v_3, \cdots .v_{n-1}\})|$$

It implies $S = \{v_1, v_3, \cdots, v_{n-1}\}$ is global-offensive alliance. If $S = \{v_1, v_3, \cdots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_1, v_3, \cdots, v_{n-1}\}$, then

$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 = 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 \not> 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|.$$

So $\{v_1, v_3, \cdots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_1, v_3, \cdots, v_{n-1}\}$ isn't global-offensive alliance. It induces $S = \{v_1, v_3, \cdots, v_{n-1}\}$ is minimal-global-offensive alliance. ■

Example 4.19.4. Consider Figure (4.3).

(i) $S_1 = \{s_1, s_3, s_5\}$ and $S_2 = \{s_2, s_4, s_6\}$ are only minimal-global-offensive alliances;

(ii) $S_2 = \{s_2, s_4, s_6\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number;

(iii) $S_1 = \{s_1, s_3, s_5\}$ is optimal such that only forms minimal-global-offensive-alliance number but not minimal-global-offensive-alliance-neutrosophic number;

(iv) $N = \{s_1, s_3\}$ isn't global-offensive alliance. Since there are three instances but only one of them is enough;

(a) First counterexample for the statement "$N = \{s_1, s_3\}$ is global-offensive alliance.";

$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 1 = 1 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 1 \not> 1 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| \not> |N_s(s_4) \cap (V \setminus N)|.$$

(b) second counterexample for the statement "$N = \{s_1, s_3\}$ is global-offensive alliance.";

$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| = 0 < 1 = |N_s(s_5) \cap (V \setminus N)|$$
$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| = 0 \not> 1 = |N_s(s_5) \cap (V \setminus N)|$$
$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| \not> |N_s(s_5) \cap (V \setminus N)|;$$

(c) third counterexample for the statement "$N = \{s_1, s_3\}$ is global-offensive alliance.";

$$\exists s_6 \in V \setminus N,\ |N_s(s_6) \cap N| = 0 < 1 = |N_s(s_6) \cap (V \setminus N)|$$
$$\exists s_6 \in V \setminus N,\ |N_s(s_6) \cap N| = 0 \not> 1 = |N_s(s_6) \cap (V \setminus N)|$$
$$\exists s_6 \in V \setminus N,\ |N_s(s_6) \cap N| \not> |N_s(s_6) \cap (V \setminus N)|.$$

(v) $\Gamma_s = 4.5$ and corresponded set is $S_2 = \{s_2, s_4, s_6\}$;

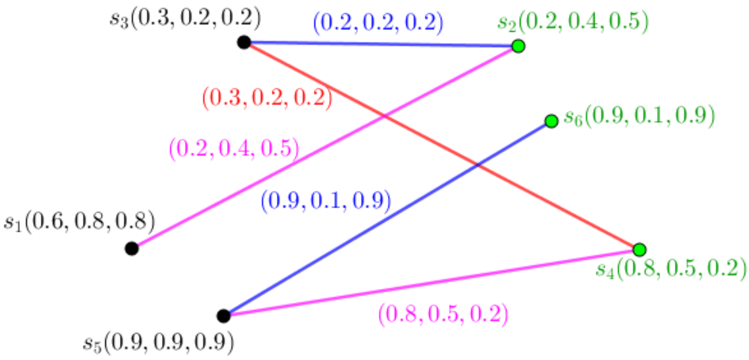

Figure 4.3: The set of black circles is minimal-global-offensive alliance. NTG3

(vi) $\Gamma = 3$ and corresponded sets are $S_1 = \{s_1, s_3, s_5\}$ and $S_2 = \{s_2, s_4, s_6\}$.

Proposition 4.19.5. *Let $NTG : (V, E, \sigma, \mu)$ be an even cycle. Then*

(i) *the set $S = \{v_2, v_4, \cdots, v_n\}$ is minimal-global-offensive alliance;*

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor$ *and corresponded sets are* $\{v_2, v_4, \cdots, v_n\}$ *and* $\{v_1, v_3, \cdots, v_{n-1}\}$;

(iii) $\Gamma_s = \min\{\Sigma_{s \in S = \{v_2, v_4, \cdots, v_n\}} \sigma(s), \Sigma_{s \in S = \{v_1, v_3, \cdots, v_{n-1}\}} \sigma(s)\}$;

(iv) *the sets $S_1 = \{v_2, v_4, \cdots, v_n\}$ and $S_2 = \{v_1, v_3, \cdots, v_{n-1}\}$ are only minimal-global-offensive alliances.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is an even cycle. Let $S = \{v_2, v_4, \cdots, v_n\}$ where for all $v_i, v_j \in \{v_2, v_4, \cdots, v_n\}$, $v_i v_j \notin E$ and $v_i, v_j \in V$.

$$v \in \{v_1, v_3, \cdots, v_{n-1}\},\ |N_s(v) \cap \{v_2, v_4, \cdots .v_n\}| = 2 > 0 =$$
$$|N_s(v) \cap \{v_1, v_3, \cdots, v_{n-1}\}|\ \forall z \in V \setminus S,\ |N_s(z) \cap S| = 2 > 0 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus \{v_2, v_4, \cdots, v_n\},\ |N_s(v) \cap \{v_2, v_4, \cdots .v_n\}| >$$
$$|N_s(v) \cap (V \setminus \{v_2, v_4, \cdots .v_n\})|$$

It implies $S = \{v_2, v_4, \cdots, v_n\}$ is global-offensive alliance. If $S = \{v_2, v_4, \cdots, v_n\} - \{v_i\}$ where $v_i \in \{v_2, v_4, \cdots, v_n\}$, then

$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 = 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 \not> 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|.$$

So $\{v_2, v_4, \cdots, v_n\} - \{v_i\}$ where $v_i \in \{v_2, v_4, \cdots, v_n\}$ isn't global-offensive alliance. It induces $S = \{v_2, v_4, \cdots, v_n\}$ is minimal-global-offensive alliance.
(ii) and (iii) are trivial.
(iv). By (i), $S_1 = \{v_2, v_4, \cdots, v_n\}$ is minimal-global-offensive alliance. Thus it's enough to show that $S_2 = \{v_1, v_3, \cdots, v_{n-1}\}$ is minimal-global-offensive alliance. Suppose $NTG : (V, E, \sigma, \mu)$ is an odd path. Let $S = \{v_1, v_3, \cdots, v_{n-1}\}$ where for all $v_i, v_j \in \{v_1, v_3, \cdots, v_{n-1}\}$, $v_i v_j \notin E$ and $v_i, v_j \in V$.

$$v \in \{v_2, v_4, \cdots, v_n\},\ |N_s(v) \cap \{v_1, v_3, \cdots .v_{n-1}\}| = 2 > 0 =$$
$$|N_s(v) \cap \{v_2, v_4, \cdots, v_n\}|\ \forall z \in V \setminus S,\ |N_s(z) \cap S| = 2 > 0 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus \{v_1, v_3, \cdots, v_{n-1}\},\ |N_s(v) \cap \{v_1, v_3, \cdots .v_{n-1}\}| >$$
$$|N_s(v) \cap (V \setminus \{v_1, v_3, \cdots .v_{n-1}\})|$$

It implies $S = \{v_1, v_3, \cdots, v_{n-1}\}$ is global-offensive alliance. If $S = \{v_1, v_3, \cdots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_1, v_3, \cdots, v_{n-1}\}$, then

$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 = 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 \not> 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|.$$

So $\{v_1, v_3, \cdots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_1, v_3, \cdots, v_{n-1}\}$ isn't global-offensive alliance. It induces $S = \{v_1, v_3, \cdots, v_{n-1}\}$ is minimal-global-offensive alliance. ■

Example 4.19.6. Consider Figure (4.4).

(i) $S_1 = \{s_1, s_3, s_5\}$ and $S_2 = \{s_2, s_4, s_6\}$ are only minimal-global-offensive alliances;

(ii) $S_2 = \{s_2, s_4, s_6\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number;

(iii) $S_1 = \{s_1, s_3, s_5\}$ is optimal such that only forms minimal-global-offensive-alliance number but not minimal-global-offensive-alliance-neutrosophic number;

(iv) $N = \{s_1, s_3\}$ isn't global-offensive alliance. Since there are three instances but only one of them is enough;

(a) First counterexample for the statement "$N = \{s_1, s_3\}$ is global-offensive alliance.";

$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 1 = 1 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 1 \not> 1 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| \not> |N_s(s_4) \cap (V \setminus N)|.$$

(b) second counterexample for the statement "$N = \{s_1, s_3\}$ is global-offensive alliance.";

$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| = 0 < 1 = |N_s(s_5) \cap (V \setminus N)|$$
$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| = 0 \not> 1 = |N_s(s_5) \cap (V \setminus N)|$$
$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| \not> |N_s(s_5) \cap (V \setminus N)|;$$

(c) third counterexample for the statement "$N = \{s_1, s_3\}$ is global-offensive alliance.";

$$\exists s_6 \in V \setminus N,\ |N_s(s_6) \cap N| = 0 < 1 = |N_s(s_6) \cap (V \setminus N)|$$
$$\exists s_6 \in V \setminus N,\ |N_s(s_6) \cap N| = 0 \not> 1 = |N_s(s_6) \cap (V \setminus N)|$$
$$\exists s_6 \in V \setminus N,\ |N_s(s_6) \cap N| \not> |N_s(s_6) \cap (V \setminus N)|.$$

(v) $\Gamma_s = 3.2$ and corresponded set is $S_2 = \{s_2, s_4, s_6\}$;

(vi) $\Gamma = 3$ and corresponded sets are $S_1 = \{s_1, s_3, s_5\}$ and $S_2 = \{s_2, s_4, s_6\}$.

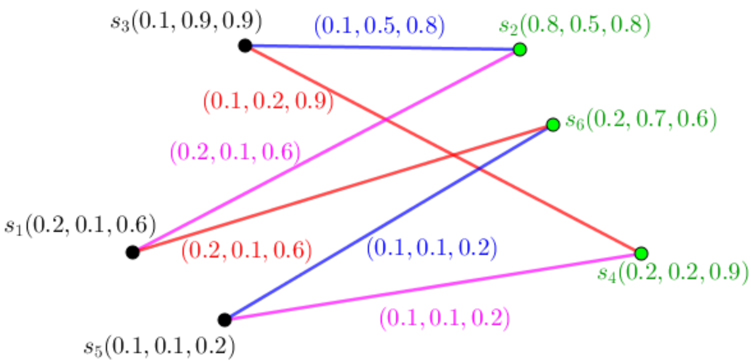

Figure 4.4: The set of black circles is minimal-global-offensive alliance. NTG4

Proposition 4.19.7. *Let $NTG : (V, E, \sigma, \mu)$ be an odd cycle. Then*

(i) *the set $S = \{v_2, v_4, \cdots, v_{n-1}\}$ is minimal-global-offensive alliance;*

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor + 1$ *and corresponded set is* $S = \{v_2, v_4, \cdots, v_{n-1}\}$;

(iii) $\Gamma_s = \min\{\Sigma_{s\in S=\{v_2,v_4,\cdots.v_{n-1}\}} \Sigma_{i=1}^3 \sigma_i(s), \Sigma_{s\in S=\{v_1,v_3,\cdots.v_{n-1}\}} \Sigma_{i=1}^3 \sigma_i(s)\}$;

(iv) *the sets $S_1 = \{v_2, v_4, \cdots .v_{n-1}\}$ and $S_2 = \{v_1, v_3, \cdots .v_{n-1}\}$ are only minimal-global-offensive alliances.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is an odd cycle. Let $S = \{v_2, v_4, \cdots, v_{n-1}\}$ where for all $v_i, v_j \in \{v_2, v_4, \cdots, v_{n-1}\}$, $v_iv_j \notin E$ and $v_i, v_j \in V$.

$$v \in \{v_1, v_3, \cdots, v_n\},\ |N_s(v) \cap \{v_2, v_4, \cdots .v_{n-1}\}| = 2 > 0 =$$
$$|N_s(v) \cap \{v_1, v_3, \cdots, v_n\}|\ \forall z \in V \setminus S,\ |N_s(z) \cap S| = 2 > 0 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus \{v_2, v_4, \cdots, v_{n-1}\},\ |N_s(v) \cap \{v_2, v_4, \cdots .v_{n-1}\}| >$$
$$|N_s(v) \cap (V \setminus \{v_2, v_4, \cdots .v_{n-1}\})|$$

It implies $S = \{v_2, v_4, \cdots, v_{n-1}\}$ is global-offensive alliance. If $S = \{v_2, v_4, \cdots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_2, v_4, \cdots, v_{n-1}\}$, then

$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 = 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 \not> 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|.$$

So $\{v_2, v_4, \cdots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_2, v_4, \cdots, v_{n-1}\}$ isn't global-offensive alliance. It induces $S = \{v_2, v_4, \cdots, v_{n-1}\}$ is minimal-global-offensive alliance.
(ii) and (iii) are trivial.
(iv). By (i), $S_1 = \{v_2, v_4, \cdots, v_{n-1}\}$ is minimal-global-offensive alliance. Thus it's enough to show that $S_2 = \{v_1, v_3, \cdots, v_{n-1}\}$ is minimal-global-offensive alliance. Suppose $NTG : (V, E, \sigma, \mu)$ is an odd cycle. Let $S = \{v_1, v_3, \cdots, v_{n-1}\}$ where for all $v_i, v_j \in \{v_1, v_3, \cdots, v_{n-1}\}$, $v_iv_j \notin E$ and $v_i, v_j \in V$.

$$v \in \{v_2, v_4, \cdots, v_n\},\ |N_s(v) \cap \{v_1, v_3, \cdots .v_{n-1}\}| = 2 > 0 =$$
$$|N_s(v) \cap \{v_2, v_4, \cdots, v_n\}|\ \forall z \in V \setminus S,\ |N_s(z) \cap S| = 2 > 0 = |N_s(z) \cap (V \setminus S)|$$

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus \{v_1, v_3, \cdots, v_{n-1}\},\ |N_s(v) \cap \{v_1, v_3, \cdots .v_{n-1}\}| >$$
$$|N_s(v) \cap (V \setminus \{v_1, v_3, \cdots .v_{n-1}\})|$$

It implies $S = \{v_1, v_3, \cdots, v_{n-1}\}$ is global-offensive alliance. If $S = \{v_1, v_3, \cdots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_1, v_3, \cdots, v_{n-1}\}$, then

$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 = 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 \not> 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|.$$

So $\{v_1, v_3, \cdots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_1, v_3, \cdots, v_{n-1}\}$ isn't global-offensive alliance. It induces $S = \{v_1, v_3, \cdots, v_{n-1}\}$ is minimal-global-offensive alliance. ■

Example 4.19.8. Consider Figure (4.5).

