Metric Number In Dimension

By Henry Garrett

WORD COUNT	23923	TIME SUBMITTED	24-MAR-2021 04:33PM
		PAPER ID	70330310

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Henry Garrett

Independent Reseracher

DrHenryGarrett@gmail.com

Twitter's ID: @DrHenryGarrett | ©B08PDK8J5G

Abstract

In this outlet, I've devised the concept of relation amid two points where these points are coming up to make situation which in that the set of objects are greed to represent the story of how to be in whatever situations when these two points have the styles of being everywhere in the highlights of the concept which are coming from the merits of these points where are eligible to make capable situation to overcome every situation when they're participant in the hugely diverse situations which mean too styles of graphs with have the name or the general results for the general situation as possible as are.

Keywords: Metric, Dimension, named graphs, unnamed graphs
AMS Subject Classification: 05C17, 05C22, 05E45, 05E14

1 Preliminary On The Concept

I'm going to refer to some books which are cited to the necessary and sufficient material which are covering the introduction and the preliminary of this outlet so look [**Ref.** [1], **Ref.** [2], **Ref.** [3], **Ref.** [4]] where **Ref.** [1] is about the textbook, **Ref.** [2] is common, **Ref.** [3] has good ideas and **Ref.** [4] is kind of disciplinary approaches in the good ways. Approach for solving problems is an obvious selection for doing research and analysis the situation which may elicit the vague perspectives which we want not to be for extracting creative and new ideas which we want to be. I simultaneously study two models. This study is based both research and discussion which the author thinks that may be useful for understanding and growing our fantasizing and reality together. The aim of this expository book is to present recent developments in the centuries-old discussion on the interrelations between several types of domination in graphs. However, the novelty even more prominent in the newly discovered simplified presentations of several older results. Domination can be seen as arising from real-world application and extracting classical results as first described by this article.The main part of this article, concerning a new domination and older one, is presented in a narrative that answers two classical questions: (i) To what extend must closing set be dominating? (ii) How strong is the assumption of domination of a closing set? In a addition, we give an overview of the results concerning domination. The problem asks how small can a subset of vertices be and contain no edges or, more generally how can small a subset of vertices be and contain other ones. Our work was as elegant as it was unexpected being a departure from the tried and true methods of this theory that had dominated the field for one fifth a century. This expository article covers all previous definitions. The inability of previous definitions in solving even one case of real-world problems due to the lack of simultaneous attentions to the worthy both of vertices and edges causing us to make the new one. The concept of domination in a variety of graphs models such as crisp, weighted and fuzzy, has been in a spotlight. We turn our attention to sets of vertices in a fuzzy graph that are so close to all vertices, in a variety of ways, and study minimum such sets and their cardinality. A natural way to introduce and motivate our subject is to view it as a real-world problem. In its most elementary form, we consider the problem of reducing waste of time in transport planning. Our goal here is to first describe the previous definitions and the results, and then to provide an overview of the flows ideas in their articles. The final outcome of this article is twofold: (i) Solving the problem of reducing waste of time in transport planning at static state; (ii) Solving and having a gentle discussions on problem of reducing waste of time in transport planning at dynamic state. Finally, we discuss the results concerning holding domination that are independent of fuzzy graphs. We close with a list of currently open problems related to this subject. Most of our exposition assumes only familiarity with basic linear algebra, polynomials, fuzzy graph theory and graph theory.

In this study, author analyzes the structure of domination in t?norm fuzzy graphs and its special case when using Tmin, as fuzzy graphs.

L.A. Zadeh introduced the concept of a fuzzy subset of a set as a way for representing uncertainty. Zadeh?s ideas stirred the interest of researchers worldwide. His ideas have been applied to a wide range of scientific areas. Theoretical mathematics has also been touched by the notion of a fuzzy subset. In 1965, Zadeh published his seminal paper ?fuzzy sets? which described fuzzy set theory and consequently fuzzy logic. The purpose of Zadeh?s paper was to develop a theory which could deal with ambiguity and imprecision of certain classes or sets in Human thinking, particularly in the domains of pattern recognition, communication of information, and observation. This theory proposed making the grade of membership of an element in a subset of a universal set a value in the closed interval [0, 1] of real numbers. Zadeh?s idea have found applications in computer science, artificial intelligence, decision analysis, information science, system science, control engineering, expert systems, pattern recognition, management science, operations research, and robotics. Theoretical mathematics has also been touched by fuzzy set theory. In the classical set theory introduced by Cantor, values of elements in a set are either 0 or 1. That is for any element, there are only two possibilities: the element is the set or it is not. Therefore, Cantor set theory cannot handle data with ambiguity and uncertainty. The ideas of fuzzy set theory have been introduced into topology, abstract algebra, geometry, graph theory, and analysis. Analytical representation of physical phenomena can be fruitful as models of reality, but are sometimes difficult to understand because they do not explain much by themselves, and may remain unclear to the non-specialist. In other words, Zadeh proposed fuzzy theory and introduced fuzzy set theory which can be considered as the phenomenon of ambiguity across all systems displaying this property and its consequences. Graph theory is one of the branches of modern mathematics having experienced a most impressive development in recent years. The origin of graph theory can be traced back to Euler?s work on the Konigsberg bridge problem (1735) which subsequently led to the concept of an Eulerian graph. The first text book on graph theory was written by D?enesKonig and published in 1936. A later text book by Frank Harary published in 1968, was enormously popular and enabled mathematicians, chemists, electrical engineers and social scientists to have common platform to dialogue with each other. Graphs are represented graphically by taking a set of points on the plane and it is desired to find some structure among the points in the form of edges containing a subset of the pair of points. Graph theory plays a vital role as far as application side is concerned. Graph theory is intimately related to many branches of mathematics including group theory, matrix theory, numerical analysis, probability, topology and combinatorics because of its diagrammatic representation and its intuitive and aesthetic appeal. One of the most interesting problems in graph theory is that of Domination Theory. The earliest ideas of dominating sets are found in the classical problems of covering chess board with minimum number of chess pieces. Nowadays domination theory ranks top among the most prominent areas of research in graph theory and combinatorics. The concept of domination in graphs, with its many variations, is now well studied in graph theory. The book by Chartrand and Lesniak includes a chapter on dom- ination. For a more thorough study of domination in graphs, see Haynes et al.. The current list of papers on domination has over 1200 entries.The theory of domination is formalized by Clauge Berge in his book ?Theory of graphs and its application? (1962). Berge mentions the strategies of keeping a number of locations under surveillance, by a set of radar station. Oystein Ore was the first person to use the term domination number in his book on Graph Theory. The theory of domination has been the nucleus of research activity in graph theory in recent times. The fastest growing area within graph theory is a study of domination and related subset problems such independence, covering, matching, decomposition and labelling. Domination boasts a host of applications to social network theory, land surveying, game theory, interconnection network, parallel computing and image processing and so on. Today, this theory gained popularity and remains as a major area of research due to the contributions of O.Ore, C.Berge, E.J.Cockayne, S.T.Hedetniemi, T.W.Haynes, R.C.Laskar, P.J.Slater, V.R.Kulli, E.Sampathkumar, S.Arumugam. Fuzzy graph theory has numerous applications in various fields like clustering analysis, database theory, network analysis, information theory, etc. Fuzzy models can be used in problems handling uncertainty to get more accurate and precise solutions. As in graphs, connectivity concepts play a key role in ap- plications related with fuzzy graphs. The fuzzy definition of fuzzy graphs was proposed by Kaufmann, from the fuzzy relations introduced by Zadeh. Although Rosenfeld introduced another elaborated definition, including fuzzy vertex and fuzzy edges. Fuzzy graphs were introduced by Rosenfeld and Yeh and Bang independently in 1975. Rosenfeld in his paper ?Fuzzy Graphs? presented the basic structural and connectivity concepts while Yeh and Bang introduced different connectivity parame- ters of a fuzzy graph and discussed their applications in the paper titled ?Fuzzy relations, Fuzzy graphs and their applications to clustering analysis?. Rosenfeld considered fuzzy relations on fuzzy sets and developed the structure of fuzzy graphs, obtaining analogues of several graph theoretical concepts. He introduced and examined such concepts as paths, connectedness and clusters, bridges, cut vertices, forests and trees. Fuzzy graphs introduced by Rosenfeld are finding an increasing number of applica- tions in modelling real time systems where the level of information inherent in the system varies with different levels of precision. Fuzzy models are becoming useful because of their aim in reducing the difference between the traditional numerical models used in engineering and sciences and the symbolic models used in expert systems and AI. After the pioneering work of Rosenfeld and Yeh and Bang in 1975, when some basic fuzzy graph theoretic concepts and applications have been indicated, several authors have been finding deeper results, and fuzzy analogues of many other graph theoretic concepts. This include fuzzy trees, fuzzy line graphs, operations on fuzzy graphs, automorphism of fuzzy graphs,

fuzzy interval graphs, cycles and cocycles of fuzzy graphs, and metric aspects in fuzzy graphs. Bhutani and Rosenfeld have introduced the concept of strong arcs. Different parameters like sum distance in fuzzy graphs and chromatic number of fuzzy graphs were discussed. The work on fuzzy graphs was also done by Akram, Samanta, Nayeem, Pramanik, Rashmanlou and Pal. P.Bhattacharya discussed some properties of fuzzy graphs and introduced the notion of eccentricity and centre in fuzzy graphs.

K.R.Bhutani introduced the concept of complete fuzzy graphs and concluded that a complete fuzzy graph has no cut nodes. Xu applied connectivity parameters of fuzzy graphs to problems in chemical structures. The concept of domination in fuzzy graphs was investigated by A.Somasundaram and S.Somasundaram. A.Somasundaram presented the concepts of independent domination, total domination, connected domination and domination in cartesian products and composition of fuzzy graphs. Somasundaram and Somasundaram discussed domination in fuzzy graphs. They defined domination using effective edges in fuzzy graph. Nagoorgani and Chandrasekharan defined domination in fuzzy graphs using strong arcs. Manjusha and Sunitha discussed some concepts in domination and total domination in fuzzy graphs using strong arcs. A. Selvam Avadayappan, G.Mahadevan, A. Mydeenbibi, T.A. Sub- ramanian, A. Nagarajan, A. Rajeswari have studied the problem of obtaining an upper bound for the sum of a domination parameter and a graph theoretic parameter and characterized the corresponding extremal graphs. Motivated by the notion of dominating sets and their applicability, we focused on introducing some dominating parameters in fuzzy graph theory. For fuzzification of the following problems, types of nodes (based on advantages) and types of connection with nodes can be assigned by different values. So the question is based on based on values on nodes and ratio of total values of adjacent ?-strong connections to total of values of adjacent connections? Chess enthusiasts in Europe considered the problem of determining the minimum number of queens that can be placed on a chess board so that all the squares are either attacked by a queen or occupied by a queen. Harary et al. explained an interesting application in voting situations using the concept of domination. A number of strategic locations are to be kept under observations. One of the important areas of applications of domination is communication network, where a dominating set represents a set of cities which, acting as transmitting stations, can transmit messages to every city in the network. Another area of application of domination is voting situations. Suppose the commander of the Army Postal services plans to set up a few post offices in an important region with minimum number of post offices to control the whole region. Now-a-day almost all schools operate school buses for trans- porting children to and from schools. Among many points, three important points to be noted are 1. The running time of a bus between school and its terminus. 2. Maximum number of students in a bus at any one time and 3. The maximum distance a student has to walk to board a school bus. Consider a computer network modeled by a 4-cube. The vertices of the 4-cube represents computers and edges represent direct communication link between two computers. So, in this model we have 16 computers or processors to which it is directly connected. The problem is to collect information from all processors and we like to do it relatively often and relatively fast. So we identify a small set of

processors called collecting processors and ask each processor to send its information to one of a small set of collecting processors. We assume that at most a one-unit delay between the time a processor sends its information and time it arrives at a nearest collector is allowed. So, we have to find an dominating set among the set of a processors. Consider the problem of locating a single fire station, police station or a similar such service facility to serve the communities. Also, we would like to locate such a service facility in one of these communities and not at an arbitrary point along the road, due to some reasons. Let Pn be a set of points in general position on the plane. The unit distance graph UDG(Pn) associated to Pn is a graph whose vertex set consists of the elements of Pn, two of which are connected if they are at distance at most one. Unit distance graphs are used to model various types of wireless networks, including cellular networks, sensor networks, ad-hoc networks and others in which the nodes represent broadcast stations with a uniform broadcast range we shall refer to networks that can be modeled using unit distance graphs as unit distance wireless networks, abbreviated as UDW networks. We first briefly illustrate our opinion. Domination are among the most fundamental concepts of graph theory. Also, domination can behave in many strange ways. For instance, besides the classical definitions of domination, there are many characterization of this concept. One of this characterization due to A. Somasundaram and S. Somasundaram, see also Refs. for further generalizations. One the contrary and quite surprisingly, there are nowhere these definitions Solving the problem of reducing waste of time in transport planning and also (separately) all others real-world problems. Somehow, a key direction of study of domination deals with trying to provide a clear structure of what the dominating set of vertices looks like. The leading theme of this expository article is to discuss the following two questions concerning fuzzy graphs

Q1: How much closing does dominating imply?

Q2: How much dominating does closing imply?

They will be addressed. The main narrative presented in these sections is independent of any results from graph theory and/or calculus. The purpose of this expository article is to provide an overview of the authors? recent series of work, in which a positive answer to the problem of reducing waste of time in transport planning for the our new definition is given. Consider a set of cities connected by communication paths, Which cities is connected to others by roads? We face with a graph model of this situation. But the cities are not same and they have different privileges in low traffic levels and this events also occur for the roads in low-cost levels. So we face with the weighted graph model, at first. These privileges are not crisp but they are vague in nature. So we don?t have a weighted graph model. In other words, we face with a fuzzy graph model, which must study the concept of domination on it. Next we turn our attention to sets of vertices in a fuzzy graph G that are close to all vertices of G, in a variety of ways, and study minimum such sets and their cardinality. In 1998, the concept of effective domination in fuzzy graphs was introduced by A. Somasundaram and S. Somasundaram as the classical problems of covering chess board with minimum number of chess pieces. In 2010, the concept of 2-strong(weak) domination in fuzzy graphs was introduced by

C. Natarajan and S.K. Ayyaswamy as the extension of strong (weak) domination in crisp graphs. In 2014, the concept of 1-strong domination in fuzzy graphs was introduced by O.T. Manjusha and M.S. Sunitha as the extension of domination in fuzzy graphs with strong edges. In 2015, the concept of 2-domination in fuzzy graphs was introduced by A. Nagoor Gani and K. Prasanna Devi as the extension of 2-domination in crisp graphs. In 2015, the concept of strong domination in fuzzy graphs was introduced by O.T. Manjusha and M.S. Sunitha as reduction of the value of old domination num- ber and extraction of classic results. In 2016, the concept of (1,2)?domination in fuzzy graphs was introduced by N. Sarala and T. Kavitha as the extension of (1,2)?domination in crisp graphs. A few researchers studied other domination variations which are based on above definitions. So we only compare our new definition with the fundamental dominations.

This problem was mentioned by Ore. According to the rules of chess a queen can, in one move, advance any number of squares horizontally, diagonally, or vertically (assuming that no other chess figure is on its way). How to place a minimum number of queens on a chessboard so that each square is controlled by at least one queen? See one of the solutions in (Fig. ??). For fuzzification of this problem, types of square (based on sensitive place in game of chess, chess pieces) and types of connection can be assigned by different values. So the question is changed to this. How to place a number of queens on a chessboard so that each square is controlled by at least one queen based on values on queens and ratio of total of values of adjacent ?-strong connections to total of values of adjacent connections? Locating Radar Stations Problem The problem was discussed by Berge. A number of strategic locations are to be kept under surveil- lance. The goal is to locate a radar for the surveillance at as few of these locations as possible. How a set of locations in which the radar stations are to be placed can be determined? For fuzzification of this problem, types of radar stations (based on power of them) and types of connection with locations can be assigned by different values. So the question is changed to this. How a set of locations in which the radar stations are to be placed can be determined based on values on radar stations and ratio of total of values of adjacent ?-strong connections to total of values of adjacent connections? Problem of Communications in a Network Suppose that there is a network of cities with communication links. How to set up transmitting sta- tions at some of the cities so that every city can receive a message from at least one of the transmitting stations? This problem was discussed in detail by Liu. For fuzzification of this problem, types of cities (based on population, structure) and types of connection with cities can be assigned by different val- ues. So the question is changed to this. How to set up transmitting stations at some of the cities so that every city can receive a message from at least one of the transmitting stations based on values on cities and ratio of total of values of adjacent ?-strong connections to total of values of adjacent connections? Nuclear Power Plants Problem A similar known problem is a nuclear power plants problem. There are various locations and an arc can be drawn from location x to location y if it is possible for a watchman stationed at x to observe a warning light located at y. How many guards are needed to observe all of the warning lights, and where should they be located? For fuzzification of this problem, types of guards (based on abilities) and types of connection with guards can be assigned by different values. So the question is changed to this. How many guards are needed to observe all of the warning lights, and where should they be located based on values on guards and ratio of total of values of adjacent ?-strong connections to total of values of adjacent connections? At present, domination is considered to be one of the fundamental concepts in graph theory and its various applications to ad hoc networks, biological networks, distributed computing, social networks and web graphs partly explain the increased interest. Such applications usually aim to select a subset of nodes that will provide some definite service such that every node in the network is ?close? to some node in the subset. The following examples show when the concept of domination can be applied in modelling real-life problems. Modelling Biological Networks Using graph theory as a modelling tool in biological networks allows the utilization of the most graph- ical invariants in such a way that it is possible to identify secondary RNA (Ribonucleic acid) motifs numerically. Those graphical invariants are variations of the domination number of a graph. The results of the research carried out show that the variations of the domination number can be used for correctly distinguishing among the trees that represent native structures and those that are not likely candidates to represent RNA. For fuzzification of this problem, types of location (based on advantages) and types of connection with locations can be assigned by different values. So the question is based on based on values on locations and ratio of total of values of adjacent ?-strong connections to total of values of adjacent connections? Modelling Social Networks Dominating sets can be used in modelling social networks and studying the dynamics of relations among numerous individuals in different domains. A social network is a social structure made of individuals (or groups of individuals), which are connected by one or more specific types of interde- pendency. The choice of initial sets of target individuals is an important problem in the theory of social networks. In the work of Kelleher and Cozzens, social networks are modelled in terms of graph theory and it was shown that some of these sets can be found by using the properties of dominating sets in graphs. For fuzzification of this problem, types of people (based on abilities) and types of connection with people can be assigned by different values. So the question is based on based on values on people and ratio of total of values of adjacent ?-strong connections to total of values of adjacent connections? Facility Location Problems The dominating sets in graphs are natural models for facility location problems in operational re- search. Facility location problems are concerned with the location of one or more facilities in a way

that optimizes a certain objective such as minimizing transportation cost, providing equitable ser- vice to customers and capturing the largest market share. For fuzzification of this problem, types of location (based on advantages) and types of connection with locations can be assigned by different values. So the question is based on based on values on locations and ratio of total of values of adjacent ?-strong connections to total of values of adjacent connections? Coding Theory The concept of domination is also applied in coding theory as discussed by Kalbfleisch, Stanton and Horton and Cockayne and Hedetniemi. If one defines a graph, the vertices of which are the n- dimensional vectors with coordinates chosen from $\{1, ..., p\}, p > 1$, and two vertices are adjacent if they differ in one coordinate, then the sets of vectors which are (n, p)-covering sets, single error cor- recting codes, or perfect covering sets are all dominating sets of the graph with determined additional properties. For fuzzification of this problem, types of codes (based on types of words, different words, same words) and types of connection with codes can be assigned by different values. So the question is based on based on values on codes and ratio of total of values of adjacent ?-strong connections to total of values of adjacent connections? Multiple Domination Problems An important role is played by multiple domination. Multiple domination can be used to construct hierarchical overlay networks in peer-to-peer applications for more efficient index searching. The hier- archical overlay networks usually serve as distributed databases for index searching, e.g. in modern file sharing and instant messaging computer network applications. Dominating sets of several kinds are used for balancing efficiency and fault tolerance as well as in the distributed construction of minimum spanning trees. Another good example of direct, important and quickly developing application of multiple domination in modern computer networks is a wireless sensor network. A wireless sensor net- work (WSN) usually consists of up to several hundred small autonomous devices to measure some physical parameters. Each device contains a processing unit and a limited memory as well as a radio transmitter and a receiver to be able to communicate with its neighbors. Also, it contains a limited power battery and is constrained in energy consumption. There is a base station, which is a special sensor node used as a sink to collect information gathered by other sensor nodes and to provide a connection between the WSN and a usual network. A routing algorithm allows the sensor nodes to self-organize into a WSN. As stated, an important goal in WSN design is to maximize the functional lifetime of a sensor network by using energy efficient distributed algorithms, networking and routing techniques. To maximize the functional lifetime, it is important to select some sensor nodes to be- have as a backbone set to support routing communications. The backbone set can be considered as a dominating set in the corresponding graph. Dominating sets of several different kinds have proved to be useful and effective for modelling backbone sets. In the recent literature, particular attention has been paid to construction of k-connected k-dominating sets in WSNs, and several probabilistic and deterministic approaches have been proposed and analyzed. The backbone set of sensor nodes should be selected as small as possible and, on the other hand, it should guarantee high efficiency and

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total of values of adjacent connections? A homogeneous WSN consists of wireless sensor devices of the same kind. All the devices have the same set of limited resources and, originally, no hierarchy is imposed on the network structure and communications. In a network of this kind, the only special sensor node is a base station. For all the other nodes, it is necessary to construct and switch the backbone sets and communications efficiently so that all the network nodes stay in operation as long as possible. Therefore, in this case, it is important to be able to construct and switch dominating sets and route communications uniformly and efficiently with respect to the energy consumption of each particular sensor node. This has to be done to optimize the functional lifetime of the whole network. Usually, a WSN is mathematically modelled as a unit or quasi-unit disk graph. These are the most natural and general graph models for a WSN. In a unit disk graph model, nodes correspond to sensor locations in the Euclidean plane and are assumed to have identical (unit) transmission ranges. An edge between two nodes means that they can communicate directly, i.e. the distance between them is at most one. A survey of known results on unit disk graphs, including algorithms for constructing dominating sets, can be found. A quasi-unit disk graph model takes into consideration possible trans- mission obstacles and is much closer to reality: we are sure to have an edge between two nodes if the distance between them is at most a parameter d, 0 ¡ d ¡ 1. If the distance between two nodes is in the range from d to 1, the existence of an edge is not specified. A description of several more restricted geometric graph models for WSN design, e.g. the related neighborhood graph, Gabriel graph, Yao graph etc., can be found. Domination is an area in graph theory with an extensive research activity. A book by Haynes, Hedet- niemi and Slater on domination published in 1998 lists 1222 articles in this area.

Short Scrutiny on Background

We introduce a new variation on the domination theme. These concepts are definitely interesting in the context of networks, as mentioned, the realization that networks are ?everywhere?, is fundamental to our modern lives. It becomes even more important now that algorithms are becoming more and more ?prevalent? in everything, too. The mathematical background of this domination are related to other theoretical concepts of fuzzy graphs, more than old definitions. Some applications, from the real-world problems, are better modeled with this definition other than old ones. In one applications, optimization of transport routes occurs such that the acceptable parts are higher than on others. In the other application, reducing waste of time in transportation planning is caused by analyzing data of its fuzzy graph model. From the transport properties, comparison of cities can be better modeled. So we can assign assets usefully or change the infrastructures of transport for reducing waste of time. We hope these concepts are useful for studying problems of mathematics and real-world which make the future better as possible. At present, domination is considered to be one of the fundamental concepts in graph theory and its various applications to ad hoc networks, biological networks, distributed computing, social networks and web graphs partly explain the increased interest. t-norm fuzzy graphs are the vast subject which have the fresh topics and many applications from the real-world problems that make the future better. So we defined domination which is a strong tools for analyzing data, on t-norm fuzzy graphs, for the first time. We hope this concept is useful for studying theoretical topics and applications on t-norm fuzzy graphs. Graph theory is one of the branches of modern mathematics having experienced a most impressive development in recent years. One of the most interesting problems in graph theory is that of Domination Theory. Nowadays domination theory ranks top among the most prominent areas of research in graph theory and combinatorics. The theory of domination has been the nucleus of research activity in graph theory in recent times. The fastest growing area within graph theory is a study of domina- tion and related subset problems such independence, covering, matching, decomposition and labelling. Domination boasts a host of applications to social network theory, land surveying, game theory, interconnection network, parallel computing and image processing and so on. Today, this theory gained popularity and remains as a major area of research. At present, domination is considered to be one of the fundamental concepts in graph theory and its various applications to ad hoc networks, biological networks, distributed computing, social networks and web graphs partly explain the increased interest. More than 1200 papers already published on domination in graphs. Without a doubt, the literature on this subject is growing rapidly, and a considerable amount of work has been dedicated to find different bounds for the domination numbers of graphs. However, from practical point of view, it was necessary to define other types of dominations. Most of these new variations required the dominating set to have additional properties. In 1965, Zadeh published his seminal paper ?fuzzy sets? as a way for representing uncertainty. In 1975, fuzzy graphs were introduced by Rosenfeld and Yeh and Bang independently as fuzzy models

which can be used in problems handling uncertainty. In 1998, the concept of domination in fuzzy graphs was introduced by A. Somasundaram and S. Somasundaram as the classical problem of cov- ering chess board with minimum number of chess pieces. They defined domination in fuzzy graph by using effective edges. The works on domination in fuzzy graphs were also done such as domination, strong domination, (1, 2)-vertex domination, 2-domination, connected domination, total domination, Independent domination, Complementary nil domination, Efficient domination, strong (weak) domi- nation and etc. In 1965, Zadeh published his seminal paper ?fuzzy sets? as a way for representing uncertainty. In 1975, fuzzy graphs were introduced by Rosenfeld and Yeh and Bang independently as fuzzy models which can be used in problems handling uncertainty. Domination as a theoretical area in graph theory was formalized by Berge in 1958, in the chapter 4 with title ? The fundamental Numbers of the theory of Graphs? (Theorem 7, p.40) and Ore (Chapter 13 , pp. 206, 207) in 1962. Since 1977, when Cockayne and Hedetniemi (Section 3, p. 249-251) presented a survey of domination results, domination theory has received considerable attention. A set S of vertices of G (Chap. 10, p. 302) is a dominating set if every vertex in V (G) ? S is adjacent to at least one vertex in S. The minimum cardinality among the dominating sets of G is called the domination number of G and is denoted by ?(G). A dominating set of cardinality ?(G) is then referred to as minimum dominationg set. Dominating sets appear to have their origins (Example 2, p. 41) in the game of chess, where the goal is to cover or dominate various squares of a chessboard by certain chess pieces. Consider a set of cities connected by communication paths, Which cities is connected to others by roads? We face with a graph model of this situation. But the cities are not same and they have different privileges in low traffic levels and this events also occur for the roads in low-cost levels. So we face with the weighted graph model, at first. These privileges are not crisp but they are vague in nature. So we don?t have a weighted graph model. In other words, we face with a fuzzy graph model, which must study the concept of domination on it. Next we turn our attention to sets of vertices in a fuzzy graph G that are close to all vertices of G, in a variety of ways, and study minimum such sets and their cardinality. In 1998, the concept of effective domination in fuzzy graphs was introduced by A. Somasundaram and S. Somasundaram as the classical problems of covering chess board with minimum number of chess pieces. In 2010, the concept of 2-strong(weak) domination in fuzzy graphs was introduced by C. Natarajan and S.K. Ayyaswamy as the extension of strong (weak) domination in crisp graphs. In 2014, the concept of 1-strong domination in fuzzy graphs was introduced by O.T. Manjusha and M. S. Sunitha as the extension of domination in fuzzy graphs with strong edges. In 2015, the concept of 2- domination in fuzzy graphs was introduced by A. Nagoor Gani and K.

Prasanna Devi as the extension of 2-domination in crisp graphs. In 2015, the concept of strong domination in fuzzy graphs was intro- duced by O.T. Manjusha and M.S. Sunitha as reduction of the value of old domination number and extraction of classic results. In 2016, the concept of (1, 2)?domination in fuzzy graphs was introduced by N. Sarala and T. Kavitha as the extension of (1, 2)?domination in crisp graphs. A few researchers studied other domination variations which are based on above definitions, e.g. connected domination, total domination, Independent domination, Complementary nil domination, Efficient domination. So

we only compare our new definition with the fundamental dominations. In a world of uncertainty, where systems are aligned in a complicated and unsuitable manner, a tradi- tional mathematical tool with its strict boundaries of truth and falsity has not implanted itself with capability of reflecting the reality. When the convolution of the real life system increases, the human ability to make scrupulous and yet significant statement about its conduct decreases. However, if a threshold is reached, precision and significance become practically exclusive characteristics in a mutual manner. As a result, our concern with the discernment of problems and efforts of solutions are of a different order than in the past. As we become aware of how much we know and how much we do not know, information and uncertainty themselves become the focus of our concern. This uncertainty will be of particular interest, leading to a different way of giving structure to the point set, known as fuzzy set.

Short Discernment on Tools

At first, we compare our new definition with previous definitions about domination in fuzzy graphs. We do this comparison on constructing both of ?number? and ?set? by attention to mathematical concepts and applications. Finally, we give mathematical definitions together some examples which are used them. From the mathematical aspects, being equivalent the ?-strong arcs with the bridges, cause which we use ?-strong arcs for constructing a ?-strong dominating set. The bridges have deeply concepts and various results in fuzzy graph theory due to their definition which show that they are important arcs. These arcs are also related to many important concepts of other fuzzy graphs areas, e.g. fuzzy forest, fuzzy tress, fuzzy cut node, fuzzy cut arcs and etc. Definition of this concept state that those arcs are changing of strength of connectedness which is very important from theoretical and applicational aspects. Because the sensitive roads are effective. These roads will change any decision about transportation in reality. These arcs are definitely interesting in the context of networks, the realization that networks are everywhere is fundamental to our modern lives. It becomes even more important now that algorithms are becoming more and more prevalent in everything too. Speaking of understanding proteins is a an example. Analyzing networks, e.g. molecular networks, food web network, protein interactions network, facebook and other dense networks, for the realization of networks are ?everywhere?. From social networks such as facebook, the world wide web and the internet to the complex interactions between proteins in the cells of your bodies, we face the challenge of understanding their structure and developments. We are also interesting in the research works in new technologies that can make the future or make the future better as possible. In reality, if we have a set of cities, then those have various roads which have various types of both of qualities and numbers. Quality of locations is different. We use ?bridges? which are sensitive paths for ?constructing the set? of dominating locations and also use ?quality of locations? together quality of sensitive paths and all paths for ?constructing the number? of domination of locations on other locations. In other words, we construct a new fuzzy graph from previous fuzzy graph model by assigning a new values with respect to summation their initial values with a fraction from values of sensitive roads to values of all roads. We want to decrease the costs. So this number must be the minimum, i.e. we must use the locations and the roads which have the less values as possible for selection of the set of interesting locations. The membership function ? on the node set of G can be constructed from the statistical data that represents value of cities with respect to population, locations of stations, facilities of stations, speed of doing works, number of stations, weather and climate, uniques properties, available different roads, number of passengers in different seasons, solving special requirements of passengers, busy time and etc. The membership function ? on the arc set of G can be constructed from the statistical data with respect to less number of crime, accidents, beauty of the roads, suitable weather, lower raining, lower block of the road, lower road events e.g. falling stones, lower snowing, high numbers of less raining days, lower number of warming days, number of emergency locations in the roads, high security in events, quality of facilities in events, lower number of block of the road due to bad weather. Now the terms ?lower, high, less, beauty, busy, quality? are vague in nature. Thus we get a fuzzy graph

model. It is interesting to note that a road is of some city to next city and a path contains some roads. Now, we opt some roads which have a highest privilege between other paths. In our terminology, we call these roads by ?-strong arcs. If these roads deleted, the maximum privilege of all paths decrease between two cities. Thus we pay attentions to these special roads. Every city outside of the set of special cities must be connected to at least one special cities by the special road. For constructing the number of this fuzzy model, we assign to each special cities, a new privilege which is obtained from summation its previous privilege with amount of power of privilege of special roads to others. Finally, we opt the set which summation of privilege of its cities are the minimum. We call it by vertex dominating set. We also get a number which state other presentation of this fuzzy model with respect to privileges of cities, privileges of all roads and privileges of all special roads. This number is called by vertex domination number. Now, we will bring the old definitions which serves as a foundation of the rest comparison with the newest. The comparison between old definitions and our new definition about domination in fuzzy graphs can be discussed by structures of terms ?dominating set?, and ?domination number?. Dominating set.: The structure of ?dominating set? only depend on the type arc which is used in constructing it. We use the type of arc which is equivalent with bridge. This type of arc in comparison to other type arcs which are used in old definition, is more useful from mathematical and applicational perspective as mentioned in the first of this section. Hence these problems cause motivation for us to changing the type of arc which construct ?dominating set?. Domination number.: ?Domination number? are introduced in old definitions, based on ei- ther the values of nodes or the values of arcs, however we defined the domination number by both of value of nodes and value of arcs. In old definitions, either the values of locations or the values of path is considered, however these parameters is simultaneously affected on any decision as mentioned in the first of this section. The many variables can be defined for the junctions in planning transportation e.g. Generation variables: Occupation in the res- idence area, Population, Residential space, Population density, Number of households, Car ownership rate, Average price of one square meter of land, Students population, Traffic zone space, Number of residential buildings, Distance to entertainment complexes. Attraction vari- ables: Occupation in the working area, Business/ Administrative/Agricultural/Industrial Land Space, Administrative building space, Number of Administrative/Business/Industrial Build- ings, Schools? space, Number of Students/Schools/Classes, Number of Universities/Students, Number of Retailers, Number and Capacity of Cinemas/Mosques/ Exhibitions, Parks/Hospi- tals. These variables can be positive or negative. Vertex weight of a node in a fuzzy graph can be useful. Another privilege of this definition can be another modeling of the situation. We can assign a new value to every junctions by its vertex weight. Now, we have a new fuzzy graph model. In this model, the roads have no values but value of junctions is more useful

in transportation planning. We can pay attentions to the cities which have higher value, for assigning assets, optimization of their routes, or planning of travel and transportation. So this motivated us to improve the definition of ?domination number?. Applications.: Reducing waste of time in transportation planning and optimization of trans- port routes are examples of importance of these concepts as mentioned. A case study on optimization of transport routes is as follows. A bicyclist may prefer a route where the ac- ceptable parts (for this study, acceptable: the output value is over 0.5; above the average; the all parts of route are ?-strong by literatures of this research work.) are higher than on other route-referring to Route 2. Hence this study case illustrates the importance of choice of roads type, ?-strong arcs, which are introduced as the acceptable parts of route in this case. Another ones, Reducing waste of time in transportation planning by using the concept of vertex domination. It is well known and generally accepted that the problem of determining the domination number of an arbitrary graph is a difficult one. Because of this, researchers have turned their attention to the study of classes of graphs for which the domination problem can be solved in polynomial time.