(i) $S_1 = \{s_1, s_3, s_4\}$ and $S_2 = \{s_2, s_4\}$ are only minimal-global-offensive alliances;

(ii) $S_2 = \{s_2, s_4\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number;

(iii) $S_1 = \{s_1, s_3, s_5\}$ is optimal such that not only doesn't form minimal-global-offensive-alliance number but also doesn't form minimal-global-offensive-alliance-neutrosophic number;

(iv) $N = \{s_1, s_3\}$ isn't global-offensive alliance. Since there are two instances but only one of them is enough;

 (a) First counterexample for the statement "$N = \{s_1, s_3\}$ is global-offensive alliance.";

$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 1 = 1 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 1 \not> 1 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| \not> |N_s(s_4) \cap (V \setminus N)|;$$

 (b) second counterexample for the statement "$N = \{s_1, s_3\}$ is global-offensive alliance.";

$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| = 0 < 1 = |N_s(s_5) \cap (V \setminus N)|$$
$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| = 0 \not> 1 = |N_s(s_5) \cap (V \setminus N)|$$
$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| \not> |N_s(s_5) \cap (V \setminus N)|.$$

(v) $\Gamma_s = 3.5$ and corresponded set is $S_2 = \{s_2, s_4\}$;

(vi) $\Gamma = 2$ and corresponded set is $S_2 = \{s_2, s_4\}$.

Proposition 4.19.9. *Let $NTG : (V, E, \sigma, \mu)$ be star. Then*

(i) *the set $S = \{c\}$ is minimal-global-offensive alliance;*

(ii) $\Gamma = 1$;

(iii) $\Gamma_s = \Sigma_{i=1}^{3} \sigma_i(c)$;

(iv) *the sets $S = \{c\}$ and $S \subset S'$ are only global-offensive alliances.*

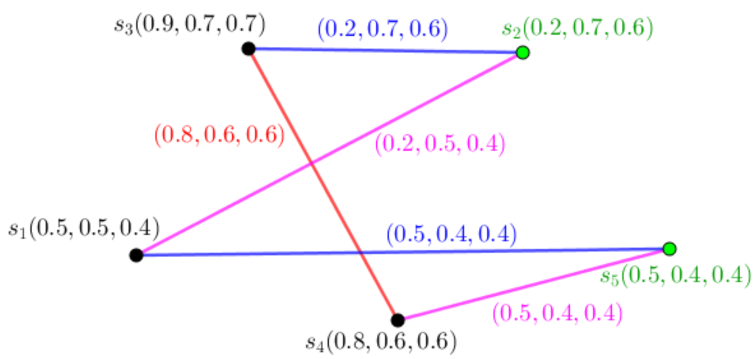

Figure 4.5: The set of black circles is minimal-global-offensive alliance.

NTG5

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a star.

$$\forall v \in V \setminus \{c\},\ |N_s(v) \cap \{c\}| = 1 > 0 = |N_s(v) \cap (V \setminus \{c\})|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| = 1 > 0 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus \{c\},\ |N_s(v) \cap \{c\}| > |N_s(v) \cap (V \setminus \{c\})|$$

It implies $S = \{c\}$ is global-offensive alliance. If $S = \{c\} - \{c\} = \emptyset$, then

$$\exists v \in V \setminus S,\ |N_s(z) \cap S| = 0 = 0 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v \in V \setminus S,\ |N_s(z) \cap S| = 0 \not> 0 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|.$$

So $S = \{c\} - \{c\} = \emptyset$ isn't global-offensive alliance. It induces $S = \{c\}$ is minimal-global-offensive alliance.
(ii) and (iii) are trivial.
(iv). By (i), $S = \{c\}$ is minimal-global-offensive alliance. Thus it's enough to show that $S \subseteq S'$ is minimal-global-offensive alliance. Suppose $NTG : (V, E, \sigma, \mu)$ is a star. Let $S \subseteq S'$.

$$\forall v \in V \setminus \{c\},\ |N_s(v) \cap \{c\}| = 1 > 0 = |N_s(v) \cap (V \setminus \{c\})|$$
$$\forall z \in V \setminus S',\ |N_s(z) \cap S'| = 1 > 0 = |N_s(z) \cap (V \setminus S')|$$
$$\forall z \in V \setminus S',\ |N_s(z) \cap S'| > |N_s(z) \cap (V \setminus S')|$$

It implies $S' \subseteq S$ is global-offensive alliance. ∎

Example 4.19.10. Consider Figure (4.6).

(i) $S = \{s_1\}$ is only minimal-global-offensive alliance;

(ii) $S = \{s_1\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number;

(iii) S' including $S = \{s_1\}$ only forms global-offensive-alliance but not minimal-global-offensive-alliance;

(iv) $N = \{s_3, s_4\}$ isn't global-offensive alliance. Since there are three instances but only one of them is enough;

(a) First counterexample for the statement "$N = \{s_3, s_4\}$ is global-offensive alliance.";

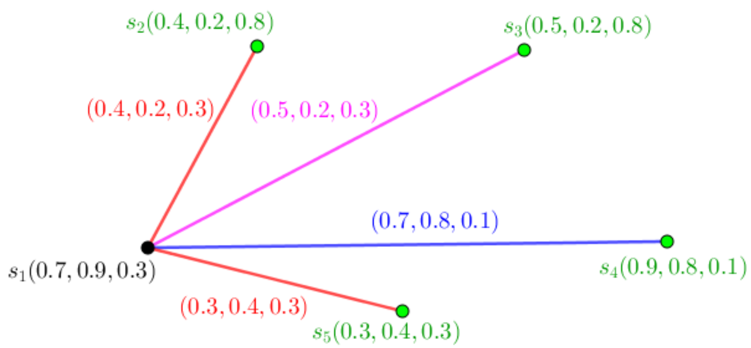

Figure 4.6: The set of black circles is minimal-global-offensive alliance.

NTG6

$$\exists s_1 \in V \setminus N,\ |N_s(s_1) \cap N| = 2 = 2 = |N_s(s_1) \cap (V \setminus N)|$$
$$\exists s_1 \in V \setminus N,\ |N_s(s_1) \cap N| = 2 \not> 2 = |N_s(s_1) \cap (V \setminus N)|$$
$$\exists s_1 \in V \setminus N,\ |N_s(s_1) \cap N| \not> |N_s(s_1) \cap (V \setminus N)|;$$

(b) second counterexample for the statement "$N = \{s_3, s_4\}$ is global-offensive alliance.";

$$\exists s_2 \in V \setminus N,\ |N_s(s_2) \cap N| = 0 < 1 = |N_s(s_2) \cap (V \setminus N)|$$
$$\exists s_2 \in V \setminus N,\ |N_s(s_2) \cap N| = 0 \not> 1 = |N_s(s_2) \cap (V \setminus N)|$$
$$\exists s_2 \in V \setminus N,\ |N_s(s_2) \cap N| \not> |N_s(s_2) \cap (V \setminus N)|;$$

(c) third counterexample for the statement "$N = \{s_3, s_4\}$ is global-offensive alliance.";

$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| = 0 < 1 = |N_s(s_5) \cap (V \setminus N)|$$
$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| = 0 \not> 1 = |N_s(s_5) \cap (V \setminus N)|$$
$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| \not> |N_s(s_5) \cap (V \setminus N)|.$$

(v) $\Gamma_s = 1.9$ and corresponded set is $S = \{s_1\}$;

(vi) $\Gamma = 1$ and corresponded set is $S = \{s_1\}$.

Proposition 4.19.11. *Let $NTG : (V, E, \sigma, \mu)$ be wheel. Then*

(i) *the set $S = \{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1) \leq n}$ is minimal-global-offensive alliance;*

(ii) $\Gamma = |\{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1) \leq n}|$;

(iii) $\Gamma_s = \Sigma_{\{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1) \leq n}} \Sigma_{i=1}^{3} \sigma_i(s)$;

(iv) *the set $\{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1) \leq n}$ is only minimal-global-offensive alliance.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a wheel. Let $S = \{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1) \leq n}$. There are either

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| = 2 > 1 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$

or

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| = 3 > 0 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$

It implies $S = \{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1)\leq n}$ is global-offensive alliance. If $S' = \{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1)\leq n} - \{z\}$ where $z \in S = \{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1)\leq n}$, then There are either

$$\forall z \in V \setminus S',\ |N_s(z) \cap S'| = 1 < 2 = |N_s(z) \cap (V \setminus S')|$$
$$\forall z \in V \setminus S',\ |N_s(z) \cap S'| < |N_s(z) \cap (V \setminus S')|$$
$$\forall z \in V \setminus S',\ |N_s(z) \cap S'| \not> |N_s(z) \cap (V \setminus S')|$$

or

$$\forall z \in V \setminus S',\ |N_s(z) \cap S'| = 1 = 1 = |N_s(z) \cap (V \setminus S')|$$
$$\forall z \in V \setminus S',\ |N_s(z) \cap S'| = |N_s(z) \cap (V \setminus S')|$$
$$\forall z \in V \setminus S',\ |N_s(z) \cap S'| \not> |N_s(z) \cap (V \setminus S')|$$

So $S' = \{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1)\leq n} - \{z\}$ where $z \in S = \{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1)\leq n}$ isn't global-offensive alliance. It induces $S = \{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1)\leq n}$ is minimal-global-offensive alliance.

(ii), (iii) and (iv) are obvious. ■

Example 4.19.12. Consider Figure (4.7).

(i) $S = \{s_1, s_3, s_5\}$ is only minimal-global-offensive alliance;

(ii) $S = \{s_1, s_3, s_5\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number;

(iii) S' including $S = \{s_2.s_4, s_5\}$ only forms global-offensive-alliance but not minimal-global-offensive-alliance;

(iv) $N = \{s_1, s_3\}$ isn't global-offensive alliance. Since there is one instance and only one instance is enough;

  (a) First counterexample for the statement "$N = \{s_1, s_3\}$ is global-offensive alliance.";

$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| = 1 = 1 = |N_s(s_5) \cap (V \setminus N)|$$
$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| = 1 \not> 1 = |N_s(s_5) \cap (V \setminus N)|$$
$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| \not> |N_s(s_5) \cap (V \setminus N)|;$$

(v) $\Gamma_s = 4.9$ and corresponded set is $S = \{s_1, s_3, s_5\}$;

(vi) $\Gamma = 3$ and corresponded set is $S = \{s_1, s_3, s_5\}$.

Proposition 4.19.13. *Let $NTG : (V, E, \sigma, \mu)$ be an odd complete. Then*

(i) *the set $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ is minimal-global-offensive alliance;*

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor + 1$;

(iii) $\Gamma_s = \min\{\Sigma_{s \in S} \Sigma_{i=1}^{3} \sigma_i(s)\}_{S=\{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}}$;

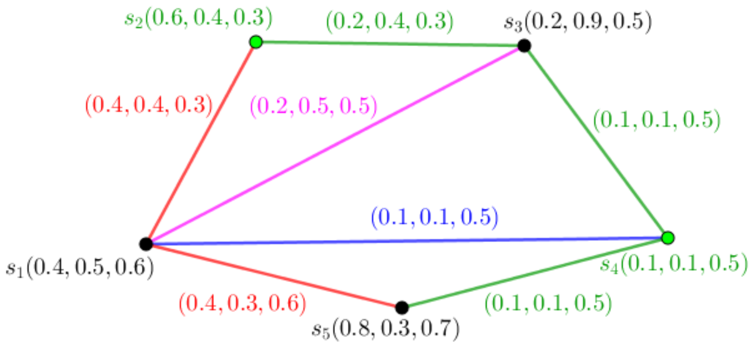

Figure 4.7: The set of black circles is minimal-global-offensive alliance. NTG7

(*iv*) *the set* $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ *is only minimal-global-offensive alliances.*

Proof. (*i*). Suppose $NTG : (V, E, \sigma, \mu)$ is odd complete. Let $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$. Thus

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| = \lfloor \tfrac{n}{2} \rfloor + 1 > \lfloor \tfrac{n}{2} \rfloor - 1 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$

It implies $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ is global-offensive alliance. If $S' = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1} - \{z\}$ where $z \in S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$, then

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| = \lfloor \tfrac{n}{2} \rfloor = \lfloor \tfrac{n}{2} \rfloor = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|$$

So $S' = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1} - \{z\}$ where $z \in S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ isn't global-offensive alliance. It induces $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ is minimal-global-offensive alliance.
(*ii*), (*iii*) and (*iv*) are obvious. ∎

Example 4.19.14. Consider Figure (4.8).