2 Results Of New Concepts

Definition 2.1. Hugely diverse situations are said to be named graphs or are called for the unnamed graphs.

Definition 2.2. In the hugely diverse situations,

- An object is said to be in the set of **Dread-greed-hunted set (DGHS)** when 'tis called for differentiating amid some couple points where are just done by the object.
- In the special case, differentiating means having different distance from the intended object so this object is gross to located the mentioned couple.

Proposition 2.3. *Every Complete Graph has* ***Dread-greed-hunted set (DGHS)*** *including all random selection of points with just only exception one point.*

Proof. Gross. □

Definition 2.4. In the hugely diverse situations,

- A line is said to be
- In the special case, differentiating means having different distance from the intended object so this object is gross to located the mentioned couple.

We provide some basic background for the paper in this section.

Definition 2.5. A binary operation $\otimes : [0,1] \times [0,1] \to [0,1]$ is a ***t*-norm** if it satisfies the following for $x, y, z, w \in [0,1]$:

1. $1 \otimes x = x$
2. $x \otimes y = y \otimes x$
3. $x \otimes (y \otimes z) = (x \otimes y) \otimes z$
4. If $w \leq x$ and $y \leq z$ then $w \otimes y \leq x \otimes z$

We concern with a t-norm fuzzy graph which is defined on a crisp graph. So we recall some basic concepts of crisp graph.

A *graph* G is a finite nonempty set of objects called *vertices* (the singular is *vertex*) together with a (possibly empty) set of unordered pairs of distinct vertices of G called *edges*. The *vertex set* of G is denoted by $V(G)$, while the *edge set* is denoted by $E(G)$.

We recall that a *fuzzy subset* of a set S is a function of S into the closed interval $[0, 1]$.

We lay down the preliminary results which recall some basic concepts of fuzzy graph. A *fuzzy graph* in is denoted by $G = (V, \sigma, \mu)$ such that $\mu(\{x, y\}) \leq \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$ where V is a vertex set, σ is a fuzzy subset of V, μ is a fuzzy relation on V and $\wedge$ denote the minimum. We call σ the fuzzy vertex set of G and μ the fuzzy edge set of G, respectively. We consider fuzzy graph G with no loops and assume that V is finite and nonempty, μ is reflexive (i.e., $\mu(\{x, x\}) = \sigma(x)$, for all x) and symmetric (i.e., $\mu(\{x, y\}) = \mu(\{y, x\})$, for all $x, y \in V$). In all the examples σ and μ is chosen suitably. In any fuzzy graph, the underlying crisp graph is denoted by $G^* = (V, E)$ where V and E are domain of σ and μ, respectively. The fuzzy graph $H = (\tau, \nu)$ is called a *partial fuzzy subgraph* of $G = (\sigma, \mu)$ if $\nu \subseteq \mu$ and $\tau \subseteq \sigma$. Similarly, the fuzzy graph $H = (\tau, \nu)$ is called a fuzzy subgraph of $G = (V, \sigma, \mu)$ induced by P in if $P \subseteq V$, $\tau(x) = \sigma(x)$ for all $x \in P$ and $\nu(\{x, y\}) = \mu(\{x, y\})$ for all $x, y \in P$. For the sake of simplicity, we sometimes call H a fuzzy subgraph of G. We say that the partial fuzzy subgraph (τ, ν) spans the fuzzy graph (σ, μ) if $\sigma = \tau$. In this case, we call (τ, ν) a spanning fuzzy subgraph of (σ, μ).

For the sake of simplicity, we sometimes write xy instead of $\{x, y\}$

A *path* P of length n in is a sequence of distinct vertices $u_0, u_1, \cdots, u_n$ such that $\mu(u_{i-1}, u_i) > 0, i = 1, 2, \cdots, n$ and the degree of membership of a weakest edge is defined as its strength. If $u_0 = u_n$ and $n \geq 3$ then P is called a cycle and P is called a *fuzzy cycle*, if it contains more than one weakest edge. The strength of a cycle is the strength of the weakest edge in it. The *strength of connectedness* between two vertices x and y in is defined as the maximum of the strengths of all paths between x and y and is denoted by $\mu^\infty_G(x, y)$.

A fuzzy graph $G = (V, \sigma, \mu)$ is *connected* in if for every x, y in V, $\mu^\infty_G(x, y) > 0$.

Definition 2.6. Let $G = (V, E)$ be a graph. Let σ be a fuzzy subset of V and μ be a fuzzy subset of E. Then (σ, μ) is called a **fuzzy subgraph** of G with respect to a t-norm $\otimes$ if for all $uv \in E, \mu(uv) \leq \sigma(u) \otimes \sigma(v)$.

Let k be a positive integer. Define $\mu^k(u, v) = \vee\{\mu(uu_1) \otimes \cdots \otimes \mu(u_{n-1}v)|P : u = u_0, u_1, \cdots, u_{n-1}, u_n = v$ is a path of length k from u to v $\}$ Let $\mu^\infty(u, v) = \vee\{\mu^k(u, v)|k \in \mathbb{N}\}$ where $\mathbb{N}$ denotes the positive integers.

In this section, we provide the main results.

Definition 2.7. Let $G = (\sigma, \mu)$ be a t-norm fuzzy graph with respect to a t-norm $\otimes$. Let $uv \in E$. We call that uv is α-strong edge if $\mu(uv) > \mu^\infty_{G-uv}(u, v)$.

Definition 2.8. Let $G = (\sigma, \mu)$ be a t-norm fuzzy graph with respect to a t-norm $\otimes$. Let $x, y \in V$. We say that x **dominates** y in G as α-strong if the edge $\{x, y\}$ is α-strong.

Definition 2.9. Let $\otimes$ be a t-norm. Let (σ, μ) be a t-norm fuzzy graph with respect to $\otimes$. A subset S of V is called a α**-strong dominating set** in G if for every $v \notin S$, there exists $u \in S$ such that u dominates v as α-strong.

Definition 2.10. Let $G = (\sigma, \mu)$ be a t-norm fuzzy graph with respect to a t-norm $\otimes$. Let S be the set of all α-strong dominating sets in G. The **vertex domination number** of G is defined as $\min_{D \in S}[\Sigma_{u \in D}(\sigma(u) + \frac{d_s(u)}{d(u)})]$ and it is denoted by $\gamma_{\mathbf{v}}(\mathbf{G})$. If $d(u) = 0$, for some $u \in V$, then we consider $\frac{d_s(u)}{d(u)}$ equal with 0. The α-strong dominating set that is correspond to $\gamma_v(G)$ is called by **vertex dominating set**. We also say $\Sigma_{u\in D}(\sigma(u)+\frac{d_s(u)}{d(u)})$, **vertex weight** of D, for every $D \in S$ and it is denoted by $\mathbf{w_v(D)}$.

Definition 2.11. Let (σ,μ) be a fuzzy graph with respect to $\otimes$. Then (σ,μ) is said to be complete with respect to $\otimes$, if for all $u,v\in V, \mu(uv)=\sigma(u)\otimes\sigma(v)$.

Proposition 2.12. *Let $G=(\sigma,\mu)$ be a complete fuzzy graph with respect to $\otimes$. Then*

1. $\mu^{\infty}_{\otimes}(u,v)=\mu(uv), \forall u,v\in V$

2. G has no cutvertices.

Corollary 2.13. *A complete t-norm fuzzy graph with respect to $\otimes$ is α-strong edgeless.*

Proof. Let (σ,μ) be a complete t-norm fuzzy graph with respect to $\otimes$. For all $u,v\in V$, $\mu^{\infty}_{\otimes}(u,v)=\mu(u,v)$ by Proposition (3.13). So for all $u,v\in V$, $\mu'^{\infty}_{\otimes}(u,v)\geq\mu(u,v)$. Hence uv is not α-strong edge. The result follows. □

It is well known and generally accepted that the problem of determining the domination number of an arbitrary graph is a difficult one. Because of this, researchers have turned their attention to the study of classes of graphs for which the domination problem can be solved in polynomial time.

Proposition 2.14 (Complete t-norm fuzzy graph)**.** *Let $G=(\sigma,\mu)$ be a complete t-norm fuzzy graph with respect to $\otimes$. Then $G=K_n$, $\gamma_v(K_n)=p$.*

Proof. Since $G=(\sigma,\mu)$ be a complete t-norm fuzzy graph with respect to $\otimes$, none of edges are α-strong by Corollary (3.14). so we have

$$\gamma_v(G)=\min_{D\in S}[\Sigma_{u\in D}\sigma(u)]=\Sigma_{u\in v}\sigma(u)=p$$

by Definition (2.10). Hence we can write $\gamma_v(K_n)=p$ by our notations. □

Proposition 2.15 (Empty t-norm fuzzy graph)**.** *Let $G=(\sigma,\mu)$ be a t-norm fuzzy graph with respect to a t-norm $\otimes$. Then $\gamma_v(G)=p$, if G be edgeless, i.e $G=\bar{K}_n$.*

Proof. Since G is edgeless, Hence V is only α-strong dominating set in G and none of arcs are α-strong. so we have $\gamma_v(G)=p$ by Definition (2.10). In other words, $\gamma_v(\bar{K}_n)=p$ by our notations. □

It is interesting to note the converse of Proposition (3.17) that does not hold.

Definition 2.16. A t-norm fuzzy graph G with respect to a t-norm $\otimes$ is said **bipartite**, if the vertex set V can be partitioned into two nonempty sets V_1 and V_2 such that $\mu(v_1v_2)=0$ if $v_1,v_2\in V_1$ or $v_1,v_2\in V_2$. Moreover, if $\mu(uv)=\sigma(u)\otimes\sigma(v)$ for all $u\in V_1$ and $v\in V_2$ then G is called a **complete bipartite *t*-norm fuzzy graph** and is denoted by K_{σ_1,σ_2}, where σ_1 and σ_2 are respectively the restrictions of σ to V_1 and V_2. In this case, If $|V_1|=1$ or $|V_2|=1$ then a complete bipartite t-norm fuzzy graph is said a **star *t*-norm fuzzy graph** which is denoted by $K_{1,\sigma}$.

Proposition 2.17. *A complete bipartite t-norm fuzzy graph is α-strong edgeless.*

Proof. Let $G=(\sigma,\mu)$ be a complete bipartite t-norm fuzzy graph with respect to a t-norm $\otimes$. Let $u\in V_1, v\in V_2$. the strength of path P from u to v is of the form $\sigma(u)\otimes\cdots\otimes\sigma(v)\leq\sigma(u)\otimes\sigma(v)=\mu(uv)$. So $\mu^{\infty}_{\otimes}(u,v)\leq\mu(uv)$. uv is a path from u to v such that $\mu(u,v)=\sigma(u)\otimes\sigma(v)$. So $\mu^{\infty}_{\otimes}(u,v)\geq\mu(uv)$. Hence $\mu^{\infty}_{\otimes}(u,v)=\mu(uv)$. So $\mu'^{\infty}_{\otimes}(u,v)\geq\mu(uv)$ that induce uv is not α-strong edge. The result follows. □

Corollary 2.18. *A star t-norm fuzzy graph has no α-strong edges.*

Proof. Obviously, the result is hold by using Proposition (3.19). ☐

Proposition 2.19 (Star t-norm fuzzy graph). *Let $G = (\sigma, \mu)$ be a star t-norm fuzzy graph with respect to a t-norm $\otimes$. Then $G = K_{1,\sigma}$ and $\gamma_v(K_{1,\sigma}) = \sigma(u)$ where u is center of G.*

Proof. Let $G = (\sigma, \mu)$ be a star t-norm fuzzy graph with respect to a t-norm $\otimes$. Let $V = \{u, v_1, v_2, \cdots, v_n\}$ such that u and v_i are center and leaves of G, for $i \leq n$, respectively. The edge $uv_i, 1 \leq i \leq n$ is omly path between u and v_i. So $\{u\}$ is vertex dominating set in G. G is α-strong edgeless by Corollary (3.21). So $\gamma_v(K_{1,\sigma}) = \sigma(u)$. ☐

Proposition 2.20 (Complete bipartite t-norm fuzzy graph). *Let $G = (\sigma, \mu)$ be a star t-norm fuzzy graph with respect to a t-norm $\otimes$ which is not star t-norm fuzzy graph. Then $G = K_{\sigma_1,\sigma_2}$ and $\gamma_v(K_{\sigma_1,\sigma_2}) = \min_{u \in V_1, v \in V_2}(\sigma(u) + \sigma(v))$.*

Proof. Let $G \neq K_{1,\sigma}$ be complete bipartite t-norm fuzzy graph with respect to $\otimes$. Then both of V_1 and V_2 include more than one vertex. In K_{σ_1,σ_2}, none of edges are α-strong by Proposition (3.19). Also, each vertex in V_1 is adjacent with all vertices in V_2 and conversely. Hence in K_{σ_1,σ_2}, the α-strong dominating sets are V_1 and V_2 and any sets containing 2 vertices, one in V_1 and other in V_2. Hence $\gamma_v(K_{\sigma_1,\sigma_2}) = \min_{u \in V_1, v \in V_2}(\sigma(u) + \sigma(v))$. So the proposition is proved. ☐

Definition 2.21 (**Ref.** [1], Definition 3.2., p.131). Let (σ, μ) be a fuzzy graph with respect to $\otimes$. Let $xy \in E$. Then xy is called a **bridge** if $\mu'^{\infty}_{\times}(u, v) < \mu^{\infty}_{\otimes}(u, v)$ for some $u, v \in V$, where $\mu'(xy) = 0$ and $\mu' = \mu$ otherwise.

Theorem 2.22 (**Ref.** [1], Theorem 3.3., p.132). *Let (σ, μ) be a fuzzy graph with respect to $\otimes$. Let $xy \in E$. Let μ' be the fuzzy subset of E such that $\mu'(xy) = 0$ and $\mu' = \mu$ otherwise. Then (3) $\Rightarrow$ (2) $\Leftrightarrow$ (1) :*

(1) *xy is a bridge with respect to $\otimes$;*

(2) *$\mu'^{\infty}_{\otimes}(x, y) < \mu(xy)$;*

(3) *xy is not a weakest edge of any cycle.*

Corollary 2.23. *Let $G = (\sigma, \mu)$ be a t-norm fuzzy graph with respect to $\otimes$. Let $xy \in E$. xy is a α-strong edge if and only if xy is a bridge.*

Proof. Obviously, The result is hold by Theorem (3.23). ☐

Definition 2.24 (**Ref.** [1], Definition 3.2., p.133). Let (σ, μ) be a fuzzy graph with respect to $\otimes$. Then an edge uv is said to be **effective**, if $\mu(uv) = \sigma(u) \otimes \sigma(v)$.

Proposition 2.25 (**Ref.** [1], proposition 3.10., p.133). *Let (σ, μ) be a fuzzy graph with respect to $\otimes$. If the edge uv is effective, then $\mu(uv) = \mu^{\infty}_{\otimes}(u, v)$.*

Corollary 2.26. *Let (σ, μ) be a fuzzy graph with respect to $\otimes$. If the edge uv is effective, then uv is not α-strong.*

Proof. Let uv be a edge of (σ, μ). So $\mu(uv) = \mu^{\infty}_{\otimes}(u, v)$ by Proposition (2.25). Hence $\mu(uv) \leq \mu^{\infty}_{\otimes}(u, v)$. It means the edge uv is not α-strong. ☐

Remark 2.27. A (crisp) graph that has no cycles is called **acyclic** or a **forest**. A connected forest is called a **tree**. A fuzzy graph is called a **forest** if the graph consisting of its nonzero edge is a forest and a **tree** if this graph is also connected. We call the fuzzy graph $G = (\sigma, \mu)$ a **fuzzy forest** if it has a partial fuzzy spanning subgraph which is a forest, where for all edges xy not in $F[\nu(xy) = 0]$, we have $\mu(xy) < \nu^{\infty}(x, y)$. In other words, if xy is in G, but not F, there is a path in F between x and y whose strength is greater than $\mu(xy)$. It is clear that a forest is a fuzzy forest.

Definition 2.28. Let $\otimes$ be a t-norm. A fuzzy graph (σ, μ) is a **fuzzy tree** with respect to $\otimes$. If (σ, μ) has a partial fuzzy spanning subgraph $F = (\tau, \nu)$ which is a tree and $\forall xy$ not in F, $\mu(xy) < \nu^{\infty}_{\otimes}(x, y)$.

Theorem 2.29. *Let $G = (\sigma, \mu)$ be a fuzzy forest with respect to $\otimes$. Then rhe edges of $F = (\tau, \nu)$ are just the bridges of G.*

Corollary 2.30. *Let $G = (\sigma, \mu)$ be a fuzzy tree with respect to $\otimes$. Then the edges of $F = (\tau, \nu)$ are just the α-strong edges of G.*

Proof. Obviously, the results follows by Theorem (3.32) and Corollary (2.23). □

Proposition 2.31. *Let $T = (\sigma, \mu)$ be a fuzzy tree with respect to $\otimes$. Then $D(T) = D(F) \cup D(S)$, where $D(T), D(F)$ and $D(S)$ are vertex dominating sets of T, F and S, respectively. S is a set of vertices which has no edge with connection to F.*

Proof. By Corollary (3.34), the edges of $F = (\tau, \nu)$ are just the α-strong edges of G. So the result follows by using Definition (2.28). □

According to some applications of t-norm fuzzy graph increasing numbers of people from Asia and Africa are seeking to enter the US illegally over the Mexican border. The vast majority of immigrants detained were from the Americas. However, a significant number were from Asian and African countries. We can obtain vertex dominating set by α-strong connections between these countries and vertex domination. In other words, We can find the countries which dominate others as α-strong from many countries which are increasing and they have a significant number. So We can study the main illegal immigration routes to the United States precisely, usefully and deeply.

Many various using of this new-born fuzzy model for solving real-world problems and urgent requirements involve introducing new concept for analyzing the situations which leads to solve them by proper, quick and efficient method based on statistical data. This gap between the model and its solution cause that we introduce nikfar domination in neutrosophic graphs as creative and effective tool for studying a few selective vertices of this model instead of all ones by using special edges. Being special selection of these edges affect to achieve quick and proper solution to these problems. Domination hasn't ever been introduced. So we don't have any comparison with another definitions. The most used graphs which have properties of being complete, empty, bipartite, tree and like stuff and they also achieve the names for themselves, are studied as fuzzy models for getting nikfar dominating set or at least becoming so close to it. We also get the relations between this special edge which plays main role in doing dominating with other special types of edges of graph like bridges. Finally, the relation between this number with other special numbers and characteristic of graph like order are discussed.

Neutrosophy as a newly-born science is a branch of philosophy that studies the origin, nature and scope of neutralities.

In 1965, Zadeh introduced "fuzzy set" by the concept of degree of truth membership In 1986, Atanassow introduced "intuitionistic fuzzy set" by adding the concept of degree of false membership to the fuzzy set In 1995, Smarandache introduced "neutrosophic set" by adding the concept of degree of indeterminate membership to the intuitionistic fuzzy set There are three different types of definitions of a neutrosophic graph Broumi et al. and Shah-Hussain introduced two different definitions of neutrosophic graph by generalizations of intuitionistic fuzzy graph Akram and shahzadi introduced neutrosophic graph by using concept of neutrosophic set They also highlighted some flaws in the definitions of Broumi et al. and Shah-Hussain. They introduced some counterexamples which state the complement of a neutrosophic graph isn't always a neutrosophic graph by using Shah-Hussain's definition of neutrosophic graph and we even have much bad situations if we used Broumi et al.'s definition of neutrosophic graph beacuse of not only we don't have complement of a neutrosophic graphs but also we don't have join of them. Moreover, they introduced binary operations cartesian product, composition, union, join, cross, lexicographic, strong product and unary operation complement along with proofs which show these operations hold neutrosophic property of graphs . In other words, the new graph is produced by these operations, is also a neutrosophic graph.

Regarding these points, we use the definition of Akram and Shahzadi as the main framework for our own study. The study behaviors of modeling is of spotlight by using few parameters. Some parameters are so close to others one. if we defined being "so close" concept properly by adding some extra properties more than existence of edge between them, we would achieve the useful tool. This tool would cause solving real-world problems by deleting useless data and focusing on a few one. This leads to the concept of domination in modeling. Domination hasn't ever been introduced on any kind of neutrosophic graphs. Regarding these points, the aim of this paper is to introduce the notion of domination in this new-born fuzzy model. It is a normal question about effects of dominations in neutrosophic graphs. From here comes the main motivation for this and in this regard, we have considered some routine and fundamental framework for studying this concept.

Domination as a theoretical area in graph theory was formalized by Berge in 1958, in the chapter 4 with title " The fundamental Numbers of the theory of Graphs" and Ore in 1962. Since 1977, when Cockayne and Hedetniemi presented a survey of domination results, domination theory has received considerable attention. A set S of vertices of G is a *dominating set* if every vertex in $V(G) - S$ is adjacent to at least one vertex in S. The minimum cardinality among the dominating sets of G is called the *domination number* of G and is denoted by $\gamma(G)$. A dominating set of cardinality $\gamma(G)$ is then referred to as *minimum dominationg set*. Dominating sets appear to have their origins in the game of chess, where the goal is to cover or dominate various squares of a chessboard by certain chess pieces.

We provide some basic background for the paper in this section.

Definition 2.32. Let V be a given set. The function $A : V \to [0, 1]$ is called a *fuzzy set* on V.

Definition 2.33. (Neutrosophic Set)

Let V be a given set. A *neutrosophic set* A in V is characterized by a truth membership function $T_A(x)$, an indeterminate membership function $I_A(x)$ and a false membership function $F_A(x)$. The functions $T_A(x), I_A(x)$, and $F_A(x)$ are fuzzy sets on V. That is, $T_A(x) : V \to [0, 1], I_A(x) : V \to [0, 1]$ and $F_A(x) : V \to [0, 1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Remark 2.34. Some special notations frequently appear in this paper. In what follows, we introduce them. Let V be a given set. For the sake of simplicity, we only use the notation E for the representation of the following set on V.
$E \subseteq \{A|A \subseteq V, |A| = 2$ It means A has only two elements$\}$, where $|A|$ means cardinality of A. By Analogous to this points, the notation E_i is corresponded to V_i.

Definition 2.35. (Neutrosophic Graph)

Let V be a given set. Also, assume E be a given set with respect to V. A neutrosophic graph is a pair $G = (A, B)$, where $A : V \to [0, 1]$ is a neutrosophic set in V and $B : E \to [0, 1]$ is a neutrosophic set in E such that

$$T_B(xy) \leq min\{T_A(x), T_A(y)\},$$

$$I_B(xy) \leq min\{I_A(x), I_A(y)\},$$

$$F_B(xy) \leq max\{F_A(x), F_A(y)\},$$

for all $\{x, y\} \in E$. V is called vertex set of G and E is called edge set of G, respectively.

Definition 2.36. (Complete Neutrosophic Graph)
Let $G = (A, B)$ be a neutrosophic graph on a given set V. G is called complete if the following conditions are satisfied:

$$T_B(xy) = min\{T_A(x), T_A(y)\},$$

$$I_B(xy) = min\{I_A(x), I_A(y)\},$$

$$F_B(xy) = max\{F_A(x), F_A(y)\},$$

for all $\{x, y\} \in E$.

Definition 2.37. (Empty Neutrosophic Graph)
Let $G = (A, B)$ be a neutrosophic graph on a given set V. G is called empty if the following conditions are satisfied:

$$T_B(xy) = I_B(xy) = F_B(xy) = 0.$$

for all $\{x, y\} \in E$.

Definition 2.38. (Bipartite Neutrosophic Graph)
Let V be a given set. A neutrosophic graph $G = (A, B)$ on V is said *bipartite* if the set V can be partitioned into two nonempty sets V_1 and V_2 such that $T_B(xy) = I_B(xy) = F_B(xy) = 0$. for all $\{x, y\} \in E_1$. or $\{x, y\} \in E_2$. Moreover, if $T_B(xy) = min\{T_A(x), T_A(y)\}, I_B(xy) = min\{I_A(x), I_A(y)\}, F_B(xy) = max\{F_A(x), F_A(y)\}$, for all $\{x, y\} \in E$ then G is called a *complete bipartite neutrosophic graph*. In this case, If either $|V_1| = 1$ or $|V_2| = 1$ then the complete bipartite neutrosophic graph is said a *star neutrosophic graph*.

Definition 2.39. (Order)
Let $G = (A, B)$ be a neutrosophic graph on a given set V. Then the real number p is called the

[a.] *T-order*, if $p = \gamma_v(G)_T = \Sigma_{u\in V} T_A(u)$.*I-order*, if $p = \gamma_v(G)_I = \Sigma_{u\in V} I_A(u)$. *F-order*, if $p = \gamma_v(G)_F = \Sigma_{u\in V} F_A(u)$. *order*, if was be either of $T-$order, $I-$order, and $F-$order.

Definition 2.40. (Bridge)
Let $G = (A, B)$ be a neutrosophic graph on a given set V. Then an edge xy in G is called the

a. *T-bridge*, if the strengths of each T-path P from x to y, not involving xy, were less than $T_B(xy)$.

b. *I-bridge*, if the strengths of each T-path P from x to y, not involving xy, were less than $T_B(xy)$.

c. *F-bridge,* if the strengths of each T-path P from x to y, not involving xy, were less than $T_B(xy)$.

d. *bridge,* if it was either of $T-$bridge, $I-$bridge, and $F-$bridge.

Definition 2.41. (Acyclic)
Let $G = (A, B)$ be a neutrosophic graph on a given set V. Then G is called the

a. *T-acyclic,* if there wasn't a T-path P from x to y, with only exception $x = y.$, for all $x \in V$.

b. *I-acyclic,* if there wasn't a I-path P from x to y, with only exception $x = y.$, for all $x \in V$.

c. *F-acyclic,* if there wasn't a F-path P from x to y, with only exception $x = y.$, for all $x \in V$.

d. *acyclic,* if it was either of $T-$acyclic, $I-$acyclic, and $F-$acyclic.

Definition 2.42. (Spanning Neutrosophic Graph)
Let $G = (A, B), G_1 = (A_1, B_1)$ be a neutrosophic graph on a given set V. Then G_1 is called the *spanning neutrosophic graph* of G if $V = V_1$ but $E_1 \subseteq E$.

Definition 2.43. (Forest)
Let $G = (A, B)$ be a neutrosophic graph on a given set V. Then G is called the

a. *T-forest,* if G was T-acyclic and there is a spanning neutrosophic graph F such that for all edge xy out of F, there is a T-path P from x to y, how whose strength greater than $T_B(xy)$.

b. *I-forest,* if G was I-acyclic and there is a spanning neutrosophic graph F such that for all edge xy out of F, there is a I-path P from x to y, how whose strength greater than $I_B(xy)$.

c. *F-forest,* if G was F-acyclic and there is a spanning neutrosophic graph F such that for all edge xy out of F, there is a F-path P from x to y, how whose strength greater than $F_B(xy)$.

d. *forest,* if it was either of neutrosophic $T-$forest, neutrosophic $I-$forest, and neutrosophic $F-$forest.

Definition 2.44. (Tree)
Let $G = (A, B)$ be a neutrosophic graph on a given set V. Then G is called the

a. *T-tree,* if G was a T-forest such that there is a T-path P from x to y, for all $x, y \in V$.

b. *I-tree,* if G was a I-forest such that there is a I-path P from x to y, for all $x, y \in V$.

c. *F-tree,* if G was a F-forest such that there is a F-path P from x to y, for all $x, y \in V$.

d. *tree,* if it was either of $T-$tree, $I-$tree, and $F-$tree.

Remark 2.45. Let V be a given set. For the sake of simplicity, we only use the notation F, p for the representation special spanning neutrosophic graph of a forest and the order a given neutrosophic graph. By Analogous to this points, the notation F_i, p_i are corresponded to G_i. Let us remind you consider three special notations in this paper by three letters. In other words, we have three correspondences for a given set, neutrosophic graph and a forest, we mean p, E_i and F_i are corresponded to G_i, V_i and G_i, respectively. Final remark is of about writing xy instead of $\{x, y\}$.

Definition 2.46. (Path)
Let $G = (A, B)$ be a neutrosophic graph on V and v_0, v_n be two given vertices such that $n \in \mathbb{N}$. Then

a. A distinct sequence of vertices $P : v_0, v_1, \cdots, v_n$ in G is called a *T-path* of length n from v_0 to v_n, if $T_B(v_iv_{i+1}) > 0$, for $i = 0, 1, \cdots, n-1$. The $\min_{i=0}^{n-1}\{T_B(v_iv_{i+1})\}$ is called the *strength* of this $T-$path and is denoted by $\mu_G(P)_T$.

b. A distinct sequence of vertices $P : v_0, v_1, \cdots, v_n$ in G is called a *I-path* of length n from v_0 to v_n, if $I_B(v_iv_{i+1}) > 0$, for $i = 0, 1, \cdots, n-1$. The $\min_{i=0}^{n-1}\{I_B(v_iv_{i+1})\}$ is called the *strength* of this $I-$path and is denoted by $\mu_G(P)_I$.

c. A distinct sequence of vertices $P : v_0, v_1, \cdots, v_n$ in G is called a *F-path* of length n from v_0 to v_n, if $F_B(v_iv_{i+1}) < 1$, for $i = 0, 1, \cdots, n-1$. The $\min_{i=0}^{n-1}\{F_B(v_iv_{i+1})\}$ is called the *strength* of this $F-$path and is denoted by $\mu_G(P)_F$.

d. A distinct sequence of vertices $P : v_0, v_1, \cdots, v_n$ in G is called a *path* of length n from v_0 to v_n, if it be $T-$path, $I-$path, and $F-$path, simultaneously. In this case, the $\min\{\mu_G(P)_T, \mu_G(P)_I, \mu_G(P)_F\}$ is called *strength* of path and is denoted by $\mu_G(P)$.

Definition 2.47. (Strength between Two Vertices)
Let $G = (A, B)$ be a neutrosophic graph on V and v_i, v_j be two given vertices such that $i > j$ and $i, j \in \mathbb{N}$. Then

a. The $\max\{\mu_G(P)_T\}$ in G is called the *T-strength* between v_i and v_j and is denoted by $\mu^\infty_G(v_i, v_j)_T$.

b. The $\max\{\mu_G(P)_I\}$ in G is called the *I-strength* between v_i and v_j is denoted by $\mu^\infty_G(v_i, v_j)_I$.

c. The $\max\{\mu_G(P)_F\}$ in G is called the *F-strength* between v_i and v_j is denoted by $\mu^\infty_G(v_i, v_j)_F$.

d. The $\max\{\mu^\infty_G(v_i, v_j)_T, \mu^\infty_G(v_i, v_j)_I, \mu^\infty_G(v_i, v_j)_F\}$ is called the *strength* between v_i and v_j in G and is denoted by $\mu^\infty_G(v_i, v_j)$.

Remark 2.48. $\mu^\infty_{G-\{xy\}}(x, y)$ is the strength between x and y in the neutrosophic graph obtained from G by deleting the edge xy. This is as the same for the notations $\mu^\infty_{G-\{xy\}}(x, y)_T, \mu^\infty_{G-\{xy\}}(x, y)_I$, and $\mu^\infty_{G-\{xy\}}(x, y)_F$.

In what follows, we will define four properties for edges. Based of these properties, we can construct various kinds of dominations in neutrosophic graphs.

Definition 2.49. (Effective Edges)
Let $G = (A, B)$ be a neutrosophic graph on V. Then An edge xy in G is called the

a. *T-effective*, if $T_B(xy) > \mu^\infty_{G-\{xy\}}(x, y)_T$.

b. *I-effective*, if $I_B(xy) > \mu^\infty_{G-\{xy\}}(x, y)_I$.

c. *F-effective*, if $F_B(xy) > \mu^\infty_{G-\{xy\}}(x, y)_F$.

d. *effective*, if it be either of $T-$effective, $I-$effective, and $F-$effective.

Let $G = (A, B)$ be a neutrosophic graph on V as figure ??. In the following table, we study the properties of edges. For example, v_2v_5 has not neither of $T-$effective, $I-$effective, $F-$effective, and effective property. The edge v_3v_4 has both of $T-$effective and $I-$effective property. So it is also effective edge. The edges

$\{v_1v_4, v_2v_4, v_3v_4, v_4v_5\}, \{v_1v_3, v_3v_4\}, \{v_1v_3, v_1v_4, v_2v_4\}$, and $\{v_1v_3, v_1v_4, v_2v_4, v_3v_4, v_4v_5\}$ have $T-$effective, $I-$effective, $F-$effective, and effective property, respectively. $\{v_2v_5, v_1v_2\}$ has no ones.