(*i*) $S_1 = \{s_1, s_2, s_3\}, S_2 = \{s_1, s_2, s_4\}, S_3 = \{s_1, s_2, s_5\}, S_4 = \{s_1, s_3, s_4\}, S_5 = \{s_1, s_3, s_5\}, S_6 = \{s_2, s_3, s_4\}, S_7 = \{s_2, s_3, s_5\}, S_8 = \{s_3, s_4, s_5\}$ are only minimal-global-offensive alliances;

(*ii*) $S_6 = \{s_2, s_3, s_4\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number;

(*iii*) $S = \{s_3, s_4, s_5\}$ only forms minimal-global-offensive-alliance number but not minimal-global-offensive-alliance-neutrosophic;

(*iv*) $N = \{s_3, s_4\}$ isn't global-offensive alliance. Since there is three instances and only one instance is enough;

(*a*) First counterexample for the statement "$N = \{s_3, s_4\}$ is global-offensive alliance.";

$$\exists s_1 \in V \setminus N,\ |N_s(s_1) \cap N| = 2 = 2 = |N_s(s_1) \cap (V \setminus N)|$$
$$\exists s_1 \in V \setminus N,\ |N_s(s_1) \cap N| = 2 \not> 2 = |N_s(s_1) \cap (V \setminus N)|$$
$$\exists s_1 \in V \setminus N,\ |N_s(s_1) \cap N| \not> |N_s(s_1) \cap (V \setminus N)|;$$

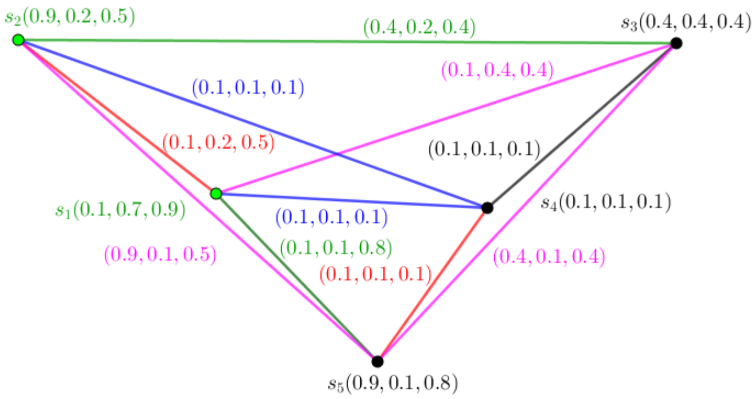

Figure 4.8: The set of black circles is minimal-global-offensive alliance. NTG10

(b) second counterexample for the statement "$N = \{s_3, s_4\}$ is global-offensive alliance.";

$$\exists s_2 \in V \setminus N,\ |N_s(s_2) \cap N| = 2 = 2 = |N_s(s_2) \cap (V \setminus N)|$$
$$\exists s_2 \in V \setminus N,\ |N_s(s_2) \cap N| = 2 \not> 2 = |N_s(s_2) \cap (V \setminus N)|$$
$$\exists s_2 \in V \setminus N,\ |N_s(s_2) \cap N| \not> |N_s(s_2) \cap (V \setminus N)|;$$

(c) third counterexample for the statement "$N = \{s_3, s_4\}$ is global-offensive alliance.".

$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| = 2 = 2 = |N_s(s_5) \cap (V \setminus N)|$$
$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| = 2 \not> 2 = |N_s(s_5) \cap (V \setminus N)|$$
$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| \not> |N_s(s_5) \cap (V \setminus N)|;$$

(v) $\Gamma_s = 3.3$ and corresponded set is $S_6 = \{s_2, s_3, s_4\}$;

(vi) $\Gamma = 3$ and corresponded sets are $S_1 = \{s_1, s_2, s_3\}, S_2 = \{s_1, s_2, s_4\}, S_3 = \{s_1, s_2, s_5\}, S_4 = \{s_1, s_3, s_4\}, S_5 = \{s_1, s_3, s_5\}, S_6 = \{s_2, s_3, s_4\}, S_7 = \{s_2, s_3, s_5\}, S_8 = \{s_3, s_4, s_5\}$ which are only minimal-global-offensive alliances.

Proposition 4.19.15. *Let $NTG : (V, E, \sigma, \mu)$ be an even complete. Then*

(i) *the set $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ is minimal-global-offensive alliance;*

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor$;

(iii) $\Gamma_s = \min\{\Sigma_{s \in S} \Sigma_{i=1}^{3} \sigma_i(s)\}_{S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}}$;

(iv) *the set $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ is only minimal-global-offensive alliances.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is even complete. Let $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$. Thus

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| = \lfloor \tfrac{n}{2} \rfloor > \lfloor \tfrac{n}{2} \rfloor - 1 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$

It implies $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ is global-offensive alliance. If $S' = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor} - \{z\}$ where $z \in S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$, then

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| = \lfloor \frac{n}{2} \rfloor - 1 < \lfloor \frac{n}{2} \rfloor + 1 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|$$

So $S' = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor} - \{z\}$ where $z \in S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ isn't global-offensive alliance. It induces $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ is minimal-global-offensive alliance.
(ii), (iii) and (iv) are obvious. ■

Example 4.19.16. Consider Figure (4.17).

(i) $S_1 = \{s_1, s_2\}, S_2 = \{s_1, s_3\}, S_3 = \{s_1, s_4\}, S_4 = \{s_2, s_3\}, S_5 = \{s_2, s_4\}, S_6 = \{s_3, s_4\}$ are only minimal-global-offensive alliances;

(ii) $S_6 = \{s_3, s_4\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number;

(iii) $S = \{s_1, s_3\}$ only forms minimal-global-offensive-alliance number but not minimal-global-offensive-alliance-neutrosophic;

(iv) $N = \{s_1\}$ isn't global-offensive alliance. Since there is three instances and only one instance is enough;

(a) First counterexample for the statement "$N = \{s_1\}$ is global-offensive alliance.";

$$\exists s_2 \in V \setminus N,\ |N_s(s_2) \cap N| = 1 < 2 = |N_s(s_2) \cap (V \setminus N)|$$
$$\exists s_2 \in V \setminus N,\ |N_s(s_2) \cap N| = 1 \not> 2 = |N_s(s_2) \cap (V \setminus N)|$$
$$\exists s_2 \in V \setminus N,\ |N_s(s_2) \cap N| \not> |N_s(s_2) \cap (V \setminus N)|;$$

(b) second counterexample for the statement "$N = \{s_1\}$ is global-offensive alliance.";

$$\exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| = 1 < 2 = |N_s(s_3) \cap (V \setminus N)|$$
$$\exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| = 1 \not> 2 = |N_s(s_3) \cap (V \setminus N)|$$
$$\exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| \not> |N_s(s_3) \cap (V \setminus N)|;$$

(c) third counterexample for the statement "$N = \{s_1\}$ is global-offensive alliance.".

$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 1 < 2 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 1 \not> 2 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| \not> |N_s(s_4) \cap (V \setminus N)|;$$

(v) $\Gamma_s = 2.3$ and corresponded set is $S_6 = \{s_3, s_4\}$;

(vi) $\Gamma = 2$ and corresponded set is $S_6 = \{s_3, s_4\}$.

4.20 Family of Neutrosophic Graphs

In this section, new definition is applied into family of some classes of neutrosophic graphs which in this family, all neutrosophic graphs have common neutrosophic vertex set. In the case of complete model, the parity of number of vertices concludes to have different results. Clarifications and demonstrations are given for every result as usual.

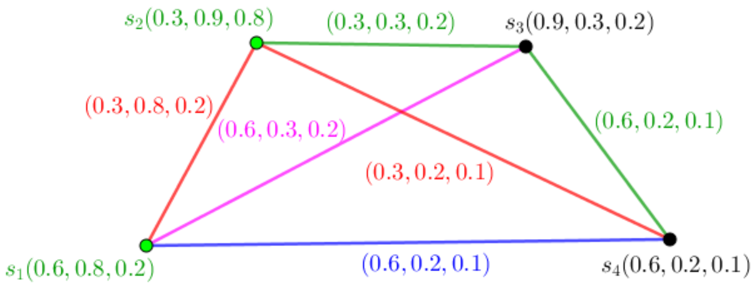

Figure 4.9: The set of black circles is minimal-global-offensive alliance. NTG8

Proposition 4.20.1. *Let $\mathcal{G}$ be a m-family of neutrosophic stars with common neutrosophic vertex set. Then*

(i) *the set $S = \{c_1, c_2, \cdots, c_m\}$ is minimal-global-offensive alliance for $\mathcal{G}$;*

(ii) $\Gamma = m$ *for $\mathcal{G}$;*

(iii) $\Gamma_s = \Sigma_{i=1}^{m}\Sigma_{j=1}^{3}\sigma_j(c_i)$ *for $\mathcal{G}$;*

(iv) *the sets $S = \{c_1, c_2, \cdots, c_m\}$ and $S \subset S'$ are only minimal-global-offensive alliances for $\mathcal{G}$.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a star.

$$\begin{gathered}\forall v \in V \setminus \{c\},\ |N_s(v) \cap \{c\}| = 1 > 0 = |N_s(v) \cap (V \setminus \{c\})| \\ \forall z \in V \setminus S,\ |N_s(z) \cap S| = 1 > 0 = |N_s(z) \cap (V \setminus S)| \\ \forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)| \\ v \in V \setminus \{c\},\ |N_s(v) \cap \{c\}| > |N_s(v) \cap (V \setminus \{c\})|\end{gathered}$$

It implies $S = \{c_1, c_2, \cdots, c_m\}$ is global-offensive alliance or $\mathcal{G}$. If $S = \{c\} - \{c\} = \emptyset$, then

$$\begin{gathered}\exists v \in V \setminus S,\ |N_s(z) \cap S| = 0 = 0 = |N_s(z) \cap (V \setminus S)| \\ \exists v \in V \setminus S,\ |N_s(z) \cap S| = 0 \not> 0 = |N_s(z) \cap (V \setminus S)| \\ \exists v \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|.\end{gathered}$$

So $S = \{c\} - \{c\} = \emptyset$ isn't global-offensive alliance for $\mathcal{G}$. It induces $S = \{c_1, c_2, \cdots, c_m\}$ is minimal-global-offensive alliance for $\mathcal{G}$.
(ii) and (iii) are trivial.
(iv). By (i), $S = \{c_1, c_2, \cdots, c_m\}$ is minimal-global-offensive alliance for $\mathcal{G}$. Thus it's enough to show that $S \subseteq S'$ is minimal-global-offensive alliance for $\mathcal{G}$. Suppose $NTG : (V, E, \sigma, \mu)$ is a star. Let $S \subseteq S'$.

$$\begin{gathered}\forall v \in V \setminus \{c\},\ |N_s(v) \cap \{c\}| = 1 > 0 = |N_s(v) \cap (V \setminus \{c\})| \\ \forall z \in V \setminus S',\ |N_s(z) \cap S'| = 1 > 0 = |N_s(z) \cap (V \setminus S')| \\ \forall z \in V \setminus S',\ |N_s(z) \cap S'| > |N_s(z) \cap (V \setminus S')|\end{gathered}$$

It implies $S' \subseteq S$ is global-offensive alliance for $\mathcal{G}$. ■

Example 4.20.2. Consider Figure (4.10).

(i) $S = \{s_1\}$ is only minimal-global-offensive alliance for $\mathcal{G}$;

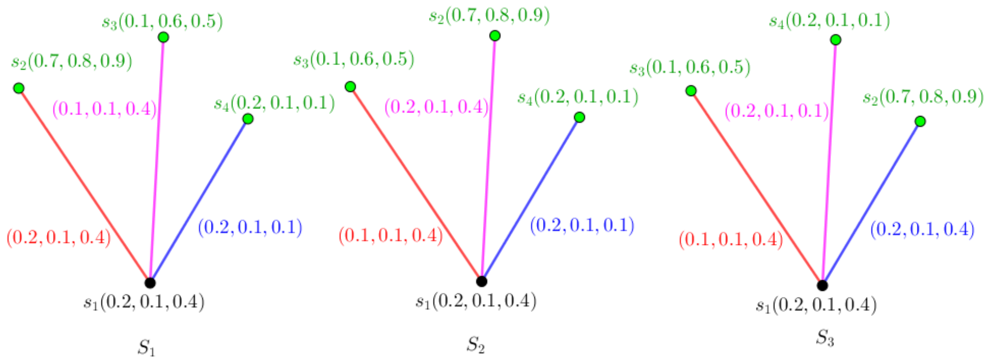

Figure 4.10: The set of black circles is minimal-global-offensive alliance. NTG9

(ii) $S = \{s_1\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number for $\mathcal{G}$;

(iii) S' including $S = \{s_1\}$ only forms global-offensive-alliance but not minimal-global-offensive-alliance for $\mathcal{G}$;

(iv) $N = \{s_3, s_4\}$ isn't global-offensive alliance for $\mathcal{G}$. Since there are two instances for every member of $\mathcal{G}$ but only one of them is enough; for every member of $\mathcal{G}$, we have same following instances;

(a) First counterexample for the statement "$N = \{s_3, s_4\}$ is global-offensive alliance for $\mathcal{G}$.";

$$\exists s_1 \in V \setminus N,\ |N_s(s_1) \cap N| = 2 = 2 = |N_s(s_1) \cap (V \setminus N)|$$
$$\exists s_1 \in V \setminus N,\ |N_s(s_1) \cap N| = 2 \not> 2 = |N_s(s_1) \cap (V \setminus N)|$$
$$\exists s_1 \in V \setminus N,\ |N_s(s_1) \cap N| \not> |N_s(s_1) \cap (V \setminus N)|;$$

(b) second counterexample for the statement "$N = \{s_3, s_4\}$ is global-offensive alliance for $\mathcal{G}$.";

$$\exists s_2 \in V \setminus N,\ |N_s(s_2) \cap N| = 0 < 1 = |N_s(s_2) \cap (V \setminus N)|$$
$$\exists s_2 \in V \setminus N,\ |N_s(s_2) \cap N| = 0 \not> 1 = |N_s(s_2) \cap (V \setminus N)|$$
$$\exists s_2 \in V \setminus N,\ |N_s(s_2) \cap N| \not> |N_s(s_2) \cap (V \setminus N)|;$$

(v) $\Gamma_s = 0.7$ and corresponded set is $S = \{s_1\}$;

(vi) $\Gamma = 1$ and corresponded set is $S = \{s_1\}$.