Edges \ Properties	$T-$effective	$I-$effective	$F-$effective	Effective
v_1v_2	×	×	×	×
v_1v_3	×	√	√	√
v_1v_4	√	×	√	√
v_2v_4	√	×	√	√
v_2v_5	×	×	×	×
v_3v_4	√	√	×	√
v_4v_5	√	×	×	√

Definition 2.50. (Nikfar Domination)
Let $G = (A, B)$ be a neutrosophic graph on V and $x, y \in V$. Then

a. We say that x *dominates* y in G as *T-effective*, if the edge xy be T-effective. A subset S of V is called the *T-effective dominating set* in G, if for every $v \in V - S$, there is $u \in S$ such that u dominates v as T-effective. The *T-nikfar weight* of x is defined by $w_v(x)_T = T_A(x) + \frac{\Sigma_{xy \text{ is a T-effective edge}} T_B(xy)}{\Sigma_{xy \text{ is a edge}} T_B(xy)}$. If $\Sigma_{xy \text{ is a edge}} T_B(xy)$, for some $x \in V$. Then we consider $\frac{\Sigma_{xy \text{ is a T-effective edge}} T_B(xy)}{\Sigma_{xy \text{ is a edge}} T_B(xy)}$ equal with 0. For any $S \subseteq V$, a natural extension of this concept to a set, is as follows. We also say the *T-nikfar weight* of S, it is defined by $w_v(S)_T = \Sigma_{u \in S}(w_v(u)_T)$. Now, let U be the set of all T-effective dominating sets in G. The *T-nikfar domination number* of G is defined as $\gamma_v(G)_T = \min_{D \in U}(w_v(D)_T)$. The T-effective dominating set that is correspond to $\gamma_v(G)_T$ is called by *T-nikfar dominating set*.

b. We say that x *dominates* y in G as *I-effective*, if the edge xy be I-effective. A subset S of V is called the *I-effective dominating set* in G, if for every $v \in V - S$, there is $u \in S$ such that u dominates v as I-effective. The *I-nikfar weight* of x is defined by $w_v(x)_I = I_A(x) + \frac{\Sigma_{xy \text{ is a I-effective edge}} I_B(xy)}{\Sigma_{xy \text{ is a edge}} I_B(xy)}$. If $\Sigma_{xy \text{ is a edge}} I_B(xy)$, for some $x \in V$. Then we consider $\frac{\Sigma_{xy \text{ is a I-effective edge}} I_B(xy)}{\Sigma_{xy \text{ is a edge}} I_B(xy)}$ equal with 0. For any $S \subseteq V$, a natural extension of this concept to a set, is as follows. We also say the *I-nikfar weight* of S, it is defined by $w_v(S)_I = \Sigma_{u \in S}(w_v(u)_I)$. Now, let U be the set of all I-effective dominating sets in G. The *I-nikfar domination number* of G is defined as $\gamma_v(G)_I = \min_{D \in U}(w_v(D)_I)$. The I-effective dominating set that is correspond to $\gamma_v(G)_I$ is called by *I-nikfar dominating set*.

c. We say that x *dominates* y in G as *F-effective*, if the edge xy be F-effective. A subset S of V is called the *F-effective dominating set* in G, if for every $v \in V - S$, there is $u \in S$ such that u dominates v as F-effective. The *F-nikfar weight* of x is defined by $w_v(x)_F = F_A(x) + \frac{\Sigma_{xy \text{ is a F-effective edge}} F_B(xy)}{\Sigma_{xy \text{ is a edge}} F_B(xy)}$. If $\Sigma_{xy \text{ is a edge}} F_B(xy)$, for some $x \in V$. Then we consider $\frac{\Sigma_{xy \text{ is a F-effective edge}} F_B(xy)}{\Sigma_{xy \text{ is a edge}} F_B(xy)}$ equal with 0. For any $S \subseteq V$, a natural extension of this concept to a set, is as follows. We also say the *F-nikfar weight* of S, it is defined by $w_v(S)_F = \Sigma_{u \in S}(w_v(u)_F)$. Now, let U be the set of all F-effective dominating sets in G. The *F-nikfar domination number* of G is defined as $\gamma_v(G)_F = \min_{D \in U}(w_v(D)_F)$. The F-effective dominating set that is correspond to $\gamma_v(G)_F$ is called by *F-nikfar dominating set*.

d. We say that x *dominates* y in G as *effective*, if the edge xy be effective. A subset S of V is called the *effective dominating set* in G, if for every $v \in V - S$, there is $u \in S$ such that u dominates v as effective. We also say the *nikfar weight* of S, it is defined by $w_v(S) = \min\{w_v(S)_T, w_v(S)_I, w_v(S)_F\}$. Now, let U be the set of all effective dominating sets in G. The *nikfar domination number* of G is defined as $\gamma_v(G) = \min_{D \in U}(w_v(D))$. The effective dominating set that is correspond to $\gamma_v(G)$ is called by *nikfar dominating set*.

Let $G = (A, B)$ be a complete neutrosophic graph on a given set V such that there is exactly one path between two given vertices, which has

a. T-strength. Then $\gamma_v(G)_T = \min_{u \in V}(T_A(u)) + 1$.

b. I-strength. Then $\gamma_v(G)_I = \min_{u \in V}(I_A(u)) + 1$.

c. F-strength. Then $\gamma_v(G)_F = \min_{u \in V}(F_A(u)) + 1$.

d. strength. Then $\gamma_v(G) = \min_{u \in V}(T_A(u), I_A(u), F_A(u)) + 1$.

Proof. (a). Let $G = (A, B)$ be a neutrosophic graph on a given set V. The T-strength of path P from u to v is of the form $T_A(u) \wedge \cdots \wedge T_A(v) \leq T_A(u) \wedge T_A(v) = T_B(uv)$. So $\mu_G^\infty(u, v)_T \leq T_B(uv)$. uv is a path from u to v such that $T_B(uv) = T_A(u) \wedge T_A(v)$. Therefore $\mu_G^\infty(u, v)_T \geq T_B(uv)$. Hence $\mu_G^\infty(u, v)_T = T_B(uv)$. Then $T_B(uv) > \mu^\infty_{G-\{xy\}}((u, v)_T$. It means that the edge uv is T-effective. All edges are T-effective and each vertex is adjacent to all other vertices. So $D = \{u\}$ is a T-effective dominating set and $\Sigma_{xy \text{ is a T-effective edge}} T_B(xy) = \Sigma_{xy \text{ is a edge}} T_B(xy)$ for each $u \in V$. The result follows.

By analogues to the proof of (a), the result is obviously hold for (b), (c), and (d). ◻

Let $G = (A, B)$ be an empty neutrosophic graph on a given set V. Then $\gamma_v(G)_T = \gamma_v(G)_I = \gamma_v(G)_F = \gamma_v(G) = p$ where p denotes the order of G.

Proof. Let G be an empty neutrosophic graph on a given set V. Hence V is only T-effective dominating set in G and there is also no T-effective edge. So by Definition 2.50(a), we have $\gamma_v(G)_T = \min_{D \in S}[\Sigma_{u \in D} T_A(u)] = \Sigma_{u \in V} T_A(u) = p$. Therefore $\gamma_v(G)_T = p$.

By analogues to the proof of $\gamma_v(G)_T = p$ and Definition 2.50, the result is obviously hold for $\gamma_v(G)_I, \gamma_v(G)_F$ and $\gamma_v(G)$. ◻

It is interesting to note that the converse of Propositions 2, does not hold.

We show that the converse of Propositions 2, does not hold. Let $G = (\sigma, \mu)$ be a fuzzy graph as Figure 1. The edges $\{v_2v_5, v_2v_4, v_3v_4, v_1v_3\}$ are T-effective, I-effective,

Figure 1. nikfar domination

F-effective, and effective and the edges $\{v_1v_4, v_1v_2, v_4v_5\}$ are neither of types of being effective. So the set $\{v_2, v_3\}$ is all types of the effective dominating set. This set is also all types of nikfar dominating set in neutrosophic graph G. Hence $\gamma_v(G) = \gamma_v(G)_T = \gamma_v(G)_I = \gamma_v(G)_F = 1.75 + 0.9 + 0.7 = 3.35 = \Sigma_{u \in V} T(u) = \Sigma_{u \in V} I(u) = \Sigma_{u \in V} F(u) = p$. Therefore G isn't an empty neutrosophic graph but $\gamma_v(G) = \gamma_v(G)_T = \gamma_v(G)_I = \gamma_v(G)_F = p$. Let $G = (A, B)$ be the complete bipartite neutrosophic graph on a given set V such that there is exactly one path between two given vertices, which has

a. T-strength. Then $\gamma_v(G)_T$ is either $T_A(u) + 1$, $u \in V$ or $\min_{u \in V_1, v \in V_2}(T_A(u) + T_A(v)) + 2$.

b. I-strength. Then $\gamma_v(G)_I$ is either $I_A(u) + 1$, $u \in V$ or $\min_{u \in V_1, v \in V_2}(I_A(u) + I_A(v)) + 2$

c. F-strength. Then $\gamma_v(G)_F$ is either $F_A(u)+1,\ u\in V$ or $\min_{u\in V_1, v\in V_2}(F_A(u)+F_A(v))+2$.

d. strength. Then $\gamma_v(G)_T$ is either $\min(T_A(u), I_A(u), F_A(u))+1,\ u\in V$ or $\min_{u\in V_1, v\in V_2}(T_A(u)+T_A(v), I_A(u)+I_A(v), F_A(u)+F_A(v))+2$.

Proof. (a). Let $G=(A,B)$ be the complete bipartite neutrosophic graph on a given set V such that there is exactly one path which has T-strength between two given vertices. By analogues to the proof of Theorem 2, all the edges are T-effective.

If G be the star neutrosophic graph with $V=\{u, v_1, v_2, \cdots, v_n\}$ such that u and v_i are the center and the leaves of G, for $1\le i\le n$, respectively. Then $\{u\}$ is the T-nikfar dominating set of G. Hence $\gamma_v(G)_T = T_A(u)+1$.

Otherwise, both of V_1 and V_2 include more than one vertex. Every vertex in V_1 is dominated by every vertices in V_2, as T-effective and conversely. Hence in G, the T-effective dominating sets are V_1 and V_2 and any set containing 2 vertices, one in V_1 and other in V_2. So $\gamma_v(G)_T = \min_{u\in V_1, v\in V_2}(T_A(u)+T_A(v))+2$. The result follows.

By analogues to the proof of (a) and Definition 2.50, the result is obviously hold for (b), (c), and (d). □

Proposition 2.51. *Let $G=(A,B)$ be a neutrosophic graph on a given set V and $xy\in E$. xy is a*

a. *T-effective edge if and only if xy is a T-bridge.*

b. *I-effective edge if and only if xy is a I-bridge.*

c. *F-effective edge if and only if xy is a F-bridge.*

d. *effective edge if and only if xy is a bridge.*

Proof. (a). Let $G=(A,B)$ be a neutrosophic graph on a given set V and $xy\in E$.

Suppose xy is a T-effective edge. By Definition 2.49(a), $T_B(xy) > \mu^{\infty}_{G-\{xy\}}(x,y)_T$. So $T_B(xy)=\mu^{\infty}_G(x,y)_T$. Therefore $\mu^{\infty}_G(x,y)_T > \mu^{\infty}_{G-\{xy\}}(x,y)_T$. It means xy is a bridge.

Suppose xy is a bridge. So $\mu^{\infty}_G(x,y)_T > \mu^{\infty}_{G-\{xy\}}(x,y)_T$. Hence $T_B(xy)=\mu^{\infty}_G(x,y)_T$. By $\mu^{\infty}_G(x,y)_T > \mu^{\infty}_{G-\{xy\}}(x,y)_T$ and $T_B(xy)=\mu^{\infty}_G(x,y)_T$, $T_B(xy) > \mu^{\infty}_{G-\{xy\}}(x,y)_T$. By Definition 2.49(a), it means xy is a T-effective edge.

Therefore the result follows.

By analogues to the proof of (a) and Definition 2.50, the result is obviously hold for (b), (c), and (d). □

Proposition 2.52. *Let $G=(\sigma,\mu)$ be a tree on a given set V. Then the edges of $F=(\tau,\nu)$ are just*

a. *the T-bridges, I-bridges, F-bridges, and bridges of G.*

b. *the T-effective, I-effective, F-effective, and effective edges of G.*

c. *constructed from the vertices of $T-$effective, $I-$effective, $F-$effective, and effective dominating sets in G. Hence $\gamma_v(G)_T=\gamma_v(F)_T, \gamma_v(G)_I=\gamma_v(F)_I, \gamma_v(G)_F=\gamma_v(F)_F$, and $\gamma_v(G)=\gamma_v(F)$.*

Proof. (a). Suppose that xy is an edge in F. If it were not a T-bridge, we would have a T-path P from x to y, not involving xy, of strength greater than $T_B(xy)$. By being special spanning neutrosophic graph F, P must involve edges not in F. Let u_1v_1 be an edge from P, which don't belong to F. u_1v_1 can be replaced by a T-path P_1 in F of strength than $T_B(uv)$. P_1 cannot involve xy. So by replacing each edge u_iv_i from P, which don't belong to F, by P_i, we can construct a T-path in F from x to y that does not involve xy. But G is T-acyclic. This is a contradiction. The latter of the proof is obvious. Therefore the result follows.

By Proposition 2.51(a), and (a), the result is obviously hold for (b).

By Definition 2.50(a), and (b), the result holds obviously for (c). □

Proposition 2.53. *For any neutrosophic graph $G = (A, B)$ on a given set V, we have*

a. $\gamma_v(G), \gamma_v(G)_T, \gamma_v(G)_I, \gamma_v(G)_F \leq p.$

b. $\gamma_v(G) + \gamma_v(\bar{G}), \gamma_v(G)_T + \gamma_v(\bar{G})_T, \gamma_v(G)_I + \gamma_v(\bar{G})_I, \gamma_v(G)_F + \gamma_v(\bar{G})_F \leq 2p.$

Let us remind you consider p as the order of this graph.

Proof. (a). By Proposition 2, there is a neutrosophic graph $G = (A, B)$ such that $\gamma_v(G)_T = \gamma_v(G)_I = \gamma_v(G)_F = \gamma_v(G) = p$. So the result follows.

(b). By implementing (a) on G and $\bar{G}$, the result is obviously hold. □

The concept of neutrosophy are used as the framework in algebraic structures and fuzzy models. There are three kinds of neutrosophic graphs. As it mentioned, we chose one kind of them as the framework. In this paper, we introduce the new tool in new-born fuzzy model for analyzing its structure. In future, we would explore other elements of this fuzzy model, e. g. binary operations, unary operations and like stuff by this tool. It's extremely effective to use other tools like coloring and relations between them. It might be our future work. Also, we would like introducing neutrosophic structures along with their properties.

oduction and Overview In this study, author analyze the structure of domination in $t-$norm fuzzy graphs and a its special case when using $T_{\min}$, as fuzzy graphs.

In **Ref.** [?], we have a real world application concerning this concept. you can refer it if you need or are interested. Some issues in **Ref.** [?], "$\cdots$ The Global Slavery Index is an annual study of world-wide slavery conditions by country published by the Walk Free Foundation. In 2016, the study estimated a total of 45.8 million people to be in some form of modern slavery in 167 countries. The report contains data for countries concerning the estimate of the prevalence of modern slavery, vulnerability measures, and an assessment of the strength of government response$\cdots$"

In this work, author always use v if the vertex is specific. Otherwise, author apply its indices, i.e. v_i. So v or v_i always refers to vertices and their twofold part refers to edge. The power "$'$" usually states that one edge is deleted.

At first, author introduce two types of a fuzzy models concerning $t-$norm. It is well known that T_{min} is a function (precisely a relation) which is greater than any $t - norm$.

"Basic Definition', '"Size", "Order", "Scalar Cardinality", "Path", "Fuzzy Cycle", "Isolate", "$\alpha-$strong", "$M-$strong", "Bridge", "Bipartite", "Star", "Complete", "Spanning Subgraph", "Fuzzy Tree" and "Operations" are introduced as preliminaries in what follows. Some concepts are not related to choosing any $t-$norm because they don't state any relation between two functions μ and σ which are depended on each other by definition of fuzzy model (precisely using $t-$norm). So in all fuzzy models can be the same.

Definition 2.54 (Definitions, Size and Order, Scalar Cardinality)**.** author introduce some elementary concepts as follows

(i) [Definitions] Let V be a nonempty finite set and $E \subseteq V \times V$. Then $G = (\sigma, \mu)$ is called **Fuzzy Graph** if $\forall v_1v_2 \in E$, $\mu(v_1v_2) = \mu(v_2v_1) \leq \min\{\sigma(v_1), \sigma(v_2)\}$. And is called an **t-norm Fuzzy Graph** if $\forall v_1v_2 \in E$, $\mu(v_1v_2) = \mu(v_2v_1) \leq T(\sigma(v_1), \sigma(v_2))$, where $\sigma : V \to [0, 1]$ and $\mu : E \to [0, 1]$ be the fuzzy sets, μ is reflexive and T is an arbitrary $t-$norm.

(*ii*) [Size and Order] The **Order** p and the **Size** q are defined $p = \Sigma_{v \in V} \sigma(v)$ and $q = \Sigma_{v_1 v_2 \in E} \mu(v_1 v_2)$.

(*iii*) [Scalar Cardinality] The **Scalar Cardinality** of S is defined to be $\Sigma_{v \in S} \sigma(v)$.

Definition 2.55 (Path, Fuzzy Cycle)**.** Let $G = (\sigma, \mu)$ be a fuzzy graph or an $t-$norm fuzzy graph.

(*i*) [Path, its Strength] A **Path** P of length n is a sequence of distinct vertices $v_0, v_1, \cdots, v_n$ such that $\mu(v_{i-1}v_i) > 0, i = 1, 2, \cdots, n$ and $T(\mu(v_0 v_1), \cdots, \mu(v_{i-1} v_i))$ is defined as its **Strength**. The **Strength of Connectedness** between two vertices v_1 and v_2 in G is defined as the maximum of the strengths of all paths between v_1 and v_2 and is denoted by $\mu^{\infty}_{G}(v_1, v_2)$.

(*ii*) [Fuzzy Cycle, its Strength] Let $v_0, v_1, \cdots, v_n$ be a path. It is called a **Fuzzy Cycle** C of length n If $v_0 = v_n, n \geq 3$ and at least the values of two edges are $T(\mu(v_0 v_1), \cdots, \mu(v_{i-1} v_i)$ which is defined as **Strength** of a fuzzy cycle.

Definition 2.56 (Types of Vertices)**.** Let $G = (\sigma, \mu)$ be a fuzzy graph or an $t-$norm fuzzy graph. A vertex v is said **isolated** if $\mu(v v_1) = 0$ for all $v \neq v_1$.

Definition 2.57 (Types of Edges)**.** Let $G = (\sigma, \mu)$ be a fuzzy graph or an $t-$norm fuzzy graph. Let $v_1 v_2 \in E$. Note that $\mu^{\infty}_{G'}(v_1, v_2)$ is the strength of connectedness between v_1 and v_2 in the fuzzy model which is obtained from G by deleting the edge $v_1 v_2$.

An edge $v_1 v_2$ in G is called

(*i*) $\alpha-$**strong** if $\mu(v_1 v_2) > \mu^{\infty}_{G'}(v_1, v_2)$ and **strong** if $\mu(v_1 v_2) \geq \mu^{\infty}_{G'}(v_1, v_2)$. The case $\mu(v_1 v_2) = \mu^{\infty}_{G'}(v_1, v_2)$, is not considered in any study of domination. The case $\mu(v_1 v_2) < \mu^{\infty}_{G'}(v_1, v_2)$ is not possible.

(*ii*) $M-$**strong** if both $\mu(v_1 v_2) = \sigma(v_1) \wedge \sigma(v_2)$ and G is a fuzzy graph or both $\mu(v_1 v_2) = T(\sigma(v_1), \sigma(v_2))$ and G is an $t-$norm fuzzy graph.

(*iii*) **bridge** if $\mu^{\infty}_{G'}(v_3, v_4) < \mu^{\infty}_{G}(v_3, v_4)$ for some $v_3, v_4 \in V$.

Definition 2.58 (Types of Models)**.** Let $G = (\sigma, \mu)$ and $G_1 = (\tau, \nu)$ be a fuzzy graph or an $t-$norm fuzzy graph. Then $G = (\sigma, \mu)$ is said to be

(*i*) **Bipartite** if V can be partitioned into two nonempty sets V_1 and V_2 such that $\mu(v_1 v_2) = 0$ if $v_1, v_2 \in V_1$ or $v_1, v_2 \in V_2$;

(*ii*) **Star** which is denoted by $K_{1,\sigma}$ If it is a bipartite and either $|V_1| = 1$ or $|V_2| = 1$ which imply that we call its corresponded vertex a **center**;

(*iii*) **Complete** if all edges be $M-$ strong. e.g., Complete bipartite fuzzy graph, Complete fuzzy graph, Complete bipartite $t-$norm fuzzy graph, Complete $t-$norm fuzzy graph.

(*iv*) has a **Spanning Subgraph** $G_1 = (\tau, \nu)$ if $\tau = \sigma$ and $\nu \subseteq \mu$.

(*v*) **Fuzzy tree** if its spanning subgraph $F = (\sigma, \tau)$ is a tree (Ref. [?]), where for all edges $v_1 v_2$ is in G but not F, we have $\mu(v_1 v_2) < \tau^{\infty}_{F}(v_1, v_2)$.

Definition 2.59 (Types of New Models)**.** If we alter min, max (precisely t-norm T_{min}, T_{max}) with an arbitrary t-norm T, we have these concepts for $t-$norm fuzzy graphs. To avoid confusion, we only write down for fuzzy graph and the analogues concepts are supposed to be obvious and we use these names for both models, fuzzy graphs and fuzzy $t-$norm graphs.

Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ be fuzzy graphs on $G^*_1 = (V_1, E_1)$ and $G^*_2 = (V_2, E_2)$, respectively. Then

(i) [Binary Operation, Complement] A **Complement** of a fuzzy graph $G_1 = (\sigma_1, \mu_1)$ is denoted by $\bar{G_1}$ and is defined to $\bar{G_1} = (\sigma_1, \bar{\mu_1})$, where $\bar{\mu_1}(v_1v_2) = \min\{\sigma_1(v_1), \sigma_1(v_2)\} - \mu_1(v_1v_2)$, for all $v_1, v_2 \in V_1$;

(ii) [Binary Operation, Cartesian Product] A **Cartesian product** $G = G_1 \times G_2$ is defined as a fuzzy graph $G = (\sigma_1 \times \sigma_2, \mu_1\mu_2)$ on $G^* = (V_1 \times V_2, E)$ where $E = \{(v, v_1)(v, v_2) | v \in V_1, v_1v_2 \in E_2\} \cup \{(v_1, v)(v_2, v)\} | v_1v_2 \in E_1, v \in V_2\}$. Fuzzy sets $\sigma_1 \times \sigma_2$ on $V_1 \times V_2$ and $\mu_1\mu_2$ on E, are defined as $(\sigma_1 \times \sigma_2)(v_1, v_2) = \min\{\sigma_1(v_1), \sigma_2(v_2)\}, \forall (v_1, v_2) \in V_1 \times V_2$ and $\forall v \in V_1, \forall v_1v_2 \in E_2, \mu_1\mu_2((v, v_1)(v, v_2)) = \min\{\sigma_1(v), \mu_2(v_1v_2)\}$ and $\forall v_1v_2 \in E_1, \forall v \in V_2, \mu_1\mu_2((v_1, v)(v_2, v)) = \min\{\mu_1(v_1v_2), \sigma_2(v)\}$;

(iii) [Binary Operation, Union] An **Union** $G = G_1 \cup G_2$ is defined as a fuzzy graph $G = (\sigma_1 \cup \sigma_2, \mu_1 \cup \mu_2)$ on $G^* = (V_1 \cup V_2, E_1 \cup E_2)$. Fuzzy sets $\sigma_1 \cup \sigma_2$ and $\mu_1 \cup \mu_2$ are defined as $(\sigma_1 \cup \sigma_2)(v) = \sigma_1(v)$ if $v \in V_1 - V_2$, $(\sigma_1 \cup \sigma_2)(v) = \sigma_2(v)$ if $v \in V_2 - V_1$, and $(\sigma_1 \cup \sigma_2)(v) = \max\{\sigma_1(v), \sigma_2(v)\}$ if $v \in V_1 \cap V_2$. Also $(\mu_1 \cup \mu_2)(v_1v_2) = \mu_1(v_1v_2)$ if $v_1v_2 \in E_1 - E_2$ and $(\mu_1 \cup \mu_2)(v_1v_2) = \mu_2(v_1v_2)$ if $v_1v_2 \in E_2 - E_1$, and $(\mu_1 \cup \mu_2)(v_1v_2) = \max\{\mu_1(v_1v_2), \mu_2(v_1v_2)\}$ if $v_1v_2 \in E_1 \cap E_2$;

(iv) [Binary Operation, Join] A **Join** $G = G_1 + G_2$ is defined as a fuzzy graph $G = (\sigma_1 + \sigma_2, \mu_1 + \mu_2)$ on $G^* = (V_1 \cup V_2, E = E_1 \cup E_2 \cup E')$ where E' is the set of all edges joining vertices of V_1 with the vertices of V_2 and we assume that $V_1 \cap V_2 = \emptyset$. Fuzzy sets $\sigma_1 + \sigma_2$ and $\mu_1 + \mu_2$ are defined as $(\sigma_1 + \sigma_2)(v) = (\sigma_1 \cup \sigma_2)(v)$ and $\forall v \in V_1 \cup V_2; (\mu_1 + \mu_2)(v_1v_2) = (\mu_1 \cup \mu_2)(v_1v_2)$ if $v_1v_2 \in E_1 \cup E_2$ and $(\mu_1 + \mu_2)(v_1v_2) = \min\{\sigma_1(v_1), \sigma_2(v_2)\}$ if $v_1v_2 \in E'$.

We choose a name for our new definition as vertex domination and we refer to others with only the name domination. To avoid confusion, we bring references if it is necessary.

Definition 2.60 (Domination: Edge, Set, Number)**.** Let $G = (\sigma, \mu)$ be a fuzzy graph or an $t-$norm fuzzy graph and $v, v_1 \in V$. Then

(i) A vertex v α-strongly **dominates** a vertex v_1 in G, if its corresponded edge vv_1 is an α-strong edge;

(ii) D is called an α**-strong dominating set** in G, if for every $v_1 \in V \setminus D$, there is $v \in D$ such that α-strongly dominates v_1.

(iii) The **weight** of D is defined by $w_v(D) = \sum_{v \in D}(\sigma(v) + \frac{\sum_{vv_1 \in S} \mu(vv_1)}{\sum_{vv_1 \in E} \mu(vv_1)})$, where $S = \{v_1v_2 \in E \mid \mu(v_1v_2) > \mu^{\infty}_{G'}(v_1, v_2)\}$.

(iv) A *vertex domination number* of G is defined as $\gamma_v(G) = \min_{D \in \mathcal{D}}\{w_v(D)\}$, where $\mathcal{D}$ is the set of all α**-strong dominating sets** in G. The α-strong dominating set that corresponds to $\gamma_v(G)$ is called by **vertex dominating set.**

We give some definitions concerning domination on fuzzy graphs. It can be extended to $t-$norm fuzzy graphs. We only use them in some examples for illustrating our concepts and do a comparison between them with ours. It is worth to note that if we alter min (precisely t-norm T_{min}) with any t-norm T, we have these concepts for $t-$norm fuzzy graphs. To avoid confusion, we only write down for fuzzy graph and the analogues concepts are supposed to be obvious.

Definition 2.61. Let $G = (\sigma, \mu)$ be a fuzzy graph, $D \subseteq V$ and $\mathcal{D}$ is a set of all dominating sets in G. Then

(i) (**A. Somasundaram and S. Somasundaram** (**Ref.** [?])) $D \subseteq V$ is said to be an **dominating set** in G, if for every $v_1 \in V \setminus D$, there exists $v \in D$ such that its corresponded edge vv_1 is an $M-$strong edge.$\gamma(G) = \min_{D \in \mathcal{D}}\{\sum_{v \in D} \sigma(v)\}$ is said to be a **domination number** of G.

(ii) (**C. Natarajan and S.K. Ayyaswamy** (**Ref.** [?])) $D \subseteq V$ is said to be a **dominating set,** if for every $v_1 \in V \setminus D$, there exists $v \in D$ such that its corresponded edge vv_1 is an $M-$strong edge and $d_e(v) = \sum_{v_2 \in N(v)} \sigma(v_2) \geq d_e(v_1) = \sum_{v_2 \in N(v_1)} \sigma(v_2)$ where for all $v \in V, N(v) = \{v_1 \in V \mid \mu(vv_1) = \min\{\sigma(v), \sigma(v_1)\}\}$. $\gamma(G) = \min_{D \in \mathcal{D}}\{\sum_{v \in D} \sigma(v)\}$ is said to be a **domination number** of G.

(iii) (**O.T. Manjusha and M.S. Sunitha** (**Ref.** [?])) $D \subseteq V$ is said to be **dominating set** if for every $v_1 \in V \setminus D$, there exists $v \in D$ such that its corresponded edge vv_1 is a strong edge. $\gamma(G) = \min_{D \in \mathcal{D}}\{\sum_{v \in D} \sigma(v)\}$ is said to be a **domination number** of G.

(iv) (**A. Nagoor Gani and K. Prasanna Devi** (**Ref.** [?])) $D \subseteq V$ is said to be **dominating set,** if for every $v_1 \in V \setminus D$, there exists two vertices like $v \in D$ such that their corresponded edges are strong edges. $\gamma(G) = \min_{D \in \mathcal{D}}\{\sum_{v \in D} \sigma(v)\}$ is said to be a **domination number;**

(v) (**O.T. Manjusha and M.S. Sunitha** (**Ref.** [?])) $D \subseteq V$ is said to be **dominating set,** if for every $v_1 \in V \setminus D$, there exists $v \in D$ such that its corresponded edge vv_1 is an strong edge. **domination number** of G is said to be $\gamma(G) = \min_{D \in \mathcal{D}}\{\sum_{v \in D} \min_{vv_1 \text{is a strong edge.}}\{\mu(v, v_1)\}\}$.

In two upcoming examples, we illustrates the concept of our definition.

Example 2.62 ($\alpha-$strong edge)**.** Let $G = (\sigma, \mu)$ be a fuzzy graph as Figure 2. Then the edges $\{v_2v_5, v_2v_4, v_3v_4, v_1v_3\}$ are α-strong and the edges $\{v_1v_4, v_1v_2, v_4v_5\}$ are not α-strong.

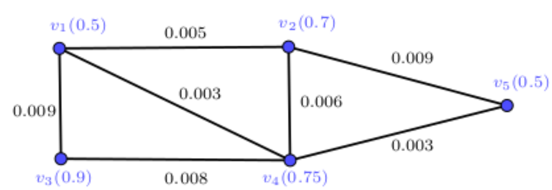

Figure 2. vertex domination

Example 2.63 (Domination)**.** Let $G = (\sigma, \mu)$ be a fuzzy graph as Figure 2. The set $S = \{v_2, v_3\}$ is an α-strong dominating set. This set is also vertex dominating set in fuzzy graph G. Hence $\gamma_v(G) = 1.75 + 0.9 + 0.7 = 3.35$. So $\gamma_v(G) = 3.35$.

In two upcoming examples, we compare our definition with others as theoretic and practical aspects.

Example 2.64 (Theoretic Aspect)**.** The following is a table consist of a brief fundamental comparison between types of domination in fuzzy graphs. There are two different types of the complete bipartite fuzzy graphs as Figures 3 and 4, which compare types of domination in fuzzy graphs.

Types of Edges	Types of Numbers	Figure 3	Figure 4
$M-$strong	Scalar cardinality	0.9	0.9
$M-$strong and $d_e(u) \geq d_e(v)$	Scalar cardinality	1.9	1.3
Strong	Scalar cardinality	0.9	0.9
$\beta-$strong	Scalar cardinality	0.9	1.5
Strong	$\Sigma_{u\in D}t(u,v)$	0.8	0.4
Our new definition	vertex weight	1.9	2.4

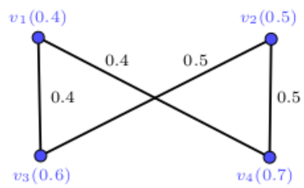

Figure 3. Comparison of Dominations

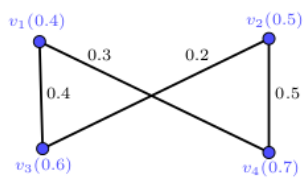

Figure 4. Comparison of Dominations with Different Values

Example 2.65 (Practical Aspect: A Comparison in Real-World Problem)**.** In this section, we introduce one practical application in related to this concept. In the following, we will try to solve this problem by previous definitions, too.

Suppose the Figure 5, the fuzzy graph model of the hypothetical condition of cities and the paths between them in a region.

Problem[reducing waste of time in transport planning] Consider a set of cities connected by communication paths. Which cities have these properties? Having low traffic levels and other cities associating with at least ones by low-cost roads.

The terms "low traffic" and "low-cost" are vague in nature. So we are faced with a fuzzy graph model. In other words, Let G be a graph which represents the roads between cities. Let the vertices denote the cities and the edges denote the roads connecting the cities. From the statistical data that represents the high traffic flow of cities and high-cost roads, the functions σ and μ on the vertex set and edge set of G can be constructed by using the standard techniques. In this fuzzy graph, a dominating set D can be interpreted as a set of cities which have low traffic and every city not in D is connected to a member in D by a low-cost road. We now look at the answer to the problem raised by using the old and the new definitions. As you can see in this model, finding the desirable cities is more important than finding the domination number. Because the numbers given for the set and each situation are compared with each others in the context of the same definition, and this number is merely to compare the different sets of cities in the context of the same definition. Therefore, speaking of the magnitude of this number is meaningless. The table below illustrates the solutions presented for this problem.

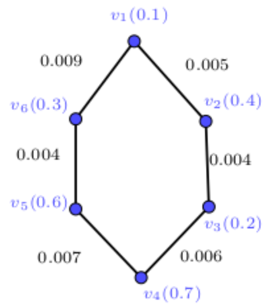

Figure 5. The exemplary scheme of road infrastructure

Definitions	Given desirable set
A. Somasundaram and S. Somasundaram (Ref. [?])	V
C. Natarajan and S.K. Ayyaswamy (Ref. [?])	V
O.T. Manjusha and M.S. Sunitha (Ref. [?])	$\{v_3, v_6\}$
A. Nagoor Gani and K. Prasanna Devi (Ref. [?])	V
O.T. Manjusha and M.S. Sunitha (Ref. [?])	$\{v_3, v_6\}$
Our new definition	$\{v_1, v_4\}$

It is obvious from the above table and Figure 5 that the desirable cities given by previous definitions, are not appropriate due to the lack of simultaneous attention to cities and roads.

We are now presenting the dynamic status of the problem. The dynamic state is the situation in which the fuzzy graph model is found over time. Since over time, changes in the values of roads are more than changes in the values of cities in the fuzzy graph model of the hypothetical condition of cities and the paths between them in a region. So values of the roads increases. Values of cities (their traffics) do not change significantly over time. Because the traffic problem is an infrastructure problem. The Figure 6 depicts the dynamic case of a fuzzy graph model. Over time, the values of the roads increases equally.

In this situation, the answer are given by the previous definitions reflects the wrong perspectives while the our new definition adapts itself well to the new situation. Previous definitions didn't use simultaneous attentions to cities and roads.

Dynamic analysis of networks in the first row of Figure 6 are the following table.