Proposition 4.20.3. *Let $\mathcal{G}$ be a m-family of odd complete graphs with common neutrosophic vertex set. Then*

(i) *the set $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ is minimal-global-offensive alliance for $\mathcal{G}$;*

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor + 1$ *for $\mathcal{G}$;*

(iii) $\Gamma_s = \min\{\Sigma_{s \in S} \Sigma_{i=1}^{3} \sigma_i(s)\}_{S=\{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}}$ *for $\mathcal{G}$;*

(iv) *the sets $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ are only minimal-global-offensive alliances for $\mathcal{G}$.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is odd complete. Let $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$. Thus

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| = \lfloor \tfrac{n}{2} \rfloor + 1 > \lfloor \tfrac{n}{2} \rfloor - 1 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$

It implies $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ is global-offensive alliance for $\mathcal{G}$. If $S' = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1} - \{z\}$ where $z \in S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$, then

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| = \lfloor \tfrac{n}{2} \rfloor = \lfloor \tfrac{n}{2} \rfloor = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|$$

So $S' = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1} - \{z\}$ where $z \in S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ isn't global-offensive alliance for $\mathcal{G}$. It induces $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ is minimal-global-offensive alliance for $\mathcal{G}$.
(ii), (iii) and (iv) are obvious. ∎

Example 4.20.4. Consider Figure (4.18).

(i) $S_1 = \{s_1, s_2, s_3\}, S_2 = \{s_1, s_2, s_4\}, S_3 = \{s_1, s_2, s_5\}, S_4 = \{s_1, s_3, s_4\}, S_5 = \{s_1, s_3, s_5\}, S_6 = \{s_2, s_3, s_4\}, S_7 = \{s_2, s_3, s_5\}, S_8 = \{s_3, s_4, s_5\}$ are only minimal-global-offensive alliances;

(ii) $S_3 = \{s_1, s_2, s_5\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number for $\mathcal{G}$;

(iii) $S_8 = \{s_3, s_4, s_5\}$ only forms minimal-global-offensive-alliance number but not minimal-global-offensive-alliance-neutrosophic for $\mathcal{G}$;

(iv) $N = \{s_1, s_2\}$ isn't global-offensive alliance. Since there is three instances and only one instance is enough for $\mathcal{G}$;

(a) First counterexample for the statement "$N = \{s_1, s_2\}$ is global-offensive alliance."for $\mathcal{G}$;

$$\exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| = 2 = 2 = |N_s(s_3 \cap (V \setminus N)|$$
$$\exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| = 2 \not> 2 = |N_s(s_3) \cap (V \setminus N)|$$
$$\exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| \not> |N_s(s_3) \cap (V \setminus N)|;$$

(b) second counterexample for the statement "$N = \{s_1, s_2\}$ is global-offensive alliance." for $\mathcal{G}$;

$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 2 = 2 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 2 \not> 2 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| \not> |N_s(s_4) \cap (V \setminus N)|;$$

(c) third counterexample for the statement "$N = \{s_1, s_2\}$ is global-offensive alliance." for $\mathcal{G}$.

$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| = 2 = 2 = |N_s(s_5) \cap (V \setminus N)|$$
$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| = 2 \not> 2 = |N_s(s_5) \cap (V \setminus N)|$$
$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| \not> |N_s(s_5) \cap (V \setminus N)|;$$

(v) $\Gamma_s = 4$ and corresponded set is $S_3 = \{s_1, s_2, s_5\}$ for $\mathcal{G}$;

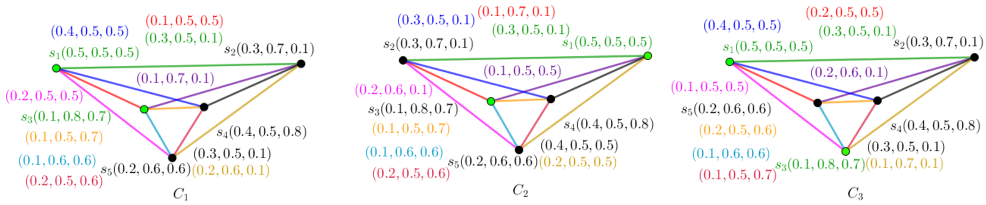

Figure 4.11: The set of black circles is minimal-global-offensive alliance.

NTG11

(vi) $\Gamma = 3$ and corresponded sets are $S_1 = \{s_1, s_2, s_3\}, S_2 = \{s_1, s_2, s_4\}, S_3 = \{s_1, s_2, s_5\}, S_4 = \{s_1, s_3, s_4\}, S_5 = \{s_1, s_3, s_5\}, S_6 = \{s_2, s_3, s_4\}, S_7 = \{s_2, s_3, s_5\}, S_8 = \{s_3, s_4, s_5\}$ which are only minimal-global-offensive alliances for $\mathcal{G}$.

Proposition 4.20.5. *Let $\mathcal{G}$ be a m-family of even complete graphs with common neutrosophic vertex set. Then*

(i) *the set $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ is minimal-global-offensive alliance for $\mathcal{G}$;*

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor$ *for $\mathcal{G}$;*

(iii) $\Gamma_s = \min\{\Sigma_{s \in S} \Sigma_{i=1}^{3} \sigma_i(s)\}_{S=\{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}}$ *for $\mathcal{G}$;*

(iv) *the sets $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ are only minimal-global-offensive alliances for $\mathcal{G}$.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is even complete. Let $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$. Thus

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| = \lfloor \tfrac{n}{2} \rfloor > \lfloor \tfrac{n}{2} \rfloor - 1 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$

It implies $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ is global-offensive alliance for $\mathcal{G}$. If $S' = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor} - \{z\}$ where $z \in S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$, then

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| = \lfloor \tfrac{n}{2} \rfloor - 1 < \lfloor \tfrac{n}{2} \rfloor + 1 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|$$

So $S' = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor} - \{z\}$ where $z \in S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ isn't global-offensive alliance for $\mathcal{G}$. It induces $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ is minimal-global-offensive alliance for $\mathcal{G}$.
(ii), (iii) and (iv) are obvious. ∎

Example 4.20.6. Consider Figure (4.12).

(i) $S_1 = \{s_1, s_2\}, S_2 = \{s_1, s_3\}, S_3 = \{s_1, s_4\}, S_4 = \{s_2, s_3\}, S_5 = \{s_2, s_4\}, S_6 = \{s_3, s_4\}$ are only minimal-global-offensive alliances for $\mathcal{G}$;

(ii) $S_1 = \{s_1, s_2\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number for $\mathcal{G}$;

(iii) $S = \{s_1, s_3\}$ only forms minimal-global-offensive-alliance number but not minimal-global-offensive-alliance-neutrosophic for $\mathcal{G}$;

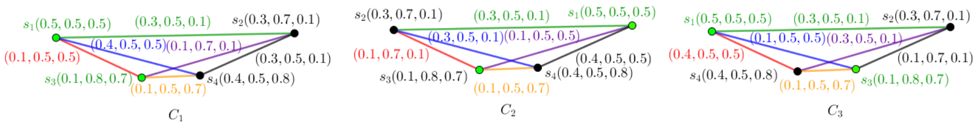

Figure 4.12: The set of black circles is minimal-global-offensive alliance.

NTG12

(iv) $N = \{s_1\}$ isn't global-offensive alliance. Since there is three instances and only one instance is enough for $\mathcal{G}$;

(a) First counterexample for the statement "$N = \{s_1\}$ is global-offensive alliance." for $\mathcal{G}$;

$$\begin{aligned} \exists s_2 \in V \setminus N,\ |N_s(s_2) \cap N| = 1 < 2 = |N_s(s_2) \cap (V \setminus N)| \\ \exists s_2 \in V \setminus N,\ |N_s(s_2) \cap N| = 1 \not\geq 2 = |N_s(s_2) \cap (V \setminus N)| \\ \exists s_2 \in V \setminus N,\ |N_s(s_2) \cap N| \not\geq |N_s(s_2) \cap (V \setminus N)|; \end{aligned}$$

(b) second counterexample for the statement "$N = \{s_1\}$ is global-offensive alliance." for $\mathcal{G}$;

$$\begin{aligned} \exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| = 1 < 2 = |N_s(s_3) \cap (V \setminus N)| \\ \exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| = 1 \not\geq 2 = |N_s(s_3) \cap (V \setminus N)| \\ \exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| \not\geq |N_s(s_3) \cap (V \setminus N)|; \end{aligned}$$

(c) third counterexample for the statement "$N = \{s_1\}$ is global-offensive alliance." for $\mathcal{G}$.

$$\begin{aligned} \exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 1 < 2 = |N_s(s_4) \cap (V \setminus N)| \\ \exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 1 \not\geq 2 = |N_s(s_4) \cap (V \setminus N)| \\ \exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| \not\geq |N_s(s_4) \cap (V \setminus N)|; \end{aligned}$$

(v) $\Gamma_s = 2.6$ and corresponded set is $S_1 = \{s_1, s_2\}$ for $\mathcal{G}$;

(vi) $\Gamma = 2$ and corresponded sets are $S_1 = \{s_1, s_2\}, S_2 = \{s_1, s_3\}, S_3 = \{s_1, s_4\}, S_4 = \{s_2, s_3\}, S_5 = \{s_2, s_4\}, S_6 = \{s_3, s_4\}$ for $\mathcal{G}$.

4.21 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are complete models which mean complete connections are formed as individual and family of complete models with common neutrosophic vertex set.

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importantance to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive section. Beyond that, sometimes sections are not the same.

Step 3. (Model) The situation is designed as a model. The model uses data to assign every section and to assign to relation amid section, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relation amid them. Table (4.5), clarifies about the assigned numbers to these situation.

Table 4.3: Scheduling concerns its Subjects and its Connections as a neutrosophic graph and its alliances in a Model.

tbl1c

Sections of NTG	n_1	$n_2 \cdots$	n_9
Values	$(0.99, 0.98, 0.55)$	$(0.74, 0.64, 0.46) \cdots$	$(0.99, 0.98, 0.55)$
Connections of NTG	E_1	E_2	E_3
Values	$(0.01, 0.01, 0.01)$	$(0.01, 0.01, 0.01)$	$(0.01, 0.01, 0.01)$

4.22 Case 1: Complete Model as Individual

Step 4. (Solution) The neutrosophic graph and its global offensive alliance as model, propose to use specific set. Every subject has connection with every given subject. Thus the connection is applied as possible and the model demonstrates full connections as possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete, the set is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of global offensive alliance when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are four subjects which are represented as Figure (4.17). This model is strong. And the study proposes using specific set of objects which is called minimal-global-offensive alliance. There are also some analyses on other sets in the way that, the clarification is gained about being special set or not. Also, in the last part, there are two numbers to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (4.17).

(i) $S_1 = \{s_1, s_2\}, S_2 = \{s_1, s_3\}, S_3 = \{s_1, s_4\}, S_4 = \{s_2, s_3\}, S_5 = \{s_2, s_4\}, S_6 = \{s_3, s_4\}$ are only minimal-global-offensive alliances;

(ii) $S_6 = \{s_3, s_4\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number;

(iii) $S = \{s_1, s_3\}$ only forms minimal-global-offensive-alliance number but not minimal-global-offensive-alliance-neutrosophic;

(iv) $N = \{s_1\}$ isn't global-offensive alliance. Since there is three instances and only one instance is enough;

- (a) First counterexample for the statement "$N = \{s_1\}$ is global-offensive alliance.";

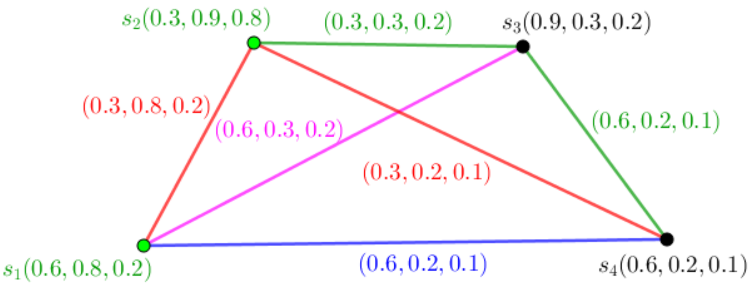

Figure 4.13: The set of black circles is minimal-global-offensive alliance.

NTG8

$$\exists s_2 \in V \setminus N,\ |N_s(s_2) \cap N| = 1 < 2 = |N_s(s_2) \cap (V \setminus N)|$$
$$\exists s_2 \in V \setminus N,\ |N_s(s_2) \cap N| = 1 \not> 2 = |N_s(s_2) \cap (V \setminus N)|$$
$$\exists s_2 \in V \setminus N,\ |N_s(s_2) \cap N| \not> |N_s(s_2) \cap (V \setminus N)|;$$

(b) second counterexample for the statement "$N = \{s_1\}$ is global-offensive alliance.";

$$\exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| = 1 < 2 = |N_s(s_3) \cap (V \setminus N)|$$
$$\exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| = 1 \not> 2 = |N_s(s_3) \cap (V \setminus N)|$$
$$\exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| \not> |N_s(s_3) \cap (V \setminus N)|;$$

(c) third counterexample for the statement "$N = \{s_1\}$ is global-offensive alliance.".

$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 1 < 2 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 1 \not> 2 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| \not> |N_s(s_4) \cap (V \setminus N)|;$$

(v) $\Gamma_s = 2.3$ and corresponded set is $S_6 = \{s_3, s_4\}$;

(vi) $\Gamma = 2$ and corresponded set is $S_6 = \{s_3, s_4\}$.