Definitions	Given desirable set
A. Somasundaram and S. Somasundaram (Ref. [?])	$V, V-\{v_6\}, V-\{v_2, v_6\}$
C. Natarajan and S.K. Ayyaswamy (Ref. [?])	$V, V-\{v_6\}, V-\{v_2, v_6\}$
O.T. Manjusha and M.S. Sunitha (Ref. [?])	$\{v_3, v_6\}, \{v_3, v_6\}, \{v_3, v_6\}$
A. Nagoor Gani and K. Prasanna Devi (Ref. [?])	$\{v_1, v_3, v_5\}, \{v_1, v_3, v_5\}, \{v_1, v_3, v_5\}$
O.T. Manjusha and M.S. Sunitha (Ref. [?])	$\{v_3, v_6\}, \{v_3, v_6\}, \{v_3, v_6\}$
Our new definition	$\{v_1, v_4\}, \{v_1, v_4\}, \{v_1, v_4\}$

Dynamic analysis of networks in the second row of Figure 6 are the following table.

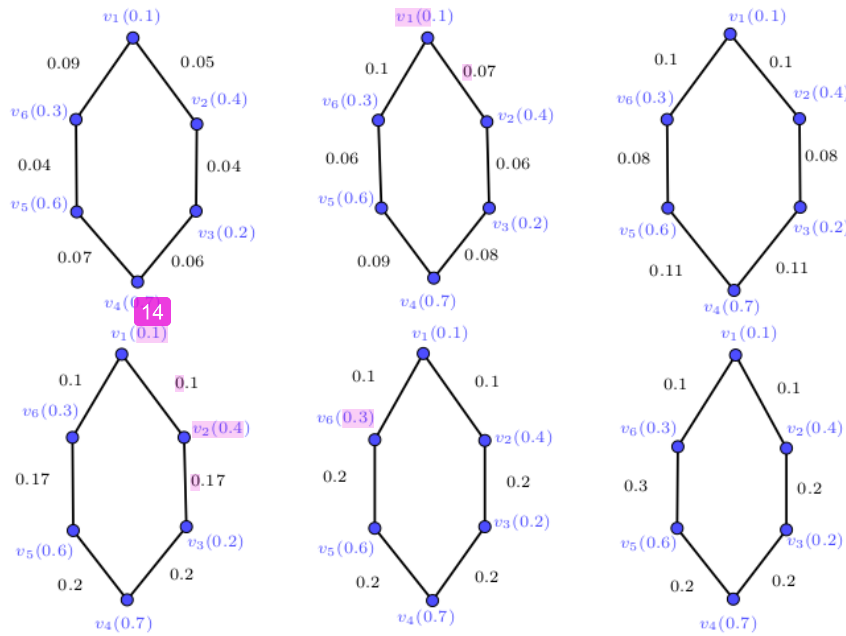

Figure 6. The dynamic scheme of road infrastructure

Definitions	Given desirable set
A. Somasundaram and S. Somasundaram (Ref. [?])	$\{v_1, v_3, v_5\}$, $\{v_1, v_3, v_5\}$, $\{v_3, v_5\}$
C. Natarajan and S.K. Ayyaswamy (Ref. [?])	$\{v_2, v_4, v_6\}$, $\{v_3, v_5, v_6\}$, $\{v_2, v_3, v_5\}$
O.T. Manjusha and M.S. Sunitha (Ref. [?])	$\{v_3, v_6\}$, $\{v_3, v_6\}$, $\{v_3, v_6\}$
A. Nagoor Gani and K. Prasanna Devi (Ref. [?])	$\{v_1, v_3, v_5\}$, $\{v_1, v_3, v_5\}$, $\{v_1, v_3, v_5\}$
O.T. Manjusha and M.S. Sunitha (Ref. [?])	$\{v_3, v_6\}$, $\{v_3, v_6\}$, $\{v_3, v_6\}$
Our new definition	$\{v_1, v_3, v_6\}$, $\{v_1, v_3, v_6\}$, $\{v_1, v_3, v_6\}$

All parts are twofold even if we don't mention, directly. I.e., all results depicts some properties about fuzzy graph and $t-$norm fuzzy graph.

It is well known and generally accepted that the problem of determining the domination number of an arbitrary fuzzy model is a difficult one. Because of this, researchers have turned their attention to the study of classes of fuzzy models for which the domination problem can be solved in polynomial time.

Proposition 2.66 (**Ref.** [?], Proposition 3.24. , pp. 135, 136)**.** *Let $G = (\sigma, \mu)$ be a complete $t-$norm fuzzy graph. Then*

(1) $\mu_G^\infty(v_1, v_2) = \mu(v_1 v_2), \forall v_1, v_2 \in V$

(2) *G has no cut vertices.*

Corollary 2.67. *Every edges in complete t-norm fuzzy graph are α-strong if $\forall v_1, v_2 \in V$, there is exactly one path with strength of $\mu^\infty(v_1, v_2)$.*

Proof. Let G be complete. For all $v_1, v_2 \in V$, $\mu_G^\infty(v_1, v_2) = \mu(v_1 v_2)$ by Proposition (3.13). So for all $v_1 v_2 \in V$, $\mu_G'^\infty(v_1, v_2) < \mu(v_1, v_2)$. Hence uv is α-strong edge. The result follows. □

Proposition 2.68. *Let $G = (\sigma, \mu)$ be complete such that $\forall v_1, v_2 \in V$, there is exactly one path with strength of $\mu^{\infty}(v_1, v_2)$. Then, every edges are α-strong.*

Proof. We prove it in two cases.

Fuzzy Graphs Let G be a complete fuzzy graph. The strength of path P from v_1 to v_2 is of the form $\min\{\sigma(v_1), \cdots \sigma(v_2)\} \leq \min\{\sigma(v_1), \sigma(v_2)\} = \mu(v_1v_2)$. So $\mu_G^{\infty}(v_1, v_2) \leq \mu(v_1v_2)$. v_1v_2 is a path from v_1 to v_2 such that $\mu(v_1v_2) = \min\{\sigma(v_1), \sigma(v_2)\}$. Therefore $\mu_G^{\infty}(v_1, v_2) \geq \mu(v_1v_2)$. Hence $\mu_G^{\infty}(v_1, v_2) = \mu(v_1v_2)$. Then $\mu(v_1v_2) > \mu_{G'}^{\infty}(v_1, v_2)$. It means that the edge v_1v_2 is α-strong. All edges are α-strong, as we wished to show. Its proof works equally well for the latter.

$t-$norm Fuzzy Graphs The strength of path P from v_1 to v_2 is of the form $T(\sigma(v_1), \cdots \sigma(v_2)) \leq T(\sigma(v_1), \sigma(v_2))$. G is complete. By regarding this point, we have $T(\sigma(v_1), \sigma(v_2)) = \mu(v_1v_2)$. Therefore, $T(\sigma(v_1), \cdots \sigma(v_2)) \leq \mu(v_1v_2)$. It means that $\mu_G^{\infty}(v_1, v_2) \leq \mu(v_1v_2)$. v_1, v_2 is a path from v_1 to v_2 such that $\mu(v_1v_2) = T(\sigma(v_1), \sigma(v_2))$. Therefore $\mu_G^{\infty}(v_1, v_2) \geq \mu(v_1v_2)$. Hence $\mu_G^{\infty}(v_1, v_2) = \mu(v_1v_2)$. Then $\mu(v_1v_2) > \mu_{G'}^{\infty}(v_1, v_2)$. It means that the edge v_1v_2 is α-strong. All edges are α-strong.

□

Corollary 2.69 (Complete)**.** *Let $G = (\sigma, \mu)$ be complete such that $\forall v_1, v_2 \in V$, there is exactly one path with strength of $\mu^{\infty}(v_1, v_2)$. Then, $\gamma_v(G) = \min_{v \in V}(\sigma(v)) + 1$.*

Proof. We prove it in two cases.

Fuzzy Graphs All edges are α-strong and each vertex is adjacent to all other vertices. So $D = \{v\}$ is an α-strong dominating set and $\sum_{vv_1 \in S} \mu(vv_1) = \sum_{vv_1 \in E} \mu(vv_1)$ for each $v \in V$, where $S = \{v_1v_2 \in E \mid \mu(v_1v_2) > \mu_{G'}^{\infty}(v_1, v_2)\}$. The result follows.

$t-$norm Fuzzy Graphs All edges are α-strong and each vertex is adjacent to all other vertices. So $D = \{v\}$ is an α-strong dominating set and $\sum_{vv_1 \in S} \mu(vv_1) = \sum_{vv_1 \in E} \mu(vv_1)$ for each $v \in V$, where $S = \{v_1v_2 \in E \mid \mu(v_1v_2) > \mu_{G'}^{\infty}(v_1, v_2)\}$. The case where equality holds is of particular interest.

□

Proposition 2.70 (Edgeless)**.** *Let $G = (\sigma, \mu)$ be an edgeless fuzzy graph. Then $\gamma_v(G) = p$, where p denotes the order of G.*

Proof. We prove it in two cases.

Fuzzy Graphs G is edgeless. Hence V is only α-strong dominating set in G and there is no α-strong edge. So by Definition, we have $\gamma_v(G) = \Sigma_{v \in V}\sigma(v) = p$.

$t-$norm Fuzzy Graphs The previous proof works equally well for this case.

□

It is interesting to note that the converse of Proposition 3.17, does not hold.

Example 2.71. We show that the converse of Proposition 3.17 does not hold. For this purpose, Let $G = (\sigma, \mu)$ be a fuzzy graph where $V = \{v_1, v_2, v_3, v_4, v_5\}, E = \{v_1v_2, v_1v_4, v_1v_3, v_2v_4, v_2v_5, v_3v_4, v_4v_5\}$ and σ, μ are fuzzy sets which are defined on V, E, respectively, as follows. For the fuzzy set σ, we have

$$\sigma(v_1) = 0.5, \sigma(v_2) = 0.7, \sigma(v_3) = 0.9, \sigma(v_4) = 0.75, \sigma(v_5) = 0.5$$

Now, for the fuzzy set μ, we have $\mu(v_1v_2) = 0.005$,

$$\mu(v_1v_4) = 0.003, \mu(v_1v_3) = 0.009, \mu(v_2v_4) = 0.006, \mu(v_2v_5) = 0.009,$$

$\mu(v_3v_4) = 0.008, \mu(v_4v_5) = 0.003$ such that $\forall v_1, v_2 \in V, \mu(v_1v_2) \leq \min\{\sigma(v_1), \sigma(v_2)\}$. The edges $\{v_2v_5, v_2v_4, v_3v_4, v_1v_3\}$ are α-strong and the edges $\{v_1v_4, v_1v_2, v_4v_5\}$ are not α-strong. So the set $\{v_2, v_3\}$ is the α-strong dominating set. This set is also vertex dominating set in fuzzy graph G. Hence $\gamma_v(G) = 1.75 + 0.9 + 0.7 = 3.35 = \Sigma_{v \in V}\sigma(v) = p$. So G is not edgeless but $\gamma_v(G) = p$.

Corollary 2.72. *Let $G = (\sigma, \mu)$ be complete bipartite such that $\forall v_1, v_2 \in V$, there is exactly one path with strength of $\mu^\infty(v_1, v_2)$. Then, every edges are α-strong.*

Proof. The proof in Proposition (3.15), works equally well for this case. □

Corollary 2.73. *Let $G = (\sigma, \mu)$ be complete star such that $\forall v_1, v_2 \in V$, there is exactly one path with strength of $\mu^\infty(v_1, v_2)$. Then, every edges are α-strong.*

Proof. The proof in Proposition (3.15), works equally well for this case. □

Corollary 2.74 (Complete Star)**.** *Let $G = (\sigma, \mu)$ be complete star such that $\forall v_1, v_2 \in V$, there is exactly one path with strength of $\mu^\infty(v_1, v_2)$. Then, $\gamma_v(G)$ is $\sigma(v) + 1$ where $v \in V$ is supposed as a center of G.*

Proof. We prove it in two cases.

Fuzzy Graphs Let $G = (\sigma, \mu)$ be a star fuzzy graph with $V = \{v, v_1, v_2, \cdots, v_n\}$ such that v is a center. Then $\{v\}$ is a vertex dominating set of G. Hence $\gamma_v(G) = \sigma(v) + 1$.

$t-$norm Fuzzy Graphs The previous proof works equally well for this case.

□

Corollary 2.75 (Complete Bipartite)**.** *Let $G = (\sigma, \mu)$ be a complete bipartite such that $\forall v_1, v_2 \in V$, there is exactly one path with strength of $\mu^\infty(v_1, v_2)$. Then $\gamma_v(G)$ is either $\sigma(v) + 1, v \in V$ or* $\min_{v_1 \in V_1, v_2 \in V_2}(\sigma(v_1) + \sigma(v_2)) + 2$.

Proof. We prove it in two cases.

Fuzzy Graphs Let $G = (\sigma, \mu)$ be a complete bipartite fuzzy graph such that $\forall v_1, v_2 \in V$, there is exactly one path with strength of $\mu^\infty(v_1, v_2)$. By Corollary (3.19), all the edges are α-strong.
If G is a complete star fuzzy graph, then by Corollary (3.21), the result follows. Otherwise, the vertex set V can be partitioned into two nonempty sets V_1 and V_2 such that both of V_1 and V_2 include more than one vertex. Every vertex in V_1 is dominated by every vertices in V_2, as α-strong and conversely. Hence in K_{σ_1, σ_2}, the α-strong dominating sets are V_1 and V_2 and any set containing 2 vertices, one in V_1 and other in V_2. So $\gamma_v(K_{\sigma_1, \sigma_2}) = \min_{v_1 \in V_1, v_2 \in V_2}(\sigma(v_1) + \sigma(v_2)) + 2$. The result follows.

$t-$norm Fuzzy Graphs Let $G = (\sigma, \mu)$ be a complete bipartite $t-$norm fuzzy graph such that $\forall v_1, v_2 \in V$, there is exactly one path with strength of $\mu^{\infty}(v_1, v_2)$. By Corollary (3.19), all the edges are α-strong.
If G is a complete star $t-$norm fuzzy graph, then by Corollary (3.21), the result follows. Otherwise, the vertex set V can be partitioned into two nonempty sets V_1 and V_2 such that both of V_1 and V_2 include more than one vertex. Every vertex in V_1 is dominated by every vertices in V_2, as α-strong and conversely. Hence in K_{σ_1,σ_2}, the α-strong dominating sets are V_1 and V_2 and any set containing 2 vertices, one in V_1 and other in V_2. So
$\gamma_v(K_{\sigma_1,\sigma_2}) = \min_{v_1 \in V_1, v_2 \in V_2}(\sigma(v_1) + \sigma(v_2)) + 2$. The result follows.

□

Theorem 2.76. *Let $G = (\sigma, \mu)$ be a fuzzy graph [**Ref.** [?], Theorem 2.4., p.21] or an $t-$norm fuzzy graph [**Ref.** [?], Theorem 3.3., p.132]. Let $v_1v_2 \in E$. Let μ' be the fuzzy subset of E such that $\mu'(xy) = 0$ and $\mu' = \mu$ otherwise. Then*

*-$t-$norm **Fuzzy Graphs:*** (3) $\Rightarrow$ (2) $\Leftrightarrow$ (1) *-**Fuzzy Graphs:*** (3) $\Leftrightarrow$ (2) $\Leftrightarrow$ (1)

(1) v_1v_2 *is a bridge;*

(2) $\mu^{\infty}_{G'}(v_1, v_2) < \mu(v_1v_2)$;

(3) v_1v_2 *is not a weakest edge of any cycle.*

Corollary 2.77. *Let $G = (\sigma, \mu)$ be a fuzzy graph or an $t-$norm fuzzy graph and $v_1v_2 \in E$. v_1v_2 is an α-strong edge if and only if v_1v_2 is a bridge.*

Proof. By Theorem 3.23, the result is obviously hold. □

Theorem 2.78. *[Fuzzy Graph: **Ref.** [?], Proposition 2.7, p.24] [$t-$norm Fuzzy Graph: **Ref.** [?], Theorem 3.30, p.137] Let $G = (\sigma, \mu)$ be a fuzzy tree. Then the edges of $F = (\tau, \nu)$ are just the bridges of G.*

Corollary 2.79. *Let $G = (\sigma, \mu)$ be a fuzzy tree. Then edges of $F = (\sigma, \tau)$ are just the α-strong edges of G.*

Proof. By Theorem 3.32 and Corollary 3.24, the result follows. □

Proposition 2.80. *Let $G = (\sigma, \mu)$ be a fuzzy tree. Then $D(T) = D(F) \cup D(S)$, where $D(T), D(F)$ and $D(S)$ are vertex dominating sets of T, F and S, respectively. S is a set of edges which has no edges with connection to F.*

Proof. By Corollary 3.26, the edges of $F = (\sigma, \tau)$ are just the α-strong edges of G. The result follows. □

In the following result, we will partition the edges of a fuzzy cycle to two types $\alpha-$strong and other one.

Proposition 2.81 (Fuzzy Cycle)**.** *Let $G = (\sigma, \mu)$ be a fuzzy cycle. All edges are $\alpha-$strong with the only exceptions of weakest edges.*

Proof. We study it in two cases.

Fuzzy Graphs By regarding the definition of a fuzzy cycle, at least two edges have minimum value between all edges. It implies two cases. The first is of weakest edges and the latter case is of $\alpha-$strong edges.

$t-$norm Fuzzy Graphs We can say about the weakest edges in $t-$norm fuzzy graphs but there is no information about their relations with strength of path which they are on it. In other words, Is $T(v_1, v_2, \cdots, v_n)$ equal with strength of weakest edges?

□

Proposition 2.82. *For any fuzzy graph $G = (\mu, \sigma)$, if there is a path which an edge v_1v_2 is only weakest edge on it, then v_1v_2 is not $\alpha-$strong edge.*

Proof. We study it in two cases.

Fuzzy Graphs There is a path which an edge v_1v_2 is only weakest edge on it. So by deleting this edge, the intended path increases the strength of connectedness between v_1 and v_2. Then v_1v_2 is not $\alpha-$strong edge.

$t-$norm Fuzzy Graphs We can say about the weakest edges in $t-$norm fuzzy graphs but there is no information about their relations with strength of path which they are on it. In other words, Is $T(v_1, v_2, \cdots, v_n)$ equal with strength of weakest edges?

□

Example 2.83. Let $G_1 = (\sigma, \mu_1)$ and $G_2 = (\sigma, \mu_2)$ be fuzzy graphs as Figures 7 and 8. Then $G_1 = (\sigma, \mu_1)$ is a fuzzy tree, but not a tree and not a fuzzy cycle while $G_2 = (\sigma, \mu_2)$ is a fuzzy cycle, but not a fuzzy tree.

In $G_1 = (\sigma, \mu_1)$, the set $S = \{v_1\}$ is an α-strong dominating set. This set is also vertex dominating set in fuzzy tree (but not a fuzzy cycle) G_1. Hence $\gamma_v(G_1) = 0.7 + 0.77 = 1.47$. So $\gamma_v(G_1) = 1.47$.

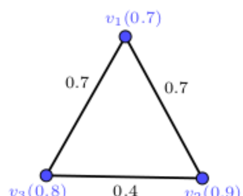

Figure 7. A Fuzzy Tree, but neither a Tree and nor a Fuzzy Cycle

In $G_2 = (\sigma, \mu_2)$, the set $S = \{v_1, v_3\}$ is an α-strong dominating set. This set is also vertex dominating set in fuzzy cycle (but not a fuzzy tree) G_2. Hence $\gamma_v(G_2) = 0.7 + 0.63 + 0.8 + 0 = 2.13$. So $\gamma_v(G_2) = 2.13$.

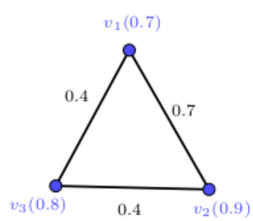

Figure 8. A Fuzzy Cycle, but not a Fuzzy Tree.

We give an upper bound for the vertex domination number, Proposition 3.31.

Proposition 2.84. *Let $G = (\sigma, \mu)$ be a fuzzy graph or an $t-$norm fuzzy graph. Then we have $\gamma_v \leq p$.*

Proof. By Proposition 3.17, the intended fuzzy graph has vertex domination number equals p. So the result follows. □

For any fuzzy graph or $t-$norm fuzzy graph, the Nordhaus-Gaddum(NG)'s result holds, (Theorem 3.32).

Theorem 2.85. *For any fuzzy graph or $t-$norm fuzzy graph $G = (\sigma, \mu)$, the Nordhaus-Gaddum result holds. In other words, we have $\gamma_v + \bar{\gamma_v} \leq 2p$.*

Proof. Let $G = (\sigma, \mu)$ be a fuzzy graph or an $t-$norm fuzzy graph. So $\bar{G}$ is also the same type. We implement Theorem 3.31, on G and $\bar{G}$. Then $\gamma_v \leq p$ and $\bar{\gamma_v} \leq p$. Hence $\gamma_v + \bar{\gamma_v} \leq 2p$. □

Definition 2.86. An α-strong dominating set D is called a *minimal* α-strong dominating set if no proper subset of D is an α-strong dominating set.

Theorem 2.87. *Let $G = (\sigma, \mu)$ be a fuzzy graph or an $t-$norm fuzzy graph, without isolated vertices. If D is a minimal α-strong dominating set then $V \setminus D$ is a α-strong dominating set.*

Proof. By attentions to all edges between two sets, which are only α-strong, the result follows. □

A domatic partition is a partition of the vertices of a graph into disjoint dominating sets. The maximum number of disjoint dominating sets in a domatic partition of a graph is called its domatic number.

Finding a domatic partition of size 1 is trivial and finding a domatic partition of size 2 (or establishing that none exists) is easy but finding a maximum-size domatic partition (i.e., the domatic number), is computationally hard. Finding domatic partition of size two in a fuzzy graph or an $t-$norm fuzzy graph G of order $n \geq 2$ is obtained by the following.

Theorem 2.88. *Every fuzzy graph or $t-$norm fuzzy graph $G = (\sigma, \mu)$, without isolated vertices, of order $n \geq 2$ has an α-strong dominating set D such that whose complement $V \setminus D$ is also an α-strong dominating set.*

Proof. For every fuzzy graph or $t-$norm fuzzy graph $G = (\sigma, \mu)$, without isolated vertices, V is an α-strong dominating set. By analogous to the proof of Theorem 3.34, we can obtain the result. □

We improve the upper bound for the vertex domination number of fuzzy graphs and $t-$norm fuzzy graphs, without isolated vertices, (Theorem 3.36).

Theorem 2.89. *For any fuzzy graph or $t-$norm fuzzy graph $G = (\sigma, \mu)$, without isolated vertices, we have $\gamma_v \leq \frac{p}{2}$.*

Proof. Let D be a minimal dominating set of G. By Theorem 3.35, $V \setminus D$ is an α-strong dominating set of G. Hence $\gamma_v(G) \leq w_v(D)$ and $\gamma_v(G) \leq w_v(V \setminus D)$.

Therefore $2\gamma_v(G) \leq w_v(D) + w_v(V \setminus D) \leq p$ which implies $\gamma_v \leq \frac{p}{2}$. Hence the proof is completed. □

We also improve Nordhaus-Gaddum (NG)'s result for fuzzy graphs or $t-$norm fuzzy graphs, without isolated vertices, (Corollary 3.37).

Corollary 2.90. *Let $G=(\sigma,\mu)$ be a fuzzy graph or an $t-$norm fuzzy graph, such that both of G and $\bar{G}$ have no isolated vertices. Then $\gamma_v+\bar{\gamma_v}\leq p$, where $\bar{\gamma_v}$ is the vertex domination number of $\bar{G}$. Moreover, the equality holds if and only if $\gamma_v=\bar{\gamma_v}=\frac{p}{2}$.*

Proof. By the Implement of Theorem 3.36, on G and $\bar{G}$, we have $\gamma_v(G)=\gamma_v\leq\frac{p}{2}$, and $\gamma_v(\bar{G})=\bar{\gamma_v}(G)=\bar{\gamma_v}\leq\frac{p}{2}$. So $\gamma_v+\bar{\gamma_v}\leq\frac{p}{2}+\frac{p}{2}=p$. Hence $\gamma_v+\bar{\gamma_v}\leq p$.

Suppose $\gamma_v=\bar{\gamma_v}=\frac{p}{2}$. Then obviously, $\gamma_v+\bar{\gamma_v}=p$. Conversely, suppose $\gamma_v+\bar{\gamma_v}\leq p$. Then we have $\gamma_v\leq\frac{p}{2}$ and $\bar{\gamma_v}\leq\frac{p}{2}$. If either $\gamma_v<\frac{p}{2}$ or $\bar{\gamma_v}<\frac{p}{2}$, then $\gamma_v+\bar{\gamma_v}<p$, which is a contradiction. Hence the only possible case is $\gamma_v=\bar{\gamma_v}=\frac{p}{2}$. □

Proposition 2.91. *Let $G=(\sigma,\mu)$ be a fuzzy graph or an $t-$norm fuzzy graph. If all edges have equal value, then G has no α-strong edge.*

Proof. By using Definition of α-strong edge, the result is hold. □

The following example illustrates this concept.

Example 2.92. In Figure 9, all edges have the same value but there is no α-strong edges in this fuzzy graph.

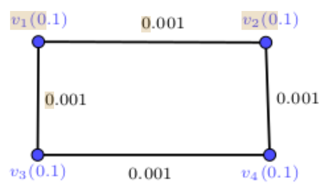

Figure 9. Identical edges and α-strong edges

We give the relationship between M-strong edges and α-strong edges, (Corollary 3.40).

Corollary 2.93. *Let $G=(\sigma,\mu)$ be a fuzzy graph or an $t-$norm fuzzy graph. If all edges are M-strong, then G has no α-strong edge.*

Proof. By Proposition 3.38, the result follows. □

We give a necessary and sufficient condition for vertex domination number which is half of order, under some specific conditions. In fact, the fuzzy graphs and $t-$norm fuzzy graphs, which their vertex domination number is half of order, are characterized under some specific conditions, (Theorem 3.41).

Theorem 2.94. *In any fuzzy graph or any $t-$norm fuzzy graph $G=(\sigma,\mu)$, such that values of vertices are equal and all edges have same values, i.e. $\forall v_1,v_2\in V,\sigma(v_1)=\sigma(v_2)$ and $\forall v_1v_2,v_3v_4\in E,\mu(v_1v_2)=\mu(v_3v_4)$. $\gamma_v=\frac{p}{2}$ if and only if for any vertex dominating set D in G, we have $|D|=\frac{n}{2}$.*

Proof. Suppose D has the conditions. By Proposition 3.38, $\forall v\in D,\sum_{vv_1\in S}\mu(vv_1)=0$ where $S=\{v_1v_2\in E\mid\mu(v_1v_2)>\mu^{\infty}_{G'}(v_1,v_2)\}$; so by using Definition, $\gamma_v(G)=\Sigma_{v\in D}\sigma(v)$. Since values of vertices are equal and $|D|=\frac{n}{2}$, we have $\gamma_v(G)=\Sigma_{v\in D}\sigma(v)=\frac{n}{2}\sigma(v)=\frac{1}{2}(n\sigma(v))=\frac{1}{2}(\Sigma_{v\in V}\sigma(v))=\frac{1}{2}(p)=\frac{p}{2}$. Hence the result is hold in this case.

Conversely, suppose $\gamma_v=\frac{p}{2}$. Let $D=\{v_1,v_2,\cdots,v_n\}$ be a vertex dominating set. By Proposition 3.38, $\forall v\in D,\sum_{vv_1\in S}\mu(vv_1)=0$ where $S=\{v_1v_2\in E\mid\mu(v_1v_2)>\mu^{\infty}_{G'}(v_1,v_2)\}$; so by using Definition, $\gamma_v(G)=\Sigma_{v\in D}\sigma(v)$. Since $\gamma_v(G)=W_v(D)$, we have $\gamma_v=\frac{p}{2}=\frac{1}{2}(\Sigma_{v\in V}\sigma(v))=\Sigma_{v\in D}\sigma(v)$. Suppose $n'\neq\frac{n}{2}$.

So $\Sigma_{i=1}^{n''}\sigma(v_i)) = 0$ which is a contradiction with $\forall v_i \in V, \sigma(v_i) > 0$. Hence $n' = \frac{n}{2}$, i.e. $|D| = n' = \frac{n}{2}$. The result is hold in this case. □

The goal of upcoming texts is to prove some results concerning operations and study some conjectures arising from it.

Proposition 2.95. *Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ be fuzzy graphs or $t-$norm fuzzy graphs. A vertex dominating set in $G_1 \cup G_2$ is $D = D_1 \cup D_2$ such that D_1 and D_2 are vertex dominating sets of G_1 and G_2, respectively. Moreover, $\gamma_v(G_1 \cup G_2) = \gamma_v(G_1) + \gamma_v(G_2)$.*

Proof. By using Definition of union, the result is obviously hold. □

Corollary 2.96. *Let $G_i = (\sigma_i, \mu_i)$ be fuzzy graphs or $t-$norm fuzzy graphs, for $i = 1, \cdots, n$. A vertex dominating set in $\cup_{i=1}^{n} G_i$ is $D = \cup_{i=1}^{n} D_i$ such that D_i are vertex dominating sets in $G_i, i = 1, \cdots, n$. Moreover, $\gamma_v(\cup_{i=1}^{n} G_i) = \Sigma_{i=1}^{n}\gamma_v(G_i)$.*

Proof. By Proposition 3.42, the result is hold. □

The concept of monotone decreasing, (Definition 3.44), are introduced.

Definition 2.97. Let $G = (\sigma, \mu)$ be a fuzzy graph or an $t-$norm fuzzy graph. A property is monotone decreasing if removing an edge, does not destroy the property.

Conjecture (Vizing). For all G and H, $\gamma(G)\gamma(H) \leq \gamma(G \times H)$.

By using α-strong edge and monotone decreasing, the result in relation with Vizing's conjecture is determined, (Theorem 3.45).

Theorem 2.98. *The Vizing's conjecture is monotone decreasing property if removed edges are $\alpha-$strong.*

Proof. Let $G = (\sigma, \mu)$ be a fuzzy graph or an $t-$norm fuzzy graph and G' be a new one which is obtained from G by removing an edge. For every $G_1 = (\sigma_1, \mu_1)$, a $G' \times G_1$ is a spanning subgraph of $G \times G_1$. So $\gamma_v(G' \times G_1) \geq \gamma_v(G \times G_1) \geq \gamma_v(G)\gamma_v(G_1) = \gamma_v(G')\gamma_v(G_1)$. Hence Vizing's conjecture is also hold for G'. Then the result follows. □

Corollary 2.99. *Suppose the Vizing's conjecture is hold. Let G_1 be a spanning subgraph of G such that $\gamma_v(G_1) = \gamma_v(G)$. Then the Vizing's conjecture is also hold for G_1.*

Proof. Let $G = (\sigma, \mu)$ be a fuzzy graph or an $t-$norm fuzzy graph and G_1 be a spanning subgraph of G such that $\gamma_v(G_1) = \gamma_v(G)$. For every $G_2 = (\sigma_2, \mu_2)$, a $G_1 \times G_2$ is a spanning subgraph of $G \times G_2$. So $\gamma_v(G_1 \times G_2) \geq \gamma_v(G \times G_2) \geq \gamma_v(G)\gamma_v(G_2) = \gamma_v(G_1)\gamma_v(G_2)$. Hence the Vizing's conjecture is also hold for G_1. So the result follows. □

Proposition 2.100. *Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ be fuzzy graphs or $t-$norm fuzzy graphs. A vertex dominating set of $G_1 + G_2$ is $D = D_1 \cup D_2$ such that D_1 and D_2 are vertex dominating sets of G_1 and G_2, respectively. Moreover, $\gamma_v(G_1 + G_2) = \gamma_v(G_1) + \gamma_v(G_2)$.*

Proof. By using Definition of join, M-strong edges between two models are not α-strong which is a weak edge changing strength of connectedness of G. □

Corollary 2.101. *Let $G_i = (\sigma_i, \mu_i)$ be fuzzy graphs or $t-$norm fuzzy graphs, for $i = 1, \cdots, n$, respectively. A vertex dominating set of $+_{i=1}^{n} G_i$ is $D = +_{i=1}^{n} D_i$ such that D_i are vertex dominating sets of G_i. Moreover, $\gamma_v(+_{i=1}^{n} G_i) = \Sigma_{i=1}^{n}\gamma_v(G_i)$.*

Proof. By Proposition 3.47, the result is hold. □

Conjecture (Gravier and Khelladi). For all G and H,

$$\gamma(G)\gamma(H) \leq 2\gamma(G+H).$$

By using α-strong edge and monotone decreasing, the result in relation with the Gravier and Khelladi's conjecture is determined, (Theorem 3.49).

Theorem 2.102. *The Gravier and Khelladi's conjecture is monotone decreasing property if removed edges are $\alpha-$strong.*

Proof. Let $G = (\sigma, \mu)$ be a fuzzy graph or $t-$norm fuzzy graph, and G' be a new one which is obtained from G by removing an edge. For every $G_1 = (\sigma_1, \mu_1)$, a $G' + G_1$ is a spanning subgraph of $G + G_1$. So
$2\gamma_v(G') + G_1) \geq 2\gamma_v(G + G_1) \geq \gamma_v(G)\gamma_v(G_1) = \gamma_v(G')\gamma_v(G_1)$. Hence the Gravier and Khelladi's conjecture is also hold for G'. Then the result follows. □

We conclude this section with some result in relation with the Gravier and Khelladi's conjecture, (Corollary 3.50).

Corollary 2.103. *Suppose the Gravier and Khelladi's conjecture is hold. Let G_1 be a spanning subgraph of G such that $\gamma_v(G_1) = \gamma_v(G)$. Then the Gravier and Khelladi's conjecture is hold for G_1*

Proof. Let $G = (\sigma, \mu)$ be a fuzzy graph or $t-$norm fuzzy graph, and G_1 be a spanning subgraph of G such that $\gamma_v(G_1) = \gamma_v(G)$. For every $G_2 = (\sigma_2, \mu_2)$, a $G_1 \times G_2$ is a spanning subgraph of $G \times G_2$. So
$2\gamma_v(G_1 + G_2) \geq 2\gamma_v(G + G_2) \geq \gamma_v(G)\gamma_v(G_2) = \gamma_v(G_1)\gamma_v(G_2)$. Hence the Gravier and Khelladi's conjecture is also hold for G_1. The result follows. □

Acknowledgments

The author thank just about everybody. The author is highly thankful to the Editor-in-Chief and the referees for their valuable comments and suggestions for improving the paper.

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