4.23 Case 2: Family of Complete Models

Step 4. (Solution) The neutrosophic graph and its global offensive alliance as model, propose to use specific set. Every subject has connection with every given subject. Thus the connection is applied as possible and the model demonstrates full connections as possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete, the set is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of global offensive alliance when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are five subjects which are represented in the formation of family of models as Figure (4.17). These models are strong in family. And the study proposes using specific set of objects which is called minimal-global-offensive alliance for this family of models. There are also some analyses on other sets in the way that, the clarification is gained about

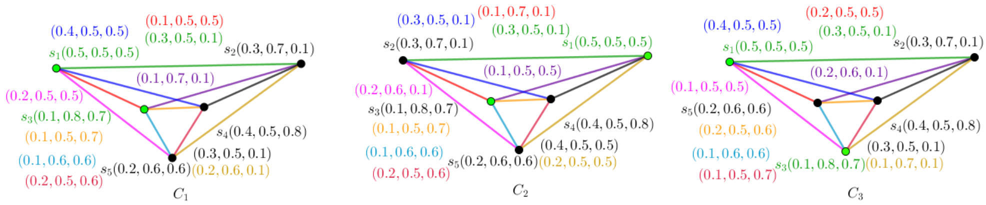

Figure 4.14: The set of black circles is minimal-global-offensive alliance. NTG11

being special set or not. Also, in the last part, there are two numbers to assign to this family of models and collection of situations to compare them with collection of situations to get more precise. Consider Figure (4.18).

(i) $S_1 = \{s_1, s_2, s_3\}, S_2 = \{s_1, s_2, s_4\}, S_3 = \{s_1, s_2, s_5\}, S_4 = \{s_1, s_3, s_4\}, S_5 = \{s_1, s_3, s_5\}, S_6 = \{s_2, s_3, s_4\}, S_7 = \{s_2, s_3, s_5\}, S_8 = \{s_3, s_4, s_5\}$ are only minimal-global-offensive alliances;

(ii) $S_3 = \{s_1, s_2, s_5\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number for $\mathcal{G}$;

(iii) $S_8 = \{s_3, s_4, s_5\}$ only forms minimal-global-offensive-alliance number but not minimal-global-offensive-alliance-neutrosophic for $\mathcal{G}$;

(iv) $N = \{s_1, s_2\}$ isn't global-offensive alliance. Since there is three instances and only one instance is enough for $\mathcal{G}$;

(a) First counterexample for the statement "$N = \{s_1, s_2\}$ is global-offensive alliance."for $\mathcal{G}$;

$$\exists s_3 \in V \setminus N, \ |N_s(s_3) \cap N| = 2 = 2 = |N_s(s_3 \cap (V \setminus N)|$$
$$\exists s_3 \in V \setminus N, \ |N_s(s_3) \cap N| = 2 \not> 2 = |N_s(s_3) \cap (V \setminus N)|$$
$$\exists s_3 \in V \setminus N, \ |N_s(s_3) \cap N| \not> |N_s(s_3) \cap (V \setminus N)|;$$

(b) second counterexample for the statement "$N = \{s_1, s_2\}$ is global-offensive alliance." for $\mathcal{G}$;

$$\exists s_4 \in V \setminus N, \ |N_s(s_4) \cap N| = 2 = 2 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N, \ |N_s(s_4) \cap N| = 2 \not> 2 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N, \ |N_s(s_4) \cap N| \not> |N_s(s_4) \cap (V \setminus N)|;$$

(c) third counterexample for the statement "$N = \{s_1, s_2\}$ is global-offensive alliance." for $\mathcal{G}$.

$$\exists s_5 \in V \setminus N, \ |N_s(s_5) \cap N| = 2 = 2 = |N_s(s_5) \cap (V \setminus N)|$$
$$\exists s_5 \in V \setminus N, \ |N_s(s_5) \cap N| = 2 \not> 2 = |N_s(s_5) \cap (V \setminus N)|$$
$$\exists s_5 \in V \setminus N, \ |N_s(s_5) \cap N| \not> |N_s(s_5) \cap (V \setminus N)|;$$

(v) $\Gamma_s = 4$ and corresponded set is $S_3 = \{s_1, s_2, s_5\}$ for $\mathcal{G}$;

(vi) $\Gamma = 3$ and corresponded sets are $S_1 = \{s_1, s_2, s_3\}, S_2 = \{s_1, s_2, s_4\}, S_3 = \{s_1, s_2, s_5\}, S_4 = \{s_1, s_3, s_4\}, S_5 = \{s_1, s_3, s_5\}, S_6 = \{s_2, s_3, s_4\}, S_7 = \{s_2, s_3, s_5\}, S_8 = \{s_3, s_4, s_5\}$ which are only minimal-global-offensive alliances for $\mathcal{G}$.

4.24 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study. Notion concerning alliance is defined in neutrosophic graphs. Neutrosophic number is also introduced. Thus,

Question 4.24.1. *Is it possible to use other types neighborhood arising from different types of edges to define new alliances?*

Question 4.24.2. *Are existed some connections amid different types of alliances in neutrosophic graphs?*

Question 4.24.3. *Is it possible to construct some classes of which have "nice" behavior?*

Question 4.24.4. *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

Problem 4.24.5. *Which parameters are related to this parameter?*

Problem 4.24.6. *Which approaches do work to construct applications to create independent study?*

Problem 4.24.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

4.25 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted.
This study uses one definition concerning global offensive alliance to study neutrosophic graphs. New neutrosophic number is introduced which is too close to the notion of neutrosophic number but it's different since it uses all values as type-summation on them. The connections of vertices which are clarified by general edges differ them from each other and put them in different categories to represent a set which is called global offensive alliance. Further studies could be about changes in the settings to compare this notion amid different settings of neutrosophic graphs theory. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (4.6), some limitations and advantages of this study are pointed out.

4.26 Global Powerful Alliance in Strong Neutrosophic Graphs

The following sections are cited as [**2**].

Table 4.4: A Brief Overview about Advantages and Limitations of this study

tbl2c

Advantages	Limitations
1. Defining Global Offensive Alliances	1. General Results
2. Applying on Strong Neutrosophic Graphs	
3. Study on Complete Models	2. Deeply More Connections
4. Applying on Individuals	
5. Applying on Family	3. Same Models in Family

4.27 Abstract

New setting is introduced to study the global powerful alliance. Global powerful alliance is about a set of vertices which are applied into the setting of neutrosophic graphs. Neighborhood has the key role to define this notion. Also, neighborhood is defined based on strong edges. Strong edge gets a framework as neighborhood and after that, too close vertices have key role to define global powerful alliance based on strong edges. The structure of set is studied and general results are obtained. Also, some classes of neutrosophic graphs excluding empty, path, star, and wheel and containing complete, cycle and r-regular-strong are investigated in the terms of set, minimal set, number, and neutrosophic number. Neutrosophic number is used in this way. It's applied to use the type of neutrosophic number in the way that, three values of a vertex are used and they've same share to construct this number. It's called "modified neutrosophic number". Summation of three values of vertex makes one number and applying it to a set makes neutrosophic number of set. This approach facilitates identifying minimal set and optimal set which forms minimal-global-powerful-alliance number and minimal-global-powerful-alliance-neutrosophic number. Two different types of sets namely global-powerful alliance and minimal-global-powerful alliance are defined. Global-powerful alliance identifies the sets in general vision but minimal-global-powerful alliance takes focus on the sets which deleting a vertex is impossible. Minimal-global-powerful-alliance number is about minimum cardinality amid the cardinalities of all minimal-global-powerful alliances in a given neutrosophic graph. New notions are applied in the settings both individual and family. Family of neutrosophic graphs has an open avenue, in the way that, the family only contains same classes of neutrosophic graphs. The results are about minimal-global-powerful alliance, minimal-global-powerful-alliance number and its corresponded sets, minimal-global-powerful-alliance-neutrosophic number and its corresponded sets, and characterizing all minimal-global-powerful alliances, minimal-t-powerful alliance, minimal-t-powerful-alliance number and its corresponded sets, minimal-t-powerful-alliance-neutrosophic number and its corresponded sets, and characterizing all minimal-t-powerful alliances. The connections amid t-powerful-alliances are obtained. The number of connected components has some relations with this new concept and it gets some results. Some classes of neutrosophic graphs behave differently when the parity of vertices are different and in this case, cycle, and complete illustrate these behaviors. Two applications concerning complete model as individual and family, under the titles of time table and scheduling conclude the results and they give more clarifications and closing remarks. In this study, there's an open way to extend these results into the family of these classes of neutrosophic graphs. The family of neutrosophic graphs aren't study deeply and with more results but it seems that analogous results are determined. Slight progress is obtained in the family of these models but there are open avenues to study family of other models as same models and different models. There's a question. How can be related to each other, two sets partitioning the vertex set of a graph? The ideas of neighborhood and neighbors based on strong edges illustrate open way to get results. A set is global powerful alliance when two sets partitioning vertex set have uniform structure. All members of set have more amount of neighbors in the set than out of set and reversely for non-members of set with less members in the way that the set is simultaneously t-offensive and (t-2)-defensive. A set is global if t=0. It leads us to the notion of global powerful alliance. Different edges make different neighborhoods but it's used one style edge titled strong edge. These notions are applied into neutrosophic graphs as individuals and family of them. Independent set as an alliance is a special set which has no neighbor inside and it implies some drawbacks for these notions. Finding special sets which are well-known, is an open way to purse this study. Special set which its members have only one neighbor inside, characterize the connected components where the cardinality of its complement is the number of connected components. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Keywords: Modified Neutrosophic Number, Global Powerful Alliance, R-

Regular-Strong

AMS Subject Classification: 05C17, 05C22, 05E45

4.28 Motivation and Contributions

In this study, there's an idea which could be considered as a motivation.

Question 4.28.1. *Is it possible to use mixed versions of ideas concerning "Global Powerful Alliance", "Modified Neutrosophic Number" and "Complete Neutrosophic Graph" to define some notions which are applied to neutrosophic graphs?*

It's motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Connections amid two vertices have key roles to assign global-powerful alliance, minimal-global-powerful alliance, minimal-global-powerful-alliance number, and minimal-global-powerful-alliance-neutrosophic number. Thus they're used to define new ideas which conclude to the structure global powerful alliance. The concept of having strong edge inspires me to study the behavior of strong edges in the way that, two types of numbers and set, e.g., global-powerful alliance, minimal-global-powerful alliance, minimal-global-powerful-alliance number, and minimal-global-powerful-alliance-neutrosophic number are the cases of study in the settings of individuals and in

settings of families. Also, there are some avenues to extend these notions.
The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In subsection "Preliminaries", new notions of global- powerful alliance, minimal-global-powerful alliance, minimal-global-powerful-alliance number, and minimal-global-powerful-alliance-neutrosophic number are introduced and are clarified as individuals. In section "Preliminaries", general sets have the key role in this way. General results are obtained and also, the results about the basic notions of global-powerful alliance are elicited. Two classes of neutrosophic graphs are studied in the terms of global-powerful alliance, minimal-global-powerful alliance, minimal-global-powerful-alliance number, and minimal-global-powerful-alliance-neutrosophic number in section "r-Regular-Strong-Neutrosophic Graph' as individuals. In section "r-Regular-Strong-Neutrosophic Graph", both numbers have applied into individuals. As a concluding result, there are three statements and remarks about r-regular-strong-neutrosophic graphs which are either cycle or complete. The clarifications are also presented in section "r-Regular-Strong-Neutrosophic Graph" for introduced results. In section "Applications in Time Table and Scheduling", two applications are posed for global-powerful alliance concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are complete as individual and uniform family. In section "Open Problems", some problems and questions for further studies are proposed. In section "Conclusion and Closing Remarks", gentle discussion about results and applications is featured. In section "Conclusion and Closing Remarks", a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

4.29 Preliminaries

In this subsection, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.
Basic idea is about the model which is used. First definition introduces basic model.

Definition 4.29.1. (Graph).
$G = (V, E)$ is called a **graph** if V is a set of objects and E is a subset of $V \times V$ (E is a set of 2-subsets of V) where V is called **vertex set** and E is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

Definition 4.29.2. (Neutrosophic Graph And Its Special Case).
$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a **neutrosophic graph** if it's graph, $\sigma_i : V \to [0, 1]$, $\mu_i : E \to [0, 1]$. We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every $v_i v_j \in E$,

$$\mu(v_i v_j) \leq \sigma(v_i) \wedge \sigma(v_j).$$

(i) : σ is called **neutrosophic vertex set**.

(ii) : μ is called **neutrosophic edge set**.

$(iii):$ $|V|$ is called **order** of NTG and it's denoted by $\mathcal{O}(NTG)$.

$(iv):$ $\Sigma_{v\in V}\sigma(v)$ is called **neutrosophic order** of NTG and it's denoted by $\mathcal{O}_n(NTG)$.

$(v):$ $|E|$ is called **size** of NTG and it's denoted by $\mathcal{S}(NTG)$.

$(vi):$ $\Sigma_{e\in E}\Sigma_{i=1}^{3}\mu_i(e)$ is called **neutrosophic size** of NTG and it's denoted by $\mathcal{S}_n(NTG)$.

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

Definition 4.29.3. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

$(i):$ a sequence of vertices $P : x_0, x_1, \cdots, x_n$ is called **path** where $x_ix_{i+1} \in E,\ i = 0, 1, \cdots, n-1$;

$(ii):$ **strength** of path $P : x_0, x_1, \cdots, x_n$ is $\bigwedge_{i=0,\cdots,n-1}\mu(x_ix_{i+1})$;

$(iii):$ **connectedness** amid vertices x_0 and x_n is

$$\mu^{\infty}(x,y) = \bigwedge_{P:x_0,x_1,\cdots,x_n}\ \bigwedge_{i=0,\cdots,n-1}\mu(x_ix_{i+1});$$

$(iv):$ a sequence of vertices $P : x_0, x_1, \cdots, x_n$ is called **cycle** where $x_ix_{i+1} \in E,\ i = 0, 1, \cdots, n-1$ and there are two edges xy and uv such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1}\mu(v_iv_{i+1})$;

$(v):$ it's **t-partite** where V is partitioned to t parts, $V_1, V_2, \cdots, V_t$ and the edge xy implies $x \in V_i$ and $y \in V_j$ where $i \neq j$. If it's complete, then it's denoted by $K_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where σ_i is σ on V_i instead V which mean $x \notin V_i$ induces $\sigma_i(x) = 0$;

$(vi):$ t-partite is **complete bipartite** if $t = 2$, and it's denoted by K_{σ_1,σ_2};

$(vii):$ complete bipartite is **star** if $|V_1| = 1$, and it's denoted by S_{1,σ_2};

$(viii):$ a vertex in V is **center** if the vertex joins to all vertices of a cycle. Then it's **wheel** and it's denoted by W_{1,σ_2};

$(ix):$ it's **complete** where $\forall uv \in V,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$;

$(x):$ it's **strong** where $\forall uv \in E,\ \mu(uv) = \sigma(u) \wedge \sigma(v)$.

The notions of neighbor and neighborhood are about some vertices which have one edge with a fixed vertex. These notions present vertices which are close to a fixed vertex as possible. Based on strong edge, it's possible to define different neighborhood as follows.

Definition 4.29.4. (Strong Neighborhood).
Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Suppose $x \in V$. Then

$$N_s(x) = \{y \in N(x) \mid \mu(xy) = \sigma(x) \wedge \sigma(y)\}.$$

New notion is defined between two types of neighborhoods for a fixed vertex. A minimal set and some numbers are introduced in this way. The next definition has main role in every results which are given in this essay.

Definition 4.29.5. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) a set S of vertices is called **t-offensive alliance** if

$$\forall a \in V \setminus S,\ |N_s(a) \cap S| - |N_s(a) \cap (V \setminus S)| > t;$$

(ii) a t-offensive alliance is called **global-offensive alliance** if $t = 0$;

(iii) a set S of vertices is called **t-defensive alliance** if

$$\forall a \in S,\ |N_s(a) \cap S| - |N_s(a) \cap (V \setminus S)| < t;$$

(iv) a t-defensive alliance is called **global-defensive alliance** if $t = 0$;

(v) a set S of vertices is called **t-powerful alliance** if it's both t-offensive alliance and (t-2)-defensive alliance;

(vi) a t-powerful alliance is called **global-powerful alliance** if $t = 0$;

(vii) $\forall S' \subseteq S$, S is global-powerful alliance but S' isn't global-powerful alliance. Then S is called **minimal-global-powerful alliance**;

$(viii)$ **minimal-global-powerful-alliance number** of NTG is

$$\bigwedge_{S \text{ is a minimal-global-powerful alliance.}} |S|$$

and it's denoted by Γ;

(ix) **minimal-global-powerful-alliance-neutrosophic number** of NTG is

$$\bigwedge_{S \text{ is a minimal-global-offensive alliance.}} \Sigma_{s \in S} \Sigma_{i=1}^{3} \sigma_i(s)$$

and it's denoted by Γ_s.

In the next result, the notions of t-defensive alliance and t-offensive alliance have been extended to present the classes of defensive alliance and offensive alliance which hold when one type of them holds for a given set of vertices.

Proposition 4.29.6. *Let $NTG : (V, E, \sigma, \mu)$ be a strong neutrosophic graph. Then following statements hold;*

(i) *if $s \geq t$ and a set S of vertices is t-defensive alliance, then S is s-defensive alliance;*

(ii) *if $s \leq t$ and a set S of vertices is t-offensive alliance, then S is s-offensive alliance.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Consider a set S of vertices is t-defensive alliance. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < t;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < t \leq s;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < s.$$

Thus S is s-defensive alliance.
(ii). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Consider a set S of vertices is t-offensive alliance. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > t;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > t \geq s;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > s.$$

Thus S is s-offensive alliance. ■

As a consequence of previous result, the relations amid a set which is both t-offensive alliance and t-defensive alliance lead us toward the notion of t-powerful alliance.

Proposition 4.29.7. *Let $NTG : (V, E, \sigma, \mu)$ be a strong neutrosophic graph. Then following statements hold;*

(i) *if $s \geq t + 2$ and a set S of vertices is t-defensive alliance, then S is s-powerful alliance;*

(ii) *if $s \leq t$ and a set S of vertices is t-offensive alliance, then S is t-powerful alliance.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Consider a set S of vertices is t-defensive alliance. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < t;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < t \leq t + 2 \leq s;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < s.$$

Thus S is (t+2)-defensive alliance. By S is s-defensive alliance and S is (s+2)-offensive alliance, S is s-powerful alliance.
(ii). Suppose $NTG : (V, E, \sigma, \mu)$ is a strong neutrosophic graph. Consider a set S of vertices is t-offensive alliance. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > t;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > t \geq s > s - 2;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > s - 2.$$

Thus S is (s-2)-defensive alliance. By S is (s-2)-defensive alliance and S is s-offensive alliance, S is s-powerful alliance. ■

4.30 r-Regular-Strong-Neutrosophic Graph

r-regular is an attribute. This property facilitates the results when the condition is about the neighbors inside fixed set to determine 2-defensive alliance and 2-offensive alliance. Also, a condition about the neighbors outside of fixed set determines some results about r-defensive alliance and r-offensive alliance.

Proposition 4.30.1. *Let $NTG : (V, E, \sigma, \mu)$ be a r-regular-strong-neutrosophic graph. Then following statements hold;*

(i) *if* $\forall a \in S,\ |N_s(a) \cap S| < \lfloor \frac{r}{2} \rfloor + 1,$ *then* $NTG : (V, E, \sigma, \mu)$ *is 2-defensive alliance;*

(ii) *if* $\forall a \in V \setminus S,\ |N_s(a) \cap S| > \lfloor \frac{r}{2} \rfloor + 1,$ *then* $NTG : (V, E, \sigma, \mu)$ *is 2-offensive alliance;*

(iii) *if* $\forall a \in S,\ |N_s(a) \cap V \setminus S| = 0,$ *then* $NTG : (V, E, \sigma, \mu)$ *is r-defensive alliance;*

(iv) *if* $\forall a \in V \setminus S,\ |N_s(a) \cap V \setminus S| = 0,$ *then* $NTG : (V, E, \sigma, \mu)$ *is r-offensive alliance.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \lfloor \tfrac{r}{2} \rfloor + 1 - (\lfloor \tfrac{r}{2} \rfloor - 1);$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \lfloor \tfrac{r}{2} \rfloor + 1 - (\lfloor \tfrac{r}{2} \rfloor - 1) < 2;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2.$$

Thus S is 2-defensive alliance.

(ii). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \lfloor \tfrac{r}{2} \rfloor + 1 - (\lfloor \tfrac{r}{2} \rfloor - 1);$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \lfloor \tfrac{r}{2} \rfloor + 1 - (\lfloor \tfrac{r}{2} \rfloor - 1) > 2;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2.$$

Thus S is 2-offensive alliance.

(iii). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < r - 0;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < r - 0 = r;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < r.$$

Thus S is r-defensive alliance.

(iv). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > r - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > r - 0 = r;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > r.$$

Thus S is r-offensive alliance.

■

2-defensive alliance and 2-offensive alliance get some results about the neighbors inside fixed set. Also, r-defensive alliance and r-offensive alliance get some results about the neighbors outside of fixed set.

Proposition 4.30.2. *Let* $NTG : (V, E, \sigma, \mu)$ *be a r-regular-strong-neutrosophic graph. Then following statements hold;*

(i) $\forall a \in S,\ |N_s(a) \cap S| < \lfloor \frac{r}{2} \rfloor + 1$ *if* $NTG : (V, E, \sigma, \mu)$ *is 2-defensive alliance;*

(ii) $\forall a \in V \setminus S,\ |N_s(a) \cap S| > \lfloor \frac{r}{2} \rfloor + 1$ *if* $NTG : (V, E, \sigma, \mu)$ *is 2-offensive alliance;*

(iii) $\forall a \in S,\ |N_s(a) \cap V \setminus S| = 0$ *if* $NTG : (V, E, \sigma, \mu)$ *is r-defensive alliance;*

(iv) $\forall a \in V \setminus S,\ |N_s(a) \cap V \setminus S| = 0$ *if* $NTG : (V, E, \sigma, \mu)$ *is r-offensive alliance.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph and 2-defensive alliance. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2 = \lfloor \frac{r}{2} \rfloor + 1 - (\lfloor \frac{r}{2} \rfloor - 1);$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \lfloor \frac{r}{2} \rfloor + 1 - (\lfloor \frac{r}{2} \rfloor - 1);$$
$$\forall t \in S,\ |N_s(t) \cap S| = \lfloor \frac{r}{2} \rfloor + 1,\ |N_s(t) \cap (V \setminus S)| = \lfloor \frac{r}{2} \rfloor - 1.$$

(ii). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph and 2-offensive alliance. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2 = \lfloor \frac{r}{2} \rfloor + 1 - (\lfloor \frac{r}{2} \rfloor - 1);$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \lfloor \frac{r}{2} \rfloor + 1 - (\lfloor \frac{r}{2} \rfloor - 1);$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| = \lfloor \frac{r}{2} \rfloor + 1,\ |N_s(t) \cap (V \setminus S) = \lfloor \frac{r}{2} \rfloor - 1.$$

(iii). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph and r-defensive alliance.

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < r;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < r = r - 0;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < r - 0;$$
$$\forall t \in S,\ |N_s(t) \cap S| = r,\ |N_s(t) \cap (V \setminus S)| = 0.$$

(iv). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph and r-offensive alliance. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > r;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > r = r - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > r - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| = r,\ |N_s(t) \cap (V \setminus S)| = 0.$$

■

As a special case, complete neutrosophic graph gets specific result excerpt from r-regular neutrosophic graph. 2-defensive alliance and 2-offensive alliance get some results about the neighbors inside fixed set depending on order. Also, $(\mathcal{O} - 1)$-defensive alliance and $(\mathcal{O} - 1)$-offensive alliance get some results about the neighbors outside of fixed set depending on order.

Proposition 4.30.3. *Let $NTG : (V, E, \sigma, \mu)$ be a r-regular-strong-neutrosophic graph which is complete. Then following statements hold;*

(i) $\forall a \in S,\ |N_s(a) \cap S| < \lfloor \frac{\mathcal{O}-1}{2} \rfloor + 1$ *if $NTG : (V, E, \sigma, \mu)$ is 2-defensive alliance;*

(ii) $\forall a \in V \setminus S,\ |N_s(a) \cap S| > \lfloor \frac{\mathcal{O}-1}{2} \rfloor + 1$ *if $NTG : (V, E, \sigma, \mu)$ is 2-offensive alliance;*

(iii) $\forall a \in S,\ |N_s(a) \cap V \setminus S| = 0$ *if $NTG : (V, E, \sigma, \mu)$ is $(\mathcal{O} - 1)$-defensive alliance;*

(iv) $\forall a \in V \setminus S,\ |N_s(a) \cap V \setminus S| = 0$ *if $NTG : (V, E, \sigma, \mu)$ is $(\mathcal{O} - 1)$-offensive alliance.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph and 2-defensive alliance. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2 = \lfloor \frac{\mathcal{O}-1}{2} \rfloor + 1 - (\lfloor \frac{\mathcal{O}-1}{2} \rfloor - 1);$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \lfloor \frac{\mathcal{O}-1}{2} \rfloor + 1 - (\lfloor \frac{\mathcal{O}-1}{2} \rfloor - 1);$$
$$\forall t \in S,\ |N_s(t) \cap S| = \lfloor \frac{\mathcal{O}-1}{2} \rfloor + 1,\ |N_s(t) \cap (V \setminus S)| = \lfloor \frac{\mathcal{O}-1}{2} \rfloor - 1.$$

(ii). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph and 2-offensive alliance. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2 = \lfloor \frac{\mathcal{O}-1}{2} \rfloor + 1 - (\lfloor \frac{\mathcal{O}-1}{2} \rfloor - 1);$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \lfloor \frac{\mathcal{O}-1}{2} \rfloor + 1 - (\lfloor \frac{\mathcal{O}-1}{2} \rfloor - 1);$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| = \lfloor \frac{\mathcal{O}-1}{2} \rfloor + 1,\ |N_s(t) \cap (V \setminus S) = \lfloor \frac{\mathcal{O}-1}{2} \rfloor - 1.$$

(iii). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph and $(\mathcal{O} - 1)$-defensive alliance.

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \mathcal{O} - 1;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \mathcal{O} - 1 = \mathcal{O} - 1 - 0;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \mathcal{O} - 1 - 0;$$
$$\forall t \in S,\ |N_s(t) \cap S| = \mathcal{O} - 1,\ |N_s(t) \cap (V \setminus S)| = 0.$$

(iv). Suppose $NTG : (V, E, \sigma, \mu)$ is a $(\mathcal{O} - 1)$-regular-strong-neutrosophic graph and r-offensive alliance. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \mathcal{O} - 1;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \mathcal{O} - 1 = \mathcal{O} - 1 - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \mathcal{O} - 1 - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| = \mathcal{O} - 1,\ |N_s(t) \cap (V \setminus S)| = 0.$$

■

As a special case of r-regular, complete is an attribute. This property facilitates the results when the condition is about the neighbors inside fixed set to determine 2-defensive alliance and 2-offensive alliance. Also, a condition about the neighbors outside of fixed set determines some results about $(\mathcal{O} - 1)$-defensive alliance and $(\mathcal{O} - 1)$-offensive alliance.

Proposition 4.30.4. *Let $NTG : (V, E, \sigma, \mu)$ be a r-regular-strong-neutrosophic graph which is complete. Then following statements hold;*

(i) *if $\forall a \in S,\ |N_s(a) \cap S| < \lfloor \frac{\mathcal{O}-1}{2} \rfloor + 1$, then $NTG : (V, E, \sigma, \mu)$ is 2-defensive alliance;*

(ii) *if $\forall a \in V \setminus S,\ |N_s(a) \cap S| > \lfloor \frac{\mathcal{O}-1}{2} \rfloor + 1$, then $NTG : (V, E, \sigma, \mu)$ is 2-offensive alliance;*

(iii) *if $\forall a \in S,\ |N_s(a) \cap V \setminus S| = 0$, then $NTG : (V, E, \sigma, \mu)$ is $(\mathcal{O} - 1)$-defensive alliance;*

(iv) *if $\forall a \in V \setminus S,\ |N_s(a) \cap V \setminus S| = 0$, then $NTG : (V, E, \sigma, \mu)$ is $(\mathcal{O} - 1)$-offensive alliance.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \lfloor \frac{\mathcal{O}-1}{2} \rfloor + 1 - (\lfloor \frac{\mathcal{O}-1}{2} \rfloor - 1);$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \lfloor \frac{\mathcal{O}-1}{2} \rfloor + 1 - (\lfloor \frac{\mathcal{O}-1}{2} \rfloor - 1) < 2;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2.$$

Thus S is 2-defensive alliance.

(ii). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \lfloor \frac{\mathcal{O}-1}{2} \rfloor + 1 - (\lfloor \frac{\mathcal{O}-1}{2} \rfloor - 1);$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \lfloor \frac{\mathcal{O}-1}{2} \rfloor + 1 - (\lfloor \frac{\mathcal{O}-1}{2} \rfloor - 1) > 2;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2.$$

Thus S is 2-offensive alliance.

(iii). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \mathcal{O} - 1 - 0;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \mathcal{O} - 1 - 0 = \mathcal{O} - 1;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \mathcal{O} - 1.$$

Thus S is $(\mathcal{O} - 1)$-defensive alliance.

(iv). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \mathcal{O} - 1 - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \mathcal{O} - 1 - 0 = \mathcal{O} - 1;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \mathcal{O} - 1.$$

Thus S is $(\mathcal{O} - 1)$-offensive alliance.

■

In next example, the concept of r-defensive alliance and r-offensive alliance are applied into a r-regular-strong-neutrosophic graph which is complete and its order is five, it means $\mathcal{O} = 5$.

Example 4.30.5. Consider Figure (4.15). In this section, 1-powerful alliance is studied in the way that more clarifications are represented.

(i) Every 3-set of vertices, e.g.,

$$S_1 = \{s_1, s_2, s_3\}, S_2 = \{s_1, s_3, s_5\}, S_3 = \{s_2, s_3, s_4\}, S_4 = \{s_3, s_4, s_5\}$$

is minimal-1-powerful alliance and it forms a minimal-1-powerful-alliance number but only $S_3 = \{s_3, s_4, s_5\}$ is optimal such that forms both minimal-1-powerful-alliance-neutrosophic number and minimal-1-powerful-alliance number;

(ii) $N = \{s_2, s_5\}$ isn't 1-powerful alliance. Since

$$\exists s_1 \in V \setminus N,\ |N_s(s_1) \cap N| - |N_s(s_1) \cap (V \setminus N)| = 2 - 2 = 0 < 1$$
$$\exists s_1 \in V \setminus N,\ |N_s(s_1) \cap N| - |N_s(s_1) \cap (V \setminus N)| = 2 - 2 = 0 \not> 1$$
$$\exists s_1 \in V \setminus N,\ |N_s(s_1) \cap N| - |N_s(s_1) \cap (V \setminus N)| \not> 1;$$

it implies $N = \{s_2, s_5\}$ isn't 1-offensive alliance. So $N = \{s_2, s_5\}$ isn't 1-powerful alliance. Also,

$$\exists s_2 \in N,\ |N_s(s_1) \cap N| - |N_s(s_1) \cap (V \setminus N)| = 1 - 3 = -2 < 1$$
$$\exists s_2 \in N,\ |N_s(s_1) \cap N| - |N_s(s_1) \cap (V \setminus N)| = 1 - 3 = -2 < 1$$
$$\exists s_2 \in N,\ |N_s(s_1) \cap N| - |N_s(s_1) \cap (V \setminus N)| < 1;$$

it implies $N = \{s_2, s_5\}$ 1-defensive alliance but $N = \{s_2, s_5\}$ isn't 1-powerful alliance.

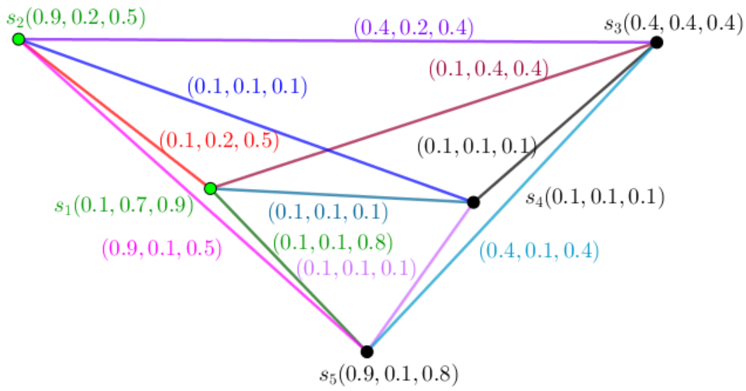

Figure 4.15: Black circles form a set which is 1-powerful alliance. NTG13

(iii) $\Gamma_s = 3.3$;

(iv) $\Gamma = 3$.

As a special case, cycle neutrosophic graph gets specific result excerpt from 2-regular neutrosophic graph. 2-defensive alliance and 2-offensive alliance get some results about the neighbors inside fixed set which their number is at most 2. Also, 2-defensive alliance and 2-offensive alliance get some results about the neighbors outside of fixed set which their number is at most 2.

Proposition 4.30.6. *Let $NTG : (V, E, \sigma, \mu)$ be a r-regular-strong-neutrosophic graph which is cycle. Then following statements hold;*

(i) *$\forall a \in S,\ |N_s(a) \cap S| < 2$ if $NTG : (V, E, \sigma, \mu)$ is 2-defensive alliance;*

(ii) *$\forall a \in V \setminus S,\ |N_s(a) \cap S| > 2$ if $NTG : (V, E, \sigma, \mu)$ is 2-offensive alliance;*

(iii) *$\forall a \in S,\ |N_s(a) \cap V \setminus S| = 0$ if $NTG : (V, E, \sigma, \mu)$ is 2-defensive alliance;*

(iv) *$\forall a \in V \setminus S,\ |N_s(a) \cap V \setminus S| = 0$ if $NTG : (V, E, \sigma, \mu)$ is 2-offensive alliance.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph and 2-defensive alliance. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2 = 2 - 0;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2;$$
$$\forall t \in S,\ |N_s(t) \cap S| < 2,\ |N_s(t) \cap (V \setminus S)| = 0.$$

(ii). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph and 2-offensive alliance. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2 = 2 - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| > 2,\ |N_s(t) \cap (V \setminus S) = 0.$$

(iii). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph and 2-defensive alliance.

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2 = 2 - 0;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2 - 0;$$
$$\forall t \in S,\ |N_s(t) \cap S| < 2,\ |N_s(t) \cap (V \setminus S)| = 0.$$

(iv). Suppose $NTG : (V, E, \sigma, \mu)$ is a 2-regular-strong-neutrosophic graph and r-offensive alliance. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2 = 2 - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2 - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| > 2,\ |N_s(t) \cap (V \setminus S)| = 0.$$

■

As a special case of r-regular, cycle is an attribute. This property facilitates the results when the condition is about the neighbors inside fixed set to determine 2-defensive alliance and 2-offensive alliance. Also, a condition about the neighbors outside of fixed set determines some results about 2-defensive alliance and 2-offensive alliance.

Proposition 4.30.7. *Let $NTG : (V, E, \sigma, \mu)$ be a r-regular-strong-neutrosophic graph which is cycle. Then following statements hold;*

(i) *if $\forall a \in S,\ |N_s(a) \cap S| < 2$, then $NTG : (V, E, \sigma, \mu)$ is 2-defensive alliance;*

(ii) *if $\forall a \in V \setminus S,\ |N_s(a) \cap S| > 2$, then $NTG : (V, E, \sigma, \mu)$ is 2-offensive alliance;*

(iii) *if $\forall a \in S,\ |N_s(a) \cap V \setminus S| = 0$, then $NTG : (V, E, \sigma, \mu)$ is 2-defensive alliance;*

(iv) *if $\forall a \in V \setminus S,\ |N_s(a) \cap V \setminus S| = 0$, then $NTG : (V, E, \sigma, \mu)$ is 2-offensive alliance.*

Proof. (i). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2 - 0;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2 - 0 = 2;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2.$$

Thus S is 2-defensive alliance.

(ii). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2 - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2 - 0 = 2;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2.$$

Thus S is 2-offensive alliance.

(iii). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2 - 0;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2 - 0 = 2;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2.$$

Thus S is 2-defensive alliance.

(iv). Suppose $NTG : (V, E, \sigma, \mu)$ is a r-regular-strong-neutrosophic graph. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2 - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2 - 0 = 2;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2.$$

Thus S is 2-offensive alliance.

■

Example 4.30.8. Consider Figure (4.16). In this section, 3-powerful alliance is studied in the way that more clarifications are represented.

(i) Every 3-set of vertices, e.g.,

$$S_1 = \{s_1, s_2, s_3\}, S_2 = \{s_1, s_3, s_5\}, S_3 = \{s_2, s_3, s_4\}, S_4 = \{s_3, s_4, s_5\}$$

is minimal-3-powerful alliance and it forms a minimal-3-powerful-alliance number but only $S_3 = \{s_3, s_4, s_5\}$ is optimal such that forms both minimal-3-powerful-alliance-neutrosophic number and minimal-3-powerful-alliance number; since

$$\exists s_3 \in S_4,\ |N_s(s_3) \cap S_4| - |N_s(s_3) \cap (V \setminus S_4)| = 1 - 1 = 0 < 3$$
$$\exists s_3 \in S_4,\ |N_s(s_3) \cap S_4| - |N_s(s_3) \cap (V \setminus S_4)| = 1 - 1 = 0 < 3$$
$$\exists s_3 \in S_4,\ |N_s(s_3) \cap S_4| - |N_s(s_3) \cap (V \setminus S_4)| < 3;$$

$$\exists s_5 \in S_4,\ |N_s(s_5) \cap S_4| - |N_s(s_5) \cap (V \setminus S_4)| = 1 - 1 = 0 < 3$$
$$\exists s_5 \in S_4,\ |N_s(s_5) \cap N| - |N_s(s_5) \cap (V \setminus S_4)| = 1 - 1 = 0 < 3$$
$$\exists s_5 \in S_4,\ |N_s(s_5) \cap S_4| - |N_s(s_5) \cap (V \setminus S_4)| < 3;$$

$$\exists s_4 \in S_4,\ |N_s(s_4) \cap S_4| - |N_s(s_4) \cap (V \setminus S_4)| = 2 - 0 = 2 < 3$$
$$\exists s_4 \in S_4,\ |N_s(s_4) \cap S_4| - |N_s(s_4) \cap (V \setminus S_4)| = 2 - 0 = 2 < 3$$
$$\exists s_4 \in S_4,\ |N_s(s_4) \cap S_4| - |N_s(s_4) \cap (V \setminus N)| < 3;$$

It implies S_4 is 3-defensive alliance. Also,

$$\exists s_1 \in V \setminus S_4,\ |N_s(s_1) \cap S_4| - |N_s(s_1) \cap (V \setminus S_4)| = 1 - 1 = 0 > -1$$
$$\exists s_1 \in V \setminus S_4,\ |N_s(s_1) \cap S_4| - |N_s(s_1) \cap (V \setminus S_4)| = 1 - 1 = 0 > -1$$
$$\exists s_1 \in V \setminus S_4,\ |N_s(s_1) \cap S_4| - |N_s(s_1) \cap (V \setminus S_4)| > -1;$$

$$\exists s_2 \in S_4,\ |N_s(s_2) \cap N| - |N_s(s_2) \cap (V \setminus N)| = 1 - 1 = 0 > -1$$
$$\exists s_2 \in N,\ |N_s(s_2) \cap N| - |N_s(s_2) \cap (V \setminus N)| = 1 - 1 = 0 > -1$$
$$\exists s_2 \in N,\ |N_s(s_2) \cap N| - |N_s(s_2) \cap (V \setminus N)| > -1;$$

It implies S_4 is (-1)-offensive alliance. S_4 isn't (-1)-powerful alliance.

(ii) Every 4-set of vertices, e.g.,

$$S_1 = \{s_1, s_2, s_3, s_4\}, S_2 = \{s_1, s_2, s_3, s_5\}, S_3 = \{s_2, s_3, s_4, s_5\}$$

is minimal-3-powerful alliance and it forms a minimal-3-powerful-alliance number but only $S = \{s_2, s_3, s_4, s_5\}$ is optimal such that forms both minimal-3-powerful-alliance-neutrosophic number and minimal-3-powerful-alliance number; since

$$\exists s_3 \in S,\ |N_s(s_3) \cap S| - |N_s(s_3) \cap (V \setminus S)| = 2 - 0 = 2 < 3$$
$$\exists s_3 \in S,\ |N_s(s_3) \cap S| - |N_s(s_3) \cap (V \setminus S)| = 2 - 0 = 0 < 3$$
$$\exists s_3 \in S,\ |N_s(s_3) \cap S| - |N_s(s_3) \cap (V \setminus S)| < 3;$$

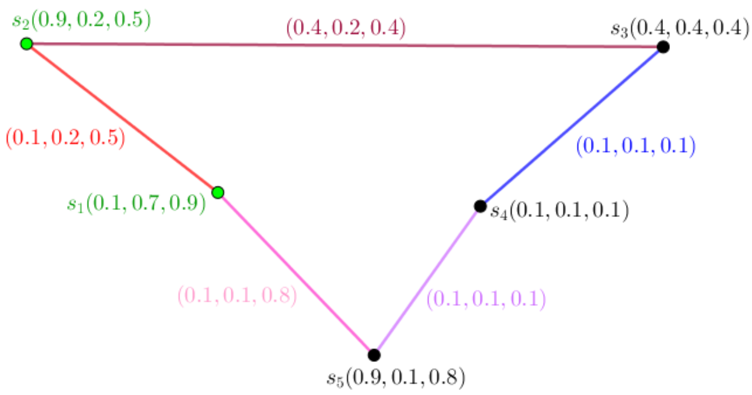

Figure 4.16: Black circles form a set which is 1-powerful alliance.

NTG14

$$\begin{gathered}\exists s_4 \in S,\ |N_s(s_4) \cap S| - |N_s(s_4) \cap (V \setminus S)| = 2 - 0 = 2 < 3 \\ \exists s_4 \in S,\ |N_s(s_4) \cap S| - |N_s(s_4) \cap (V \setminus S)| = 2 - 0 = 2 < 3 \\ \exists s_4 \in S,\ |N_s(s_4) \cap S| - |N_s(s_4) \cap (V \setminus S)| < 3;\end{gathered}$$

$$\begin{gathered}\exists s_5 \in S,\ |N_s(s_5) \cap S| - |N_s(s_5) \cap (V \setminus S)| = 1 - 1 = 0 < 3 \\ \exists s_5 \in S,\ |N_s(s_5) \cap S| - |N_s(s_5) \cap (V \setminus S)| = 1 - 1 = 0 < 3 \\ \exists s_5 \in S,\ |N_s(s_5) \cap S| - |N_s(s_5) \cap (V \setminus S)| < 3;\end{gathered}$$

$$\begin{gathered}\exists s_2 \in S,\ |N_s(s_2) \cap S| - |N_s(s_2) \cap (V \setminus S)| = 1 - 1 = 0 < 3 \\ \exists s_2 \in S,\ |N_s(s_2) \cap S| - |N_s(s_2) \cap (V \setminus S)| = 1 - 1 = 0 < 3 \\ \exists s_2 \in S,\ |N_s(s_2) \cap S| - |N_s(s_2) \cap (V \setminus S)| < 3;\end{gathered}$$

it implies S is 3-defensive alliance. Also,

$$\begin{gathered}\exists s_1 \in V \setminus S,\ |N_s(s_1) \cap S| - |N_s(s_1) \cap (V \setminus S)| = 2 - 0 = 2 > 1 \\ \exists s_1 \in V \setminus S,\ |N_s(s_1) \cap S| - |N_s(s_1) \cap (V \setminus S)| = 2 - 0 = 2 > 1 \\ \exists s_1 \in V \setminus S,\ |N_s(s_1) \cap S| - |N_s(s_1) \cap (V \setminus S)| > 1;\end{gathered}$$

it implies S_4 is 1-offensive alliance. S_4 isn't 1-powerful alliance.

(iii) Γ_s isn't well-defined;

(iv) Γ isn't well-defined.

4.31 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are complete models which mean complete connections are formed as individual and family of complete models with common neutrosophic vertex set.
Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importantance to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive section. Beyond that, sometimes sections are not the same.

Step 3. (Model) The situation is designed as a model. The model uses data to assign every section and to assign to relation amid section, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There's one restriction in that, the numbers amid two sections are at least the number of the relation amid them. Table (4.5), clarifies about the assigned numbers to these situation.

Table 4.5: Scheduling concerns its Subjects and its Connections as a neutrosophic graph and its alliances in a Model.

tbl1c

Sections of NTG	n_1	$n_2 \cdots$	n_9
Values	$(0.99, 0.98, 0.55)$	$(0.74, 0.64, 0.46) \cdots$	$(0.99, 0.98, 0.55)$
Connections of NTG	E_1	E_2	E_3
Values	$(0.01, 0.01, 0.01)$	$(0.01, 0.01, 0.01)$	$(0.01, 0.01, 0.01)$

4.32 Case 1: Complete Model as Individual

Step 4. (Solution) The neutrosophic graph and its global offensive alliance as model, propose to use specific set. Every subject has connection with every given subject. Thus the connection is applied as possible and the model demonstrates full connections as possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete, the set is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of global offensive alliance when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are four subjects which are represented as Figure (4.17). This model is strong. And the study proposes using specific set of objects which is called minimal-global-offensive alliance. There are also some analyses on other sets in the way that, the clarification is gained about being special set or not. Also, in the last part, there are two numbers to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (4.17).

(*i*) $S_1 = \{s_1, s_2\}, S_2 = \{s_1, s_3\}, S_3 = \{s_1, s_4\}, S_4 = \{s_2, s_3\}, S_5 = \{s_2, s_4\}, S_6 = \{s_3, s_4\}$ are only minimal-global-offensive alliances;

(*ii*) $S_6 = \{s_3, s_4\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number;

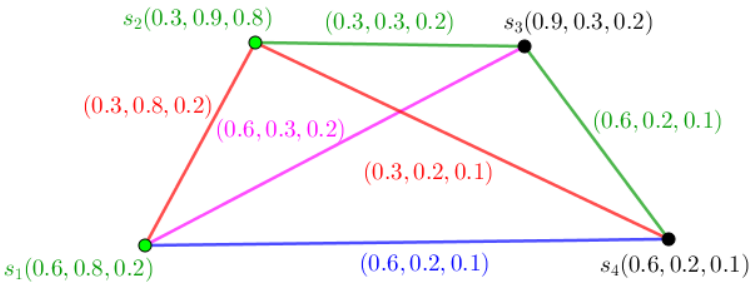

Figure 4.17: The set of black circles is minimal-global-offensive alliance. NTG8

(*iii*) $S = \{s_1, s_3\}$ only forms minimal-global-offensive-alliance number but not minimal-global-offensive-alliance-neutrosophic;

(*iv*) $N = \{s_1\}$ isn't global-offensive alliance. Since there is three instances and only one instance is enough;

(*a*) First counterexample for the statement "$N = \{s_1\}$ is global-offensive alliance.";

$$\exists s_2 \in V \setminus N,\ |N_s(s_2) \cap N| = 1 < 2 = |N_s(s_2) \cap (V \setminus N)|$$
$$\exists s_2 \in V \setminus N,\ |N_s(s_2) \cap N| = 1 \not> 2 = |N_s(s_2) \cap (V \setminus N)|$$
$$\exists s_2 \in V \setminus N,\ |N_s(s_2) \cap N| \not> |N_s(s_2) \cap (V \setminus N)|;$$

(*b*) second counterexample for the statement "$N = \{s_1\}$ is global-offensive alliance.";

$$\exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| = 1 < 2 = |N_s(s_3) \cap (V \setminus N)|$$
$$\exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| = 1 \not> 2 = |N_s(s_3) \cap (V \setminus N)|$$
$$\exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| \not> |N_s(s_3) \cap (V \setminus N)|;$$

(*c*) third counterexample for the statement "$N = \{s_1\}$ is global-offensive alliance.".

$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 1 < 2 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 1 \not> 2 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| \not> |N_s(s_4) \cap (V \setminus N)|;$$

(*v*) $\Gamma_s = 2.3$ and corresponded set is $S_6 = \{s_3, s_4\}$;

(*vi*) $\Gamma = 2$ and corresponded set is $S_6 = \{s_3, s_4\}$.

4.33 Case 2: Family of Complete Models

Step 4. (Solution) The neutrosophic graph and its global offensive alliance as model, propose to use specific set. Every subject has connection with every given subject. Thus the connection is applied as possible and the model demonstrates full connections as possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete, the set is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of global offensive alliance when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are five subjects which are represented in the formation of family of models as Figure (4.17). These models are strong in family. And the study proposes using specific set of objects which is called minimal-global-offensive alliance for this family of models. There are also some analyses on other sets in the way that, the clarification is gained about being special set or not. Also, in the last part, there are two numbers to assign to this family of models and collection of situations to compare them with collection of situations to get more precise. Consider Figure (4.18).

(*i*) $S_1 = \{s_1, s_2, s_3\}, S_2 = \{s_1, s_2, s_4\}, S_3 = \{s_1, s_2, s_5\}, S_4 = \{s_1, s_3, s_4\}, S_5 = \{s_1, s_3, s_5\}, S_6 = \{s_2, s_3, s_4\}, S_7 = \{s_2, s_3, s_5\}, S_8 = \{s_3, s_4, s_5\}$ are only minimal-global-offensive alliances;

(*ii*) $S_3 = \{s_1, s_2, s_5\}$ is optimal such that forms both minimal-global-offensive-alliance-neutrosophic number and minimal-global-offensive-alliance number for $\mathcal{G}$;

(*iii*) $S_8 = \{s_3, s_4, s_5\}$ only forms minimal-global-offensive-alliance number but not minimal-global-offensive-alliance-neutrosophic for $\mathcal{G}$;

(*iv*) $N = \{s_1, s_2\}$ isn't global-offensive alliance. Since there is three instances and only one instance is enough for $\mathcal{G}$;

(*a*) First counterexample for the statement "$N = \{s_1, s_2\}$ is global-offensive alliance."for $\mathcal{G}$;

$$\exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| = 2 = 2 = |N_s(s_3 \cap (V \setminus N)|$$
$$\exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| = 2 \not> 2 = |N_s(s_3) \cap (V \setminus N)|$$
$$\exists s_3 \in V \setminus N,\ |N_s(s_3) \cap N| \not> |N_s(s_3) \cap (V \setminus N)|;$$

(*b*) second counterexample for the statement "$N = \{s_1, s_2\}$ is global-offensive alliance." for $\mathcal{G}$;

$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 2 = 2 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| = 2 \not> 2 = |N_s(s_4) \cap (V \setminus N)|$$
$$\exists s_4 \in V \setminus N,\ |N_s(s_4) \cap N| \not> |N_s(s_4) \cap (V \setminus N)|;$$

(*c*) third counterexample for the statement "$N = \{s_1, s_2\}$ is global-offensive alliance." for $\mathcal{G}$.

$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| = 2 = 2 = |N_s(s_5) \cap (V \setminus N)|$$
$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| = 2 \not> 2 = |N_s(s_5) \cap (V \setminus N)|$$
$$\exists s_5 \in V \setminus N,\ |N_s(s_5) \cap N| \not> |N_s(s_5) \cap (V \setminus N)|;$$

(*v*) $\Gamma_s = 4$ and corresponded set is $S_3 = \{s_1, s_2, s_5\}$ for $\mathcal{G}$;

(*vi*) $\Gamma = 3$ and corresponded sets are $S_1 = \{s_1, s_2, s_3\}, S_2 = \{s_1, s_2, s_4\}, S_3 = \{s_1, s_2, s_5\}, S_4 = \{s_1, s_3, s_4\}, S_5 = \{s_1, s_3, s_5\}, S_6 = \{s_2, s_3, s_4\}, S_7 = \{s_2, s_3, s_5\}, S_8 = \{s_3, s_4, s_5\}$ which are only minimal-global-offensive alliances for $\mathcal{G}$.

4.34 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study.

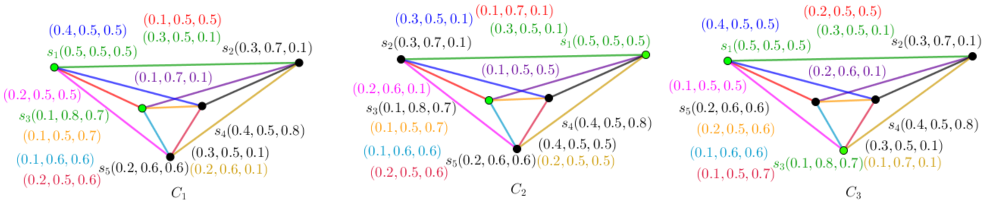

Figure 4.18: The set of black circles is minimal-global-offensive alliance.

NTG11

Notion concerning alliance is defined in neutrosophic graphs. Neutrosophic number is also introduced. Thus,

Question 4.34.1. *Is it possible to use other types neighborhood arising from different types of edges to define new alliances?*

Question 4.34.2. *Are existed some connections amid different types of alliances in neutrosophic graphs?*

Question 4.34.3. *Is it possible to construct some classes of which have "nice" behavior?*

Question 4.34.4. *Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?*

Problem 4.34.5. *Which parameters are related to this parameter?*

Problem 4.34.6. *Which approaches do work to construct applications to create independent study?*

Problem 4.34.7. *Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?*

4.35 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted.
This study uses one definition concerning global powerful alliance to study neutrosophic graphs. New neutrosophic number is introduced which is too close to the notion of neutrosophic number but it's different since it uses all values as type-summation on them. The connections of vertices which are clarified by general edges differ them from each other and put them in different categories to represent a set which is called global powerful alliance. Further studies could be about changes in the settings to compare this notion amid different settings of neutrosophic graphs theory. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table (4.6), some limitations and advantages of this study are pointed out.

Table 4.6: A Brief Overview about Advantages and Limitations of this study

tbl2c

Advantages	Limitations
1. Defining Global Powerful Alliances	1. General Results
2. Applying on Strong Neutrosophic Graphs	
3. Study on Complete Models	2. Study On Classes
4. Applying on Individuals	
5. Applying on Family	3. Same Models in Family

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1 [1] Henry Garrett, "*Global Offensive Alliance in Strong Neutrosophic Graphs*", Preprints 2022, 2022010429 (doi: 10.20944/preprints202201.0429.v1).

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3 [3] Henry Garrett, "*Three Types of Neutrosophic Alliances based on Connectedness and (Strong) Edges*", Preprints 2022, 2022010239 (doi: 10.20944/preprints202201.0239.v1).

This book is based on neutrosophic graph theory which is designed to study different types of coloring in that graphs to get new ideas and new results. The results concern specific classes of neutrosophic graphs. New notions are defined in the comparable structures on these models to understand the behaviors of these models according to the notions.

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